

Supersymmetry on the run: LHC & Dark Matter



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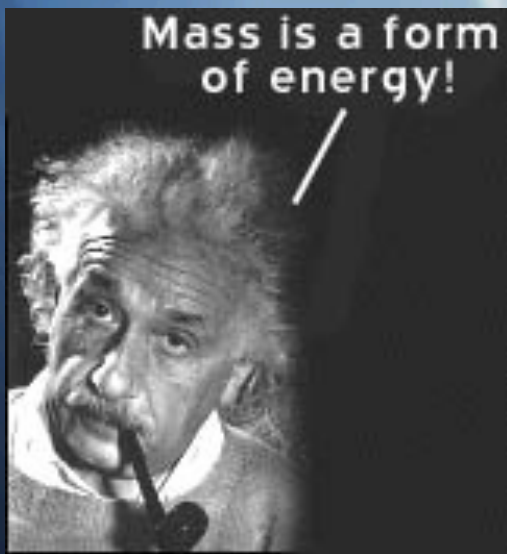


Outline

- Basics of SUSY
- MSSM
- SUSY Searches at LHC
- SUSY in the sky (DM)

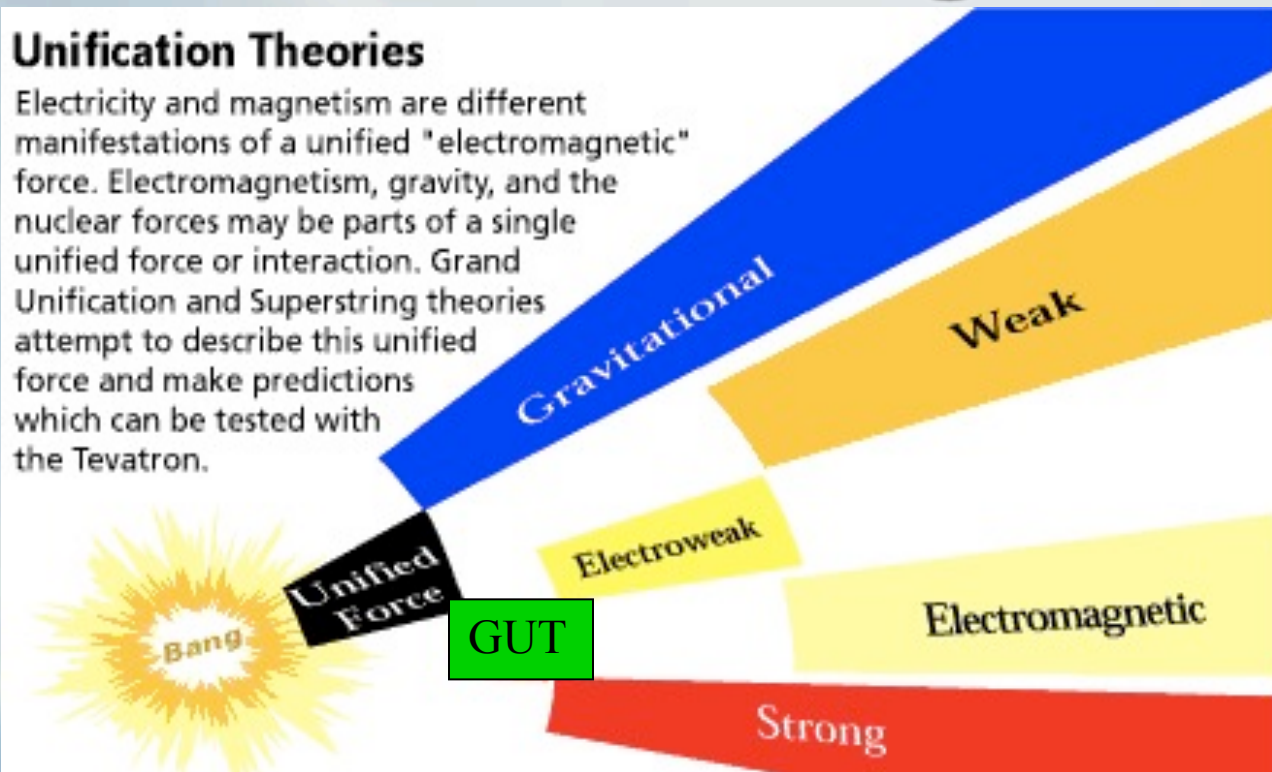
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Unification Paradigm



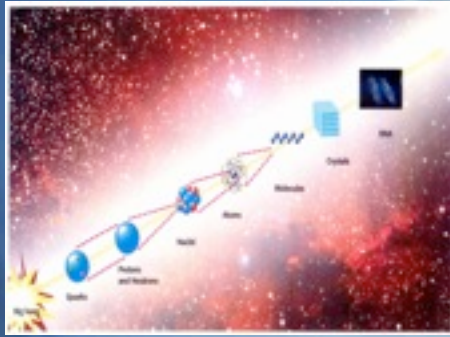
Unification Theories

Electricity and magnetism are different manifestations of a unified "electromagnetic" force. Electromagnetism, gravity, and the nuclear forces may be parts of a single unified force or interaction. Grand Unification and Superstring theories attempt to describe this unified force and make predictions which can be tested with the Tevatron.



- Unification of strong, weak and electromagnetic interactions within Grand Unified Theories is the new step in unification of all forces of Nature
- Creation of a unified theory of everything based on string paradigm seems to be possible





What is SUSY?

- Supersymmetry is a boson-fermion symmetry that is aimed to unify all forces including gravity within a single framework

$Q |boson\rangle = \dots |fermion\rangle$

$[b, \{a, c\}] = 2\delta^{ij} (\sigma^\mu)_{\alpha\beta} P_\mu$

- Modern supersymmetry in particle physics are based on a paradigm, though low energy manifestations of SUSY can be found (?) at modern colliders and in non-accelerator experiments

First papers in 1971-1972
 No evidence in particle physics yet

Motivation of SUSY in Particle Physics

- Unification with Gravity
- Unification of gauge couplings
- Solution of the hierarchy problem
- Dark matter in the Universe
- (Super)string consistency

Motivation of SUSY in Particle Physics

- Unification with Gravity

$$Q | boson \rangle = | fermion \rangle \quad Q | fermion \rangle = | boson \rangle$$

$$spin\ 2 \Rightarrow spin\ 3/2 \Rightarrow spin\ 1 \Rightarrow spin\ 1/2 \Rightarrow spin\ 0$$

Unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with the other interactions

$$\{Q_\alpha^i, \bar{Q}_\beta^j\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\beta} P_\mu \Rightarrow \{\delta_\varepsilon, \bar{\delta}_{\bar{\varepsilon}}\} = 2(\varepsilon\sigma^\mu\bar{\varepsilon})P_\mu$$

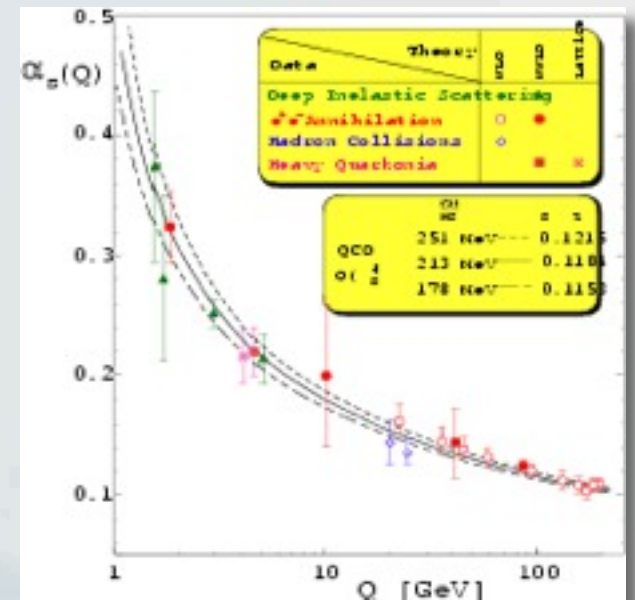
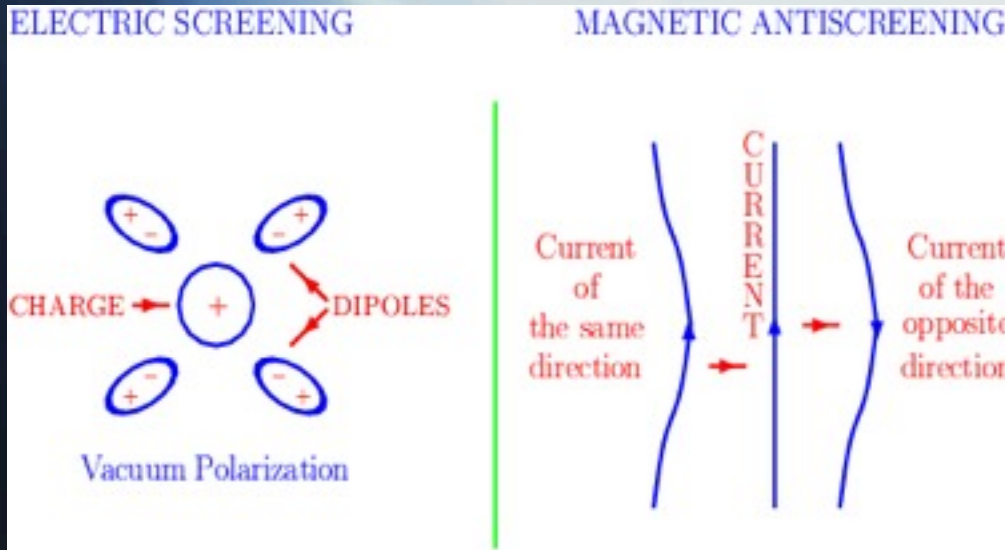
$\varepsilon = \varepsilon(x)$ local coordinate transf. \Rightarrow (super)gravity

Local supersymmetry = general relativity !

Motivation of SUSY in Particle Physics

- Unification of gauge couplings

Low Energy		⇒ High Energy	
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	⇒ G_{GUT} (or $G^n + \text{symm}$)
gluons	W, Z	photon	⇒ gauge bosons
quarks	leptons		⇒ fermions
g_3	g_2	g_1	⇒ g_{GUT}



$$\alpha_i = \alpha_i \left(\frac{Q^2}{\Lambda^2} \right) = \alpha_i(\text{distance})$$

Running of the strong coupling

Motivation of SUSY

RG Equations $\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \alpha_i / 4\pi = g_i^2 / 16\pi^2, \quad t = \log(Q^2 / \mu^2)$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Unification of the Coupling Constants
in the SM and in the MSSM

Input

$$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$$

$$\sin^2 \theta_{\overline{MS}} = 0.23146 \pm 0.00017$$

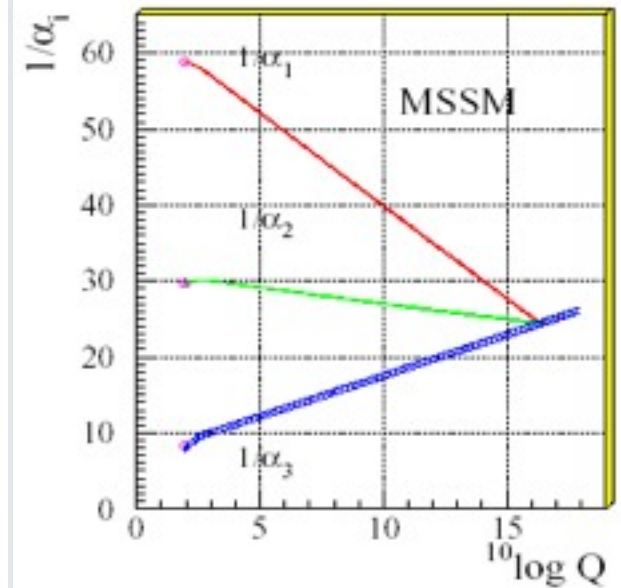
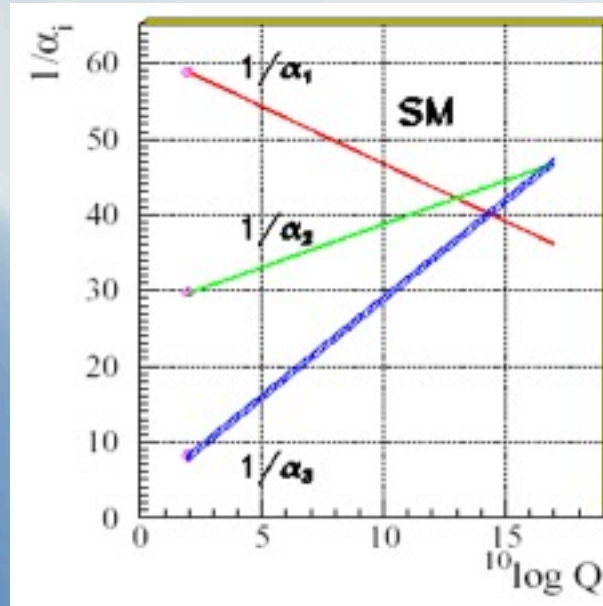
$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

Output

$$M_{SUSY} = 10^{3.4 \pm 0.9 \pm 0.4} \text{ GeV}$$

$$M_{GUT} = 10^{15.8 \pm 0.3 \pm 0.1} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = 26.3 \pm 1.9 \pm 1.0$$



SUSY yields unification!

Motivation of SUSY

- Solution of the Hierarchy Problem

$$m_H \sim v \sim 10^2 \text{ GeV}$$

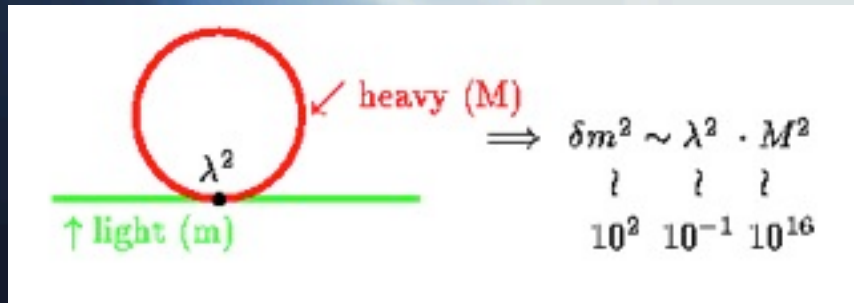
$$m_\Sigma \sim V \sim 10^{16} \text{ GeV}$$

$$\frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

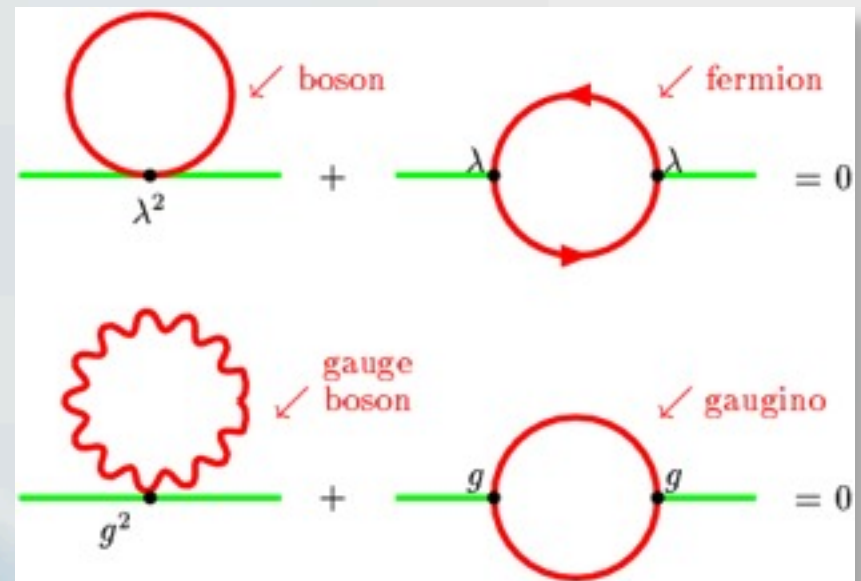
$$m_\Sigma$$

Cancellation of quadratic terms

Destruction of the hierarchy by radiative corrections



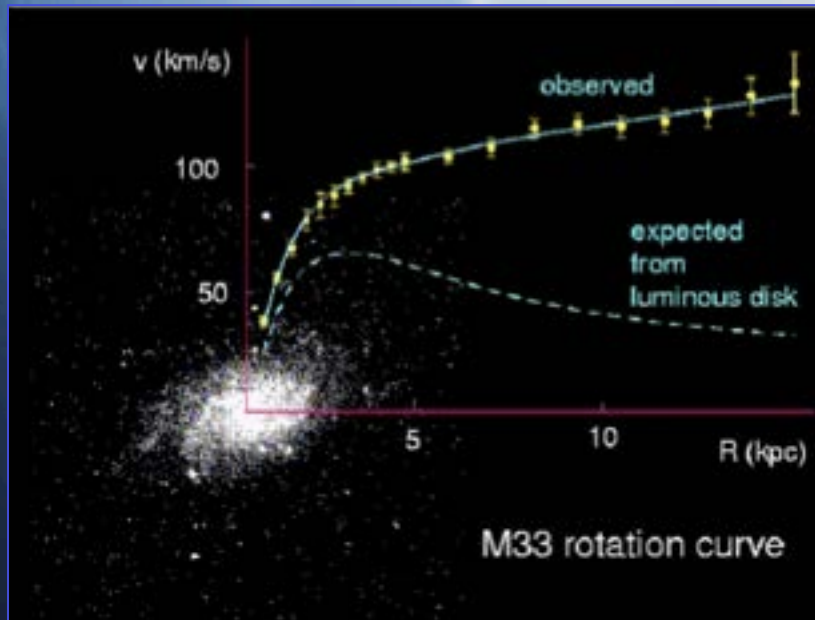
SUSY may also explain the origin of the hierarchy due to radiative mechanism



$$\sum_{\text{bosons}} m^2 = \sum_{\text{fermions}} m^2$$

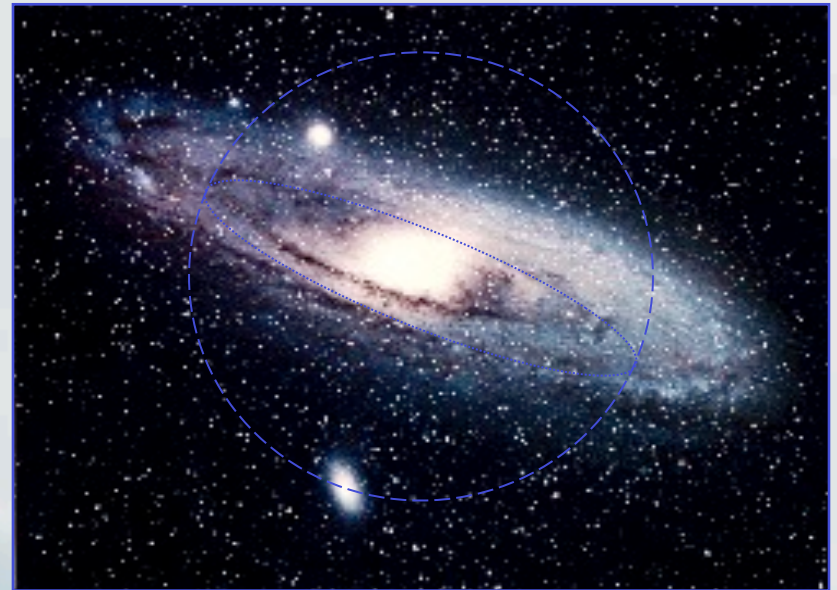
Motivation of SUSY

- Dark Matter in the Universe



The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.

Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter



SUSY provides a candidate for the Dark matter – a stable neutral particle

Cosmological Constraints

New precise cosmological data

$$\Omega h^2 = 1 \quad \longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$



- Supernova Ia explosion
- CMBR thermal fluctuations (measured by WMAP)

Dark Matter in the Universe:



Hot DM
(not favoured by galaxy formation)

Cold DM
(rotation curves of Galaxies)

SUSY

Superalgebra

(Super) Algebra

Lorentz Algebra

$$[P_\mu, P_\nu] = 0, [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho} P_\sigma - g_{\mu\sigma} P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho} M_{\mu\sigma} - g_{\nu\sigma} M_{\mu\rho} - g_{\mu\rho} M_{\nu\sigma} + g_{\mu\sigma} M_{\nu\rho}),$$

SUSY Algebra

$$[Q_\alpha^i, P_\mu] = [\bar{Q}_{\dot{\alpha}}^i, P_\mu] = 0,$$

$$[Q_\alpha^i, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i, [\bar{Q}_{\dot{\alpha}}^i, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}^i (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}},$$

$$\{Q_\alpha^i, \bar{Q}_{\dot{\beta}}^j\} = 2\delta^{ij} (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; i, j = 1, 2, \dots, N.$$

Superspace

$$x_\mu \rightarrow x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$$

Grassmannian
parameters

$$\alpha, \dot{\alpha} = 1, 2$$

$$\theta_\alpha^2 = 0, \bar{\theta}_{\dot{\alpha}}^2 = 0$$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial \theta_\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$Q_\alpha^2 = 0, \bar{Q}_{\dot{\alpha}}^2 = 0$$

Supertranslation

$$x_\mu \rightarrow x_\mu + i\theta_\mu \bar{\xi} - i\xi_\mu \bar{\theta},$$

$$\theta \rightarrow \theta + \xi,$$

$$\bar{\theta} \rightarrow \bar{\theta} + \bar{\xi}$$

Quantum States

Quantum states: Vacuum = $|E, \lambda\rangle$ $Q|E, \lambda\rangle = 0$

$$[Q_\alpha^i, P_\mu] = [\bar{Q}_\alpha^i, P_\mu] = 0$$

\swarrow Energy \nwarrow helicity

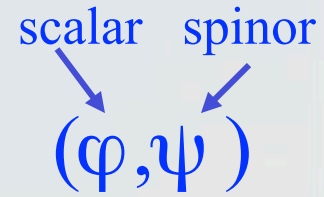
State	Expression	# of states
vacuum	$ E, \lambda\rangle$	1
1-particle	$\bar{Q}_i E, \lambda\rangle = E, \lambda + 1/2\rangle$	$\binom{N}{1} = N$
2-particle	$\bar{Q}_i \bar{Q}_j E, \lambda\rangle = E, \lambda + 1\rangle$	$\binom{N}{2} = \frac{N(N-1)}{2}$
...
N-particle	$\bar{Q}_1 \bar{Q}_2 \dots \bar{Q}_N E, \lambda\rangle = E, \lambda + N/2\rangle$	$\binom{N}{N} = 1$

Total # of states: $\sum_{k=0}^N \binom{N}{k} = 2^N = 2^{N-1} \text{ bosons} + 2^{N-1} \text{ fermions}$

SUSY Multiplets

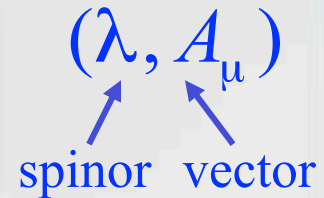
Chiral multiplet $N = 1, \lambda = 0$

helicity	-1/2	0	1/2
# of states	1	2	1



Vector multiplet $N = 1, \lambda = 1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1



Members of a supermultiplet are called **superpartners**

N=4	SUSY YM	helicity	-1	-1/2	0	1/2	1				
	$\lambda = -1$	# of states	1	4	6	4	1				
N=8	SUGRA	helicity	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2
	$\lambda = -2$	# of states	1	8	28	56	70	56	28	8	1

$$N \leq 4S$$

spin

$$N \leq 4$$

For renormalizable theories (YM)

$$N \leq 8$$

For (super)gravity

Simplest (N=1) SUSY Multiplets

Bosons and Fermions come in pairs

(φ, ψ)

(λ, A_μ)

(\tilde{g}, g)

Spin 0

Spin 1/2

Spin 1/2

Spin 1

Spin 3/2

Spin 2

scalar

chiral
fermion

majorana
fermion

vector

gravitino

graviton

Minimal Supersymmetric Standard Model (MSSM)

- SUSY: # of fermions = # of bosons N=1 SUSY: (φ, ψ) (λ, A_μ)
- SM: 28 bosonic d.o.f. & 90 (96) fermionic d.o.f.

There are no particles in the SM that can be superpartners

SUSY associates known bosons with new fermions and known fermions with new bosons

- Even number of the Higgs doublets – min = 2
Cancellation of axial anomalies (in each generation)

$$Tr Y^3 = 3\left(\frac{1}{27} + \frac{1}{27} - \frac{64}{27} + \frac{8}{27}\right) - 1 - 1 + 8 = 0$$

colour
 u_L
 d_L
 u_R
 d_R
 ν_L
 e_L
 e_R

Higgsinos

$$-1 + 1 = 0$$

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Particle Content of the MSSM

Superfield	Bosons	Fermions	$SU_c(3)$	$SU_L(2)$	$U_Y(1)$			
<i>Gauge</i>								
G^a	gluon g^a	gluino \tilde{g}^a	8	1	0			
V^k	Weak $W^k (W^\pm, Z)$	wino, zino $\tilde{w}^k (\tilde{w}^\pm, \tilde{z})$	1	3	0			
V'	Hypercharge $B(\gamma)$	bino $\tilde{b}(\tilde{\gamma})$	1	1	0			
<i>Matter</i>								
L_i	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1	
E_i				$\tilde{E}_i = \tilde{e}_R$	$E_i = e_R$	1	1	2
Q_i	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	1/3	
U_i				$\tilde{U}_i = \tilde{u}_R$	$U_i = u_R^c$	3*	1	-4/3
D_i				$\tilde{D}_i = \tilde{d}_R$	$D_i = d_R^c$	3*	1	2/3
<i>Higgs</i>								
H_1	Higgses	H_1	higgsinos	\tilde{H}_1	1	2	-1	
H_2				H_2	\tilde{H}_2	1	2	1

How to write SUSY Lagrangians

- 1st step

Take your favorite Lagrangian written in terms of fields

- 2nd step

Replace *Field* $(\varphi, \psi, A_\mu) \Rightarrow$ *Superfield* (Φ, V)

- 3rd step

Replace

$$\textit{Action} = \int d^4x L(x) \quad \Rightarrow \quad \int d^4x d^4\theta L(x, \theta, \bar{\theta})$$

Grassmannian integration in superspace

$$\int d\theta_\alpha = 0, \quad \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$$

The MSSM Lagrangian

$$L = L_{gauge} + L_{Yukawa} + L_{SoftBreaking}$$

The Yukawa Superpotential

Superfields

$$W_R = y_U Q_L H_2 U_R + y_D Q_L H_1 D_R + y_L L_L H_1 E_R + \mu H_1 H_2$$

Yukawa couplings

Higgs mixing term

$$W_{NR} = \lambda_L L_L L_L E_R + \lambda'_L L_L Q_L D_R + \mu' L_L H_2 + \lambda_B U_R D_R D_R$$

Violate:

Lepton number

Baryon number

$$\lambda_L, \lambda'_L < 10^{-6}, \quad \lambda_B < 10^{-9}$$

These terms are forbidden in the SM

R-parity

$$R = (-)^{3(B-L)+2S}$$

The Usual Particle : $R = +1$
SUSY Particle : $R = -1$

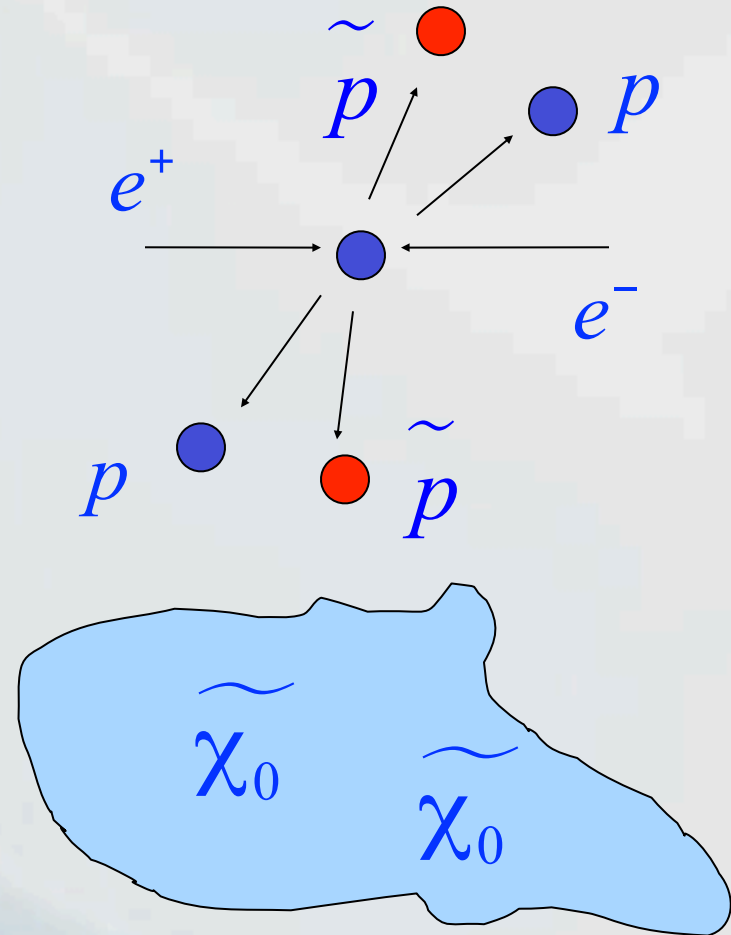
B - Baryon Number
L - Lepton Number
S - Spin

The consequences:

- The superpartners are created in pairs
- The lightest superparticle is stable



- The lightest superparticle (LSP) should be neutral - the best candidate is neutralino (photino or higgsino) $\tilde{\chi}_0$
- It can survive from the Big Bang and form the Dark matter in the Universe



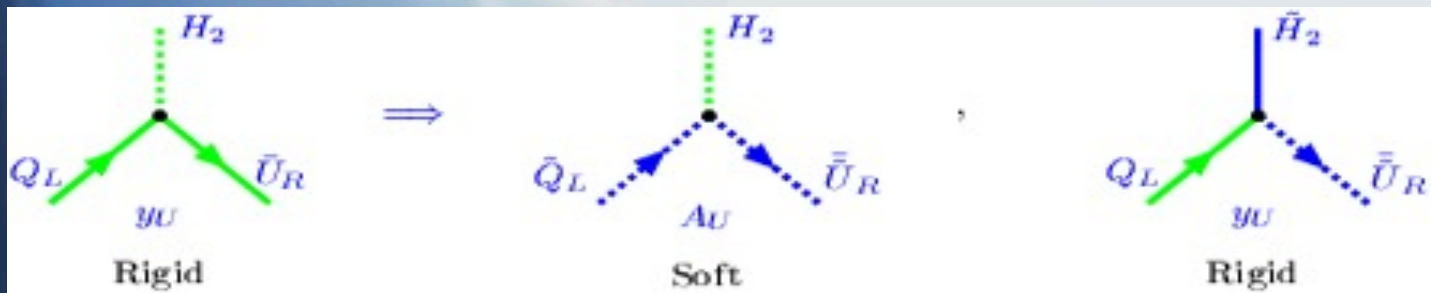
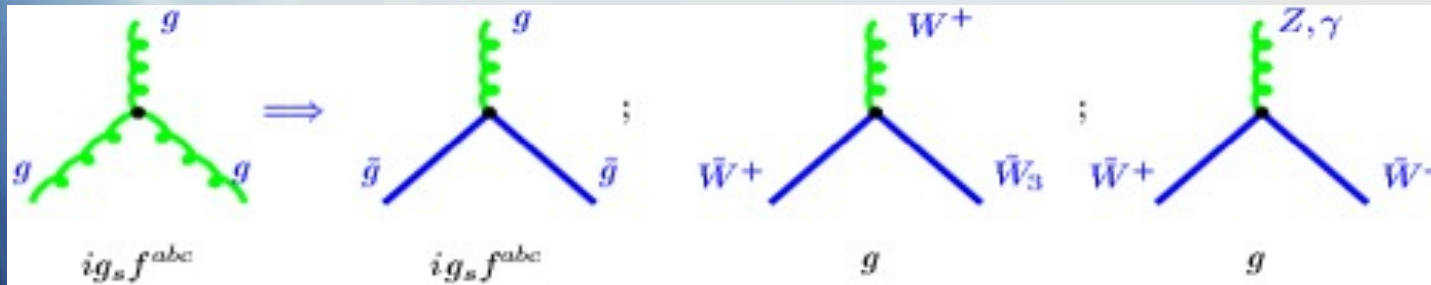
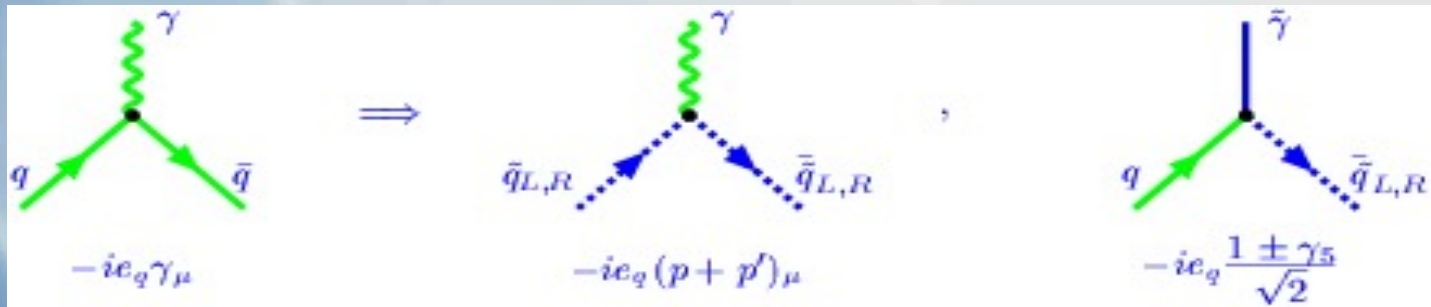
Interactions in the MSSM

SM



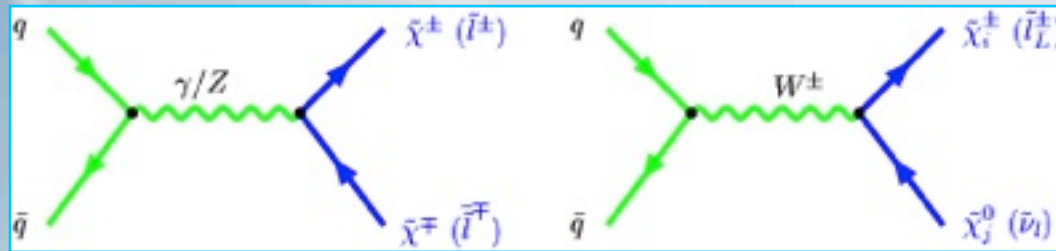
MSSM

Vertices

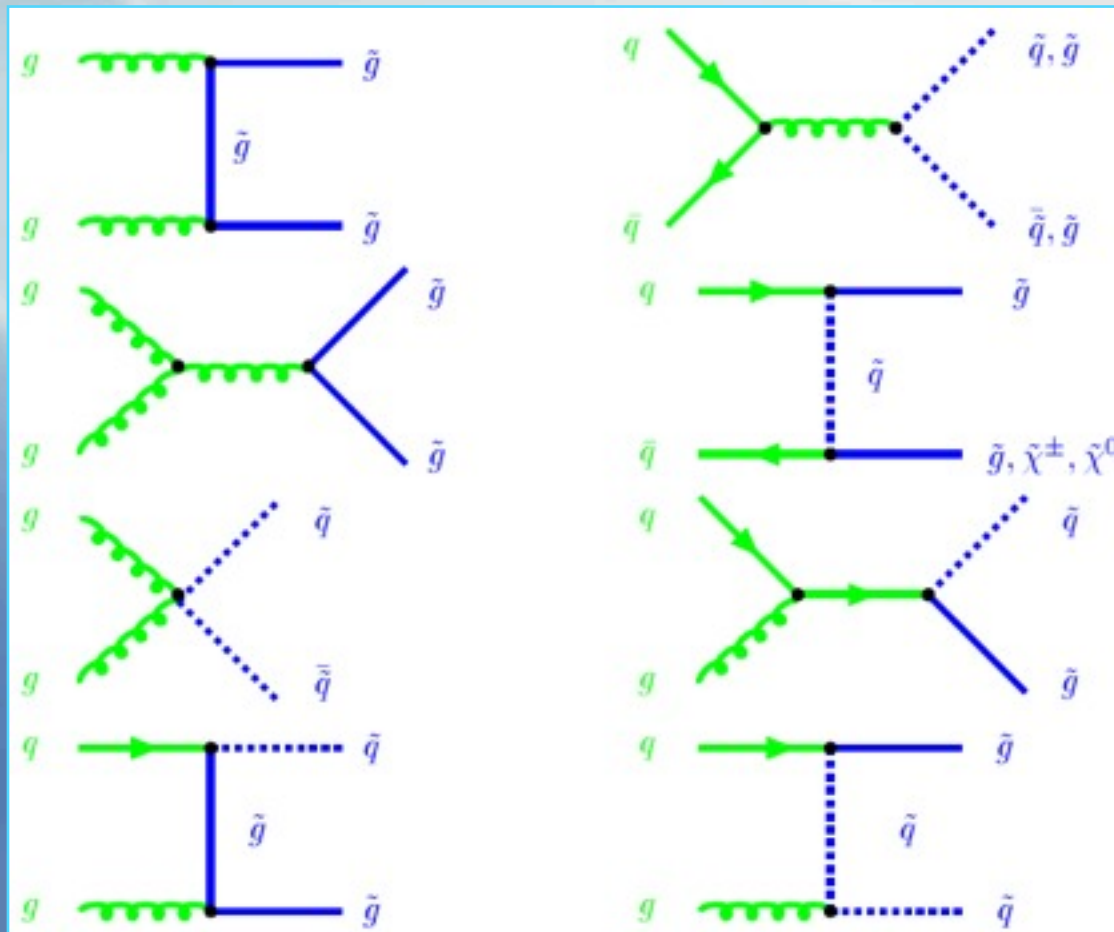


Superpartners Production at LHC

Annihilation



Quark-gluon Fusion



Decay of Superpartners

squarks

$$\tilde{q}_{L,R} \rightarrow q + \tilde{\chi}_i^0$$

$$\tilde{q}_L \rightarrow q' + \tilde{\chi}_i^\pm$$

$$\tilde{q}_{L,R} \rightarrow q + \tilde{g}$$

sleptons

$$\tilde{l} \rightarrow l + \tilde{\chi}_i^0$$

$$\tilde{l}_L \rightarrow \nu_l + \tilde{\chi}_i^\pm$$

chargino

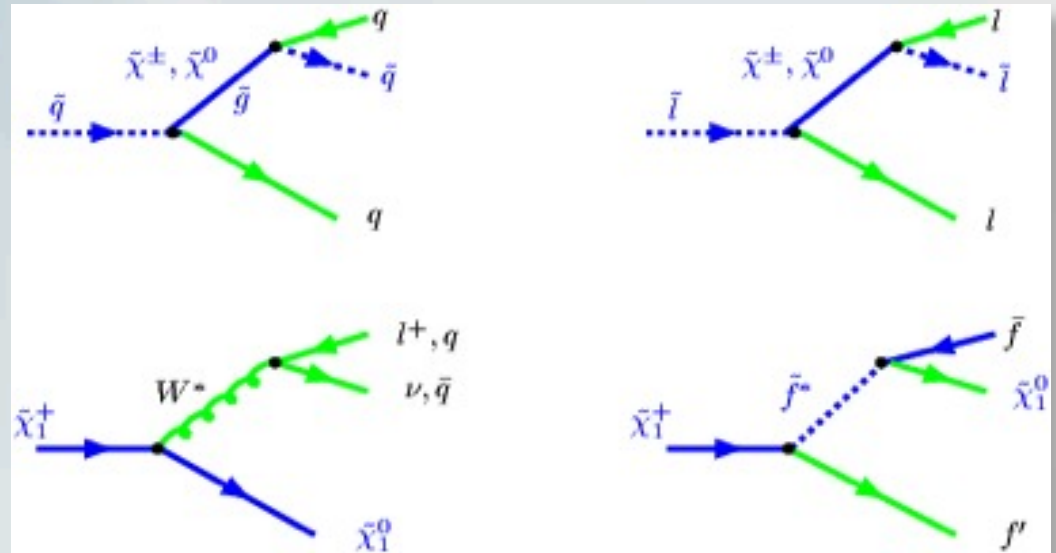
$$\tilde{\chi}_i^\pm \rightarrow e + \nu_e + \tilde{\chi}_i^0$$

$$\tilde{\chi}_i^\pm \rightarrow q + \bar{q}' + \tilde{\chi}_i^0$$

gluino

$$\tilde{g} \rightarrow q + \bar{q} + \tilde{\gamma}$$

$$\tilde{g} \rightarrow g + \tilde{\gamma}$$



neutralino

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + l^+ + l^-$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + q + \bar{q}'$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^\pm + l^\pm + \nu_l$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + \nu_l + \bar{\nu}_l$$

Final states

$$l^+ l^- + \cancel{E}_T$$

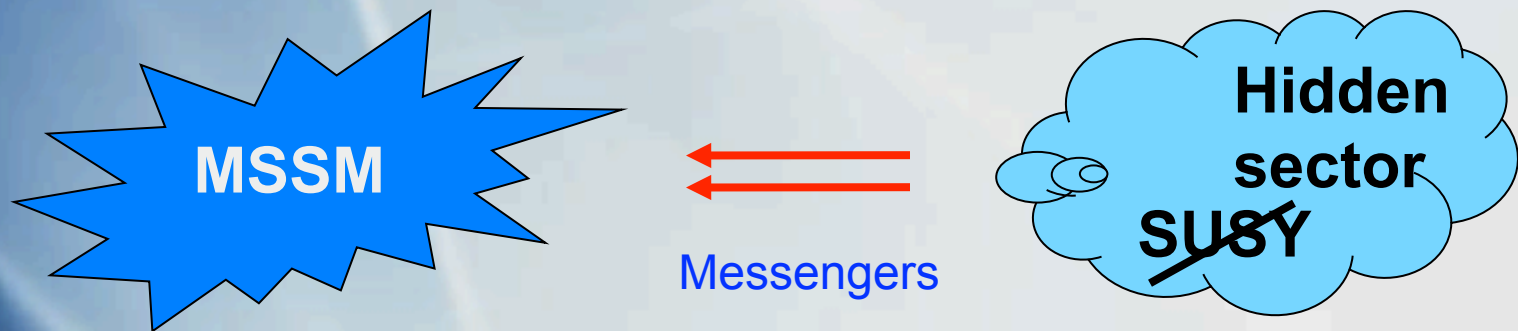
$$2 \text{ jets} + \cancel{E}_T$$

$$\gamma + \cancel{E}_T$$

$$\cancel{E}_T$$

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Soft SUSY Breaking



Gravitons, gauge, gauginos, etc

Breaking via F and D terms in a hidden sector

$$-L_{Soft} = \sum_{\alpha} M_{\alpha} \tilde{\lambda}_{\alpha} \tilde{\lambda}_{\alpha} + \sum_i m_{0i}^2 |A_i|^2 + \sum_{ijk} A_{ijk} A_i A_j A_k + \sum_{ij} B_{ij} A_i A_j$$

gauginos
scalar fields

Over 100 of free parameters !

MSSM Parameter Space

- Three gauge couplings
- Three (four) Yukawa matrices
- The Higgs mixing parameter
- Soft SUSY breaking terms

mSUGRA Universality hypothesis (gravity is colour and flavour blind):
Soft parameters are equal at Planck (GUT) scale

$$-L_{Soft} = A\{y_t Q_L H_2 U_R + y_b Q_L H_1 D_R + y_L L_L H_1 E_R\} + B\mu H_1 H_2 + m_0^2 \sum_i |\varphi_i|^2 + \frac{1}{2} M_{1/2} \sum_\alpha \widetilde{\lambda}_\alpha \widetilde{\lambda}_\alpha$$

Five universal soft parameters:

$$A, m_0, M_{1/2}, B \leftrightarrow \tan\beta = v_2 / v_1 \quad \text{and} \quad \mu$$

versus

$$m \quad \text{and} \quad \lambda$$

in the SM
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Mass Spectrum (spin=1/2)

$$L_{\text{gaugino-Higgsino}} = -\frac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \frac{1}{2} \bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.)$$

Chargino

$$\psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix}$$

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix}$$



$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix}$$

Neutralino

$$(\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0)$$



$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}$$

$$M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta \sin W & M_Z \sin \beta \sin W \\ 0 & M_2 & M_Z \cos \beta \cos W & -M_Z \sin \beta \cos W \\ -M_Z \cos \beta \sin W & M_Z \cos \beta \cos W & 0 & -\mu \\ M_Z \sin \beta \sin W & -M_Z \sin \beta \cos W & -\mu & 0 \end{pmatrix}$$

Mass Spectrum (spin=0)

$$\tilde{m}_t^2 = \begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & \tilde{m}_{tR}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$$\tilde{m}_b^2 = \begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu \tan \beta) \\ m_b(A_b - \mu \tan \beta) & \tilde{m}_{bR}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

$$\begin{aligned} \tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2) \cos 2\beta, \\ \tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2) \cos 2\beta, \end{aligned}$$

$$\begin{aligned} \tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2) \cos 2\beta, \\ \tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2) \cos 2\beta. \end{aligned}$$

$$\tilde{m}_\tau^2 = \begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu \tan \beta) \\ m_\tau(A_\tau - \mu \tan \beta) & \tilde{m}_{\tau R}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix}$$

SUSY Higgs Bosons

SM

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} = \begin{pmatrix} v + \frac{S + iP}{\sqrt{2}} \\ H^- \end{pmatrix} = \exp\left(i\frac{\vec{\xi}\vec{\sigma}}{2}\right) \begin{pmatrix} v + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$4=2+2=3+1$

$$H \rightarrow H' = \exp\left(i\frac{\vec{\alpha}\vec{\sigma}}{2}\right)H \xrightarrow{(\vec{\alpha}=-\vec{\xi})} H' = \begin{pmatrix} v + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

MSSM

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix}, H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_2^+ \\ v_2 + \frac{S_2 + iP_2}{\sqrt{2}} \end{pmatrix}$$

$8=4+4=3+5$

$v_1^2 + v_2^2 = v^2, v_2/v_1 \equiv \tan\beta$

The Higgs Potential

$$V_{tree}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^+ H_2|^2$$

At the GUT scale: $m_1^2 = m_2^2 = \mu_0^2 + m_0^2$, $m_3^2 = -B\mu_0$

Minimization

$$\frac{1}{2} \frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_1 = 0,$$

$$\frac{1}{2} \frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 - \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_2 = 0.$$

$$\langle H_1 \rangle \equiv v_1 = v \cos \beta, \quad \langle H_2 \rangle \equiv v_2 = v \sin \beta,$$

No SSB in SUSY theory !



Solution

$$v^2 = \frac{4(m_1^2 - m_2^2 \tan^2 \beta)}{(g^2 + g'^2)(\tan^2 \beta - 1)},$$

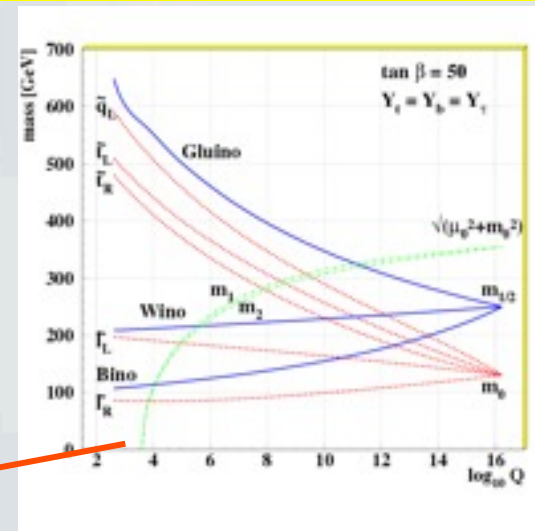
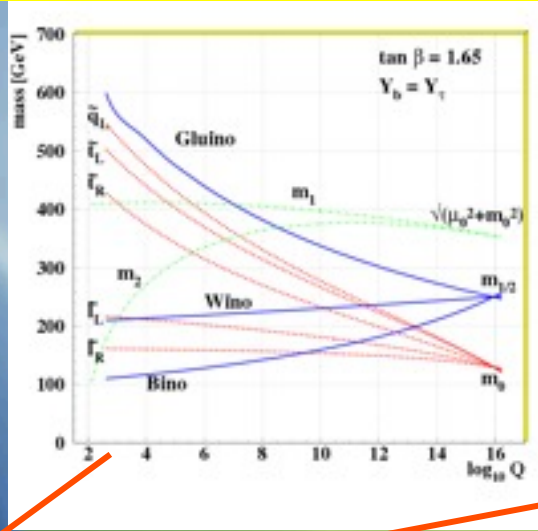
$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}$$

At the GUT scale

$$v^2 = -\frac{4}{g^2 + g'^2} m^2 < 0$$

Radiative EW Symmetry Breaking

Due to RG controlled running of the mass terms from the Higgs potential they may change sign and trigger the appearance of non-trivial minimum leading to spontaneous breaking of EW symmetry - this is called Radiative EWSB



$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1}$$



$$\mu^2$$

For given $\tan \beta$
 m_0 and $m_{1/2}$

$$m_{H_1} \sim m_{H_2} \sim m_0 \sim 1 \text{ TeV}$$

Soft SUSY parameters



$$\mu \sim 1 \text{ TeV}$$

Hard SUSY parameter

μ - problem

Higgs Boson's Masses

$$M^{odd} = \frac{\partial^2 V}{\partial P_i \partial P_j} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix} m_3^2$$

$$M^{even} = \frac{\partial^2 V}{\partial S_i \partial S_j} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} m_3^2 + \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix} M_Z^2 \cos \beta \sin \beta$$

$$M^{ch} = \frac{\partial^2 V}{\partial H_i^+ \partial H_j^-} \Big|_{H_i=v_i} = \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix} (m_3^2 + M_W^2 \cos \beta \sin \beta)$$

$$G^0 = -\cos \beta P_1 + \sin \beta P_2$$

Goldstone boson $\rightarrow Z_0$

$$A = \sin \beta P_1 + \cos \beta P_2$$

Neutral CP = -1 Higgs

$$G^+ = -\cos \beta (H_1^-)^* + \sin \beta H_2^+$$

Goldstone boson $\rightarrow W^+$

$$H^+ = \sin \beta (H_1^-)^* + \cos \beta H_2^+$$

Charged Higgs

$$h = -\sin \alpha S_1 + \cos \alpha S_2$$

SM Higgs boson CP = 1

$$H = \cos \alpha S_1 + \sin \alpha S_2$$

Extra heavy Higgs boson

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}$$

The Higgs Bosons Masses

CP-odd neutral Higgs A

CP-even charged Higgses H^\pm

CP-even neutral Higgses h,H

$$m_A^2 = m_1^2 + m_2^2$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

$$M_W^2 = \frac{g^2}{2} v^2$$

$$M_Z^2 = \frac{g^2 + g'^2}{2} v^2$$

$$m_{h,H}^2 = \frac{1}{2} [m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}]$$

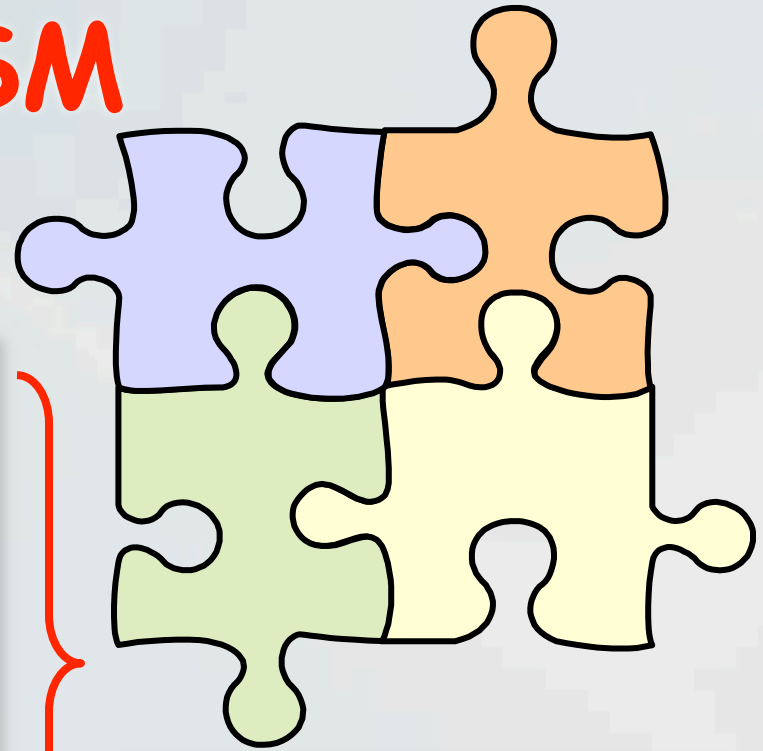
$$m_h \approx M_Z |\cos 2\beta| < M_Z ! \quad \Rightarrow \quad \text{Radiative corrections}$$

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{m_{t_1}^{\sim 2} m_{t_2}^{\sim 2}}{m_t^4} + 2 \text{ loops}$$

Constrained MSSM

Requirements:

- Unification of the gauge couplings
- Radiative EW Symmetry Breaking
- Heavy quark and lepton masses
- Rare decays ($b \rightarrow s\gamma$, $b \rightarrow \mu\mu$)
- Anomalous magnetic moment of muon
- LSP is neutral
- Amount of the Dark Matter
- Experimental limits from direct search



$$A_0, m_0, M_{1/2}, \mu, \tan \beta$$

Allowed region
in the parameter
space of the MSSM

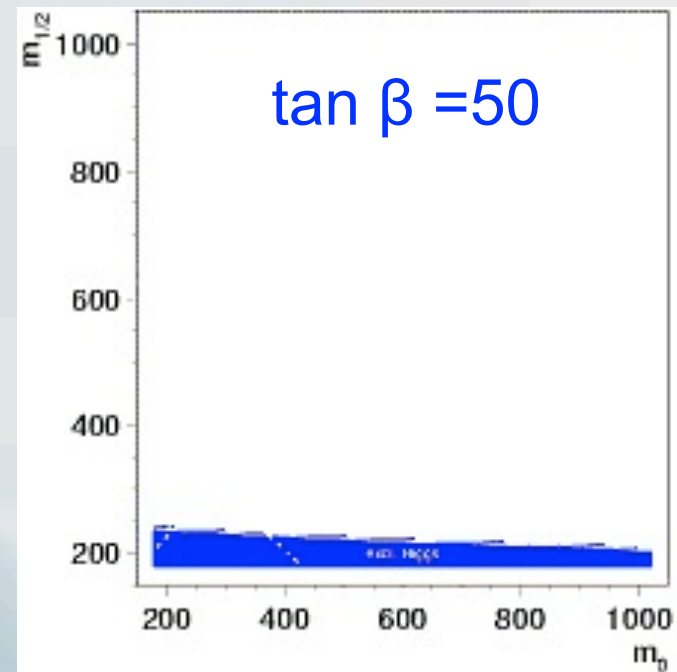
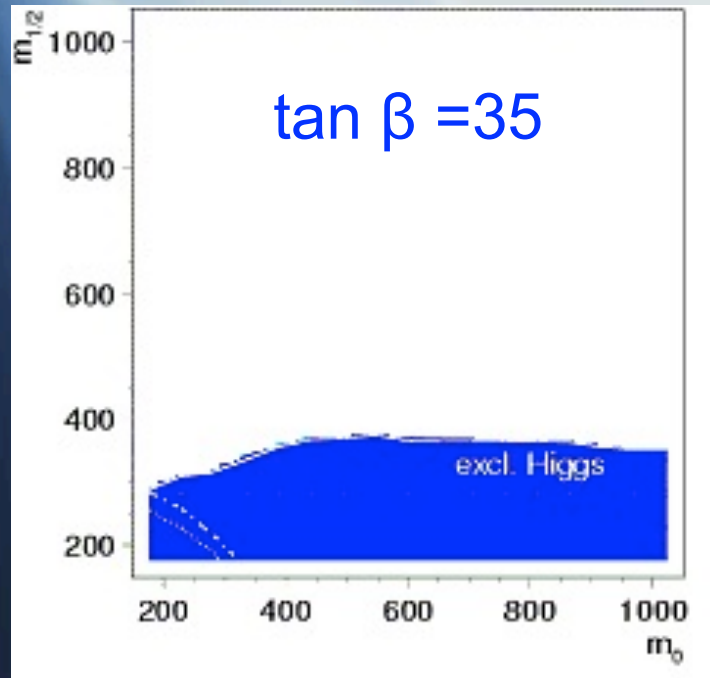
$$100 \text{ GeV} \leq m_0, M_{1/2}, \mu \leq 2 \text{ TeV}$$
$$-3m_0 \leq A_0 \leq 3m_0, 1 \leq \tan \beta \leq 70$$

Constrained MSSM (Choice of constraints)

Experimental lower limits on Higgs and superparticle masses

Regions excluded by Higgs experimental limits provided by LEP2

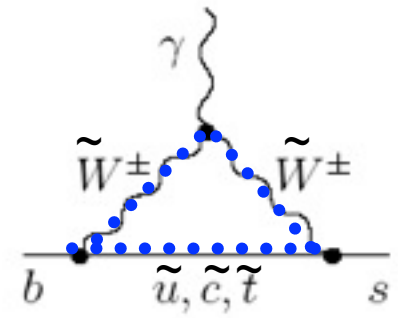
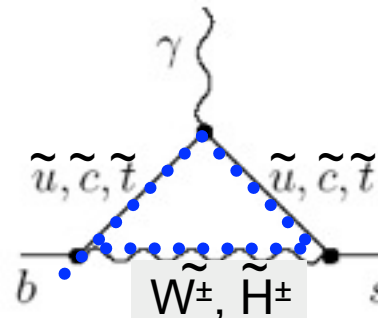
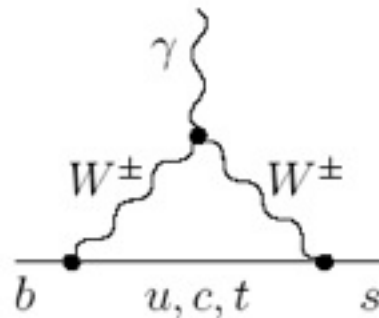
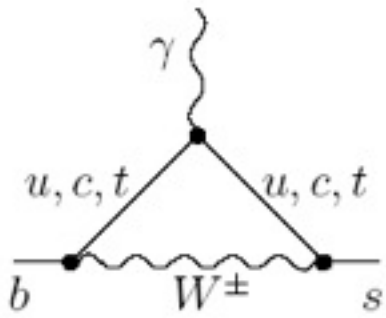
$$m_{Higgs} \geq 114.3 \text{ GeV}$$



B → s γ decay rate

Standard Model

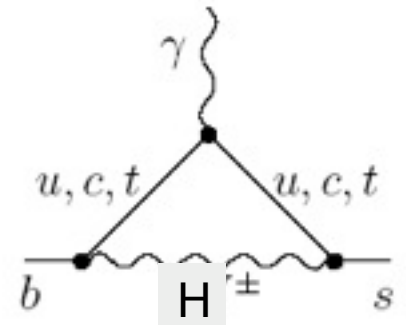
MSSM



SM: $\mathcal{B}(B \rightarrow X_s \gamma) = (3.28 \pm 0.33) \times 10^{-4}$.

MSSM

$$\mathcal{BR}(b \rightarrow s \gamma)|_{\chi^\pm} \propto \mu A_t \tan \beta f(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{\chi}^\pm}) \frac{m_b}{v(1 + \Delta m_b)}$$



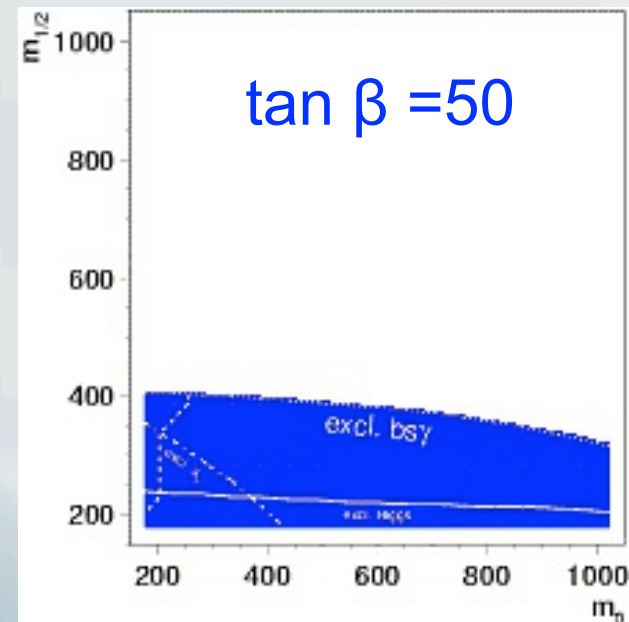
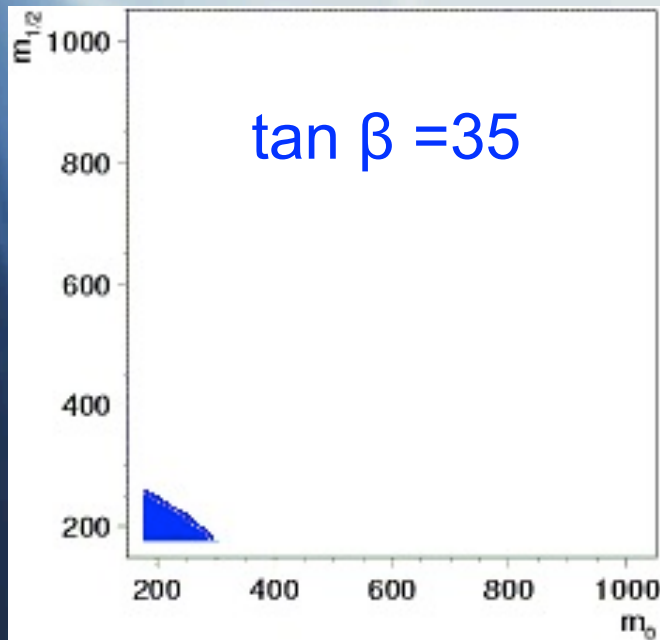
Experiment $\mathcal{B}(B \rightarrow X_s \gamma) = (3.43 \pm 0.36) \times 10^{-4}$ 34

Constrained MSSM (Choice of constraints)

Data on rare processes branching ratios

$$B(B \rightarrow X_s \gamma) = (3.43 \pm 0.36) \times 10^{-4}$$

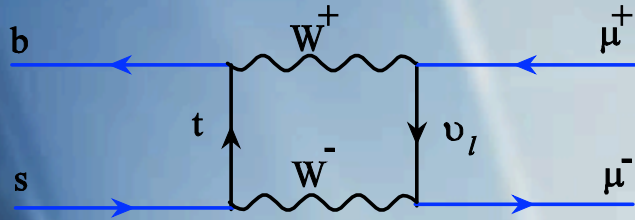
Regions excluded by experimental limits (for large $\tan\beta$)



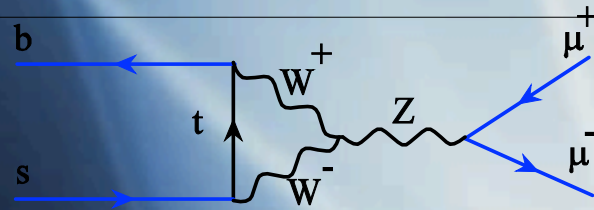
33

Rare Decay $B_s \rightarrow \mu^+ \mu^-$

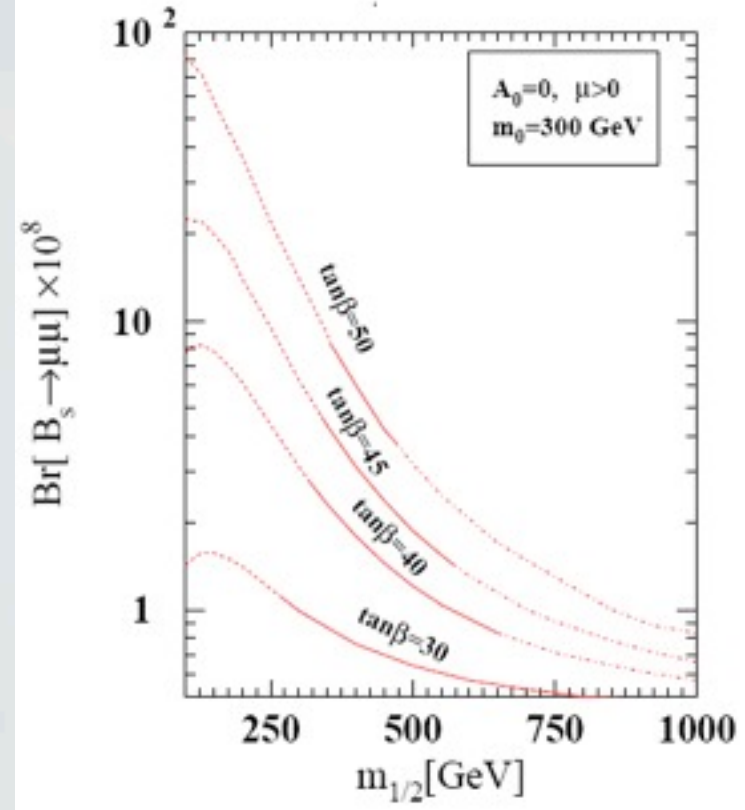
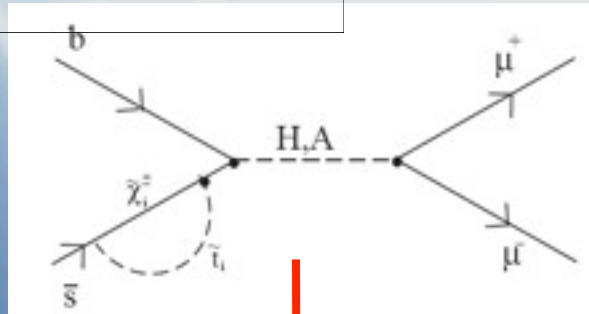
SM



SM: $Br = 3.5 \cdot 10^{-9}$
 Ex: $< 4.5 \cdot 10^{-8}$



Main SUSY contribution



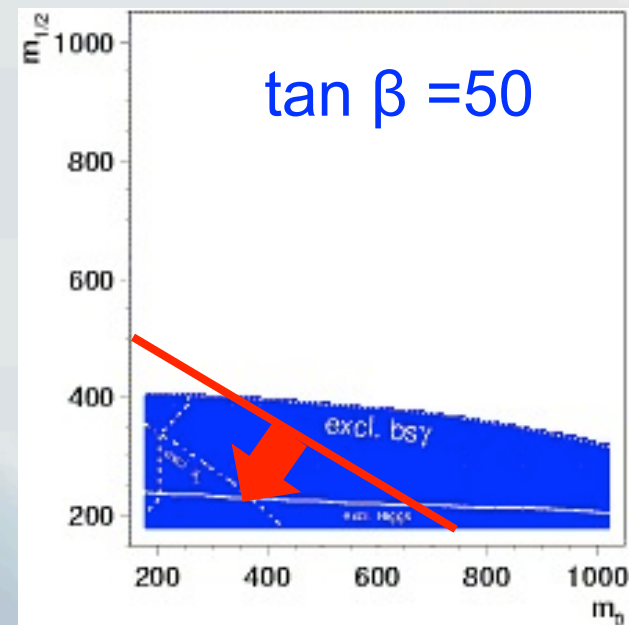
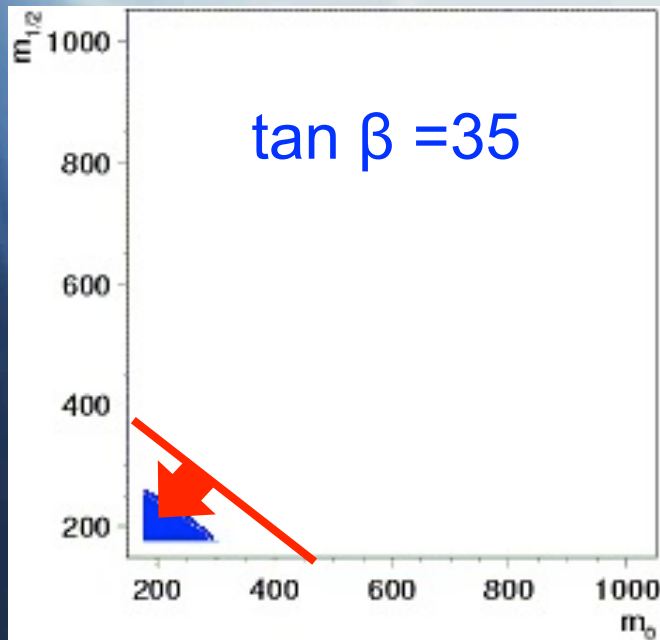
$$Br[B_s \rightarrow \mu^+ \mu^-] \sim \left| \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left(\frac{\tan^3 \beta}{4 \sin^2 \theta_W} \right) \left(\frac{m_b m_\mu m_t \mu}{M_W^2 M_A^2} \right) \frac{\sin 2\theta_i}{2} \left(\frac{m_{\tilde{t}_1}^2 \log \left[\frac{m_{\tilde{t}_1}^2}{\mu^2} \right]}{\mu^2 - m_{\tilde{t}_1}^2} - \frac{m_{\tilde{t}_2}^2 \log \left[\frac{m_{\tilde{t}_2}^2}{\mu^2} \right]}{\mu^2 - m_{\tilde{t}_2}^2} \right) \right|^2$$

Constrained MSSM (Choice of constraints)

Data on rare processes branching ratios

$$B(B_s \rightarrow \mu^+ \mu^-) < 3.7 \times 10^{-7}$$

Regions excluded by experimental limits (for large $\tan\beta$)



Anomalous magnetic moment

$$a_{\mu}^{exp} = 11\,659\,202(14)(6) \cdot 10^{-10}$$

$$a_{\mu}^{SM} = 11\,659\,159.6(6.7) \cdot 10^{-10}$$

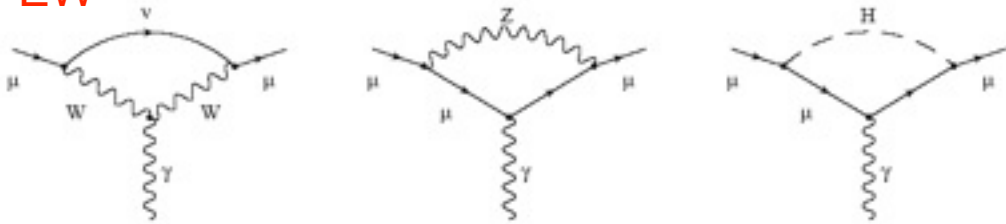
$$a_{\mu}^{exp} - a_{\mu}^{SM} = (27 \pm 10) \cdot 10^{-10}$$

$$a_{\mu}^{QED} = 11\,658\,470.56(0.29) \cdot 10^{-10}$$

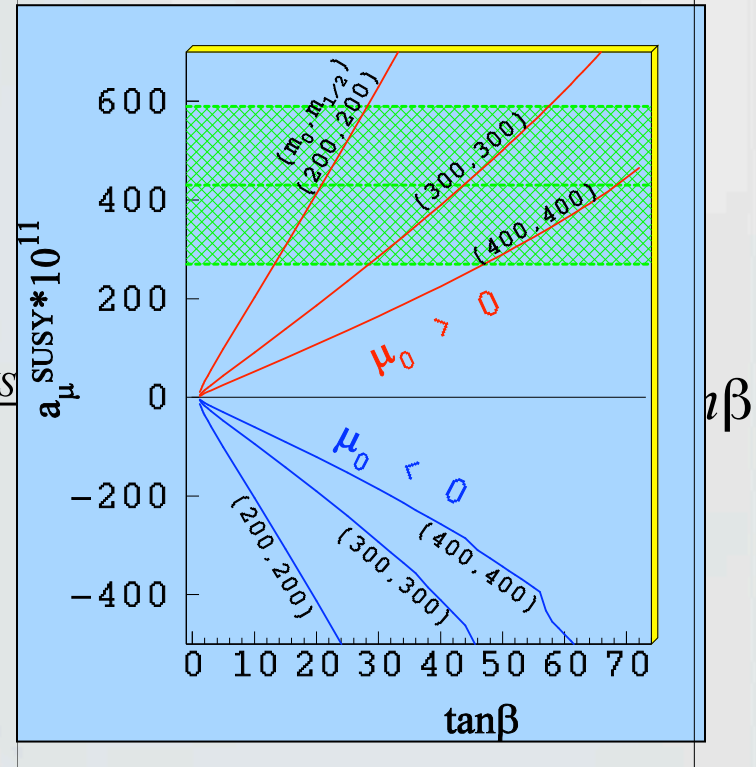
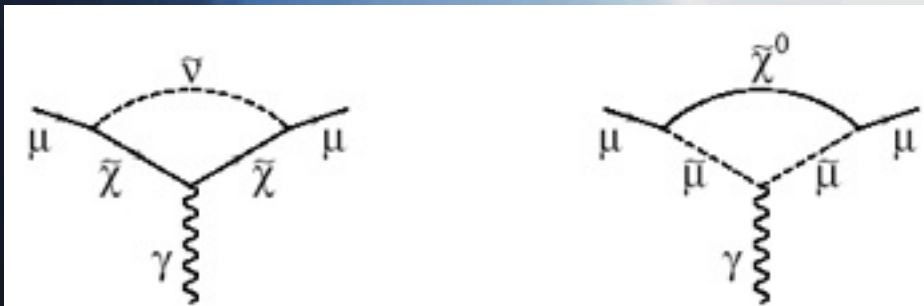
$$a_{\mu}^{weak} = 15.1(0.4) \cdot 10^{-10}$$

$$a_{\mu}^{hadr} = 673.9(6.7) \cdot 10^{-10}$$

EW



$$|a_{\mu}^{SUSY}| \simeq \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W} \frac{m_{\mu}^2}{M_{SUSY}^2} \tan\beta \left(1 - \frac{4\alpha}{\pi} \log \frac{M_{SUSY}}{m_{\mu}}\right)$$

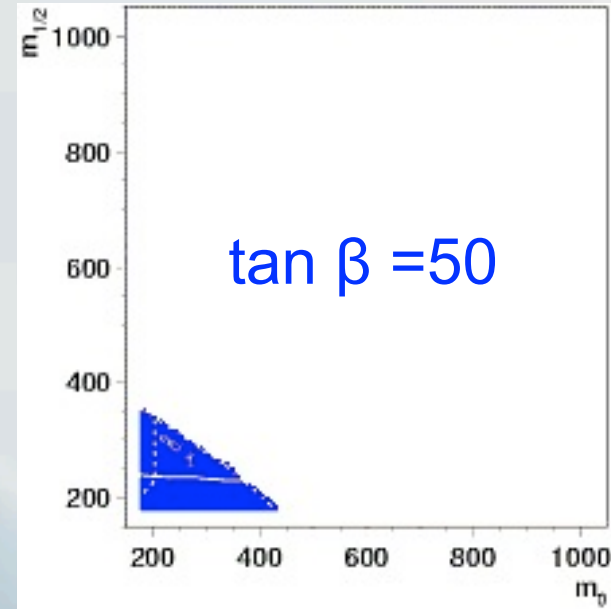
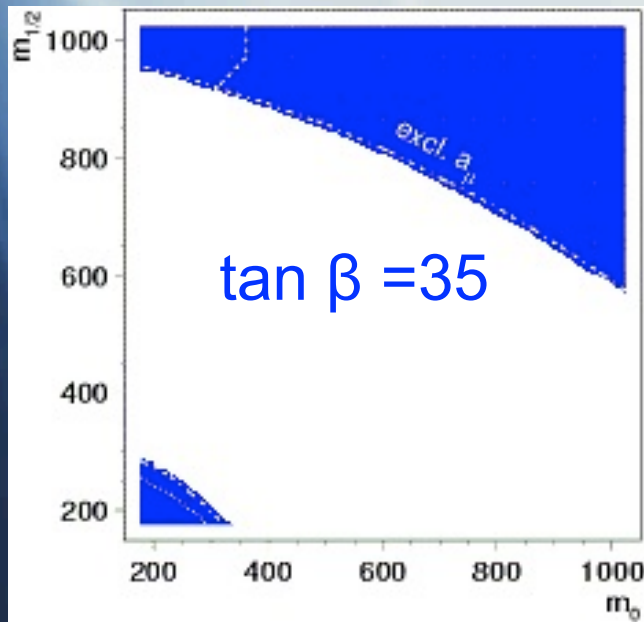


Constrained MSSM (Choice of constraints)

Muon anomalous magnetic moment

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (27 \pm 10) \cdot 10^{-10}$$

Regions excluded by muon amm constraint

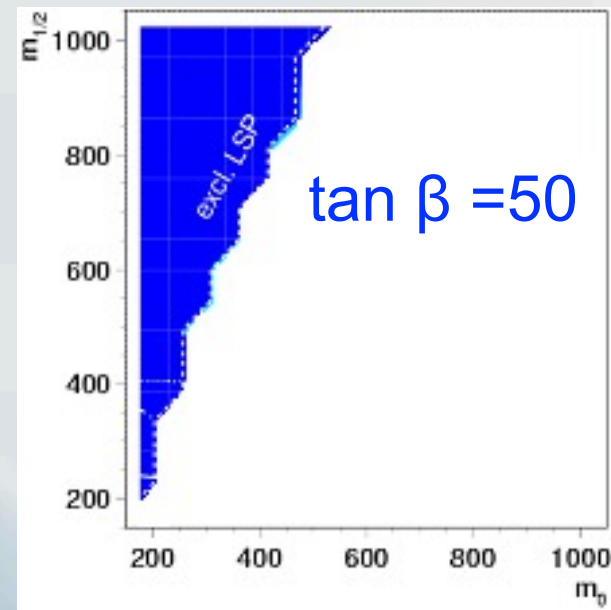
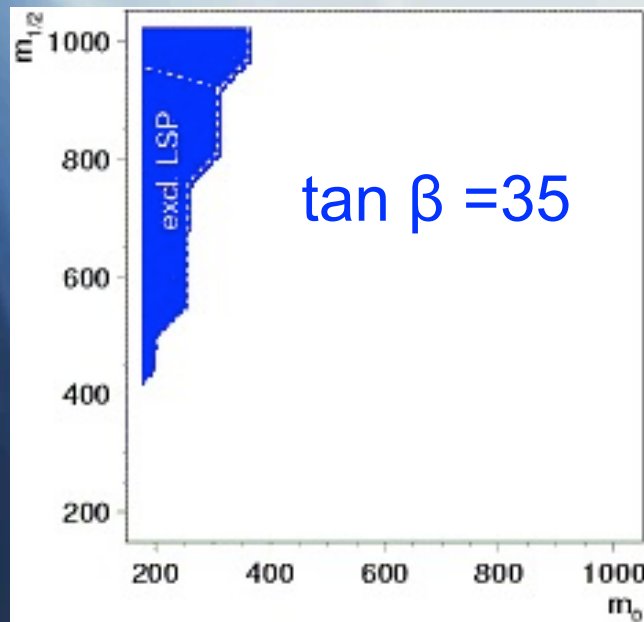


Constrained MSSM (Choice of constraints)

The lightest supersymmetric particle (LSP) is neutral.

This constraint is a consequence of R -parity conservation requirement

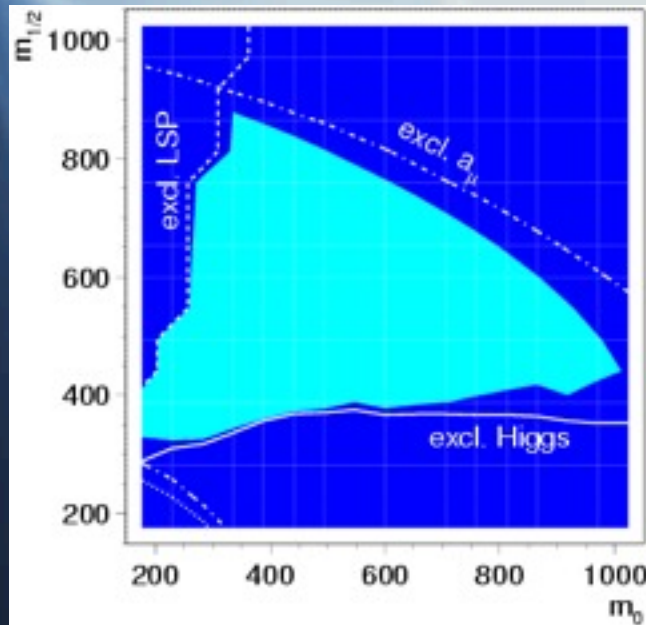
Regions excluded by LSP constraint



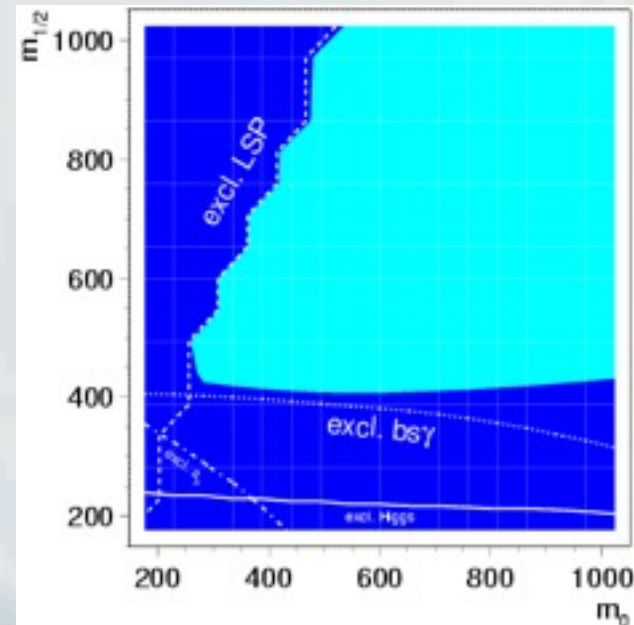
Favoured regions of parameter space (mSUGRA)

Pre-WMAP allowed regions in the parameter space.

From the Higgs searches $\tan \beta > 4$, from a_μ measurements $\mu > 0$

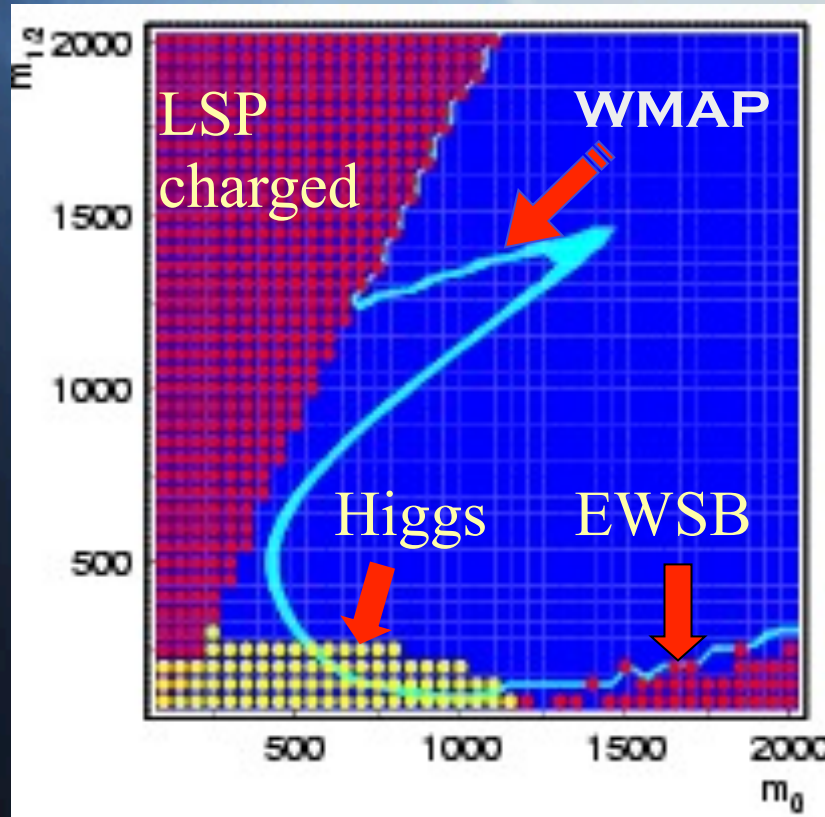


$\tan \beta = 35$



$\tan \beta = 50$

Allowed regions after WMAP



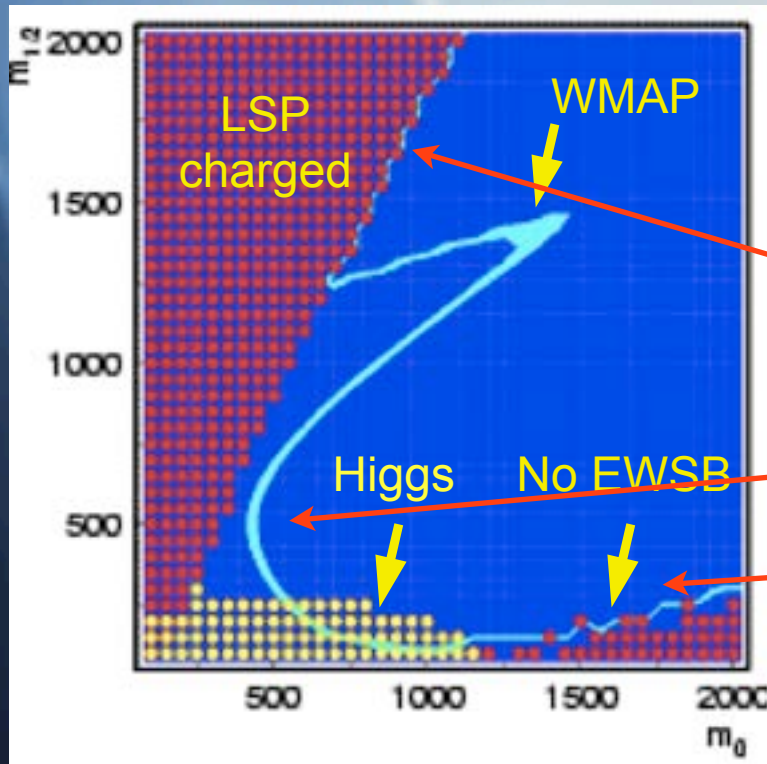
$\tan \beta = 50$

In allowed region one fulfills all the constraints simultaneously and has the suitable amount of the dark matter

Narrow allowed region enables one to predict the particle spectra and the main decay patterns

Phenomenology essentially depends on the region of parameter space and has direct influence on the strategy of SUSY searches

Long-lived Superparticles



The reason for long life-time is mass degeneracy with the LSP

Long-lived $\tilde{\tau}$

Long-lived \tilde{t}

Long-lived $\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$

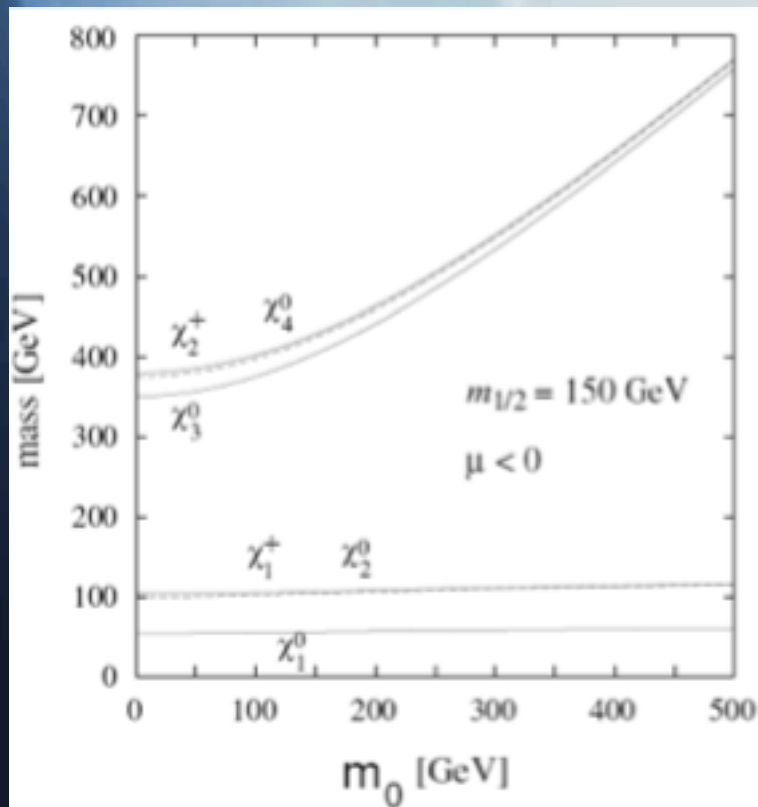
Life-time $\sim 10^{-10}$ sec, $M \sim 100$ GeV

Decays in a secondary vertex or flies through detector

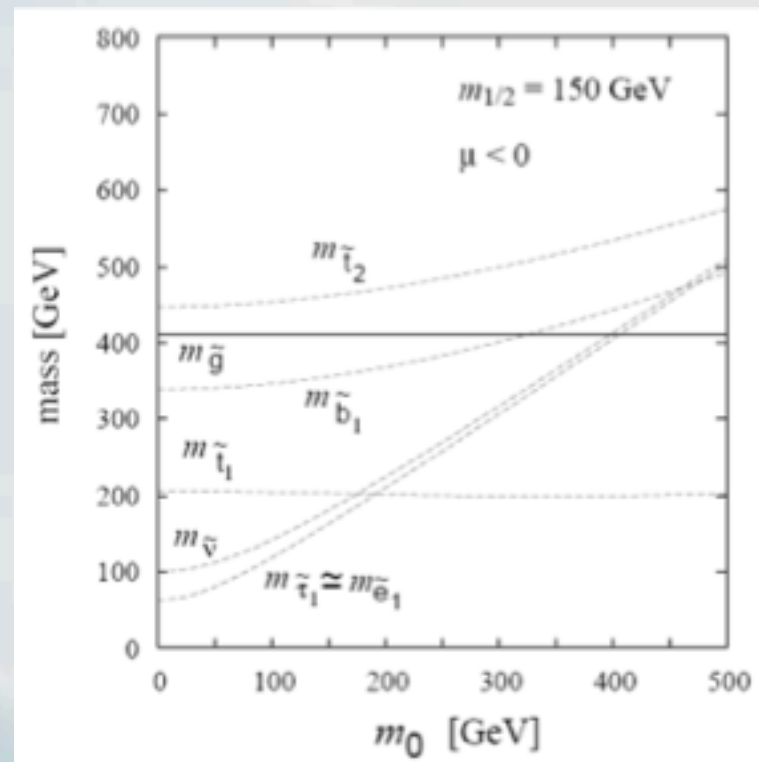
Requires fine-tuning of parameters !

SUSY Masses in MSSM

Gauginos+Higgsinos



Squarks and Sleptons

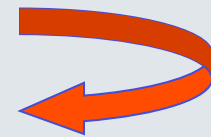


The Lightest Superparticle

		<u>property</u>	<u>signature</u>
• <u>Gravity mediation</u>	LSP = $\tilde{\chi}_1^0$	stable	jets/leptons + \cancel{E}_T
• <u>Gauge mediation</u>	LSP = \tilde{G}	stable	\cancel{E}_T
	NLSP =		
	$\left\{ \begin{array}{l} \tilde{\chi}_1^0 \\ \tilde{l}_R \end{array} \right.$	$\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}, h \tilde{G}, Z \tilde{G}$	photons/jets + \cancel{E}_T
		$\tilde{l}_R \rightarrow \tau \tilde{G}$	lepton + \cancel{E}_T
• <u>Anomaly mediation</u>	LSP =		
	$\left\{ \begin{array}{l} \tilde{\chi}_1^0 \\ \tilde{\nu}_L \end{array} \right.$	stable	
		stable	lepton + \cancel{E}_T
• <u>R-parity violation</u>	LSP is unstable \rightarrow SM particles		

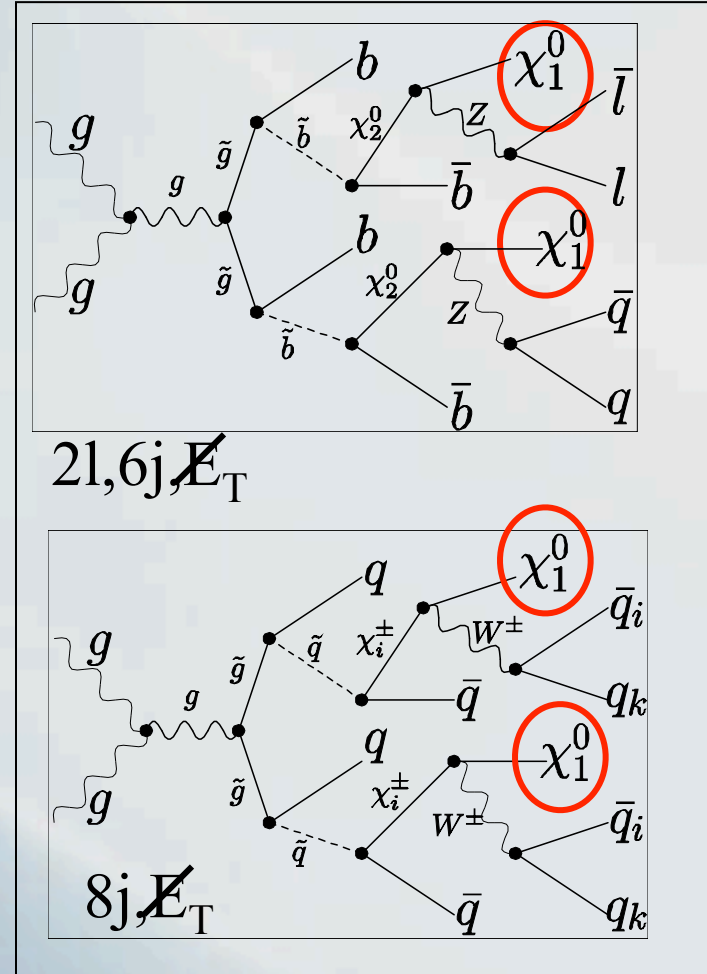
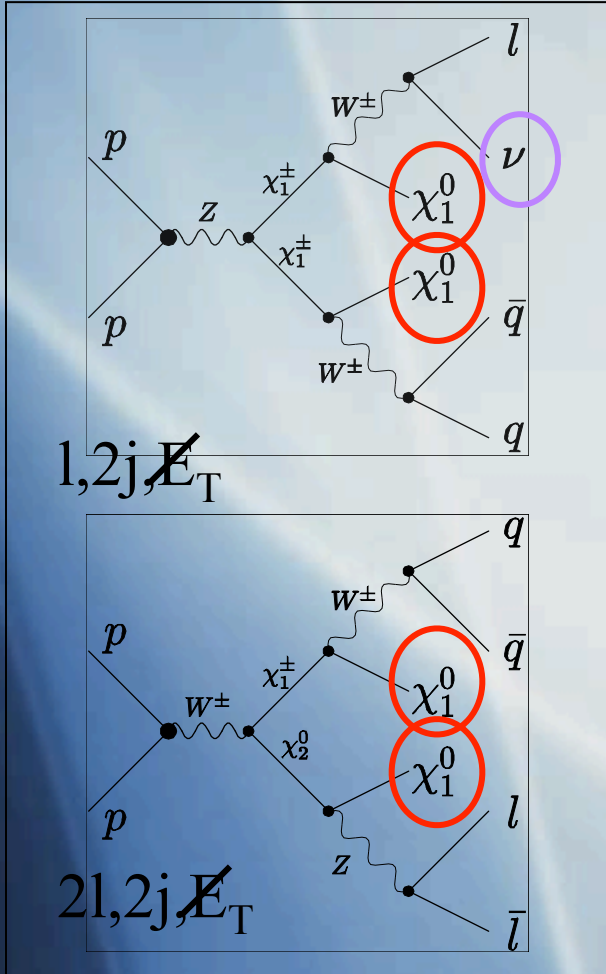
Rare decays

Neutrinoless double β decay



Creation and Decay of Superpartners in Cascade Processes @ LHC

weak int's

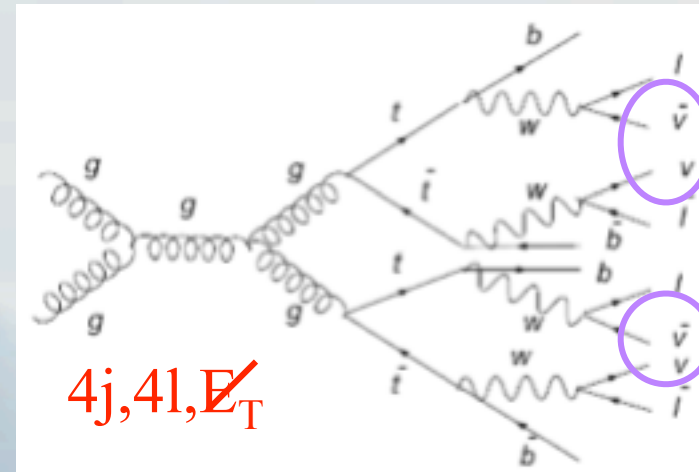
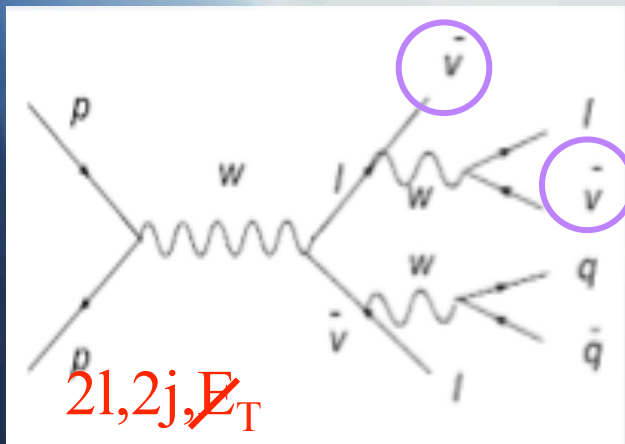
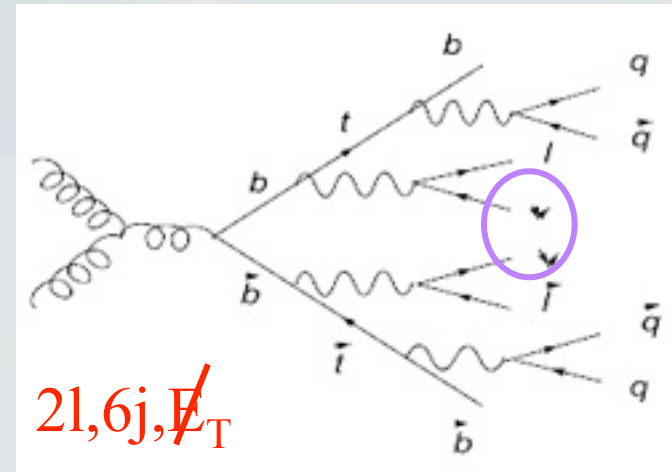
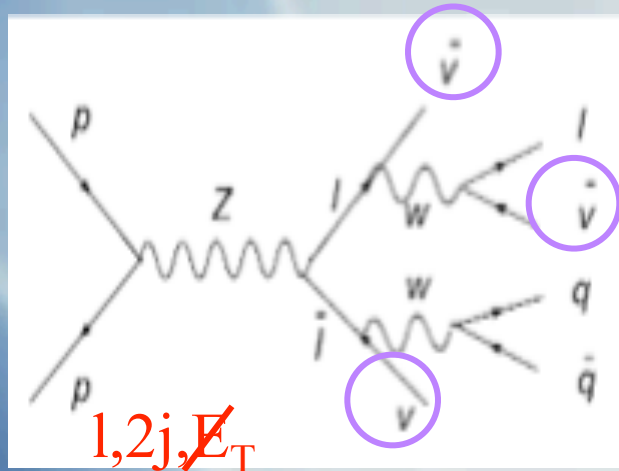


Strong int's

Typical SUSY signature: Missing Energy and Transverse Momentum

Background Processes of the SM for creation of Superpartners

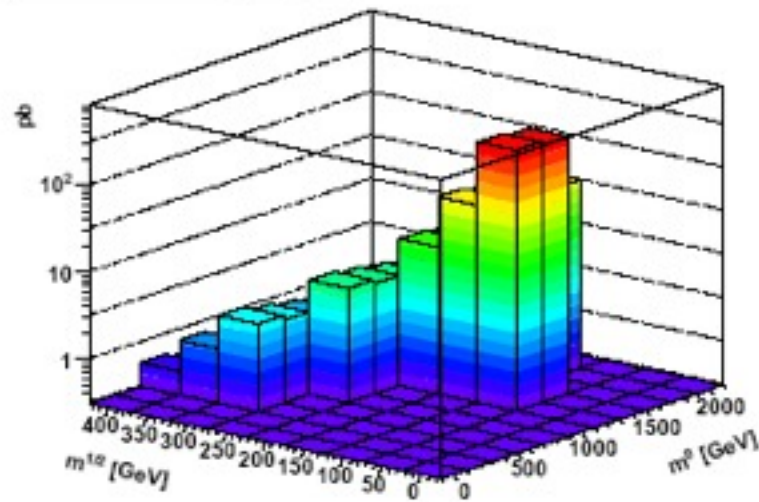
weak int's



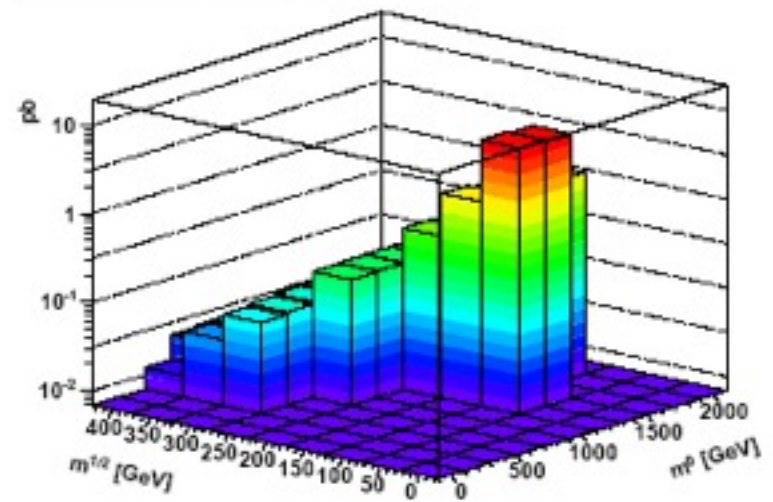
Strong int's

Cross-Sections at LHC

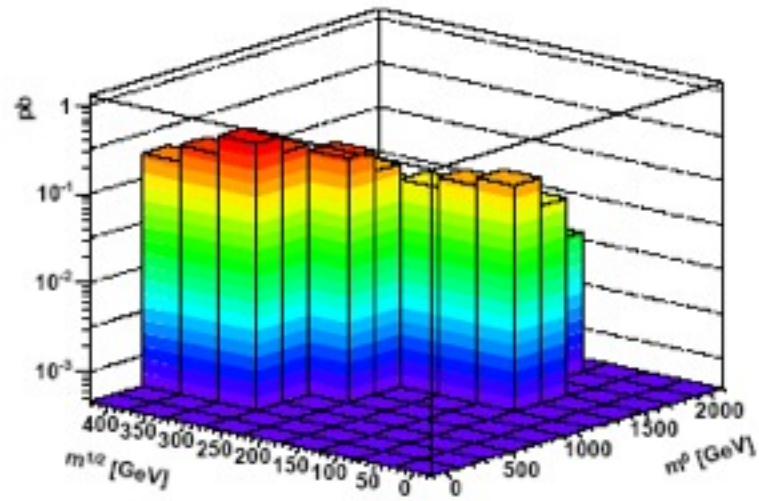
cross section p-p to $\tilde{g}\tilde{g}$



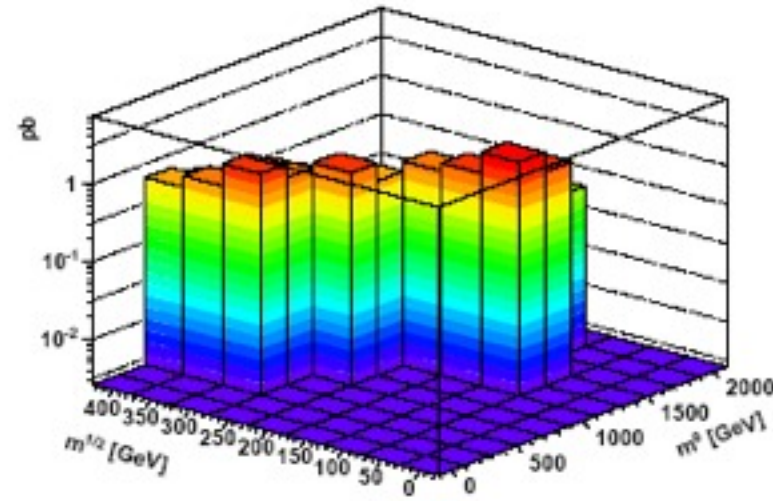
cross section p-p to $\tilde{\chi}_1^+ \tilde{\chi}_2^0$



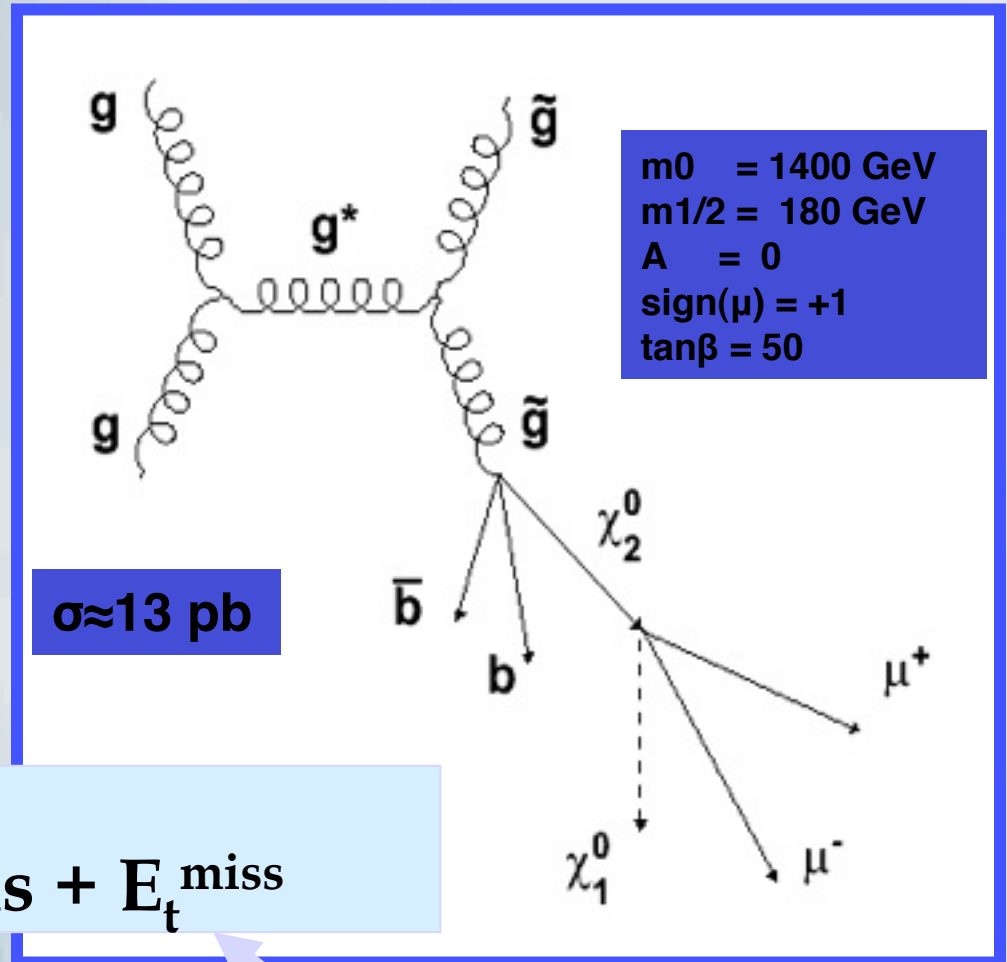
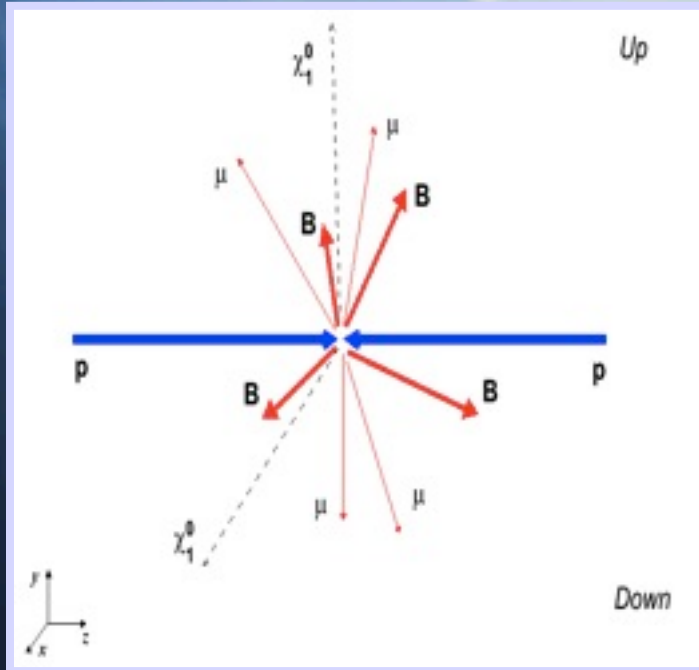
cross section p-p to $\tilde{u}_L \tilde{u}_R$



cross section p-p to $\tilde{u}_L \tilde{g}$



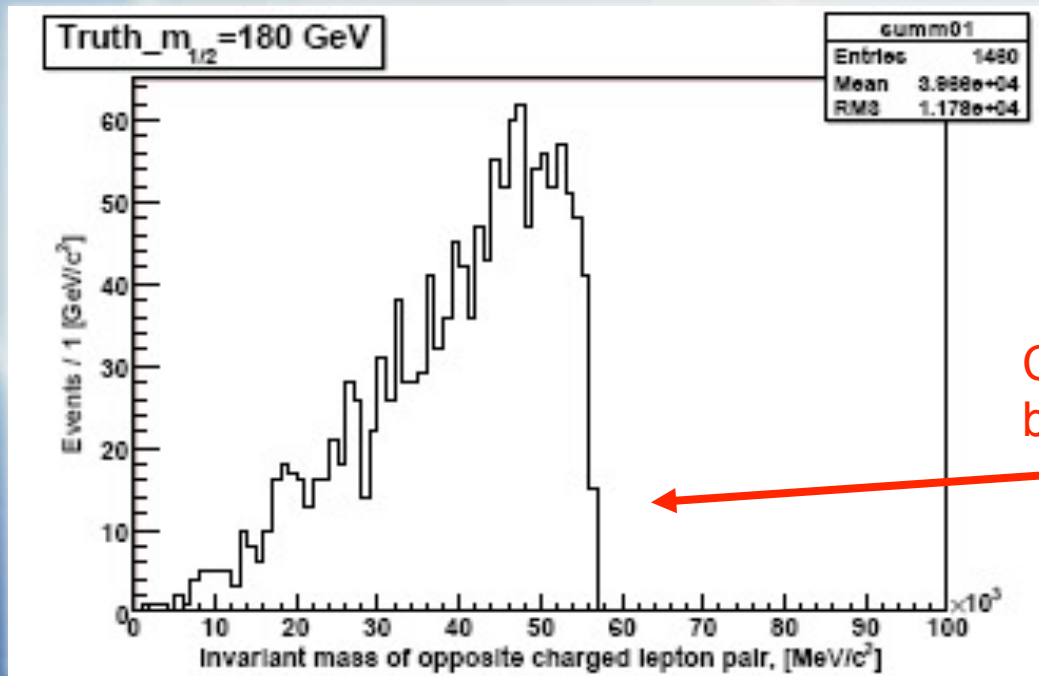
SUSY Production AT LHC



SIGNATURE:
4 b-jets + 4 muons + E_t^{miss}

LARGE!

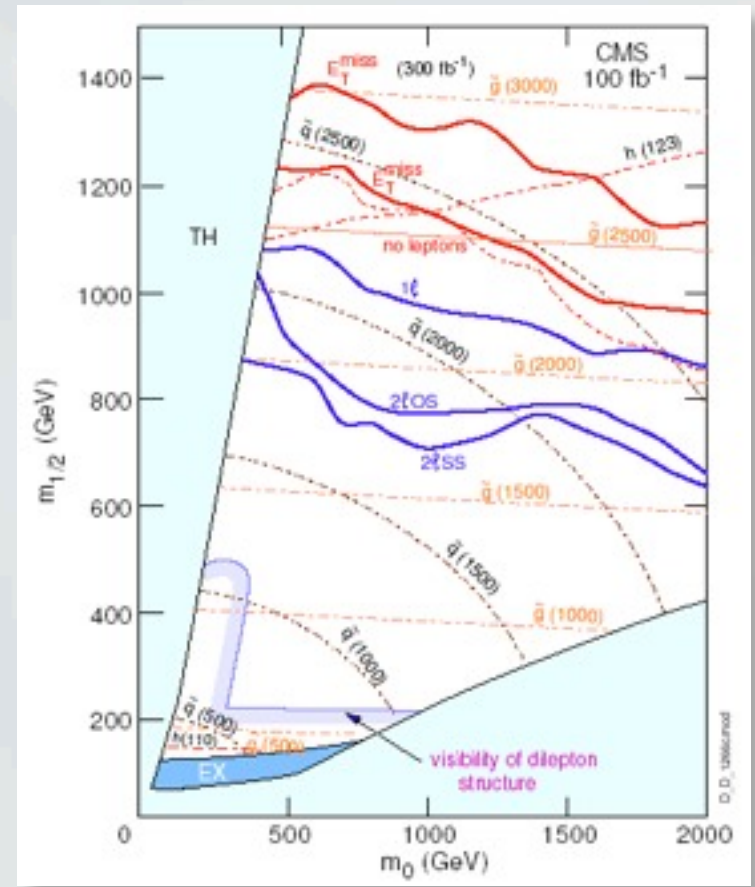
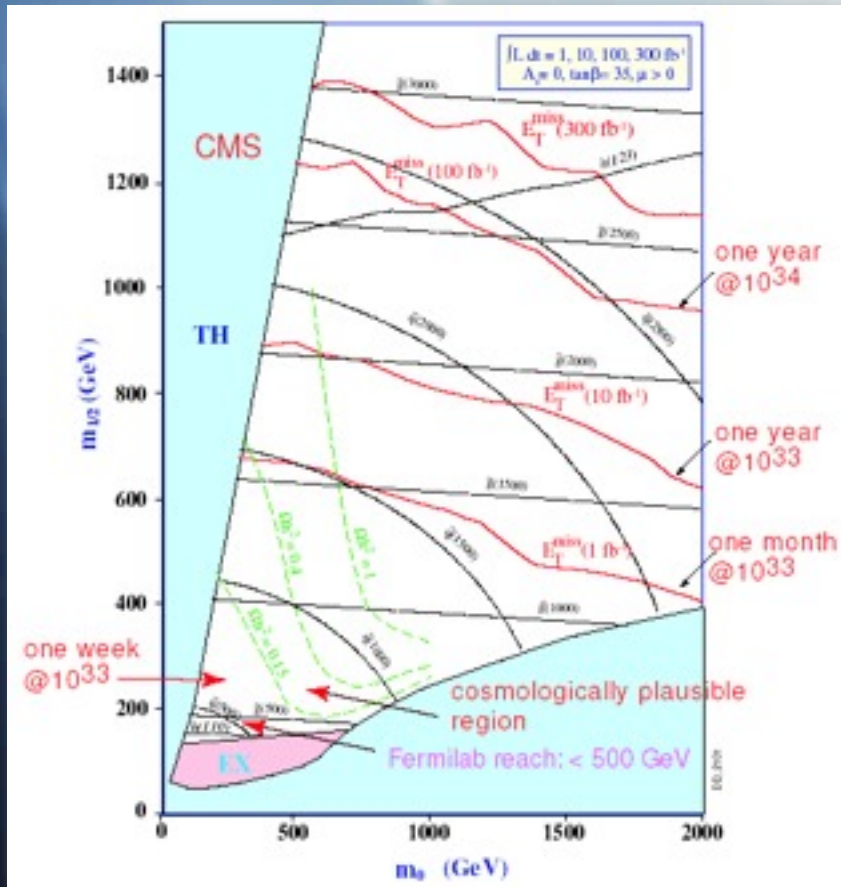
SUSY Signal @ LHC



Important to distinguish true from false events

Invariant mass distribution of OS charged lepton pair

Search for Supersymmetry @ LHC



5 σ reach in jets + \cancel{E}_T channel

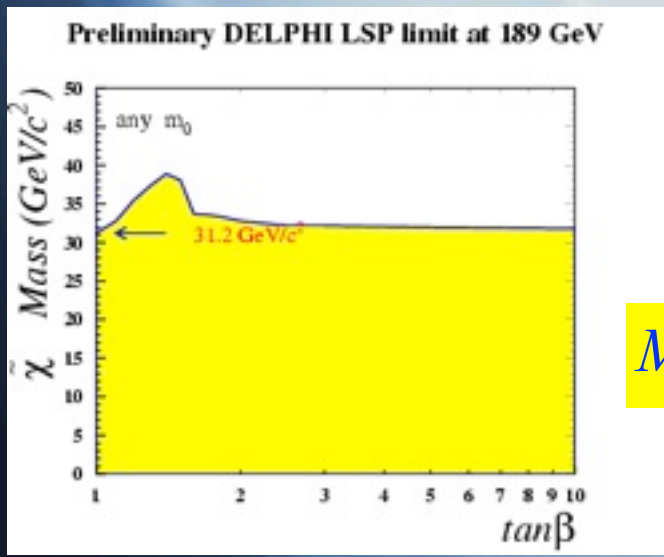
Reach limits for various channels at 100 fb^{-1}

SUSY Dark Matter

Neutralino = SUSY candidate for the cold Dark Matter
 Neutralino = the Lightest Superparticle (LSP) = WIMP

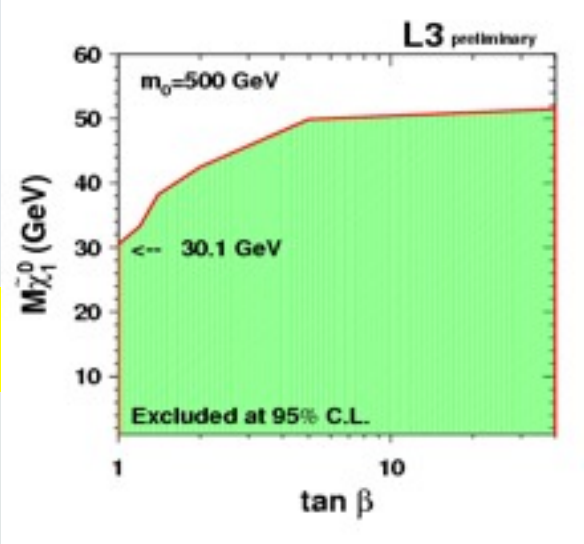
$$\tilde{\chi}^0 = N_1 \tilde{\gamma} + N_2 \tilde{z} + N_3 \tilde{H}_1^0 + N_4 \tilde{H}_2^0$$

photino zino higgsino higgsino



$$M_{\tilde{\chi}}^{\text{exp}} \geq 40 \text{ GeV}$$

$$M_{\tilde{\chi}}^{\text{theor}} = 40 \div 400 \text{ GeV}$$



$$R = (-1)^{3(B-L)+2S}$$

$$R_p = +1, R_{\tilde{p}} = -1$$



- Superparticles are created in pairs
- The lightest superparticle is stable

Dark Matter Detection



Direct detection



No convincing evidence so far
Hope for new results soon



Indirect detection

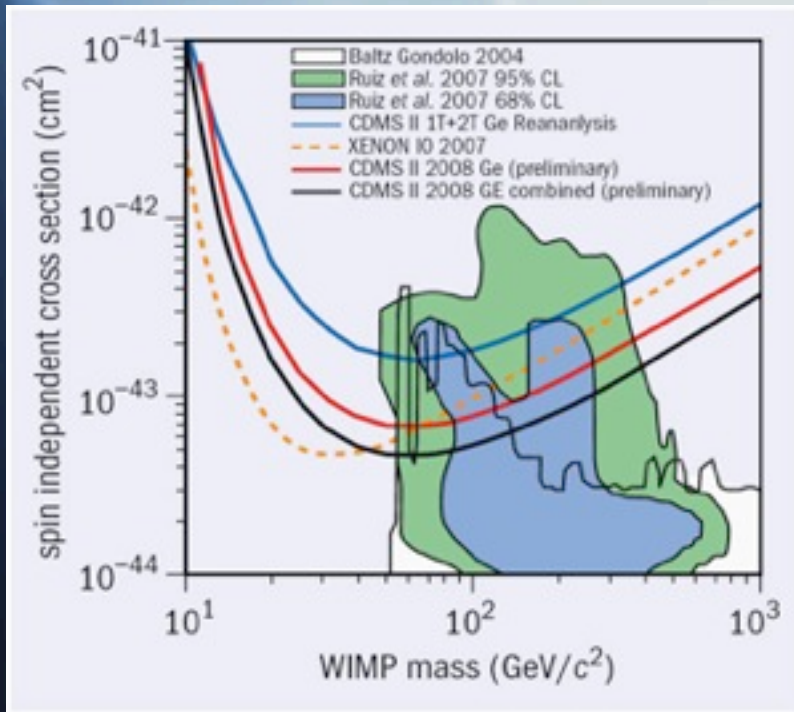
- EGRET -> GLAST(FERMI)
Diffuse Gamma Rays
- HEAT, AMS01 -> PAMELA
Positrons in Cosmic Rays
- BESS -> AMS02
Antiprotons in Cosmic Rays



Evidence of DM annihilation!

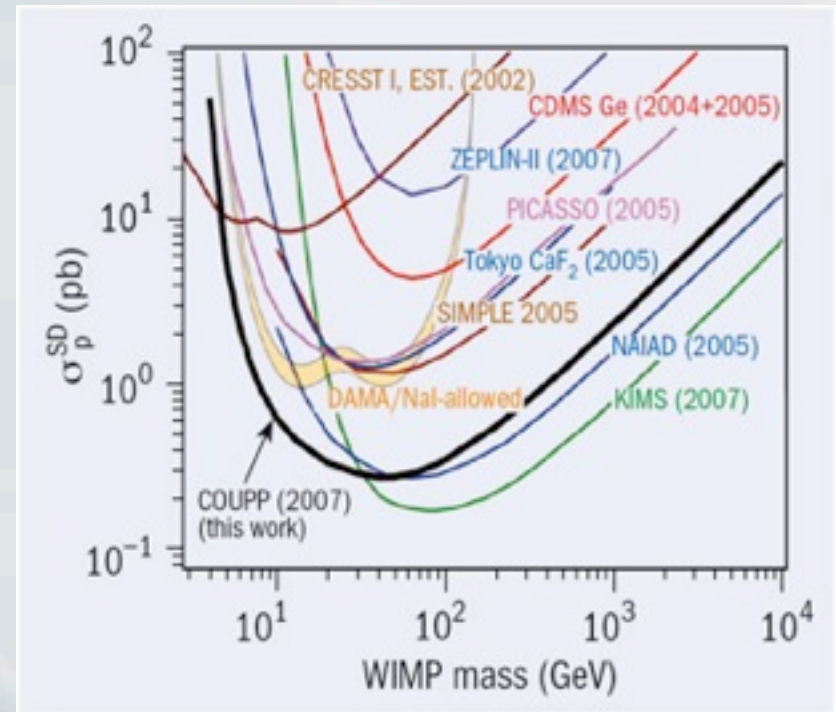
Recent Results on Direct Detection

Spin Independent



The Chicagoland Observatory for Underground Particle Physics (COUPP)

Spin Dependent



Cryogenic Dark Matter Search (CDMS)

Dark Matter Annihilation

Annihilation products from dark matter annihilation:

Gamma rays
(EGRET, FERMI)

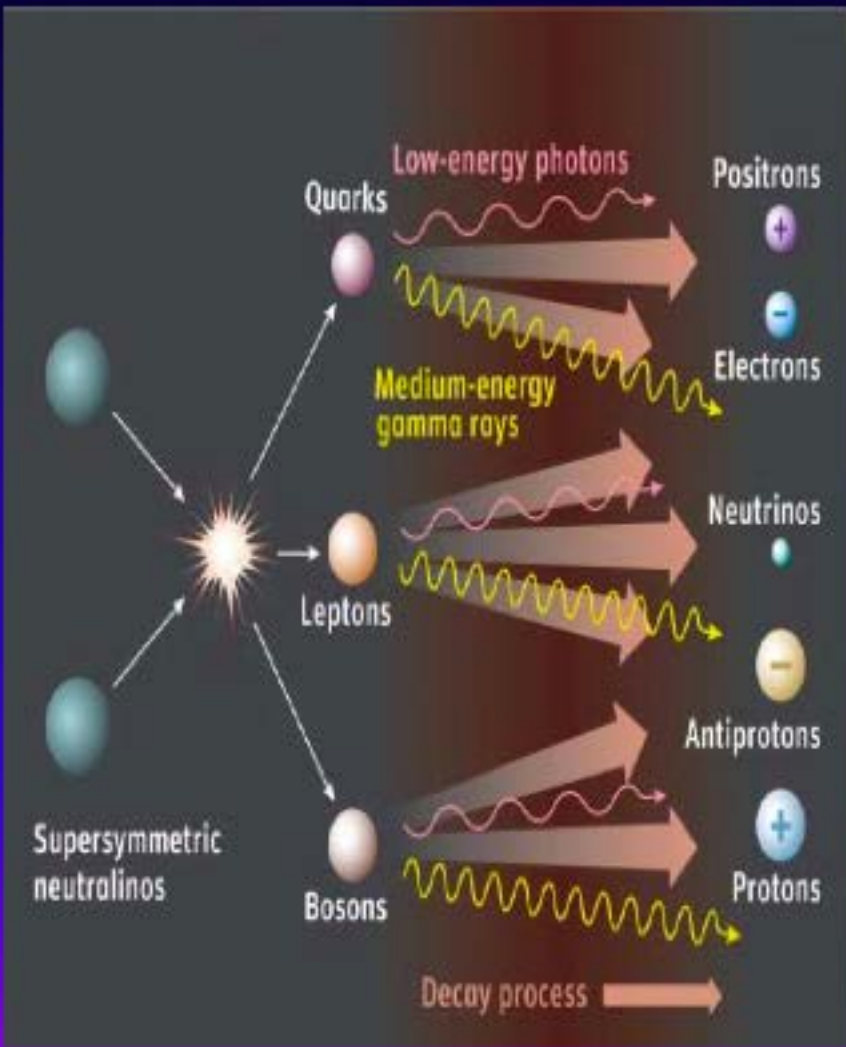
Positrons (PAMELA)

Antiprotons (PAMELA)

$e^+ + e^-$
(ATIC, FERMI, HESS, PAMELA)

Neutrinos (Icecube, no results yet)

e^- , p down in cosmic rays?



Why WIMP?

Boltzman Equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - \langle \sigma v \rangle (n_\chi^2 - n_{\chi,eq}^2),$$

Hubble constant

$$H = \dot{R} / R$$

Relic Abundance

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_c} \approx \frac{2 \cdot 10^{-27} \text{ cm}^3 \text{ sec}^{-1}}{\langle \sigma v \rangle}$$

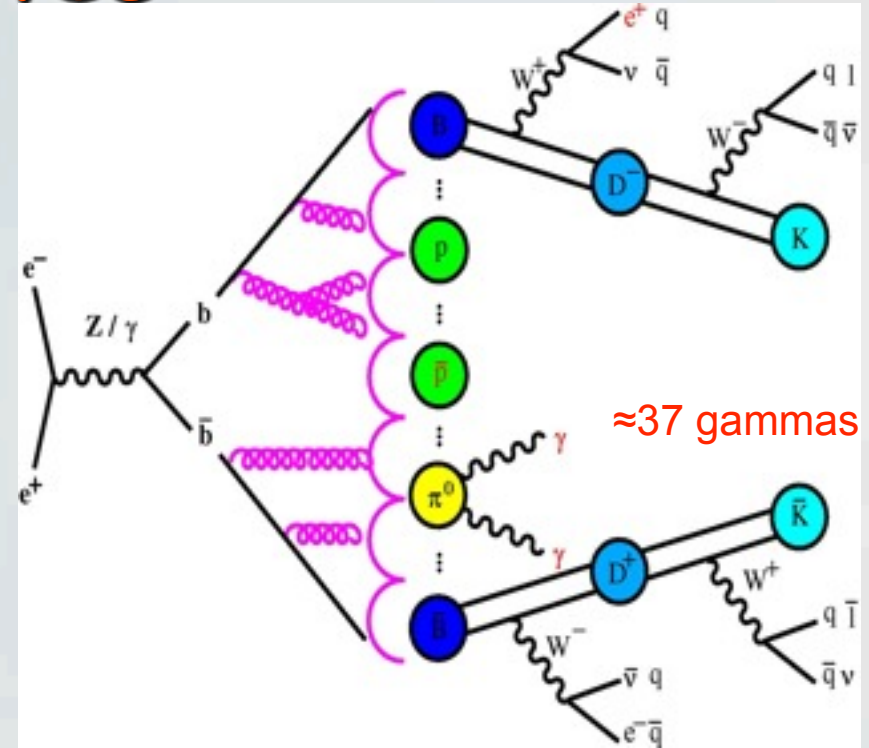
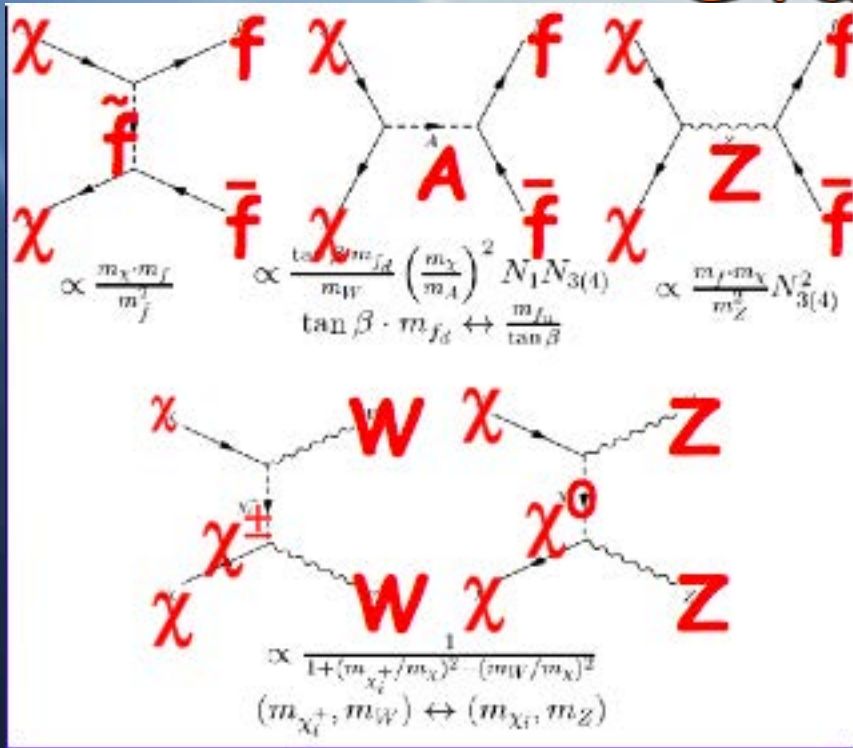
$$\Omega_\chi h^2 \sim 0.113 \pm 0.009,$$

$$v \sim 300 \text{ km / sec}$$

$$\sigma \sim 10^{-34} \text{ cm}^2 = 100 \text{ pb}$$

Typical EW x-section

DM Neutralino Annihilation Final States

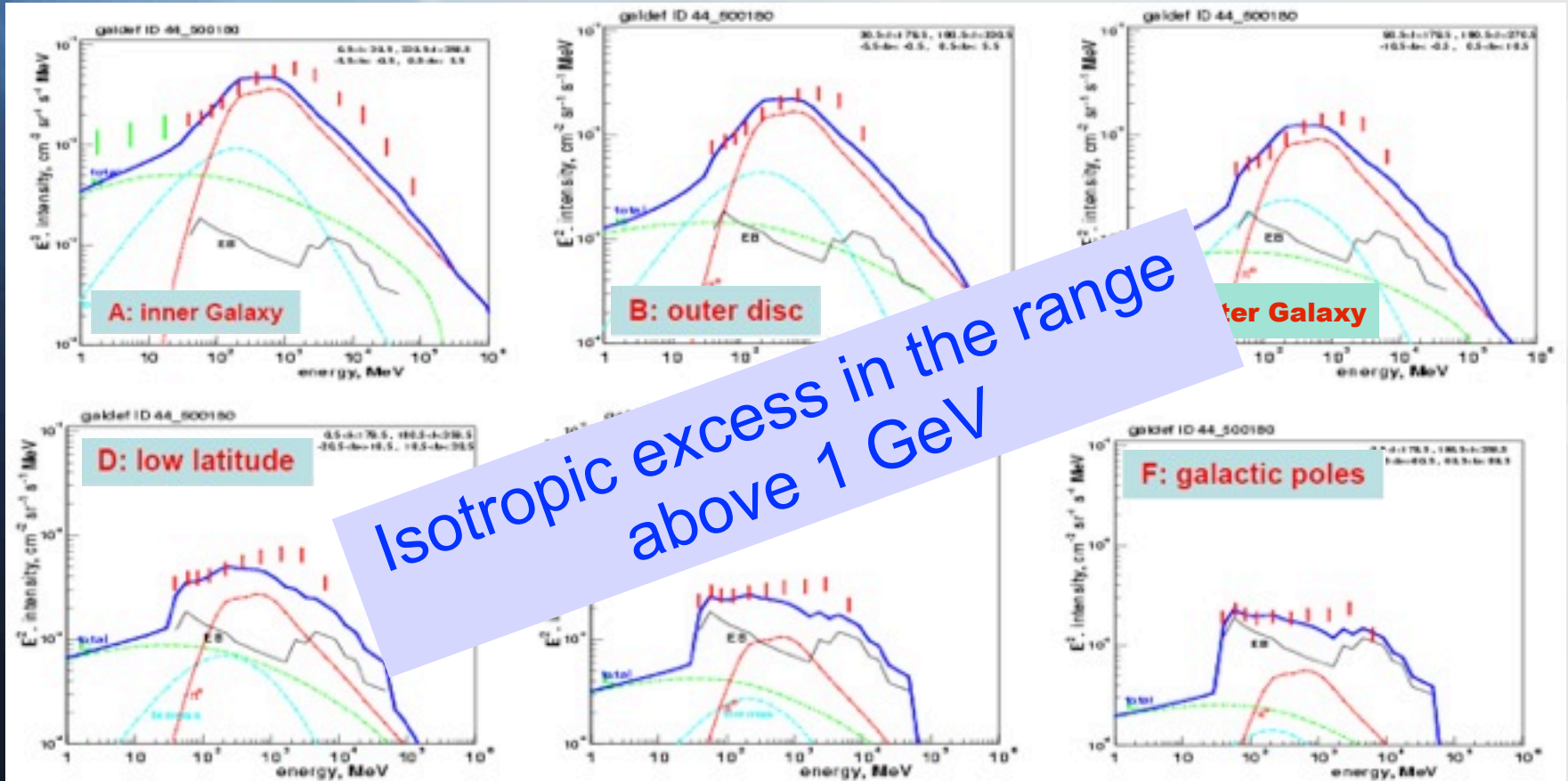


Dominant

$\chi + \chi \Rightarrow A \Rightarrow b \bar{b}$ quark pair
 Sum of diagrams should yield
 $\langle \sigma v \rangle = 2 \cdot 10^{-26} \text{ cm}^3/\text{s}$ to get
 correct relic density

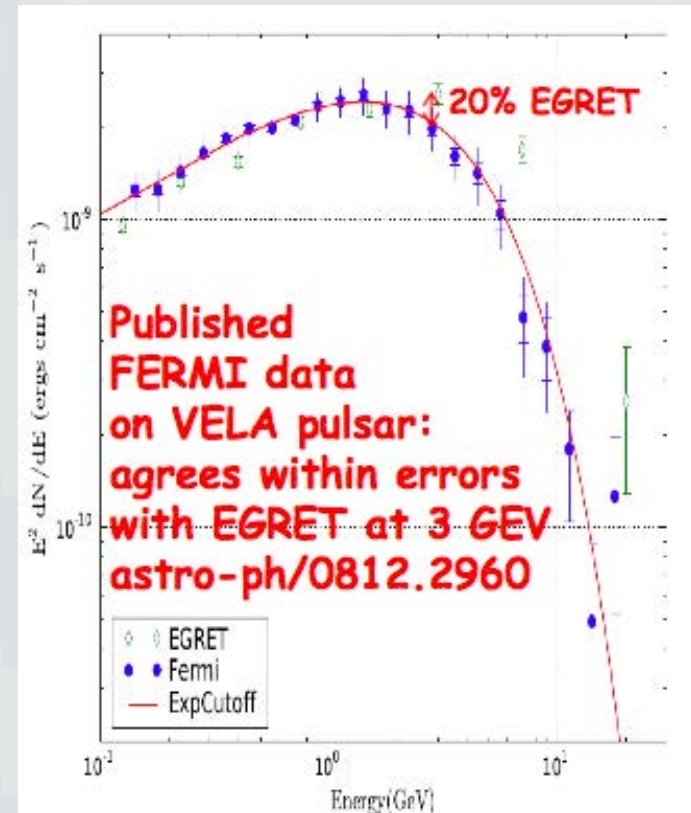
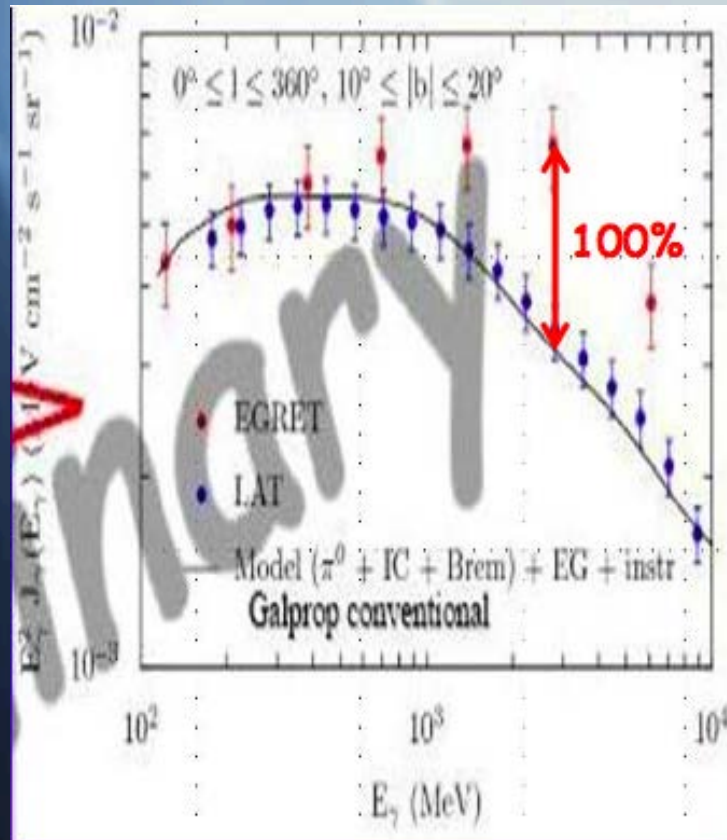
Quark-fragmentation known!
 Hence spectra of positrons,
 gammas and antiprotons known!
 Relative amount of γ, p, e^+ known
 as well.

Analysis of EGRET Data in 6 Sky Directions



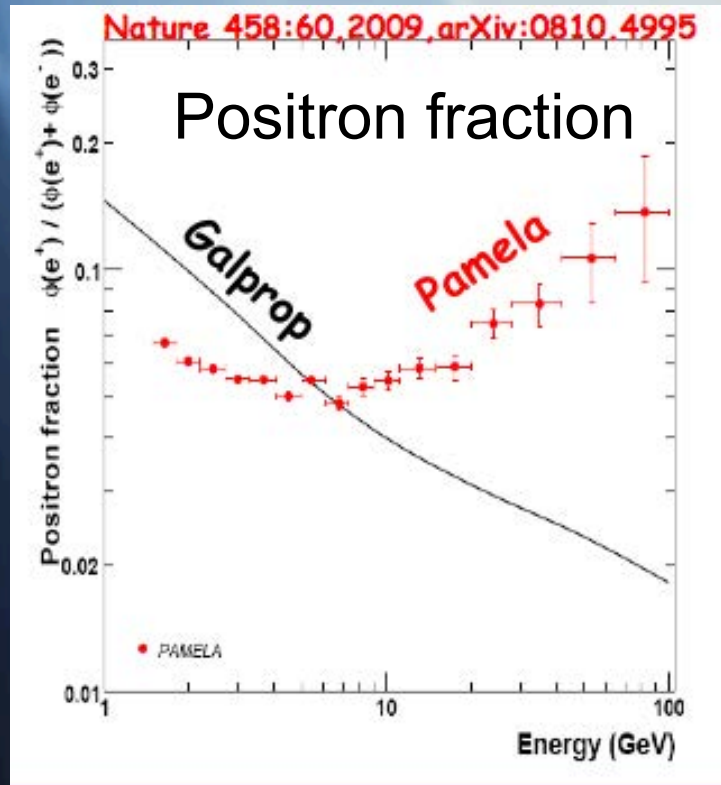
Total χ^2 for all regions: 28/36 \Rightarrow Prob.=0.8 Excess above background $> 10\sigma$

Diffuse gamma rays from FERMI

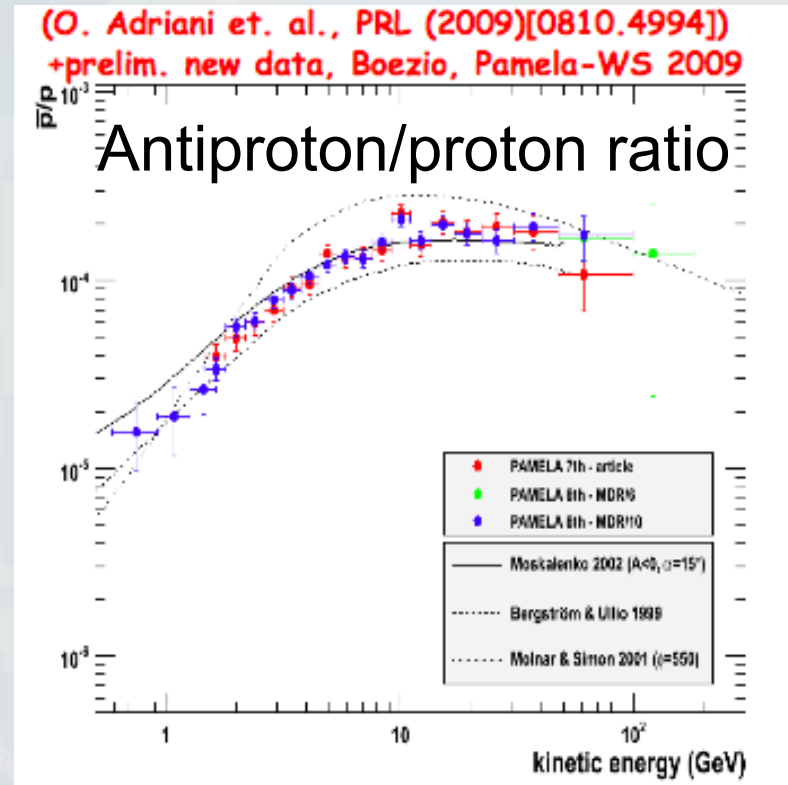


Why diffuse spectrum disagrees 100% with EGRET at 3 GeV while VELA spectrum agrees with EGRET at 3 GeV within 20%?

PAMELA: positron and antiproton measurements



Positrons: excess



Antiprotons: NO excess

Interpretation of PAMELA Excess

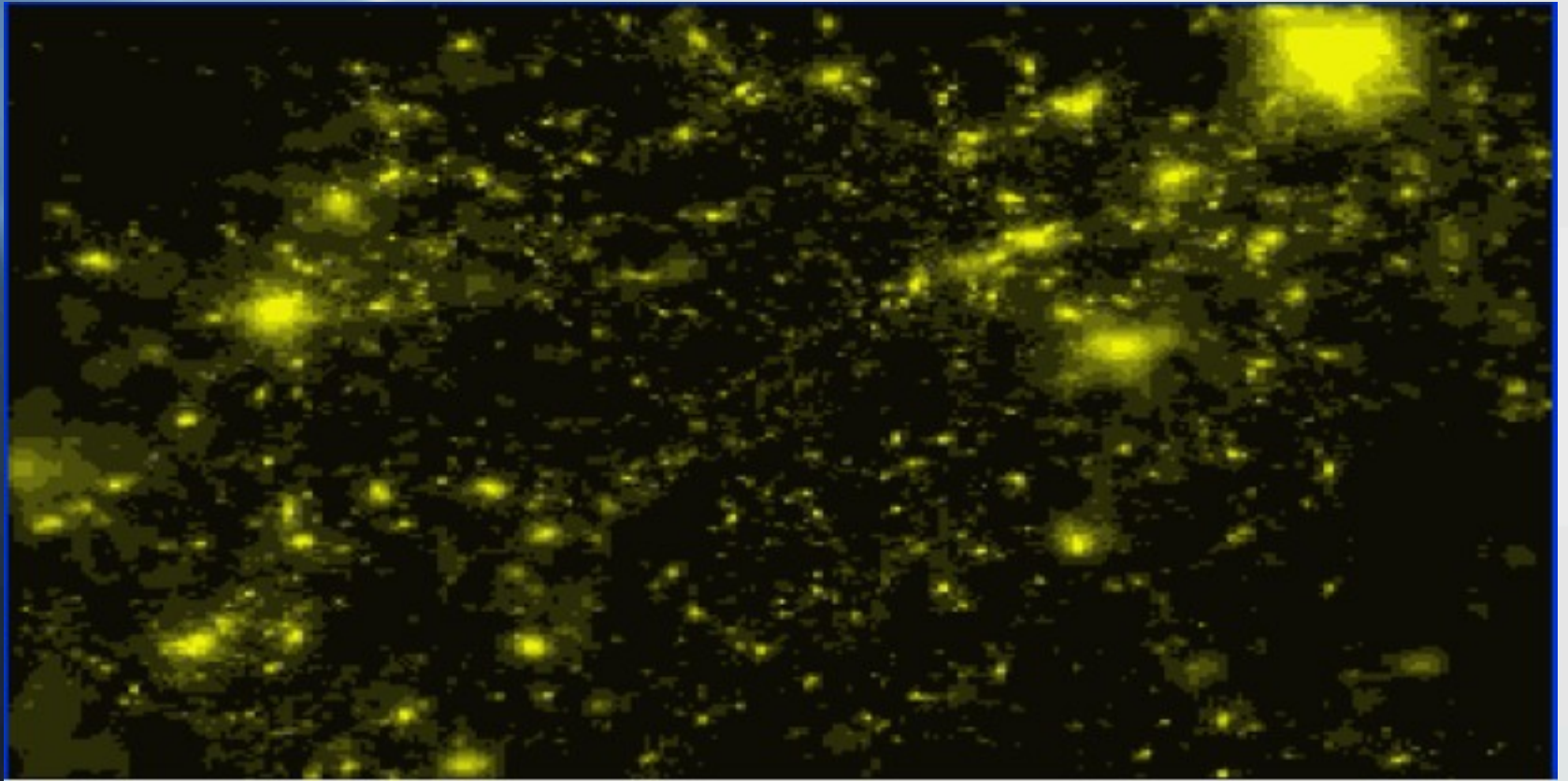
Many possibilities:

- **Background from hadronic showers with large electromagnetic component** -> $a_{p \rightarrow \pi^0}$

- **astrophysical sources**
 - ❖ pulsars -> a_{pulsar}
 - ❖ positron acceleration in SNR -> a_{sec}
 - ❖ locality of sources -> a_{SNR}

- **dark matter annihilation** -> a_{DMA}
 - ❖ leptophilic?
 - ❖ bound states?
 - ❖ Kaluza-Klein

Clustering of of Dark Matter



Cluster size: \approx Solar system?

$M_{\min} \cong 10^{-8} - 10^{-6} M_{\odot}$

Steeply falling mass spectrum.

Boost factor $\sim \langle \rho^2 \rangle / \langle \rho \rangle^2 \sim 20 - 2000$

From fit: $B \approx 100$ for WIMP of 60 GeV

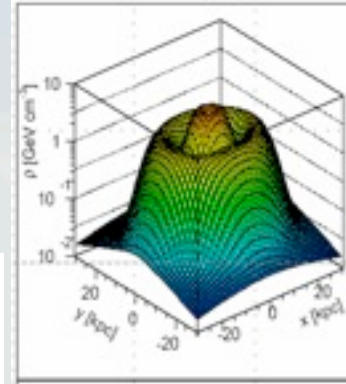
Clumps with M_{\min} \rightarrow dominant contribution \rightarrow MANY clumps in given direction \rightarrow same boostfactor in all directions

Fitted Halo Parameters

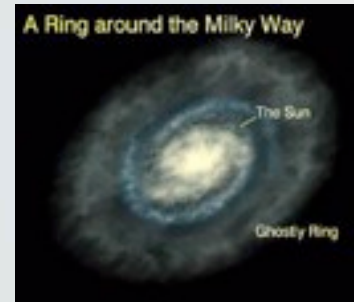
Gamma Ray Flux: ($\langle\sigma v\rangle$ from WMAP)

$$\phi_{\chi}(E, \psi) = \frac{\langle\sigma v\rangle}{4\pi} \sum_f \frac{dN_f}{dE} b_f \int_{\text{line of sight}} B_l \frac{1}{2} \frac{\langle\rho_{\chi}^2\rangle}{M_{\chi}^2} dl_{\psi}$$

$$\rho_{\chi}(\vec{r}) = \rho_0 \left(\frac{R_0}{\tilde{r}}\right)^{\gamma} \left[\frac{1 + \left(\frac{\tilde{r}}{a}\right)^{\alpha}}{1 + \left(\frac{R_0}{a}\right)^{\alpha}}\right]^{\frac{\gamma-\beta}{\alpha}} + \sum_{n=1}^N \rho_n \exp\left(-\frac{(\tilde{r}_{gc} - Rn)^2}{2\sigma_{Rn}^2} - \frac{(z_n)^2}{2\sigma_{zn}^2}\right)$$



Enhancement of rings over $1/r^2$ profile 2 and 7, respectively.
Mass in rings 1.6 and 0.3% of total DM



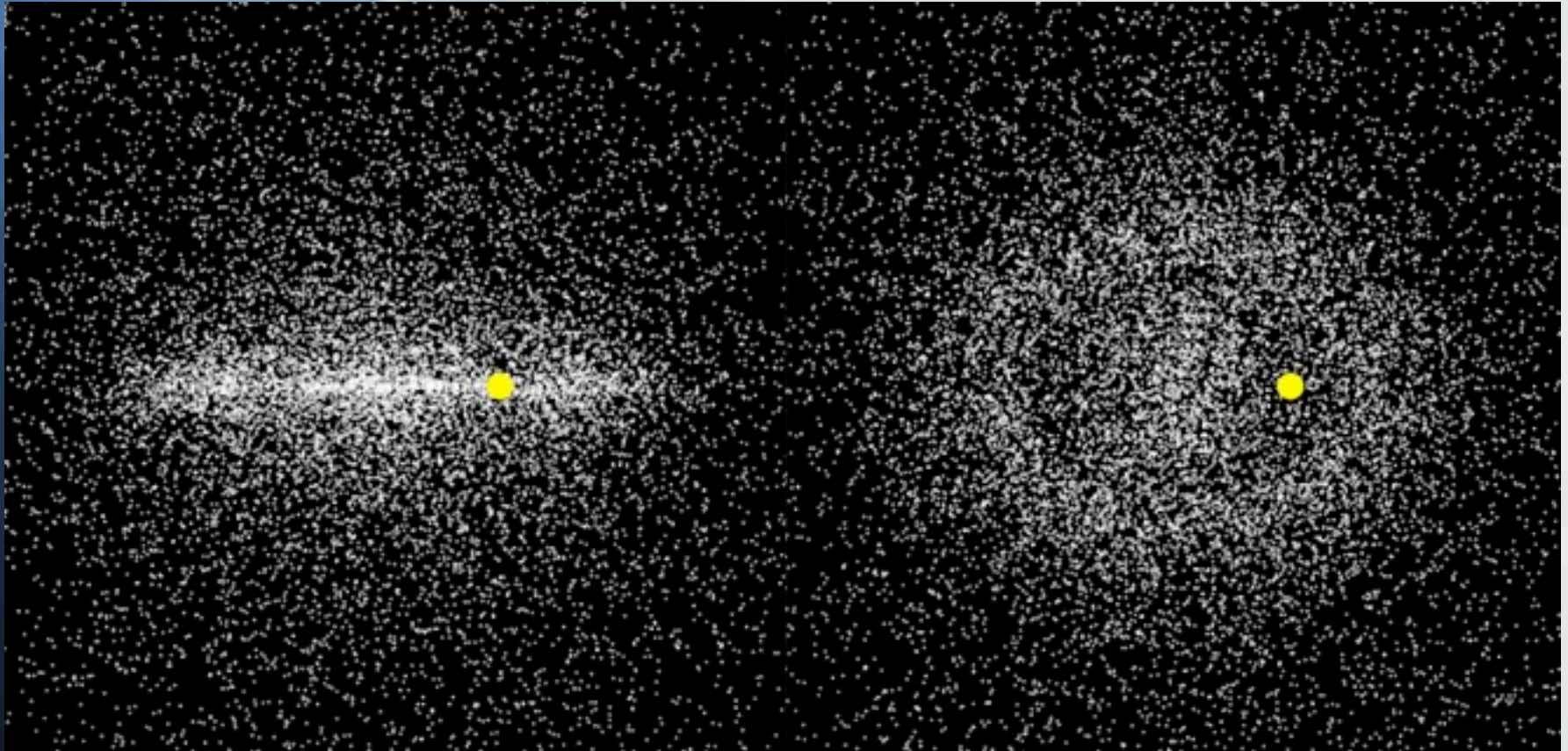
14 kpc coincides with ring of stars at 14-18 kpc due to infall of dwarf galaxy

4 kpc coincides with ring of neutral hydrogen molecules!

Parameter	Value	Parameter	Value
α	2	R_a	4.3 kpc
β	2	$\sigma_{R,a}$	3.4 kpc
γ	0	$\sigma_{z,a}$	0.3 kpc
R_0	8.5 kpc	ρ_b	2.3 GeV cm^{-3}
a	4 kpc	R_b	14 kpc
ρ_0	0.47 GeV cm^{-3}	$\sigma_{R,b}$	2.1 kpc
ρ_a	3.3 GeV cm^{-3}	$\sigma_{z,b}$	1.3 kpc
b/a	0.9	c/a	0.8

W.de Boer, C.Sander, V.Zhukov, A.Gladyshev, and D.Kazakov, A&A 444 (2005)17

Halo Density on Scale of 30 Kpc



Side view

	Isother.	NFW
R200	295 kpc	145 kpc
DM:	$3 \cdot 10^{12} M_{\odot}$	$3 \cdot 10^{11} M_{\odot}$
Vis.:	$6 \cdot 10^{10} M_{\odot}$	
Outer Ring:	$3 \cdot 10^{10} M_{\odot}$	
Inner Ring:	$3 \cdot 10^9 M_{\odot}$	

Top view

64

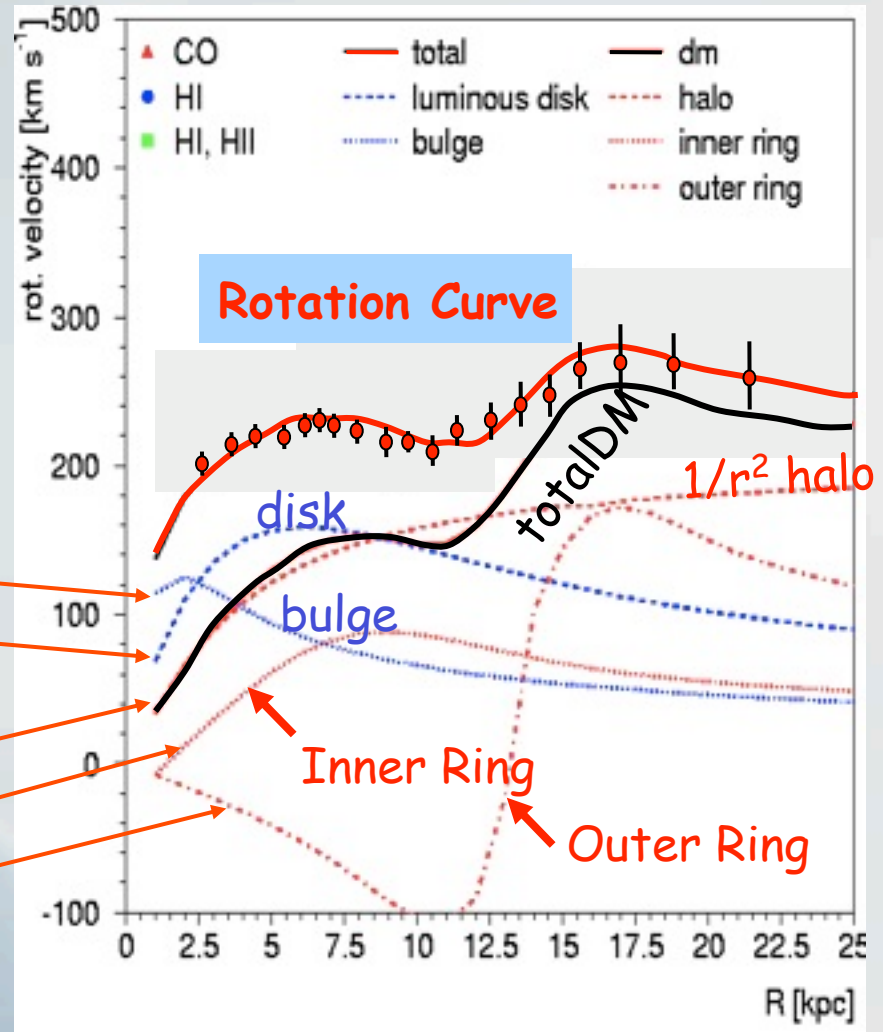
Rotation Curve for the Milky Way



Contributions to the rotation curve of the Milky Way from

Visible bulge
Visible disk

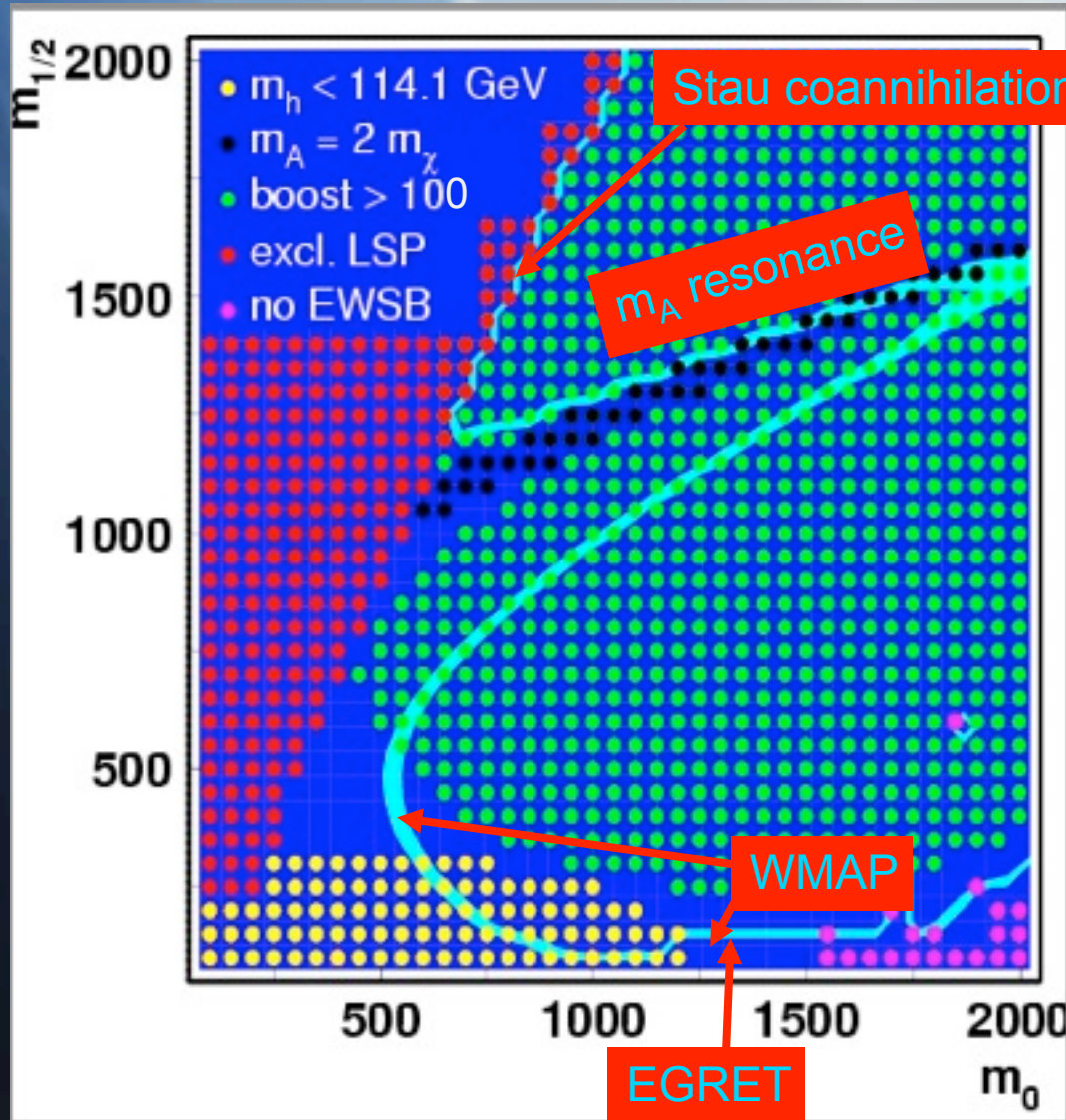
Dark halo
Inner dark ring
Outer dark ring



Normalize to solar velocity of 220 km/s

R [kpc]⁵

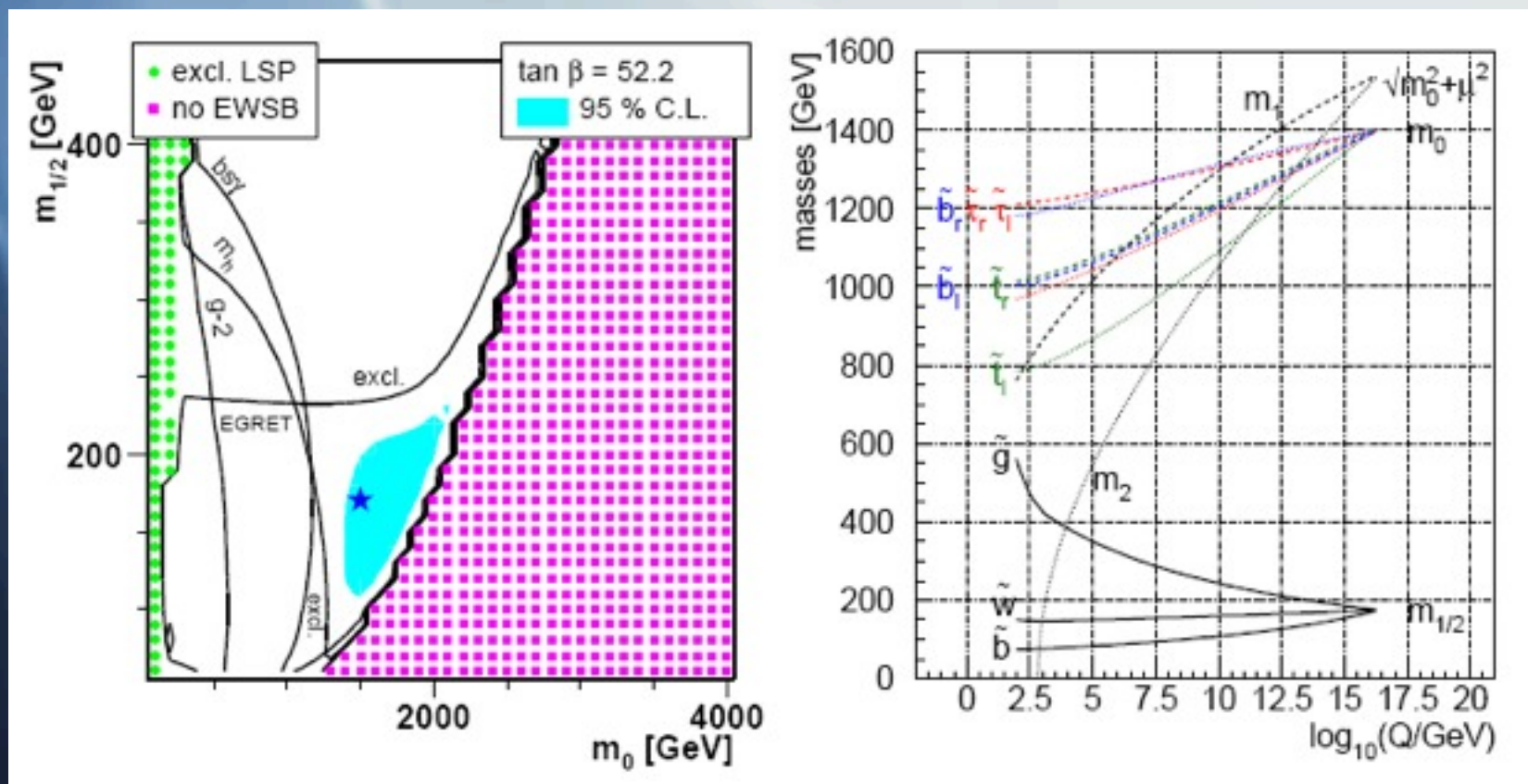
Allowed SUSY Parameter



MSUGRA can fulfill all constraints from WMAP, LEP, $b \rightarrow s\gamma$, $g-2$ and EGRET simultaneously, if DM is neutralino with mass in range 50-100 GeV and squarks and sleptons are $O(1 \text{ TeV})$
 m_0 common spin 0 mass
 $m_{1/2}$ common spin $\frac{1}{2}$ mass
 $\tan\beta = v_2/v_1$

High $\tan\beta$ solution
 $\tan\beta = 50$

EGRET Point and Mass Spectrum



W.de Boer et al PL B636 (2006)13

SUSY DM: $m_{\chi^0} \sim 60 \text{ GeV}$

$m_{\chi^\pm} \sim 120 \text{ GeV}$

PHYSICS PROBLEMS

Cosmologists:

What is CDM and Dark Energy made of?

Astrophysicists:

What is the origin of excess of diffuse Galactic Gamma Rays?

Particle physicists :

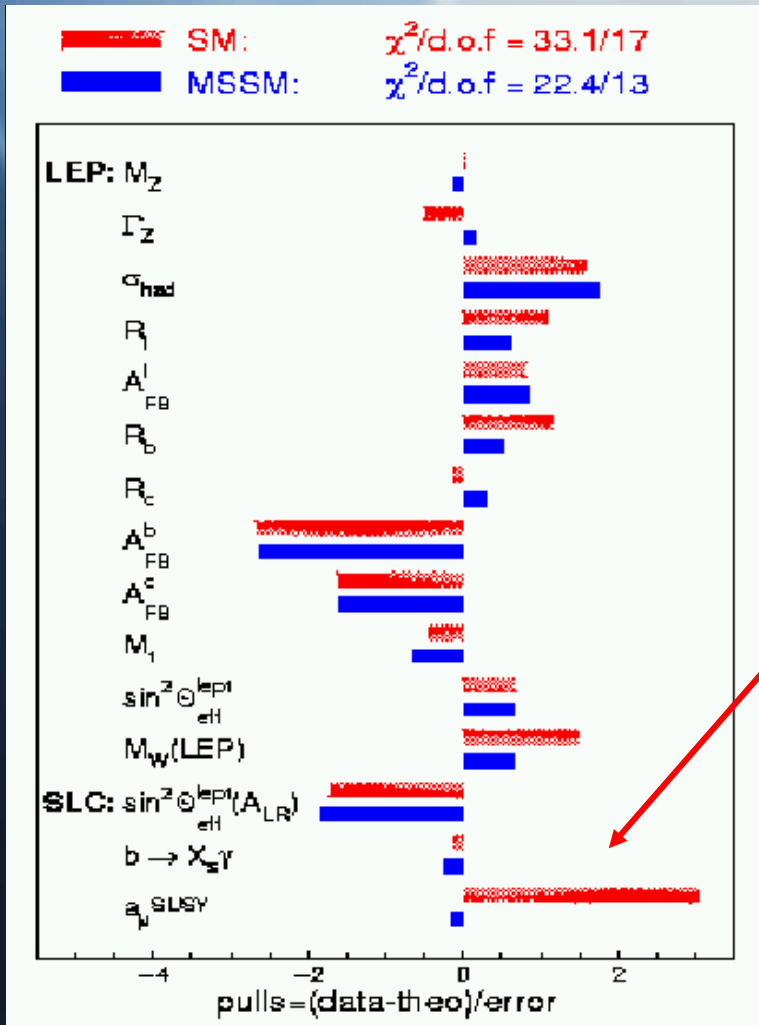
Where are the Supersymmetric Particles?

Astronomers:

Why a change of slope in the galactic rotation curve at $1.1 R_0$?

Solution: DM is made of WIMPs which are SUSY particles distributed in Halo of our Galaxy with a mass around 70 GeV

MSSM versus SM



• *MSSM is as good as SM*

• $B \rightarrow s \gamma$ } Better than in the SM
 • a_μ }

Ω_{DM} Is not described in the SM but naturally predicted by MSSM

SUSY: Pros and Cons

Pro :

- Provides natural framework for unification with gravity
- Leads to gauge coupling unification (GUT)
- Solves the hierarchy problem
- Is a solid quantum field theory
- Provides natural candidate for the WIMP cold DM
- Predicts new particles and thus generates new job positions

Contra :

Does not shed new light on the problem of

- Quark and lepton mass spectrum
- Quark and lepton mixing angles
- the origin of CP violation
- Number of flavours
- Baryon asymmetry of the Universe

Doubles the number of particles