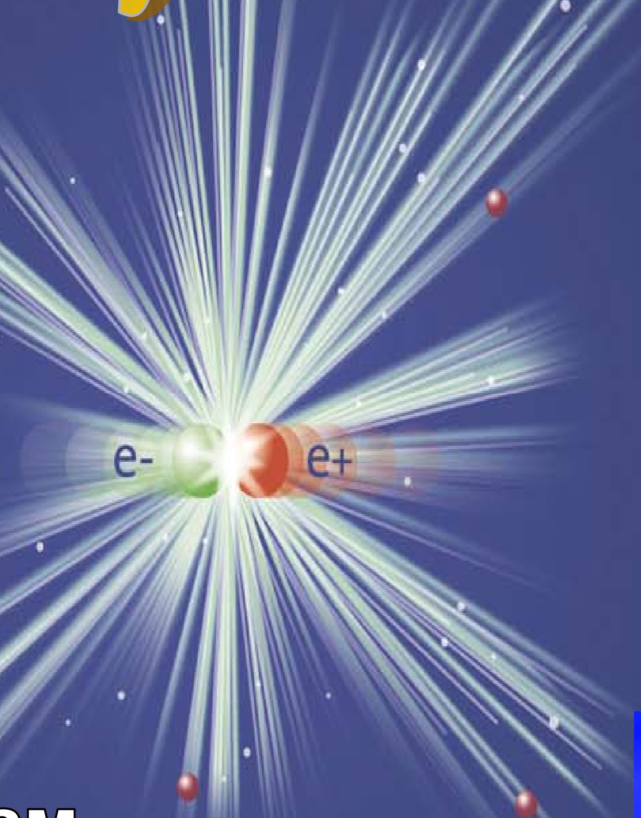
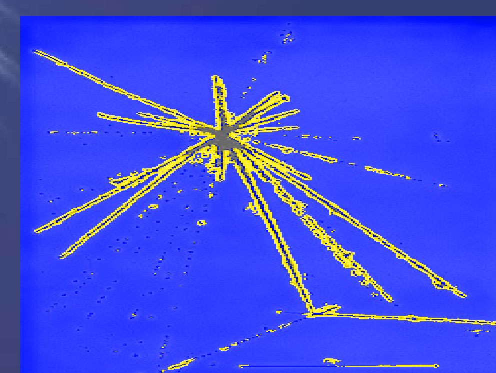
Supersymmetry in Particle Physics

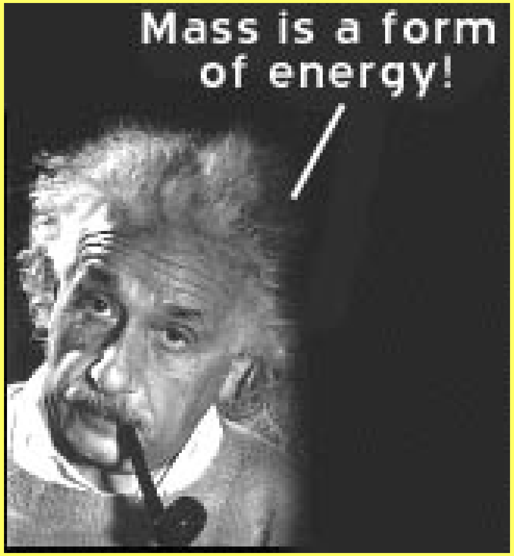


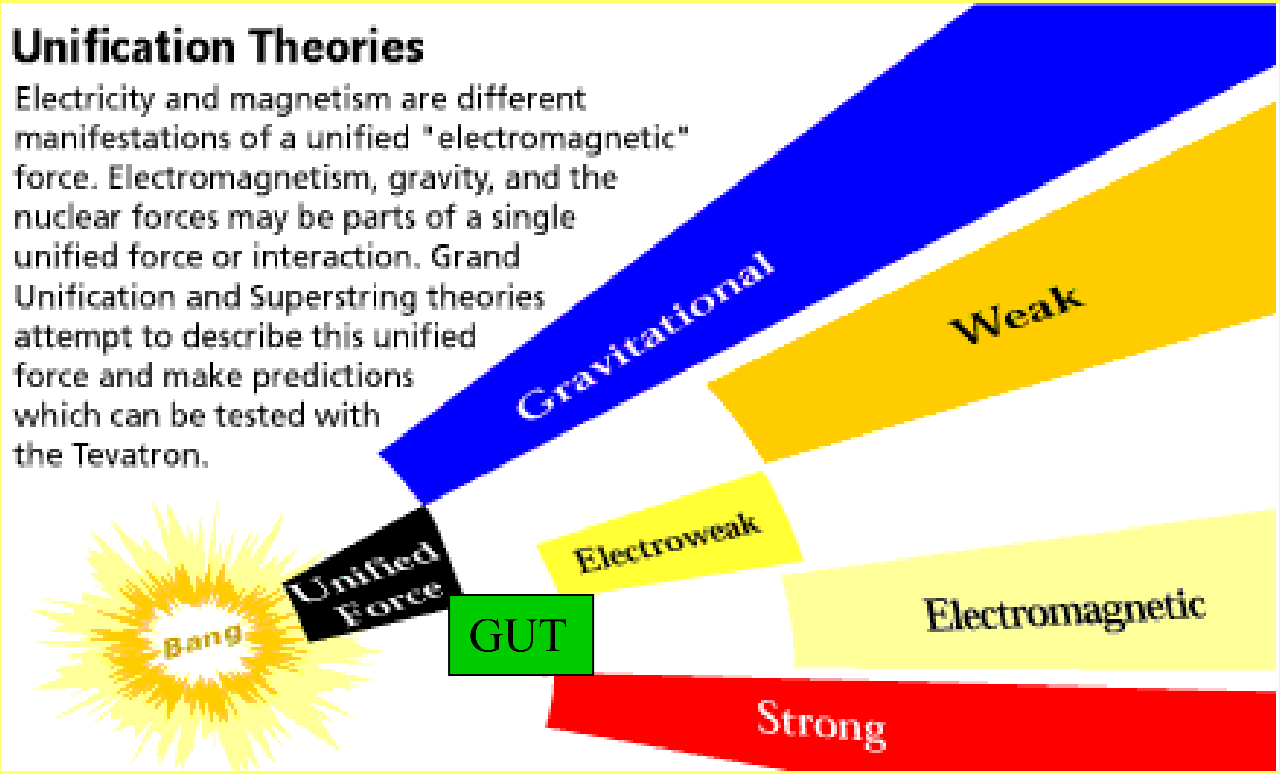
Outline

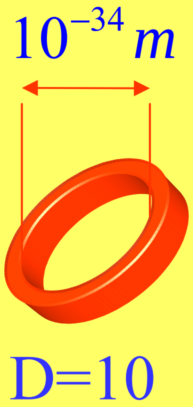
1. What is SUSY
2. Basics of SUSY
3. The MSSM
4. Constrained MSSM
5. SUSY Searches
6. SUSY DM



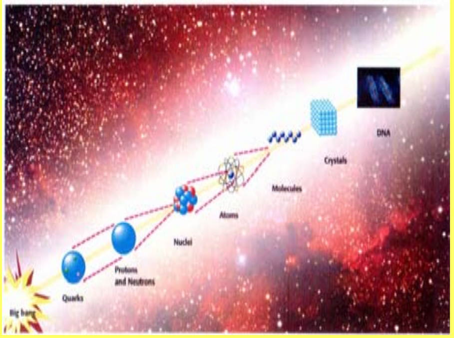
Unification Paradigm



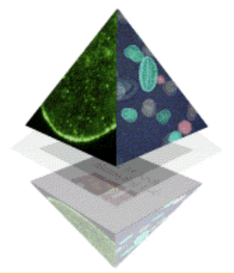




- Unification of strong, weak and electromagnetic interactions within Grand Unified Theories is the new step in unification of all forces of Nature
- Creation of a unified theory of everything based on string paradigm seems to be possible



What is SUSY?



• **Supersymmetry** is a boson-fermion sy
that is aimed to unify all forces in N
gravity within a singe frame

$Q\,|\,boson>=|\;fe$ ... $on>$

$[b,b]$ ... $\delta^{ij}(\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu}$

First papers in 1971-1972

No evidence in particle physics yet

• Modern ... y in particle physics
are based o ... gin, though low energy
manifestation ... SY can be found (?) at modern
colliders and in non-accelerator experiments

Motivation of SUSY in Particle Physics

- **Unification with Gravity**
- Unification of gauge couplings
- Solution of the hierarchy problem
- Dark matter in the Universe
- Superstrings

$$Q\,|\,boson> = |\,fermion> \qquad Q\,|\,fermion> = |\,boson>$$

$$spin\ 2 \rightarrow\ spin\ 3/2\ \rightarrow\ spin\ 1\ \rightarrow\ spin\ 1/2\ \rightarrow\ spin\ 0$$

Unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with the other interactions

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu \quad \Rightarrow \quad \{\delta_\varepsilon, \bar\delta_{\bar\varepsilon}\} = 2(\varepsilon\sigma^\mu\bar\varepsilon)P_\mu$$

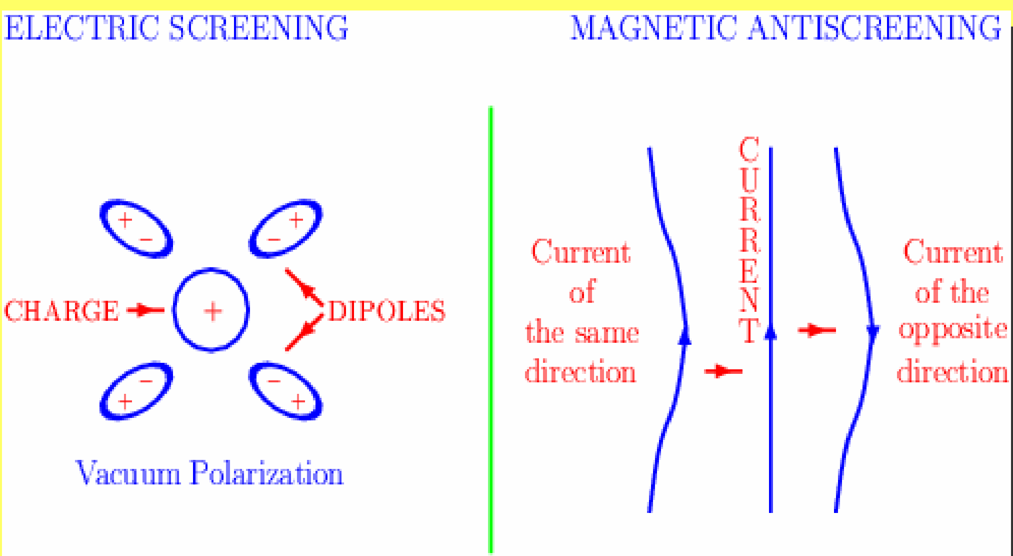
$$\varepsilon = \varepsilon(x)\ \ \text{local coordinate transf.} \ \Rightarrow\ \text{(super)gravity}$$

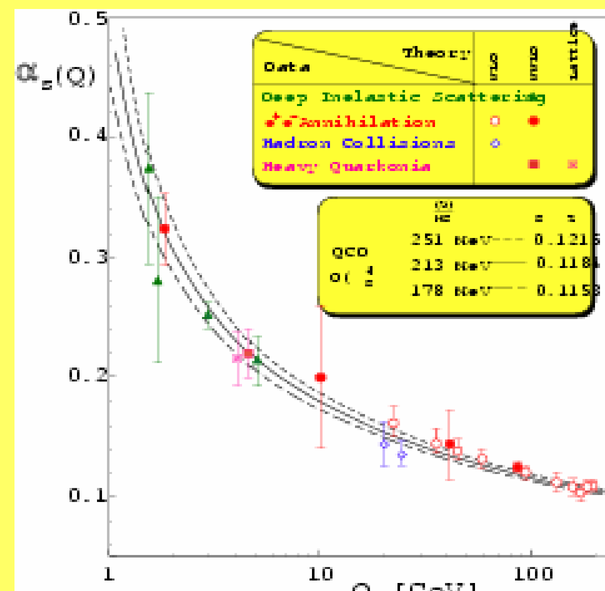
Local supersymmetry = general relativity !

Motivation of SUSY in Particle Physics

- Unification of gauge couplings

	Low Energy			High Energy
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	$\Rightarrow$	G_{GUT} (or G^n + symm)
gluons	W, Z	*photon*	$\Rightarrow$	*gauge bosons*
quarks	*leptons*		$\Rightarrow$	*fermions*
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}





$$\alpha_i = \alpha_i\left(\frac{Q^2}{\Lambda^2}\right) = \alpha_i(\text{distance})$$

Running of the strong coupling

Motivation of SUSY

RG Equations $$\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \alpha_i / 4\pi = g_i^2 / 16\pi^2, \quad \text{t=log}(\text{Q}^2 / \mu^2)$$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Unification of the Coupling Constants
in the SM and in the MSSM

Input

$$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$$

$$\sin^2 \theta_{\overline{MS}} = 0.23146 \pm 0.00017$$

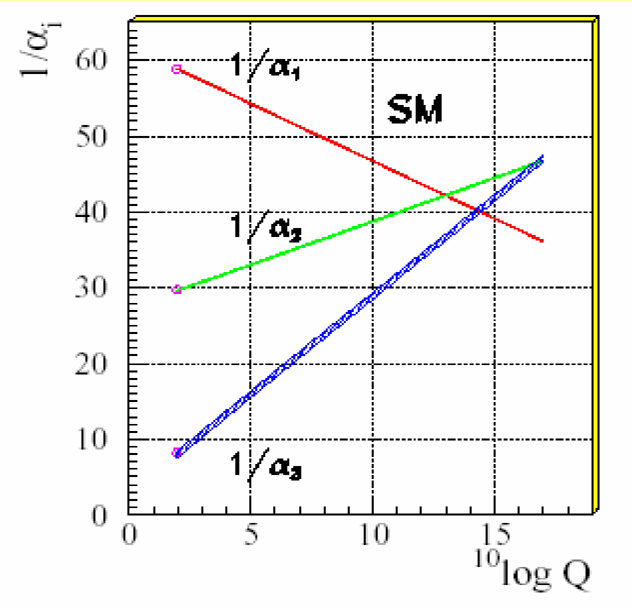
$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

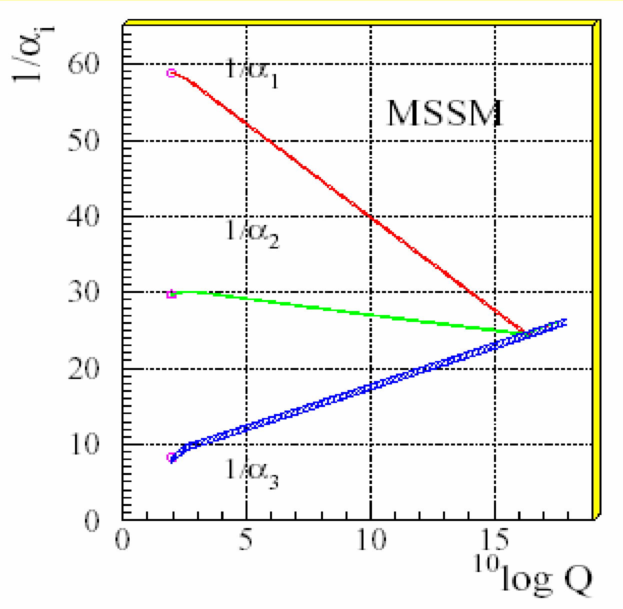
Output

$$M_{SUSY} = 10^{3.4\pm0.9\pm0.4} \text{ GeV}$$

$$M_{GUT} = 10^{15.8\pm0.3\pm0.1} \text{ GeV}$$

$$\alpha_{\text{GUT}}^{-1} = 26.3 \pm 1.9 \pm 1.0$$





SUSY yields unification!

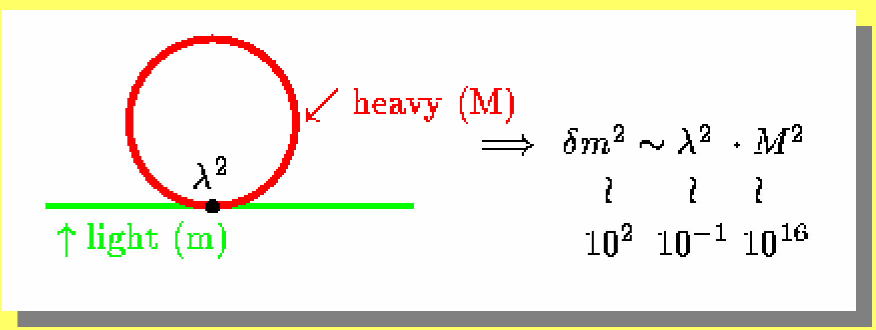
Motivation of SUSY

- Solution of the Hierarchy Problem

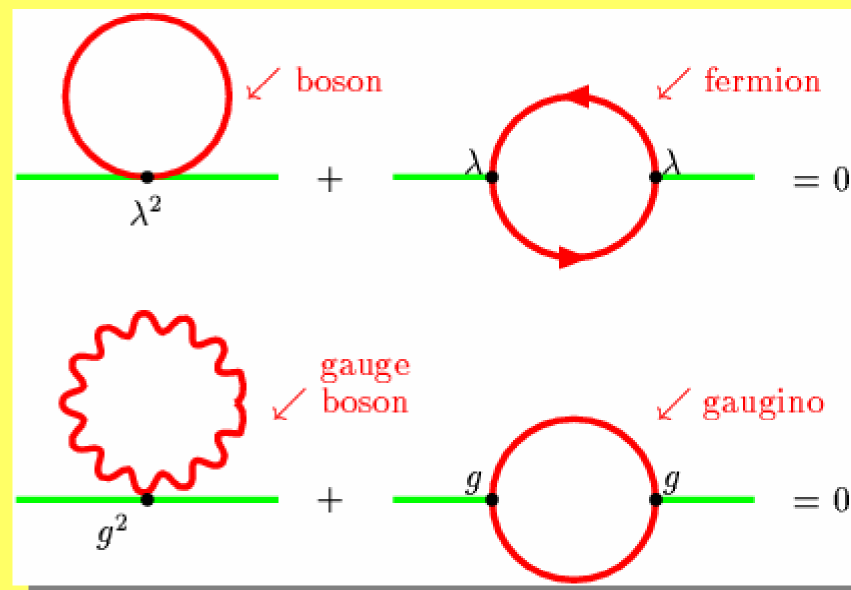
$$m_H \;\sim\; v \;\sim\; 10^2 \;\text{GeV} \qquad \frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

$$m_\Sigma \sim V \sim 10^{16} \;\text{GeV}$$

Destruction of the hierarchy by radiative corrections



Cancellation of quadratic terms

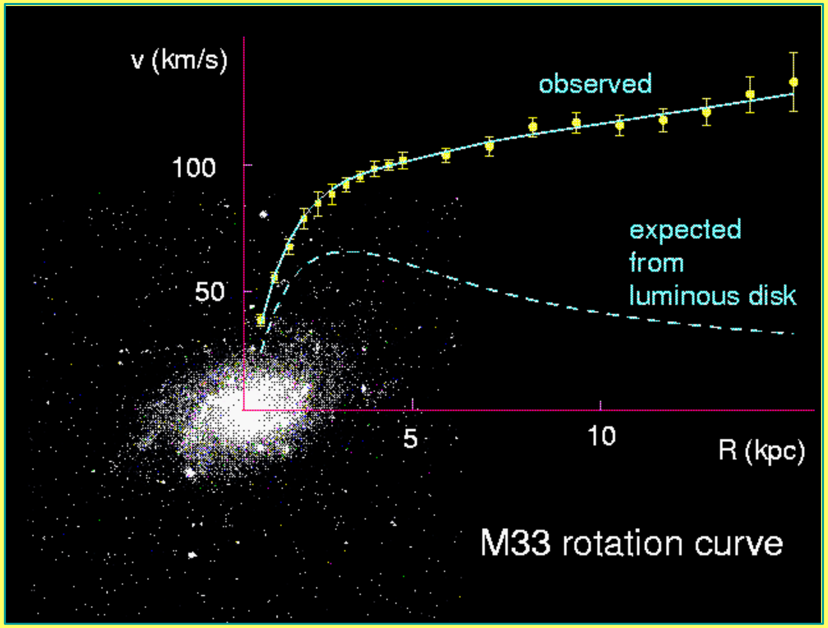


SUSY may also explain the origin of the hierarchy due to radiative mechanism

$$\sum_{bosons} m^2 = \sum_{fermions} m^2$$

Motivation of SUSY

- Dark Matter in the Universe



The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.

Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter

SUSY provides a candidate for the Dark matter – a stable neutral particle

Cosmological Constraints

New precise cosmological data

$$\Omega h^2 = 1 \quad \Longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$

- Supernova Ia explosion
- CMBR thermal fluctuations

(recent news from WMAP)

Dark Matter in the Universe:

Hot DM
(not favoured by
galaxy formation)

Cold DM
(rotation curves
of Galaxies)

SUSY

Superalgebra

$(Super)\ Algebra$

Lorentz Algebra

$$[P_\mu, P_\nu] = 0,\ [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$$

The only possible graded Lie algebra that mixes integer and half-integer spins and changes statistics

Superspace

$$x_\mu \to x_\mu, \theta_\alpha, \bar{\theta}_{\dot\alpha}$$

Grassmannian parameters

$$\alpha, \dot\alpha = 1, 2$$

$$\vartheta_\alpha^2 = 0,\ \bar{\vartheta}_{\dot\alpha}^2 = 0$$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial \vartheta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_\mu$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial \bar{\vartheta}_{\dot\alpha}} + i\theta_\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$Q_\alpha^2 = 0,\ \bar{Q}_{\dot\alpha}^2 = 0$$

Supertranslation

$$x_\mu \to x_\mu + i\theta\sigma_\mu\bar{\xi} - i\xi\sigma_\mu\bar{\theta},$$

$$\theta \to \theta + \xi,$$

$$\bar{\theta} \to \bar{\theta} + \bar{\xi}$$

Quantum States

Quantum states: Vacuum = $|E,\lambda>$ $\quad Q|E,\lambda>=0$

Energy helicity

$$[Q^i_\alpha, P_\mu] = [\overline{Q}^i_{\dot\alpha}, P_\mu] = 0$$

State	Expression	# of states
vacuum	$\vert E,\lambda>$	1
1-particle	$\overline{Q}_i \vert E,\lambda> = \vert E,\lambda+1/2>$	$\binom{N}{1}=N$
2-particle	$\overline{Q}_i\overline{Q}_j \vert E,\lambda> = \vert E,\lambda+1>$	$\binom{N}{2}=\frac{N(N-1)}{2}$
...	...	...
N-particle	$\overline{Q}_1\overline{Q}_2\ldots\overline{Q}_N \vert E,\lambda> = \vert E,\lambda+N/2>$	$\binom{N}{N}=1$

Total # of states: $\sum_{k=0}^{N}\binom{N}{k} = 2^N = 2^{N-1} bosons + 2^{N-1}\ fermions$

SUSY Multiplets

Chiral multiplet $N=1,\ \lambda=0$

helicity	-1/2	0	1/2
# of states	1	2	1

scalar spinor

(φ, ψ)

Vector multiplet $N=1,\ \lambda=1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1

(λ, A_μ)

spinor vector

Members of a supermultiplet are called **superpartners**

N=4	SUSY YM	helicity	-1 –1/2 0 1/2 1
	λ = -1	# of states	1 4 6 4 1
N=8	SUGRA	helicity	-2 –3/2 –1 –1/2 0 1/2 1 3/2 2
	λ = -2	# of states	1 8 28 56 70 56 28 8 1

$\boxed{N \leq 4S}$ ← spin

$N \leq 4$ For renormalizable theories (YM)

$N \leq 8$ For (super)gravity

Simplest (N=1) SUSY Multiplets

Bosons and Fermions come in pairs

$$(\varphi,\psi) \qquad (\lambda, A_\mu) \qquad (\tilde{g}, g)$$

Spin 0	Spin 1/2	Spin 1/2	Spin 1	Spin 3/2	Spin 2
scalar	chiral fermion	majorana fermion	vector	gravitino	graviton

SUSY Transformation

N=1 SUSY Chiral supermultiplet:

parameter of SUSY transformation (spinor)

spin=0 → $$\delta_\varepsilon A = \sqrt{2}\varepsilon\psi,$$

spin=1/2 → $$\delta_\varepsilon \psi = i\sqrt{2}\sigma^\mu \bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F,$$

$$\delta_\varepsilon F = i\sqrt{2}\bar{\varepsilon}\sigma^\mu \partial_\mu \psi$$

Auxiliary field

(unphysical d.o.f. needed to close SYSY algebra)

SUSY multiplets ➡ Superfiled in Superspace

$$(y = x + i\theta\sigma\bar{\theta})$$

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

$$= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$$

$$+\sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)$$

superfield

Expansion over grassmannian parameter

component fields

$$\boxed{\theta^2 = \theta_1\theta_2, \quad \theta_1^2 = \theta_2^2 = 0!}$$

Gauge superfields

Gauge superfield

$$V(x,\theta,\bar\theta) = C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + i\theta\theta M(x) - i\bar\theta\bar\theta M^+(x)$$
$$-\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\bar\lambda(x) + i\bar\sigma^\mu\partial_\mu\chi(x)] - i\bar\theta\bar\theta\theta[\lambda(x) + i\sigma^\mu\partial_\mu\bar\chi(x)]$$
$$+\tfrac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \tfrac{1}{2}\Box C(x)]$$

Field strength tensor

$$W_\alpha = -\tfrac{1}{4}\bar D^2 e^V D_\alpha e^{-V}$$

Gauge transformation

$$V \to V + \Phi + \bar\Phi$$

$$C \to C + A + A^*$$
$$\chi \to \chi - i\sqrt{2}\psi$$
$$M \to M - 2iF$$
$$v_\mu \to v_\mu - i\partial_\mu(A - A^*)$$
$$\lambda \to \lambda$$
$$D \to D$$

Wess-Zumino gauge

$$C = \chi = M = 0$$

physical fields

Covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\vartheta_\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\vartheta^{\dot\alpha}\partial_\mu$$
$$\bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta_\alpha} - i\theta_{\dot\alpha}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \tfrac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma^\mu D_\mu\bar\lambda$$

How to write SUSY Lagrangians

1st step

Take your favorite Lagrangian written in terms of fields

2nd step

Replace $$Field\ (\varphi, \psi, A_\mu) \Rightarrow Superfield\ (\Phi, V)$$

3rd step

Replace

$$Action = \int d^4x\, L(x) \quad \Rightarrow \quad \int d^4x\, d^4\theta\, L(x, \theta, \bar{\theta})$$

Grassmannian integration in superspace $\int d\theta_\alpha = 0, \ \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$