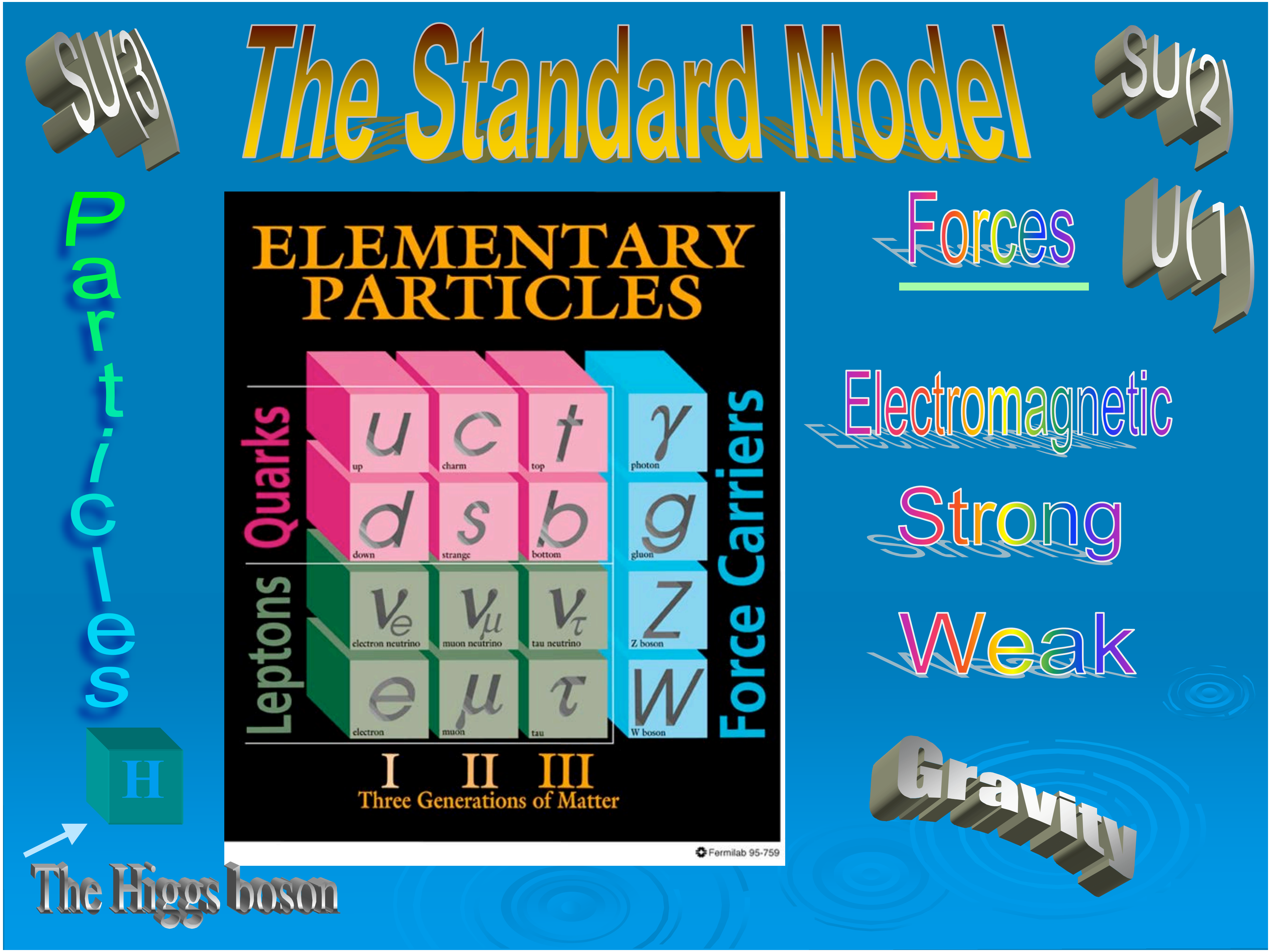
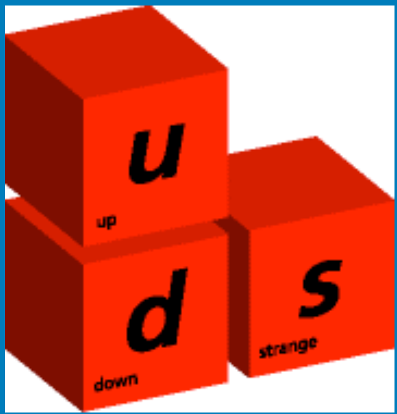
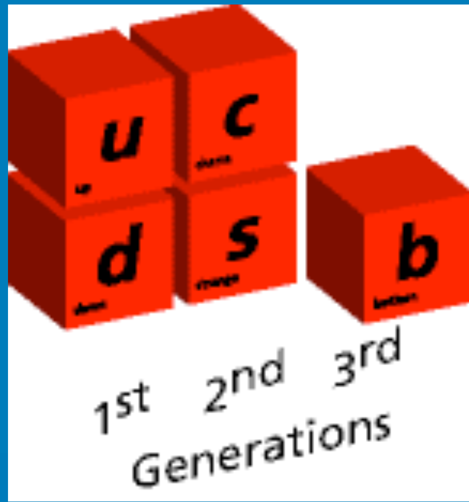


SU(3)
The Standard Model
SU(2)
U(1)
Particles
ELEMENTARY PARTICLES
Quarks
u up
c charm
t top
d down
s strange
b bottom
Leptons
ν_e electron neutrino
ν_μ muon neutrino
ν_τ tau neutrino
e electron
μ muon
τ tau
Force Carriers
γ photon
g gluon
Z Z boson
W W boson
I II III
Three Generations of Matter
Fermilab 95-759
H
The Higgs boson
Forces
Electromagnetic
Strong
Weak
Gravity

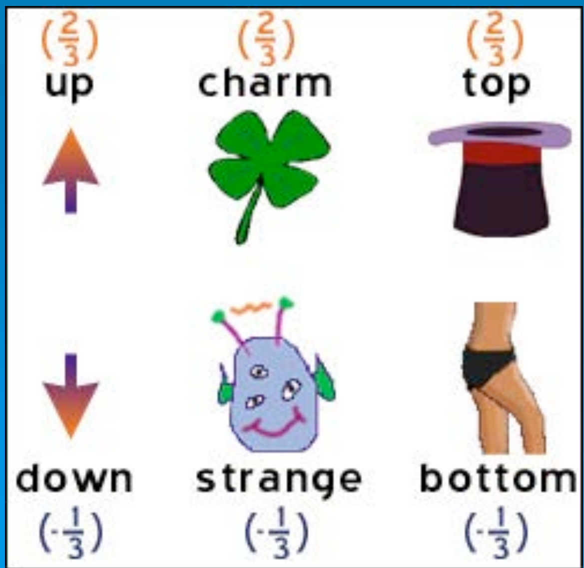
Quarks – “the building blocks of the Universe”



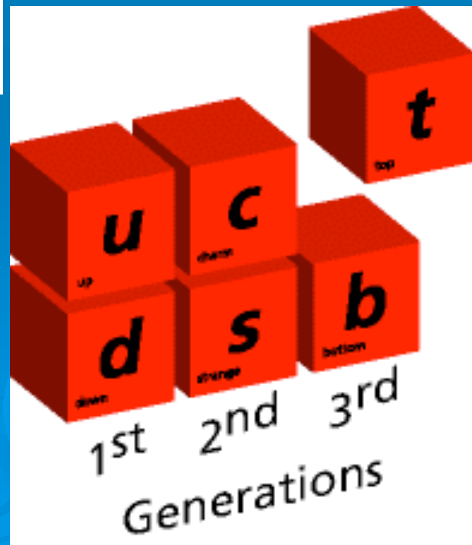
Charm came as surprise but completed the picture



The number of quarks increased with discoveries of new particles and have reached 6



For unknown reasons Nature created 3 copies (generations) of quarks and leptons



Discovery History

u	c 1974	t 1995	ν_e 1956	ν_μ 1963	ν_τ 2000
d	s 1947	b 1977	e 1895	μ 1936	τ 1975

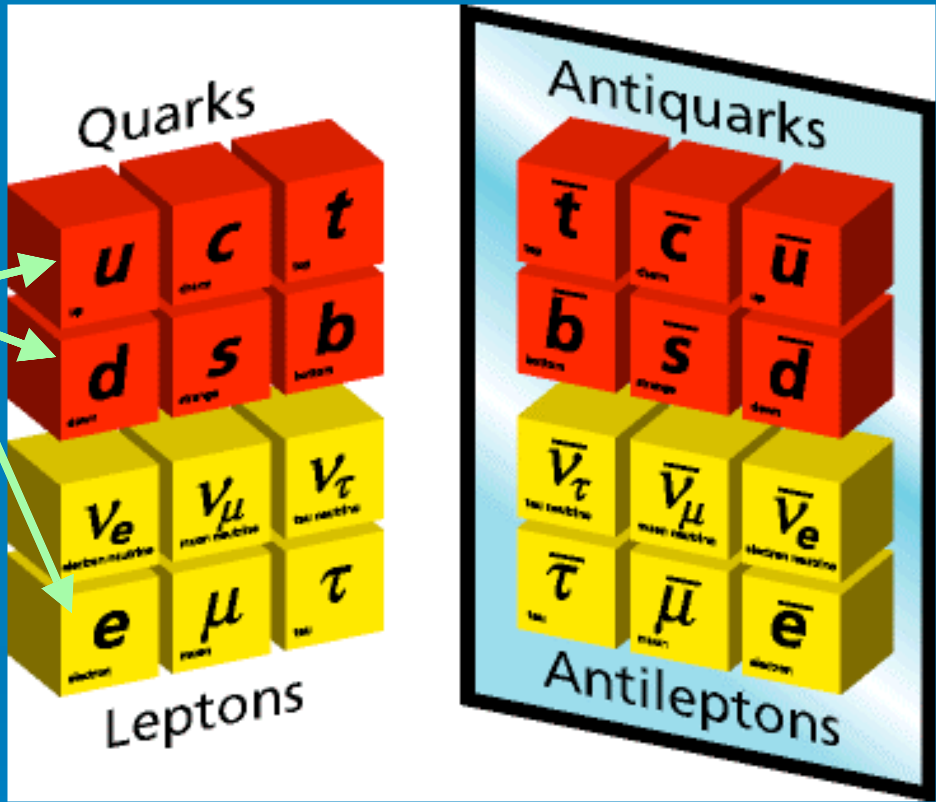
six quarks

six leptons

Now we have a beautiful pattern of three pairs of quarks and three pairs of leptons. They are shown here with their year of discovery.

Matter and Antimatter

The first generation is what we are made of

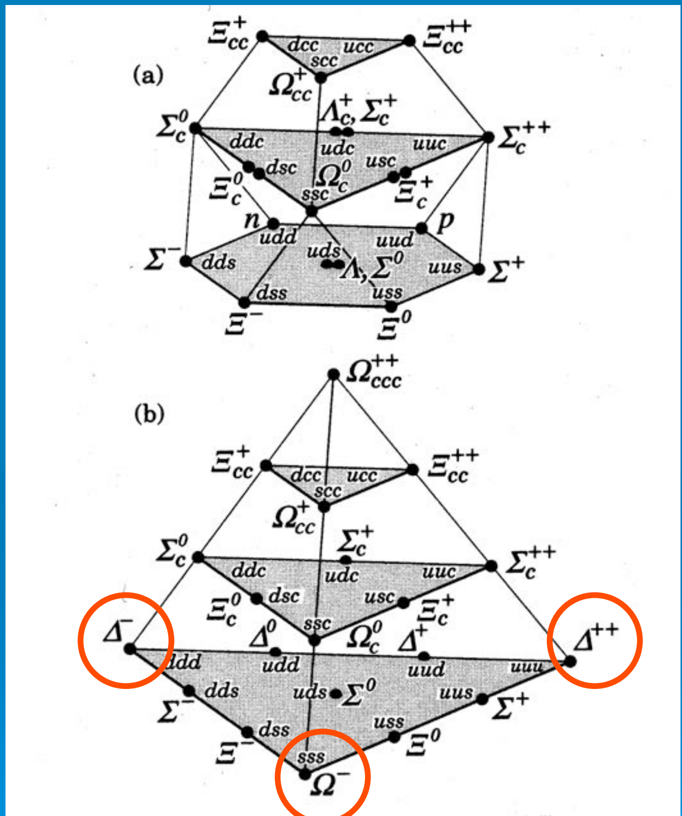


Antimatter was created together with matter during the “Big bang”

Antiparticles are created at accelerators in ensemble with particles but the visible Universe does not contain antimatter

Quark's Colour

Baryons are "made" of quarks



$$\Delta^{-}(d\uparrow d\uparrow d\uparrow)$$

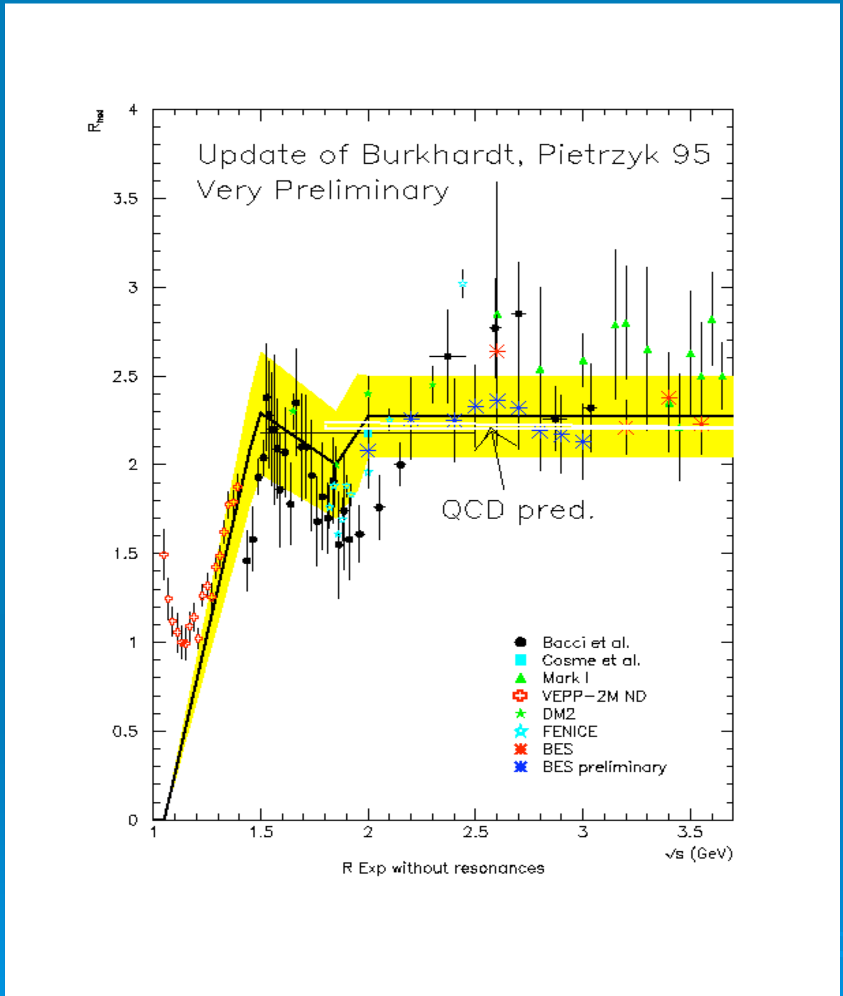
$$\Omega^{-}(s\uparrow s\uparrow s\uparrow) \qquad ?$$

$$\Delta^{++}(u\uparrow u\uparrow u\uparrow)$$

To avoid Pauli principle veto one can antisymmetrize the wave function introducing a new quantum number - "colour", so that

$$\Delta^{-} = \varepsilon^{ijk}(d_i\uparrow d_j\uparrow d_k\uparrow)$$

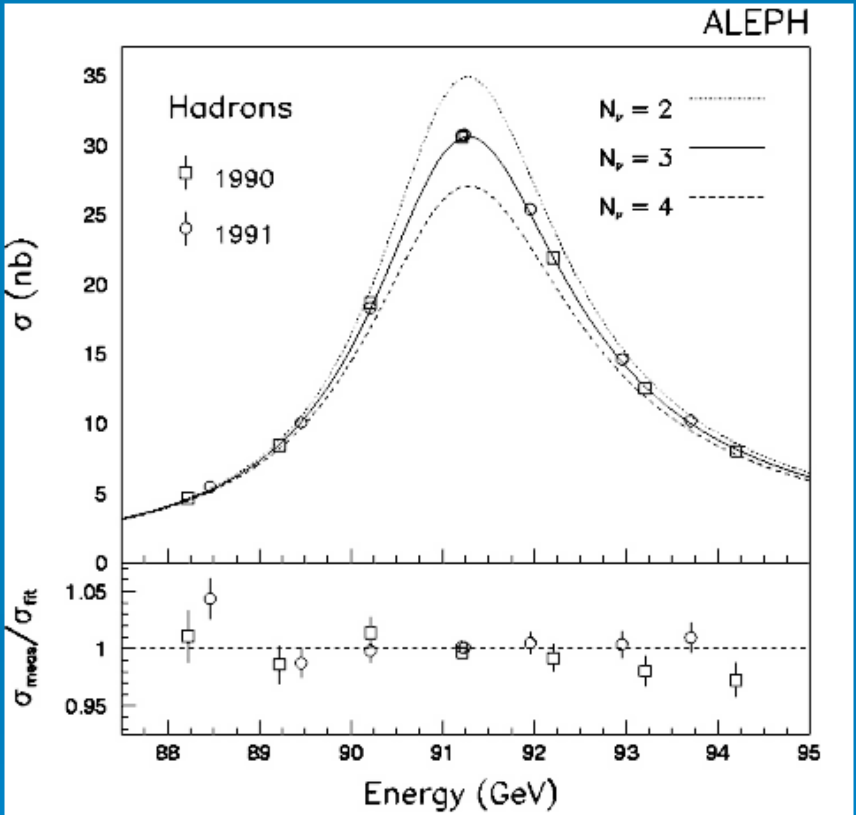
The Number of Colours



- The x-section of electron-positron annihilation into hadrons is proportional to the number of quark colours. The fit to experimental data at various colliders at different energies gives

$$N_c = 3.06 \pm 0.10$$

The Number of Generations



$N_g = 2.982 \pm 0.013$

- Z-line shape obtained at LEP depends on the number of flavours and gives the number of (light) neutrinos or (generations) of the Standard Model

Quantum Numbers of Matter

- Quarks

$$Q_L = \begin{pmatrix} up \\ down \end{pmatrix}_L$$

$$U_R = up_R$$

$$D_R = down_R$$

	SU(3)$_c$	SU(2)$_L$	U$_Y$(1)
Q_L	3	2	$1/3$
U_R	3	1	$4/3$
D_R	3	1	$-2/3$

triplets

doublets

singlets

V-A currents in weak interactions

- Leptons

$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$N_R = \nu_R \ ?$$

$$E_R = e_R$$

$\frac{1}{2}$, $-\frac{1}{2}$ ← T_3

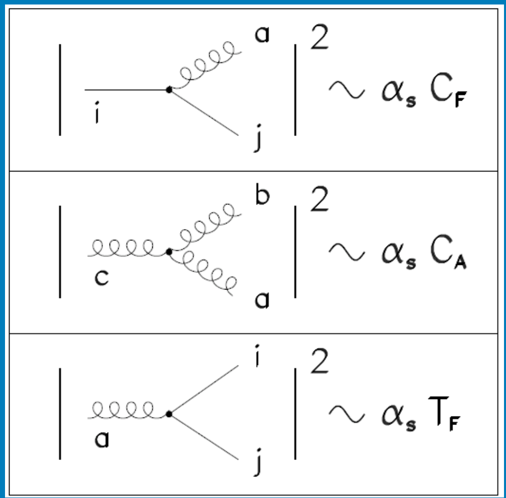
0, 0 ← T_3

	SU(3)$_c$	SU(2)$_L$	U$_Y$(1)
L_L	1	2	-1
N_R	1	1	0
E_R	1	1	2

Electric charge

$$Q = T_3 + Y/2$$

The group structure of the SM



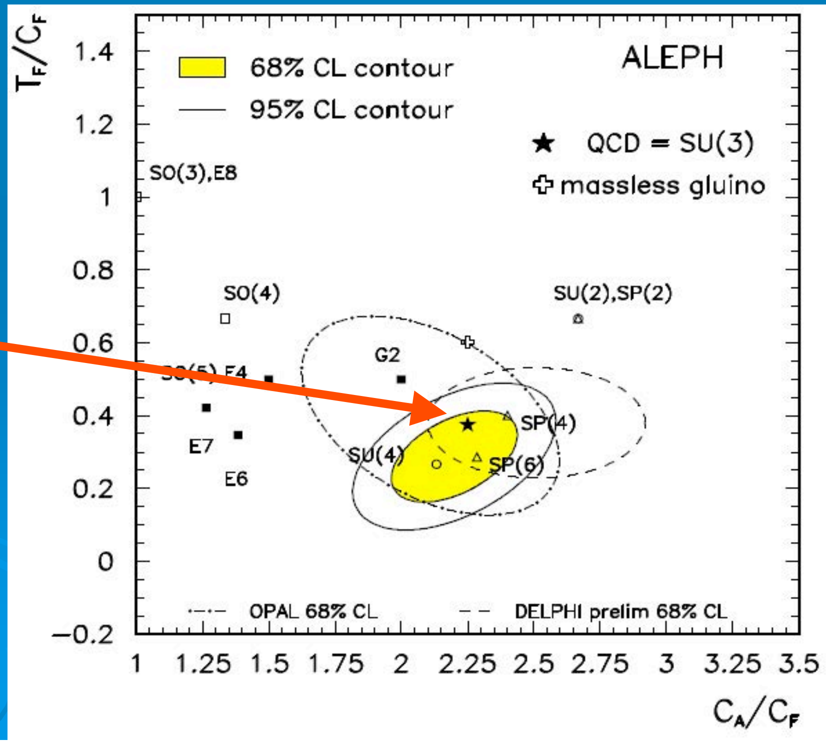
$$\sum_{a=1}^{N_A} (T^a T^{\dagger a})_{ij} = \delta_{ij} C_F \quad , \quad \sum_{i,j=1}^{N_F} T^a_{ij} T^{\dagger b}_{ji} = \delta^{ab} T_F \quad , \quad \sum_{a,b=1}^{N_A} f^{abc} f^{*abd} = \delta^{cd} C_A$$

Casimir Operators

For SU(N)

$$C_A = N_C \quad , \quad C_F = \frac{N_C^2 - 1}{2N_C} \quad , \quad T_F = 1/2$$

QCD analysis definitely singles out the SU(3) group as the symmetry group of strong interactions



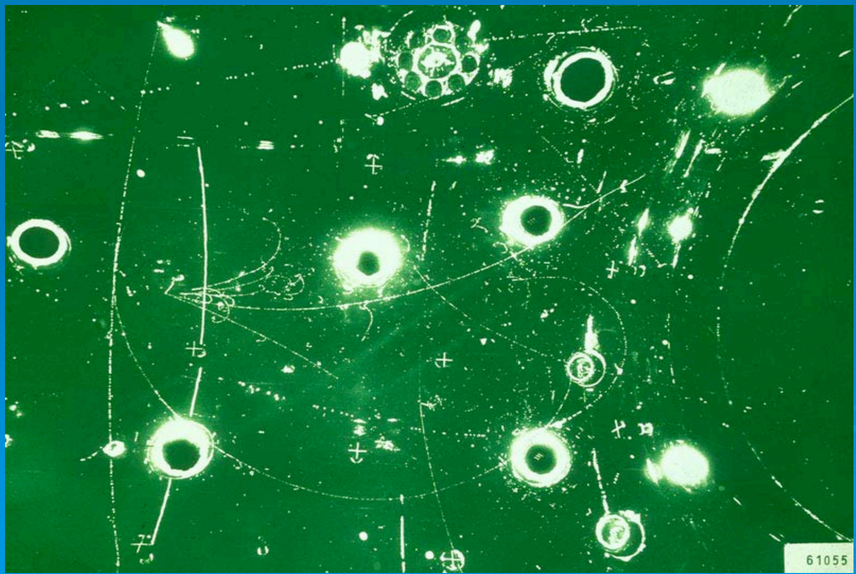
Electro-weak sector of the SM

SU(2) x U(1) versus O(3)

3 gauge bosons 1 gauge boson 3 gauge bosons

After spontaneous symmetry breaking one has

3 massive gauge bosons (W^+ , W^- , Z^0) and 1 massless (γ)	⟺	2 massive gauge bosons (W^+ , W^-) and 1 massless (γ)



- Discovery of neutral currents was a crucial test of the gauge model of weak interactions at CERN in 1973
- The heavy photon gives the neutral current without flavour violation

Gauge Invariance

Gauge transformation $\psi_i(x) \to U_{ij}(x)\psi_j = \exp[i\alpha^a(x)T_{ij}^a]\psi_j$ $a = 1, 2, \ldots, N$

$\bar{\psi}_i(x) \to \bar{\psi}_j U^+_{ji}(x)$ matrix parameter matrix $U^+ U = 1$

Fermion Kinetic term $i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) \to i\bar{\psi}(x)U^+(x)\gamma^\mu \partial_\mu \left(U(x)\psi(x)\right)$

$$= i\bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x) + \underline{\bar{\psi}(x)\gamma^\mu U^+(x)\partial_\mu U(x)\psi(x)}$$

Covariant derivative $\partial_\mu \to D_\mu = \partial_\mu I - gA_\mu^a T^a = \partial_\mu \hat{I} - g A_\mu$ ← Gauge field

$A_\mu(x) \to U(x)A_\mu(x)U^+(x) + \frac{1}{g}\partial_\mu U(x)U^+(x)$ ⟶ $D_\mu \psi(x) \to U(x)D_\mu \psi(x)$

Gauge invariant kinetic term $\boxed{i\bar{\psi}(x)\gamma^\mu D_\mu \psi(x)}$

$[D_\mu, D_\nu] = G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]$ $G_{\mu\nu}(x) \to U(x)G_{\mu\nu}(x)U^+(x)$

Gauge field kinetic term $\boxed{-\frac{1}{4} Tr\ G_{\mu\nu} G^{\mu\nu}}$ Field strength tensor

Lagrangian of the SM

$$SU_c(3)\otimes SU_L(2)\otimes U_Y(1)$$

$$L = L_{gauge} + L_{Yukawa} + L_{Higgs}$$

$$L_{gauge} = -\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \tfrac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B_{\mu\nu}$$
$$+i\overline{L}_\alpha\gamma^\mu D_\mu L_\alpha + i\overline{Q}_\alpha\gamma^\mu D_\mu Q_\alpha + i\overline{E}_\alpha\gamma^\mu D_\mu E_\alpha$$
$$+i\overline{U}_\alpha\gamma^\mu D_\mu U_\alpha + i\overline{D}_\alpha\gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger(D_\mu H)$$

$$L_{Yukawa} = y^L_{\alpha\beta}\overline{L}_\alpha E_\beta H + y^D_{\alpha\beta}\overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{H}$$

$$L_{Higgs} = -V = m^2 H^\dagger H - \tfrac{\lambda}{2}(H^\dagger H)^2$$

$$\tilde{H} = i\tau_2 H^\dagger$$

α,β=1,2,3 - generation index

Fermion Masses in the SM

Direct mass terms are forbidden due to $SU(2)_L$ invariance !

Dirac Spinors — left — right — Dirac conjugated — Charge conjugated

$$\psi,\ \psi_L = \frac{1-\gamma^5}{2}\psi,\ \psi_R = \frac{1+\gamma^5}{2}\psi\ ,\bar{\psi} = \psi^+\gamma^0,\ \psi^c = C\gamma^0\psi = i\gamma^2\psi^*$$

Lorenz invariant Mass terms

$SU_L(2)$

$$\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

SU(2) doublet — SU(2) singlet

$$\bar{\psi}_L\psi_L = \bar{\psi}_R\psi_R = 0$$

$SU_L(2)$ & $U_Y(1)$

$$\bar{\psi}^c_L\psi_L + \bar{\psi}_L\psi^c_L$$

$U_Y(1)$

$$\bar{\psi}^c_R\psi_R + \bar{\psi}_R\psi^c_R$$

Unless Q=0, Y=0

$$\bar{\nu}^c_R\nu_R$$

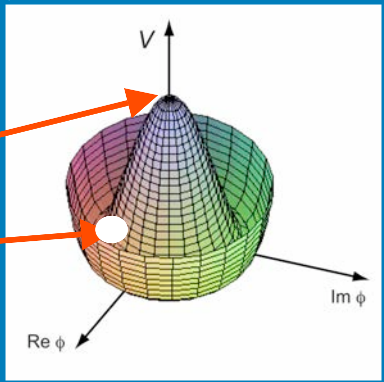
Majorana mass term

Spontaneous Symmetry Breaking

$$SU_c(3)\otimes SU_L(2)\otimes U_Y(1)\rightarrow SU_c(3)\otimes U_{EM}(1)$$

Introduce a scalar field with quantum numbers: (1,2,1) $H=\begin{pmatrix}H^+\\H^0\end{pmatrix}$

With potential $V=-m^2H^\dagger H+\frac{\lambda}{2}(H^\dagger H)^2$



Unstable maximum

Stable minimum

At the minimum

v.e.v. scalar

$$H=\begin{pmatrix}H^+\\H^0\end{pmatrix}=\begin{pmatrix}H^+\\ \mathrm{v}+\frac{S+iP}{\sqrt{2}}\end{pmatrix}=\exp\!\left(i\frac{\vec{\xi}\vec{\sigma}}{2}\right)\begin{pmatrix}0\\ \mathrm{v}+\frac{S}{\sqrt{2}}\end{pmatrix}$$

pseudoscalar

Gauge transformation

Higgs boson

$$H\longrightarrow H'=\exp\!\left(i\frac{\vec{\alpha}\vec{\sigma}}{2}\right)H\xrightarrow{(\vec{\alpha}=-\vec{\xi})}H'=\begin{pmatrix}0\\ \mathrm{v}+\frac{\mathbf{h}}{\sqrt{2}}\end{pmatrix}$$

The Higgs Mechanism

Q: What happens with missing d.o.f. (massless goldstone bosons P,H⁺ or $\vec{\xi}$) ?

A: They become longitudinal d.o.f. of the gauge bosons W_μ^i, i=1,2,3

Gauge transformation

$$W_\mu \rightarrow e^{i\alpha^a\sigma^a} W_\mu e^{-i\alpha^a\sigma^a} + \frac{1}{g}\partial_\mu\left(e^{i\alpha^a\sigma^a}\right)e^{-i\alpha^a\sigma^a}$$

$$\alpha^a = -\xi^a$$

Longitudinal components

Higgs field kinetic term

$$\left|D_\mu H\right|^2 = \left|\partial_\mu H - \tfrac{g}{2}W_\mu H - \tfrac{g'}{2}B_\mu H\right|^2 \longleftarrow H = \begin{pmatrix}0\\ \mathrm{v}\end{pmatrix}$$

$$\rightarrow \tfrac{1}{4}(0\ \mathrm{v})\begin{pmatrix}\mathrm{g}W_\mu^3 + g'B_\mu & \sqrt{2}\mathrm{g}W_\mu^- \\ \sqrt{2}\mathrm{g}W_\mu^+ & -\mathrm{g}W_\mu^3 + g'B_\mu\end{pmatrix}\begin{pmatrix}\mathrm{g}W_\mu^3 + g'B_\mu & \sqrt{2}\mathrm{g}W_\mu^- \\ \sqrt{2}\mathrm{g}W_\mu^+ & -\mathrm{g}W_\mu^3 + g'B_\mu\end{pmatrix}\begin{pmatrix}0\\ \mathrm{v}\end{pmatrix}$$

$$\Rightarrow \frac{g^2}{2}\mathrm{v}^2 W_\mu^+ W_\mu^- + \frac{1}{4}\mathrm{v}^2(-gW_\mu^3 + g'B_\mu)^2$$

$$M_W^2 = \tfrac{1}{2}g^2\mathrm{v}^2$$
$$M_Z^2 = \tfrac{1}{2}(g^2+g'^2)\mathrm{v}^2$$

$$\tan\theta_W = g'/g$$
$$M_\gamma = 0$$

$$W_\mu^\pm = \frac{W_\mu^1 \mp W_\mu^2}{\sqrt{2}}$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$
$$\gamma_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

The Higgs Boson and Fermion Masses

$$H = \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix} \Rightarrow V = -m^2 H^\dagger H + \tfrac{\lambda}{2}(H^\dagger H)^2$$

$$\Rightarrow V = -\frac{\lambda v^4}{2} + \lambda v^2 h^2 + \frac{\lambda v}{\sqrt{2}} h^3 + \frac{\lambda}{8} h^4 \qquad v^2 = m^2/\lambda$$

$$m_h = \sqrt{2} m = \sqrt{2\lambda} v$$

$$L_{Yukawa} = y^E_{\alpha\beta} \overline{L}_\alpha E_\beta H + y^D_{\alpha\beta} \overline{Q}_\alpha D_\beta H + y^U_{\alpha\beta} \overline{Q}_\alpha U_\beta H$$

α, β =1,2,3 - generation index

Dirac fermion mass

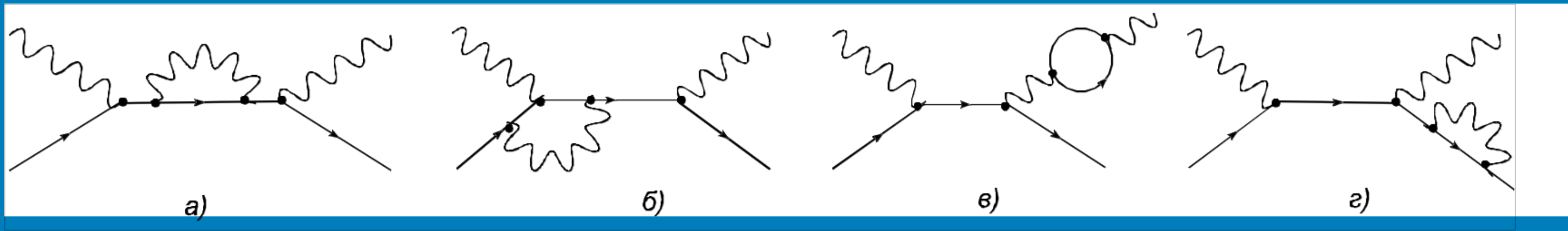
$$M_i^u = Diag(y^u_{\alpha\beta})v, \quad M_i^d = Diag(y^d_{\alpha\beta})v, \quad M_i^l = Diag(y^l_{\alpha\beta})v$$

$$y^N_{\alpha\beta} \overline{L}_\alpha N_\beta H \to M_i^{\nu} = Diag(y^N_{\alpha\beta})v$$

Dirac neutrino mass

The Running Couplings

Radiative Corrections $\sim\alpha\,(\log \Lambda^2/p^2 + \text{fin.part})$ UV divergence



Renormalization operation

$$\alpha_{Bare}(\Lambda) = Z(\Lambda/\mu)\;\alpha_R(\mu)$$

UV cutoff — Renormalization scale

Renormalization constant

$$Z(\Lambda/\mu) = 1 - b\alpha Log(\Lambda^2/\mu^2) + \ldots$$

Subtraction of UV div

$$\alpha Log(\Lambda^2/p^2) - \alpha Log(\Lambda^2/\mu^2) = \alpha Log(\mu^2/p^2)$$

Finite

$$\boxed{\alpha_R(\mu)} \qquad \mu^2 \frac{d\alpha}{d\mu^2} = \beta(\alpha), \qquad \beta(\alpha) = -\mu^2 \frac{d}{d\mu^2} Log\, Z(\Lambda/\mu)$$

Running coupling

Renormalization Group

Observable $\quad R_{PT}(Q^2/\mu^2, \alpha(\mu)) = \alpha(1 + b\alpha Log(Q^2/\mu^2) + O(\alpha^2))$

RG Eq.

$$\mu^2 \frac{d}{d\mu^2} R = (\mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\mu^2 \frac{d\alpha}{d\mu^2}}_{\beta(\alpha)} \frac{\partial}{\partial \alpha}) R = 0$$

Solution to RG eq.

$$R_{RG}(Q^2/\mu^2, \alpha(\mu)) = R_{PT}(1, \overline{\alpha}(Q^2/\mu^2, \alpha))$$

$$Q^2 \frac{d\overline{\alpha}}{dQ^2} = \beta(\overline{\alpha})$$

Effective coupling

Solution to RG eq. sums up an infinite series of the leading Logs coming from Feynman diagrams

$$R_{PT}(1, \overline{\alpha}) = \overline{\alpha}, \quad \overline{\alpha} = \frac{\alpha}{1 - b\alpha Log(Q^2/\mu^2)} \approx \alpha(1 + b\alpha Log(Q^2/\mu^2) + \ldots)$$

Asymptotic Freedom and Infrared Slavery

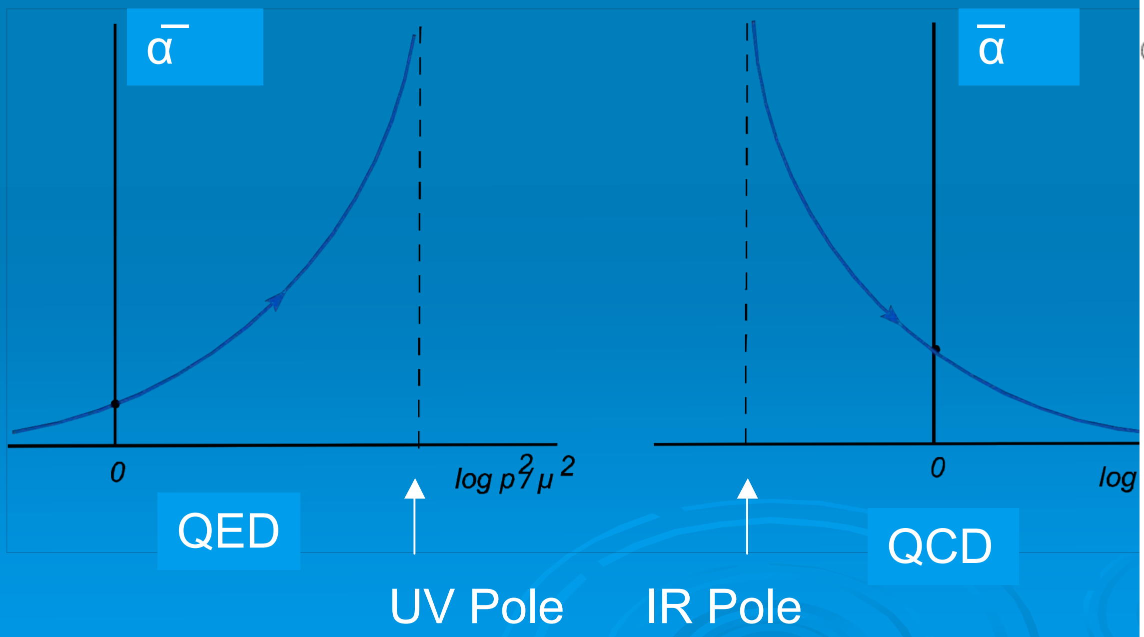
One-loop order

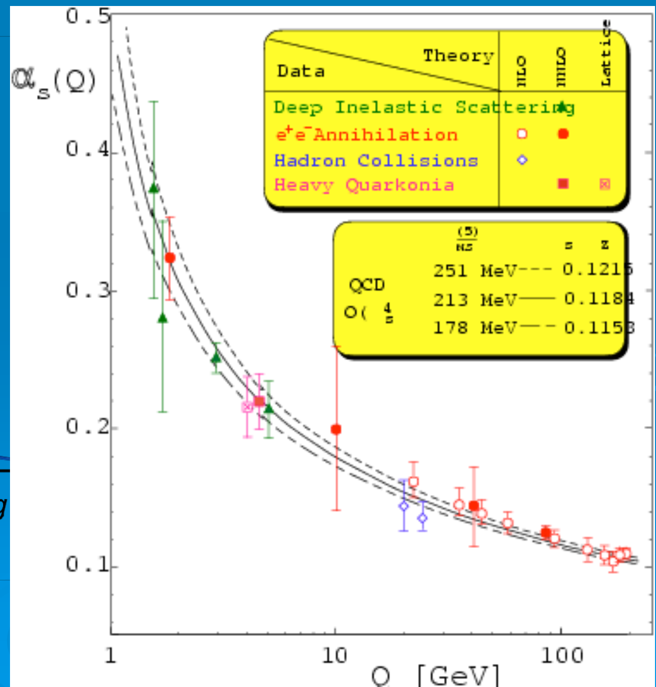
$$\beta(\alpha) = b\alpha^2$$

→ $4/3\, n_f$ QED

→ $-11+2/3\, n_f$ QCD

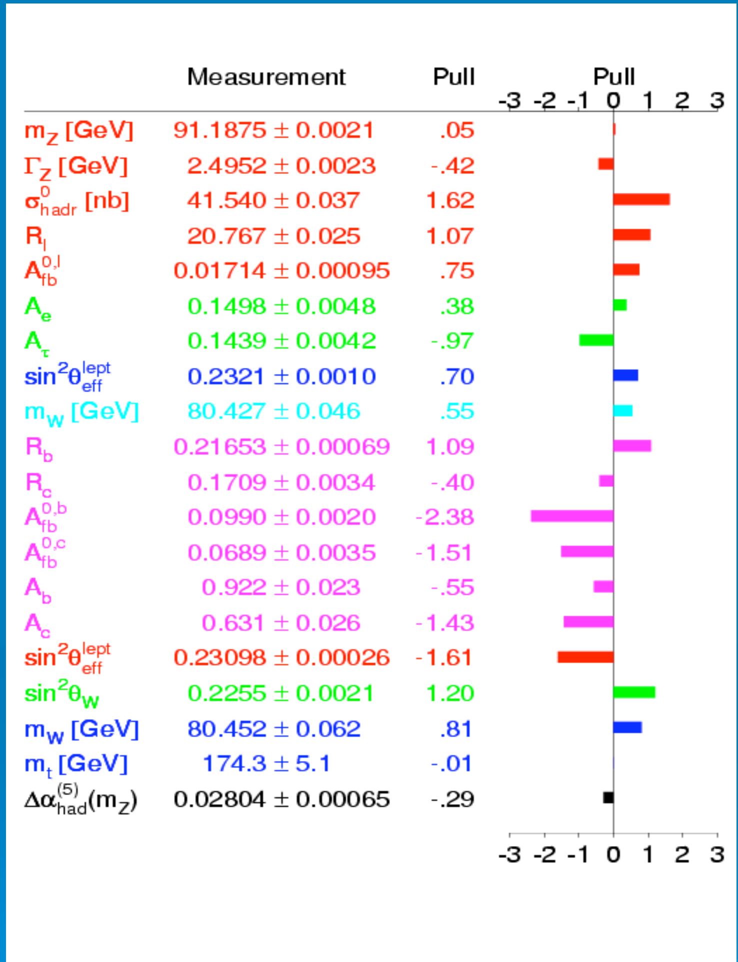
$$\bar{\alpha} = \frac{\alpha}{1 - b\alpha Log(Q^2/\mu^2)}$$



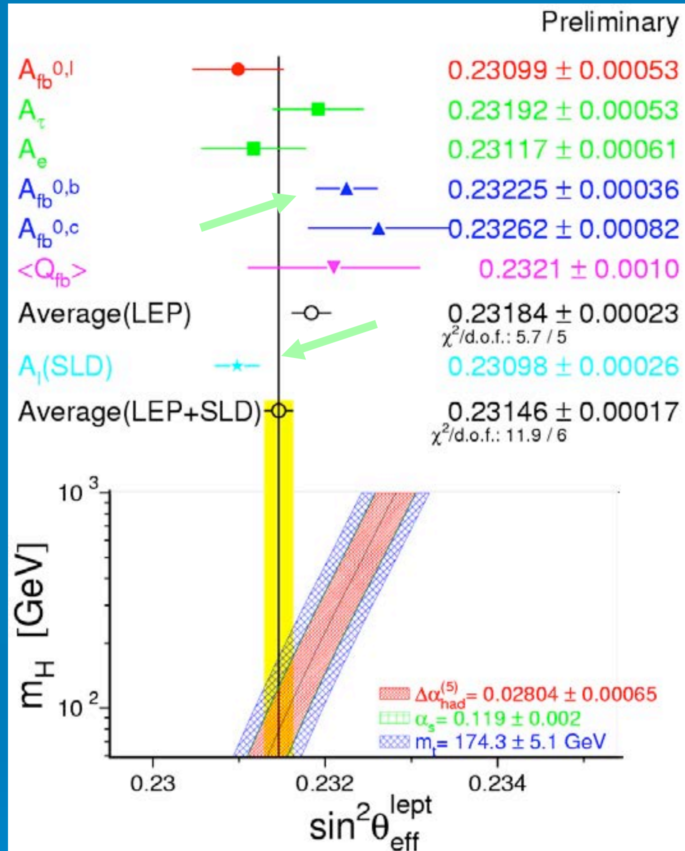


Comparison with Experiment

Global Fit to Data



Higgs Mass Constraint

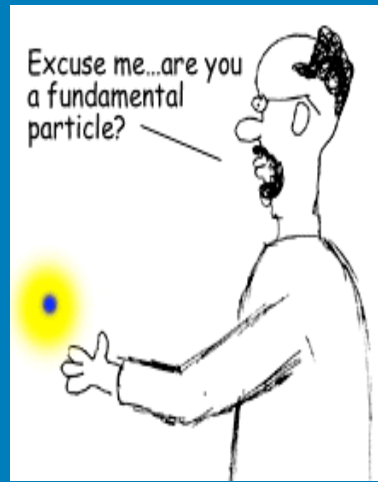


Remarkable agreement of ALL the data with the SM predictions - precision tests of radiative corrections and the SM

Though the values of $\sin \vartheta w$ extracted from different experiments are in good agreement, two most precise measurements from hadron and lepton asymmetries disagree by 3σ

The SM and Beyond

The problems of the SM:



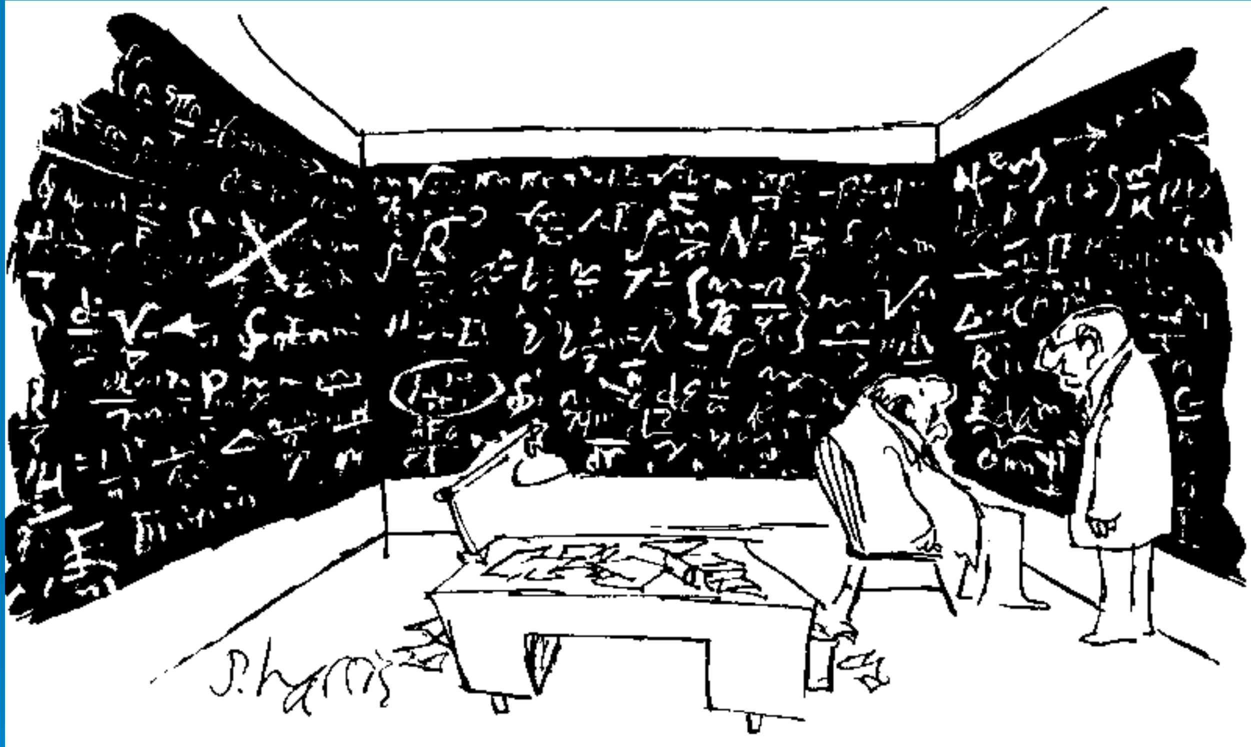
- Inconsistency at high energies due to Landau poles
- Large number of free parameters
- Still unclear mechanism of EW… y breaking
- CP-violation is not und…
- The origin of the m… unclear
- Flavour mixing and … of generations is arbitrary
- Formal unification of … rong and electroweak interactions

Where is the Dark matter?

The way beyond the SM:

- The SAME fields with NEW interactions and NEW fields ⇨ GUT, SUSY, String, ED
- NEW fields with NEW interactions ⇨ Compositeness, Technicolour, preons

We like elegant solutions



"Whatever happened to *elegant* solutions?"