Extra Dimensions of Space-Time

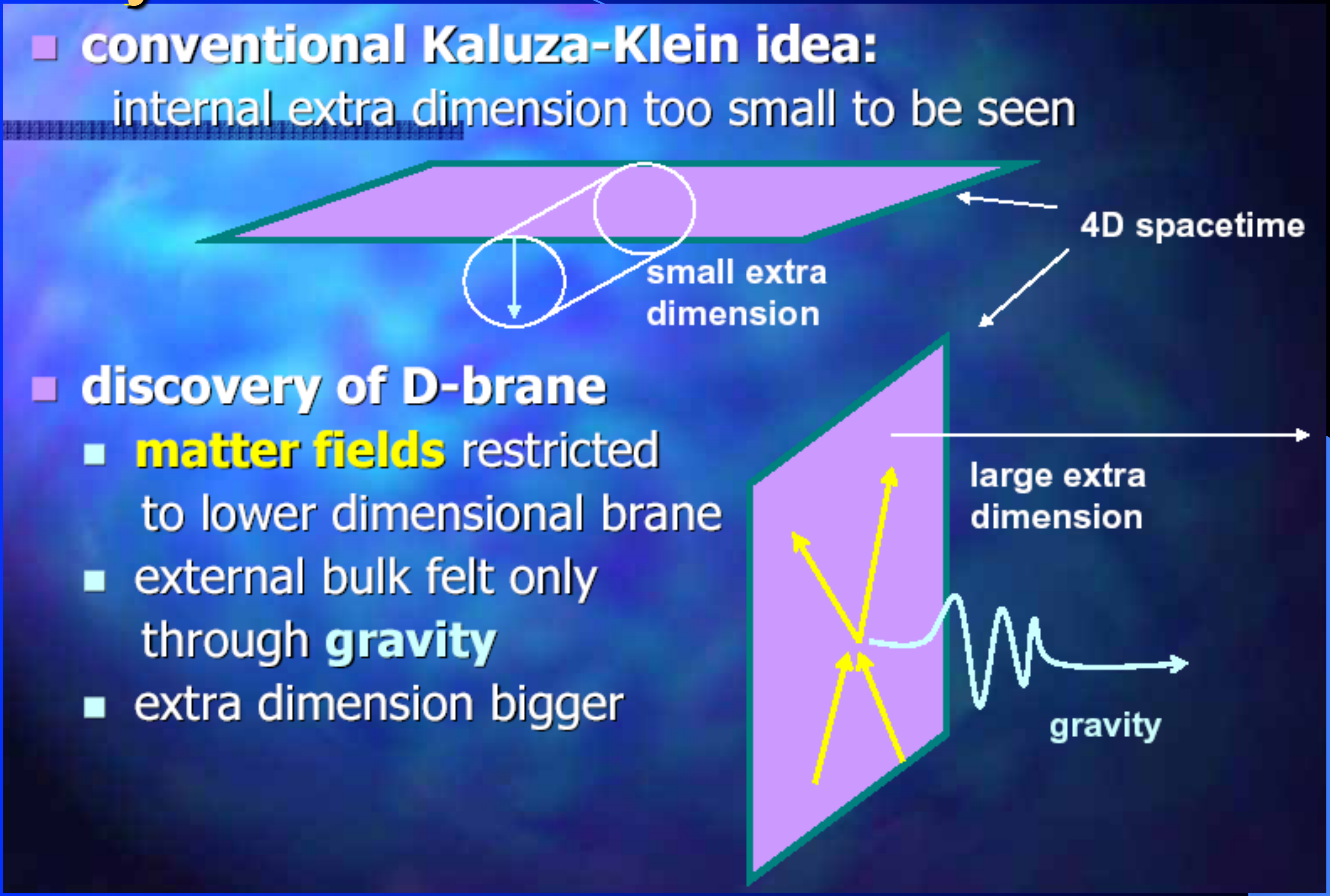
Motivation:

String theory suffers conformal anomaly that makes theory inconsistent --> get rid of it

Conformal anomaly ~ (D-26) for a bosonic string
~ (D-10) for a fermionic string

- Is it possible that we actually leave in D > 4?
- Do we have experimental evidence for D=4, D> 4?

Why don’t we see extra dimensions



- **conventional Kaluza-Klein idea:**
  internal extra dimension too small to be seen
- **discovery of D-brane**
  - **matter fields** restricted to lower dimensional brane
  - external bulk felt only through **gravity**
  - extra dimension bigger

Kaluza-Klein Approach

Pseudo-Euclidean space

$$\boxed{E_{4+d} = M_4 \times K_d}$$

compact space

Minkowski space

Metrics

$$\boxed{ds^2 = G_{MN}(X)dX^M dX^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(x,y)dy^m dy^n}$$

Fields

$$\Phi(x,y) = \sum_{n=0} \phi^{(n)}(x)Y_n(y)$$

Eigenfunctions of Laplace operator on internal space K_d

K-K modes

Masses

$$\boxed{m_n^2 = m^2 + \frac{n_1^2 + n_2^2 + \ldots + n_d^2}{R^2}}$$

Radius of the compact space

Couplings

$$\boxed{g_{(4)} = \frac{g_{(4+d)}}{V_{(d)}}}$$

$$V_{(d)} : R^d$$

Multidimensional Gravity

Action

$$S_E = \int d^{4+d} X \sqrt{-\hat{G}} \frac{1}{16\pi G_{N(4+d)}} R^{(4+d)}[\hat{G}_{MN}]$$

K-K Expansion

$$S_E = \int d^4 x \sqrt{-g} \left\{ \frac{1}{16\pi G_{N(4)}} R^{(4)}[g_{\mu\nu}^{(0)}] + non - zero\ KK\ \text{modes} \right\}$$

Newton constant

$$\boxed{G_{N(4)} = \frac{1}{V_d} G_{N(4+d)}}$$

$$V = R^d$$

Plank Mass

$$M_{Pl} = (G_{N(4)})^{(-1/2)} \longleftrightarrow M = (G_{N(4+d)})^{(-\frac{1}{d+2})}$$

Reduction formula

$$\boxed{M_{Pl}^2 = V_d M^{d+2}}$$

Low Scale Gravity

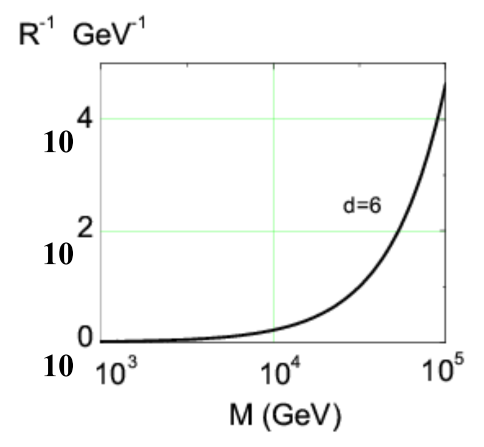
$$M_{Pl}^{2} = R^{d} M^{2+d} \Rightarrow \mathrm{R} \sim \frac{1}{M}\left(\frac{M_{Pl}}{M}\right)^{2/d}$$

$$M \sim 1\ \mathrm{TeV} \Rightarrow \mathrm{R} \sim 10^{30/d-17}\ \mathrm{cm}$$

$$d = 2 \quad R \sim 0.1\ mm \quad \mathrm{R}^{-1} \sim 10^{-3}\ \mathrm{e}V$$

$$d = 3 \quad R \sim 10^{-7}\ cm \quad \mathrm{R}^{-1} \sim 100\ \mathrm{e}V$$

$$d = 6 \quad R \sim 10^{-12}\ cm \quad \mathrm{R}^{-1} \sim 10\ \mathrm{Me}V$$



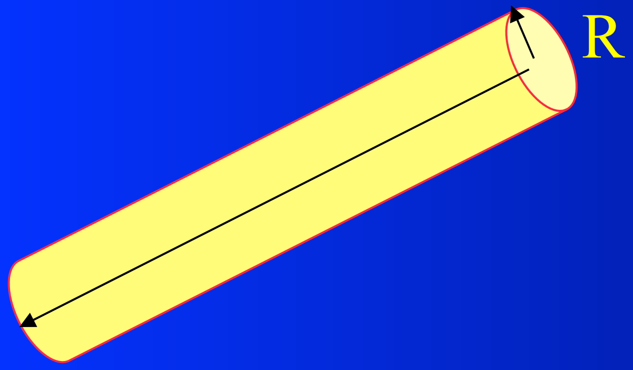
Modified Newton potential

$$V(r) = G_{N(4)} m_1 m_2 \sum_{n} \tfrac{1}{r} e^{-m_n r} = G_{N(4)} m_1 m_2 \left(\tfrac{1}{r} + \sum_{n \neq 0} \tfrac{1}{r} e^{-|n| r / R} \right)$$

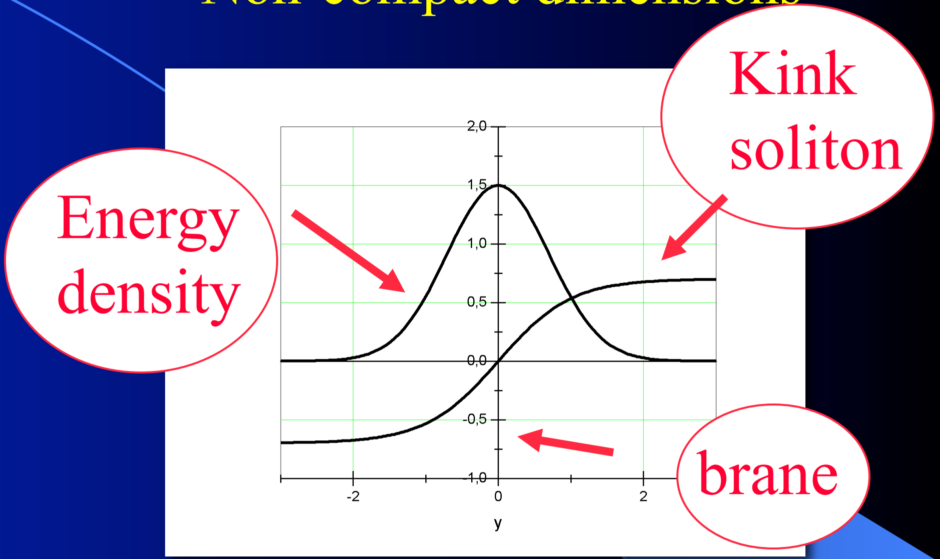
➡
$$\begin{cases} V(r) = G_{N(4)} \dfrac{m_1 m_2}{r}, & \mathrm{r} \gg \mathrm{R} \\ V(r) = G_{N(4+d)} \dfrac{m_1 m_2}{r^{d+1}} (2\pi)^{\mathrm{d}} \dfrac{\Gamma(\frac{\mathrm{d}+1}{2})}{\Gamma(\frac{1}{2})}, & \mathrm{r} = \ \mathrm{R} \end{cases}$$

Brane World

Compact Dimensions

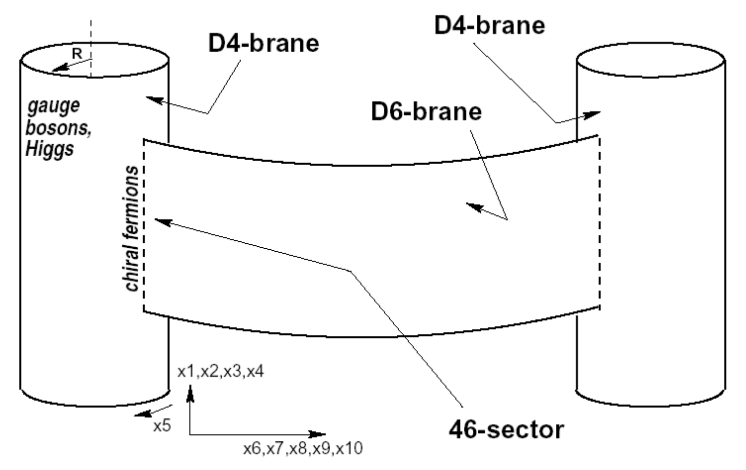


Non-compact dimensions


Energy density

Kink soliton

brane



Space-time of Type I superstring

Localization on the brane

(Potential well)


D4-brane

SM

Bulk

New

D4-brane

The ADD Model

$$G_{MN} = \eta_{MN} + \frac{2}{M^{1+d/2}} \hat{h}_{MN}(x,y)$$

metric

$$\hat{h}_{MN}(x,y) = \sum_n h^{(n)}_{MN}(x) \frac{1}{\sqrt{V_d}} \mathrm{e}^{-\mathrm{i}\, \mathrm{n_m} y^m / R}$$

K-K gravitons

Interactions with the fields on the brane

$$S_{\mathrm{int}} = \int d^{4+d}\hat{x}\sqrt{-\hat{G}}\,\hat{T}_{MN}\hat{h}^{MN}(x,y) \Rightarrow \sum_n \int d^4x \frac{1}{M_{Pl}} T^{\mu\nu} h_{\mu\nu}{}^{(n)}(x)$$

The # of KK gravitons with masses $m_n \le E \le M$

$$\mathrm{N}(E) = S_{d-1}\sum_{n=0}^{ER} n^{d-1} \approx S_{d-1}\int_0^{ER} n^{d-1} dn = \frac{2\pi^{d/2}}{\Gamma(d+1)} R^d E^d$$

Emission rate $\boxed{\sim \frac{1}{M_{Pl}^2} \mathrm{N}(E) \sim \frac{E^d}{M^{d+2}}}$

Particle content of ADD model

(4+d)-dimensional picture:

- (4+d)-dimensional massless graviton + matter

4-dimensional picture

- 1 massless graviton $G^{(0)}$ (spin 2) + matter
- KK tower of massive gravitons $G^{(n)}$ (spin 2)
- (d-1) KK spin 1 decoupling fields
- $(d^2 - d - 2)/2$ KK tower of real scalar decoupling fields $(d \geq 2)$
- KK tower of scalar fields (zero mode – radion)

The SM fields are localized on the brane,
while gravitons propagate in the bulk

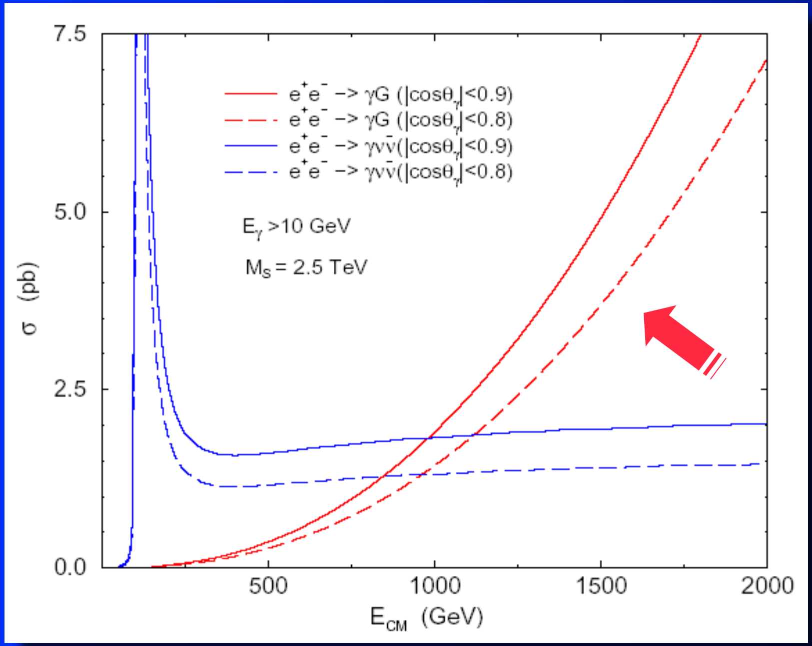
The “gravitational” coupling is $\sim 1/M^{1+d/2}$

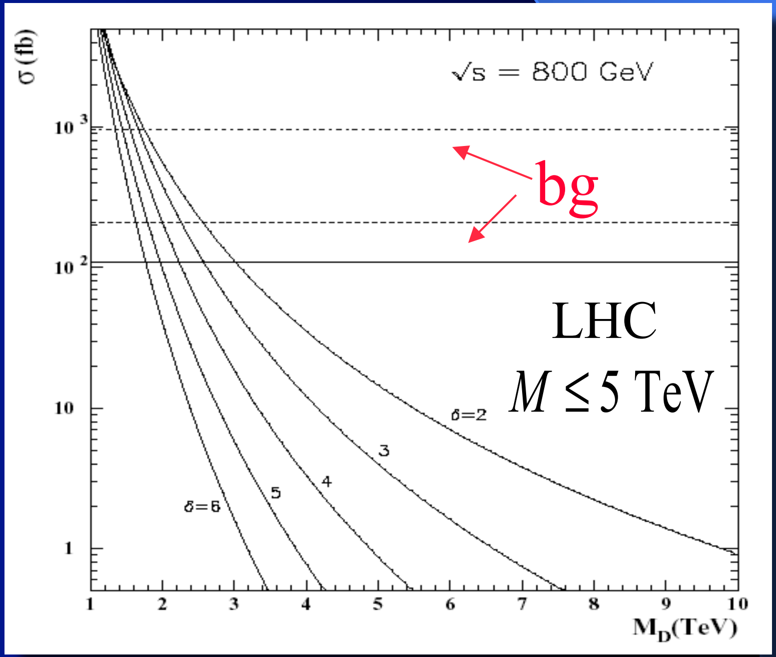
HEP Phenomenology

New phenomena: graviton emission & virtual graviton exchange

- KK states production $e^+e^- \rightarrow \gamma G^{(n)}$ $(e^+e^- \rightarrow \nu\bar{\nu}\gamma)$

$$\boxed{\frac{d^2\sigma}{dtdm} \sim S_{d-1}\frac{M_{Pl}^2}{M^{d+2}}m^{d-1}\frac{d\sigma_m}{dt} \sim \frac{1}{M^{d+2}}}$$

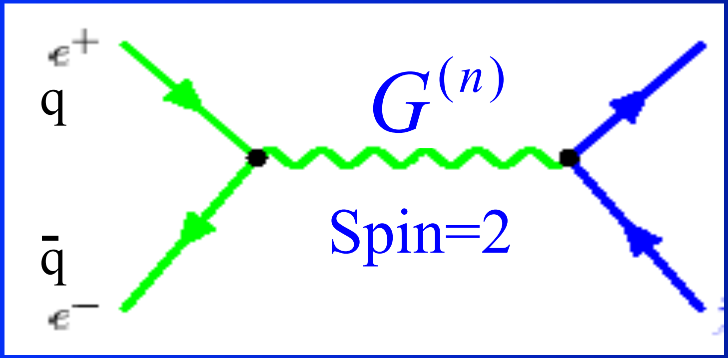


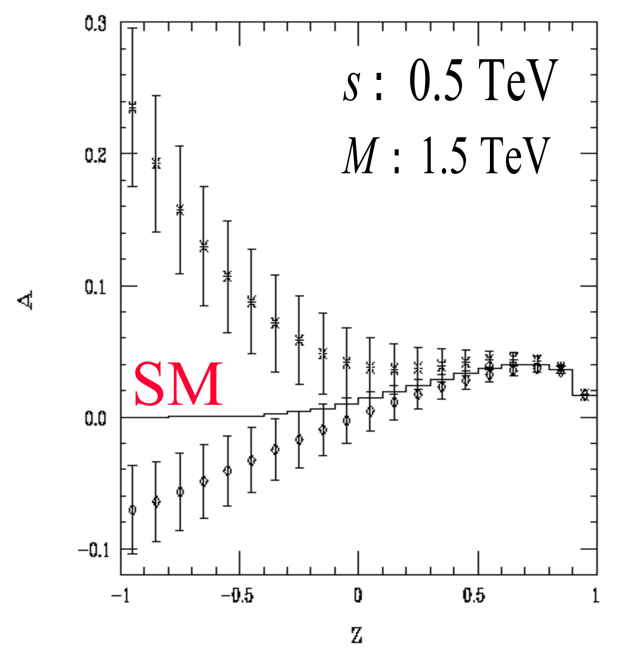


HEP Phenomenology II

- Virtual graviton exchange $e^+e^- \to G^{(n)} \to f\,\bar{f}\ (HH, gg)$

$$\boxed{A = \frac{1}{M_{Pl}^2}\sum_n \left\{ T_{\mu\nu}\frac{P^{\mu\nu}P^{\rho\sigma}}{s-m_n^2}T_{\rho\sigma} + \sqrt{\frac{3(d-1)}{d+2}}\,\frac{T_\mu^\mu T_\nu^\nu}{s-m_n^2} \right\}}$$





Angular distribution

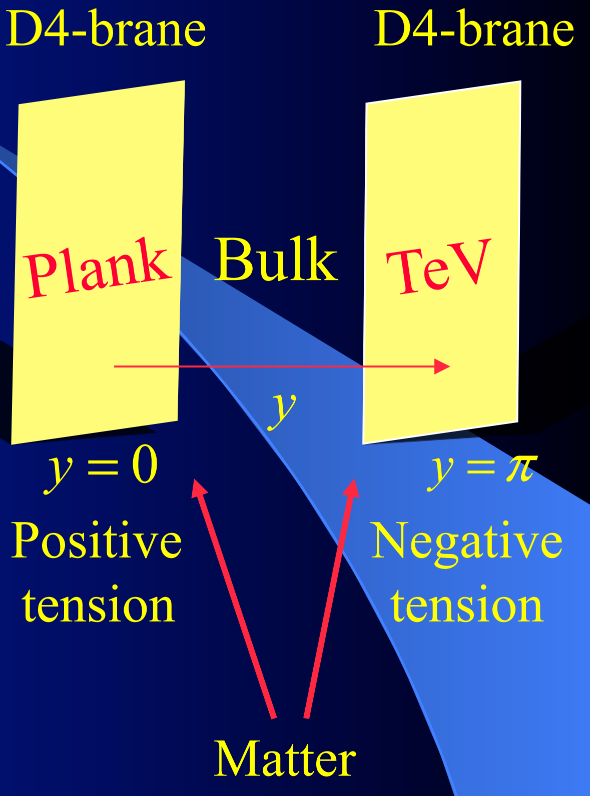
$$S = \frac{1}{M_{Pl}^2}\sum_n \frac{1}{s-m_n^2} \approx \frac{1}{M_{Pl}^2} S_{d-1}\frac{M_{Pl}^2}{M^{d+2}}\int_0^\Lambda \frac{m^{d-1}dm}{s-m^2}$$

$$= \frac{S_{d-1}}{2M^4}\left\{ i\pi\left(\frac{s}{M^2}\right)^{d/2-1} + \sum_{k=1}^{[(d-1)/2]} c_k\left(\frac{s}{M^2}\right)^{k-1}\left(\frac{\Lambda}{M}\right)^{d-2k} \right\}$$

Randall-Sandrum Models

$$E_5 = M_4 \otimes S^1 / Z_2$$

$$\boxed{\begin{aligned} S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-\hat{G}}\{2M^3 R^{(5)}[\hat{G}_{MN}] + \Lambda\} \\ + \int_{B_1} d^4x \sqrt{-g^{(1)}}(L_1 - \tau_1) + \int_{B_2} d^4x \sqrt{-g^{(2)}}(L_2 - \tau_2) \end{aligned}}$$



D4-brane

D4-brane

Plank

Bulk

TeV

y

$y = 0$

$y = \pi$

Positive tension

Negative tension

Matter

Metric $ds^2 = \underset{\substack{\uparrow \\ \text{warp factor}}}{e^{-2\sigma(y)}}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$

$\sigma(y) = k|y|$

$$\tau_1 = -\tau_2 = 24M^3k, \quad \Lambda = 24M^3k^2$$

Perturbed Metric $ds^2 = e^{-2\sigma(y)}(\eta_{\mu\nu} + \overset{\text{graviton}}{h_{\mu\nu}}(x,y))dx^\mu dx^\nu + (1+\overset{\text{radion}}{\varphi}(x))dy^2$

Randall-Sandrum Model cont’d

Brane 1

- Massless graviton
- massive K-K gravitons

$$m_n = \beta_n k e^{-\pi k R}$$

- massless radion

Brane 2

Wrap factor $\boxed{e^{-2\sigma(\pi R)}}$

Hierarchy Problem !

$$\boxed{M_{Pl}^2 = \frac{M^3}{k}(e^{2k\pi R} - 1)}$$

$$S_{eff} = \frac{1}{2}\int_{B_2} d^4 z \left[\frac{1}{M_{Pl}} h_{\mu\nu}^{(0)} T^{\mu\nu} - \sum_{n=1} \frac{\omega_n}{\Lambda_\pi} h_{\mu\nu}^{(n)} T^{\mu\nu} - \frac{1}{\Lambda_\pi \sqrt{3}} T_\mu^\mu \right]$$

$$\Lambda_\pi = M_{Pl} e^{-k\pi R} : 1 \text{ TeV}$$

- Massless graviton $\longrightarrow \sim M_{Pl}$
- massive K-K gravitons $m_n = \beta_n k \longrightarrow \sim \Lambda_\pi$
- massless radion

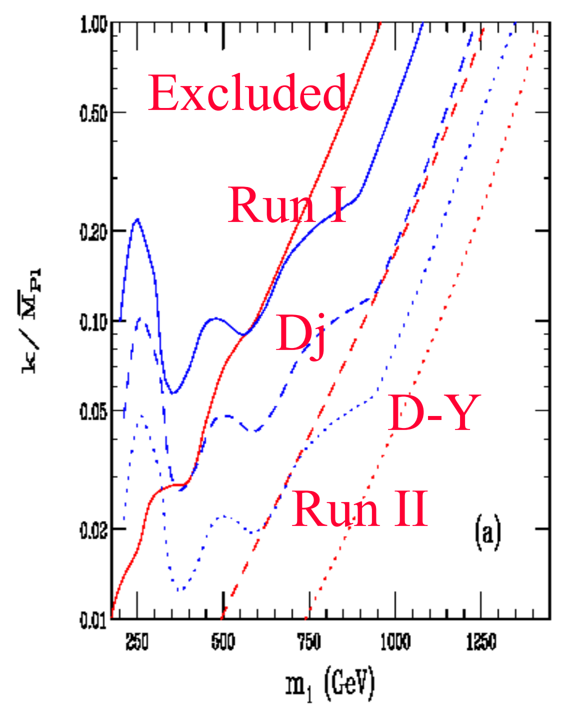
HEP Phenomenology

The first KK graviton mode M ~ 1 TeV

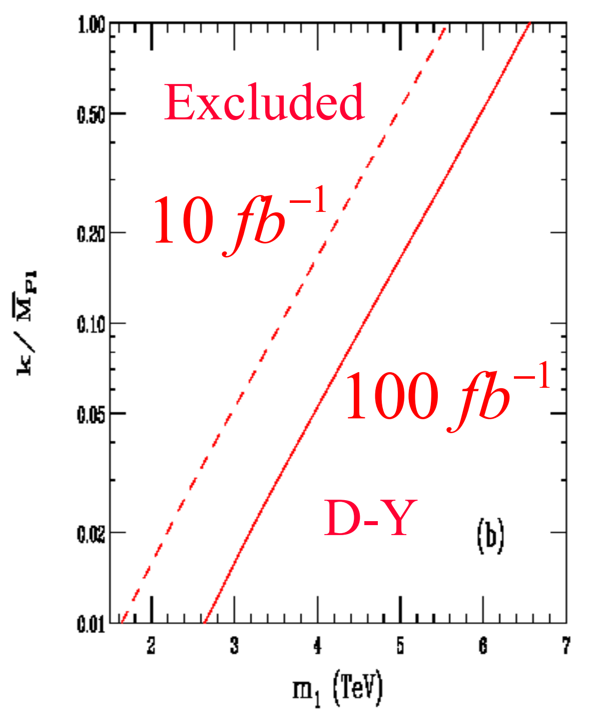
- Drell-Yan process $q\bar{q} \rightarrow G^{(1)} \rightarrow l^+l^-, \; gg \rightarrow G^{(1)} \rightarrow l^+l^-$
- Excess in dijet process $q\bar{q}, gg \rightarrow G^{(1)} \rightarrow q\bar{q}, gg$

Exclusion plots for resonance production

$$\eta = (k / M_{Pl})e^{k\pi R}$$



Tevatron



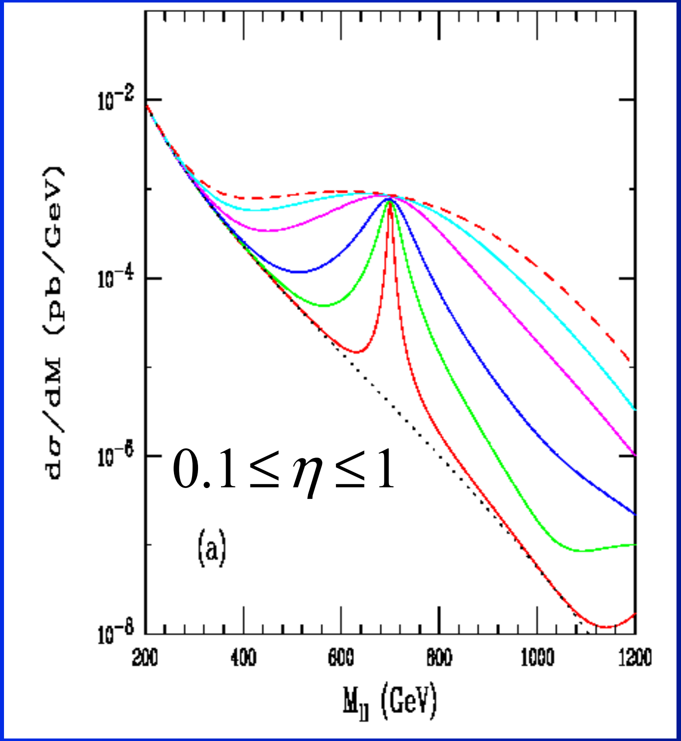
LHC

HEP Phenomenology II

$$\eta = (k / M_{Pl}) e^{k\pi R}$$

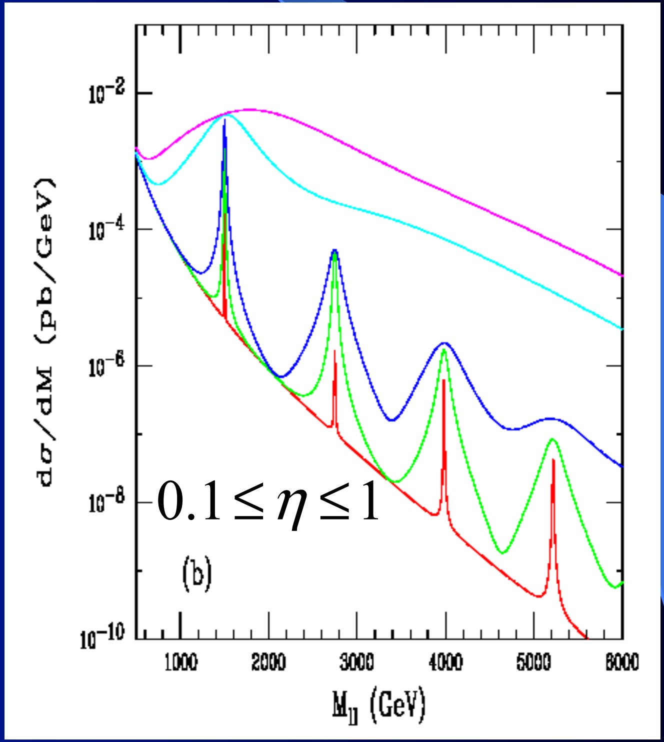
The x-section of D-Y production

First KK mode



Tevatron (M ~ 700 GeV)

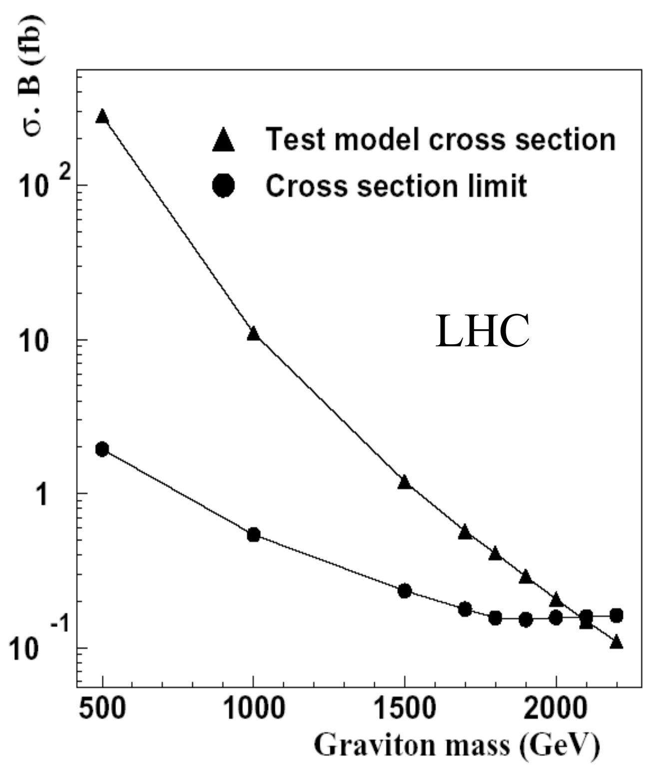
First and subsequent KK modes



LHC (M ~ 1500 GeV)

HEP Phenomenology III

$pp \rightarrow G^{(1)} \rightarrow e^{+}e^{-}$

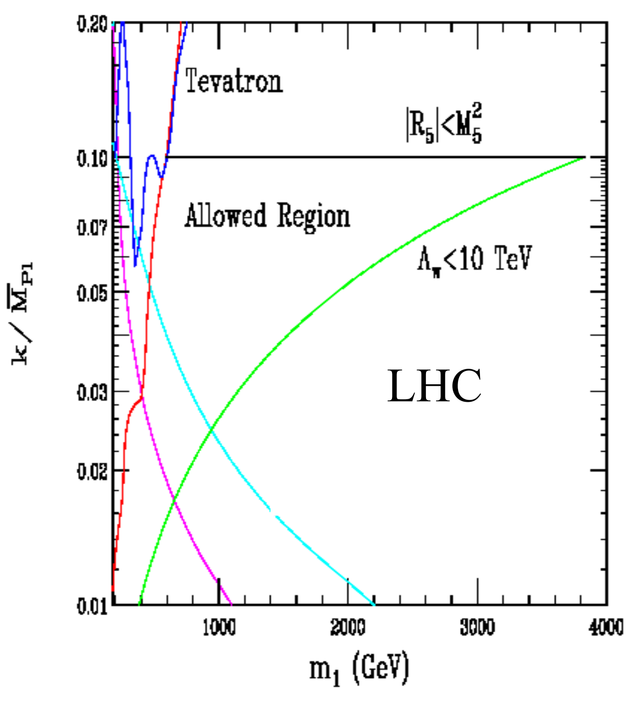


Angular dependence

$$spin\ 0 \Rightarrow \ \mathrm{f}(\theta)=1,\ \mathrm{spin}\ 1 \Rightarrow \ \mathrm{f}(\theta)=1+\cos^{2}\theta$$

$$q\bar{q} \rightarrow G^{(1)} \rightarrow l^{+}l^{-},\ \mathrm{f}(\theta)=1-\cos^{4}\theta$$

$$\mathrm{gg} \rightarrow G^{(1)} \rightarrow l^{+}l^{-},\ \mathrm{f}(\theta)=1-3\cos^{2}\theta+4\cos^{4}\theta$$



ED Conclusion

ADD Model

- The M_{EW}/M_{PL} hierarchy is replaced by

$$\frac{R^{-1}}{M} \sim \left(\frac{M}{M_{Pl}} \right)^{2/d} \sim 10^{-30/d}$$

- For M small enough it can be checked at modern and future colliders
- For d=2 cosmological bounds on M are high (> 100 TeV), but for d>2 are mild
- Preictions of modification of the Newton's law may be checked

RS Model

- The M_{EW}/M_{PL} hierarchy is solved without new hierarchy
- A large part of parameter space will be studied in future collider experiments
- With the mechanism of radion stabilization the model is viable
- Cosmological scenarios are consistent (except the cosmological constant problem)

What comes beyond the Standard Model ?