EUROPEAN SCHOOL OF HIGH-ENERGY PHYSICS

SANT FELIU DE GUÍXOLS, NEAR BARCELONA, SPAIN
30 MAY-12 JUNE 2004

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Field Theory & the Standard Model
A. Pich, *IFIC, València*
Beyond the Standard Model
D. Kazakov, *JINR, Dubna*
QCD
M. Seymour, *Univ. Manchester & CERN, Geneva*
Flavour Physics & CP Violation
A.J. Buras, *Technical University, Munich*
Neutrinos
C. Gonzalez-Garcia, *Stony Brook Univ. & IFIC, València*
Cosmology and Astrophysics
J. Garcia-Bellido, *Universidad Autónoma, Madrid*
Quark deconfinement & Nuclear collisions
H. Satz, *Universität Bielefeld*
Accelerators
K. Hubner, *CERN, Geneva*

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V. Bednyakov
JINR, Dubna
A. Donini
U. Autónoma, Madrid
T. Feldmann
CERN, Geneva
R. Fleischer
CERN, Geneva
A. Santamaria
IFIC, València
O. Vives
CERN, Geneva

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R. Pascual - *Universidad Autònoma de Barcelona*

ENQUIRIES AND CORRESPONDENCE

Mrs Danielle Métral
CERN Schools of Physics
CERN/DSU
CH-1211 GENEVA 23
Switzerland
Tel +41 22 767 9141
Fax +41 22 782 3011
Physics.school@cern.ch

Mrs Tatyana Donskova
International Department
Joint Institute for Nuclear Research
RU-141980 Dubna, Moscow Region
Russia
Tel +7 096 21 63448
Fax +7 09621 65599
phs@jinr.ru

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Latest information and application forms can be found at the URL:

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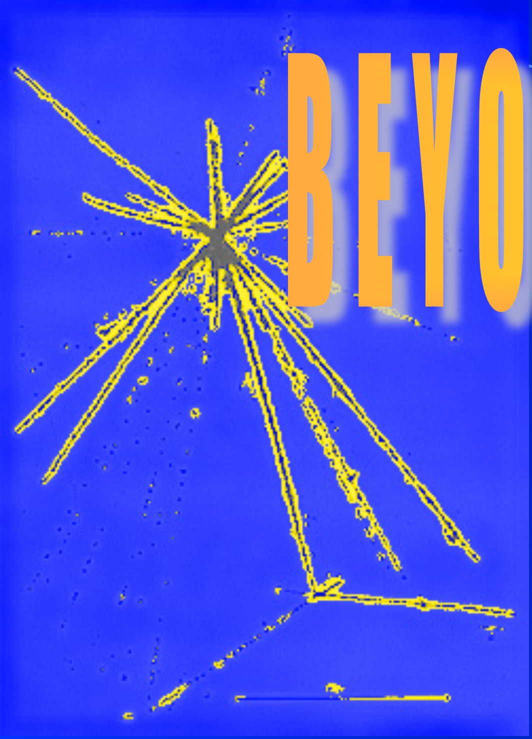
Closing date for applications 18 January 2004



BEYOND THE STANDARD MODEL

Dmitri Kazakov
JINR/ITEP

Outline

Part I Supersymmetry

1. What is SUSY
2. Motivation of SUSY
3. Basics of SUSY
4. The MSSM
5. Constrained MSSM
6. SUSY searches

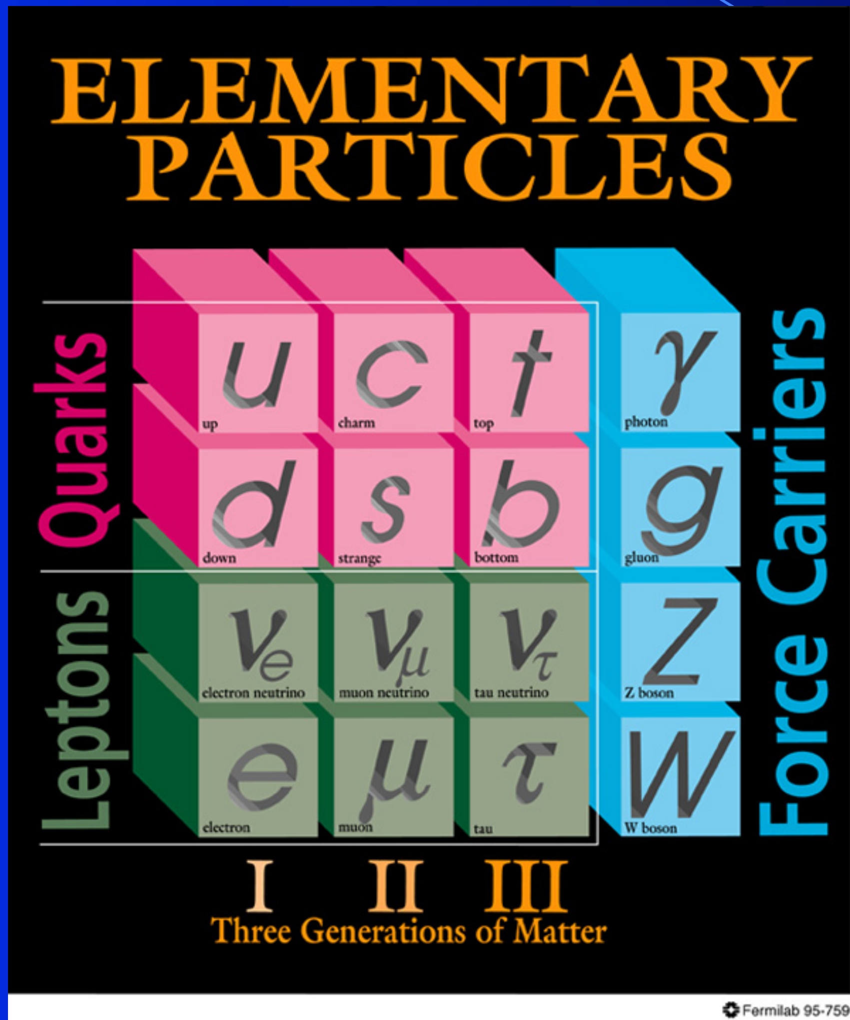
Part II Extra Dimensions

1. The main idea
2. Kaluza-Klein Approach
3. Brane-world models
4. Possible experimental signatures of ED

SU(3)

The Standard Model

Particles



H

The Higgs boson

Forces

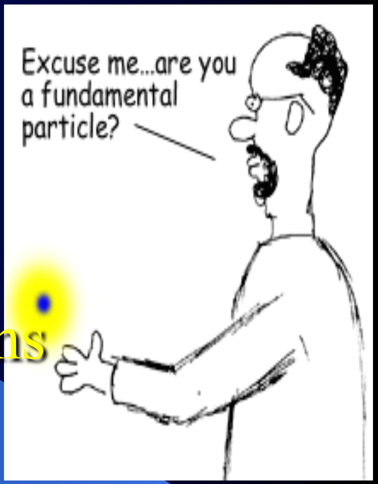
Electromagnetic

Strong

Weak

Gravity

The SM and Beyond



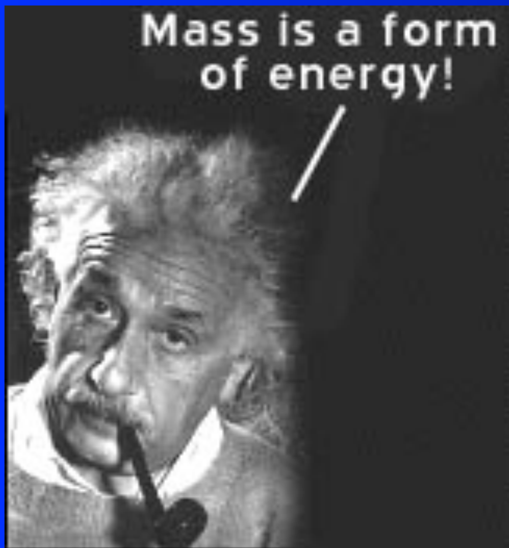
The problems of the SM:

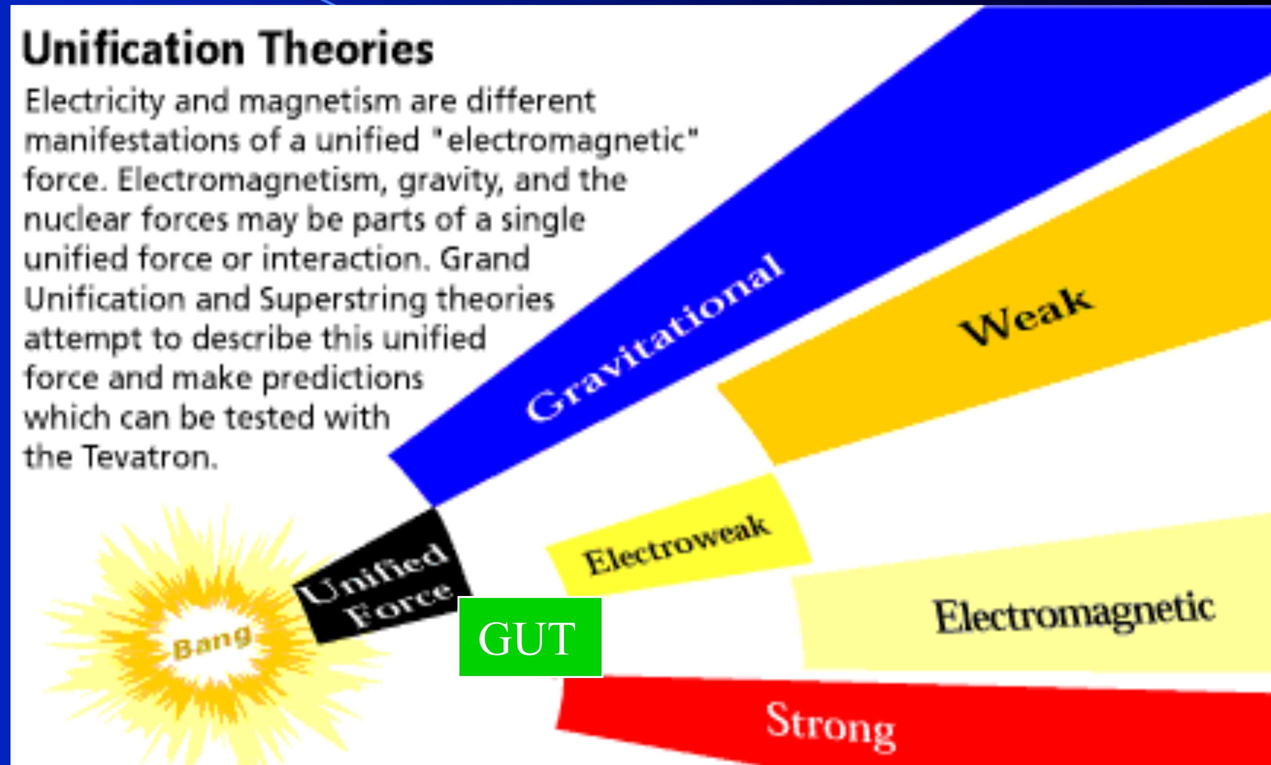
- Inconsistency at high energies due to Landau pole
- Large number of free parameters
- Formal unification of strong and electroweak interactions
- Still unclear mechanism of EW symmetry breaking
- CP-violation is not understood
- Flavour mixing and the number of generations is arbitrary
- The origin of the mass spectrum in unclear

The way beyond the SM:

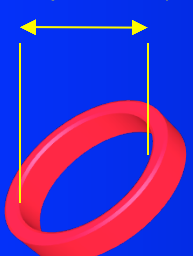
- The SAME fields with NEW interactions ⇒ GUT, SUSY, String
- NEW fields with NEW interactions ⇒ Compositeness, Technicolour, preons

Grand Unified Theories





$10^{-34}\,m$

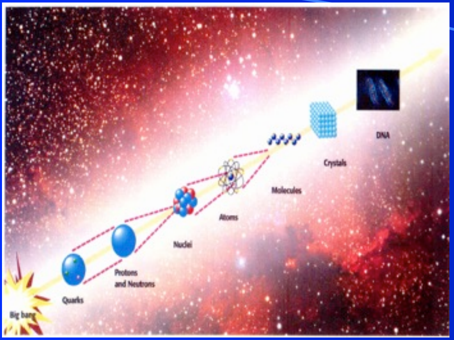


D=10

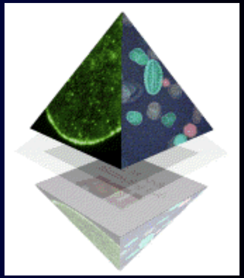
- Unification of strong, weak and electromagnetic interactions within Grand Unified Theories is the new step in unification of all forces of Nature
- Creation of a unified theory of everything based on string paradigm seems to be possible

PART I : SUPERSYMMETRY

1. What is SUSY
2. Motivation of SUSY
3. Basics of SUSY
4. The MSSM
5. Constrained MSSM
6. SUSY searches



What is SUSY



- **Supersymmetry** is a boson-fermion symmetry that is aimed to unify all forces in Nature including gravity within a singe framework

$$Q\,|\,boson> = |\,fermion> \qquad Q\,|\,fermion> = |\,boson>$$

$$[b,b]=0, \quad \{f,f\}=0 \;\Rightarrow \qquad \{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

- Modern views on supersymmetry in particle physics are based on string paradigm, though low energy manifestations of SUSY can be found (?) at modern colliders and in non-accelerator experiments

Motivation of SUSY in Particle Physics

- Unification with Gravity
- Unification of gauge couplings
- Solution of the hierarchy problem
- Dark matter in the Universe
- Superstrings

$$spin\ 2\ \rightarrow\ spin\ 3/2\ \rightarrow\ spin\ 1\ \rightarrow\ spin\ 1/2\ \rightarrow\ spin\ 0$$

Unification of matter (fermions) with forces (bosons) naturally arises
from an attempt to unify gravity with the other interactions

$$\{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu \quad \Rightarrow \quad \{\delta_\varepsilon, \bar\delta_{\bar\varepsilon}\} = 2(\varepsilon\sigma^\mu\bar\varepsilon)P_\mu$$

$\varepsilon = \varepsilon(x)$ local coordinate transformation.

Local translation =
general relativity !

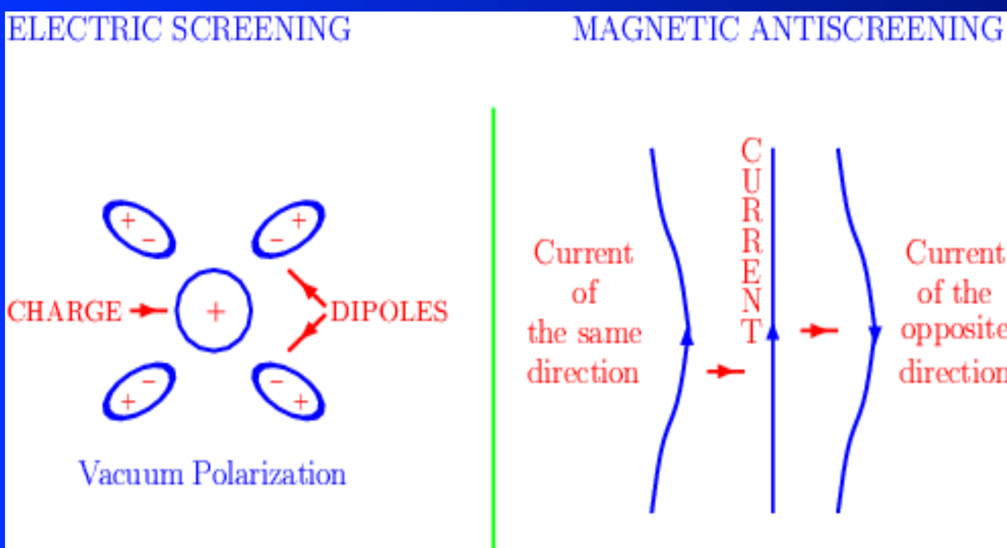
Supertranslation

$$x_\mu \rightarrow x_\mu + i\theta\sigma_\mu\bar\xi - i\xi\sigma_\mu\bar\theta,$$
$$\theta \rightarrow \theta + \xi,$$
$$\bar\theta \rightarrow \bar\theta + \bar\xi$$

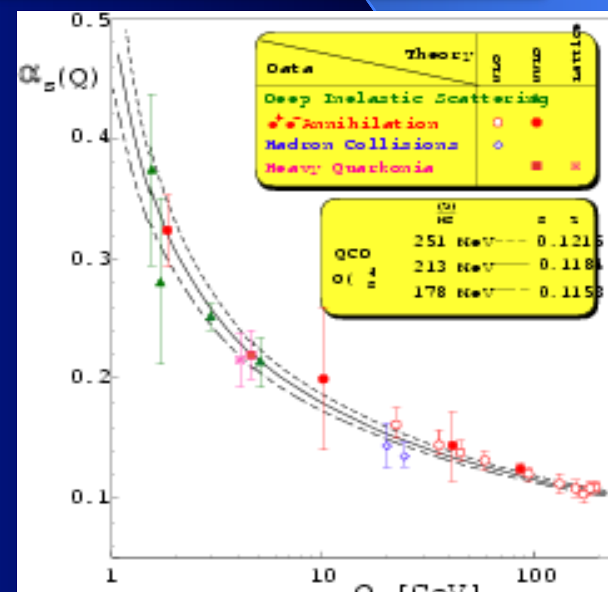
Motivation of SUSY in Particle Physics

- Unification of gauge couplings

	Low Energy		$\Rightarrow$	*High Energy*
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	$\Rightarrow$	G_{GUT} (or G^n + symm)
gluons	W, Z	*photon*	$\Rightarrow$	*gauge bosons*
quarks	*leptons*		$\Rightarrow$	*fermions*
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}



$$\alpha_i = \alpha_i(\frac{Q^2}{\Lambda^2}) = \alpha_i(\text{distance})$$



Running of the strong coupling

Motivation of SUSY

RG Equations $\frac{d\tilde{\alpha}_i}{dt} = b_i\tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \alpha_i / 4\pi = g_i^2 / 16\pi^2, \quad \text{t=log}(Q^2 / \mu^2)$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Input

$$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$$

$$\sin^2\theta_{\overline{MS}} = 0.23146 \pm 0.00017$$

$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

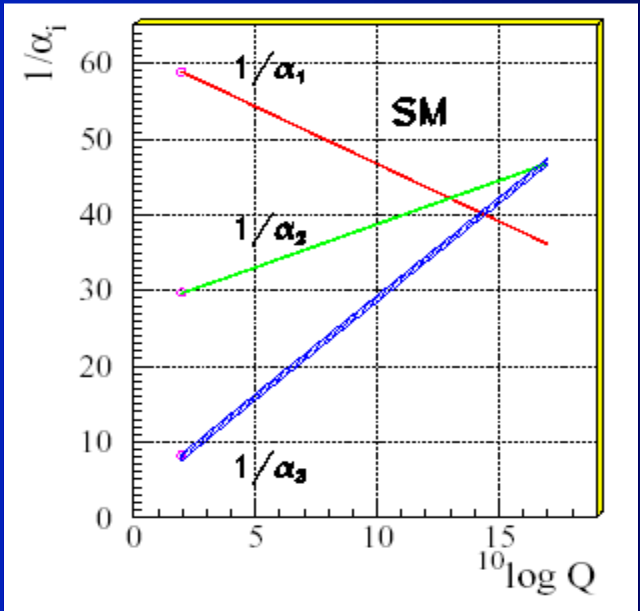
Output

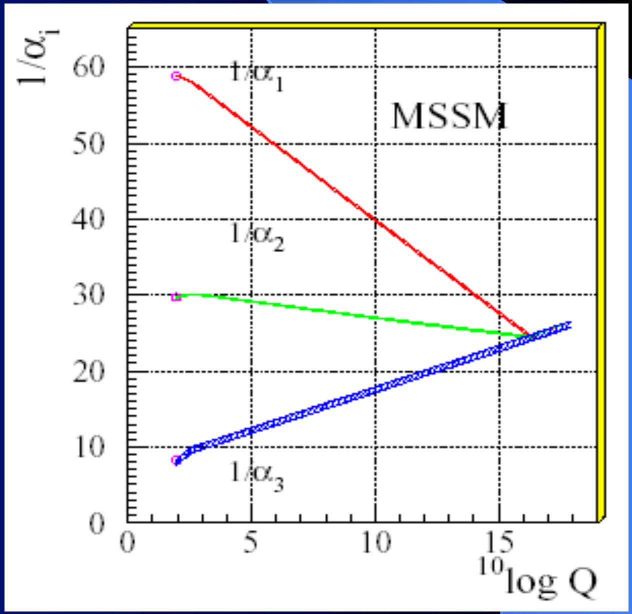
$$M_{SUSY} = 10^{3.4\pm0.9\pm0.4} \text{ GeV}$$

$$M_{GUT} = 10^{15.8\pm0.3\pm0.1} \text{ GeV}$$

$$\alpha_{\text{GUT}}^{-1} = 26.3 \pm 1.9 \pm 1.0$$

Unification of the Coupling Constants in the SM and in the MSSM





SUSY yields unification!

Motivation of SUSY

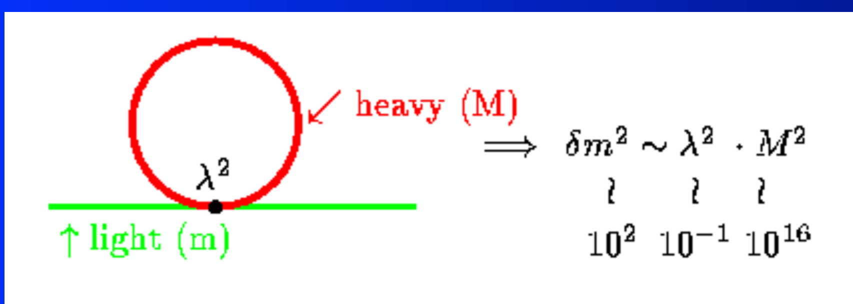
- Solution of the Hierarchy Problem

$$m_H \sim \mathrm{v} \sim 10^2 \text{ GeV}$$

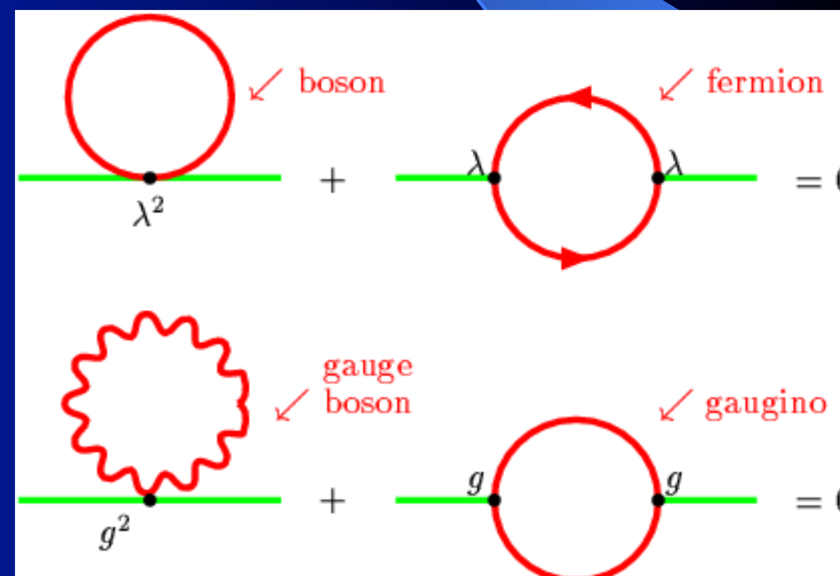
$$\mathrm{m}_\Sigma \sim \mathrm{V} \sim 10^{16} \text{ GeV}$$

$$\frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

Destruction of the hierarchy by radiative corrections



Cancellation of quadratic terms

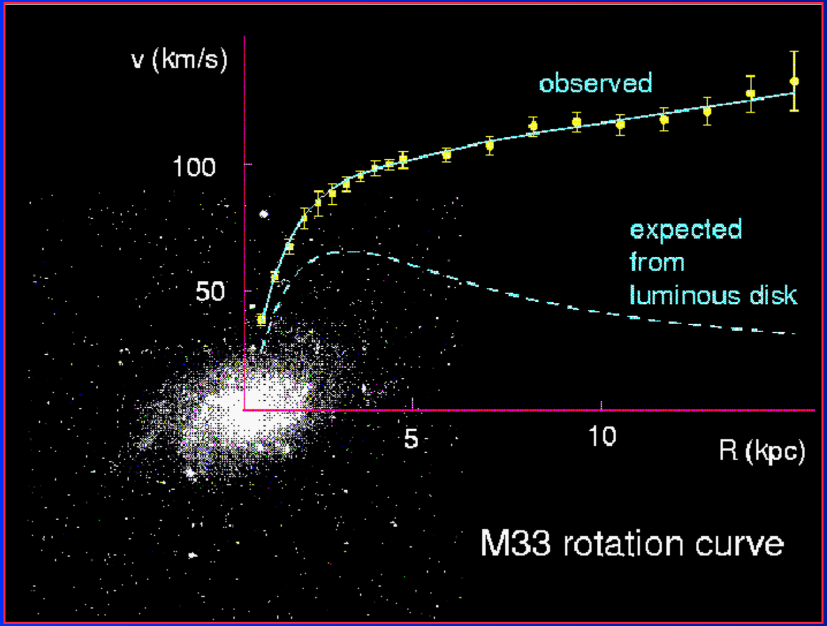


SUSY may also explain the origin of the hierarchy due to radiative mechanism

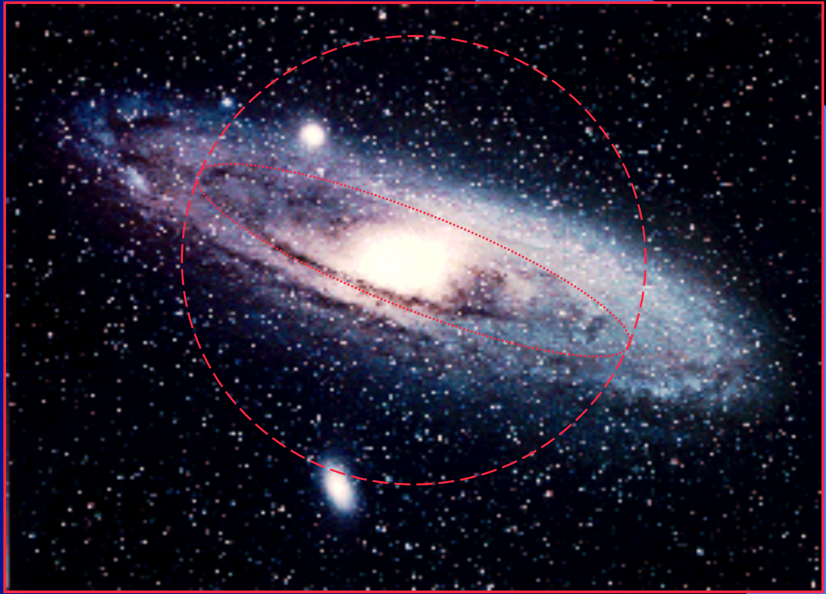
$$\sum_{bosons} m^2 = \sum_{fermions} m^2$$

Motivation of SUSY

- Dark Matter in the Universe



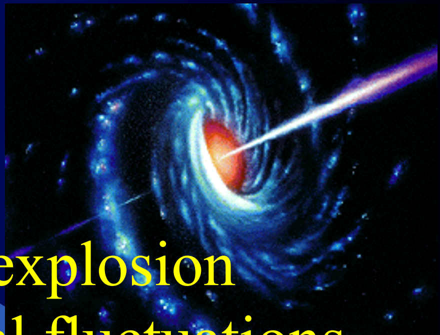
The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.



Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter

SUSY provides a candidate for the Dark matter – a stable neutral particle

Cosmological Constraints



New precise cosmological data

$$\Omega h^2 = 1 \quad \longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$

- Supernova Ia explosion
- CMBR thermal fluctuations

(news from WMAP)

Dark Matter in the Universe:

→ Hot DM (not favoured by galaxy formation)

→ Cold DM (rotation curves of Galaxies) ← SUSY

Supersymmetry

$$(Super)\ Algebra$$

$$[P_\mu, P_\nu] = 0,\ [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$$

$$[B_r, B_s] = iC^t_{rs}B_t,\ \ [B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,$$

$$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,$$

$$[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta,\ [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},$$

$$[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha,\ [\bar{Q}^i_{\dot\alpha}, B_r] = -\bar{Q}^j_{\dot\alpha}(b_r)_j^i,$$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

$$\{Q^i_\alpha, Q^j_\beta\} = 2\varepsilon_{\alpha\beta}Z^{ij},\ Z^{ij} = Z^\dagger_{ij}, Z_{ij} = a^r_{ij}b_r,$$

$$\{\bar{Q}^i_{\dot\alpha}, \bar{Q}^j_{\dot\beta}\} = -2\varepsilon_{\dot\alpha\dot\beta}Z^{ij},\ [Z_{ij}, anything] = 0,$$

$$\alpha, \dot\alpha, \beta, \dot\beta = 1,2;\ i, j = 1,2,\ldots,N.$$

Superspace

$$x_\mu \to x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha}$$

$$\alpha, \dot\alpha = 1,2$$

Grassmannian parameters $\quad \vartheta^2_\alpha = 0,\ \bar\vartheta^2_{\dot\alpha} = 0$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial\vartheta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\vartheta_{\dot\alpha}} + i\theta_\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$Q^2_\alpha = 0,\ \bar{Q}^2_{\dot\alpha} = 0$$

This is the only possible graded Lie algebra that mixes integer and half-integer spins and changes statistics

Basics of SUSY

Quantum states: Vacuum = $|E,\lambda>$ $\quad Q|E,\lambda>=0$

Energy helicity

$$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot{\alpha}}, P_\mu] = 0$$

State	Expression	# of states
vacuum	$\|E,\lambda>$	1
1-particle	$\bar{Q}_i\|E,\lambda>=\|E,\lambda+1/2>$	$\binom{N}{1}=N$
2-particle	$\bar{Q}_i\bar{Q}_j\|E,\lambda>=\|E,\lambda+1>$	$\binom{N}{2}=\frac{N(N-1)}{2}$
…	…	…
N-particle	$\bar{Q}_1\bar{Q}_2\ldots\bar{Q}_N\|E,\lambda>=\|E,\lambda+N/2>$	$\binom{N}{N}=1$

Total # of states $\displaystyle\sum_{k=0}^{N}\binom{N}{k}=2^N=2^{N-1}\,bosons+2^{N-1}\,fermions$

SUSY Multiplets

Chiral multiplet $N=1,\ \lambda=0$

helicity	-1/2	0	1/2
# of states	1	2	1

(φ, ψ) — scalar, spinor

Vector multiplet $N=1,\ \lambda=1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1

(λ, A_μ) — spinor, vector

Members of a supermultiplet are called **superpartners**

Extended SUSY multiplets

N=4	SUSY YM	helicity			-1	–1/2	0	1/2	1		
	λ = -1	# of states			1	4	6	4	1		
N=8	SUGRA	helicity	-2	–3/2	–1	–1/2	0	1/2	1	3/2	2
	λ = -2	# of states	1	8	28	56	70	56	28	8	1

$N \le 4S$ ← spin

$N \le 4$ For renormalizable theories (YM)

$N \le 8$ For (super)gravity

Matter Superfields

$F(x,\theta,\bar{\theta})$ - general superfield –reducible representation

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

chiral superfield: $\bar{D}\Phi = 0$

$(y = x + i\theta\sigma\bar{\theta})$

$$= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$$

$$+ \sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)$$

spin=0

spin=1/2

auxiliary

component fields

SUSY transformation

$$\delta_\varepsilon A = \sqrt{2}\varepsilon\psi,$$
$$\delta_\varepsilon\psi = i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F,$$
$$\delta_\varepsilon F = i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi$$

Superpotential

$$W(\Phi) = W(A + \sqrt{2}\theta\psi + \theta\theta F)$$
$$= W(A) + \frac{\partial W}{\partial A}\sqrt{2}\theta\psi + \theta\theta(\frac{\partial W}{\partial A}F - \frac{1}{2}\frac{\partial^2 W}{\partial A^2}\psi\psi)$$

F-component is a total derivative ⇒ $\Phi|_{\theta\theta}$ is SUSY invariant

Gauge superfields

$V = V^{+}$ real superfield

$$V(x,\theta,\bar{\theta}) = C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta M(x) - i\bar{\theta}\bar{\theta}M^{+}(x)$$
$$-\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}[\bar{\lambda}(x) + i\bar{\sigma}^{\mu}\partial_{\mu}\chi(x)] - i\bar{\theta}\bar{\theta}\theta[\lambda(x) + i\sigma^{\mu}\partial_{\mu}\bar{\chi}(x)]$$
$$+\tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \tfrac{1}{2}\mathrm{W}C(x)]$$

Gauge transformation

$$V \to V + \Phi + \bar{\Phi}$$

$C \to C + A + A^{*}$

$\chi \to \chi - i\sqrt{2}\psi$

$M \to M - 2iF$

$v_{\mu} \to v_{\mu} - i\partial_{\mu}(A - A^{*})$

$\lambda \to \lambda$

$D \to D$

Wess-Zumino gauge

$$C = \chi = M = 0$$

physical fields

Covariant derivatives

$$D_{\alpha} = \frac{\partial}{\partial\vartheta_{\alpha}} + i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}\partial_{\mu}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\alpha}} - i\theta_{\dot{\alpha}}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}$$

Field strength tensor

$$W_{\alpha} = -\tfrac{1}{4}\bar{D}^{2}e^{V}D_{\alpha}e^{-V}$$

$$W_{\alpha} = -i\lambda_{\alpha} + \theta_{\alpha}D - \tfrac{i}{2}(\sigma^{\mu}\bar{\sigma}^{\nu}\theta)_{\alpha}F_{\mu\nu} + \theta^{2}\sigma^{\mu}D_{\mu}\bar{\lambda}$$

SUSY Lagrangians

Superfields

$$L = \Phi_i^{+}\Phi\,\big|_{\theta\theta\bar{\theta}\bar{\theta}} + \big[\big(\lambda_i \Phi_i + \tfrac{1}{2} m_{ij}\Phi_i\Phi_j + \tfrac{1}{3} y_{ijk}\Phi_i\Phi_j\Phi_k\big)\big|_{\theta\theta} + h.c.\big]$$

Components

$$L = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i + F_i^* F_i$$ ← no derivatives

$$+\ \big[\lambda_i F_i + m_{ij}\big(A_i F_j - \tfrac{1}{2}\psi_i\psi_j\big) + y_{ijk}\big(A_i A_j F_k - \psi_i\psi_j A_k\big) + h.c.\big]$$

Constraint $$\frac{\delta L}{\delta F_k} = F_k^* + \lambda_k + m_{ik} A_i + y_{ijk} A_i A_j = 0 \quad \Rightarrow \quad F_k$$

$$L = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \tfrac{1}{2} m_{ij}\psi_i\psi_j - \tfrac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$

$$-\ y_{ijk}\psi_i\psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j)$$

$$V = F_k^* F_k$$

Superfield Lagrangians

$$Action=\int d^4x\,L \quad \Rightarrow \quad \int d^4x\, d^4\theta\, L$$

Grassmannian integration in superspace $\int d\theta_\alpha = 0,\ \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$

Matter fields

$$L=\int d^2\theta d^2\bar{\theta}\,\Phi_i^+\Phi_i+\int d^2\theta\,(\lambda_i\Phi_i+\tfrac{1}{2}m_{ij}\Phi_i\Phi_j+\tfrac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k)+h.c.]$$

Superpotential

Gauge fields

$$L=\tfrac{1}{4}\int d^2\theta\, W^\alpha W_\alpha+\int d^2\bar{\theta}\,\bar{W}^{\dot\alpha}\bar{W}_{\dot\alpha}=\tfrac{1}{2}D^2-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-i\lambda\sigma^\mu D_\mu\bar{\lambda}$$

Gauge transformation $\Phi\to e^{-ig\Lambda}\Phi,\ \Phi^+\to\Phi^+e^{ig\Lambda^+},\ V\to V+i(\Lambda-\Lambda^+)$

Gauge invariant interaction $\Phi^+\Phi\to\Phi^+e^{gV}\Phi$

Gauge Invariant SUSY Lagrangian

Superfields

$$L_{SUSY\ YM} = \tfrac{1}{4}\int d^2\theta\ \mathrm{Tr}(W^{\alpha}W_{\alpha}) + \tfrac{1}{4}\int d^2\bar\theta\ \mathrm{Tr}(\overline{W}^{\alpha}\overline{W}_{\alpha})$$
$$+\int d^2\theta d^2\bar\theta\ \overline{\Phi}_{ia}(e^{gV})^a_b\Phi^b_i + \int d^2\theta\ W(\Phi_i) + \int d^2\bar\theta\ \overline{W}(\overline{\Phi}_i)$$

Components

$$L_{SUSY\ YM} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^a\sigma^\mu D_\mu\bar\lambda^a + \tfrac{1}{2}D^aD^a$$
$$+(\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger(\partial_\mu A_i - igv^a_\mu T^a A_i) - i\bar\psi_i\bar\sigma^\mu(\partial_\mu\psi_i - igv^a_\mu T^a\psi_i)$$
$$-D^a g A_i^\dagger T^a A_i - i\sqrt{2}gA_i^\dagger T^a\lambda^a\psi_i + i\sqrt{2}g\bar\psi_i T^a\bar\lambda^a A_i + F_i^\dagger F_i$$
$$+\frac{\partial W}{\partial A_i}F_i + \frac{\partial \overline{W}}{\partial A_i^\dagger}F_i^\dagger - \tfrac{1}{2}\frac{\partial^2 W}{\partial A_i\partial A_j}\psi_i\psi_j - \tfrac{1}{2}\frac{\partial^2 \overline{W}}{\partial A_i^\dagger\partial A_j^\dagger}\bar\psi_i\bar\psi_j$$

Potential

$$D^a = -gA_i^\dagger T^a A_i,\quad F_i = -\frac{\partial W}{\partial A_i} \quad\rightarrow\quad V = \tfrac{1}{2}D^aD^a + F_i^\dagger F_i$$

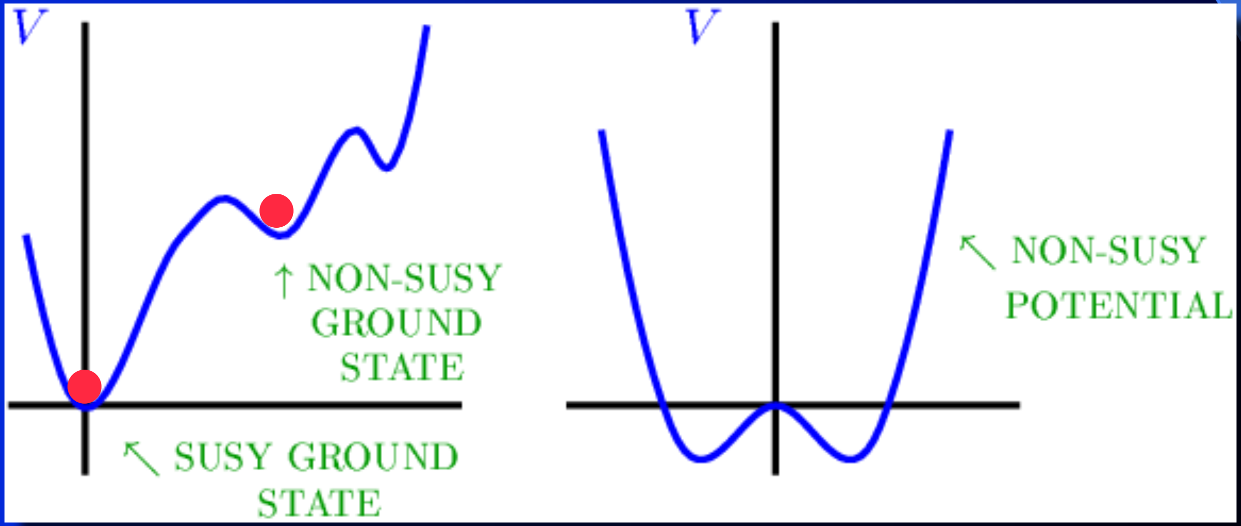
Spontaneous Breaking of SUSY

Energy $E = <0|H|0>$

$$\{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta} P_\mu$$

$$E = \frac{1}{4}\sum_{\alpha=1,2} <0|\{Q^i_\alpha, \overline{Q}^j_\alpha\}|0> = \frac{1}{4}\sum_\alpha |Q_\alpha|0>|^2 \geq 0$$

$E = <0|H|0> \neq 0$ if and only if $Q_\alpha|0> \neq 0$



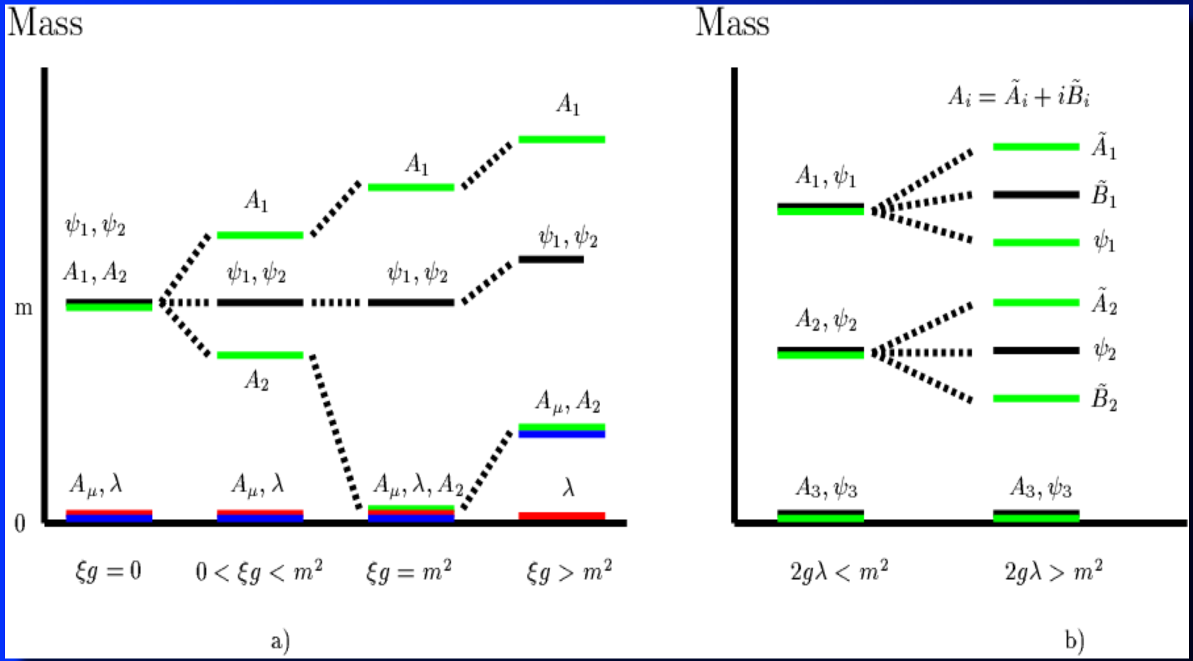
Mechanism of SUSY Breaking

- Fayet-Iliopoulos (D-term) mechanism (in Abelian theory)

$$\Delta L = \xi V|_{\theta\theta\bar{\theta}\bar{\theta}} = \xi \int d^4\theta \; V = \xi D \neq 0$$

- O'Raifertaigh (F-term) mechanism

$$W(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2$$



D-term F-term

$$F_1^* = mA_2 + 2gA_1A_2$$

$$F_2^* = mA_1$$

$$F_3^* = \lambda + gA_1^2$$

$$\Rightarrow \quad \langle F_i \rangle \neq 0$$

$$\sum_{bosons} m_i^2 = \sum_{fermions} m_i^2$$

Minimal Supersymmetric Standard Model (MSSM)

- SUSY: # of fermions = # of bosons   N=1 SUSY: (φ,ψ) (λ, A_μ)
- SM: 28 bosonic d.o.f. & 90 (96) fermionic d.o.f.

There are no particles in the SM that can be superpartners

SUSY associates known bosons with new fermions and known fermions with new bosons

- Even number of the Higgs doublets – min = 2

Cancellation of axial anomalies (in each generation)

$$Tr\ Y^3 = 3(\tfrac{1}{27}+\tfrac{1}{27}-\tfrac{64}{27}+\tfrac{8}{27})-1-1+8=0$$

$$\uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow$$

colour u_L d_L u_R d_R ν_L e_L e_R

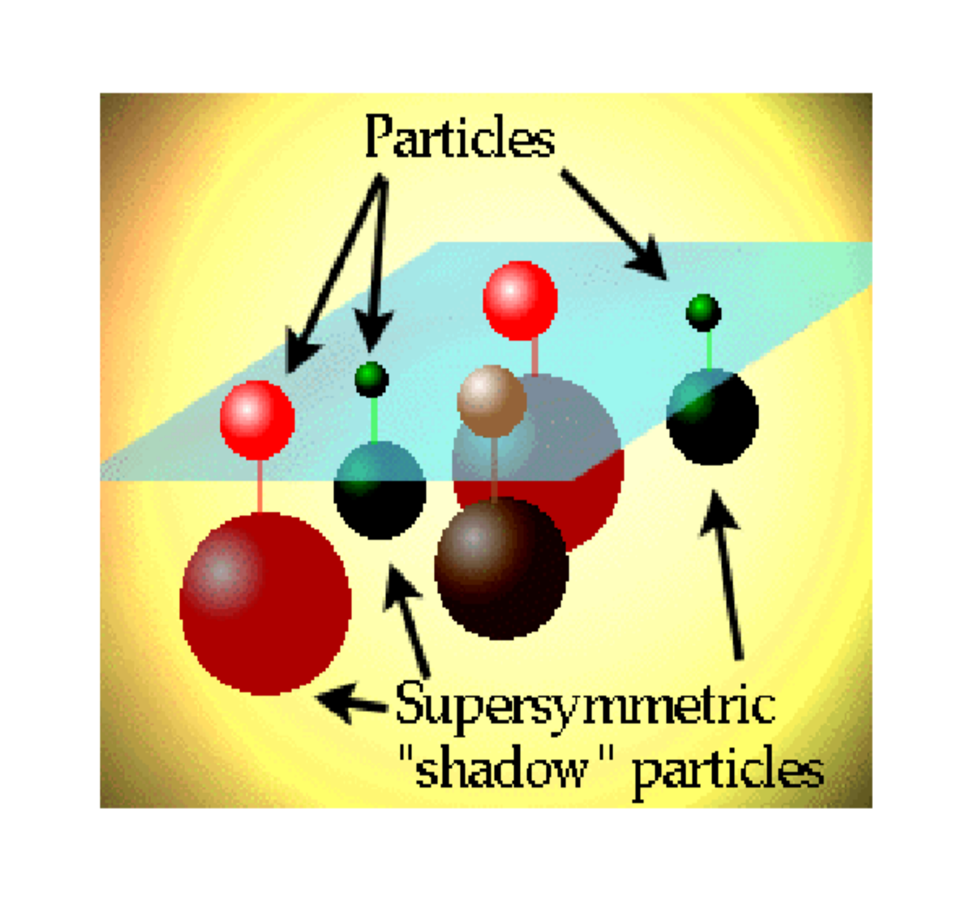
Higgsinos

-1+1=0

Particle Content of the MSSM

Superfield	*Bosons*	*Fermions*	$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge					
G^a	*gluon* g^a	*gluino* $\tilde{\mathrm{g}}^a$	8	1	0
V^k	*Weak* $\mathrm{W}^k(W^\pm, Z)$	*wino, zino* $\tilde{w}^k(\tilde{w}^\pm, \tilde{z})$	1	3	0
V'	*Hypercharge* $\mathrm{B}(\gamma)$	*bino* $\tilde{b}(\tilde{\gamma})$	1	1	0
Matter					
L_i	sleptons $\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons $L_i = (\nu, e)_L$	1	2	–1
E_i	sleptons $\tilde{E}_i = \tilde{e}_R$	leptons $E_i = e_R$	1	1	2
Q_i	squarks $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks $Q_i = (u, d)_L$	3	2	1/3
U_i	squarks $\tilde{U}_i = \tilde{u}_R$	quarks $U_i = u_R^c$	3^*	1	–4/3
D_i	squarks $\tilde{D}_i = \tilde{d}_R$	quarks $D_i = d_R^c$	3^*	1	2/3
Higgs					
H_1	Higgses H_1	higgsinos $\tilde{H}_1$	1	2	–1
H_2	Higgses H_2	higgsinos $\tilde{H}_2$	1	2	1

SUSY Shadow World



One half is observed!

One half is NOT observed!

The MSSM Lagrangian

$$L = L_{gauge} + L_{Yukawa} + L_{SoftBreaking}$$

The Yukawa Superpotential

superfields

$$W_R = y_U Q_L H_2 U_R + y_D Q_L H_1 D_R + y_L L_L H_1 E_R + \mu H_1 H_2$$

Yukawa couplings   Higgs mixing term

$$W_{NR} = \lambda_L L_L L_L E_R + \lambda_L^{'} L_L Q_L D_R + \mu^{'} L_L H_2 + \lambda_B U_R D_R D_R$$

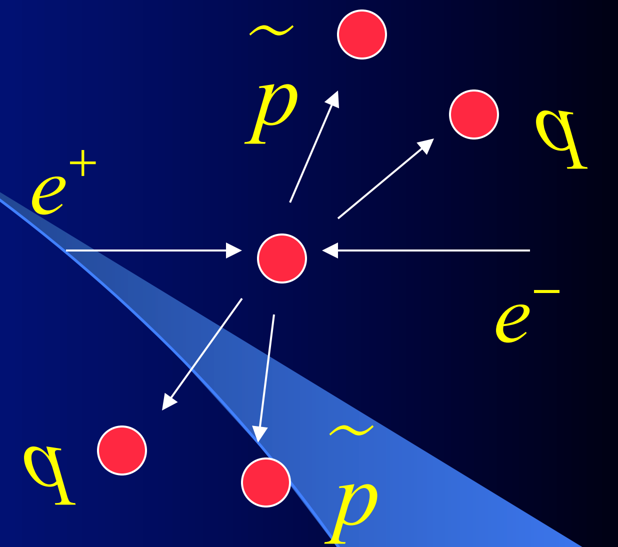
These terms are forbidden in the SM

R-parity $R = (-)^{3(B-L)+2S}$

The Usual Particle : R = + 1
SUSY Particle : R = - 1

B - Baryon Number
L - Lepton Number
S - Spin

R-parity Conservation

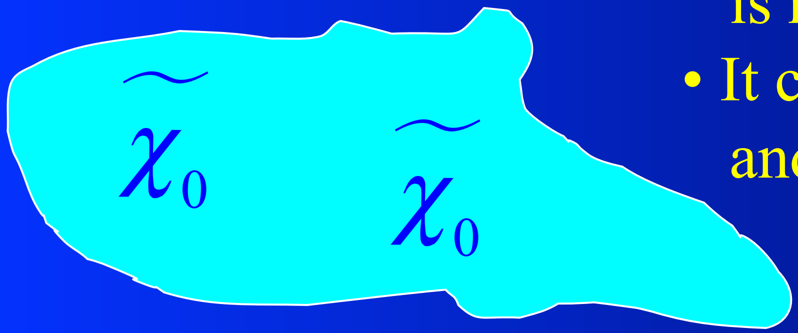


The consequences:

- The superpartners are created in pairs
- The lightest superparticle is stable

Physical output:

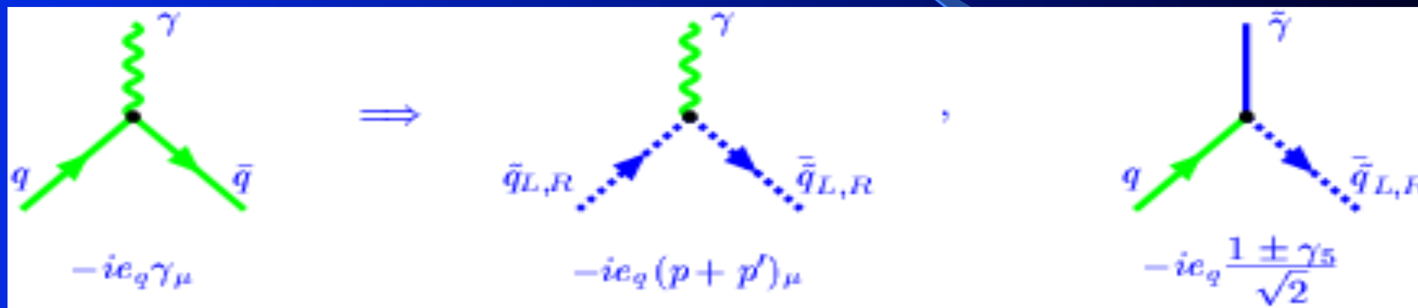
- The lightest superparticle (LSP) should be neutral - the best candidate is neutralino (photino or higgsino) $\tilde{\chi}_0$
- It can survive from the Big Bang and form the Dark matter in the Universe

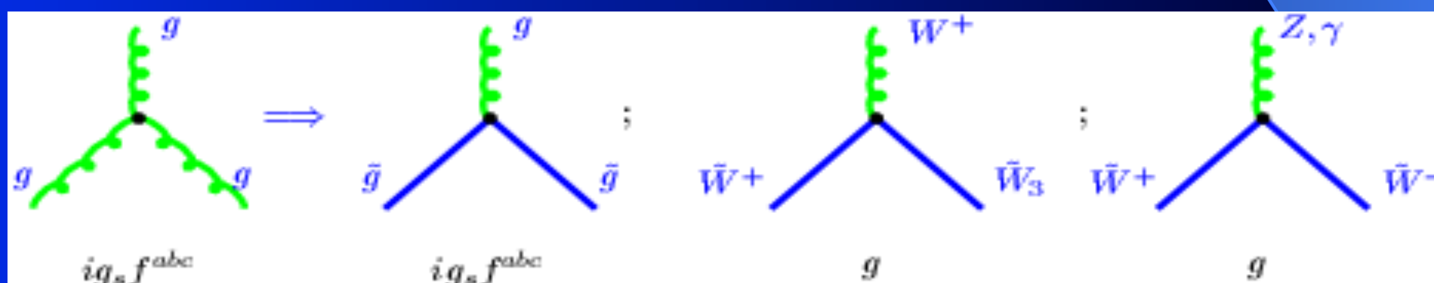


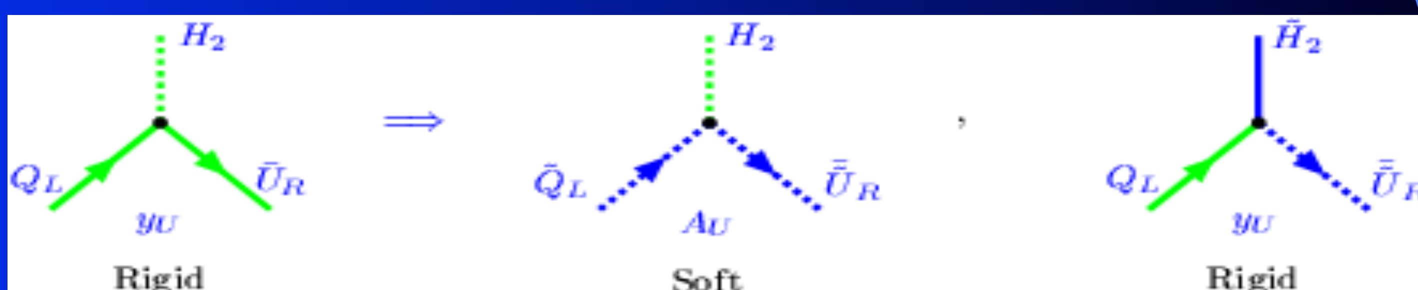
Interactions in the MSSM

SM → **MSSM**

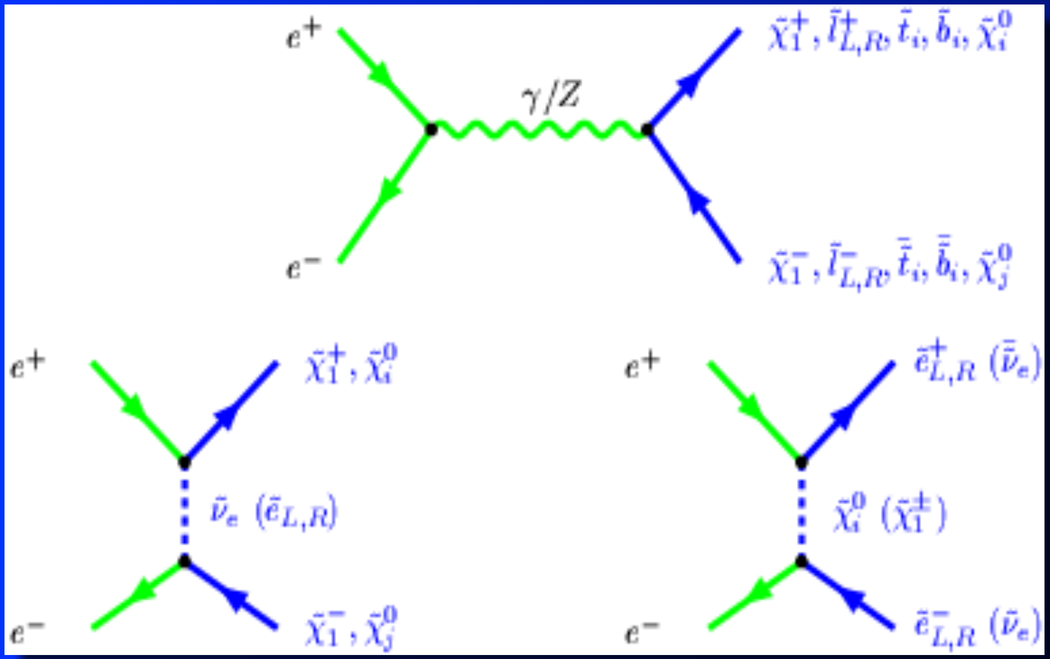
Vertices



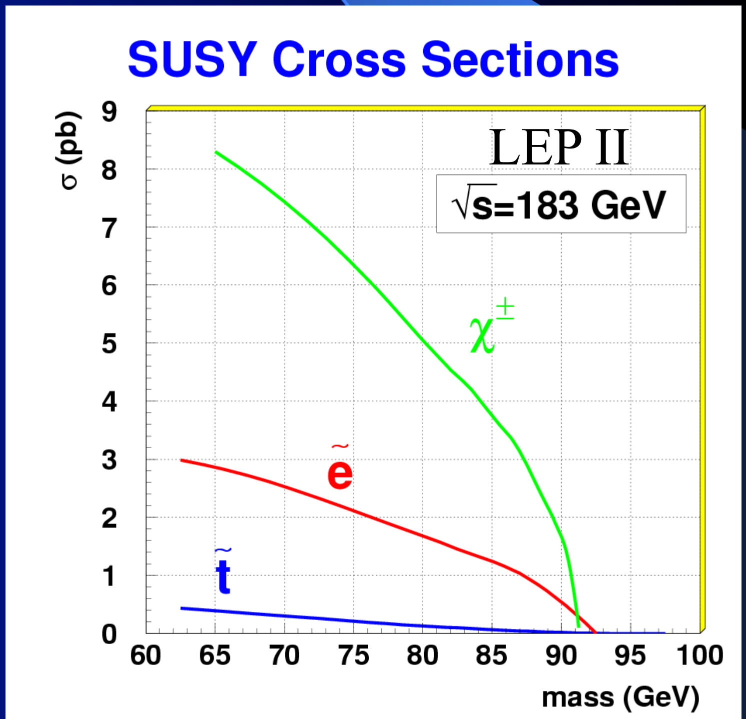




Creation of Superpartners at e^+e^- colliders



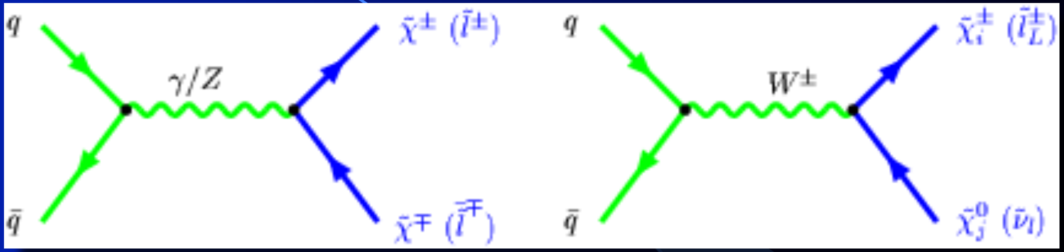
$$m_{\text{sparticle}}^{\text{max}} \leq \frac{\sqrt{s}}{2}$$



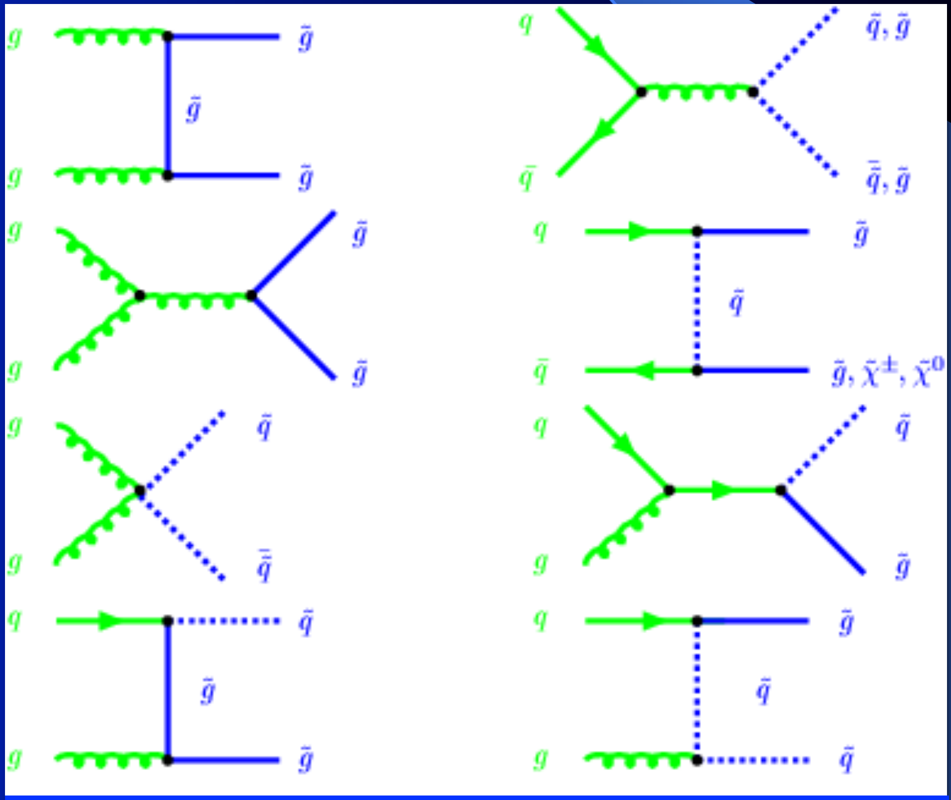
Experimental signature:
missing energy and transverse momentum

SUSY Production at Hadron Colliders

Annihilation channel



Gluon fusion, qq scattering and qg scattering channels



No new data so far due to insufficient luminosity at the Tevatron

Decay of Superpartners

squarks

$$\tilde{q}_{L,R} \rightarrow q + \tilde{\chi}_i^0$$

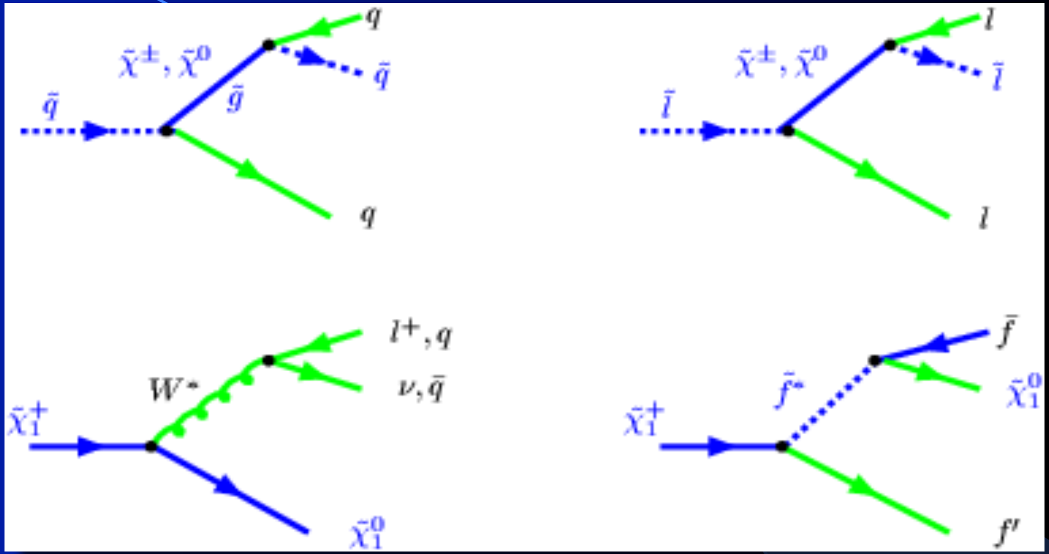
$$\tilde{q}_L \rightarrow q' + \tilde{\chi}_i^\pm$$

$$\tilde{q}_{L,R} \rightarrow q + \tilde{g}$$

sleptons

$$\tilde{l} \rightarrow l + \tilde{\chi}_i^0$$

$$\tilde{l}_L \rightarrow \nu_l + \tilde{\chi}_i^\pm$$



chargino

$$\tilde{\chi}_i^\pm \rightarrow e + \nu_e + \tilde{\chi}_i^0$$

$$\tilde{\chi}_i^\pm \rightarrow q + \bar{q}' + \tilde{\chi}_i^0$$

gluino

$$\tilde{g} \rightarrow q + \bar{q} + \tilde{\gamma}$$

$$\tilde{g} \rightarrow g + \tilde{\gamma}$$

neutralino

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + l^+ + l^-$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + q + \bar{q}'$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^\pm + l^\pm + \nu_l$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + \nu_l + \bar{\nu}_l$$

Final sates

$l^+ l^- + \not{E}_T$

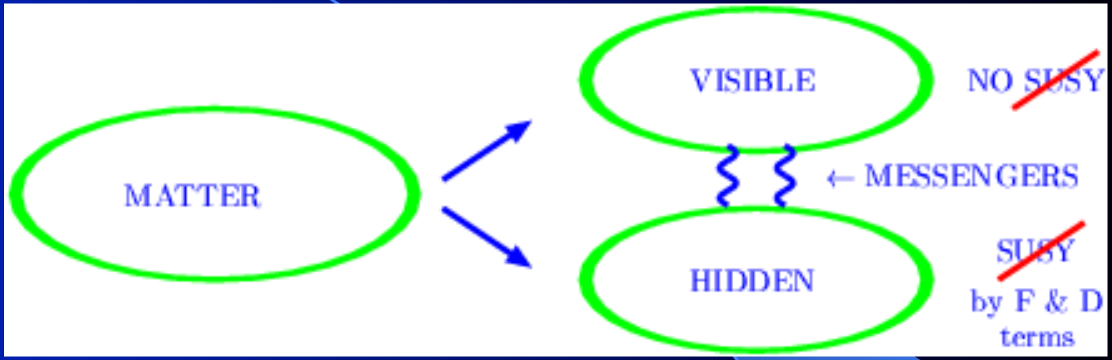
2 jets $+ \not{E}_T$

$\gamma + \not{E}_T$

$\not{E}_T$

Soft SUSY Breaking

Hidden sector scenario:



four scenarios:

1. Gravity mediation
2. Gauge mediation
3. Anomaly mediation
4. Gaugino mediation

<u>SUGRA</u> S-dilaton, T-moduli $< F_T > \neq 0, < F_S > \neq 0$

$$M_{\not{SUSY}} \sim \frac{< F_T >}{M_{PL}} + \frac{< F_S >}{M_{PL}} \sim m_{3/2} \longleftarrow$$ gravitino mass $:$ 1 TeV

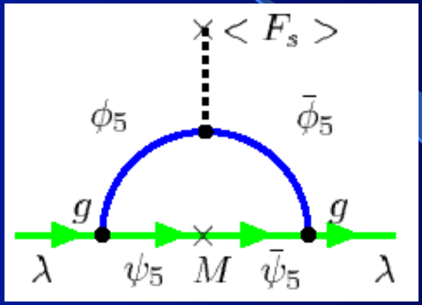
$$L_{soft} = -\sum_i m_i^2 \,| A_i |^2 - \sum_i M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i) - \mathrm{B} W^{(2)}(A) - \mathrm{A} W^{(3)}(A)$$

$$m_i^2 \sim \mathrm{B} \sim m_{3/2}^2,\; M_i \sim \mathrm{A} \sim m_{3/2}$$

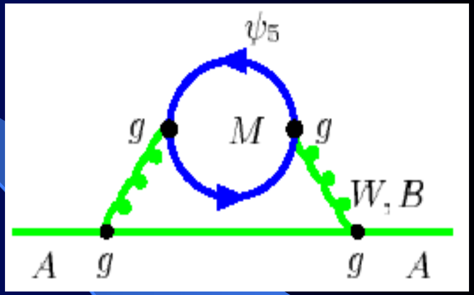
Soft SUSY Breaking Cont'd

Gauge mediation Scalar singlet S $< S >= M \quad < F_S > \neq 0$

Messenger Φ $W: \; S\Phi^{+}\Phi$



gaugino



squark

$$m_{\tilde{G}} \sim \frac{< F_S >}{M_{PL}} \frac{M}{M_{PL}} \sim 10^{-14} \frac{M}{[GeV]}$$

gravitino mass

- LSP=gravitino

$$M_i \sim c_i N \frac{\alpha_i}{4\pi} \frac{< F_S >}{M} \qquad m_i^2 \sim \left(\frac{< F_S >}{M_{PL}} \right)^2 N \left(\frac{\alpha_i}{4\pi} \right)^2$$

Anomaly mediation Results from conformal anomaly = β function

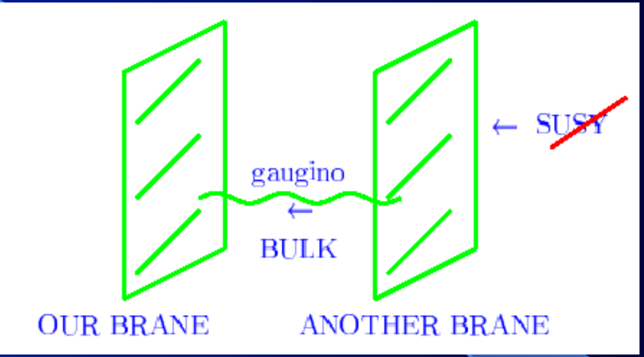
$$M_i \sim b_i \frac{\alpha_i(\Lambda)}{4\pi} \frac{< F_{T,S} >}{M_{PL}} \sim b_i \alpha_i m_{3/2}$$

$$m_i^2 \sim b_i^2 \alpha_i^2 m_{3/2}^2$$

- $M_1 : M_2 : M_3 = b_1 : b_2 : b_3$
- LSP=slepton

Soft SUSY Breaking Cont'd

<u>Gaugino mediation</u>



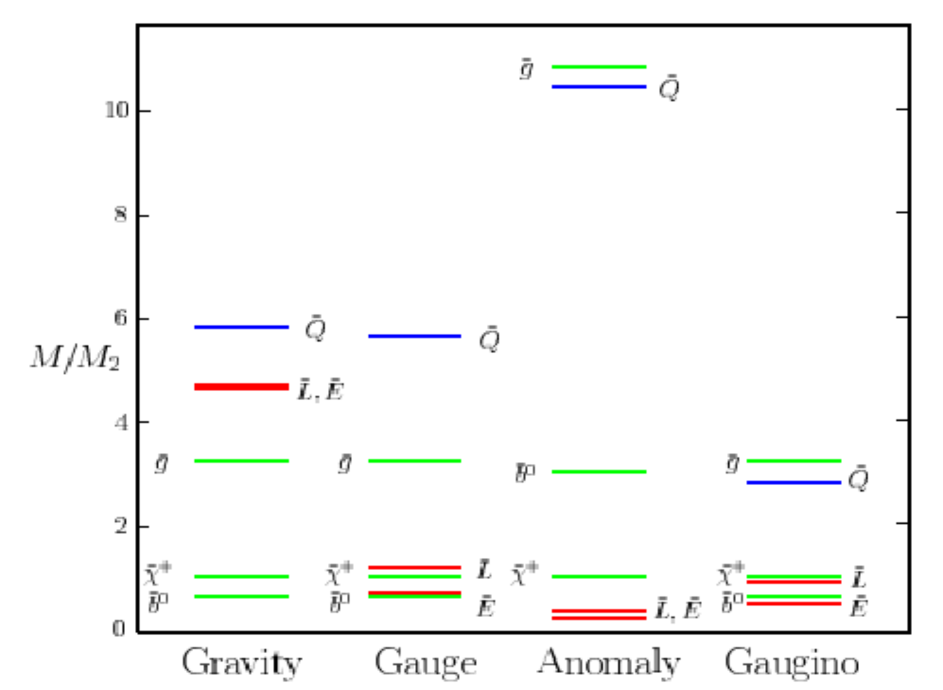
All scenarios produce soft SUSY breaking terms

Soft = operators of dimension ≤ 4

<u>Net result of SUSY breaking</u>

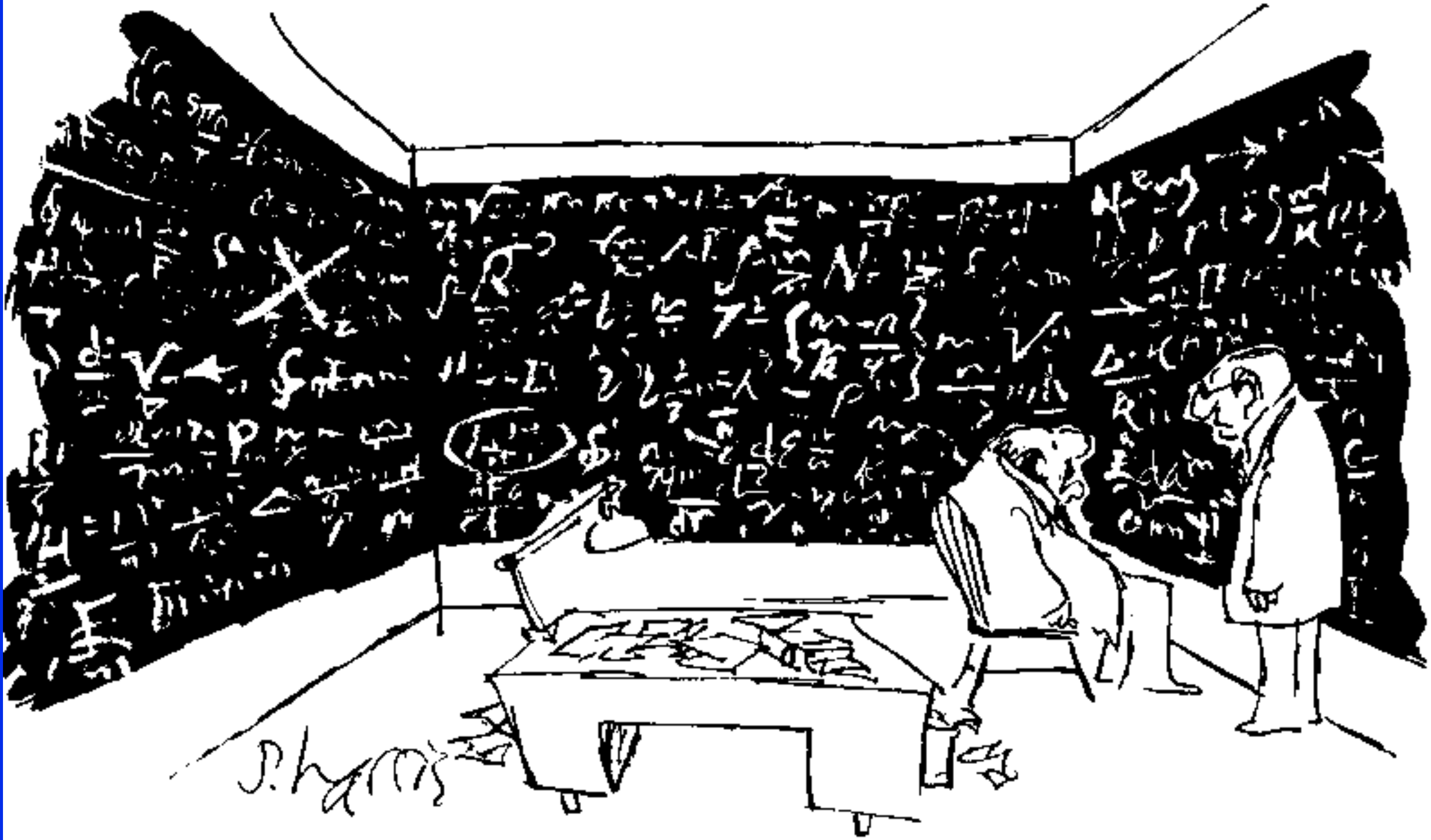
$$-L_{Soft} = \sum_i m_{0i}^2 |A_i|^2 + \sum_\alpha M_i \widetilde{\lambda}_i \widetilde{\lambda}_i + \sum_{ijk} \mathrm{A}_{ijk} A_i A_j A_k + \sum_{ij} \mathrm{B}_{ij} A_i A_j$$

scalar fileds gauginos



SUSY spectra for various mediation mechanisms

We like elegant solutions



"Whatever happened to *elegant* solutions?"

Parameter Space of the MSSM

- Three gauge coupligs α_i, i=1,2,3
- Three (four) Yukawa matrices $y_{ab}^k,\ k = U, D, L, (E)$
- The Higgs mixing parameter μ
- Soft SUSY breaking terms

SUGRA   Universality hypothesis: soft terms are universal and repeat the Yukawa potential

$$-L_{Soft} = A\{y_t Q_L H_2 U_R + y_b Q_L H_1 D_R + y_L L_L H_1 E_R\} + B\mu H_1 H_2 + m_0^2 \sum_i |\varphi_i|^2 + \tfrac{1}{2} M_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha$$

Five universal soft parameters: $A,\ m_0,\ M_{1/2},\ B \leftrightarrow \tan\beta$ and μ

versus m and λ in the SM

Mass Spectrum

$$L_{gaugino-Higgsino} = -\tfrac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \tfrac{1}{2}\bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.)$$

Chargi

$$\psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} \qquad M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} \Rightarrow \begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix}$$

Neutralino

$$\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$$

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} \quad M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin W & M_Z \sin\beta \sin W \\ 0 & M_2 & M_Z \cos\beta \cos W & -M_Z \sin\beta \cos W \\ -M_Z \cos\beta \sin W & M_Z \cos\beta \cos W & 0 & -\mu \\ M_Z \sin\beta \sin W & -M_Z \sin\beta \cos W & -\mu & 0 \end{pmatrix}$$

Squarks & Sleptons

Mass Spectrum

$$\tilde{m}_t^2 = \begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & \tilde{m}_{tR}^2 \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$$\tilde{m}_b^2 = \begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \tilde{m}_{bR}^2 \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

$$\tilde{m}_{tL}^2 = \tilde{m}_Q^2 + m_t^2 + \tfrac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{tR}^2 = \tilde{m}_U^2 + m_t^2 - \tfrac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{bL}^2 = \tilde{m}_Q^2 + m_b^2 - \tfrac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{bR}^2 = \tilde{m}_D^2 + m_b^2 + \tfrac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{\tau L}^2 = \tilde{m}_L^2 + m_\tau^2 - \tfrac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{\tau R}^2 = \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta.$$

$$\tilde{m}_\tau^2 = \begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & \tilde{m}_{\tau R}^2 \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix}$$

SUSY Higgs Bosons

SM

4=2+2=3+1

$$H=\begin{pmatrix} H^0 \\ H^- \end{pmatrix}=\begin{pmatrix} v+\frac{S+iP}{\sqrt{2}} \\ H^- \end{pmatrix}=\exp\left(i\frac{\vec{\xi}\vec{\sigma}}{2}\right)\begin{pmatrix} v+\frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$H \rightarrow H'=\exp\left(i\frac{\vec{\alpha}\vec{\sigma}}{2}\right)H \xrightarrow{(\vec{\alpha}=-\vec{\xi})} H'=\begin{pmatrix} v+\frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

MSSM

8=4+4=3+5

$$H_1=\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}=\begin{pmatrix} v_1+\frac{S_1+iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix},\quad H_2=\begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}=\begin{pmatrix} H_2^+ \\ v_2+\frac{S_2+iP_2}{\sqrt{2}} \end{pmatrix},$$

$v_1^2+v_2^2=v^2$, $v_2/v_1 \equiv \tan\beta$

$G^0=-\cos\beta P_1+\sin\beta P_2$	*Goldstone boson* $\rightarrow Z_0$
$A=\sin\beta P_1+\cos\beta P_2$	*Neutral* $CP=-1$ *Higgs*
$G^+=-\cos\beta(H_1^-)^*+\sin\beta H_2^+$	*Goldstone boson* $\rightarrow W^+$
$H^+=\sin\beta(H_1^-)^*+\cos\beta H_2^+$	*Charged Higgs*
$h=-\sin\alpha S_1+\cos\alpha S_2$	SM *Higgs boson* $CP=1$
$H=\cos\alpha S_1+\sin\alpha S_2$	*Extra heavy Higgs boson*

$$\tan 2\alpha=\tan 2\beta\frac{m_A^2+m_Z^2}{m_A^2-m_Z^2}$$

The Higgs Potential

$$V_{tree}(H_1,H_2)=m_1^2\,|H_1|^2+m_2^2\,|H_2|^2-m_3^2(H_1H_2+h.c.)$$
$$+\frac{g^2+g'^2}{8}(|H_1|^2-|H_2|^2)^2+\frac{g^2}{2}|H_1^+H_2|^2$$

At the GUT scale: $m_1^2=m_2^2=\mu_0^2+m_0^2,\ m_3^2=-B\mu_0$

Minimization

$$\frac{1}{2}\frac{\delta V}{\delta H_1}=m_1^2v_1-m_3^2v_2+\frac{g^2+g'^2}{4}(v_1^2-v_2^2)v_1=0,$$
$$\frac{1}{2}\frac{\delta V}{\delta H_2}=m_2^2v_2-m_3^2v_1-\frac{g^2+g'^2}{4}(v_1^2-v_2^2)v_2=0.$$
$$<H_1>\equiv v_1=v\cos\beta,\quad <H_2>\equiv v_2=v\sin\beta,$$

Solution

$$v^2=\frac{4(m_1^2-m_2^2\tan^2\beta)}{(g^2+g'^2)(\tan^2\beta-1)},$$
$$\sin 2\beta=\frac{2m_3^2}{m_1^2+m_2^2}$$

At the GUT scale

$$v^2=-\frac{4}{g^2+g'^2}m^2<0$$

No SSB in SUSY theory !

Renormalization Group Eqns

$$\tilde{\alpha}_i \equiv \frac{g_i^2}{16\pi^2} = \frac{\alpha_i}{4\pi}, \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad t = \log(M_{GUT}^2 / Q^2)$$

$$i = 1,2,3 \qquad k = U, D, L$$

The couplings

$$\dot{\tilde{\alpha}}_i = -b_i \tilde{\alpha}_i^2, \qquad b_i^{MSSM} = (\tfrac{33}{5}, 1, -3)$$

$$\dot{Y}_U = Y_U(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{13}{15}\tilde{\alpha}_1 - 6Y_U - Y_D),$$

$$\dot{Y}_D = Y_D(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{7}{15}\tilde{\alpha}_1 - Y_U - 6Y_D - Y_L),$$

$$\dot{Y}_L = Y_L(3\tilde{\alpha}_2 + \tfrac{9}{5}\tilde{\alpha}_1 - 3Y_D - 4Y_L),$$

$$\dot{M}_i = b_i \tilde{\alpha}_i M_i,$$

$$\dot{A}_U = -(\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{13}{15}\tilde{\alpha}_1 M_1) - 6Y_U A_U - Y_D A_D,$$

$$\dot{A}_D = -(\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{7}{15}\tilde{\alpha}_1 M_1) - Y_U A_U - 6Y_D A_D - Y_L A_L,$$

$$\dot{A}_L = -(3\tilde{\alpha}_2 M_2 + \tfrac{9}{5}\tilde{\alpha}_1 M_1) - 3Y_D A_D - 4Y_L A_L,$$

$$\dot{B} = -3(\tilde{\alpha}_2 M_2 + \tfrac{1}{5}\tilde{\alpha}_1 M_1) - 3Y_U A_U - 3Y_D A_D - Y_L A_L,$$

$$\dot{\mu} = -\mu^2(3\tilde{\alpha}_2 + \tfrac{3}{5}\tilde{\alpha}_1 - 3Y_U - 3Y_D - Y_L)$$

Soft Terms

RG Eqns for the Soft Masses

$$\dot{\tilde{m}}_Q^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \tfrac{1}{15}\tilde{\alpha}_1 M_1^2 - Y_t(\Sigma_t + A_t^2) - Y_b(\Sigma_b + A_b^2)]$$

$$\dot{\tilde{m}}_U^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + \tfrac{16}{15}\tilde{\alpha}_1 M_1^2 - 2Y_t(\Sigma_t + A_t^2)]$$

$$\dot{\tilde{m}}_D^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + \tfrac{4}{15}\tilde{\alpha}_1 M_1^2 - 2Y_b(\Sigma_b + A_b^2)]$$

$$\dot{\tilde{m}}_L^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - Y_\tau(\Sigma_\tau + A_\tau^2)]$$

$$\dot{\tilde{m}}_E^2 = -[\tfrac{12}{5}\tilde{\alpha}_1 M_1^2 - 2Y_\tau(\Sigma_\tau + A_\tau^2)]$$

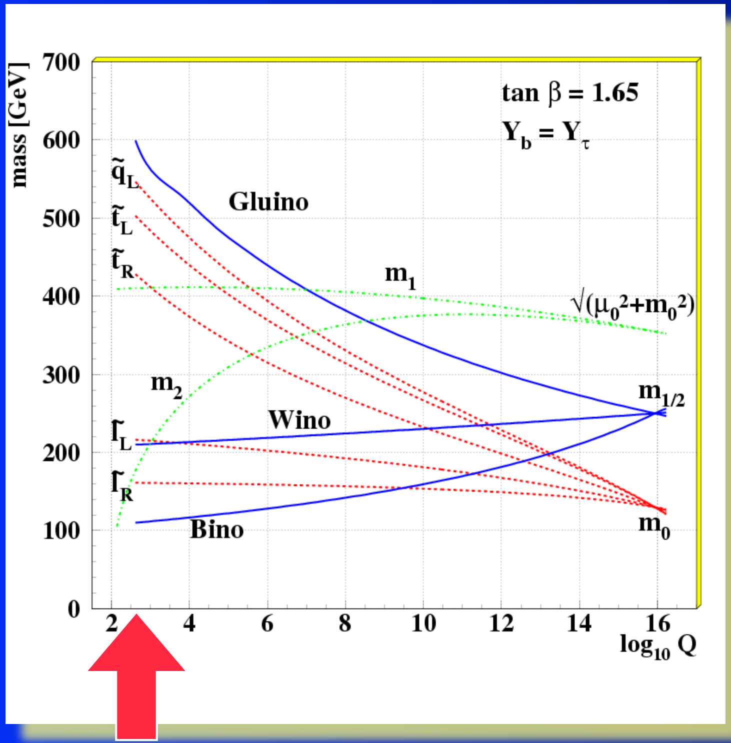
$$\dot{\tilde{m}}_{H_1}^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_b(\Sigma_b + A_b^2) - Y_\tau(\Sigma_\tau + A_\tau^2)]$$

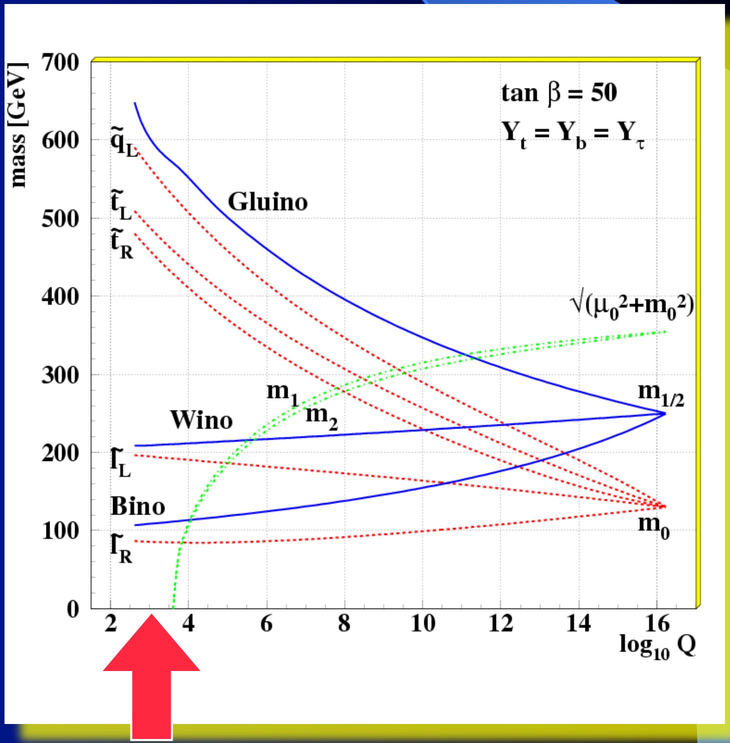
$$\dot{\tilde{m}}_{H_2}^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_t(\Sigma_t + A_t^2)]$$

$$\Sigma_t = m_Q^2 + m_U^2 + m_{H_2}^2,\ \Sigma_b = m_Q^2 + m_D^2 + m_{H_1}^2,\ \Sigma_\tau = m_L^2 + m_E^2 + m_{H_1}^2$$

Radiative EW Symmetry Breaking

Due to RG controlled running of the mass terms from the Higgs potential they may change sign and trigger the appearance of non-trivial minimum leading to spontaneous breaking of EW symmetry - this is called Radiative EWSB





The Higgs Bosons Masses

CP-odd neutral Higgs A $m_A^2 = m_1^2 + m_2^2$ $M_W^2 = \frac{g^2}{2} v^2$

CP-even charged Higgses $H_\pm$ $m_{H^\pm}^2 = m_A^2 + M_W^2$ $M_Z^2 = \frac{g^2 + g'^2}{2} v^2$

CP-even neutral Higgses h,H

$$m_{h,H}^2 = \frac{1}{2}[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}]$$

$$m_h \approx M_Z |\cos 2\beta| < M_Z \ !$$

⇒ Radiative corrections

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4} + 2\ loops$$

Constrained MSSM

Requirements:

- Unification of the gauge couplings
- Radiative EW Symmetry Breaking
- Heavy quark and lepton masses
- Rare decays (b -> sγ)
- Anomalous magnetic moment of muon
- LSP is neutral
- Amount of the Dark Matter
- Experimental limits from direct search

Allowed region in the parameter space of the MSSM

$$A_0, m_0, M_{1/2}, \mu, \tan\beta$$

Parameter space:

$$100\ Gev < m_0, M_{1/2}, \mu < 2\ Tev$$
$$-3m_0 < A_0 < 3m_0,\ 1 < \tan\beta < 70$$

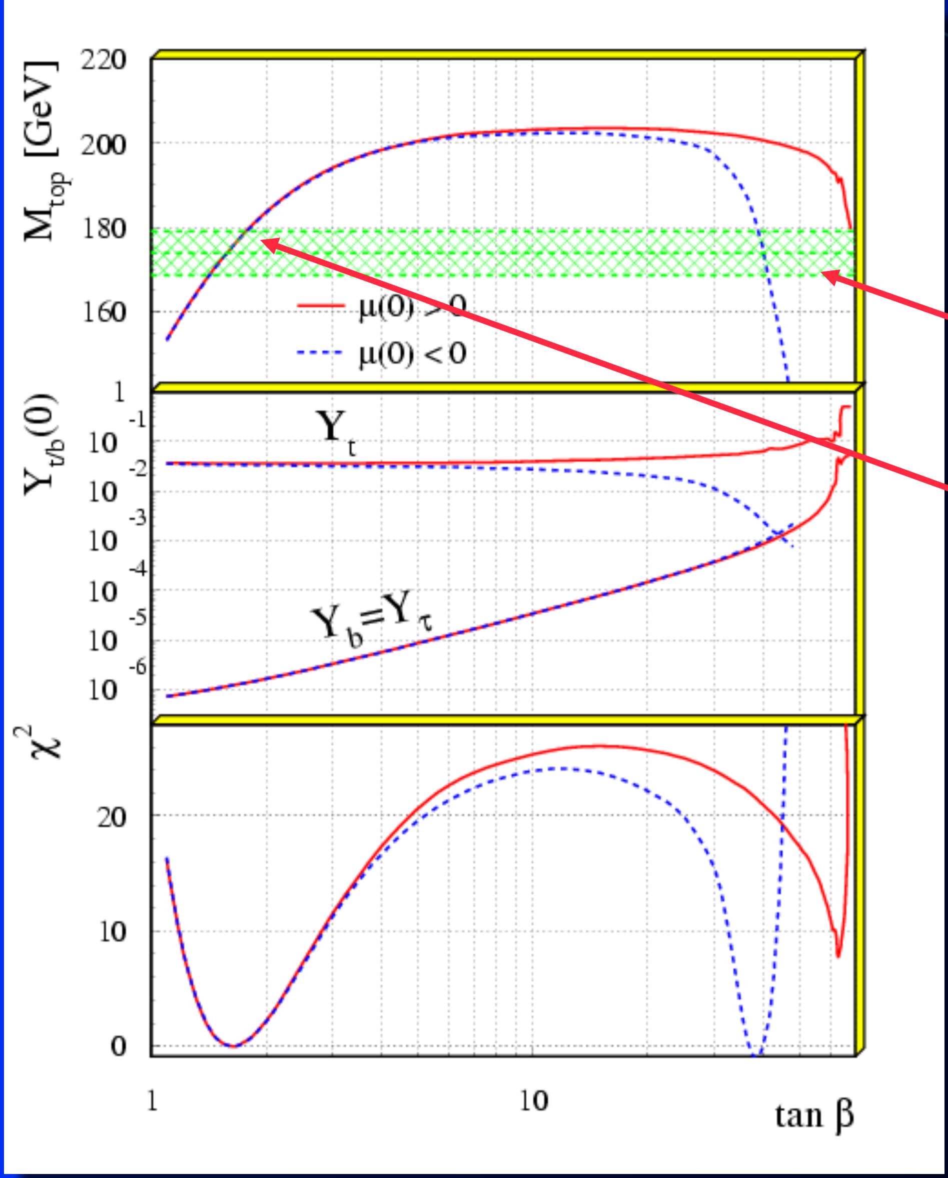
SUSY Fits

$$\chi^2 = \sum_{i=1}^{3} \frac{(\alpha_i^{-1}(M_Z) - \alpha_{MSSMi}^{-1}(M_Z))^2}{\sigma_i^2}$$
$$+ \frac{(M_Z - 91.18)^2}{\sigma_Z^2} + \frac{(M_t - 174)^2}{\sigma_t^2}$$
$$+ \frac{(M_b - 4.94)^2}{\sigma_b^2} + \frac{(M_\tau - 1.7771)^2}{\sigma_\tau^2}$$
$$+ \frac{(\mathrm{Br}(b \to s\gamma) - 3.14\times 10^{-4})^2}{\sigma^2(b \to s\gamma)}$$
$$+ \frac{(\Omega h^2 - 1)^2}{\sigma_\Omega^2} \quad (\text{for } \Omega h^2 > 1)$$
$$+ \frac{(\tilde{M} - \tilde{M}_{exp})^2}{\sigma_{\tilde{M}}^2} \quad (\text{for } \tilde{M} < \tilde{M}_{exp})$$
$$+ \frac{(\tilde{m}_{LSP} - \tilde{m}_\chi)^2}{\sigma_{LSP}^2} \quad (\text{for } \tilde{m}_{LSP} \text{ charged})$$

Minimize χ^2

Exp.input data	Fit low tanβ	Parameters high tanβ
$\alpha_1, \alpha_2, \alpha_3$	M_{GUT}, α_{GUT}	M_{GUT}, α_{GUT}
m_t	$Y_t^0, Y_b^0 = Y_\tau^0$	$Y_t^0 = Y_b^0 = Y_\tau^0$
m_b	$m_0, m_{1/2}$	$m_0, m_{1/2}$
m_τ	$\tan\beta$	$\tan\beta$
M_Z	μ	μ
$b \to s\gamma$	(A_0)	A_0
$\tau_{Universe}$		

Low and High tanβ Solutions



Requirements:

- EWSB
- bτ unification

Low tanβ solution

High tanβ solution

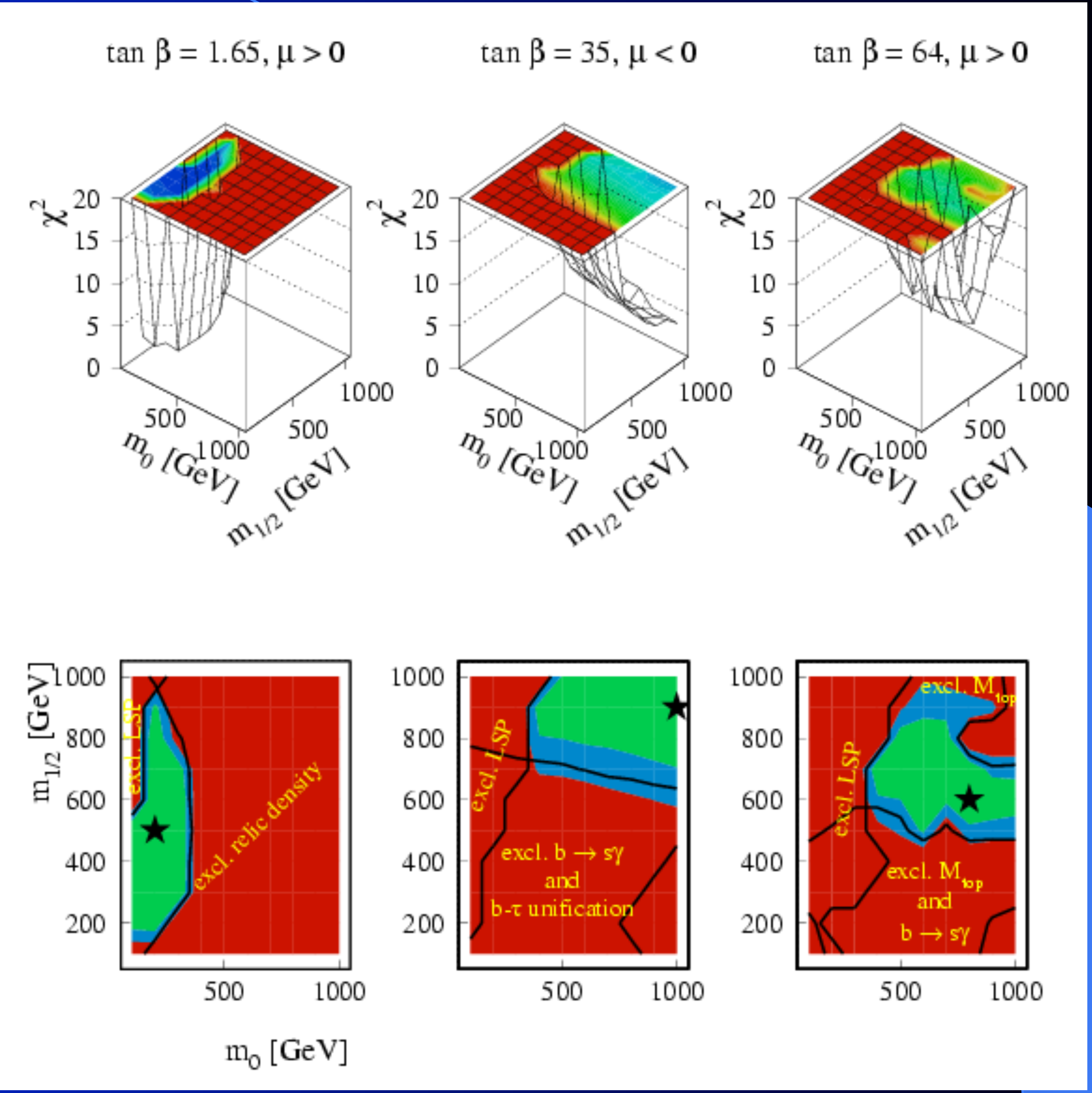
- bτ unification is the consequence of GUT
- Non working for the light generations

Allowed Regions in Parameter Space

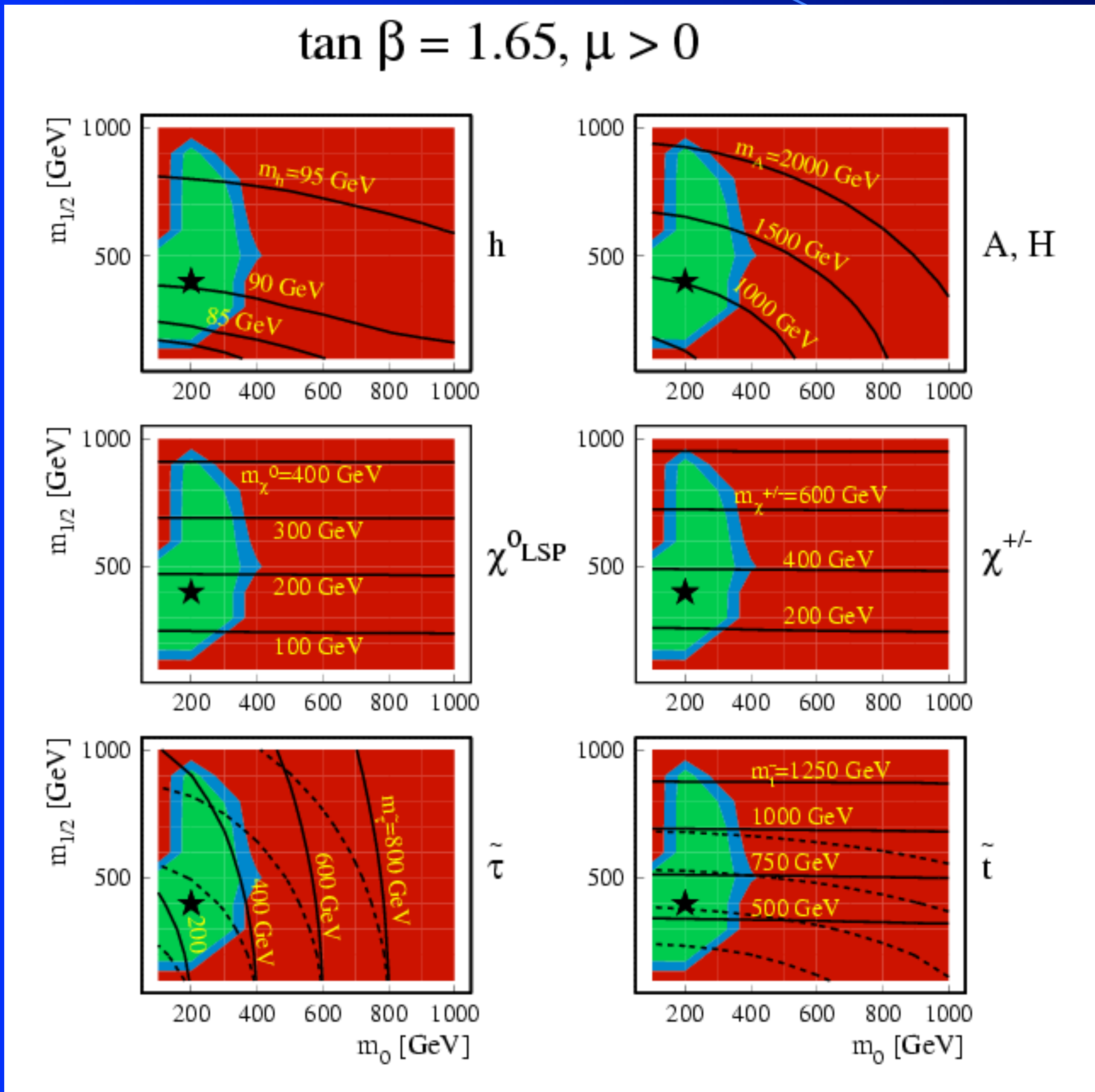
All the requirements are fulfilled simultaneously !

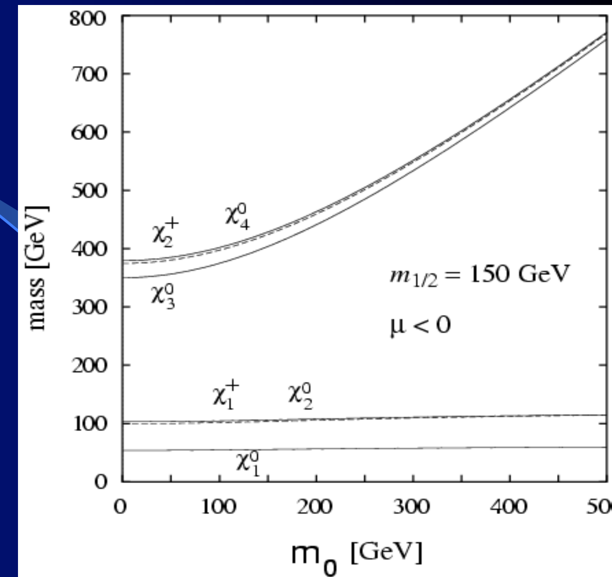
- μ is defined from the EWSB
- $A_0 = 0$

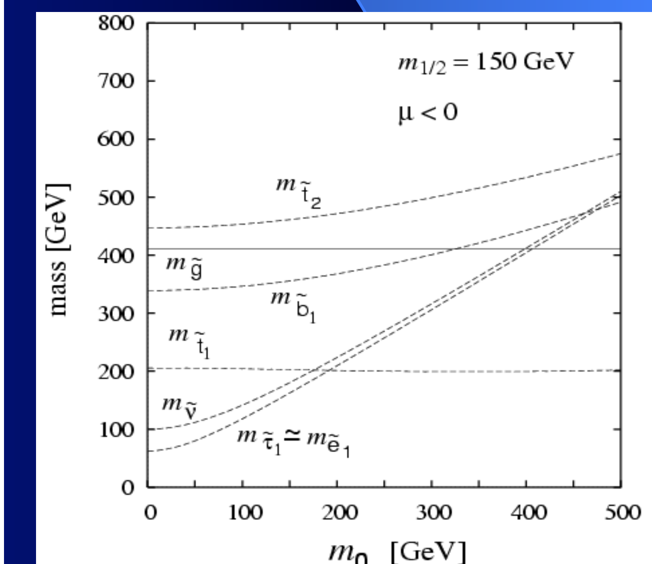
- is the best fit value



Masses of Superpartners







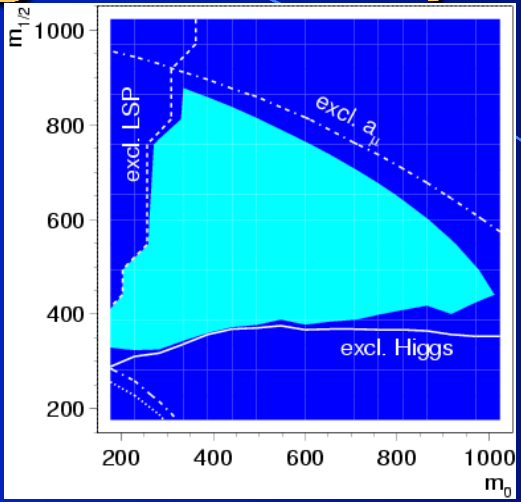
Allowed regions of parameter space

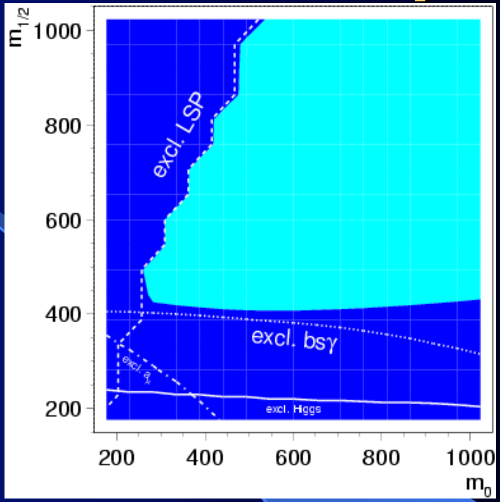
- $\tan\beta > 4$

From the Higgs searches

- $\mu > 0$

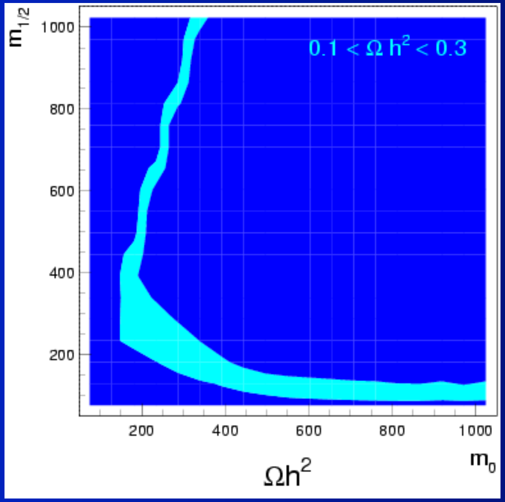
From a_μ measurement

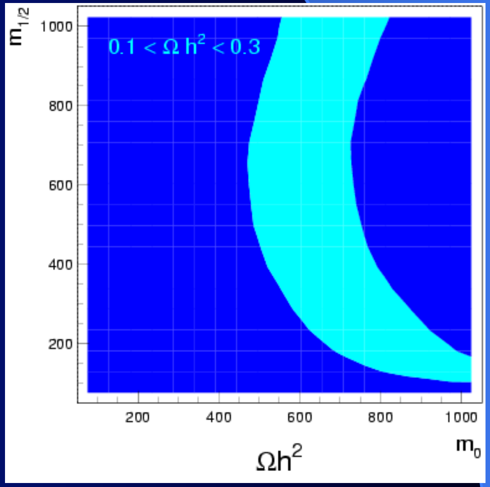




Fit to all constraints

In allowed region one fulfills all the constraints simultaneously and has the suitable amount of the dark matter





$\tan\beta = 35$ Fit to Dark Matter constraint $\tan\beta = 50$

Mass Spectrum in CMSSM

Fitted SUSY Parameters

Symbol	Low tan β	High tan β
Tan β	1.71	35.0
m_0	200	600
$m_{1/2}$	500	400
μ(0)	1084	-558
A(0)	0	0
$1/\alpha_{GUT}$	24.8	24.8
M_{GUT}	$1.6 \cdot 10^{16}$	$1.6 \cdot 10^{16}$

SUSY Masses in GeV

Symbol	Low tan β	High tan β
$\tilde{\chi}_1^0(\tilde{B}), \tilde{\chi}_2^0(\tilde{W}^3)$	214, 413	170, 322
$\tilde{\chi}_3^0(\tilde{H}_1), \tilde{\chi}_4^0(\tilde{H}_2)$	1028, 1016	481, 498
$\tilde{\chi}_1^\pm(\tilde{W}^\pm), \tilde{\chi}_2^\pm(\tilde{H}^\pm)$	413, 1026	322, 499
$\tilde{g}$	1155	950
$\tilde{e}_L, \tilde{e}_R$	303, 270	663, 621
$\tilde{\nu}_L$	290	658
$\tilde{q}_L, \tilde{q}_R$	1028, 936	1040, 1010
$\tilde{\tau}_1, \tilde{\tau}_2$	279, 403	537, 634
$\tilde{b}_1, \tilde{b}_2$	953, 1010	835, 915
$\tilde{t}_1, \tilde{t}_2$	727, 1017	735, 906
h, H	95, 1344	119, 565
A, $H^\pm$	1340, 1344	565, 571

The Lightest Superparticle

		property	signature
• Gravity mediation	$\mathrm{LSP} = \tilde{\chi}_1^0$	stable	jets/leptons $+ \not\!\!E_T$
• Gauge mediation	$\mathrm{LSP} = \tilde{G}$	stable	$\not\!\!E_T$
	$\mathrm{NLSP} = \tilde{\chi}_1^0$	$\tilde{\chi}_1^0 \to \gamma\tilde{G}, h\tilde{G}, Z\tilde{G}$	photons/jets $+ \not\!\!E_T$
	$\mathrm{NLSP} = \tilde{l}_R$	$\tilde{l}_R \to \tau\tilde{G}$	lepton $+ \not\!\!E_T$
• Anomaly mediation	$\mathrm{LSP} = \tilde{\chi}_1^0$	stable	lepton $+ \not\!\!E_T$
	$\mathrm{LSP} = \tilde{\nu}_L$	stable	

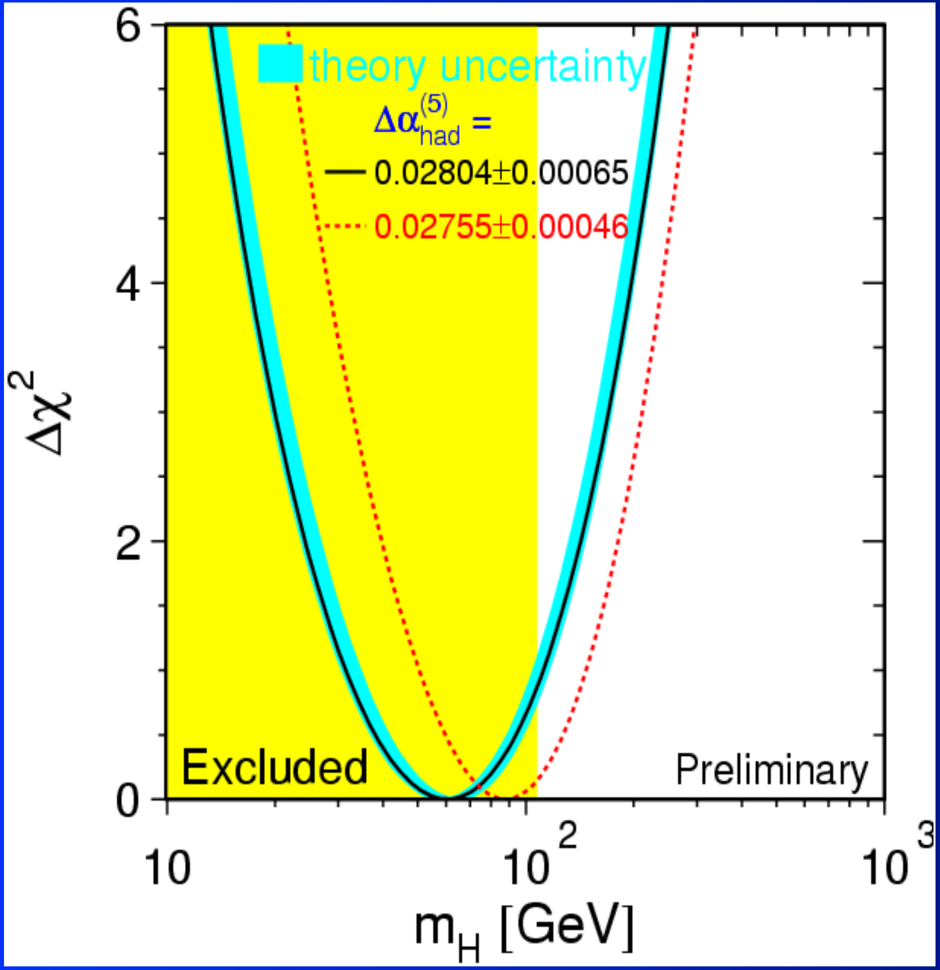
• R-parity violation LSP is unstable ➔ SM particles

Rare decays
Neutrinoless double β decay

• Modern limit

$$M_{LSP} \geq 40\ \mathrm{GeV}$$

The Higgs Mass Limit

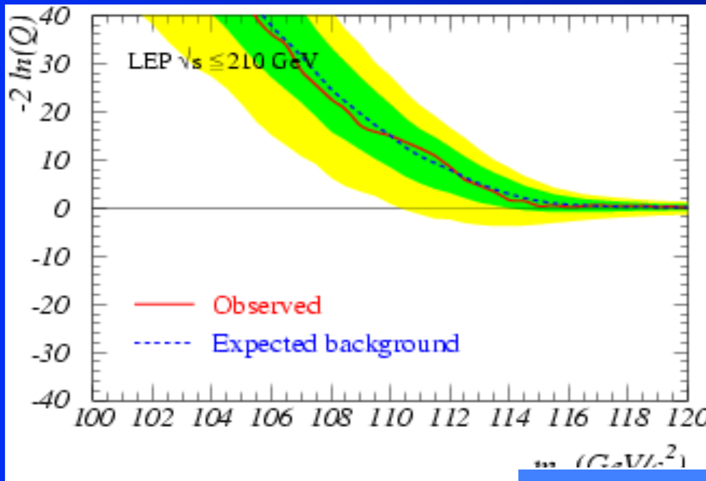


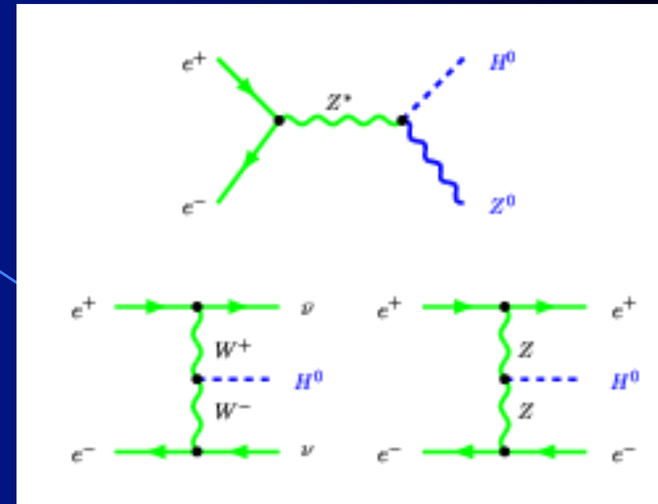
- Indirect limit from radiative corrections
- Direct limit from Higgs non-observation at LEP II (CERN)

$113 < m_H < 200$ GeV

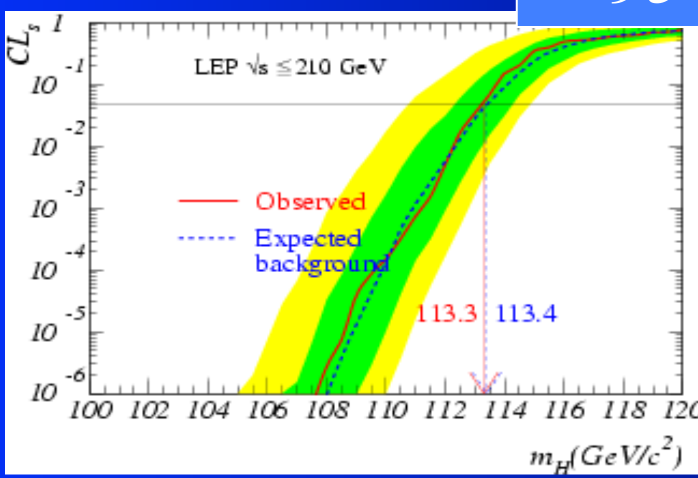
At 95 % C.L.

Higgs Searches

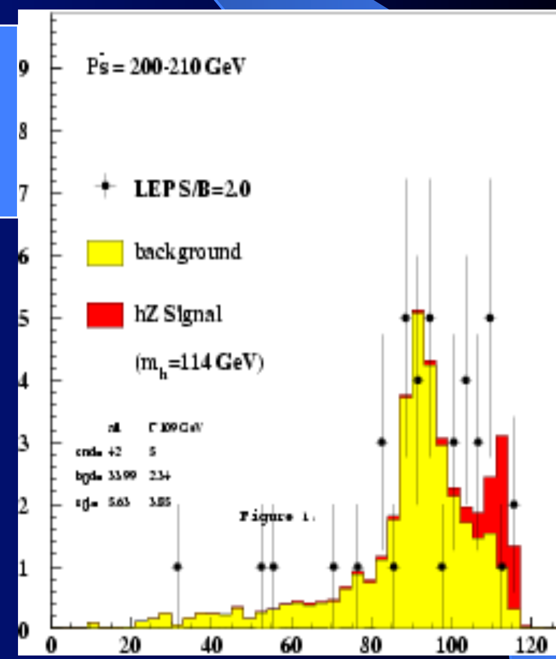




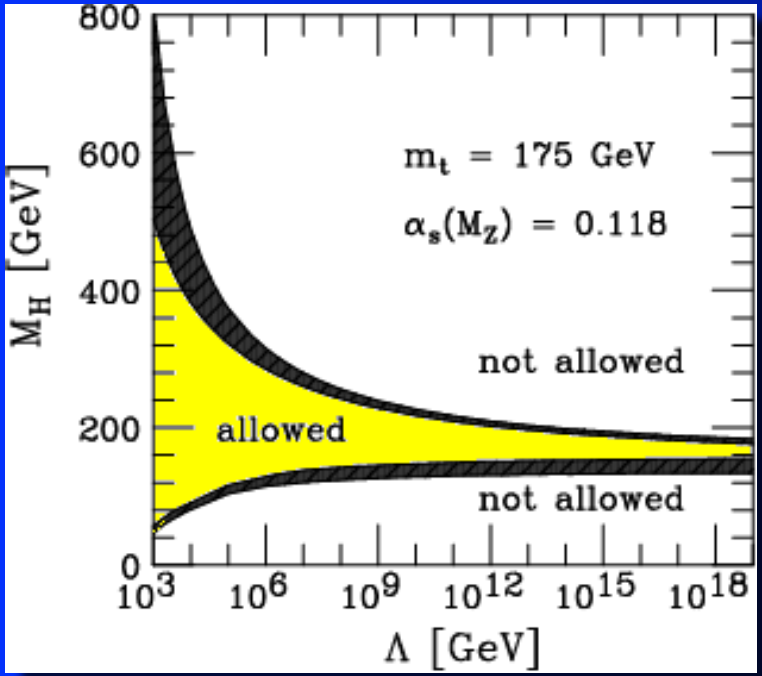
$m_H \geq 113.4$ GeV at 95 % C.L.



114 -115 GeV Event

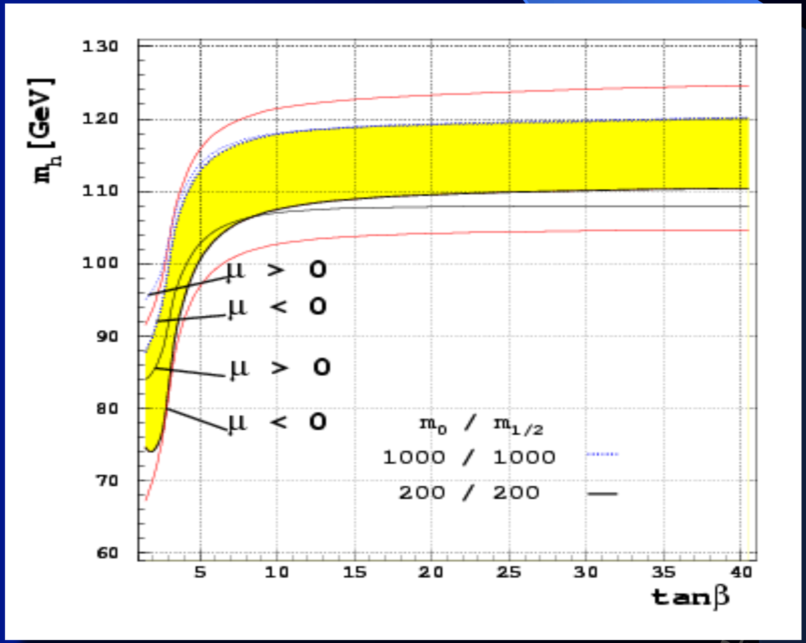


The Higgs Mass Limit (Theory)



- The SM Higgs

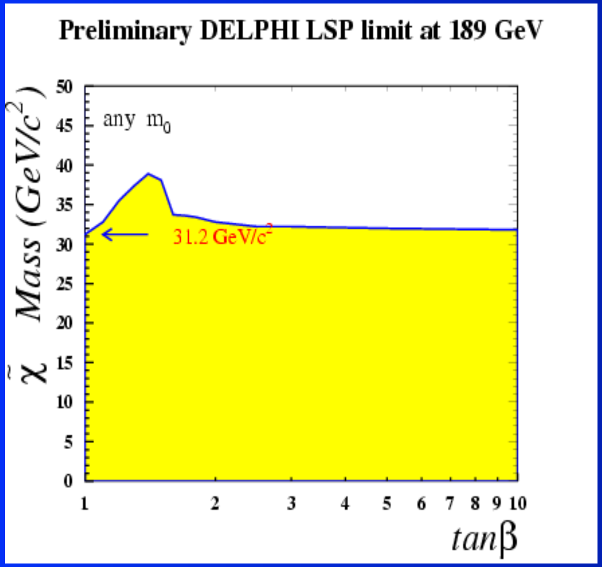
$m_H \geq 134$ GeV

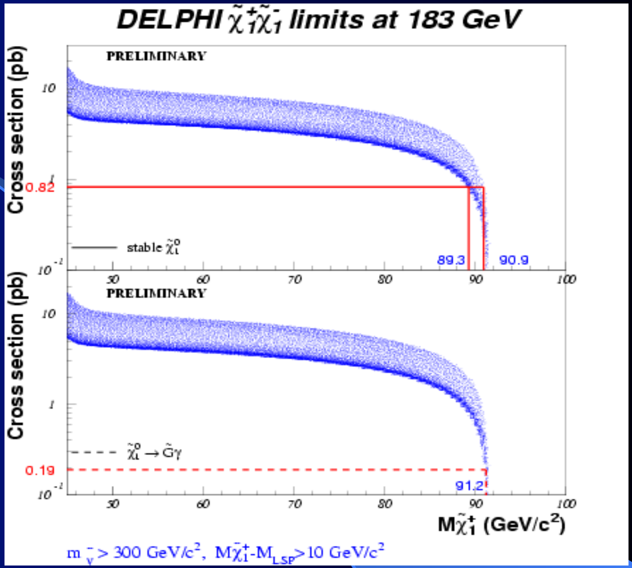


SUSY Higgs

$m_H \leq 130$ GeV

SUSY Searches at LEP



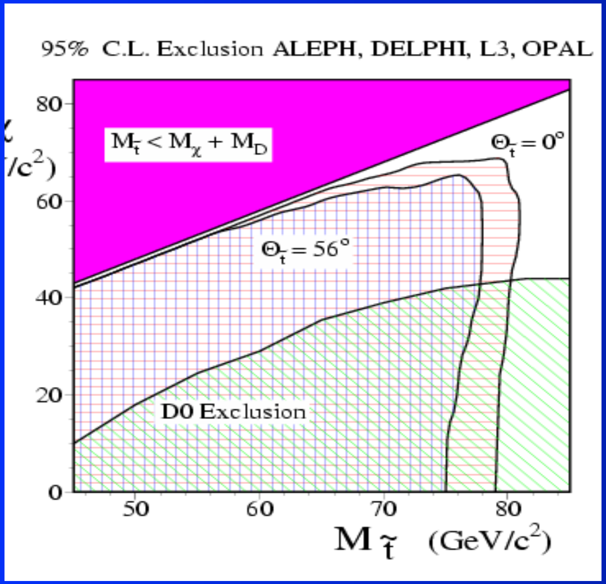


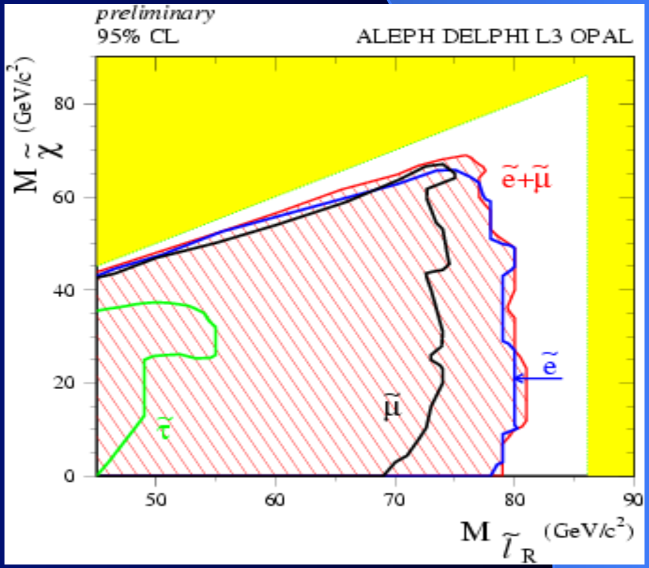
neutralinos

$\tilde{M}\chi 0 \geq$ 40 GeV

charginos

$\tilde{m}\chi + \geq$ 100 GeV





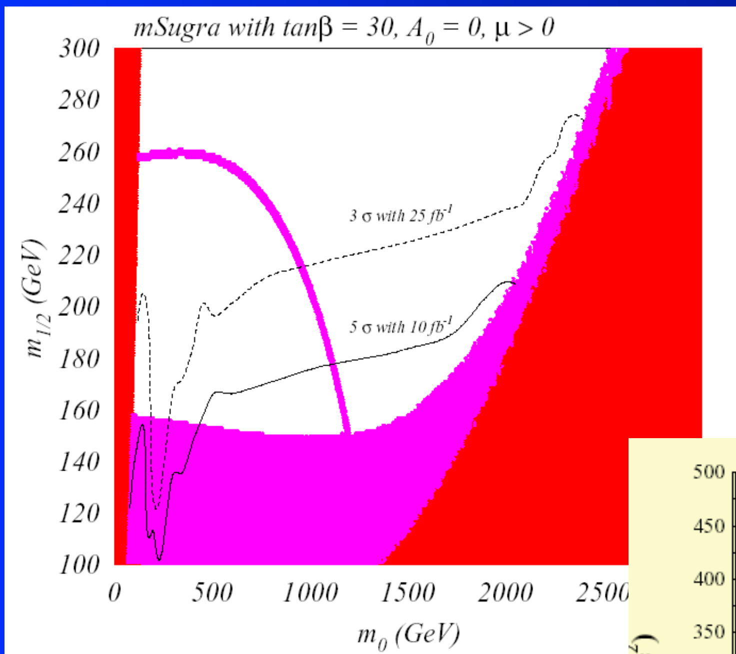
squarks

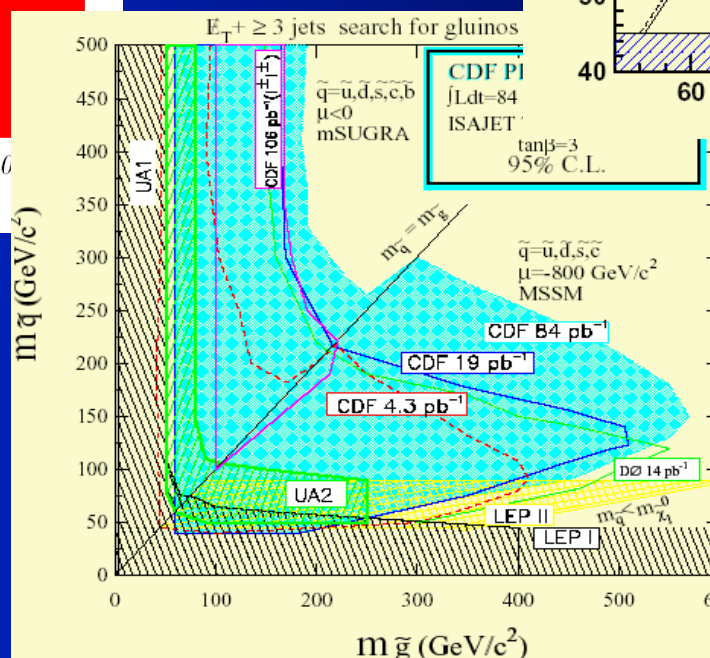
sleptons

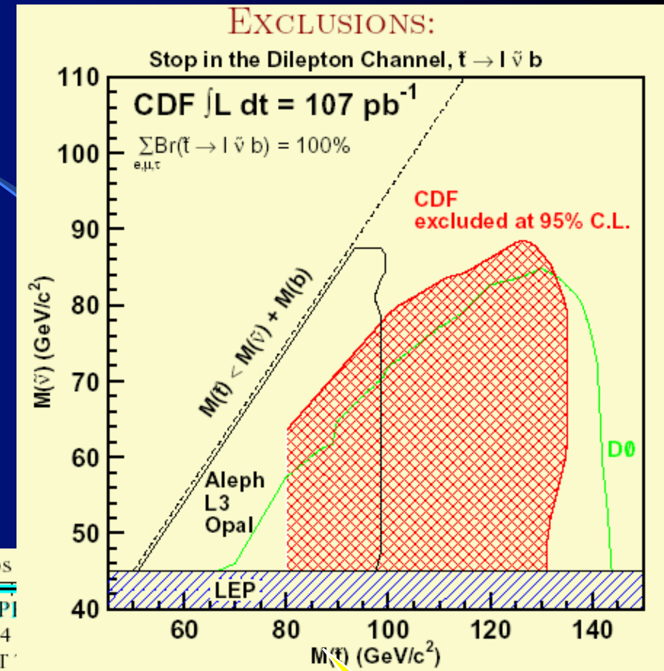
$\tilde{m}_l \geq$ 100 GeV

SUSY Searches at Tevatron

The reach of Tevatron in $m_0 / m_{1/2}$ plane







Exclusion:
World's Best Limits

Mq̃ ≥ 300 GeV

mg̃ ≥ 195 GeV

Dilepton Channel

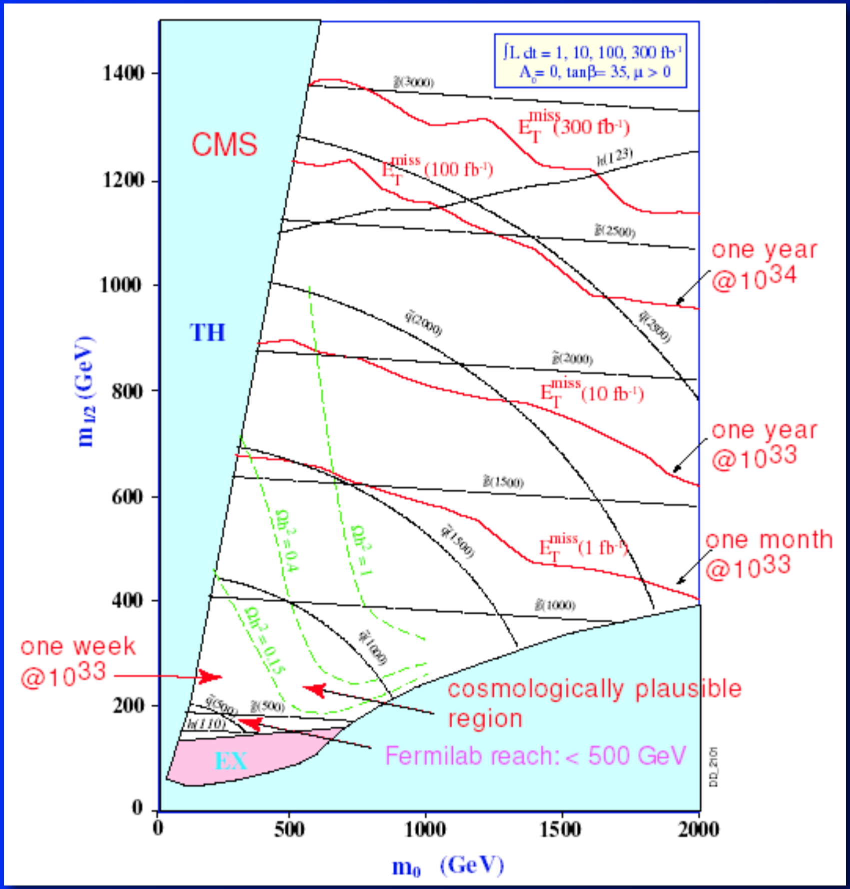
3 jet channel

Tevatron Discovery Reach

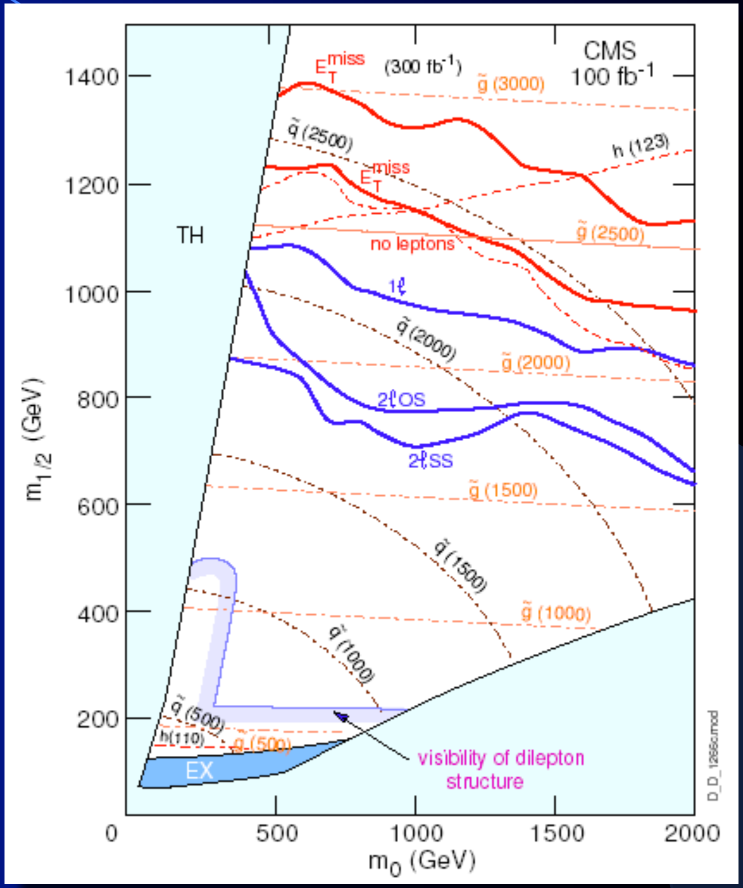
Decay ($Br = 100\%$)	Subsequent Decay	Final State of $\tilde{b}_1\bar{\tilde{b}}_1$ or $\tilde{t}_1\bar{\tilde{t}}_1$	Discovery Reach in $M_{\tilde{t}_1}$ or $M_{\tilde{b}_1}$ @20 fb^{-1}	(Run I)
$\tilde{b}_1 \to b\tilde{\chi}_1^0$		$bb\not{E}_T$	260 GeV$/c^2$	(146 GeV$/c^2$ [26])
$\tilde{t}_1 \to c\tilde{\chi}_1^0$		$cc\not{E}_T$	220 GeV$/c^2$	(116 GeV$/c^2$ [26])
$\tilde{t}_1 \to bl\tilde{\nu}$	$\tilde{\nu} \to \nu\tilde{\chi}_1^0$	$\ell^+\ell^- b\not{E}_T$	240 GeV$/c^2$	(140 GeV$/c^2$ [28])
$\tilde{t}_1 \to bl\nu\tilde{\chi}_1^0$		$\ell^+\ell^- b\not{E}_T$	-	(129 GeV$/c^2$ [28])
$\tilde{t}_1 \to b\tilde{\chi}_1^\pm$	$\tilde{\chi}_1^\pm \to W^{(*)}\tilde{\chi}_1^0$	$\ell bj\not{E}_T$ and $\ell^+\ell^- j\not{E}_T$	210 GeV$/c^2$	(-)
$\tilde{t}_1 \to bW\tilde{\chi}_1^0$		$\ell bj\not{E}_T$	190 GeV$/c^2$	(-)

NLSP	Decay Mode	$c\tau$	Prod.	Key Final State(s)	Discovery Reach @30 fb^{-1}
Bino $\tilde{\chi}_1^0$	$\gamma + \tilde{G}$	p	all	$\gamma\gamma\not{E}_T + X$	340 GeV$/c^2$ ($\tilde{\chi}_1^\pm$)
		d	all	$\gamma_d jj\not{E}_T$ or $\gamma\gamma\not{E}_T + X$	300 GeV$/c^2$ ($\tilde{\chi}_1^\pm$) ($c\tau = 50$ cm)
Higgsino $\tilde{\chi}_1^0$	$(h, Z, \gamma) + \tilde{G}$	p	all	$(hh, h\gamma, hZ, Z\gamma, ZZ, \gamma\gamma)\not{E}_T + X$	220 GeV$/c^2$ ($\tilde{\chi}_1^\pm$)
		d	all	$\delta_{ip} < 0$ for $h \to bb$, $Z \to \ell^+\ell^-$	–
		d	all	$\gamma_d + X$	–
$\tilde{\tau}$	$\tau + \tilde{G}$	p	all	$\ell\ell\ell j\not{E}_T$, $\ell^\pm\ell^\pm jj\not{E}_T$, $\tau_h\tau_h\not{E}_T$	230 GeV$/c^2$ ($\tilde{\chi}_1^\pm$) 120 GeV$/c^2$ ($\tilde{\tau}_1$)
		ll	all	$\mu(dE/dx) + \ell\ell(M_{\ell\ell} > 150$ GeV$/c^2)$	420 GeV$/c^2$ ($\tilde{\chi}_1^\pm$) 210 GeV$/c^2$ ($\tilde{\tau}$)
		ll	all	$\mu(dE/dx) + X$	180 GeV$/c^2$ ($\tilde{\tau}$)
		ll	all	$\mu(dE/dx$+TOF$) + X$	210 GeV$/c^2$ ($\tilde{\tau}$)
$\tilde{t}_1$	$(c, bW) + \tilde{G}$	p	$\tilde{t}_1\tilde{t}_1$	$cc\not{E}_T$ or $\ell + jets + \not{E}_T$	175 GeV$/c^2$ ($\tilde{t}_1$)

SUSY Searches at LHC



5 σ reach in jets + $\not\!\!E_T$ channel



Reach limits for various channels at 100 fb^{-1}

Superparticles

The SPDG is an international collaboration that reviews Sparticle Physics and related areas of Astrophysics, and compiles/analyzes data on particle properties. SPDG products are distributed to 130,000 physicists, teachers, and other interested people. The Review of Sparticle Physics is the most cited publication in particle physics during the last twenty years. Plots of SPDG statistics are available.

Mirror sites: USA (LBNL) Brazil CERN Italy (Genova) Japan (KEK) Russia (Novosibirsk) Russia (Protvino) UK (Durham)

Review of Sparticle Physics Charts, Educational materials, Sparticle Adventure Information and Databases
US-HEPFOLK Sparticle Physics: Twenty Years of Discoveries Home Pages of major HEP labs

The Review of Sparticle Physics

C. Caso *et al.* The European Physical Journal **C103** (2018) 1 (2018 Authors)

- 2019 — **2019 Web update of Reviews, Tables, Plots** — New November 2, 2019
- 2019 — **2019 Web update of Sparticle Listings** — New July 6, 2019
- 2018 — **2018 Summary Tables and Conservation Laws**
- **2018 Reviews, Tables, Plots (incl. Intro. Text)** — Superseded by 2019 Web Version
- **2018 Sparticle Listings (published version)** — Superseded by 2019 Web Version

- Errata (last changed January 18, 2020)
- Archived WWW editions: 2017 2016 2015
- Descriptions of the Summary Tables, Reviews, Listings, etc.
- Ordering information and list of products
- 2018 Authors and Directory of Sparticle Data Group Authors, Associates, and Advisors
- Computer-readable files — masses, widths, cross-sections, etc., including Palm Pilot XXII files.
- Encoder tools (for SPDG collaborators)

Discovery of the new world of SUSY

Back to 60's

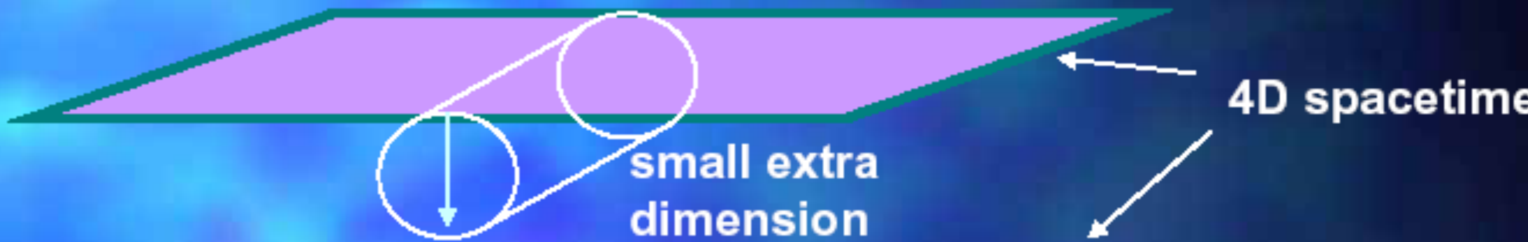
New discoveries every year

PART II: EXTRA DIMENSIONS

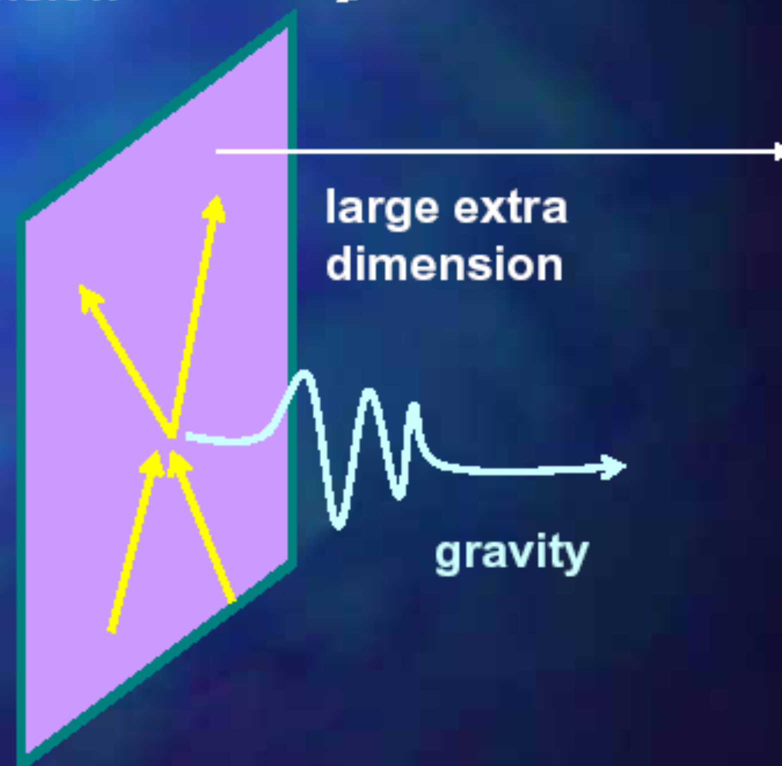
1. The main idea
2. Kaluza-Klein Approach
3. Brane-world models
4. Possible experimental signatures of ED

Why don't we see extra dimensions

- **conventional Kaluza-Klein idea:**
  internal extra dimension too small to be seen



- **discovery of D-brane**
  - **matter fields** restricted to lower dimensional brane
  - external bulk felt only through **gravity**
  - extra dimension bigger



Kaluza-Klein Approach

$$E_{4+d} = M_4 \times K_d$$

Pseudo-Euclidean space — Minkowski space — compact space

Metrics

$$ds^2 = G_{MN}(X)dX^M dX^N = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(x,y)dy^m dy^n$$

Fields

$$\Phi(x,y) = \sum_{n=0} \phi^{(n)}(x) Y_n(y)$$

Eigenfunctions of Laplace operator on internal space K_d

K-K modes

Masses

$$m_n^2 = m^2 + \frac{n_1^2 + n_2^2 + \ldots + n_d^2}{R^2}$$

Radius of the compact space

Couplings

$$g_{(4)} = \frac{g_{(4+d)}}{V_{(d)}} \qquad V_{(d)} \sim R^d$$

Multidimensional Gravity

Action

$$S_E = \int d^{4+d} X \sqrt{-\hat{G}} \frac{1}{16\pi G_{N(4+d)}} R^{(4+d)}[\hat{G}_{MN}]$$

K-K Expansion

$$S_E = \int d^4 x \sqrt{-g} \left\{ \frac{1}{16\pi G_{N(4)}} R^{(4)}[g^{(0)}_{\mu\nu}] + non - zero\ KK\ \text{modes} \right\}$$

Newton constant

$$\boxed{G_{N(4)} = \frac{1}{V_d} G_{N(4+d)}} \qquad V = R^d$$

Plank Mass

$$M_{Pl} = (G_{N(4)})^{(-1/2)} \longleftrightarrow M = (G_{N(4+d)})^{(-\frac{1}{d+2})}$$

Reduction formula

$$\boxed{M_{Pl}^2 = V_d M^{d+2}}$$

Low Scale Gravity

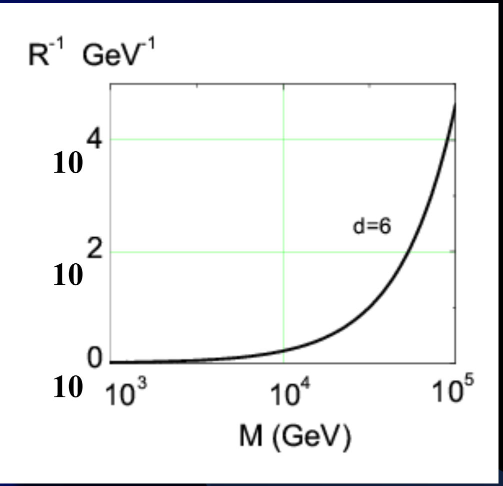
$$M_{Pl}^2 = R^d M^{2+d} \Rightarrow \mathrm{R} \sim \frac{1}{M}\left(\frac{M_{Pl}}{M}\right)^{2/d}$$

$$M \sim 1\ \mathrm{TeV} \Rightarrow \mathrm{R} \sim 10^{30/d-17}\ \mathrm{cm}$$

$$d = 2 \quad R \sim 0.1\ mm \quad \mathrm{R}^{-1} \sim 10^{-3}\ \mathrm{e}V$$

$$d = 3 \quad R \sim 10^{-7}\ cm \quad \mathrm{R}^{-1} \sim 100\ \mathrm{e}V$$

$$d = 6 \quad R \sim 10^{-12}\ cm \quad \mathrm{R}^{-1} \sim 10\ \mathrm{Me}V$$



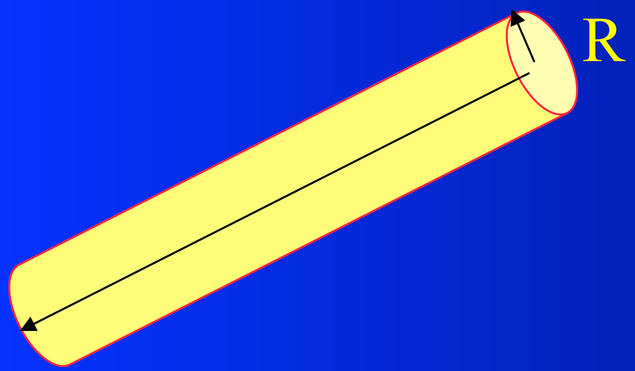
Modified Newton potential

$$V(r) = G_{N(4)} m_1 m_2 \sum_n \tfrac{1}{r} e^{-m_n r} = G_{N(4)} m_1 m_2 \left(\tfrac{1}{r} + \sum_{n \neq 0} \tfrac{1}{r} e^{-|n| r / R} \right)$$

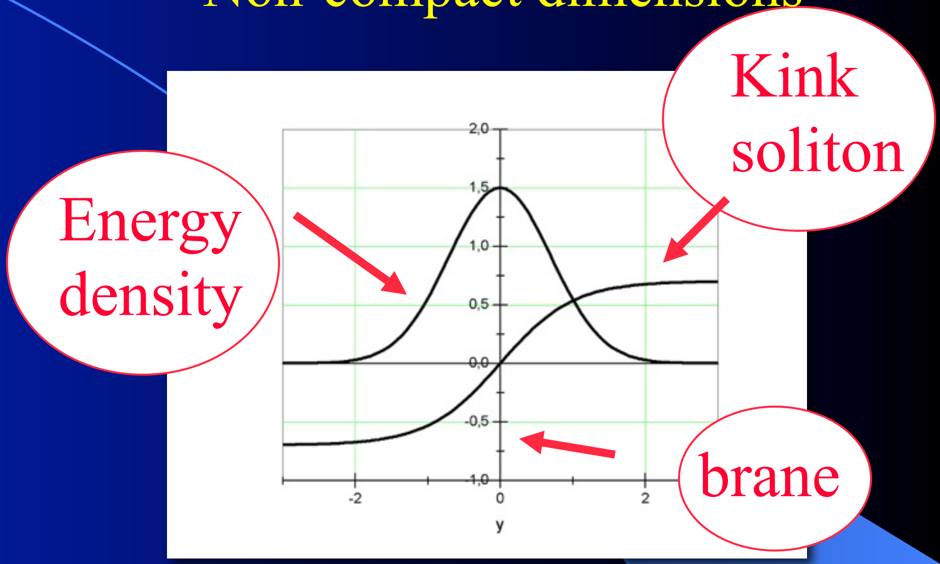
$$\begin{cases} V(r) = G_{N(4)} \dfrac{m_1 m_2}{r}, & \mathrm{r} \gg \mathrm{R} \\ V(r) = G_{N(4+d)} \dfrac{m_1 m_2}{r^{d+1}} (2\pi)^{\mathrm{d}} \dfrac{\Gamma(\frac{\mathrm{d}+1}{2})}{\Gamma(\frac{1}{2})}, & \mathrm{r} = \mathrm{R} \end{cases}$$

Brane World

Compact Dimensions

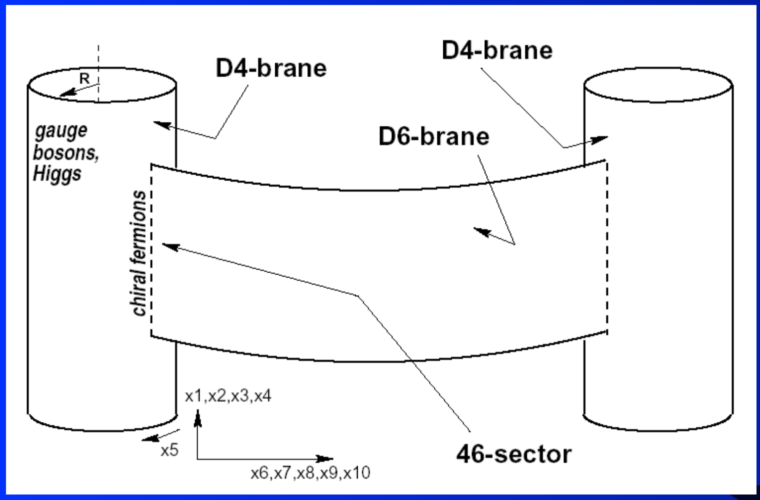


Non-compact dimensions



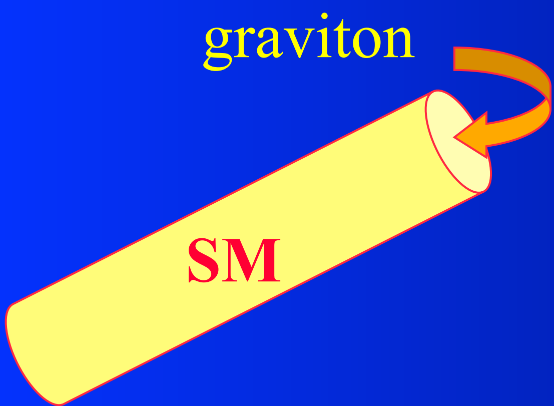
Localization on the brane

(Potential well)



Space-time of Type I superstring

The ADD Model



$$G_{MN} = \eta_{MN} + \frac{2}{M^{1+d/2}} \hat{h}_{MN}(x, y)$$

metric

$$\hat{h}_{MN}(x, y) = \sum_{n} h_{MN}^{(n)}(x) \frac{1}{\sqrt{V_d}} \mathrm{e}^{-\mathrm{i}\, \mathrm{n_m} y^m / R}$$

K-K gravitons

Interactions with the fields on the brane

$$S_{\text{int}} = \int d^{4+d} \hat{x} \sqrt{-\hat{G}} \hat{T}_{MN} \hat{h}^{MN}(x, y) \Rightarrow \sum_{n} \int d^4 x \frac{1}{M_{Pl}} T^{\mu\nu} h_{\mu\nu}^{(n)}(x)$$

The # of KK gravitons with masses $m_n \le E \le M$

$$\mathrm{N}(E) = S_{d-1} \sum_{n=0}^{ER} n^{d-1} \approx S_{d-1} \int_{0}^{ER} n^{d-1} dn = \frac{2\pi^{d/2}}{\Gamma(d+1)} R^d E^d$$

Emission rate $: \frac{1}{M_{Pl}^2} \mathrm{N}(E) : \frac{E^d}{M^{d+2}}$

Particle content of ADD model

(4+d)-dimensional picture:

- (4+d)-dimensional massless graviton + matter

4-dimensional picture

- 1 massless graviton $G^{(0)}$ (spin 2) + matter
- KK tower of massive gravitons $G^{(n)}$ (spin 2)
- (d-1) KK spin 1 decoupling fields
- $(d^2-d-2)/2$ KK tower of real scalar decoupling fields $(d \geq 2)$
- KK tower of scalar fields (zero mode – radion)

The SM fields are localized on the brane, while gravitons propagate in the bulk

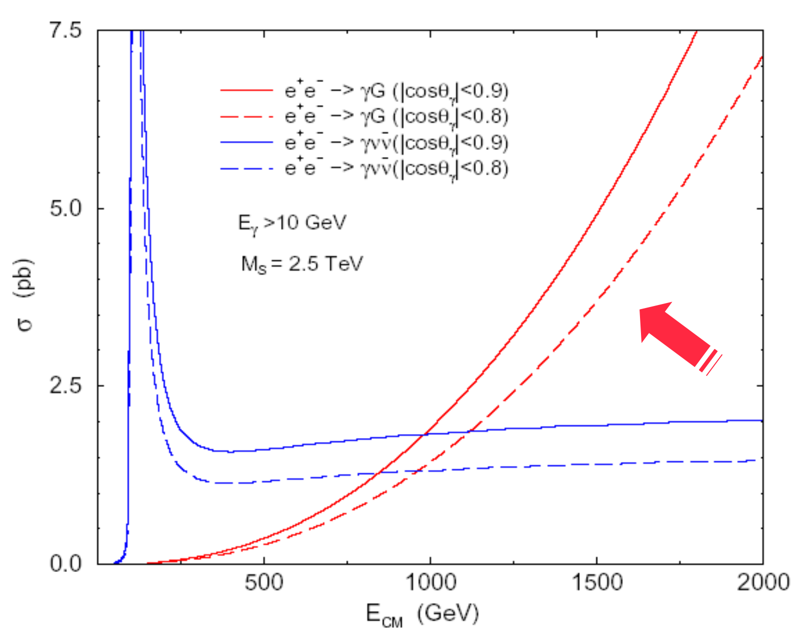
The "gravitational" coupling is $\sim 1/M^{1+d/2}$

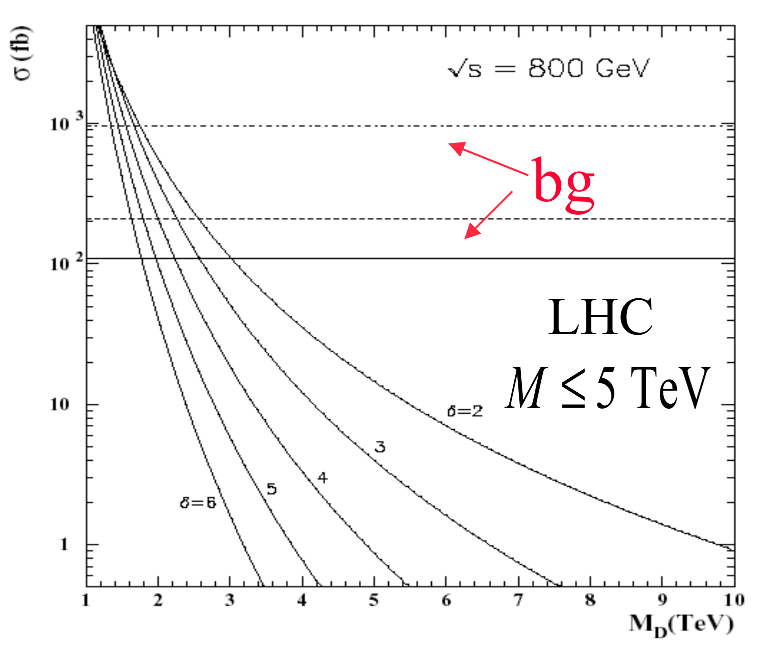
HEP Phenomenology

New phenomena: graviton emission & virtual graviton exchange

- KK states production $e^+e^- \to \gamma G^{(n)}$ $(e^+e^- \to \nu\bar{\nu}\gamma)$

$$\frac{d^2\sigma}{dtdm} \sim S_{d-1} \frac{M_{Pl}^2}{M^{d+2}} m^{d-1} \frac{d\sigma_m}{dt} \sim \frac{1}{M^{d+2}}$$

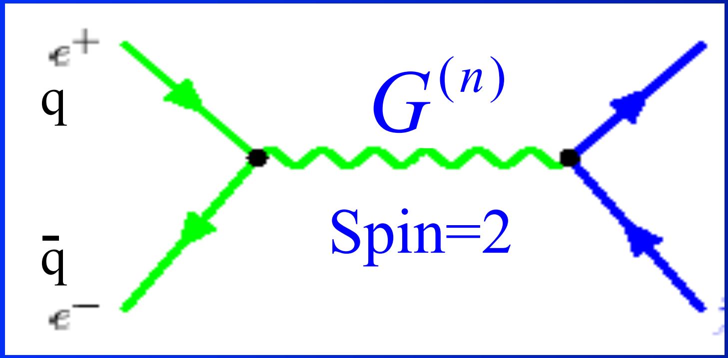




HEP Phenomenology II

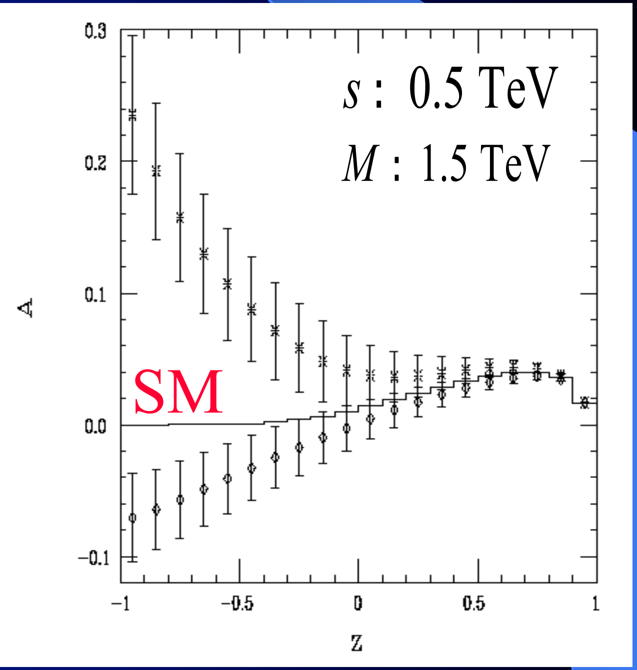
- Virtual graviton exchange $e^+e^- \to G^{(n)} \to f\bar{f}\ (HH, gg)$

$$A = \frac{1}{M_{Pl}^2}\sum_n \left\{ T_{\mu\nu}\frac{P^{\mu\nu}P^{\rho\sigma}}{s-m_n^2}T_{\rho\sigma} + \sqrt{\frac{3(d-1)}{d+2}}\frac{T_\mu^\mu T_\nu^\nu}{s-m_n^2} \right\}$$



$$S = \frac{1}{M_{Pl}^2}\sum_n \frac{1}{s-m_n^2} \approx \frac{1}{M_{Pl}^2} S_{d-1}\frac{M_{Pl}^2}{M^{d+2}}\int_0^\Lambda \frac{m^{d-1}dm}{s-m^2}$$

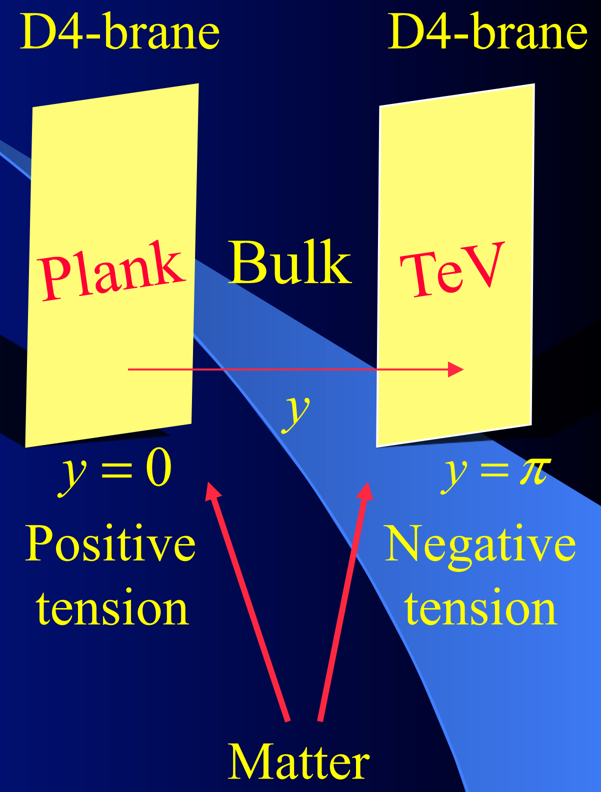
$$= \frac{S_{d-1}}{2M^4}\left\{ i\pi\left(\frac{s}{M^2}\right)^{d/2-1} + \sum_{k=1}^{[(d-1)/2]} c_k\left(\frac{s}{M^2}\right)^{k-1}\left(\frac{\Lambda}{M}\right)^{d-2k} \right\}$$



Angular distribution

Randall-Sandrum Models

$$E_5 = M_4 \otimes S^1 / Z_2$$



$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-\hat{G}} \{2M^3 R^{(5)}[\hat{G}_{MN}] + \Lambda\}$$
$$+ \int_{B_1} d^4x \sqrt{-g^{(1)}} (L_1 - \tau_1) + \int_{B_2} d^4x \sqrt{-g^{(2)}} (L_2 - \tau_2)$$

Metric $\quad ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$

$\sigma(y) = k\,|\,y\,|$ warp factor

$$\tau_1 = -\tau_2 = 24M^3 k, \quad \Lambda = 24M^3 k^2$$

graviton radion

Perturbed Metric $\quad ds^2 = e^{-2\sigma(y)} (\eta_{\mu\nu} + h_{\mu\nu}(x,y)) dx^\mu dx^\nu + (1 + \varphi(x)) dy^2$

Randall-Sandrum Model cont'd

Brane 1

- Massless graviton
- massive K-K gravitons

$$m_n = \beta_n k e^{-\pi k R}$$

- massless radion

Brane 2

Wrap factor $e^{-2\sigma(\pi R)}$

Hierarchy Problem !

$$M_{Pl}^2 = \frac{M^3}{k}\left(e^{2k\pi R} - 1\right)$$

$$S_{eff} = \frac{1}{2}\int_{B_2} d^4 z \left[\frac{1}{M_{Pl}} h_{\mu\nu}^{(0)} T^{\mu\nu} - \sum_{n=1} \frac{\omega_n}{\Lambda_\pi} h_{\mu\nu}^{(n)} T^{\mu\nu} - \frac{1}{\Lambda_\pi \sqrt{3}} T_\mu^\mu \right]$$

$$\Lambda_\pi = M_{Pl} e^{-k\pi R} : 1 \text{ TeV}$$

- Massless graviton $\longrightarrow \sim M_{Pl}$
- massive K-K gravitons $m_n = \beta_n k \longrightarrow \sim \Lambda_\pi$
- massless radion

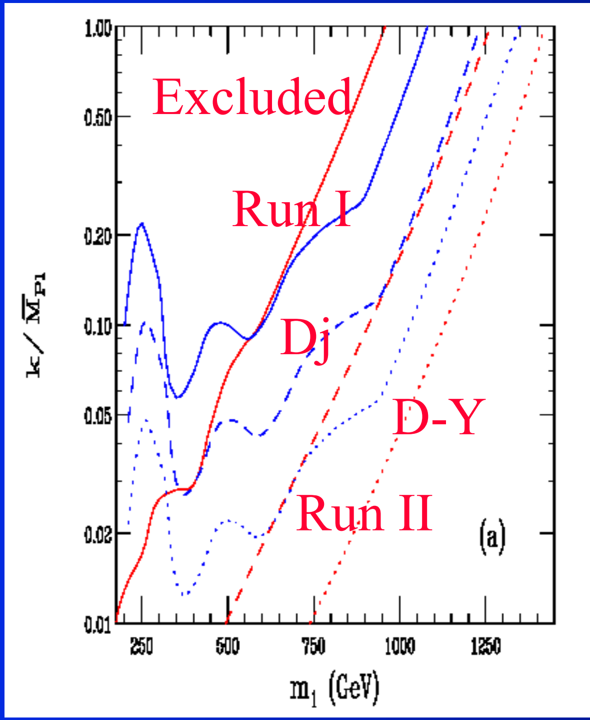
HEP Phenomenology

The first KK graviton mode M ~ 1 TeV

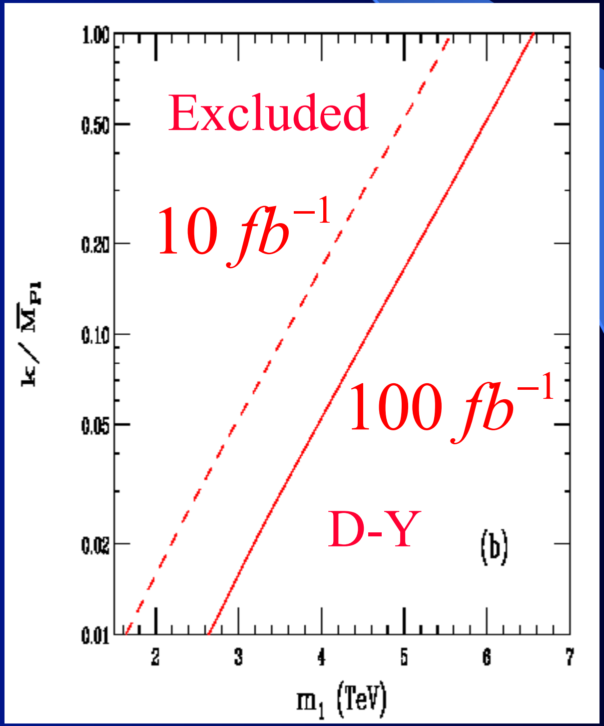
- Drell-Yan process $q\bar{q} \to G^{(1)} \to l^+l^-,\ gg \to G^{(1)} \to l^+l^-$
- Excess in dijet process $q\bar{q}, gg \to G^{(1)} \to q\bar{q}, gg$

Exclusion plots for resonance production

$$\eta = (k / M_{Pl}) e^{k\pi R}$$



Tevatron



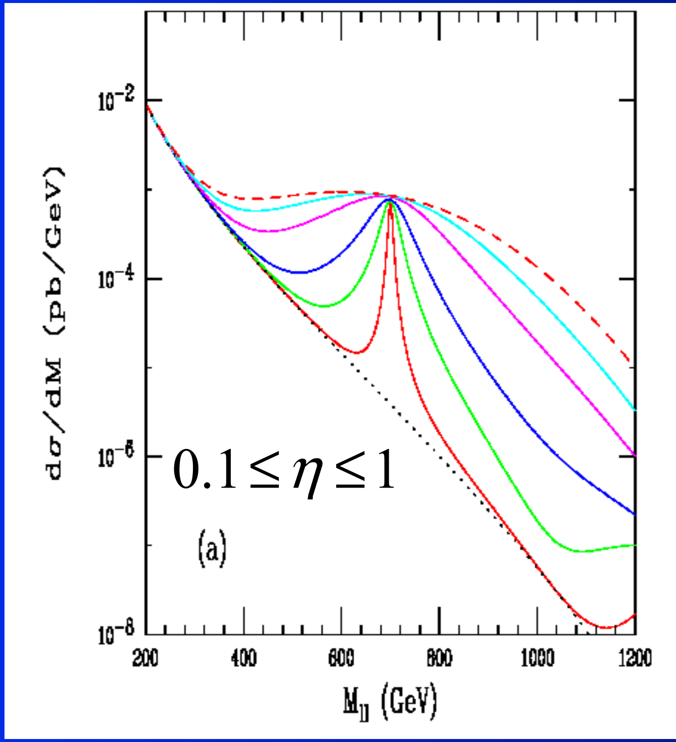
LHC

HEP Phenomenology II

$\eta = (k / M_{Pl}) e^{k\pi R}$

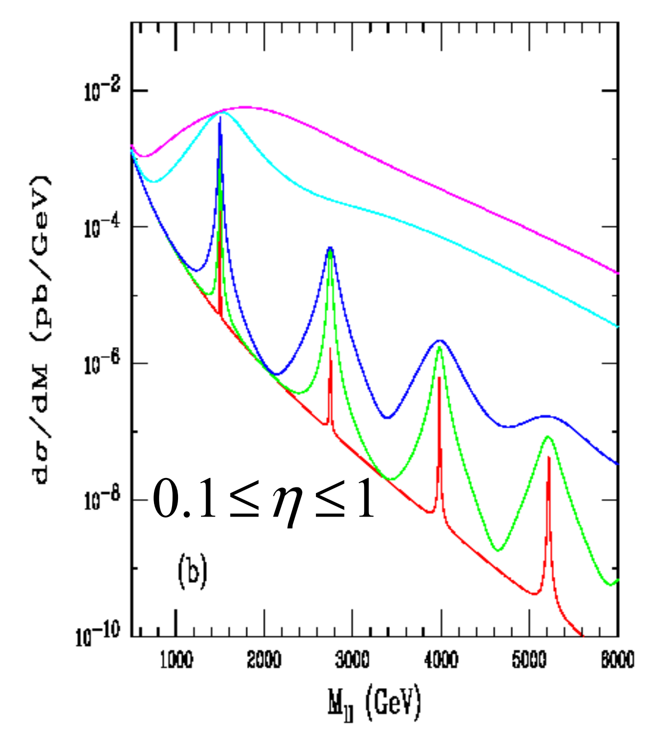
The x-section of D-Y production

First KK mode



Tevatron (M ~ 700 GeV)

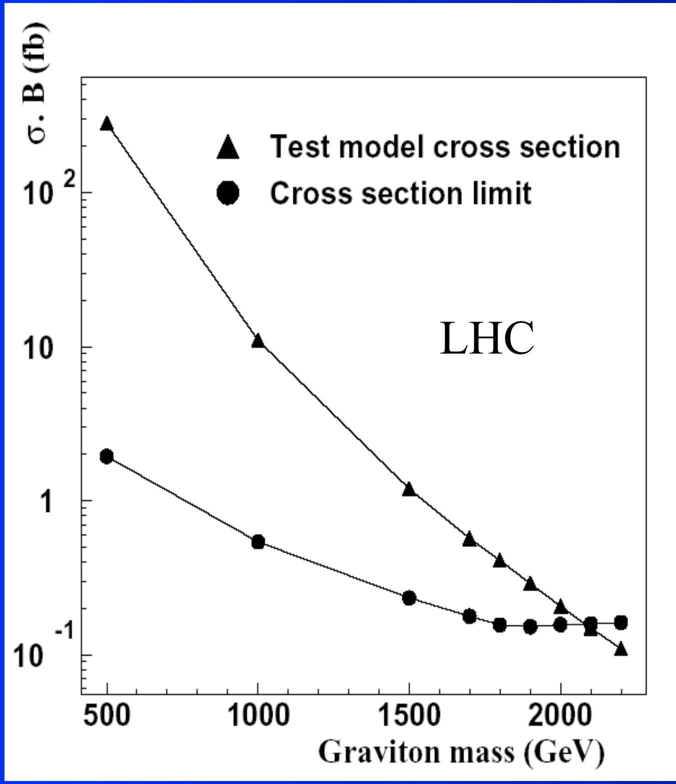
First and subsequent KK modes



LHC (M ~ 1500 GeV)

HEP Phenomenology III

$pp \rightarrow G^{(1)} \rightarrow e^+e^-$

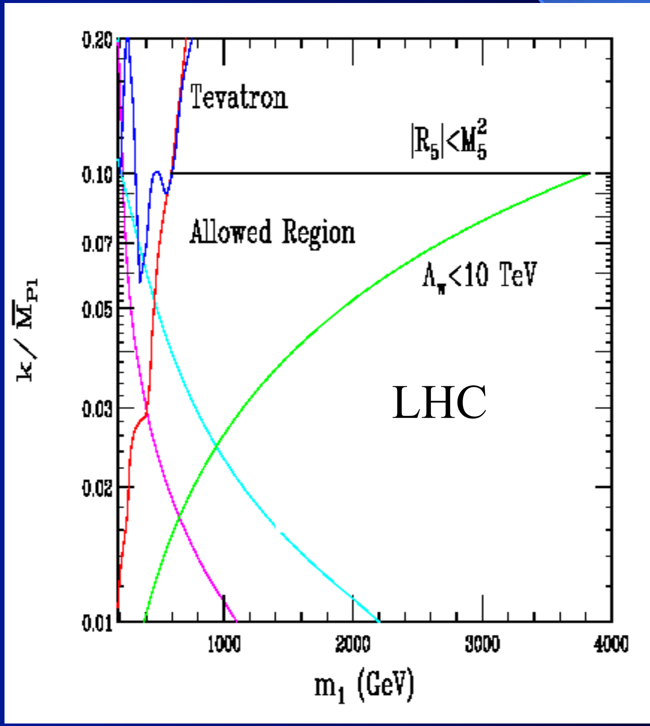


Angular dependence

$$spin\ 0 \Rightarrow \ \mathrm{f}(\theta)=1, \text{ spin } 1 \Rightarrow \ \mathrm{f}(\theta)=1+\cos^2\theta$$

$$q\bar{q} \rightarrow G^{(1)} \rightarrow l^+l^-, \ \mathrm{f}(\theta)=1-\cos^4\theta$$

$$\mathrm{gg} \rightarrow G^{(1)} \rightarrow l^+l^-, \ \mathrm{f}(\theta)=1-3\cos^2\theta+4\cos^4\theta$$



ED Conclusion

ADD Model

- The M_{EW}/M_{PL} hierarchy is replaced by

$$\frac{R^{-1}}{M} \sim \left(\frac{M}{M_{Pl}}\right)^{2/d} \sim 10^{-30/d}$$

- The scheme is viable
- For M small enough it can be checked at modern and future colliders
- For d=2 cosmological bounds on M are high (> 100 TeV), but for d>2 are mild

RS Model

- The M_{EW}/M_{PL} hierarchy is solved without new hierarchy
- A large part of parameter space will be studied in future collider experiments
- With the mechanism of radion stabilization the model is viable
- Cosmological scenarios are consistent (except the cosmological constant problem)

What comes beyond the Standard Model ?