SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

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Abstract The present lectures contain an introduction to supersymmetry, a new symmetry that relates bosons and fermions, in particle physics. The motivation to introduce supersymmetry is discussed. The main notions of supersymmetry are introduced. The supersymmetric extension of the Standard Model - the Minimal Supersymmetric Standard Model - is considered in more detail. Phenomenological features of the MSSM as well as possible experimental signatures of SUSY are described.

1. Introduction: What is supersymmetry

Supersymmetry is a *boson-fermion* symmetry that is aimed to unify all forces in Nature including gravity within a singe framework [1]-[4]. Modern views on supersymmetry in particle physics are based on string paradigm, though the low energy manifestations of SUSY can be possibly found at modern colliders and in non-accelerator experiments.

Supersymmetry emerged from the attempts to generalize the Poincaré algebra to mix representations with different spin [1]. It happened to be a problematic task due to the no-go theorems preventing such generalizations [5]. The way out was found by introducing the so-called graded Lie algebras, i.e. adding the anti-commutators to the usual commutators of the Lorentz algebra. Such a generalization, described below, appeared to be the only possible one within relativistic field theory.

If Q is a generator of SUSY algebra, then acting on a boson state it produces a fermion one and vice versa

$$\bar{Q}|boson>=|fermion> \quad \text{and} \quad Q|fermion>=|boson> .$$

Since bosons commute with each other and fermions anticommute, one immediately finds that SUSY generators should also anticommute, they must be *fermionic*, i.e. they must change the spin by a half-odd amount and change the statistics. Indeed, the key element of SUSY algebra is

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha,\dot{\alpha}} P_\mu, \tag{1.1}$$

where Q and $\bar{Q}$ are SUSY generators and P_μ is the generator of translation, the four-momentum.

In what follows we describe SUSY algebra in more detail and construct its representations which are needed to build a SUSY generalization of the Standard Model of fundamental interactions. Such a generalization is based on a softly broken SUSY quantum filed theory and contains the SM as a low energy theory.

Supersymmetry promises to solve some problems of the SM and of Grand Unified Theories. In what follows we describe supersymmetry as a nearest option for the new physics on a TeV scale.

2. Motivation of SUSY in particle physics

2.1 Unification with gravity

The *general idea* is a unification of all forces of Nature including quantum gravity. However, the graviton has spin 2, while the other gauge bosons (photon, gluons, W and Z weak bosons) have spin 1. Therefore, they correspond to different representations of the Poincaré algebra. To mix them one can use supersymmetry transformations. Starting with the graviton state of spin 2 and acting by SUSY generators we get the following chain of states:

$$\textit{spin } 2 \quad\to\quad \textit{spin } 3/2 \quad\to\quad \textit{spin } 1 \quad\to\quad \textit{spin } 1/2 \quad\to\quad \textit{spin } 0.$$

Thus, a partial unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with other interactions.

Taking infinitesimal transformations $\delta_\epsilon = \epsilon^\alpha Q_\alpha,\ \bar{\delta}_{\bar{\epsilon}} = \bar{Q}_{\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}}$, and using eq.(1.1) one gets

$$\{\delta_\epsilon, \bar{\delta}_{\bar{\epsilon}}\} = 2(\epsilon\sigma^\mu\bar{\epsilon})P_\mu, \tag{2.1}$$

where ϵ is a transformation parameter. Choosing ϵ to be local, i.e. a function of a space-time point $\epsilon = \epsilon(x)$, one finds from eq.(2.1) that an anticommutator of two SUSY transformations is a local coordinate translation. And a theory which is invariant under local coordinate transformation is General Relativity. Thus, making SUSY local, one naturally obtains General Relativity, or a theory of gravity, or supergravity [2].

2.2 Unification of gauge couplings

According to the Grand Unification *hypothesis*, gauge symmetry increases with energy [6]. All known interactions are different branches of a unique interaction associated with a simple gauge group. The unification (or splitting) occurs at high energy. To reach this goal one has to consider how the couplings change with energy. This is described by the renormalization group equations. In the SM the strong and weak couplings associated with non-Abelian gauge groups decrease with energy, while the electromagnetic one associated with the Abelian group on the contrary increases. Thus, it becomes possible that at some energy scale they become equal.

After the precise measurement of the $SU(3) \times SU(2) \times U(1)$ coupling constants, it has become possible to check the unification numerically. The three coupling constants to be compared are

$$\begin{aligned} \alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W), \\ \alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W, \\ \alpha_3 &= g_s^2/(4\pi) \end{aligned} \tag{2.2}$$

where g', g and g_s are the usual $U(1)$, $SU(2)$ and $SU(3)$ coupling constants and α is the fine structure constant. The factor of 5/3 in the definition of α_1 has been included for proper normalization of the generators.

In the modified minimal subtraction ($\overline{MS}$) scheme, the world averaged values of the couplings at the Z^0 energy are obtained from a fit to the LEP and Tevatron data [7]:

$$\begin{aligned} \alpha^{-1}(M_Z) &= 128.978 \pm 0.027 \\ \sin^2\theta_{\overline{MS}} &= 0.23146 \pm 0.00017 \\ \alpha_s &= 0.1184 \pm 0.0031, \end{aligned} \tag{2.3}$$

that gives

$$\alpha_1(M_Z) = 0.017, \quad \alpha_2(M_Z) = 0.034, \quad \alpha_3(M_Z) = 0.118 \pm 0.003. \tag{2.4}$$

Assuming that the SM is valid up to the unification scale, one can then use the known RG equations for the three couplings. In the leading order they are:

$$\frac{d\tilde{\alpha}_i}{dt} = b_i\tilde{\alpha}_i^2, \qquad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi}, \qquad t = log(\frac{Q^2}{\mu^2}), \tag{2.5}$$

where for the SM the coefficients are $b_i = (41/10, -19/6, -7)$.

The solution to eq.(2.5) is very simple

$$\frac{1}{\tilde{\alpha}_i(Q^2)} = \frac{1}{\tilde{\alpha}_i(\mu^2)} - b_i log(\frac{Q^2}{\mu^2}). \tag{2.6}$$

The result is demonstrated in Fig.1 showing the evolution of the inverse of the couplings as a function of the logarithm of energy. In this presentation, the evolution becomes a straight line in first order. The second order corrections are small and do not cause any visible deviation from a straight line. Fig.1 clearly demonstrates that within the SM the coupling constant unification at a single point is impossible. It is excluded by more than 8 standard deviations. This result means that the unification can only be obtained if new physics enters between the electroweak and the Planck scales.

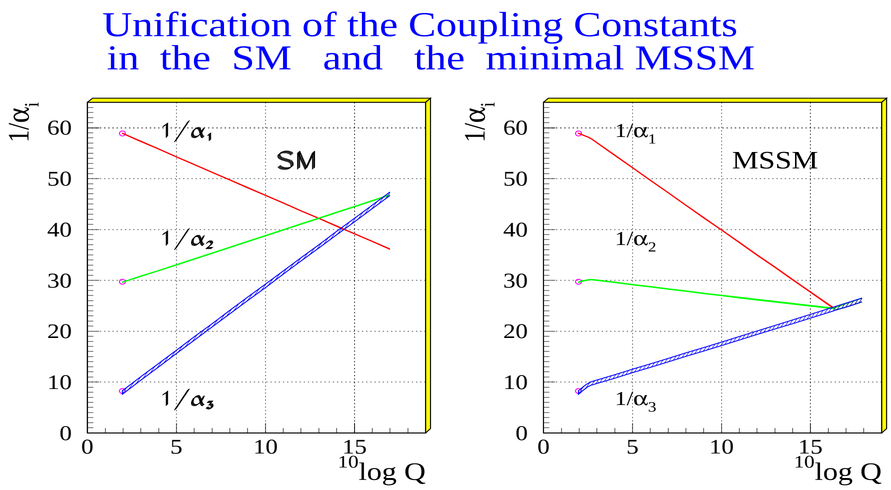

Figure 1. Evolution of the inverse of the three coupling constants in the Standard Model (left) and in the supersymmetric extension of the SM (MSSM) (right).

In the SUSY case, the slopes of the RG evolution curves are modified. The coefficients b_i in eq.(2.5) now are $b_i = (33/5, 1, -3)$. The SUSY particles are assumed to effectively contribute to the running of the coupling constants only for energies above the typical SUSY mass scale. It turns out that within the SUSY model a perfect unification can be obtained as is shown in Fig.1. From the fit requiring unification one finds for the break point M_{SUSY} and the unification point M_{GUT} [8]

$$\begin{aligned} M_{SUSY} &= 10^{3.4\pm0.9\pm0.4}\ GeV, \\ M_{GUT} &= 10^{15.8\pm0.3\pm0.1}\ GeV, \\ \alpha_{GUT}^{-1} &= 26.3 \pm 1.9 \pm 1.0, \end{aligned} \tag{2.7}$$

The first error originates from the uncertainty in the coupling constant, while the second one is due to the uncertainty in the mass splittings between the SUSY particles.

This observation was considered as the first "evidence" for supersymmetry, especially since M_{SUSY} was found in the range preferred by the fine-tuning arguments.

2.3 Solution of the hierarchy problem

The appearance of two different scales $V \gg v$ in a GUT theory, namely, M_W and M_{GUT}, leads to a very serious problem which is called the *hierarchy problem*. There are two aspects of this problem.

The first one is the very existence of the hierarchy. To get the desired spontaneous symmetry breaking pattern, one needs

$$\begin{array}{ccccc} m_H & \sim & v & \sim & 10^2 \ \text{GeV} \\ m_\Sigma & \sim & V & \sim & 10^{16} \ \text{GeV} \end{array} \qquad \frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1, \tag{2.8}$$

where H and Σ are the Higgs fields responsible for the spontaneous breaking of the $SU(2)$ and the GUT groups, respectively. The question arises of how to get so small number in a natural way.

The second aspect of the hierarchy problem is connected with the preservation of a given hierarchy. Even if we choose the hierarchy like eq.(2.8) the radiative corrections will destroy it! To see this, consider the radiative correction to the light Higgs mass given by the Feynman diagram shown in Fig.2. This correction proportional to the mass squared

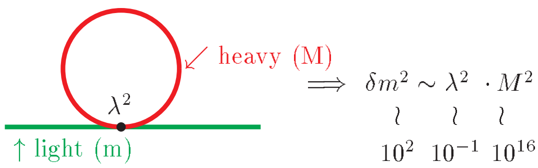

Figure 2. Radiative correction to the light Higgs boson mass

of the heavy particle, obviously, spoils the hierarchy if it is not cancelled. This very accurate cancellation with a precision $\sim 10^{-14}$ needs a fine tuning of the coupling constants.

The only known way of achieving this kind of cancellation of quadratic terms (also known as the cancellation of the quadratic divergencies) is supersymmetry. Moreover, SUSY automatically cancels quadratic corrections in all orders of PT. This is due to the contributions of superpartners of ordinary particles. The contribution from boson loops cancels those from the fermion ones because of an additional factor (-1) coming from Fermi statistics, as shown in Fig.3. One can see here two types

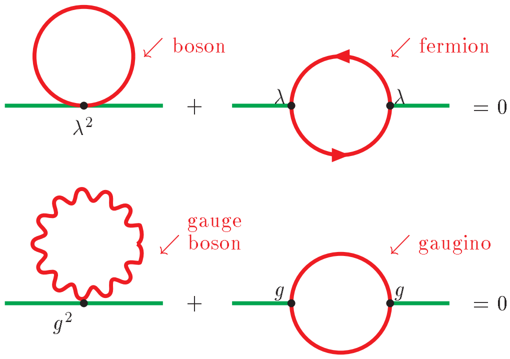

Figure 3. Cancellation of quadratic terms (divergencies)

of contribution. The first line is the contribution of the heavy Higgs boson and its superpartner. The strength of interaction is given by the Yukawa coupling λ. The second line represents the gauge interaction proportional to the gauge coupling constant g with the contribution from the heavy gauge boson and heavy gaugino.

In both the cases the cancellation of quadratic terms takes place. This cancellation is true up to the SUSY breaking scale, M_{SUSY}, which should not be very large (≤ 1 TeV) to make the fine-tuning natural. Indeed, let us take the Higgs boson mass. Requiring for consistency of perturbation theory that the radiative corrections to the Higgs boson mass do not exceed the mass itself gives

$$\delta M_h^2 \sim g^2 M_{SUSY}^2 \sim M_h^2. \tag{2.9}$$

So, if $M_h \sim 10^2$ GeV and $g \sim 10^{-1}$, one needs $M_{SUSY} \sim 10^3$ GeV in order that the relation (2.9) is valid. Thus, we again get the same rough estimate of $M_{SUSY} \sim 1$ TeV as from the gauge coupling unification above.

That is why it is usually said that supersymmetry solves the hierarchy problem. We show below how SUSY can also explain the origin of the hierarchy.

2.4 Astrophysics and Cosmology

The shining matter is not the only one in the Universe. Considerable amount consists of the so-called dark matter. The direct evidence for the presence of the dark matter are the rotation curves of galaxies (see Fig.4). To explain these curves one has to assume the existence of galactic halo made of non-shining matter which takes part in gravitational interaction. According to the latest data [9] the matter content of the Universe is

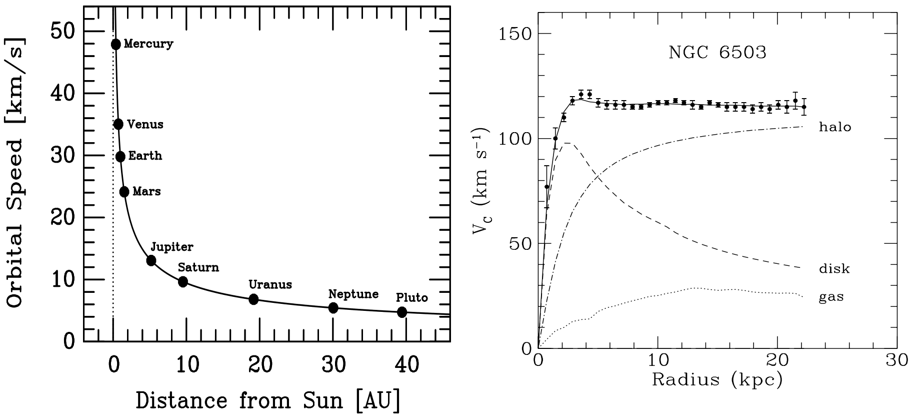

Figure 4. Roration curves for the solar system and galaxy

the following:

$$\Omega h^2 = 1 \quad \Leftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%, \quad \Omega_{DarkMatter} \approx 23\%, \quad \Omega_{Baryon} \approx 4\%$$

There are two possible types of the dark matter: the hot one, consisting of light relativistic particles and the cold one, consisting of massive weakly interacting particles (WIMPs). The hot dark matter might consist of neutrinos, however, this has problems with galaxy formation. As for the cold dark matter, it has no candidates within the SM. At the same time, SUSY provides an excellent candidate for the cold dark matter, namely neutralino, the lightest superparticle.

2.5 Beyond GUTs: superstring

Another motivation for supersymmetry follows from even more radical changes of basic ideas related to the ultimate goal of construction of consistent unified theory of everything. At the moment the only viable conception is the superstring theory [10]. In the superstring theory, strings are considered as fundamental objects, closed or open, and are nonlocal in nature. Ordinary particles are considered as string excitation modes. String interactions, which are local, generate proper interactions of usual particles, including gravitational ones.

To be consistent, the string theory should be conformal invariant in D-dimensional target space and have a stable vacuum. The first requirement is valid in classical theory but may be violated by quantum anomalies. Cancellation of quantum anomalies takes place when space-time dimension of a target space equals to a critical one which is $D_c = 26$ for bosonic string and $D_c = 10$ for a fermionic one.

The second requirement is that the massless string excitations (the particles of the SM) are stable. This assumes the absence of tachyons, the states with imaginary mass, which can be guaranteed only in supersymmetric string theories!

3. Basics of supersymmetry

3.1 Algebra of SUSY

Combined with the usual Poincaré and internal symmetry algebra the Super-Poincaré Lie algebra contains additional SUSY generators Q^i_α and $\bar{Q}^i_{\dot{\alpha}}$ [3]

$$
\begin{aligned}
&[P_\mu, P_\nu] = 0,\\
&[P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),\\
&[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),\\
&[B_r, B_s] = iC^t_{rs}B_t,\\
&[B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,\\
&[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot{\alpha}}, P_\mu] = 0,\\
&[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta, \quad [\bar{Q}^i_{\dot{\alpha}}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot{\beta}}(\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}},\\
&[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha, \quad [\bar{Q}^i_{\dot{\alpha}}, B_r] = -\bar{Q}^j_{\dot{\alpha}}(b_r)^i_j,\\
&\{Q^i_\alpha, \bar{Q}^j_{\dot{\beta}}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu,\\
&\{Q^i_\alpha, Q^j_\beta\} = 2\epsilon_{\alpha\beta}Z^{ij}, \quad Z_{ij} = a^r_{ij}b_r, \quad Z^{ij} = Z^+_{ij},\\
&\{\bar{Q}^i_{\dot{\alpha}}, \bar{Q}^j_{\dot{\beta}}\} = -2\epsilon_{\dot{\alpha}\dot{\beta}}Z^{ij}, \quad [Z_{ij}, anything] = 0,\\
&\alpha, \dot{\alpha} = 1,2 \quad i,j = 1,2,\ldots,N.
\end{aligned}
\tag{3.1}
$$

Here P_μ and $M_{\mu\nu}$ are four-momentum and angular momentum operators, respectively, B_r are the internal symmetry generators, Q^i and $\bar{Q}^i$ are the spinorial SUSY generators and Z_{ij} are the so-called central charges; $\alpha, \dot{\alpha}, \beta, \dot{\beta}$ are the spinorial indices. In the simplest case one has one spinor generator Q_α (and the conjugated one $\bar{Q}_{\dot{\alpha}}$) that corresponds to an ordinary or N=1 supersymmetry. When $N > 1$ one has an extended supersymmetry.

A natural question arises: how many SUSY generators are possible, i.e. what is the value of N? To answer this question, consider massless states. Let us start with the ground state labeled by energy and helicity, i.e. projection of a spin on the direction of momenta, and let it be annihilated by Q_i

$$\text{Vacuum} = |E, \lambda>, \qquad Q_i|E, \lambda >= 0.$$

Then one and more particle states can be constructed with the help of a creation operators as

State	Expression	# of States
vacuum	$\lvert E,\lambda>$	1
$1-$ particle state	$\bar{Q}_i\lvert E,\lambda>=\lvert E,\lambda+1/2>_i$	$\left(\begin{array}{c}N\\1\end{array}\right)=N$
$2-$ particle state	$\bar{Q}_i\bar{Q}_j\lvert E,\lambda>=\lvert E,\lambda+1>_{ij}$	$\left(\begin{array}{c}N\\2\end{array}\right)=\frac{N(N-1)}{2}$
...	...	...
$N-$ particle state	$\bar{Q}_1\bar{Q}_2...\bar{Q}_N\lvert E,\lambda>=\lvert E,\lambda+\frac{N}{2}>$	$\left(\begin{array}{c}N\\N\end{array}\right)=1$

Total # of states: $\sum_{k=0}^{N}\left(\begin{array}{c}N\\k\end{array}\right)=2^N=2^{N-1}$ bosons $+\ 2^{N-1}$ fermions.
The energy E is not changed, since according to (3.1) the operators $\bar{Q}_i$ commute with the Hamiltonian.

Thus, one has a sequence of bosonic and fermionic states and the total number of bosons equals that of fermions. This is a generic property of any supersymmetric theory. However, in CPT invariant theories the number of states is doubled, since CPT transformation changes the sign of helicity. Hence, in CPT invariant theories, one has to add the states with opposite helicity to the above mentioned ones.

Consider some examples. Let us take $N=1$ and $\lambda=0$. Then one has the following set of states:

$$N=1\quad\lambda=0\qquad\begin{array}{lcc}\text{helicity} & 0 & 1/2\\ \\ \#\text{ of states} & 1 & 1\end{array}\quad\stackrel{CPT}{\Longrightarrow}\quad\begin{array}{lcc}\text{helicity} & 0 & -1/2\\ \\ \#\text{ of states} & 1 & 1\end{array}$$

Hence, a complete $N=1$ multiplet is

$$\begin{array}{llccc}N=1 & \text{helicity} & -1/2 & 0 & 1/2\\ & \#\text{ of states} & 1 & 2 & 1\end{array}$$

which contains one complex scalar and one spinor with two helicity states.

This is an example of the so-called self-conjugated multiplet. There are also self-conjugated multiplets with $N>1$ corresponding to extended supersymmetry. Two particular examples are the $N=4$ super Yang-Mills multiplet and the $N=8$ supergravity multiplet

$$\begin{array}{lllccccc}N=4 & \text{SUSY YM} & \text{helicity} & -1 & -1/2 & 0 & 1/2 & 1\\ & \lambda=-1 & \#\text{ of states} & 1 & 4 & 6 & 4 & 1\end{array}$$

$N=8$	SUGRA	-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$	2
	$\lambda=-2$	1	8	28	56	70	56	28	8	1

One can see that the multiplets of extended supersymmetry are very rich and contain a vast number of particles.

The constraint on the number of SUSY generators comes from a requirement of consistency of the corresponding QFT. The number of supersymmetries and the maximal spin of the particle in the multiplet are related by

$$N \leq 4S,$$

where S is the maximal spin. Since the theories with spin greater than 1 are non-renormalizable and the theories with spin greater than 5/2 have no consistent coupling to gravity, this imposes a constraint on the number of SUSY generators

$$\begin{array}{ll} N \leq 4 & \text{for renormalizable theories (YM),} \\ N \leq 8 & \text{for (super)gravity.} \end{array}$$

In what follows, we shall consider simple supersymmetry, or $N = 1$ supersymmetry, contrary to extended supersymmetries with $N > 1$. In this case, one has two types of supermultiplets: the so-called chiral multiplet with $\lambda = 0$, which contains two physical states (ϕ, ψ) with spin 0 and 1/2, respectively, and the vector multiplet with $\lambda = 1/2$, which also contains two physical states (λ, A_μ) with spin 1/2 and 1, respectively.

3.2 Superspace and superfields

An elegant formulation of supersymmetry transformations and invariants can be achieved in the framework of superspace [4]. Superspace differs from the ordinary Euclidean (Minkowski) space by adding of two new coordinates, θ_α and $\bar{\theta}_{\dot{\alpha}}$, which are Grassmannian, i.e. anticommuting, variables

$$\{\theta_\alpha, \theta_\beta\} = 0, \quad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0, \quad \theta_\alpha^2 = 0, \quad \bar{\theta}_{\dot{\alpha}}^2 = 0, \quad \alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2.$$

Thus, we go from space to superspace

$$\begin{array}{ccc} Space & \Rightarrow & Superspace \\ x_\mu & & x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}} \end{array}$$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x, \theta, \bar{\theta}) = e^{i\left(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q}\right)}. \tag{3.2}$$

It leads to a supertranslation in superspace

$$\begin{array}{rcl} x_\mu & \to & x_\mu + i\theta\sigma_\mu\bar{\varepsilon} - i\varepsilon\sigma_\mu\bar{\theta}, \\ \theta & \to & \theta + \varepsilon, \quad \bar{\theta} \to \bar{\theta} + \bar{\varepsilon}, \end{array} \tag{3.3}$$

where ε and $\bar{\varepsilon}$ are Grassmannian transformation parameters. From eq.(3.3) one can easily obtain the representation for the supercharges (3.1) acting on the superspace

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \tag{3.4}$$

To define the fields on a superspace, consider representations of the Super-Poincaré group (3.1) [3]. The simplest one is a scalar superfield $F(x,\theta,\bar{\theta})$ which is SUSY invariant. Its Taylor expansion in θ and $\bar{\theta}$ has only several terms due to the nilpotent character of Grassmannian parameters. However, this superfield is a reducible representation of SUSY. To get an irreducible one, we define a *chiral* superfield which obeys the equation

$$\bar{D}_{\dot{\alpha}}F = 0, \qquad \text{where } \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\bar{\theta}}_{\dot{\alpha}}} - i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu \tag{3.5}$$

is a superspace covariant derivative.

For the chiral superfield Grassmannian Taylor expansion looks like $(y = x + i\theta\sigma\bar{\theta})$

$$\begin{array}{rcl} \Phi(y,\theta) & = & A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ & & = A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) \\ & + & \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x). \end{array} \tag{3.6}$$

The coefficients are ordinary functions of x being the usual fields. They are called the *components* of a superfield. In eq.(3.6) one has 2 bosonic (complex scalar field A) and 2 fermionic (Weyl spinor field ψ) degrees of freedom. The component fields A and ψ are called the *superpartners.* The field F is an *auxiliary* field, it has the "wrong" dimension and has no physical meaning. It is needed to close the algebra (3.1). One can get rid of the auxiliary fields with the help of equations of motion.

Thus, a superfield contains an equal number of bosonic and fermionic degrees of freedom. Under SUSY transformation they convert into one another

$$\begin{array}{rcl} \delta_\varepsilon A & = & \sqrt{2}\varepsilon\psi, \\ \delta_\varepsilon\psi & = & i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\ \delta_\varepsilon F & = & i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi. \end{array} \tag{3.7}$$

Notice that the variation of the F-component is a total derivative, i.e. it vanishes when integrated over the space-time.

One can also construct an antichiral superfield Φ^+ obeying the equation

$$D_\alpha \Phi^+ = 0, \quad \text{with } D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i(\sigma^\mu \bar{\theta})_\alpha \partial_\mu .$$

The product of chiral (antichiral) superfields Φ^2, Φ^3, etc is also a chiral (antichiral) superfield, while the product of chiral and antichiral ones $\Phi^+\Phi$ is a general superfield.

For any arbitrary function of chiral superfields one has

$$\begin{aligned} \mathcal{W}(\Phi_i) &= \mathcal{W}(A_i + \sqrt{2}\theta\psi_i + \theta\theta F) \qquad (3.8) \\ &= \mathcal{W}(A_i) + \frac{\partial \mathcal{W}}{\partial A_i}\sqrt{2}\theta\psi_i + \theta\theta\left(\frac{\partial \mathcal{W}}{\partial A_i}F_i - \frac{1}{2}\frac{\partial^2 \mathcal{W}}{\partial A_i \partial A_j}\psi_i\psi_j\right). \end{aligned}$$

The $\mathcal{W}$ is usually referred to as a superpotential which replaces the usual potential for the scalar fields.

To construct the gauge invariant interactions, one needs a real vector superfield $V = V^+$. It is not chiral but rather a general superfield with the following Grassmannian expansion:

$$\begin{aligned} V(x,\theta,\bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \frac{i}{2}\theta\theta[M(x) + iN(x)] \\ &- \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] - \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}[\lambda(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)] \\ &- i\bar{\theta}\bar{\theta}\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}\Box C(x)]. \qquad (3.9) \end{aligned}$$

The physical degrees of freedom corresponding to a real vector superfield V are the vector gauge field v_μ and the Majorana spinor field λ. All other components are unphysical and can be eliminated. Indeed, under the Abelian (super)gauge transformation the superfield V is transformed as

$$V \quad \to \quad V + \Phi + \Phi^+,$$

where Φ and Φ^+ are some chiral superfields. In components it looks like

$$\begin{aligned} C &\to C + A + A^*, \\ \chi &\to \chi - i\sqrt{2}\psi, \\ M + iN &\to M + iN - 2iF, \\ v_\mu &\to v_\mu - i\partial_\mu(A - A^*), \qquad (3.10) \\ \lambda &\to \lambda, \\ D &\to D, \end{aligned}$$

and corresponds to ordinary gauge transformations for physical components. According to eq.(3.10), one can choose a gauge (the Wess-Zumino gauge) where $C = \chi = M = N = 0$, leaving one with only physical degrees of freedom except for the auxiliary field D. In this gauge

$$\begin{aligned} V &= -\theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \\ V^2 &= -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}v_\mu(x)v^\mu(x), \\ V^3 &= 0, \quad etc. \end{aligned} \tag{3.11}$$

One can define also a field strength tensor (as analog of $F_{\mu\nu}$ in gauge theories)

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V}, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2 e^V \bar{D}_\alpha e^{-V}, \tag{3.12}$$

which is a polynomial in the Wess-Zumino gauge. (Here Ds are the supercovariant derivatives.)

The strength tensor is a chiral superfield

$$\bar{D}_{\dot{\beta}} W_\alpha = 0, \quad D_\beta \bar{W}_{\dot{\alpha}} = 0.$$

In the Wess-Zumino gauge it is a polynomial over component fields:

$$W_\alpha = T^a\left(-i\lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F^a_{\mu\nu} + \theta^2(\sigma^\mu D_\mu\bar{\lambda}^a)_\alpha\right), \tag{3.13}$$

where

$$F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu + f^{abc}v^b_\mu v^c_\nu, \quad D_\mu\bar{\lambda}^a = \partial\bar{\lambda}^a + f^{abc}v^b_\mu\bar{\lambda}^c.$$

In Abelian case eqs.(3.12) are simplified and take form

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2\bar{D}_\alpha V.$$

3.3 Construction of SUSY Lagrangians

Let us start with the Lagrangian which has no local gauge invariance. In the superfield notation SUSY invariant Lagrangians are the polynomials of superfields. Having in mind that for component fields one should have ordinary terms and the above mentioned property of SUSY invariance of the highest dimension components of a superfield, the general SUSY invariant Lagrangian has the form

$$\mathcal{L} = \Phi^+_i\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k)|_{\theta\theta} + h.c.]. \tag{3.14}$$

Hereafter the vertical line means the corresponding term of a Taylor expansion.

The first term is a kinetic term. It contains both the chiral and antichiral superfields Φ_i and Φ_i^+, respectively, and is a function of Grassmannian parameters θ and $\bar{\theta}$. Being expanded over θ and $\bar{\theta}$ it leads to the usual kinetic terms for the corresponding component fields.

The terms in the bracket form the superpotential. It is composed of the chiral fields only (plus the hermitian conjugated counterpart composed of antichiral superfields) and is a chiral superfield. Since the products of a chiral superfield and antichiral one produce a general superfield, they are not allowed in a superpotential. The last coefficient of its expansion over the parameter θ is supersymmetrically invariant and gives the usual potential after getting rid of the auxiliary fields.

The Lagrangian (3.14) can be written in a much more elegant way in superspace. The same way as an ordinary action is an integral over space-time of Lagrangian density, in supersymmetric case the action is an integral over the superspace. The space-time Lagrangian density then is [3, 4]

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\ \Phi_i^+\Phi_i + \int d^2\theta\ [\lambda_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k] + h.c. \tag{3.15}$$

where the first part is a kinetic term and the second one is a superpotential $\mathcal{W}$. Here instead of taking the proper components we use integration over the superspace according to the rules of Grassmannian integration [11]

$$\int d\theta_\alpha = 0, \qquad \int \theta_\alpha\ d\theta_\beta = \delta_{\alpha\beta}.$$

Performing explicit integration over the Grassmannian parameters, we get from eq.(3.15)

$$\begin{aligned} \mathcal{L} &= i\partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + A_i^*\Box A_i + F_i^*F_i \qquad (3.16)\\ &+ [\lambda_i F_i + m_{ij}(A_iF_j - \frac{1}{2}\psi_i\psi_j) + y_{ijk}(A_iA_jF_k - \psi_i\psi_jA_k) + h.c.]. \end{aligned}$$

The last two terms are the interaction ones. To obtain a familiar form of the Lagrangian, we have to solve the constraints

$$\frac{\partial\mathcal{L}}{\partial F_k^*} = F_k + \lambda_k^* + m_{ik}^*A_i^* + y_{ijk}^*A_i^*A_j^* = 0, \tag{3.17}$$

$$\frac{\partial\mathcal{L}}{\partial F_k} = F_k^* + \lambda_k + m_{ik}A_i + y_{ijk}A_iA_j = 0. \tag{3.18}$$

Expressing the auxiliary fields F and F^* from these equations, one finally gets

$$\begin{aligned} \mathcal{L} &= i\partial_\mu\bar{\psi}_i\bar{\sigma}^\mu\psi_i + A_i^*\Box A_i - \frac{1}{2}m_{ij}\psi_i\psi_j - \frac{1}{2}m_{ij}^*\bar{\psi}_i\bar{\psi}_j \\ & -y_{ijk}\psi_i\psi_j A_k - y_{ijk}^*\bar{\psi}_i\bar{\psi}_j A_k^* - V(A_i, A_j), \end{aligned} \tag{3.19}$$

where the scalar potential $V = F_k^* F_k$. We will return to the discussion of the form of the scalar potential in SUSY theories later.

Consider now the gauge invariant SUSY Lagrangians. They should contain gauge invariant interaction of the matter fields with the gauge ones and the kinetic term and the self-interaction of the gauge fields.

Let us start with the gauge field kinetic terms. In the Wess-Zumino gauge one has

$$W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda\sigma^\mu D_\mu\bar{\lambda} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^2 + i\frac{1}{4}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma}, \tag{3.20}$$

where $D_\mu = \partial_\mu + ig[v_\mu,]$ is the usual covariant derivative and the last, the so-called topological θ term, is the total derivative.

The gauge invariant Lagrangian now has a familiar form

$$\begin{aligned} \mathcal{L} &= \frac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \frac{1}{4}\int d^2\bar{\theta}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} \\ &= \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\bar{\lambda}. \end{aligned} \tag{3.21}$$

To obtain a gauge-invariant interaction with matter chiral superfields, consider their gauge transformation (Abelian)

$$\Phi \ \to\ e^{-ig\Lambda}\Phi, \quad \Phi^+ \ \to\ \Phi^+ e^{ig\Lambda^+}, \quad V \ \to\ V + i(\Lambda - \Lambda^+),$$

where Λ is a gauge parameter (chiral superfield).

It is clear now how to construct both the SUSY and gauge invariant kinetic term (compare with the covariant derivative in a usual gauge theory)

$$\Phi_i^+\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \Rightarrow \Phi_i^+ e^{gV}\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{3.22}$$

A complete SUSY and gauge invariant Lagrangian then looks like

$$\begin{aligned} \mathcal{L}_{inv} &= \frac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \frac{1}{4}\int d^2\bar{\theta}\ \bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}} + \int d^2\theta d^2\bar{\theta}\ \Phi_i^+ e^{gV}\Phi_i \\ &+ \int d^2\theta\ (\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}y_{ijk}\Phi_i\Phi_j\Phi_k) + h.c. \end{aligned} \tag{3.23}$$

The non-Abelian generalization is straightforward

$$\begin{aligned} \mathcal{L}_{SUSY\ YM} &= \frac{1}{4}\int d^2\theta\ Tr(W^\alpha W_\alpha) + \frac{1}{4}\int d^2\bar{\theta}\ Tr(\bar{W}^\alpha\bar{W}_\alpha) \qquad (3.24) \\ &+ \int d^2\theta d^2\bar{\theta}\ \bar{\Phi}_{ia}(e^{gV})^a_b\Phi^b_i + \int d^2\theta\ \mathcal{W}(\Phi_i) + \int d^2\bar{\theta}\ \bar{\mathcal{W}}(\bar{\Phi}_i), \end{aligned}$$

where $\mathcal{W}$ is a superpotential, which should be invariant under the group of symmetry of a particular model.

In terms of component fields the above Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{SUSY\ YM} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a + \frac{1}{2}D^aD^a \qquad (3.25)\\ &+ (\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger(\partial_\mu A_i - igv^a_\mu T^a A_i) - i\bar{\psi}_i\bar{\sigma}^\mu(\partial_\mu\psi_i - igv^a_\mu T^a\psi_i)\\ &- D^aA^\dagger_i T^a A_i - i\sqrt{2}A^\dagger_i T^a\lambda^a\psi_i + i\sqrt{2}\bar{\psi}_i T^a A_i\bar{\lambda}^a + F^\dagger_i F_i\\ &+ \frac{\partial\mathcal{W}}{\partial A_i}F_i + \frac{\partial\bar{\mathcal{W}}}{\partial A^\dagger_i}F^\dagger_i - \frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\bar{\mathcal{W}}}{\partial A^\dagger_i\partial A^\dagger_j}\bar{\psi}_i\bar{\psi}_j.\end{aligned}$$

Integrating out the auxiliary fields D^a and F_i, one reproduces the usual Lagrangian.

3.4 The scalar potential

Contrary to the SM, where the scalar potential is arbitrary and is defined only by the requirement of the gauge invariance, in supersymmetric theories it is completely defined by the superpotential. It consists of the contributions from the D-terms and F-terms. The kinetic energy of the gauge fields (recall eq.(3.21) yields the $1/2D^aD^a$ term, and the matter-gauge interaction (recall eq.(3.23) yields the $gD^aT^a_{ij}A^*_iA_j$ one. Together they give

$$\mathcal{L}_D = \frac{1}{2}D^aD^a + gD^aT^a_{ij}A^*_iA_j. \qquad (3.26)$$

The equation of motion reads

$$D^a = -gT^a_{ij}A^*_iA_j. \qquad (3.27)$$

Substituting it back into eq.(3.26) yields the D-term part of the potential

$$\mathcal{L}_D = -\frac{1}{2}D^aD^a \quad \Longrightarrow V_D = \frac{1}{2}D^aD^a, \qquad (3.28)$$

where D is given by eq.(3.27).

The F-term contribution can be derived from the matter field self-interaction eq.(3.16). For a general type superpotential W one has

$$\mathcal{L}_F = F^*_iF_i + (\frac{\partial W}{\partial A_i}F_i + h.c.). \qquad (3.29)$$

Using the equations of motion for the auxiliary field F_i

$$F^*_i = -\frac{\partial W}{\partial A_i} \qquad (3.30)$$

yields

$$\mathcal{L}_F = -F_i^* F_i \qquad \Longrightarrow V_F = F_i^* F_i, \tag{3.31}$$

where F is given by eq.(3.30). The full potential is the sum of the two contributions

$$V = V_D + V_F. \tag{3.32}$$

Thus, the form of the Lagrangian is practically fixed by symmetry requirements. The only freedom is the field content, the value of the gauge coupling g, Yukawa couplings y_{ijk} and the masses. Because of the renormalizability constraint $V \leq A^4$ the superpotential should be limited by $\mathcal{W} \leq \Phi^3$ as in eq.(3.15). All members of a supermultiplet have the same masses, i.e. bosons and fermions are degenerate in masses. This property of SUSY theories contradicts the phenomenology and requires supersymmetry breaking.

3.5 Spontaneous breaking of SUSY

Since supersymmetric algebra leads to mass degeneracy in a supermultiplet, it should be broken to explain the absence of superpartners at modern energies. There are several ways of supersymmetry breaking. It can be broken either explicitly or spontaneously. Performing SUSY breaking one has to be careful not to spoil the cancellation of quadratic divergencies which allows one to solve the hierarchy problem. This is achieved by spontaneous breaking of SUSY.

Apart from non-supersymmetric theories in SUSY models the energy is always nonnegative definite. Indeed, according to quantum mechanics

$$E = < 0|\ H\ |0>$$

and due to SUSY algebra eq.(3.1) $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$, taking into account that $tr(\sigma^\mu P_\mu) = 2P_0$, one gets

$$E = \frac{1}{4} \sum_{\alpha=1,2} < 0|\{Q_\alpha, \bar{Q}_\alpha\}|0> = \frac{1}{4} \sum_\alpha |Q_\alpha|0>|^2 \geq 0.$$

Hence

$$E = < 0|\ H\ |0> \neq 0 \quad if\ and\ only\ if \quad Q_\alpha|0> \neq 0.$$

Therefore, supersymmetry is spontaneously broken, i.e. vacuum is not invariant ($Q_\alpha|0> \neq 0$), *if and only if* the minimum of the potential is positive (*i.e.* $E > 0$) .

Spontaneous breaking of supersymmetry is achieved in the same way as the electroweak symmetry breaking. One introduces the field whose vacuum expectation value is nonzero and breaks the symmetry. However, due to a special character of SUSY, this should be a superfield whose auxiliary F and D components acquire nonzero v.e.v.'s. Thus, among possible spontaneous SUSY breaking mechanisms one distinguishes the F and D ones.

i) Fayet-Iliopoulos (D-term) mechanism [12].
In this case the, the linear D-term is added to the Lagrangian

$$\Delta\mathcal{L} = \xi V|_{\theta\theta\bar{\theta}\bar{\theta}} = \xi \int d^4\theta\ V. \tag{3.33}$$

It is gauge and SUSY invariant by itself; however, it may lead to spontaneous breaking of both of them depending on the value of ξ. We show in Fig.5a the sample spectrum for two chiral matter multiplets. The

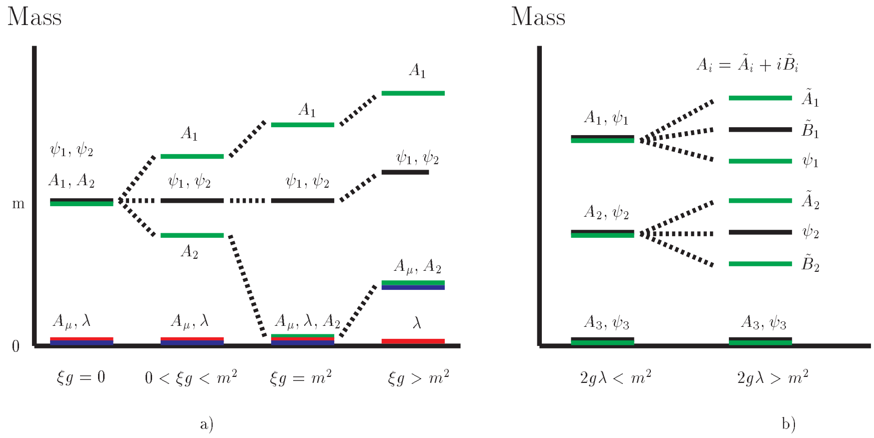

Figure 5. Spectrum of spontaneously broken SUSY theories

drawback of this mechanism is the necessity of $U(1)$ gauge invariance. It can be used in SUSY generalizations of the SM but not in GUTs.

The mass spectrum also causes some troubles since the following sum rule is always valid

$$\sum_{bosonic\ states} m_i^2 = \sum_{fermionic\ states} m_i^2, \tag{3.34}$$

which is bad for phenomenology.

ii) O'Raifeartaigh (F-term) mechanism [13].
In this case, several chiral fields are needed and the superpotential should be chosen in a way that trivial zero v.e.v.s for the auxiliary F-fields be absent. For instance, choosing the superpotential to be

$$\mathcal{W}(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2,$$

one gets the equations for the auxiliary fields

$$\begin{aligned} F_1^* &= mA_2 + 2gA_1A_3, \\ F_2^* &= mA_1, \\ F_3^* &= \lambda + gA_1^2, \end{aligned}$$

which have no solutions with $< F_i >= 0$ and SUSY is spontaneously broken. The sample spectrum is shown in Fig.5b.

The drawbacks of this mechanism is a lot of arbitrariness in the choice of potential. The sum rule (3.34) is also valid here.

Unfortunately, none of these mechanisms explicitly works in SUSY generalizations of the SM. None of the fields of the SM can develop nonzero v.e.v.s for their F or D components without breaking $SU(3)$ or $U(1)$ gauge invariance since they are not singlets with respect to these groups. This requires the presence of extra sources of spontaneous SUSY breaking, which we consider below. They are based, however, on the same F and D mechanisms.

4. SUSY generalization of the Standard Model. The MSSM

As has been already mentioned, in SUSY theories the number of bosonic degrees of freedom equals that of fermionic. At the same time, in the SM one has 28 bosonic and 90 fermionic degrees of freedom (with massless neutrino, otherwise 96). So the SM is to a great extent non-supersymmetric. Trying to add some new particles to supersymmetrize the SM, one should take into account the following observations:

• There are no fermions with quantum numbers of the gauge bosons;

• Higgs fields have nonzero v.e.v.s; hence they cannot be superpartners of quarks and leptons since this would induce spontaneous violation of baryon and lepton numbers;

• One needs at least two complex chiral Higgs multiplets to give masses to Up and Down quarks.

The latter is due to the form of a superpotential and chirality of matter superfields. Indeed, the superpotential should be invariant under the $SU(3) \times SU(2) \times U(1)$ gauge group. If one looks at the Yukawa interaction in the Standard Model, one finds that it is indeed $U(1)$ invariant since the sum of hypercharges in each vertex equals zero. In the last term this is achieved by taking the conjugated Higgs doublet $\tilde{H} = i\tau_2 H^\dagger$ instead of H. However, in SUSY H is a chiral superfield and hence a superpotential, which is constructed out of chiral fields, can contain only H but not $\tilde{H}$ which is an antichiral superfield.

Another reason for the second Higgs doublet is related to chiral anomalies. It is known that chiral anomalies spoil the gauge invariance and, hence, the renormalizability of the theory. They are canceled in the SM between quarks and leptons in each generation. However, if one introduces a chiral Higgs superfield, it contains higgsinos, which are chiral fermions, and contain anomalies. To cancel them one has to add the second Higgs doublet with the opposite hypercharge. Therefore, the Higgs sector in SUSY models is inevitably enlarged, it contains an even number of doublets.

Conclusion: In SUSY models supersymmetry associates *known* bosons with *new* fermions and *known* fermions with *new* bosons.

4.1 The field content

Consider the particle content of the Minimal Supersymmetric Standard Model [14]. According to the previous discussion, in the minimal version we double the number of particles (introducing a superpartner to each particle) and add another Higgs doublet (with its superpartner).

Thus, the characteristic feature of any supersymmetric generalization of the SM is the presence of superpartners (see Fig.6) [15]. If supersymmetry is exact, superpartners of ordinary particles should have the same masses and have to be observed. The absence of them at modern energies is believed to be explained by the fact that their masses are very heavy, that means that supersymmetry should be broken. Hence, if the energy of accelerators is high enough, the superpartners will be created.

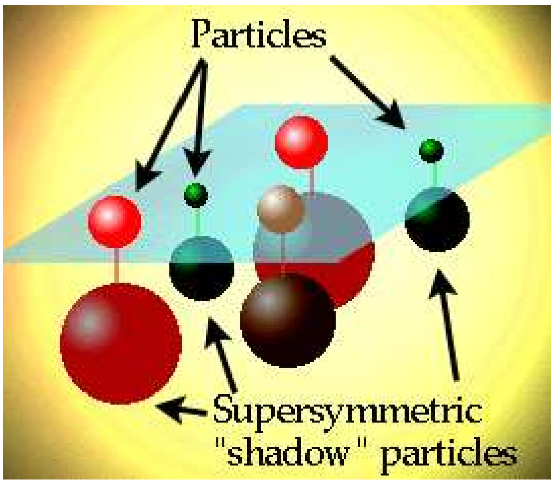

Figure 6. The shadow world of SUSY particles

The particle content of the MSSM then appears as

Particle Content of the MSSM

Superfield	Bosons		Fermions		$SU(3)$	$SU(2$	$U_Y(1)$
Gauge							
$\mathbf{G^a}$	gluon	g^a	gluino	$\tilde{g}^a$	8	0	0
$\mathbf{V^k}$	Weak	W^k $(W^\pm, Z)$	wino, zino	$\tilde{w}^k$ $(\tilde{w}^\pm, \tilde{z})$	1	3	0
$\mathbf{V'}$	Hypercharge	B (γ)	bino	$\tilde{b}(\tilde{\gamma})$	1	1	0
Matter							
$\mathbf{L_i}$	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1
$\mathbf{E_i}$		$\tilde{E}_i = \tilde{e}_R$		$E_i = e_R$	1	1	2
$\mathbf{Q_i}$	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	$1/3$
$\mathbf{U_i}$		$\tilde{U}_i = \tilde{u}_R$		$U_i = u_R^c$	3^*	1	$-4/3$
$\mathbf{D_i}$		$\tilde{D}_i = \tilde{d}_R$		$D_i = d_R^c$	3^*	1	$2/3$
Higgs							
$\mathbf{H_1}$	Higgses	H_1	higgsinos	$\tilde{H}_1$	1	2	-1
$\mathbf{H_2}$		H_2		$\tilde{H}_2$	1	2	1

Hereafter, tilde denotes a superpartner of an ordinary particle.

The presence of an extra Higgs doublet in SUSY model is a novel feature of the theory. In the MSSM one has two doublets with the quantum numbers (1,2,-1) and (1,2,1), respectively:

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix}, \ H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_2^+ \\ v_2 + \frac{S_2 + iP_2}{\sqrt{2}} \end{pmatrix},$$

where v_i are the vacuum expectation values of the neutral components.

Hence, one has 8=4+4=5+3 degrees of freedom. As in the case of the SM, 3 degrees of freedom can be gauged away, and one is left with 5 physical states compared to 1 in the SM. Thus, in the MSSM, as actually in any of two Higgs doublet models, one has five physical Higgs bosons: two CP-even neutral, one CP-odd neutral and two charged. We consider the mass eigenstates below.

4.2 Lagrangian of the MSSM

The Lagrangian of the MSSM consists of two parts; the first part is SUSY generalization of the Standard Model, while the second one represents the SUSY breaking as mentioned above.

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{Breaking}, \tag{4.1}$$

where

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa} \tag{4.2}$$

and

$$\mathcal{L}_{Gauge} = \sum_{SU(3),SU(2),U(1)} \frac{1}{4}\left(\int d^2\theta\ TrW^{\alpha}W_{\alpha} + \int d^2\bar{\theta}\ Tr\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right)$$
$$+ \sum_{Matter} \int d^2\theta d^2\bar{\theta}\ \Phi_i^{\dagger} e^{g_3\hat{V}_3 + g_2\hat{V}_2 + g_1\hat{V}_1}\Phi_i, \quad (4.3)$$

$$\mathcal{L}_{Yukawa} = \int d^2\theta\ (\mathcal{W}_R + \mathcal{W}_{NR}) + h.c. \quad (4.4)$$

The index R in a superpotential refers to the so-called R-parity [16] which adjusts a "+" charge to all the ordinary particles and a "−" charge to their superpartners. The first part of $\mathcal{W}$ is R-symmetric

$$W_R = \epsilon_{ij}(y^U_{ab}Q^j_aU^c_bH^i_2 + y^D_{ab}Q^j_aD^c_bH^i_1 + y^L_{ab}L^j_aE^c_bH^i_1 + \mu H^i_1H^j_2), \quad (4.5)$$

where $i,j = 1,2,3$ are the $SU(2)$ and $a,b = 1,2,3$ are the generation indices; colour indices are suppressed. This part of the Lagrangian almost exactly repeats that of the SM except that the fields are now the superfields rather than the ordinary fields of the SM. The only difference is the last term which describes the Higgs mixing. It is absent in the SM since there is only one Higgs field there.

The second part is R-nonsymmetric

$$W_{NR} = \epsilon_{ij}(\lambda^L_{abd}L^i_aL^j_bE^c_d + \lambda^{L\prime}_{abd}L^i_aQ^j_bD^c_d + \mu'_aL^i_aH^j_2) + \lambda^B_{abd}U^c_aD^c_bD^c_d. \quad (4.6)$$

These terms are absent in the SM. The reason is very simple: one can not replace the superfields in eq.(4.6) by the ordinary fields like in eq.(4.5) because of the Lorentz invariance. These terms have a different property, they violate either lepton (the first 3 terms in eq.(4.6)) or baryon number (the last term). Since both effects are not observed in Nature, these terms must be suppressed or be excluded. One can avoid such terms if one introduces special symmetry called the R-symmetry. This is the global $U(1)_R$ invariance

$$U(1)_R: \quad \theta \to e^{i\alpha}\theta, \quad \Phi \to e^{in\alpha}\Phi, \quad (4.7)$$

which is reduced to the discrete group Z_2, called the R-parity. The R-parity quantum number is given by $R = (-1)^{3(B-L)+2S}$ for particles with spin S. Thus, all the ordinary particles have the R-parity quantum number equal to $R = +1$, while all the superpartners have R-parity quantum number equal to $R = -1$. The R-parity obviously forbids the W_{NR} terms. However, it may well be that these terms are present, though experimental limits on the couplings are very severe

$$\lambda^L_{abc},\ \ \lambda^{L\prime}_{abc} < 10^{-4}, \qquad \lambda^B_{abc} < 10^{-9}.$$

4.3 Properties of interactions

If one assumes that the R-parity is preserved, then the interactions of superpartners are essentially the same as in the SM, but two of three particles involved into an interaction at any vertex are replaced by superpartners. The reason for it is the R-parity. Conservation of the R-parity has two consequences

- the superpartners are created in pairs;
- the lightest superparticle (LSP) is stable. Usually it is photino $\tilde{\gamma}$, the superpartner of a photon with some admixture of neutral higgsino.

Typical vertices are shown in Figs.7. The tilde above a letter denotes the corresponding superpartner. Note that the coupling is the same in all the vertices involving superpartners.

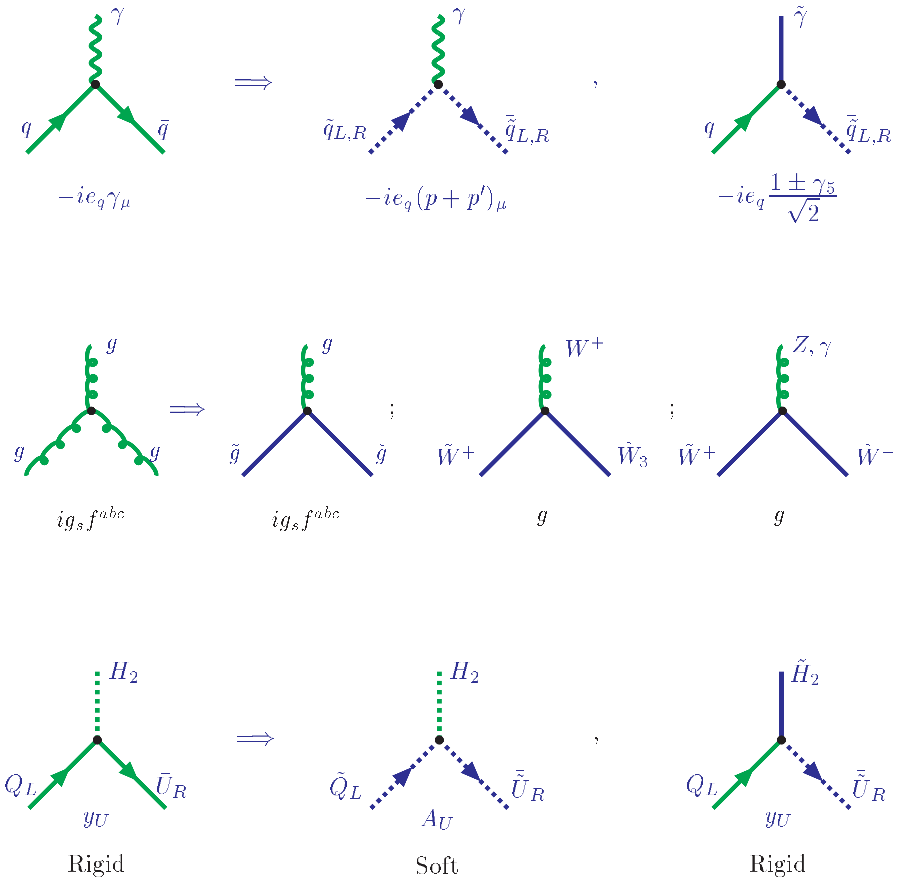

Figure 7. Gauge-matter interaction, Gauge self-interaction and Yukawa-type interaction

4.4 Creation and decay of superpartners

The above-mentioned rule together with the Feynman rules for the SM enables one to draw diagrams describing creation of superpartners. One of the most promising processes is the e^+e^- annihilation (see Fig.8).

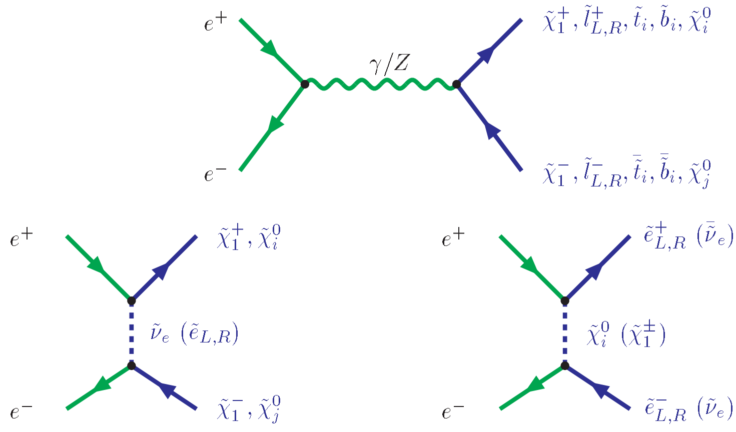

Figure 8. Creation of superpartners

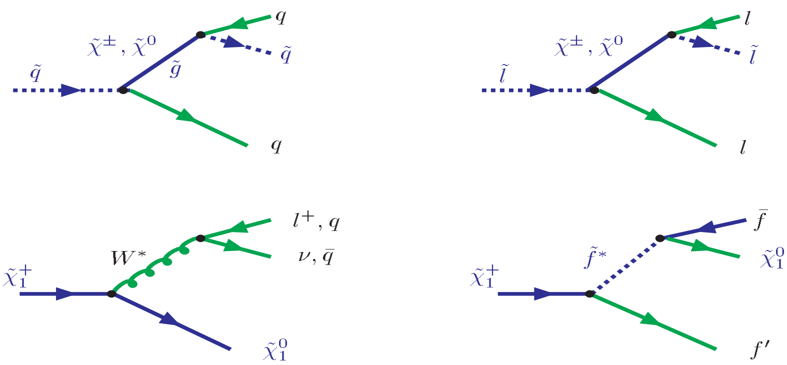

Figure 9. Decay of superpartners

The usual kinematic restriction is given by the c.m. energy $m^{max}_{sparticle} \leq \frac{\sqrt{s}}{2}$. Similar processes take place at hadron colliders with electrons and positrons being replaced by quarks and gluons.

Creation of superpartners can be accompanied by creation of ordinary particles as well. We consider various experimental signatures for e^+e^-

and hadron colliders below. They crucially depend on SUSY breaking pattern and on the mass spectrum of superpartners.

The decay properties of superpartners also depend on their masses. For the quark and lepton superpartners the main processes are shown in Fig.9.

When the R-parity is conserved, new particles will eventually end up giving neutralinos (the lightest superparticle) whose interactions are comparable to those of neutrinos and they leave undetected. Therefore, their signature would be missing energy and transverse momentum. Thus, if supersymmetry exists in Nature and if it is broken somewhere below 1 TeV, then it will be possible to detect it in the nearest future.

5. Breaking of SUSY in the MSSM

Since none of the fields of the MSSM can develop non-zero v.e.v. to break SUSY without spoiling the gauge invariance, it is supposed that spontaneous supersymmetry breaking takes place via some other fields. The most common scenario for producing low-energy supersymmetry breaking is called the *hidden sector* one [17]. According to this scenario, there exist two sectors: the usual matter belongs to the "visible" one, while the second, "hidden" sector, contains fields which lead to breaking of supersymmetry. These two sectors interact with each other by exchange of some fields called *messengers*, which mediate SUSY breaking from the hidden to the visible sector. There might be various types of messenger fields: gravity, gauge, etc. The hidden sector is the weakest part of the MSSM. It contains a lot of ambiguities and leads to uncertainties of the MSSM predictions considered below.

So far there are known four main mechanisms to mediate SUSY breaking from a hidden to a visible sector:

- Gravity mediation (SUGRA) [18];
- Gauge mediation [19];
- Anomaly mediation [20];
- Gaugino mediation [21].

All four mechanisms of soft SUSY breaking are different in details but are common in results. Predictions for the sparticle spectrum depend on the mechanism of SUSY breaking. For comparison of four above-mentioned mechanisms we show in Fig.10 the sample spectra as the ratio to the gaugino mass M_2 [22].

In what follows, to calculate the mass spectrum of superpartners, we need an explicit form of SUSY breaking terms. For the MSSM and

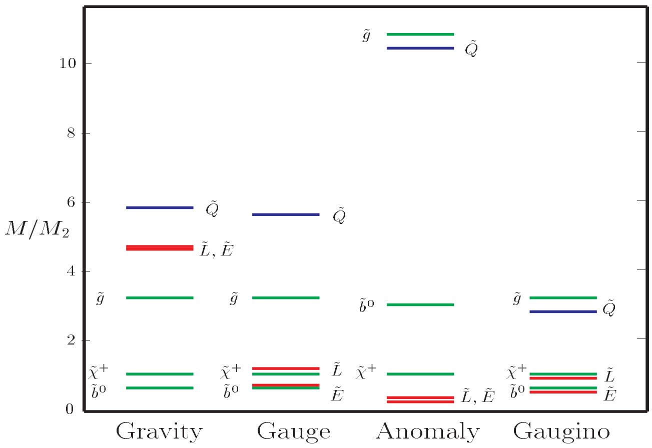

Figure 10. Superparticle spectra for various mediation mechanisms

without the R-parity violation one has

$$
\begin{aligned}
-\mathcal{L}_{Breaking} &= \sum_i m_{0i}^2 |\varphi_i|^2 + \left(\frac{1}{2} \sum_\alpha M_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha + B H_1 H_2 \right. \\
&+ \left. A^U_{ab} \tilde{Q}_a \tilde{U}^c_b H_2 + A^D_{ab} \tilde{Q}_a \tilde{D}^c_b H_1 + A^L_{ab} \tilde{L}_a \tilde{E}^c_b H_1 + h.c. \right),
\end{aligned} \tag{5.1}
$$

where we have suppressed the $SU(2)$ indices. Here φ_i are all scalar fields, $\tilde{\lambda}_\alpha$ are the gaugino fields, $\tilde{Q}, \tilde{U}, \tilde{D}$ and $\tilde{L}, \tilde{E}$ are the squark and slepton fields, respectively, and $H_{1,2}$ are the SU(2) doublet Higgs fields.

Eq.(5.1) contains a vast number of free parameters which spoils the prediction power of the model. To reduce their number, we adopt the so-called *universality* hypothesis, i.e., we assume the universality or equality of various soft parameters at a high energy scale, namely, we put all the spin 0 particle masses to be equal to the universal value m_0, all the spin 1/2 particle (gaugino) masses to be equal to $m_{1/2}$ and all the cubic and quadratic terms, proportional to A and B, to repeat the structure of the Yukawa superpotential (4.5). This is an additional requirement motivated by the supergravity mechanism of SUSY breaking. Universality is not a necessary requirement and one may consider nonuniversal soft terms as well. However, it will not change the qualitative picture presented below; so for simplicity, in what follows we consider the universal boundary conditions. In this case, eq.(5.1) takes the form

$$
-\mathcal{L}_{Breaking} = m_0^2 \sum_i |\varphi_i|^2 + \left(\frac{1}{2} m_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha \right. \tag{5.2}
$$

$$+ \ A[y^U_{ab}\tilde{Q}_a\tilde{U}^c_b H_2 + y^D_{ab}\tilde{Q}_a\tilde{D}^c_b H_1 + y^L_{ab}\tilde{L}_a\tilde{E}^c_b H_1] + B[\mu H_1 H_2] + h.c.\Big) ,$$

The soft terms explicitly break supersymmetry. As will be shown later, they lead to the mass spectrum of superpartners different from that of ordinary particles. Remind that the masses of quarks and leptons remain zero until $SU(2)$ invariance is spontaneously broken.

5.1 The soft terms and the mass formulas

There are two main sources of the mass terms in the Lagrangian: the D terms and soft ones. With given values of $m_0, m_{1/2}, \mu, Y_t, Y_b, Y_\tau, A$, and B one can construct the mass matrices for all the particles. Knowing them at the GUT scale, one can solve the corresponding RG equations, thus linking the values at the GUT and electroweak scales. Substituting these parameters into the mass matrices, one can predict the mass spectrum of superpartners [23, 24].

Gaugino-higgsino mass terms. The mass matrix for gauginos, the superpartners of the gauge bosons, and for higgsinos, the superpartners of the Higgs bosons, is nondiagonal, thus leading to their mixing. The mass terms look like

$$\mathcal{L}_{Gaugino-Higgsino} = -\frac{1}{2}M_3\bar{\lambda}_a\lambda_a - \frac{1}{2}\bar{\chi}M^{(0)}\chi - (\bar{\psi}M^{(c)}\psi + h.c.), \quad (5.3)$$

where $\lambda_a, a = 1, 2, \ldots, 8$, are the Majorana gluino fields and

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}^0_1 \\ \tilde{H}^0_2 \end{pmatrix}, \quad \psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} \quad (5.4)$$

are, respectively, the Majorana neutralino and Dirac chargino fields.

The neutralino mass matrix is

$$M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin_W & M_Z\sin\beta\sin_W \\ 0 & M_2 & M_Z\cos\beta\cos_W & -M_Z\sin\beta\cos_W \\ -M_Z\cos\beta\sin_W & M_Z\cos\beta\cos_W & 0 & -\mu \\ M_Z\sin\beta\sin_W & -M_Z\sin\beta\cos_W & -\mu & 0 \end{pmatrix}, \quad (5.5)$$

where $\tan\beta = v_2/v_1$ is the ratio of two Higgs v.e.v.s and $\sin_W = \sin\theta_W$ is the usual sinus of the weak mixing angle. The physical neutralino masses $M_{\tilde{\chi}^0_i}$ are obtained as eigenvalues of this matrix after diagonalization.

For charginos one has

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix}. \quad (5.6)$$

This matrix has two chargino eigenstates $\tilde{\chi}^{\pm}_{1,2}$ with mass eigenvalues

$$\begin{aligned} M^2_{1,2} &= \frac{1}{2}\Big[M_2^2 + \mu^2 + 2M_W^2 \\ &\mp \sqrt{(M_2^2-\mu^2)^2 + 4M_W^4\cos^2 2\beta + 4M_W^2(M_2^2+\mu^2+2M_2\mu\sin 2\beta)}\Big]. \end{aligned} \tag{5.7}$$

Squark and slepton masses. Non-negligible Yukawa couplings cause a mixing between the electroweak eigenstates and the mass eigenstates of the third generation particles. The mixing matrices for $\tilde{m}_t^2, \tilde{m}_b^2$ and $\tilde{m}_\tau^2$ are

$$\begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t-\mu\cot\beta) & \tilde{m}_{tR}^2 \end{pmatrix}, \tag{5.8}$$

$$\begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b-\mu\tan\beta) & \tilde{m}_{bR}^2 \end{pmatrix}, \tag{5.9}$$

$$\begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau-\mu\tan\beta) & \tilde{m}_{\tau R}^2 \end{pmatrix} \tag{5.10}$$

with

$$\begin{aligned} \tilde{m}_{tL}^2 &= \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta, \\ \tilde{m}_{tR}^2 &= \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta, \\ \tilde{m}_{bL}^2 &= \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta, \\ \tilde{m}_{bR}^2 &= \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta, \\ \tilde{m}_{\tau L}^2 &= \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta, \\ \tilde{m}_{\tau R}^2 &= \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta \end{aligned}$$

and the mass eigenstates are the eigenvalues of these mass matrices. For the light generations the mixing is negligible.

The first terms here ($\tilde{m}^2$) are the soft ones, which are calculated using the RG equations starting from their values at the GUT (Planck) scale. The second ones are the usual masses of quarks and leptons and the last ones are the D terms of the potential.

5.2 The Higgs potential

As has already been mentioned, the Higgs potential in the MSSM is totally defined by superpotential (and the soft terms). Due to the structure of $\mathcal{W}$ the Higgs self-interaction is given by the D-terms while the F-terms contribute only to the mass matrix. The tree level potential is

$$\begin{aligned} V_{tree}(H_1, H_2) &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + h.c.) \\ &+ \frac{g^2 + g^{'2}}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+H_2|^2, \quad (5.11) \end{aligned}$$

where $m_1^2 = m_{H_1}^2 + \mu^2, m_2^2 = m_{H_2}^2 + \mu^2$. At the GUT scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, $m_3^2 = -B\mu_0$. Notice that the Higgs self-interaction coupling in eq.(5.10) is fixed and defined by the gauge interactions as opposed to the SM.

The potential (5.10), in accordance with supersymmetry, is positive definite and stable. It has no nontrivial minimum different from zero. Indeed, let us write the minimization condition for the potential (5.10)

$$\frac{1}{2}\frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2 + g'^2}{4}(v_1^2 - v_2^2)v_1 = 0, \quad (5.12)$$

$$\frac{1}{2}\frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 + \frac{g^2 + g'^2}{4}(v_1^2 - v_2^2)v_2 = 0, \quad (5.13)$$

where we have introduced the notation

$$< H_1 >\equiv v_1 = v\cos\beta, \ < H_2 >\equiv v_2 = v\sin\beta, \ v^2 = v_1^2 + v_2^2, \ \tan\beta \equiv \frac{v_2}{v_1}.$$

Solution of eqs.(5.12),(5.13) can be expressed in terms of v^2 and $\sin 2\beta$

$$v^2 = \frac{4(m_1^2 - m_2^2\tan^2\beta)}{(g^2 + g'^2)(\tan^2\beta - 1)}, \quad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}. \quad (5.14)$$

One can easily see from eq.(5.14) that if $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, v^2 happens to be negative, i.e. the minimum does not exist. In fact, real positive solutions to eqs.(5.12),(5.13) exist only if the following conditions are satisfied:

$$m_1^2 + m_2^2 > 2m_3^2, \quad m_1^2 m_2^2 < m_3^4, \quad (5.15)$$

which is not the case at the GUT scale. This means that spontaneous breaking of the $SU(2)$ gauge invariance, which is needed in the SM to give masses for all the particles, does not take place in the MSSM.

This strong statement is valid, however, only at the GUT scale. Indeed, going down with energy, the parameters of the potential (5.10) are renormalized. They become the "running" parameters with the energy scale dependence given by the RG equations. The running of the parameters leads to a remarkable phenomenon known as *radiative spontaneous symmetry breaking* to be discussed below.

Provided conditions (5.15) are satisfied, the mass matrices at the tree level are
CP-odd components P_1 and P_2 :

$$\mathcal{M}^{odd} = \left.\frac{\partial^2 V}{\partial P_i \partial P_j}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} m_3^2, \tag{5.16}$$

CP-even neutral components S_1 and S_2:

$$\mathcal{M}^{ev} = \left.\frac{\partial^2 V}{\partial S_i \partial S_j}\right| = \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} m_3^2 + \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} M_Z \frac{\sin 2\beta}{2}, \tag{5.17}$$

Charged components H^- and H^+:

$$\mathcal{M}^{ch} = \left.\frac{\partial^2 V}{\partial H_i^+ \partial H_j^-}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} (m_3^2 + M_W \cos\beta \sin\beta). \tag{5.18}$$

Diagonalizing the mass matrices, one gets the mass eigenstates:

$$\left\{ \begin{array}{lcll} G^0 & = & -\cos\beta P_1 + \sin\beta P_2, & \textit{Goldstone boson} \ \to Z_0, \\ A & = & \sin\beta P_1 + \cos\beta P_2, & \textit{Neutral } CP = -1 \textit{ Higgs}, \end{array} \right.$$

$$\left\{ \begin{array}{lcll} G^+ & = & -\cos\beta (H_1^-)^* + \sin\beta H_2^+, & \textit{Goldstone boson} \ \to W^+, \\ H^+ & = & \sin\beta (H_1^-)^* + \cos\beta H_2^+, & \textit{Charged Higgs}, \end{array} \right.$$

$$\left\{ \begin{array}{lcll} h & = & -\sin\alpha S_1 + \cos\alpha S_2, & \textit{SM Higgs boson } CP = 1, \\ H & = & \cos\alpha S_1 + \sin\alpha S_2, & \textit{Extra heavy Higgs boson}, \end{array} \right.$$

where the mixing angle α is given by

$$\tan 2\alpha = \tan 2\beta \left(\frac{m_A^2 + M_Z^2}{m_A^2 - M_Z^2} \right).$$

The physical Higgs bosons acquire the following masses [14]:

$$\begin{array}{rl} \text{CP-odd neutral Higgs } \ A: & m_A^2 = m_1^2 + m_2^2, \\ \text{Charge Higgses } \ H^\pm: & m_{H^\pm}^2 = m_A^2 + M_W^2, \end{array} \tag{5.19}$$

CP-even neutral Higgses H, h:

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \right], \tag{5.20}$$

where, as usual,

$$M_W^2 = \frac{g^2}{2} v^2, \quad M_Z^2 = \frac{g^2 + g'^2}{2} v^2.$$

This leads to the once celebrated SUSY mass relations

$$\begin{aligned} m_{H^\pm} \geq M_W, \quad m_h \leq m_A \leq M_H, \\ m_h \leq M_Z|\cos 2\beta| \leq M_Z, \quad m_h^2 + m_H^2 = m_A^2 + M_Z^2. \end{aligned} \tag{5.21}$$

Thus, the lightest neutral Higgs boson happens to be lighter than the Z boson, which clearly distinguishes it from the SM one. Though we do not know the mass of the Higgs boson in the SM, there are several indirect constraints leading to the lower boundary of $m_h^{SM} \geq 135$ GeV. After including the radiative corrections, the mass of the lightest Higgs boson in the MSSM, m_h, however increases. We consider it in more detail below.

5.3 Renormalization group analysis

To calculate the low energy values of the soft terms, we use the corresponding RG equations. The one-loop RG equations for the rigid MSSM couplings are [25]

$$\begin{aligned} \frac{d\tilde\alpha_i}{dt} &= b_i \tilde\alpha_i^2, \qquad t \equiv \log Q^2/M_{GUT}^2 \\ \frac{dY_U}{dt} &= -Y_L\left(\frac{16}{3}\tilde\alpha_3 + 3\tilde\alpha_2 + \frac{13}{15}\tilde\alpha_1 - 6Y_U - Y_D\right), \\ \frac{dY_D}{dt} &= -Y_D\left(\frac{16}{3}\tilde\alpha_3 + 3\tilde\alpha_2 + \frac{7}{15}\tilde\alpha_1 - Y_U - 6Y_D - Y_L\right), \\ \frac{dY_L}{dt} &= -Y_L\left(3\tilde\alpha_2 + \frac{9}{5}\tilde\alpha_1 - 3Y_D - 4Y_L\right), \end{aligned} \tag{5.22}$$

where we use the notation $\tilde\alpha = \alpha/4\pi = g^2/16\pi^2, \ Y = y^2/16\pi^2$.

For the soft terms one finds

$$\begin{aligned} \frac{dM_i}{dt} &= b_i \tilde\alpha_i M_i. \\ \frac{dA_U}{dt} &= \frac{16}{3}\tilde\alpha_3 M_3 + 3\tilde\alpha_2 M_2 + \frac{13}{15}\tilde\alpha_1 M_1 + 6Y_U A_U + Y_D A_D, \\ \frac{dA_D}{dt} &= \frac{16}{3}\tilde\alpha_3 M_3 + 3\tilde\alpha_2 M_2 + \frac{7}{15}\tilde\alpha_1 M_1 + 6Y_D A_D + Y_U A_U + Y_L A_L, \\ \frac{dA_L}{dt} &= 3\tilde\alpha_2 M_2 + \frac{9}{5}\tilde\alpha_1 M_1 + 3Y_D A_D + 4Y_L A_L, \\ \frac{dB}{dt} &= 3\tilde\alpha_2 M_2 + \frac{3}{5}\tilde\alpha_1 M_1 + 3Y_U A_U + 3Y_D A_D + Y_L A_L. \\ \frac{d\tilde m_Q^2}{dt} &= -\Big[(\frac{16}{3}\tilde\alpha_3 M_3^2 + 3\tilde\alpha_2 M_2^2 + \frac{1}{15}\tilde\alpha_1 M_1^2) \end{aligned}$$

$$
\begin{aligned}
& - \quad Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2) - Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2)\Big] , \\
\frac{d\tilde{m}_U^2}{dt} &= -\left[(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{15}\tilde{\alpha}_1 M_1^2) - 2Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2)\right] , \\
\frac{d\tilde{m}_D^2}{dt} &= -\left[(\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{4}{15}\tilde{\alpha}_1 M_1^2) - 2Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2)\right] , \\
\frac{d\tilde{m}_L^2}{dt} &= -\left[3(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2) - Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2)\right] , \\
\frac{d\tilde{m}_E^2}{dt} &= -\left[(\frac{12}{5}\tilde{\alpha}_1 M_1^2) - 2Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2)\right] , \\
\frac{d\mu^2}{dt} &= -\mu^2\left[3(\tilde{\alpha}_2 + \frac{1}{5}\tilde{\alpha}_1) - (3Y_U + 3Y_D + Y_L)\right] , \qquad (5.23) \\
\frac{dm_{H_1}^2}{dt} &= -\Big[3(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2) - 3Y_D(\tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2 + A_D^2) \\
& \quad -Y_L(\tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2 + A_L^2)\Big] , \\
\frac{dm_{H_2}^2}{dt} &= -\left[3(\tilde{\alpha}_2 M_2^2 + \frac{1}{5}\tilde{\alpha}_1 M_1^2) - 3Y_U(\tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2 + A_U^2)\right] .
\end{aligned}
$$

Having all the RG equations, one can now find the RG flow for the soft terms. Taking the initial values of the soft masses at the GUT scale in the interval between $10^2 \div 10^3$ GeV consistent with the SUSY scale suggested by unification of the gauge couplings (2.7) leads to the RG flow of the soft terms shown in Fig.11. [23, 24]

One should mention the following general features common to any choice of initial conditions:

i) The gaugino masses follow the running of the gauge couplings and split at low energies. The gluino mass is running faster than the others and is usually the heaviest due to the strong interaction.

ii) The squark and slepton masses also split at low energies, the stops (and sbottoms) being the lightest due to relatively big Yukawa couplings of the third generation.

iii) The Higgs masses (or at least one of them) are running down very quickly and may even become negative.

Typical dependence of the mass spectra on the initial conditions (m_0) is also shown in Fig.12 [26]. For a given value of $m_{1/2}$ the masses of the lightest particles are practically independent of m_0, while the heavier ones increase with it monotonically. One can see that the lightest neutralinos and charginos as well as the stop squark may be rather light.

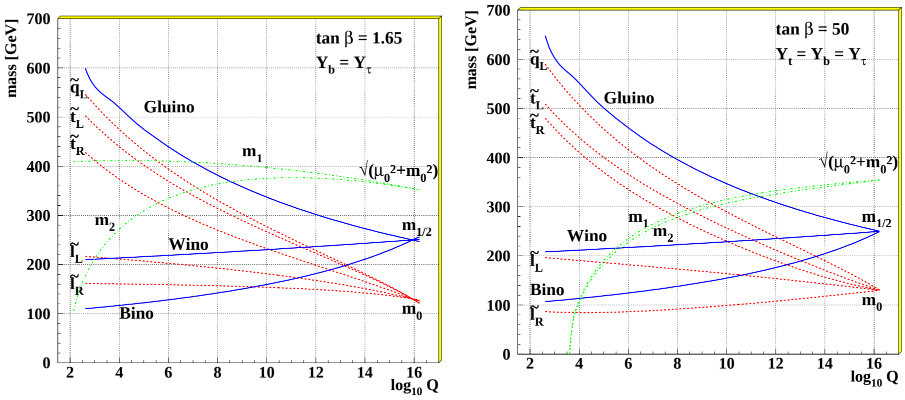

Figure 11. An example of evolution of sparticle masses and soft supersymmetry breaking parameters $m_1^2 = m_{H_1}^2 + \mu^2$ and $m_2^2 = m_{H_2}^2 + \mu^2$ for low (left) and high (right) values of $\tan\beta$

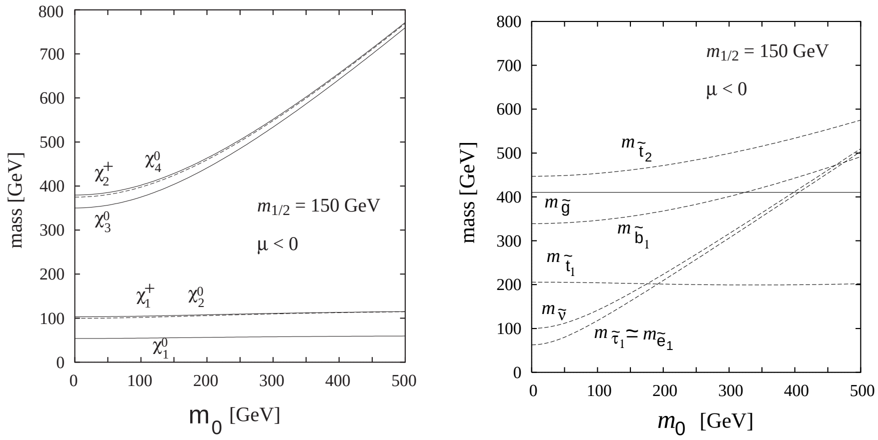

Figure 12. The masses of sparticles as functions of the initial value m_0

5.4 Radiative electroweak symmetry breaking

The running of the Higgs masses leads to the phenomenon known as *radiative electroweak symmetry breaking.* Indeed, one can see in Fig.11 that m_2^2 (or both m_1^2 and m_2^2) decreases when going down from the GUT scale to the M_Z scale and can even become negative. As a result, at some value of Q^2 the conditions (5.15) are satisfied, so that the nontrivial minimum appears. This triggers spontaneous breaking of the $SU(2)$ gauge invariance. The vacuum expectations of the Higgs fields acquire nonzero values and provide masses to quarks, leptons and $SU(2)$ gauge bosons, and additional masses to their superpartners.

In this way one also obtains the explanation of why the two scales are so much different. Due to the logarithmic running of the parameters, one needs a long "running time" to get m_2^2 (or both m_1^2 and m_2^2) to be negative when starting from a positive value of the order of $M_{SUSY} \sim 10^2 \div 10^3$ GeV at the GUT scale.

6. Constrained MSSM

6.1 Parameter space of the MSSM

The Minimal Supersymmetric Standard Model has the following free parameters: i) three gauge couplings α_i; ii) three matrices of the Yukawa couplings y^i_{ab}, where $i = L, U, D$; iii) the Higgs field mixing parameter μ; iv) the soft supersymmetry breaking parameters. Compared to the SM there is an additional Higgs mixing parameter, but the Higgs self-coupling, which is arbitrary in the SM, is fixed by supersymmetry. The main uncertainty comes from the unknown soft terms.

With the universality hypothesis one is left with the following set of 5 free parameters defining the mass scales

$$\mu,\ m_0,\ m_{1/2},\ A \text{ and } B \leftrightarrow \tan\beta = \frac{v_2}{v_1}.$$

While choosing parameters and making predictions, one has two possible ways to proceed:

i) take the low-energy parameters like superparticle masses $\tilde{m}_{t1}, \tilde{m}_{t2}, m_A$, $\tan\beta$, mixings X_{stop}, μ, etc. as input and calculate cross-sections as functions of these parameters.

ii) take the high-energy parameters like the above mentioned 5 soft parameters as input, run the RG equations and find the low-energy values. Now the calculations can be carried out in terms of the initial parameters. The experimental constraints are sufficient to determine these parameters, albeit with large uncertainties.

6.2 The choice of constraints

When subjecting constraints on the MSSM, perhaps, the most remarkable fact is that all of them can be fulfilled simultaneously. In our analysis we impose the following constraints on the parameter space of the MSSM:

• Gauge coupling constant unification;

This is one of the most restrictive constraints, which we have discussed in Sect 2. It fixes the scale of SUSY breaking of an order of 1 TeV.

• M_Z from electroweak symmetry breaking;
Radiative EW symmetry breaking (see eq.(5.14)) defines the mass of the Z-boson

$$M_Z^2 = 2\frac{m_1^2 - m_2^2 \tan^2\beta}{\tan^2\beta - 1}. \tag{6.1}$$

This condition determines the value of μ for given values of m_0 and $m_{1/2}$.
• Yukawa coupling constant unification;
The masses of top, bottom and τ can be obtained from the low energy values of the running Yukawa couplings via

$$m_t = y_t\ v\sin\beta, \quad m_b = y_b\ v\cos\beta, \quad m_\tau = y_\tau\ v\cos\beta. \tag{6.2}$$

They can be translated to the pole masses with account taken of the radiative corrections. The requirement of bottom-tau Yukawa coupling unification strongly restricts the possible solutions in m_t versus $\tan\beta$ plane [27] as it can be seen from Fig.13.

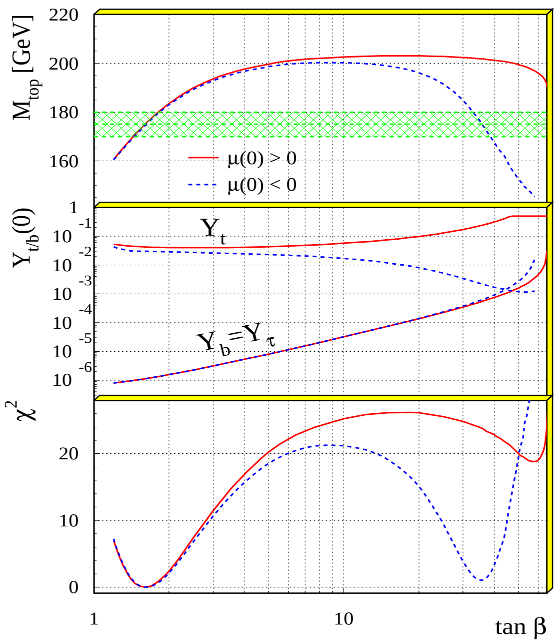

Figure 13. The upper part shows the top quark mass as a function of $\tan\beta$ for $m_0 = 600$ GeV, $m_{1/2} = 400$ GeV. The middle part shows the corresponding values of the Yukawa couplings at the GUT scale and the lower part of the χ^2 values.

• Precision measurement of decay rates;
We take the branching ratio $BR(b \to s\gamma)$ which has been measured by the CLEO [28] collaboration and later by ALEPH [29] and yields

the world average of $BR(b \to s\gamma) = (3.14 \pm 0.48) \cdot 10^{-4}$. The Standard Model contribution to this process gives slightly lower result, thus leaving window for SUSY. This requirement imposes severe restrictions on the parameter space, especially for the case of large $\tan\beta$.

• Anomalous magnetic moment of muon.

Recent measurement of the anomalous magnetic moment indicates small deviation from the SM of the order of 2 σ. The deficiency may be easily filled with SUSY contribution, which is proportional to μ. This requires positive sign of μ that kills have of the parameter space of the MSSM [30].

• Experimental lower limits on SUSY masses;

SUSY particles have not been found so far and from the searches at LEP one knows the lower limit on the charged lepton and chargino masses of about half of the centre of mass energy [31]. The lower limit on the neutralino masses is smaller. There exist also limits on squark and gluino masses from the hadron colliders [32]. These limits restrict the minimal values for the SUSY mass parameters.

• Dark Matter constraint;

Recent very precise astrophysical data restrict the amount of the Dark matter in the Universe up to 23%. Assuming $h_0 > 0.4$ one finds that the contribution of each relic particle species χ has to obey $\Omega_\chi h_0^2 \sim 0.1 \div 0.3$,. This serves as a very severe bound on SUSY parameters [33].

Having in mind the above mentioned constraints one can find the most probable region of the parameter space by minimizing the χ^2 function [24]. We first choose the value of the Higgs mixing parameter μ from the requirement of radiative EW symmetry breaking, then we take the values of $\tan\beta$ from the requirement of Yukawa coupling unification (see Fig.13). One finds two possible solutions: low $\tan\beta$ solution corresponding to $\tan\beta \approx 1.7$ and high $\tan\beta$ solution corresponding to $\tan\beta \approx 30 \div 60$.

What is left are the values of the soft parameters A, m_0 and $m_{1/2}$. However, the role of the trilinear coupling A is not essential. In what follows, we consider the plane $m_0, m_{1/2}$ and find the allowed region in this plane. Each point at this plane corresponds to a fixed set of parameters and allows one to calculate the spectrum, the cross-sections, etc.

We present the allowed regions of the parameter space for low and high $\tan\beta$ scenarios in Fig.14. This plot demonstrates the role of various constraints in the χ^2 function. The contours enclose domains by the particular constraints used in the analysis [34].

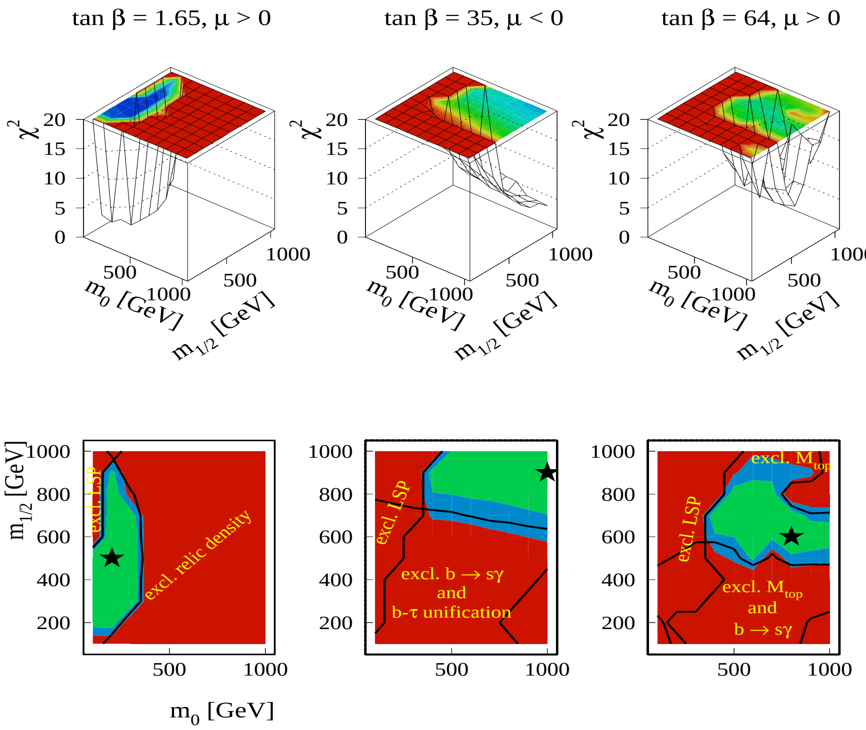

Figure 14. The χ^2-distribution for low and high $\tan\beta$ solutions. The different shades in the projections indicate steps of $\Delta\chi^2 = 4$. The stars indicate the optimum solution. Contours enclose domains by the particular constraints used in the analysis.

6.3 The mass spectrum of superpartners

When the parameter set is fixed, one can calculate the mass spectrum of superpartners. Below we show the predicted mass spectrum corresponding to the best fit values indicated by stars in Fig.14 (see Table 1) [24].

6.4 Experimental signatures at e^+e^- colliders

Experiments are finally beginning to push into a significant region of supersymmetry parameter space. We know the sparticles and their couplings, but we do not know their masses and mixings. Given the mass spectrum one can calculate the cross-sections and consider the possibilities of observing new particles at modern accelerators. Otherwise, one can get restrictions on unknown parameters.

We start with e^+e^- colliders and, first of all, with LEP II. In the leading order creation of superpartners is given by the diagrams shown in Fig.8 above. For a given center of mass energy the cross-sections depend

SUSY masses in [GeV]		
Symbol	low $\tan\beta$	high $\tan\beta$
$\tilde{\chi}_1^0(\tilde{B})$, $\tilde{\chi}_2^0(\tilde{W}^3)$	214, 413	170, 322
$\tilde{\chi}_3^0(\tilde{H}_1)$,$\tilde{\chi}_4^0(\tilde{H}_2)$	1028, 1016	481, 498
$\tilde{\chi}_1^\pm(\tilde{W}^\pm)$, $\tilde{\chi}_2^\pm(\tilde{H}^\pm)$	413, 1026	322, 499
$\tilde{g}$	1155	950
$\tilde{e}_L$, $\tilde{e}_R$	303, 270	663, 621
$\tilde{\nu}_L$	290	658
$\tilde{q}_L$, $\tilde{q}_R$	1028, 936	1040, 1010
$\tilde{\tau}_1$, $\tilde{\tau}_2$	279, 403	537, 634
$\tilde{b}_1$, $\tilde{b}_2$	953, 1010	835, 915
$\tilde{t}_1$, $\tilde{t}_2$	727, 1017	735, 906
h, H	95, 1344	119, 565
A, $H^\pm$	1340, 1344	565, 571

Table 1. Values of the SUSY mass spectra for the low and high $\tan\beta$ solutions.

on the mass of created particles and vanish at the kinematic boundary. Experimental signatures are defined by the decay modes which vary with the mass spectrum. The main ones are summarized below.

Production	Key Decay Modes	Signatures
• $\tilde{l}_{L,R}\tilde{l}_{L,R}$	$\tilde{l}_R^\pm \to l^\pm\tilde{\chi}_i^0 \searrow$ cascade	acomplanar pair of
	$\tilde{l}_L^\pm \to l^\pm\tilde{\chi}_i^0 \nearrow$ decays	charged leptons $+ \not{E}_T$
• $\tilde{\nu}\tilde{\nu}$	$\tilde{\nu} \to l^\pm\tilde{\chi}_1^0$	$\not{E}_T$
• $\tilde{\chi}_1^\pm\tilde{\chi}_1^\pm$	$\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 l^\pm\nu,\ \tilde{\chi}_1^0 q\bar{q}'$	isol lept + 2 jets $+ \not{E}_T$
	$\tilde{\chi}_1^\pm \to \tilde{\chi}_2^0 f\bar{f}'$	pair of acomplanar
	$\tilde{\chi}_1^\pm \to l\tilde{\nu}_l \to l\nu_l\tilde{\chi}_1^0$	leptons $+ \not{E}_T$
	$\tilde{\chi}_1^\pm \to \nu_l\tilde{l} \to \nu_l l\tilde{\chi}_1^0$	4 jets $+ \not{E}_T$
• $\tilde{\chi}_i^0\tilde{\chi}_j^0$	$\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 X, \tilde{\chi}_j^0 \to \tilde{\chi}_1^0 X'$	$X = \nu_l\bar{\nu}_l$ invisible
		$= \gamma, 2l, 2$ jets
		$2l + \not{E}_T, l + 2j + \not{E}_T$
• $\tilde{t}_i\tilde{t}_j$	$\tilde{t}_1 \to c\tilde{\chi}_1^0$	2 jets $+ \not{E}_T$
	$\tilde{t}_1 \to b\tilde{\chi}_1^\pm \to bf\bar{f}'\tilde{\chi}_1^0$	2 b jets + 2 leptons $+ \not{E}_T$
		2 b jets + lepton $+ \not{E}_T$

•	$\tilde{b}_i\tilde{b}_j$	$\tilde{b}_i \to b\tilde{\chi}_1^0$	2 b jets + $\not{E}_T$
		$\tilde{b}_i \to b\tilde{\chi}_2^0 \to bf\bar{f}'\tilde{\chi}_1^0$	2 b jets + 2 leptons + $\not{E}_T$
			2 b jets + 2 jets + $\not{E}_T$

A characteristic feature of all possible signatures is the missing energy and transverse momenta, which is a trade mark of a new physics.

Numerous attempts to find superpartners at LEP II gave no positive result thus imposing the lower bounds on their masses [31]. Typical LEP II limits on the masses of superpartners are

$$\begin{array}{lll} m_{\chi_1^0} > 40\ GeV & m_{\tilde{e}_{L,R}} > 105\ GeV & m_{\tilde{t}} > 90\ GeV \\ m_{\chi_1^\pm} > 100\ GeV & m_{\tilde{\mu}_{L,R}} > 100\ GeV & m_{\tilde{b}} > 80\ GeV \\ & m_{\tilde{\tau}_{L,R}} > 80\ GeV & \end{array} \tag{6.3}$$

6.5 Experimental signatures at hadron colliders

Experimental signatures at hadron colliders are similar to those at e^+e^- machines; however, here one has much wider possibilities. Besides the usual annihilation channel identical to e^+e^- one with the obvious replacement of electrons by quarks (see Fig.8), one has numerous processes of gluon fusion, quark-antiquark and quark-gluon scattering (see Fig.15).

Experimental SUSY signatures at the Tevatron (and LHC) are

	Production	Key Decay Modes		Signatures
•	$\tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{g}\tilde{q}$	$\tilde{g} \to q\bar{q}\tilde{\chi}_1^0$ $q\bar{q}'\tilde{\chi}_1^\pm$ $g\tilde{\chi}_1^0$	$m_{\tilde{q}} > m_{\tilde{g}}$	$\not{E}_T$ + multijets (+leptons)
		$\tilde{q} \to q\tilde{\chi}_i^0$ $\tilde{q} \to q'\tilde{\chi}_i^\pm$	$m_{\tilde{g}} > m_{\tilde{q}}$	
•	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0$	$\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 l^\pm\nu,\ \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 ll$		Trilepton + $\not{E}_T$
		$\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 q\bar{q}', \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 ll,$		Dilepton + jet + $\not{E}_T$
•	$\tilde{\chi}_1^+\tilde{\chi}_1^-$	$\tilde{\chi}_1^+ \to l\tilde{\chi}_1^0 l^\pm\nu$		Dilepton + $\not{E}_T$
•	$\tilde{\chi}_i^0\tilde{\chi}_i^0$	$\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 X, \tilde{\chi}_i^0 \to \tilde{\chi}_1^0 X'$		$\not{E}_T$ + Dilept+(jets)+lept
•	$\tilde{t}_1\tilde{t}_1$	$\tilde{t}_1 \to c\tilde{\chi}_1^0$		2 acollinear jets + $\not{E}_T$
		$\tilde{t}_1 \to b\tilde{\chi}_1^\pm, \tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 q\bar{q}'$		single lepton + $\not{E}_T$ + $b's$
		$\tilde{t}_1 \to b\tilde{\chi}_1^\pm, \tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 l^\pm\nu,$		Dilepton + $\not{E}_T$ + $b's$

- $\tilde{l}\tilde{l}, \tilde{l}\tilde{\nu}, \tilde{\nu}\tilde{\nu}$ $\tilde{l}^{\pm} \to l \pm \tilde{\chi}_i^0, \tilde{l}^{\pm} \to \nu_l \tilde{\chi}_i^{\pm}$ Dilepton $+ \not{E}_T$
  $\tilde{\nu} \to \nu \tilde{\chi}_1^0$ Single lept $+ \not{E}_T +$ jets
  $\not{E}_T$

Note again the characteristic missing energy and transverse momenta events. Contrary to e^+e^- colliders, at hadron machines the background is extremely rich and essential.

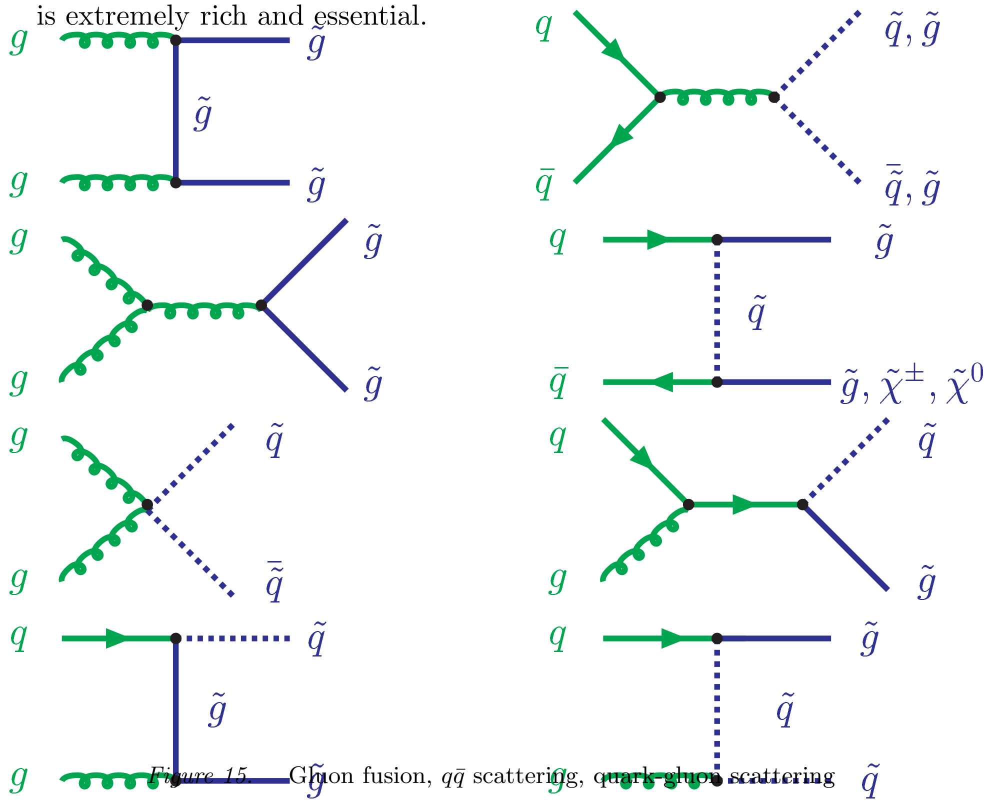

Figure 15. Gluon fusion, $q\bar{q}$ scattering, quark-gluon scattering

6.6 The lightest superparticle

One of the crucial questions is the properties of the lightest superparticle. Different SUSY breaking scenarios lead to different experimental signatures and different LSP.

• Gravity mediation

In this case, the LSP is the lightest neutralino $\tilde{\chi}_1^0$, which is almost 90% photino for a low $\tan\beta$ solution and contains more higgsino admixture for high $\tan\beta$. The usual signature for LSP is missing energy; $\tilde{\chi}_1^0$ is stable and is the best candidate for the cold dark matter in the Universe.

Typical processes, where the LSP is created, end up with jets + $\not{E}_T$, or leptons + $\not{E}_T$, or both jest + leptons + $\not{E}_T$.

• Gauge mediation

In this case the LSP is the gravitino $\tilde{G}$ which also leads to missing energy. The actual question here is what the NLSP, the next lightest particle, is. There are two possibilities:

i) $\tilde{\chi}_1^0$ is the NLSP. Then the decay modes are: $\tilde{\chi}_1^0 \to \gamma\tilde{G},\ h\tilde{G},\ Z\tilde{G}$. As a result, one has two hard photons + $\not{E}_T$, or jets + $\not{E}_T$.

ii) $\tilde{l}_R$ is the NLSP. Then the decay mode is $\tilde{l}_R \to \tau\tilde{G}$ and the signature is a charged lepton and the missing energy.

• Anomaly mediation

In this case, one also has two possibilities:

i) $\tilde{\chi}_1^0$ is the LSP and wino-like. It is almost degenerate with the NLSP.

ii) $\tilde{\nu}_L$ is the LSP. Then it appears in the decay of chargino $\tilde{\chi}^+ \to \tilde{\nu}l$ and the signature is the charged lepton and the missing energy.

• R-parity violation

In this case, the LSP is no longer stable and decays into the SM particles. It may be charged (or even colored) and may lead to rare decays like neutrinoless double β-decay, etc.

Experimental limits on the LSP mass follow from non-observation of the corresponding events. Modern lower limit is around 40 GeV .

7. The Higgs boson mass in the MSSM

One of the hottest topics in the SM now is the search for the Higgs boson. It is also a window to a new physics. Below we consider properties of the Higgs boson in the MSSM.

It has already been mentioned that in the MSSM the mass of the lightest Higgs boson is predicted to be less than the Z-boson mass. This is, however, the tree level result and the masses acquire the radiative corrections. With account taken of the one-loop radiative corrections the lightest Higgs mass is

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4}. \quad (7.1)$$

One finds that the one-loop correction is positive and increases the mass value. Two loop corrections have the opposite effect but are smaller [36].

The Higgs mass depends mainly on the following parameters: the top mass, the squark masses, the mixing in the stop sector and $\tan\beta$. The maximum Higgs mass is obtained for large $\tan\beta$, for a maximum value of the top and squark masses and a minimum value of the stop mixing.

The lightest Higgs boson mass m_h is shown as a function of $\tan\beta$ in Fig. 16 [35]. The shaded band corresponds to the uncertainty from the stop mass and stop mixing for $m_t = 175$ GeV. The upper and lower lines correspond to m_t=170 and 180 GeV, respectively.

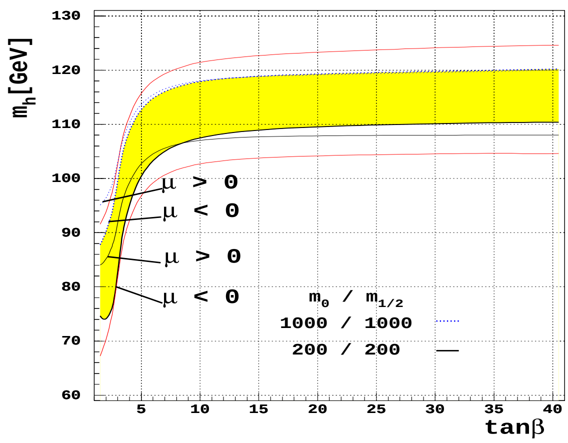

Figure 16. The mass of the lightest Higgs boson in the MSSM as a function of $\tan\beta$

Combining all the uncertainties the results for the Higgs mass in the CMSSM can be summarized as follows:

• The low $\tan\beta$ scenario ($\tan\beta < 3.3$) of the CMSSM is excluded by the lower limit on the Higgs mass of 113.3 GeV [7].

• For the high $\tan\beta$ scenario the Higgs mass is found to be [35]:

$$m_h = 115 \pm 3 \text{ (stopm) } \pm 1.5 \text{ (stopmix) } \pm 2 \text{ (theory) } \pm 5 \text{ (topm) GeV},$$

where the errors are the estimated standard deviations around the central value.

However, these SUSY limits on the Higgs mass may not be so restricting if non-minimal SUSY models are considered. However, more sophisticated models do not change the generic feature of SUSY theories, the presence of the light Higgs boson.

8. Perspectives of SUSY observation

With the LEP shut down, further attempts to discover supersymmetry are connected with the Tevatron and LHC hadron colliders.

Tevatron

Tevatron Run II has the c.m. energy of 2 TeV with planned luminosity almost 10 times greater than in RUN I. However, since it is a hadron collider, not the full energy goes into collision taken away by those quarks in a proton that do not take part in the interaction. Due to a severe background, this collider needs time to reach the integrated luminosity required for SUSY discovery.

We show in Table 2 [37] the discovery reach of the Tevatron for squarks of the third generation. Modern exclusion areas are also shown in plots in Fig.17 [38]. One can see that they are still far from the expected masses given in Table 1.

Decay ($Br = 100\%$)	Subsequent Decay	Final State of $\tilde{b}_1\bar{\tilde{b}}_1$ or $\tilde{t}_1\bar{\tilde{t}}_1$	Discovery Reach @20 $\mathrm{fb}^{(}-1)$	(Run I)
$\tilde{b}_1 \to b\tilde{\chi}_1^0$		$bb\not{E}_T$	260 GeV/c^2	(146 GeV/c^2)
$\tilde{t}_1 \to c\tilde{\chi}_1^0$		$cc\not{E}_T$	220 GeV/c^2	(116 GeV/c^2)
$\tilde{t}_1 \to bl\tilde{\nu}$	$\tilde{\nu} \to \nu\tilde{\chi}_1^0$	$l^+l^-b\not{E}_T$	240 GeV/c^2	(140 GeV/c^2)
$\tilde{t}_1 \to bl\tilde{\nu}\tilde{\chi}_1^0$		$l^+l^-b\not{E}_T$	-	(129 GeV/c^2)
$\tilde{t}_1 \to b\tilde{\chi}_1^\pm$	$\tilde{\chi}_1^\pm \to W^{(*)}\tilde{\chi}_1^0$	$l^+l^-b\not{E}_T$;	210 GeV/c^2	(-)
$\tilde{t}_1 \to bW\tilde{\chi}_1^0$		$l^+l^-bj\not{E}_T$	190 GeV/c^2	(-)

Table 2. Discovery reaches on $M_{\tilde{b}}$ and $M_{\tilde{t}}$ expected in Run II.

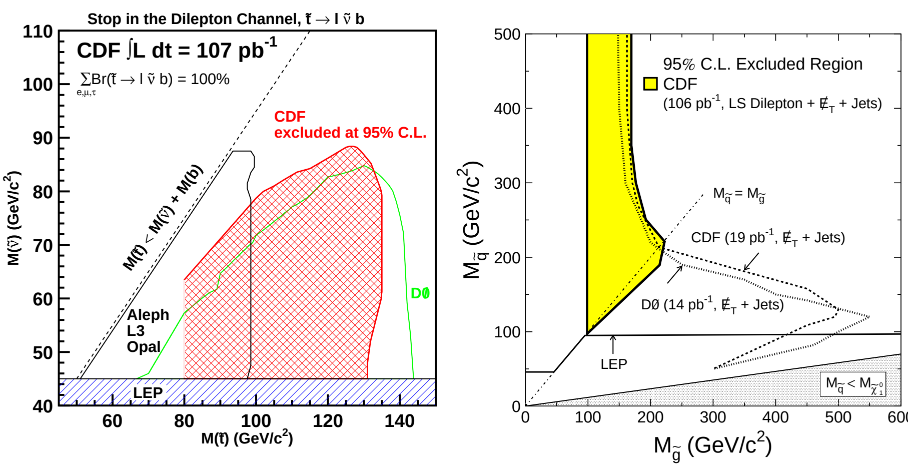

Figure 17. Exclusion plots for squarks and sneutrinos (left) and squarks and gluino (right) at Tevatron

LHC

The LHC hadron collider is the ultimate machine for a new physics at the TeV scale. Its c.m. energy is planned to be 14 TeV with very high luminosity up to a few hundred fb^{-1}. The LHC is supposed to cover the wide range of parameters of the MSSM (see Fig.18 [39]) and discover the superpartners with the masses below 2 TeV [40]. This will be a crucial test for the MSSM and the low energy supersymmetry.

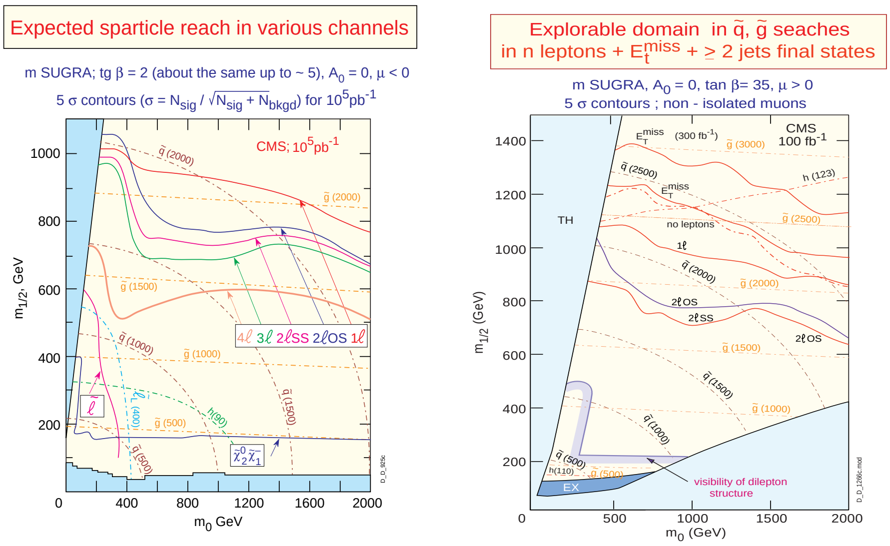

Figure 18. Expected sparticle reach at LHC

9. Conclusion

Supersymmetry is now the most popular extension of the Standard Model. It promises us that new physics is round the corner at a TeV scale to be exploited at new machines of this decade. If our expectations are correct, very soon we will face new discoveries, the whole world of supersymmetric particles will show up and the table of fundamental particles will be enlarged in increasing rate. This would be a great step in understanding the microworld.

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References

[1] Y. A. Golfand and E. P. Likhtman, *JETP Letters* **13** (1971) 452; D. V. Volkov and V. P. Akulov, *JETP Letters* **16** (1972) 621; J. Wess and B. Zumino, *Phys. Lett.* **B49** (1974) 52.

[2] P. Fayet and S. Ferrara, *Phys. Rep.* **32** (1977) 249; M. F. Sohnius, *Phys. Rep.* **128** (1985) 41; H. P. Nilles, *Phys. Rep.* **110** (1984) 1; H. E. Haber and G. L. Kane, *Phys. Rep.* **117** (1985) 75; A. B. Lahanas and D. V. Nanopoulos, *Phys. Rep.* **145** (1987) 1.

[3] J. Wess and J. Bagger, *"Supersymmetry and Supergravity"*, Princeton Univ. Press, 1983.

[4] S. J. Gates, M. Grisaru, M. Roček and W. Siegel, *"Superspace or One Thousand and One Lessons in Supersymmetry"*, Benjamin & Cummings, 1983.

[5] S. Coleman and J .Mandula, *Phys.Rev.* **159** (1967) 1251.

[6] G. G. Ross, *"Grand Unified Theories"*, Benjamin & Cummings, 1985.

[7] D. E. Groom *et al.*, "Review of Particle Physics", Eur. Phys. J. **C15** (2000) 1.

[8] U. Amaldi, W. de Boer and H. Fürstenau, *Phys. Lett.* **B260** (1991) 447.

[9] C.L. Bennett et al., 2003, ApJS, 148, 1

[10] M. B. Green, J. H. Schwarz and E. Witten, *"Superstring Theory"*, Cambridge, UK: Univ. Press, 1987. *Cambridge Monographs On Mathematical Physics.*

[11] F. A. Berezin, *"The Method of Second Quantization"*, Moscow, Nauka, 1965.

[12] P. Fayet and J. Illiopoulos, *Phys. Lett.* **B51** (1974) 461.

[13] L. O'Raifeartaigh, *Nucl. Phys.* **B96** (1975) 331.

[14] H. E. Haber, *"Introductory Low-Energy Supersymmetry"*, Lectures given at TASI 1992, (SCIPP 92/33, 1993), hep-ph/9306207.
D. I. Kazakov, *"Beyond the Standard Model (In search of supersymmetry)"*, Lectures at the European school on high energy physics, CERN-2001-003, hep-ph/0012288.

[15] http://atlasinfo.cern.ch/Atlas/documentation/EDUC/physics14.html

[16] P.Fayet, *Nucl. Phys.* **B90**(1975) 104; A.Salam and J.Srathdee, *Nucl. Phys.* **B87**(1975) 85.

[17] L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359; S. K. Soni and H. A. Weldon, *Phys. Lett.* **B126** (1983) 215; I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B256** (1985) 557.

[18] H. P. Nilles, *Phys. Lett.* **B115** (1982) 193; A. H. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970; *Nucl. Phys.* **B227** (1983) 121; R. Barbieri, S. Ferrara and C. A. Savoy, *Phys. Lett.* **B119** (1982) 343.

[19] M. Dine and A. E. Nelson, *Phys. Rev.* **D48** (1993) 1277, M. Dine, A. E. Nelson and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362.

[20] L. Randall and R. Sundrum, *Nucl. Phys.* **B557** (1999) 79; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, *JHEP*, **9812** (1998) 027.

[21] D. E. Kaplan, G. D. Kribs and M. Schmaltz, *Phys. Rev.* **D62** (2000) 035010; Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, *JHEP*, **0001** (2000) 003.

[22] M. E. Peskin, *"Theoretical summary lecture for EPS HEP99"*, hep-ph/0002041.

[23] G. G. Ross and R. G. Roberts, *Nucl. Phys.* **B377** (1992) 571.
V. Barger, M. S. Berger and P. Ohmann, *Phys. Rev.* **D47** (1993) 1093.

[24] W. de Boer, R. Ehret and D. Kazakov, *Z. Phys.* **C67** (1995) 647;
W. de Boer et al., *Z. Phys.* **C71** (1996) 415.

[25] L. E. Ibáñez, C. Lopéz and C. Muñoz, *Nucl. Phys.* **B256** (1985) 218.

[26] W. Barger, M. Berger, P. Ohman, *Phys. Rev.* **D49** (1994) 4908.

[27] V.Barger, M.S. Berger, P.Ohmann and R.Phillips, *Phys. Lett.* **B314** (1993) 351.
P. Langacker and N. Polonsky, *Phys. Rev.* **D49** (1994) 1454.
S. Kelley, J. L. Lopez and D.V. Nanopoulos, *Phys. Lett.* **B274** (1992) 387.

[28] S.Ahmed et al. (CLEO Collaboration), CLEO CONF 99/10, hep-ex/9908022.

[29] R. Barate et al. (ALEPH Collaboration), *Phys. Lett.* **B429** (1998) 169.

[30] W.de Boer, M.Huber, C.Sander, D.I.Kazakov, *Phys.Lett.* **B515** (2001) 283.

[31] ALEPH Collaboration, *Phys.Lett.* **B499** (2001) 67.

[32] S. Abel *et al.* [SUGRA Working Group Collaboration], *Report of the SUGRA working group for run II of the Tevatron*, hep-ph/0003154.

[33] M. Drees and M. M. Nojiri, *Phys. Rev.* **D47** (1993) 376;
J. L. Lopez, D. V. Nanopoulos and H. Pois, *Phys. Rev.* **D47** (1993) 2468;
P. Nath and R. Arnowitt, *Phys. Rev. Lett.* **70** (1993) 3696.

[34] W.de Boer, H.J.Grimm, A.Gladyshev, D.Kazakov, *Phys. Lett.* **B438** (1998) 281.

[35] W.de Boer, M.Huber, A.Gladyshev, D.Kazakov, *Eur.Phys.J.* **C20** (2001) 689.

[36] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Lett.* **B455** (1999) 179; *Eur. Phys. J.* **C9** (1999) 343.

[37] T.Kamon, hep-ex/0301019, Proc. of IX Int. Conf. "SUSY-01", WS 2001, p.196.

[38] CDF Collaboration (D. Acosta et al.), *Phys.Rev.Lett.* **90** (2003) 251801; CDF Collaboration (T. Affolder et al.), *Phys.Rev.Lett.* **87** (2003) 251803.

[39] http://CMSinfo.cern.ch/Welcome.html/CMSdocuments/CMSplots

[40] N.V.Krasnikov and V.A.Matveev, "Search for new physics at LHC", hep-ph/0309200.