



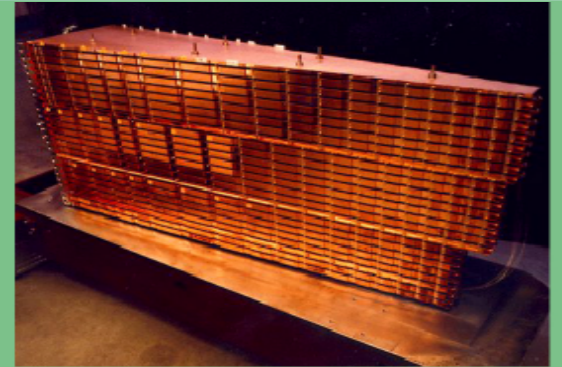
Cargese Institute of Scientific Studies

Particle Physics and Cosmology: the Interface



Cargese School
of Particle Physics and Cosmology

CARGESE,
August 4-16th 2003



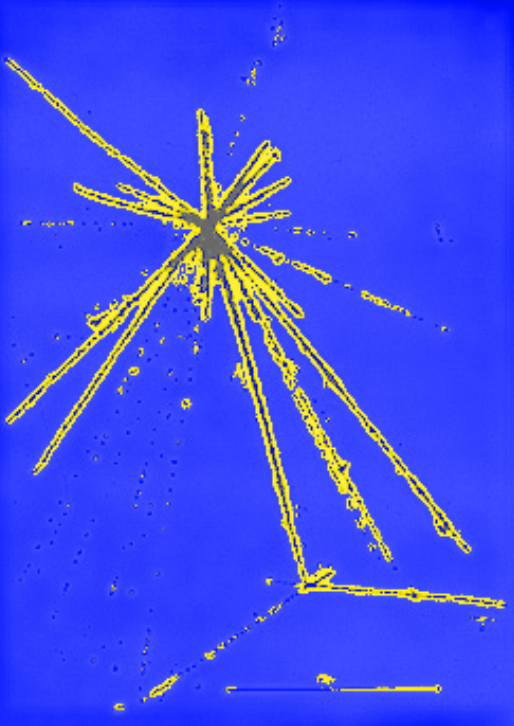
SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

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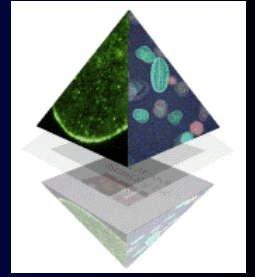
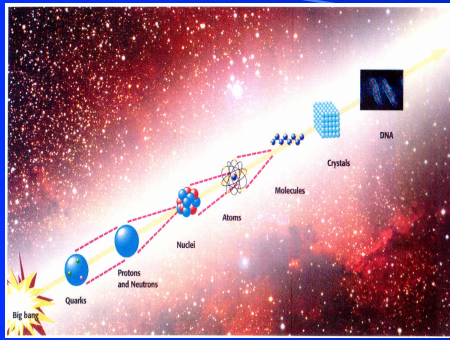
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Outline

1. What is SUSY
2. Motivation of SUSY
3. Basics of SUSY
4. The MSSM
5. Constrained MSSM
6. SUSY Searches



What is SUSY



- **Supersymmetry** is a boson-fermion symmetry that is aimed to unify all forces in Nature including gravity within a single framework
- Modern views on supersymmetry in particle physics are based on string paradigm, though low energy manifestations of SUSY can be found (?) at modern colliders and in non-accelerator experiments
- The concepts of Quantum Field Theory allow SUSY without any restrictions
- There is no direct confirmation of SUSY but this does not stop theorists due to extreme beauty of SUSY models

Supersymmetry

(Super) Algebra

$$[P_\mu, P_\nu] = 0, [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$$

$$[B_r, B_s] = iC_{rs}^t B_t, [B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,$$

$$[Q_\alpha^i, P_\mu] = [\bar{Q}_{\dot{\alpha}}^i, P_\mu] = 0,$$

$$[Q_\alpha^i, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i, [\bar{Q}_{\dot{\alpha}}^i, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}^i (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}},$$

$$[Q^i, B_r] = (h^r)_i Q^j, [\bar{Q}^i, B_r] = -\bar{Q}^j (h^r)_j^i,$$

$$\{Q_\alpha^i, \bar{Q}_{\dot{\beta}}^j\} = 2\delta^{ij} (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$\{Q_\alpha^i, Q_\beta^j\} = 2\varepsilon_{\alpha\beta} Z^{ij}, Z^{ij} = Z_{ij}^\dagger, Z_{ij} = a_{ij}^r b_r,$$

$$\{\bar{Q}_{\dot{\alpha}}^i, \bar{Q}_{\dot{\beta}}^j\} = -2\varepsilon_{\dot{\alpha}\dot{\beta}} Z^{ij}, [Z_{ij}, \text{anything}] = 0,$$

$$\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; i, j = 1, 2, \dots, N.$$

Superspace

$$x_\mu \rightarrow x_\mu, \theta, \bar{\theta}$$

Group Element

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta} \bar{Q})}$$

Supertranslation

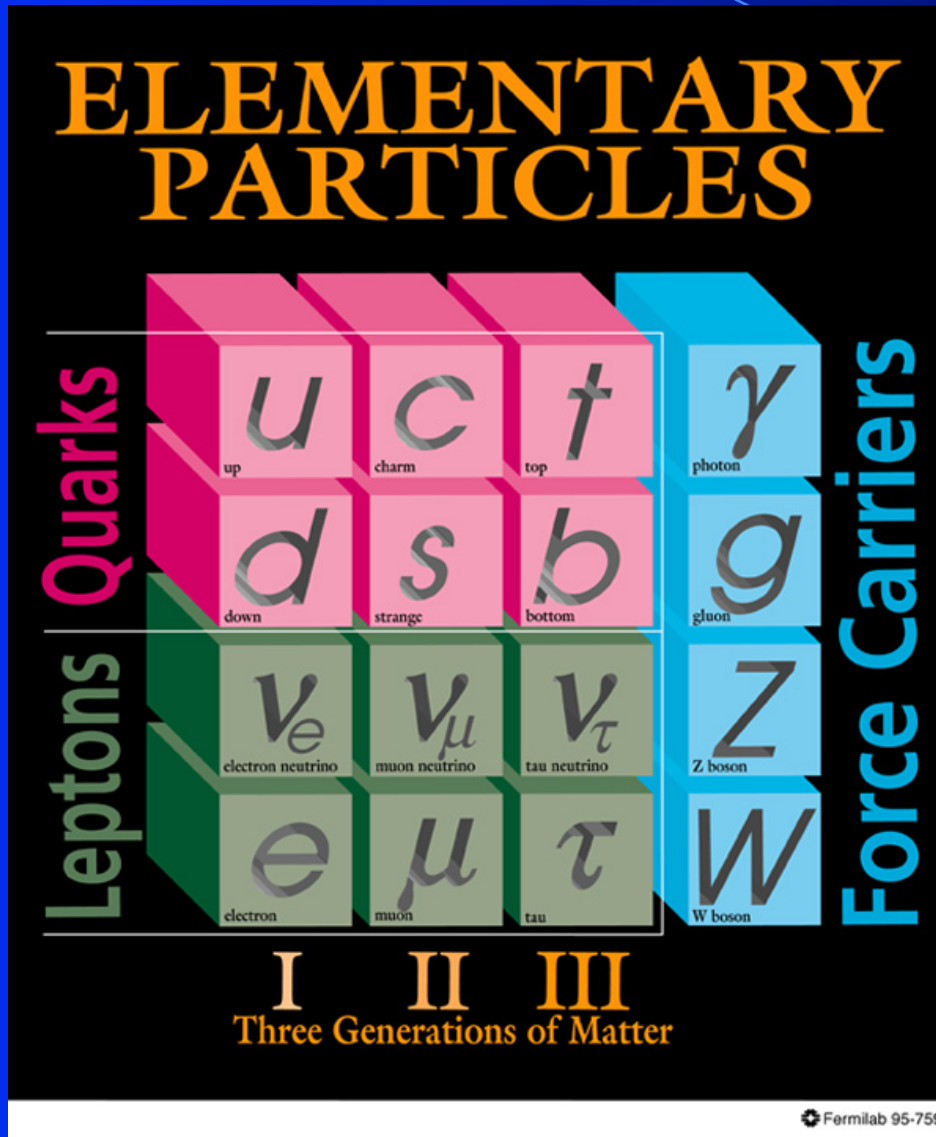
$$x_\mu \rightarrow x_\mu + i\theta\sigma_\mu \bar{\xi} - i\xi\sigma_\mu \bar{\theta},$$

$$\theta \rightarrow \theta + \xi,$$

$$\bar{\theta} \rightarrow \bar{\theta} + \bar{\xi}$$

Local translation =
general relativity !

The Standard Model



- Fits the experimental data very well
- Has too many free parameters and unexplained features
- The Higgs boson is still missing

Lagrangian of the SM

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$$

$$L = L_{\text{gauge}} + L_{\text{Yukawa}} + L_{\text{Higgs}}$$

$$\begin{aligned} L_{\text{gauge}} = & -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{4} W_{\mu\nu}^i W_{\mu\nu}^i - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \\ & + i\bar{L}_\alpha \gamma^\mu D_\mu L_\alpha + i\bar{Q}_\alpha \gamma^\mu D_\mu Q_\alpha + i\bar{E}_\alpha \gamma^\mu D_\mu E_\alpha \\ & + i\bar{U}_\alpha \gamma^\mu D_\mu U_\alpha + i\bar{D}_\alpha \gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger (D_\mu H) \end{aligned}$$

$$L_{\text{Yukawa}} = y_{\alpha\beta}^L \bar{L}_\alpha E_\beta H + y_{\alpha\beta}^D \bar{Q}_\alpha D_\beta H + y_{\alpha\beta}^U \bar{Q}_\alpha U_\beta \tilde{H}$$

$$L_{\text{Higgs}} = -V = m^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2$$

$$\tilde{H} = i\tau_2 H^\dagger$$

Spontaneous Symmetry Breaking

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \rightarrow SU_c(3) \otimes U_{EM}(1)$$

$$\langle H \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad v = \frac{m}{\sqrt{\lambda}}$$

$$m_H = \sqrt{2}m = \sqrt{2\lambda}v$$

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$$

$$\gamma_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

$$M_W = \frac{1}{2}gv$$

$$M_Z = M_W / \cos\theta_W$$

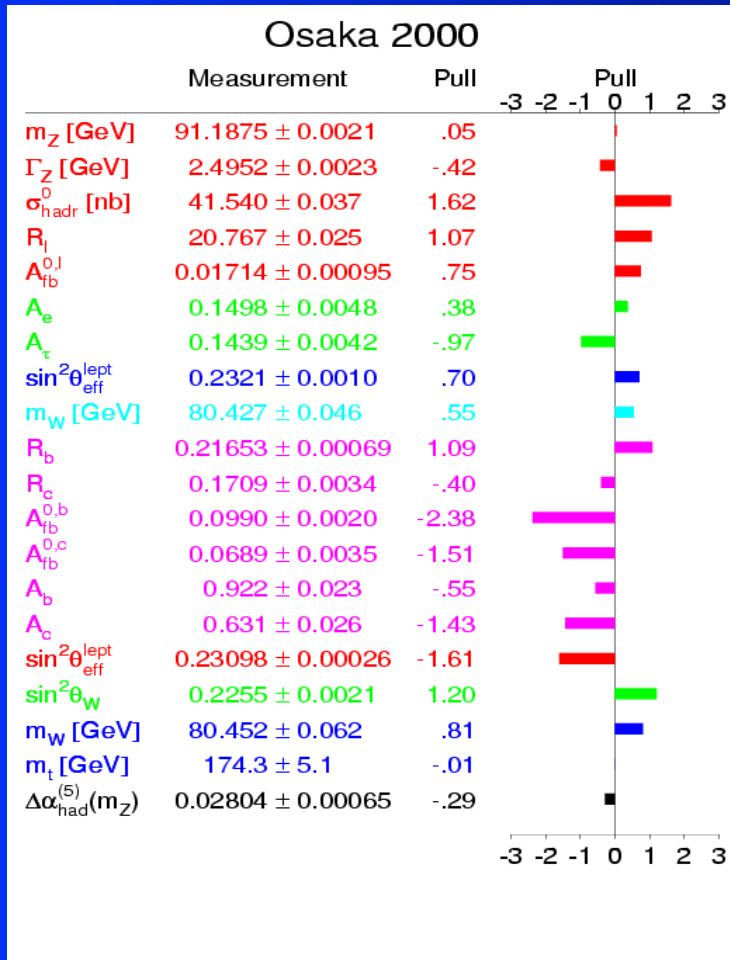
$$\tan\theta_W = g' / g$$

$$M_\gamma = 0$$

$$M_{\alpha\beta}^u = y_{\alpha\beta}^u v, \quad M_{\alpha\beta}^d = y_{\alpha\beta}^d v, \quad M_{\alpha\beta}^l = y_{\alpha\beta}^l v$$

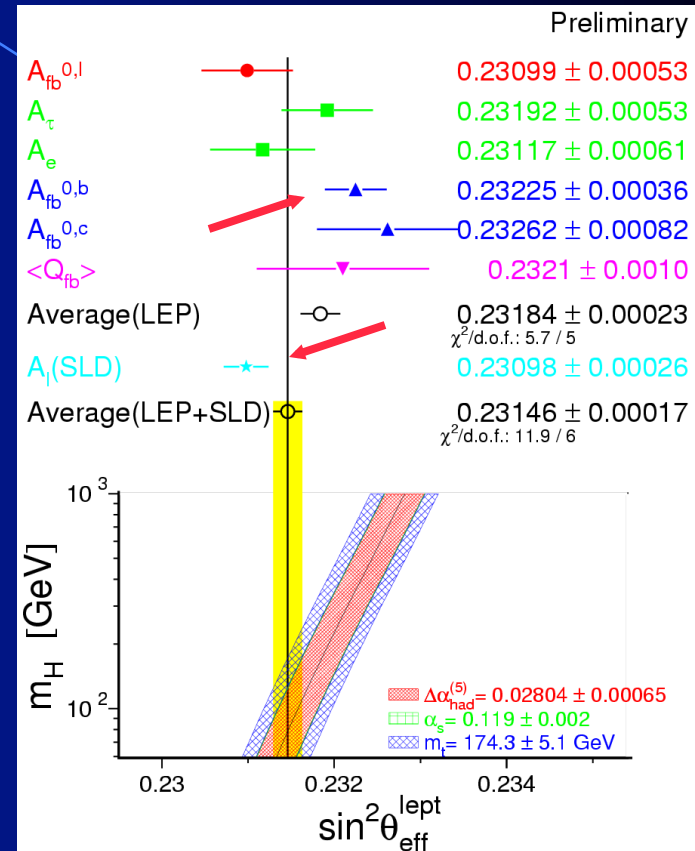
Comparison with Experiment

Global Fit to Data



Remarkable agreement of ALL the data with the SM predictions - precision tests of radiative corrections and the SM

Higgs Mass Constraint

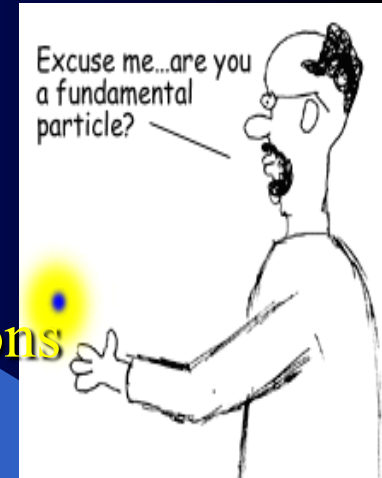


Though the values of $\sin^2\theta_W$ extracted from different experiments are in good agreement, two most precise measurements from hadron and lepton asymmetries disagree by 3σ

The SM and Beyond

The problems of the SM:

- Inconsistency at high energies due to Landau pole
- Large number of free parameters
- Formal unification of strong and electroweak interactions
- Still unclear mechanism of EW symmetry breaking
- CP-violation is not understood
- Flavour mixing and the number of generations is arbitrary
- The origin of the mass spectrum is unclear



The way beyond the SM:

- The SAME fields with NEW interactions → GUT, SUSY, String
- NEW fields with NEW interactions → Compositeness, Technicolour, preons

Motivation of SUSY in Particle Physics

- Unification with Gravity
- Unification of gauge couplings
- Solution to the hierarchy problem
- Dark matter in the Universe
- Superstrings $\rightarrow spin\ 2 \rightarrow spin\ 3/2 \rightarrow spin\ 1 \rightarrow spin\ 1/2 \rightarrow spin\ 0$

Unification of matter (fermions) with forces (bosons) naturally arises

from an attempt to unify gravity with the other interactions

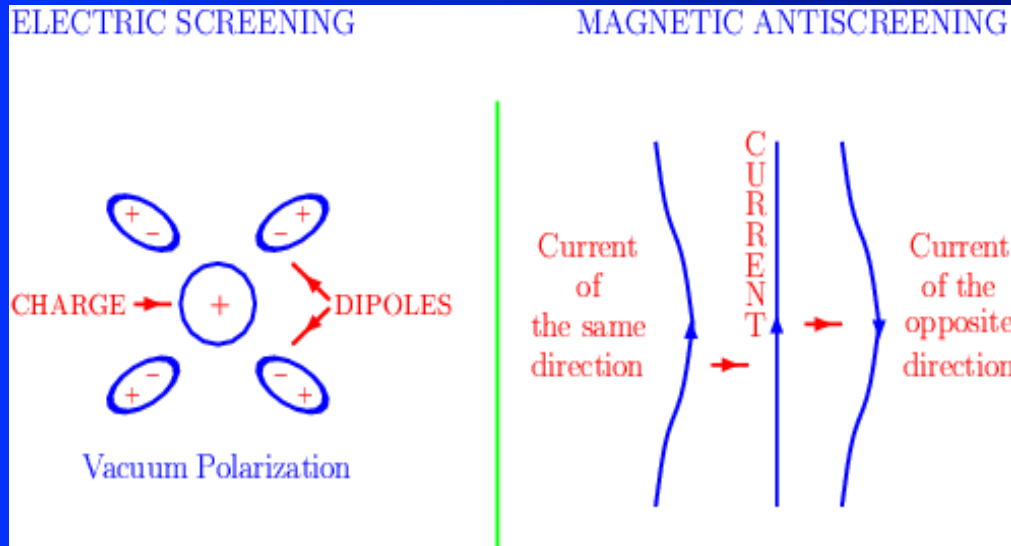
$$\{Q_{\alpha}^i, \bar{Q}_{\dot{\beta}}^j\} = 2\delta^{ij}(\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu} \Rightarrow \{\delta_{\epsilon}, \bar{\delta}_{\bar{\epsilon}}\} = 2(\epsilon\sigma^{\mu}\bar{\epsilon})P_{\mu}$$

$\epsilon = \epsilon(x)$ local coordinate transf. \Rightarrow (super)gravity

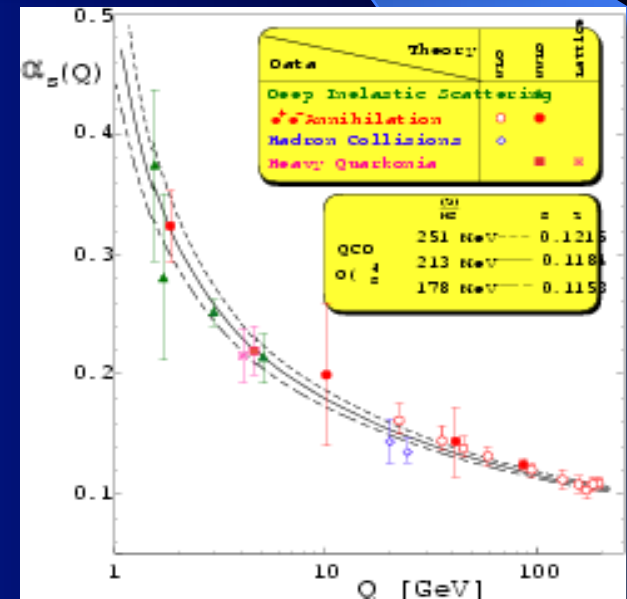
Motivation of SUSY in Particle Physics

- Unification of gauge couplings

Low Energy		⇒ High Energy	
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	⇒ G_{GUT} (or $G^n + \text{symm}$)
<i>gluons</i>	<i>W, Z</i>	<i>photon</i>	⇒ <i>gauge bosons</i>
<i>quarks</i>	<i>leptons</i>		⇒ <i>fermions</i>
g_3	g_2	g_1	⇒ g_{GUT}



$$\alpha_i = \alpha_i \left(\frac{Q^2}{\Lambda^2} \right) = \alpha_i(\text{distance})$$



Running of the strong coupling

Motivation of SUSY

RG Equations $\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2$, $\tilde{\alpha}_i = \alpha_i / 4\pi = g_i^2 / 16\pi^2$, $t = \log(Q^2 / \mu^2)$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Unification of the Coupling Constants
in the SM and in the MSSM

Input

$$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$$

$$\sin^2 \theta_{MS} = 0.23146 \pm 0.00017$$

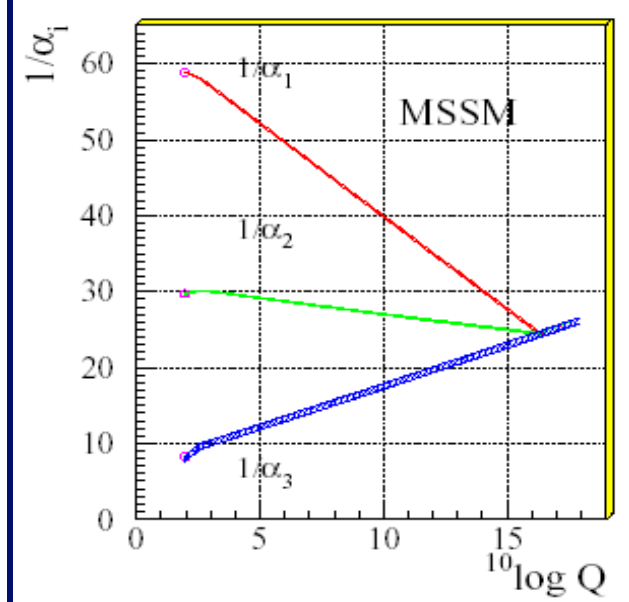
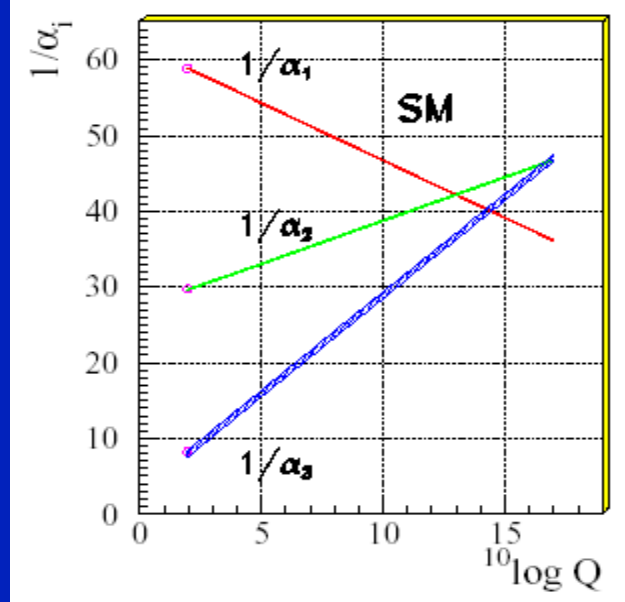
$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

Output

$$M_{SUSY} = 10^{3.4 \pm 0.9 \pm 0.4} \text{ GeV}$$

$$M_{GUT} = 10^{15.8 \pm 0.3 \pm 0.1} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = 26.3 \pm 1.9 \pm 1.0$$



SUSY yields unification!

Motivation of SUSY

- Solution of the Hierarchy Problem

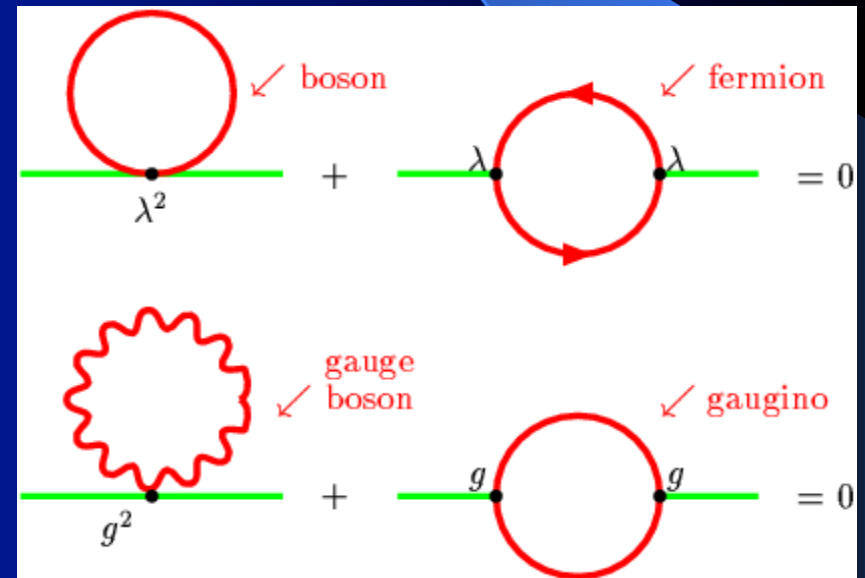
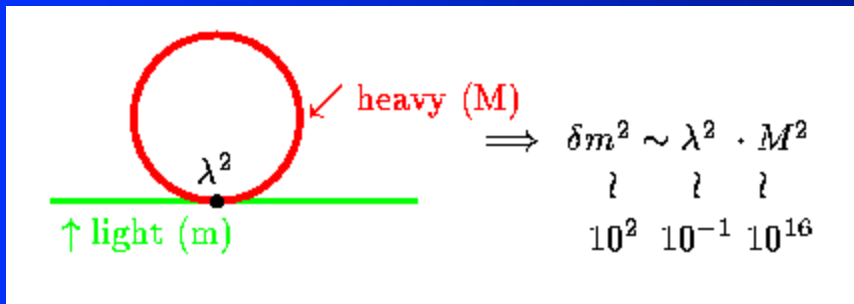
$$m_H \sim v \sim 10^2 \text{ GeV}$$

$$m_\Sigma \sim V \sim 10^{16} \text{ GeV}$$

$$\frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

Cancellation of quadratic terms

Destruction of the hierarchy by radiative corrections

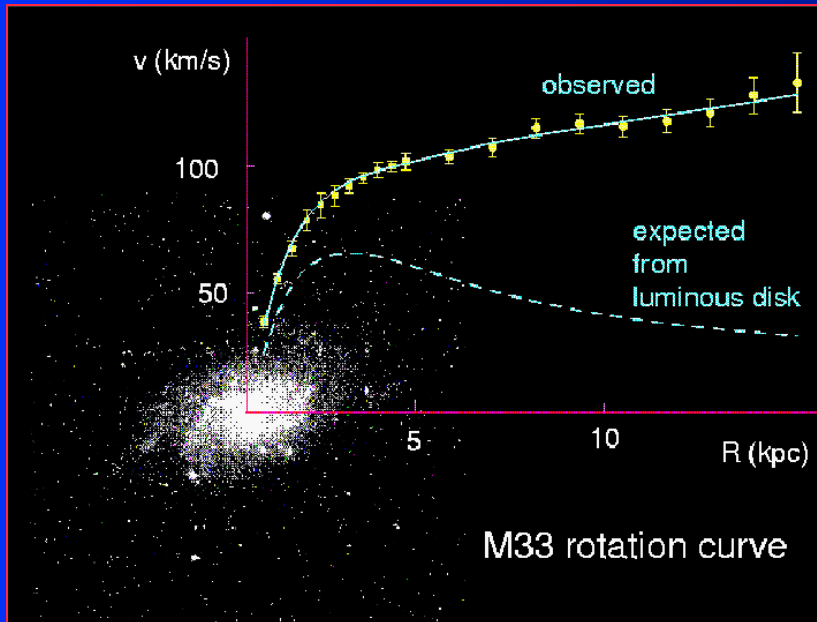


SUSY may also explain the origin of the hierarchy due to radiative mechanism

$$\sum_{bosons} m^2 = \sum_{fermions} m^2$$

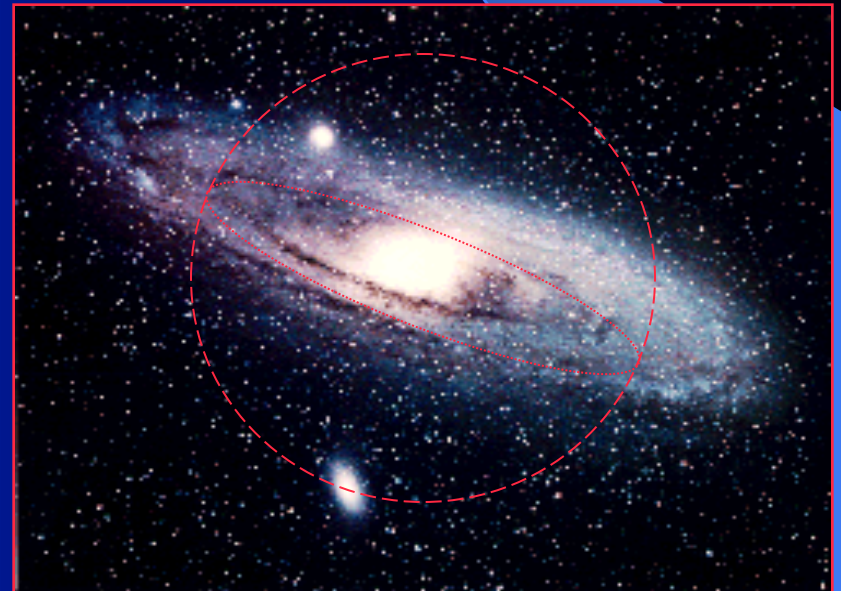
Motivation of SUSY

- Dark Matter in the Universe



Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter

The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.



SUSY provides a candidate for the Dark matter – a stable neutral particle¹⁴

Cosmological Constraints

New precise cosmological data

$$\Omega h^2 = 1 \quad \longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$

Dark Matter in the Universe:



- Supernova Ia explosion
- CMBR thermal fluctuations
(recent news from WMAP)



Hot DM
(not favoured by
galaxy formation)

Cold DM
(rotation curves
of Galaxies)

SUSY

Supersymmetry

(Super) Algebra

$$[P_\mu, P_\nu] = 0, [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho} P_\sigma - g_{\mu\sigma} P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho} M_{\mu\sigma} - g_{\nu\sigma} M_{\mu\rho} - g_{\mu\rho} M_{\nu\sigma} + g_{\mu\sigma} M_{\nu\rho}),$$

$$[B_r, B_s] = iC_{rs}^t B_t, [B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,$$

$$[Q_\alpha^i, P_\mu] = [\bar{Q}_{\dot{\alpha}}^i, P_\mu] = 0,$$

$$[Q_\alpha^i, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta^i, [\bar{Q}_{\dot{\alpha}}^i, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}_{\dot{\beta}}^i (\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}^{\dot{\beta}},$$

$$[Q_\alpha^i, B_r] = (b_r)^i_j Q_\alpha^j, [\bar{Q}_{\dot{\alpha}}^i, B_r] = -\bar{Q}_{\dot{\alpha}}^j (b_r)^i_j,$$

$$\{Q_\alpha^i, \bar{Q}_{\dot{\beta}}^j\} = 2\delta^{ij} (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu,$$

$$\{Q_\alpha^i, Q_\beta^j\} = 2\varepsilon_{\alpha\beta} Z^{ij}, Z^{ij} = Z_{ij}^\dagger, Z_{ij} = a_{ij}^r b_r,$$

$$\{\bar{Q}_{\dot{\alpha}}^i, \bar{Q}_{\dot{\beta}}^j\} = -2\varepsilon_{\dot{\alpha}\dot{\beta}} Z^{ij}, [Z_{ij}, \text{anything}] = 0,$$

$$\alpha, \dot{\alpha}, \beta, \dot{\beta} = 1, 2; i, j = 1, 2, \dots, N.$$

Superspace

$$x_\mu \rightarrow x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$$

$$\alpha, \dot{\alpha} = 1, 2$$

Grassmannian

$$\text{parameters } \vartheta_\alpha^2 = 0, \bar{\vartheta}_{\dot{\alpha}}^2 = 0$$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial \vartheta_\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\vartheta}_{\dot{\alpha}}} + i\theta_\alpha \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu$$

$$Q_\alpha^2 = 0, \bar{Q}_{\dot{\alpha}}^2 = 0$$

This is the only possible graded Lie algebra that mixes integer and half-integer spins and changes statistics

Basics of SUSY

Quantum states: Vacuum = $|E, \lambda\rangle$ $Q|E, \lambda\rangle = 0$

$$[Q_\alpha^i, P_\mu] = [\bar{Q}_\alpha^i, P_\mu] = 0$$

↑ Energy ↑ helicity

State	Expression	# of states
vacuum	$ E, \lambda\rangle$	1
1-particle	$\bar{Q}_i E, \lambda\rangle = E, \lambda + 1/2\rangle$	$\binom{N}{1} = N$
2-particle	$\bar{Q}_i \bar{Q}_j E, \lambda\rangle = E, \lambda + 1\rangle$	$\binom{N}{2} = \frac{N(N-1)}{2}$
...
N-particle	$\bar{Q}_1 \bar{Q}_2 \dots \bar{Q}_N E, \lambda\rangle = E, \lambda + N/2\rangle$	$\binom{N}{N} = 1$

Total # of states $\sum_{k=0}^N \binom{N}{k} = 2^N = 2^{N-1} \text{ bosons} + 2^{N-1} \text{ fermions}$

SUSY Multiplets

Chiral multiplet $N = 1, \lambda = 0$

helicity	-1/2	0	1/2
# of states	1	2	1

scalar spinor
 (φ, ψ)

Vector multiplet $N = 1, \lambda = 1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1

spinor vector
 (λ, A_μ)

Members of a supermultiplet are called **superpartners**

Extended SUSY multiplets

N=4	SUSY YM	helicity	-1	-1/2	0	1/2	1				
		# of states	1	4	6	4	1				
N=8	SUGRA	helicity	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2
		# of states	1	8	28	56	70	56	28	8	1

$N \leq 4S$ ← spin

$N \leq 4$

For renormalizable theories (YM)

$N \leq 8$

For (super)gravity

Matter Superfields

$F(x, \theta, \bar{\theta})$ - general superfield –reducible representation

$$\Phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \quad \leftarrow \text{chiral superfield: } \bar{D}\Phi = 0$$

$$(y = x + i\theta\sigma\bar{\theta})$$

$$= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}WA(x)$$

component fields

$$\text{spin}=0 \quad + \sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)$$

spin=1/2

auxiliary

SUSY transformation

$$\delta_\varepsilon A = \sqrt{2}\varepsilon\psi,$$

$$\delta_\varepsilon\psi = i\sqrt{2}\sigma^\mu\bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F,$$

$$\delta_\varepsilon F = i\sqrt{2}\bar{\varepsilon}\sigma^\mu\partial_\mu\psi$$

Superpotential

$$W(\Phi) = W(A + \sqrt{2}\theta\psi + \theta\theta F)$$

$$= W(A) + \frac{\partial W}{\partial A}\sqrt{2}\theta\psi + \theta\theta\left(\frac{\partial W}{\partial A}F - \frac{1}{2}\frac{\partial^2 W}{\partial A^2}\psi\psi\right)$$

F-component is a total derivative $\rightarrow \Phi|_{\theta\theta}$ is SUSY invariant

Gauge superfields

$V = V^+$ real superfield

$$V(x, \theta, \bar{\theta}) = C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta\theta M(x) - i\bar{\theta}\bar{\theta}M^+(x) \\ - \theta\sigma^\mu\bar{\theta}v_\mu(x) + i\theta\theta\bar{\theta}[\bar{\lambda}(x) + i\bar{\sigma}^\mu\partial_\mu\chi(x)] - i\bar{\theta}\bar{\theta}\theta[\lambda(x) + i\sigma^\mu\partial_\mu\bar{\chi}(x)] \\ + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) + \frac{1}{2}WC(x)]$$

Gauge transformation

$$V \rightarrow V + \Phi + \bar{\Phi}$$

$$C \rightarrow C + A + A^*$$

$$\chi \rightarrow \chi - i\sqrt{2}\psi$$

$$M \rightarrow M - 2iF$$

$$v_\mu \rightarrow v_\mu - i\partial_\mu(A - A^*)$$

$$\lambda \rightarrow \lambda$$

$$D \rightarrow D$$

Wess-Zumino gauge

$$C = \chi = M = 0$$

physical fields

Covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}^\mu\bar{\vartheta}^{\dot{\alpha}}\partial_\mu$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta_{\dot{\alpha}\alpha}\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu$$

Field strength tensor

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^V D_\alpha e^{-V}$$

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma_{20}^\mu D_\mu\bar{\lambda}$$

SUSY Lagrangians

Superfields

$$L = \Phi_i^+ \Phi_i |_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k) |_{\theta\theta} + h.c.]$$

Components

$$L = i\partial_{\mu} \bar{\psi}_i \bar{\sigma}^{\mu} \psi_i + A_i^* W A_i + F_i^* F_i \quad \leftarrow \text{no derivatives}$$

$$+ [\lambda_i F_i + m_{ij} (A_i F_j - \frac{1}{2} \psi_i \psi_j) + y_{ijk} (A_i A_j F_k - \psi_i \psi_j A_k) + h.c.]$$

Constraint $\frac{\delta L}{\delta F_k} = F_k^* + \lambda_k + m_{ik} A_i + y_{ijk} A_i A_j = 0 \quad \rightarrow \quad F_k$

$$L = i\partial_{\mu} \bar{\psi}_i \bar{\sigma}^{\mu} \psi_i + A_i^* W A_i - \frac{1}{2} m_{ij} \psi_i \psi_j - \frac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$

$$- y_{ijk} \psi_i \psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j)$$

$$V = F_k^* F_k$$

Superfield Lagrangians

$$Action = \int d^4x L \quad \Rightarrow \quad \int d^4x d^4\theta L$$

Grassmannian integration in superspace

$$\int d\theta_\alpha = 0, \quad \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$$

Matter fields

$$L = \int d^2\theta d^2\bar{\theta} \Phi_i^+ \Phi_i + \int d^2\theta \underbrace{(\lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k)} + h.c.]$$

Gauge fields

Superpotential

$$L = \frac{1}{4} \int d^2\theta W^\alpha W_\alpha + \int d^2\bar{\theta} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} = \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda \sigma^\mu D_\mu \bar{\lambda}$$

Gauge transformation $\Phi \rightarrow e^{-ig\Lambda}, \quad \Phi^+ \rightarrow \Phi^+ e^{ig\Lambda^+}, \quad V \rightarrow V + i(\Lambda - \Lambda^+)$

Gauge invariant interaction $\Phi^+ \Phi \rightarrow \Phi^+ e^{gV} \Phi$

Gauge Invariant SUSY Lagrangian

Superfields

$$L_{SUSY\ YM} = \frac{1}{4} \int d^2\theta \operatorname{Tr}(W^\alpha W_\alpha) + \frac{1}{4} \int d^2\theta \operatorname{Tr}(\bar{W}^\alpha \bar{W}_\alpha) \\ + \int d^2\theta d^2\bar{\theta} \bar{\Phi}_{ia} (e^{gV})^a_b \Phi_i^b + \int d^2\theta W(\Phi_i) + \int d^2\bar{\theta} \bar{W}(\bar{\Phi}_i)$$

Components

$$L_{SUSY\ YM} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \bar{\lambda}^a + \frac{1}{2} \underline{D^a D^a} \\ + (\partial_\mu A_i - igv_\mu^a T^a A_i)^\dagger (\partial_\mu A_i - igv_\mu^a T^a A_i) - i\bar{\psi}_i \sigma^\mu (\partial_\mu \psi_i - igv_\mu^a T^a \psi_i) \\ - \underline{D^a} g A_i^\dagger T^a A_i - i\sqrt{2} g A_i^\dagger T^a \lambda^a \psi_i + i\sqrt{2} g \bar{\psi}_i T^a \bar{\lambda}^a A_i + \underline{F_i^\dagger F_i} \\ + \frac{\partial W}{\partial A_i} \underline{F_i} + \frac{\partial \bar{W}}{\partial A_i^\dagger} \underline{F_i^\dagger} - \frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 \bar{W}}{\partial A_i^\dagger \partial A_j^\dagger} \bar{\psi}_i \bar{\psi}_j$$

Potential

$$D^a = -g A_i^\dagger T^a A_i, \quad F_i = -\frac{\partial W}{\partial A_i} \quad \rightarrow \quad V = \frac{1}{2} D^a D^a + F_i^\dagger F_i$$

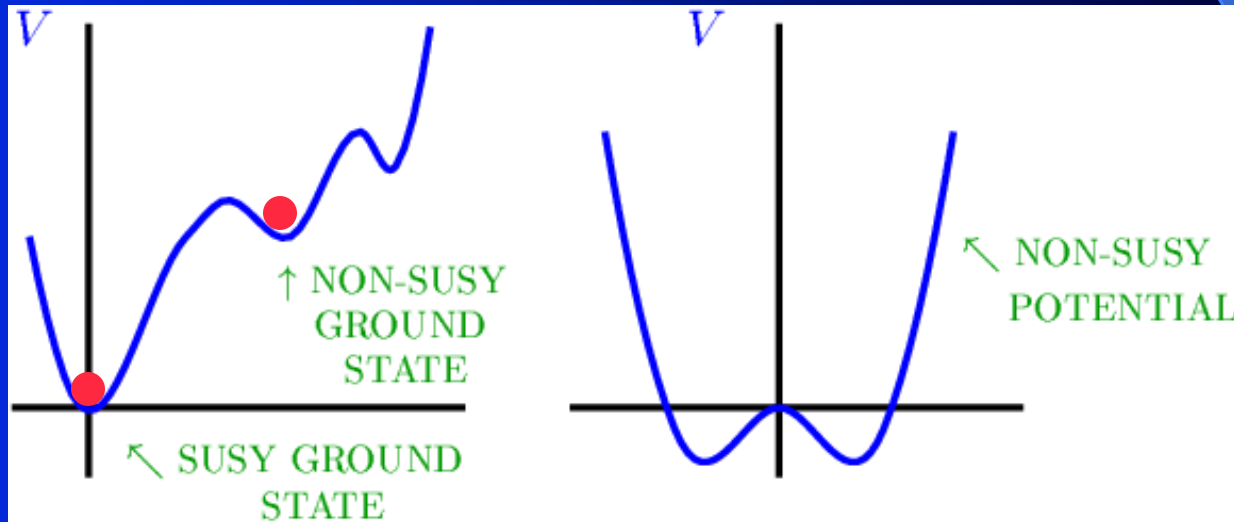
Spontaneous Breaking of SUSY

Energy $E = \langle 0 | H | 0 \rangle$

$$\{Q_\alpha^i, \bar{Q}_\beta^j\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\beta} P_\mu$$

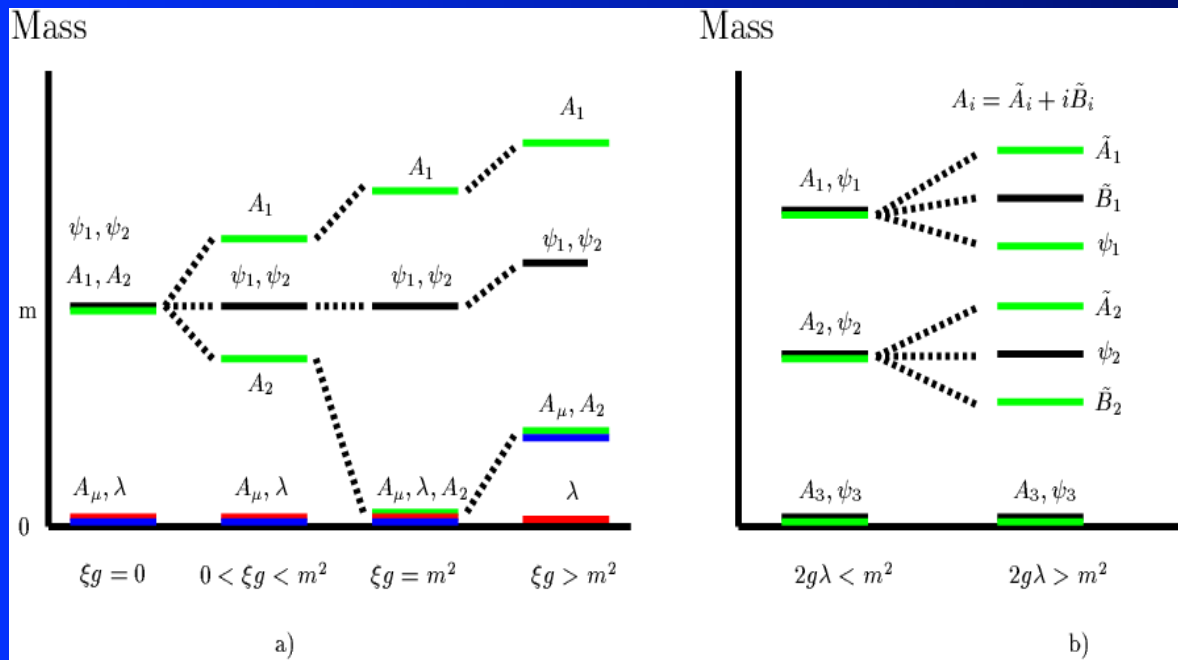
$$E = \frac{1}{4} \sum_{\alpha=1,2} \langle 0 | \{Q_\alpha^i, \bar{Q}_\alpha^j\} | 0 \rangle = \frac{1}{4} \sum_{\alpha} |Q_\alpha | 0 \rangle|^2 \geq 0$$

$$E = \langle 0 | H | 0 \rangle \neq 0 \quad \text{if and only if} \quad Q_\alpha | 0 \rangle \neq 0$$



Mechanism of SUSY Breaking

- Fayet-Iliopoulos (D-term) mechanism (in Abelian theory) $\Delta L = \xi V |_{\theta\theta\bar{\theta}\bar{\theta}} = \xi \int d^4\theta V = \xi D \neq 0$
- O’Raifeartaigh (F-term) mechanism $W(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2$



D-term

F-term

$$F_1^* = mA_2 + 2gA_1A_2$$

$$F_2^* = mA_1$$

$$F_3^* = \lambda + gA_1^2$$

$$\Rightarrow \langle F_i \rangle \neq 0$$

$$\sum_{\text{bosons}} m_i^2 = \sum_{\text{fermions}} m_i^2$$

Minimal Supersymmetric Standard Model (MSSM)

- SUSY: # of fermions = # of bosons N=1 SUSY: (φ, ψ) (λ, A_μ)
- SM: 28 bosonic d.o.f. & 90 (96) fermionic d.o.f.

There are no particles in the SM that can be superpartners

SUSY associates known bosons with new fermions and known fermions with new bosons

- Even number of the Higgs doublets – min = 2

Cancellation of axial anomalies (in each generation)

$$\text{Tr } Y^3 = 3\left(\frac{1}{27} + \frac{1}{27} - \frac{64}{27} + \frac{8}{27}\right) - 1 - 1 + 8 = 0$$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

colour u_L d_L u_R d_R ν_L e_L e_R

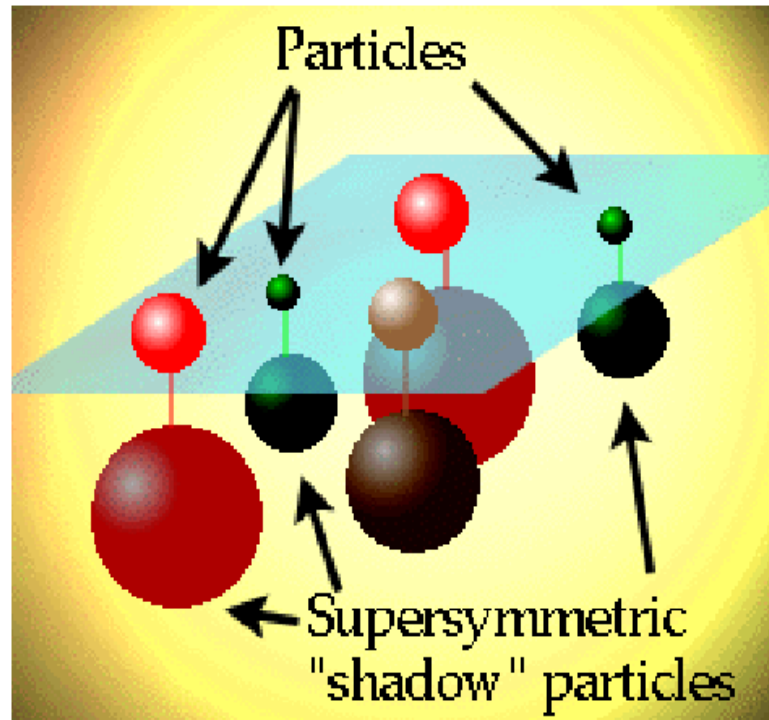
Higgsinos

$$-1 + 1 = 0$$

Particle Content of the MSSM

Superfield	Bosons	Fermions	$SU_c(3)$	$SU_L(2)$	$U_Y(1)$		
<i>Gauge</i>							
G^a	gluon g^a	gluino \tilde{g}^a	8	0	0		
V^k	Weak $W^k (W^\pm, Z)$	wino, zino $\tilde{w}^k (\tilde{w}^\pm, \tilde{z})$	1	3	0		
V'	Hypercharge $B(\gamma)$	bino $\tilde{b}(\tilde{\gamma})$	1	1	0		
<i>Matter</i>							
L_i	sleptons	$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons	$L_i = (\nu, e)_L$	1	2	-1
E_i				$E_i = e_R$	1	1	2
Q_i	squarks	$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks	$Q_i = (u, d)_L$	3	2	1/3
U_i				$U_i = u_R^c$	3*	1	-4/3
D_i				$D_i = d_R^c$	3*	1	2/3
<i>Higgs</i>							
H_1	Higgses	H_1	higgsinos	\tilde{H}_1	1	2	-1
H_2				H_2	\tilde{H}_2	1	2

SUSY Shadow World



One half is observed!



One half is NOT observed!

The MSSM Lagrangian

$$L = L_{gauge} + L_{Yukawa} + L_{SoftBreaking}$$

The Yukawa Superpotential

$$W_R = y_U Q_L H_2 U_R + y_D Q_L H_1 D_R + y_L L_L H_1 E_R + \mu H_1 H_2$$

superfields

Yukawa couplings

Higgs mixing term

$$W_{NR} = \lambda_L L_L L_L E_R + \lambda'_L L_L Q_L D_R + \mu' L_L H_2 + \lambda_B U_R D_R D_R$$

R-parity $R = (-)^{3(B-L)+2S}$

The Usual Particle : $R = + 1$

SUSY Particle : $R = - 1$

B - Baryon Number

L - Lepton Number

S - Spin

These terms are forbidden in the SM

R-parity Conservation

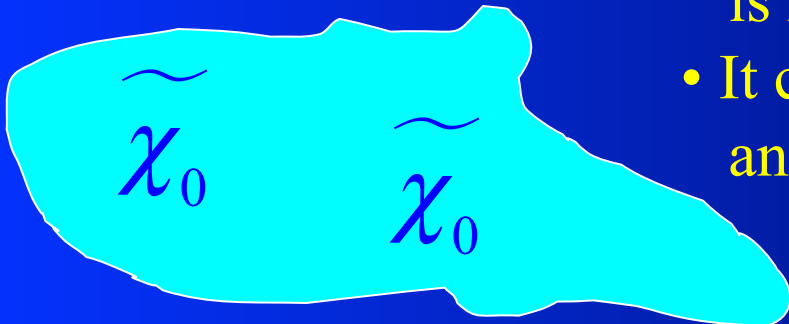
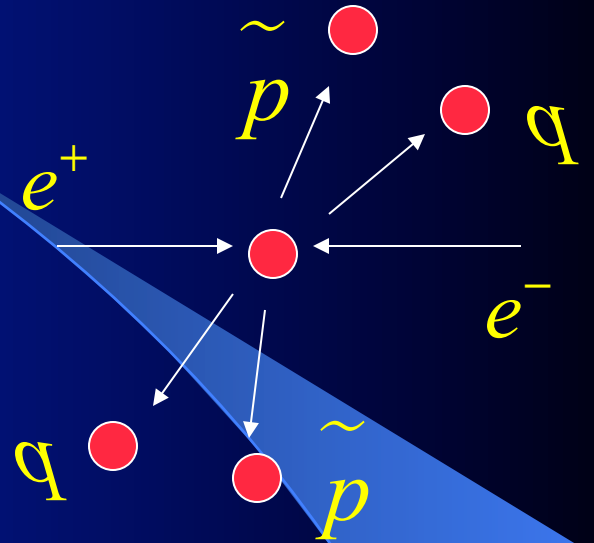
The consequences:

- The superpartners are created in pairs
- The lightest superparticle is stable



Physical output:

- The lightest superparticle (LSP) should be neutral - the best candidate is neutralino (photino or higgsino)
- It can survive from the Big Bang and form the Dark matter in the Universe

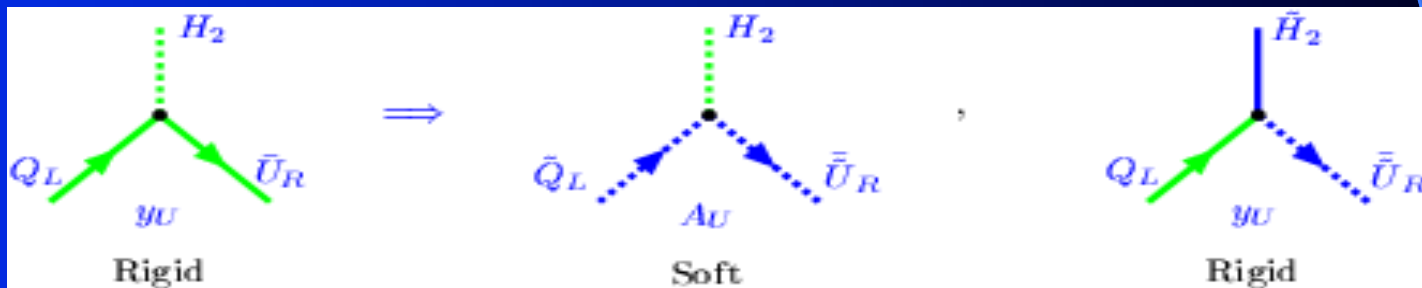
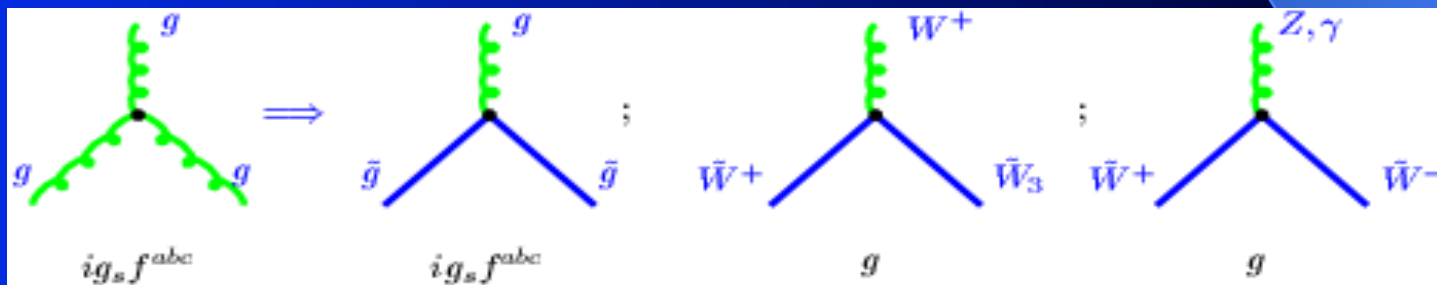
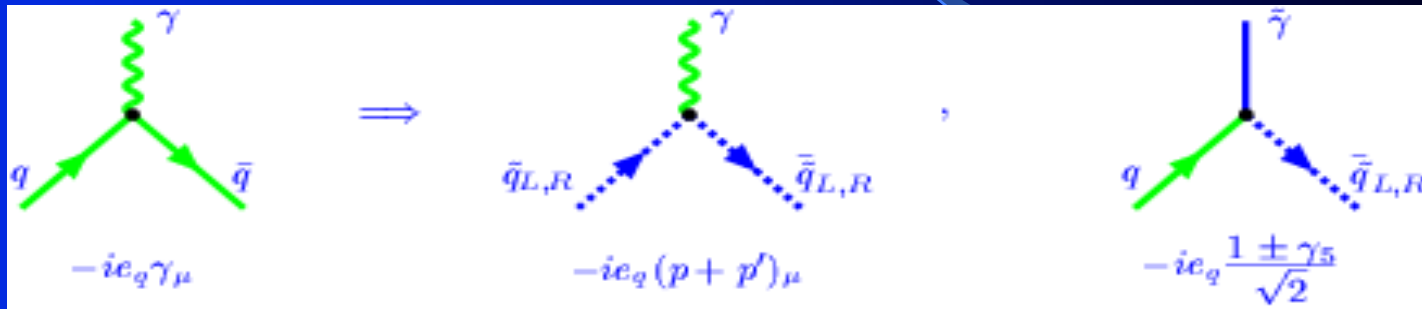


Interactions in the MSSM

SM

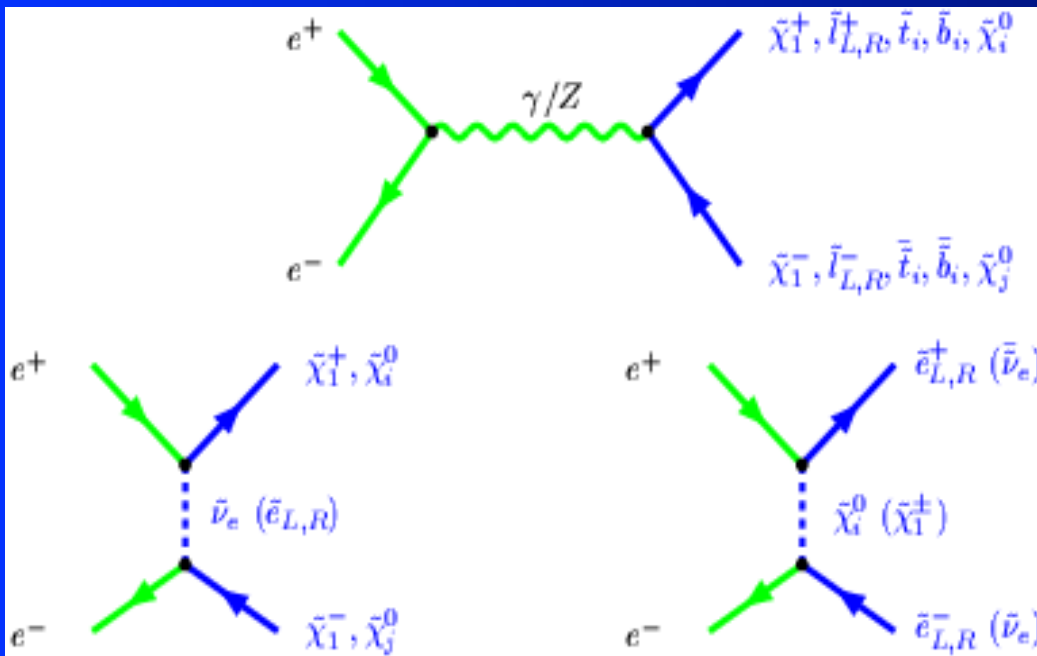


MSSM



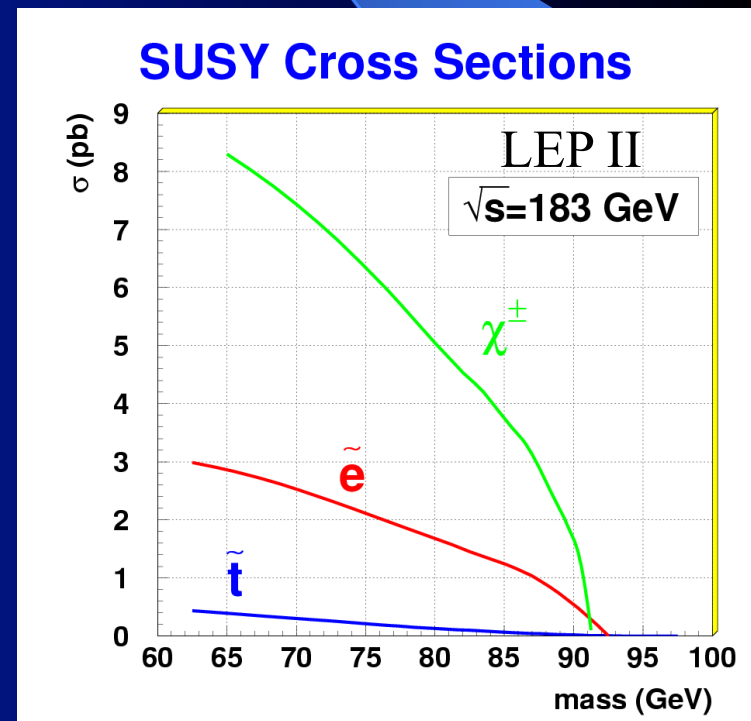
SM \rightarrow MSSM

Creation of Superpartners at e^+e^- colliders



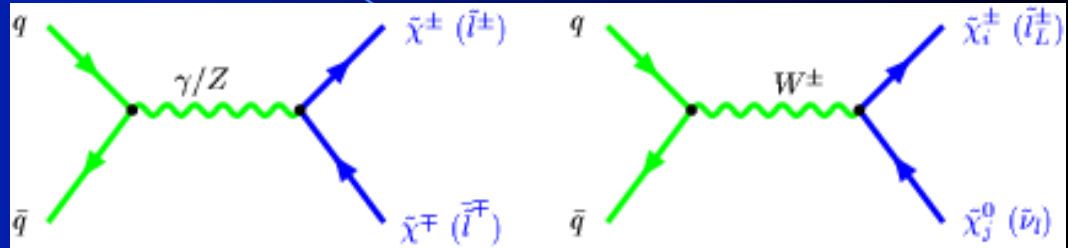
$$m_{\text{sparticle}}^{\text{max}} \leq \frac{\sqrt{s}}{2}$$

Experimental signature:
missing energy and transverse momentum



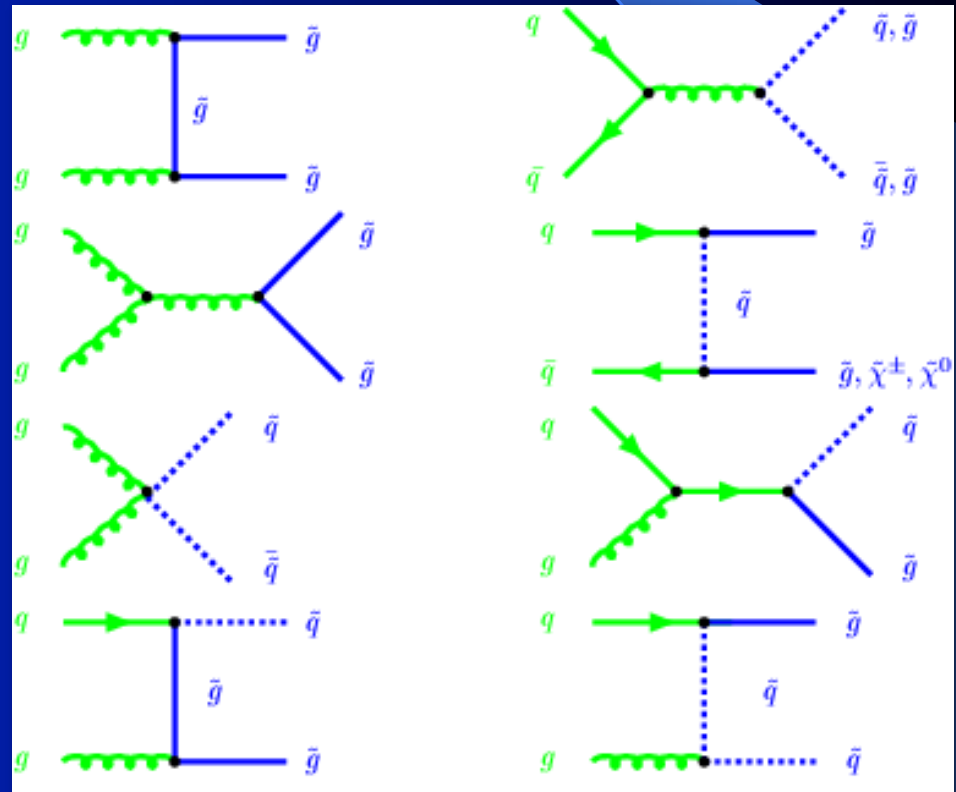
SUSY Production at Hadron Colliders

Annihilation channel



Gluon fusion, qq scattering and qg scattering channels

No new data so far due to insufficient luminosity at the Tevatron



Decay of Superpartners

squarks

$$\tilde{q}_{L,R} \rightarrow q + \tilde{\chi}_i^0$$

$$\tilde{q}_L \rightarrow q' + \tilde{\chi}_i^\pm$$

$$\tilde{q}_{L,R} \rightarrow q + g$$

sleptons

$$\tilde{l} \rightarrow l + \tilde{\chi}_i^0$$

$$\tilde{l}_L \rightarrow \nu_l + \tilde{\chi}_i^\pm$$

chargino

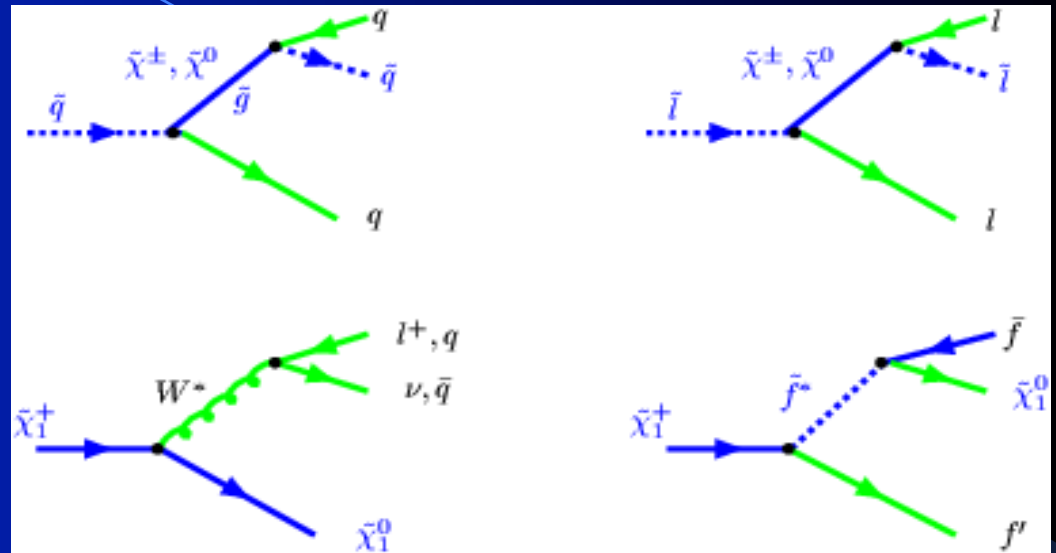
$$\tilde{\chi}_i^\pm \rightarrow e + \nu_e + \tilde{\chi}_i^0$$

$$\tilde{\chi}_i^\pm \rightarrow q + \bar{q}' + \tilde{\chi}_i^0$$

gluino

$$\tilde{g} \rightarrow q + \bar{q} + \tilde{\gamma}$$

$$\tilde{g} \rightarrow g + \tilde{\gamma}$$



neutralino

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + l^+ + l^-$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + q + \bar{q}'$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + l^\pm + \nu_l$$

$$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_1^0 + \nu_l + \bar{\nu}_l$$

Final states

$$l^+ l^- + \cancel{E}_T$$

$$2 \text{ jets} + \cancel{E}_T$$

$$\gamma + \cancel{E}_T$$

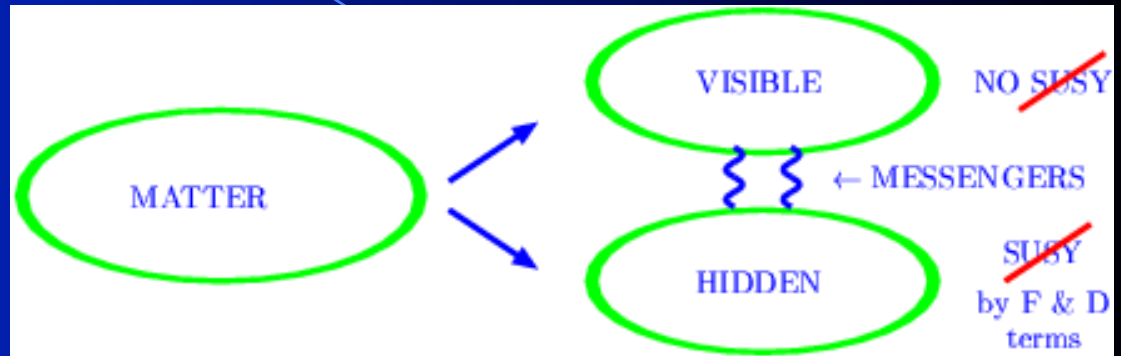
$$\cancel{E}_T$$

Soft SUSY Breaking

Hidden sector scenario:

four scenarios:

1. Gravity mediation
2. Gauge mediation
3. Anomaly mediation
4. Gaugino mediation



SUGRA S-dilaton, T-moduli $\langle F_T \rangle \neq 0, \langle F_S \rangle \neq 0$

$$M_{SUSY} \sim \frac{\langle F_T \rangle}{M_{PL}} + \frac{\langle F_S \rangle}{M_{PL}} \sim m_{3/2} \quad \leftarrow \quad \text{gravitino mass} \quad \sim 1 \text{ TeV}$$

$$L_{soft} = -\sum_i m_i^2 |A_i|^2 - \sum_i M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i) - BW^{(2)}(A) - AW^{(3)}(A)$$

$$m_i^2 \sim B \sim m_{3/2}^2, \quad M_i \sim A \sim m_{3/2}$$

Soft SUSY Breaking Cont'd

Gauge mediation

Scalar singlet S

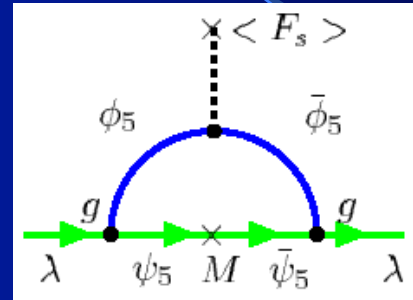
$$\langle S \rangle = M \quad \langle F_S \rangle \neq 0$$

Messenger Φ $W : S\Phi^+\Phi$

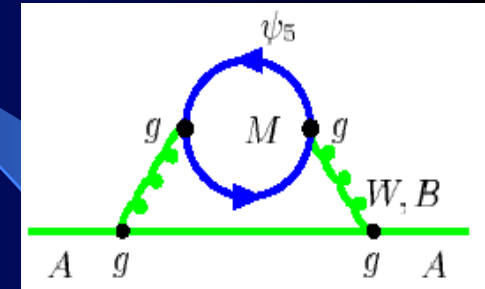
$$m_{\tilde{G}} \sim \frac{\langle F_S \rangle}{M_{PL}} \frac{M}{M_{PL}} \sim 10^{-14} \frac{M}{[GeV]}$$

gravitino mass

- LSP=gravitino



gaugino



squark

$$M_i \sim c_i N \frac{\alpha_i \langle F_S \rangle}{4\pi M} \quad m_i^2 \sim \left(\frac{\langle F_S \rangle}{M_{PL}} \right)^2 N \left(\frac{\alpha_i}{4\pi} \right)^2$$

Anomaly mediation

Results from conformal anomaly = β function

$$M_i \sim b_i \frac{\alpha_i(\Lambda)}{4\pi} \frac{\langle F_{T,S} \rangle}{M_{PL}} \sim b_i \alpha_i m_{3/2}$$

$$m_i^2 \sim b_i^2 \alpha_i^2 m_{3/2}^2$$

- $M_1 : M_2 : M_3 = b_1 : b_2 : b_3$

- LSP=slepton

Soft SUSY Breaking Cont'd

Gaugino mediation

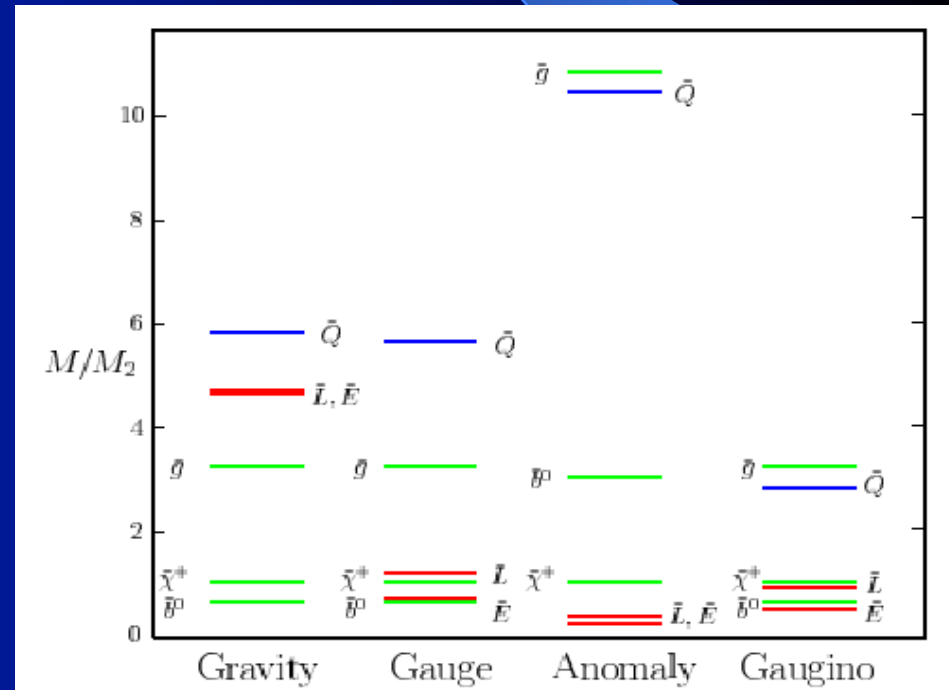
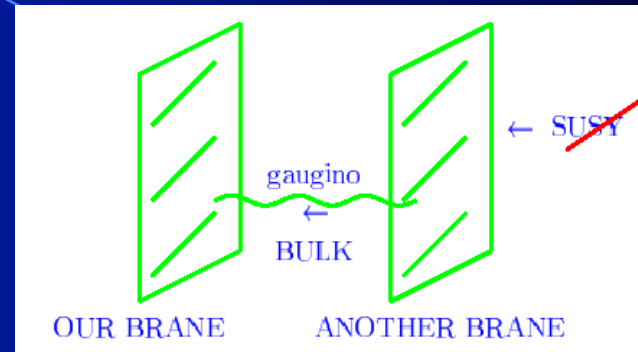
All scenarios produce soft SUSY breaking terms

Soft = operators of dimension ≤ 4

Net result of SUSY breaking

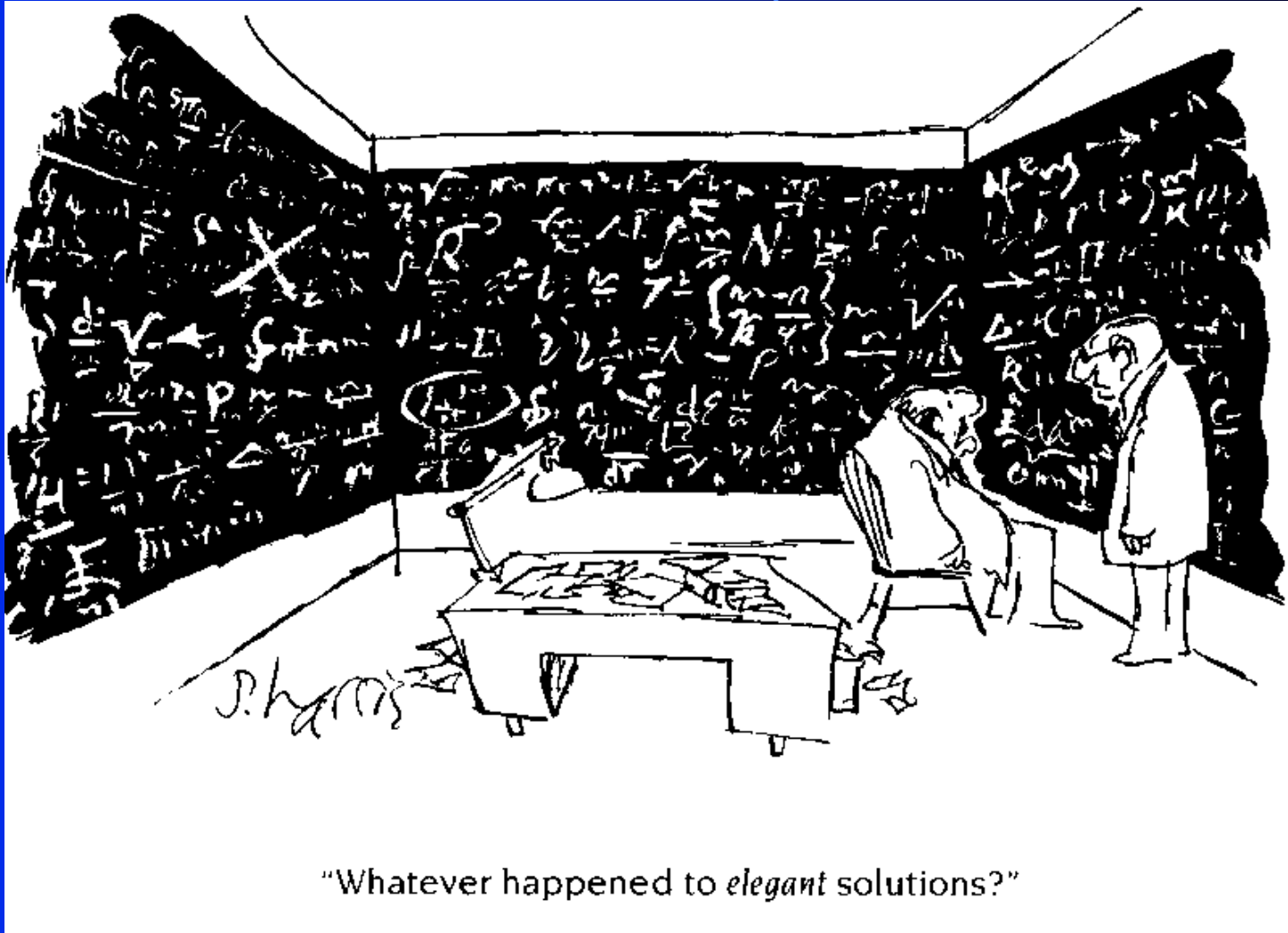
$$\begin{aligned}
 -L_{Soft} = & \sum_i m_{0i}^2 |A_i|^2 + \sum_\alpha M_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha \\
 & + \sum_{ijk} A_{ijk} A_i A_j A_k + \sum_{ij} B_{ij} A_i A_j
 \end{aligned}$$

↑ scalar fields
↑ gauginos



SUSY spectra for various mediation mechanisms

We like elegant solutions



Parameter Space of the MSSM

- Three gauge couplings $\alpha_i, i=1,2,3$
- Three (four) Yukawa matrices $y_{ab}^k, k=U, D, L, (E)$
- The Higgs mixing parameter μ
- Soft SUSY breaking terms

SUGRA Universality hypothesis: soft terms are universal and repeat the Yukawa potential

$$-L_{Soft} = A\{y_t Q_L H_2 U_R + y_b Q_L H_1 D_R + y_L L_L H_1 E_R\} + B\mu H_1 H_2 + m_0^2 \sum_i |\varphi_i|^2 + \frac{1}{2} M_{1/2} \sum_\alpha \widetilde{\lambda}_\alpha \widetilde{\lambda}_\alpha$$

Five universal soft parameters:

$$A, m_0, M_{1/2}, B \leftrightarrow \tan\beta \quad \text{and} \quad \mu$$

versus

$$m \quad \text{and} \quad \mathcal{Y}$$

in the SM

Mass Spectrum

$$L_{\text{gaugino-Higgsino}} = -\frac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \frac{1}{2} \bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.)$$

$$\psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix}$$

$$M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix}$$



$$\begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix}$$

$$\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0$$

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z \cos \beta \sin W & M_Z \sin \beta \sin W \\ 0 & M_2 & M_Z \cos \beta \cos W & -M_Z \sin \beta \cos W \\ -M_Z \cos \beta \sin W & M_Z \cos \beta \cos W & 0 & -\mu \\ M_Z \sin \beta \sin W & -M_Z \sin \beta \cos W & -\mu & 0 \end{pmatrix}$$

Mass Spectrum

$$\tilde{m}_t^2 = \begin{pmatrix} \tilde{m}_{tL}^2 & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & \tilde{m}_{tR}^2 \end{pmatrix}$$

$$\tilde{m}_b^2 = \begin{pmatrix} \tilde{m}_{bL}^2 & m_b(A_b - \mu \tan \beta) \\ m_b(A_b - \mu \tan \beta) & \tilde{m}_{bR}^2 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

$$\tilde{m}_{tL}^2 = \tilde{m}_Q^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{tR}^2 = \tilde{m}_U^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{bL}^2 = \tilde{m}_Q^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{bR}^2 = \tilde{m}_D^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{\tau L}^2 = \tilde{m}_L^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde{m}_{\tau R}^2 = \tilde{m}_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta.$$

$$\tilde{m}_\tau^2 = \begin{pmatrix} \tilde{m}_{\tau L}^2 & m_\tau(A_\tau - \mu \tan \beta) \\ m_\tau(A_\tau - \mu \tan \beta) & \tilde{m}_{\tau R}^2 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix}$$

SUSY Higgs Bosons

SM

$$4=2+2=3+1$$

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} = \begin{pmatrix} v + \frac{S + iP}{\sqrt{2}} \\ H^- \end{pmatrix} = \exp\left(i\frac{\vec{\xi}\vec{\sigma}}{2}\right) \begin{pmatrix} v + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$H \rightarrow H' = \exp\left(i\frac{\vec{\alpha}\vec{\sigma}}{2}\right)H \xrightarrow{(\vec{\alpha}=-\vec{\xi})} H' = \begin{pmatrix} v + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

MSSM

$$8=4+4=3+5$$

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_2^+ \\ v_2 + \frac{S_2 + iP_2}{\sqrt{2}} \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2, \quad v_2/v_1 \equiv \tan\beta$$

$$G^0 = -\cos\beta P_1 + \sin\beta P_2$$

Goldstone boson $\rightarrow Z_0$

$$A = \sin\beta P_1 + \cos\beta P_2$$

Neutral CP = -1 Higgs

$$G^+ = -\cos\beta (H_1^-)^* + \sin\beta H_2^+$$

Goldstone boson $\rightarrow W^+$

$$H^+ = \sin\beta (H_1^-)^* + \cos\beta H_2^+$$

Charged Higgs

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}$$

$$h = -\sin\alpha S_1 + \cos\alpha S_2$$

SM Higgs boson CP = 1

$$H = \cos\alpha S_1 + \sin\alpha S_2$$

Extra heavy Higgs boson

The Higgs Potential

$$V_{tree}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) \\ + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^+ H_2|^2$$

At the GUT scale: $m_1^2 = m_2^2 = \mu_0^2 + m_0^2$, $m_3^2 = -B\mu_0$

Minimization

$$\frac{1}{2} \frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_1 = 0,$$

$$\frac{1}{2} \frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 - \frac{g^2 + g'^2}{4} (v_1^2 - v_2^2) v_2 = 0.$$

$$\langle H_1 \rangle \equiv v_1 = v \cos \beta, \quad \langle H_2 \rangle \equiv v_2 = v \sin \beta,$$

Solution

$$v^2 = \frac{4(m_1^2 - m_2^2 \tan^2 \beta)}{(g^2 + g'^2)(\tan^2 \beta - 1)},$$

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}$$

At the GUT scale

$$v^2 = -\frac{4}{g^2 + g'^2} m^2 < 0$$

No SSB in SUSY theory !

Renormalization Group Eqns

$$\tilde{\alpha}_i \equiv \frac{g_i^2}{16\pi^2} = \frac{\alpha_i}{4\pi}, \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad t = \log(M_{GUT}^2 / Q^2)$$

$$i = 1, 2, 3 \quad k = U, D, L$$

The couplings

$$\dot{\tilde{\alpha}}_i = -b_i \tilde{\alpha}_i^2, \quad b_i^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

$$\dot{Y}_U = Y_U \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{15} \tilde{\alpha}_1 - 6Y_U - Y_D\right),$$

$$\dot{Y}_D = Y_D \left(\frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{15} \tilde{\alpha}_1 - Y_U - 6Y_D - Y_L\right),$$

$$\dot{Y}_L = Y_L \left(3\tilde{\alpha}_2 + \frac{9}{5} \tilde{\alpha}_1 - 3Y_D - 4Y_L\right),$$

$$\dot{M}_i = b_i \tilde{\alpha}_i M_i,$$

$$\dot{A}_U = -\left(\frac{16}{3} \tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{13}{15} \tilde{\alpha}_1 M_1\right) - 6Y_U A_U - Y_D A_D,$$

$$\dot{A}_D = -\left(\frac{16}{3} \tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{7}{15} \tilde{\alpha}_1 M_1\right) - Y_U A_U - 6Y_D A_D - Y_L A_L,$$

$$\dot{A}_L = -\left(3\tilde{\alpha}_2 M_2 + \frac{9}{5} \tilde{\alpha}_1 M_1\right) - 3Y_D A_D - 4Y_L A_L,$$

$$\dot{B} = -3\left(\tilde{\alpha}_2 M_2 + \frac{1}{5} \tilde{\alpha}_1 M_1\right) - 3Y_U A_U - 3Y_D A_D - Y_L A_L,$$

$$\dot{\mu} = -\mu^2 \left(3\tilde{\alpha}_2 + \frac{3}{5} \tilde{\alpha}_1 - 3Y_U - 3Y_D - Y_L\right)$$

Soft Terms

RG Eqns for the Soft Masses

$$\dot{\tilde{m}}_Q^2 = -\left[\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \frac{1}{15}\tilde{\alpha}_1 M_1^2 - Y_t(\Sigma_t + A_t^2) - Y_b(\Sigma_b + A_b^2)\right]$$

$$\dot{\tilde{m}}_U^2 = -\left[\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{16}{15}\tilde{\alpha}_1 M_1^2 - 2Y_t(\Sigma_t + A_t^2)\right]$$

$$\dot{\tilde{m}}_D^2 = -\left[\frac{16}{3}\tilde{\alpha}_3 M_3^2 + \frac{4}{15}\tilde{\alpha}_1 M_1^2 - 2Y_b(\Sigma_b + A_b^2)\right]$$

$$\dot{\tilde{m}}_L^2 = -\left[3\tilde{\alpha}_2 M_2^2 + \frac{3}{5}\tilde{\alpha}_1 M_1^2 - Y_\tau(\Sigma_\tau + A_\tau^2)\right]$$

$$\dot{\tilde{m}}_E^2 = -\left[\frac{12}{5}\tilde{\alpha}_1 M_1^2 - 2Y_\tau(\Sigma_\tau + A_\tau^2)\right]$$

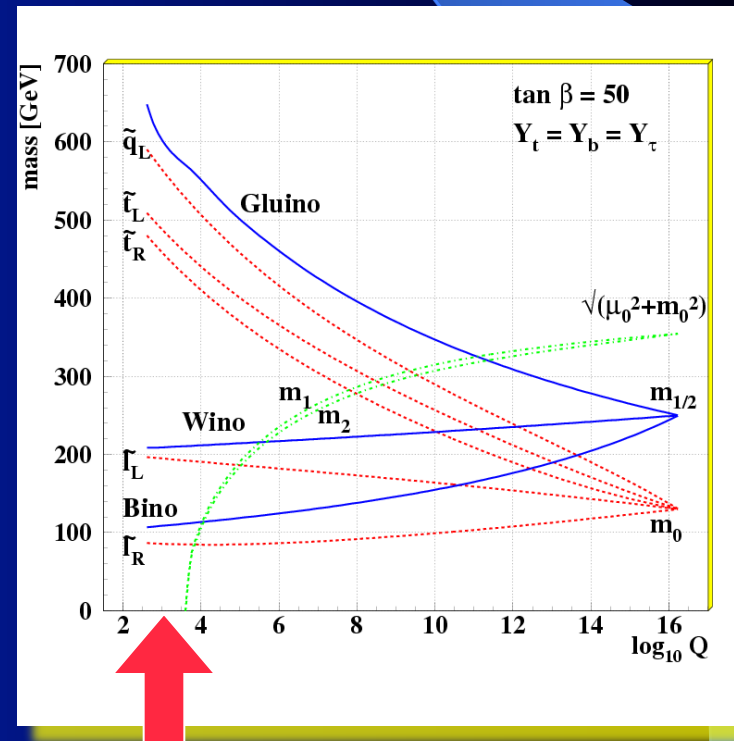
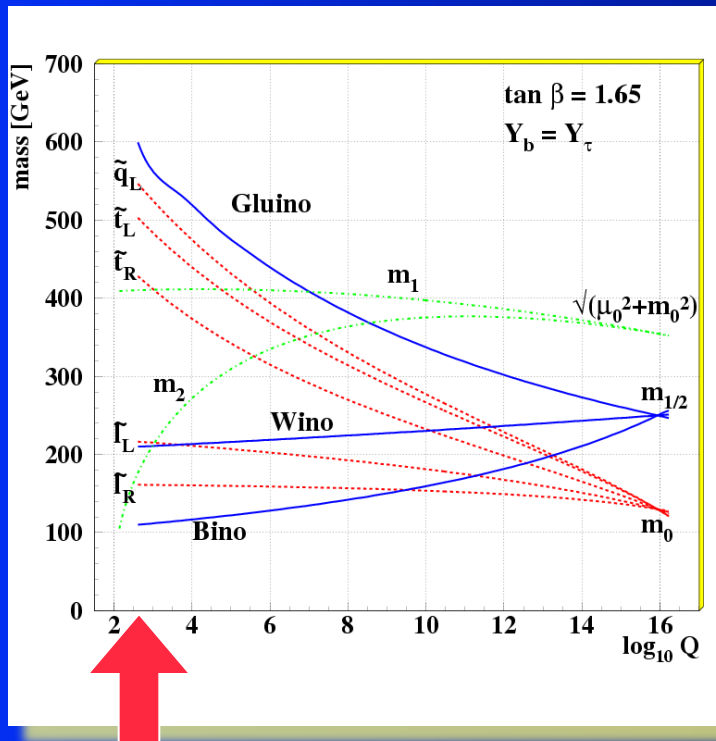
$$\dot{\tilde{m}}_{H_1}^2 = -\left[3\tilde{\alpha}_2 M_2^2 + \frac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_b(\Sigma_b + A_b^2) - Y_\tau(\Sigma_\tau + A_\tau^2)\right]$$

$$\dot{\tilde{m}}_{H_2}^2 = -\left[3\tilde{\alpha}_2 M_2^2 + \frac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_t(\Sigma_t + A_t^2)\right]$$

$$\Sigma_t = m_Q^2 + m_U^2 + m_{H_2}^2, \Sigma_b = m_Q^2 + m_D^2 + m_{H_1}^2, \Sigma_\tau = m_L^2 + m_E^2 + m_{H_1}^2$$

Radiative EW Symmetry Breaking

Due to RG controlled running of the mass terms from the Higgs potential they may change sign and trigger the appearance of non-trivial minimum leading to spontaneous breaking of EW symmetry - this is called Radiative EWSB



The Higgs Bosons Masses

CP-odd neutral Higgs A $m_A^2 = m_1^2 + m_2^2$ $M_W^2 = \frac{g^2}{2} v^2$

CP-even charged Higgses H_{\pm} $m_{H^{\pm}}^2 = m_A^2 + M_W^2$ $M_Z^2 = \frac{g^2 + g'^2}{2} v^2$

CP-even neutral Higgses h,H

$$m_{h,H}^2 = \frac{1}{2} [m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}]$$

$m_h \approx M_Z |\cos 2\beta| < M_Z ! \Rightarrow$ Radiative corrections

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{m_{t_1}^{\sim 2} m_{t_2}^{\sim 2}}{m_t^4} + 2 \text{ loops}$$

Constrained MSSM

Requirements:

- Unification of the gauge couplings
- Radiative EW Symmetry Breaking
- Heavy quark and lepton masses
- Rare decays ($b \rightarrow s\gamma$)
- Anomalous magnetic moment of muon
- LSP is neutral
- Amount of the Dark Matter
- Experimental limits from direct search

Allowed region
in the parameter
space of the MSSM

$$A_0, m_0, M_{1/2}, \mu, \tan \beta$$

Parameter space:

$$100 \text{ GeV} < m_0, M_{1/2}, \mu < 2 \text{ TeV} \\ -3m_0 < A_0 < 3m_0, 1 < \tan \beta < 70$$

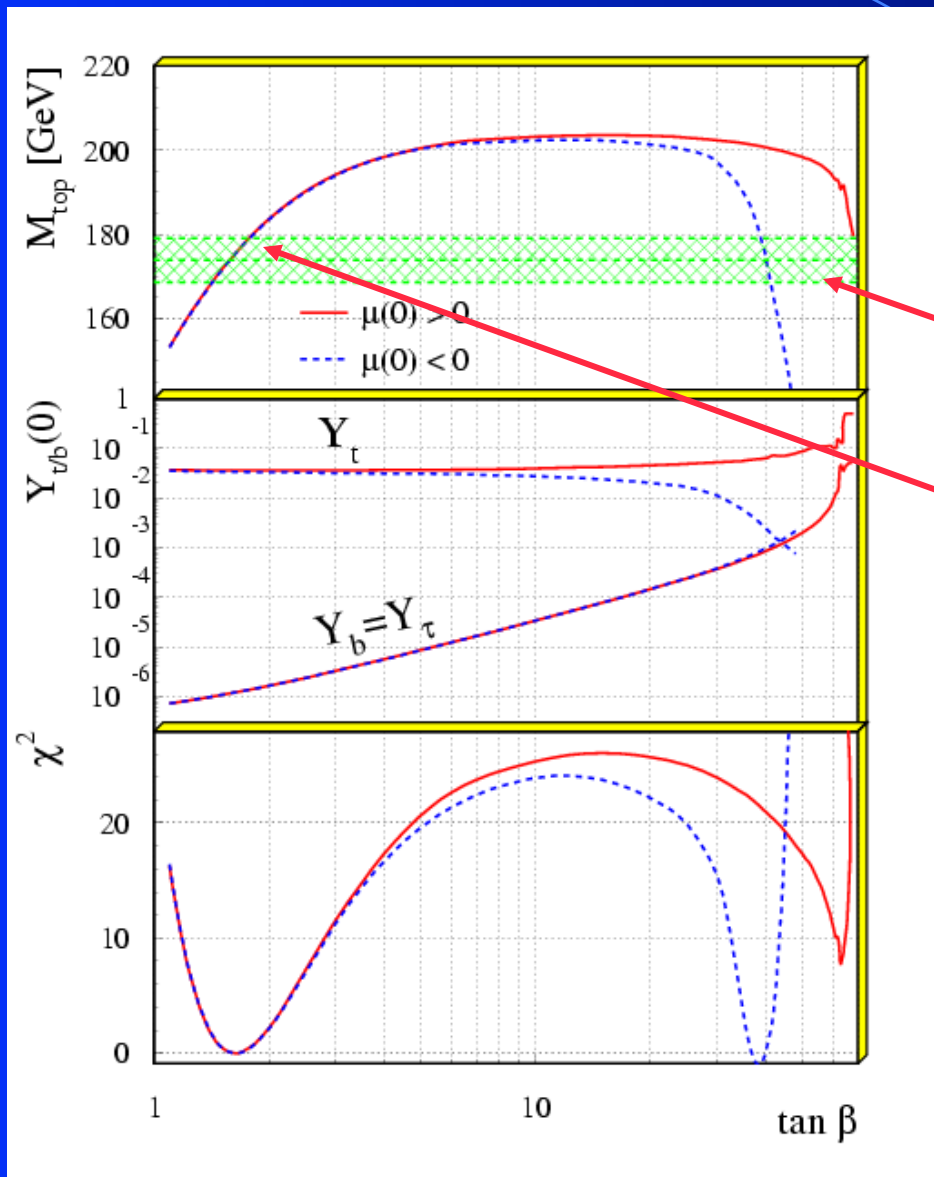
SUSY Fits

$$\begin{aligned}
 \chi^2 = & \sum_{i=1}^3 \frac{(\alpha_i^{-1}(M_Z) - \alpha_{MSSMi}^{-1}(M_Z))^2}{\sigma_i^2} \\
 & + \frac{(M_Z - 91.18)^2}{\sigma_Z^2} + \frac{(M_t - 174)^2}{\sigma_t^2} \\
 & + \frac{(M_b - 4.94)^2}{\sigma_b^2} + \frac{(M_\tau - 1.7771)^2}{\sigma_\tau^2} \\
 & + \frac{(\text{Br}(b \rightarrow s\gamma) - 3.14 \times 10^{-4})^2}{\sigma^2(b \rightarrow s\gamma)} \\
 & + \frac{(\Omega h^2 - 1)^2}{\sigma_\Omega^2} \quad (\text{for } \Omega h^2 > 1) \\
 & + \frac{(\tilde{M} - \tilde{M}_{\text{exp}})^2}{\sigma_{\tilde{M}}^2} \quad (\text{for } \tilde{M} < \tilde{M}_{\text{exp}}) \\
 & + \frac{(\tilde{m}_{\text{LSP}} - \tilde{m}_\chi)^2}{\sigma_{\text{LSP}}^2} \quad (\text{for } \tilde{m}_{\text{LSP}} \text{ charged})
 \end{aligned}$$

Minimize χ^2

Exp. input data	Fit	Parameters
	low $\tan\beta$	high $\tan\beta$
$\alpha_1, \alpha_2, \alpha_3$	M_{GUT}, α_{GUT}	M_{GUT}, α_{GUT}
m_t	$Y_t^0, Y_b^0 = Y_\tau^0$	$Y_t^0 = Y_b^0 = Y_\tau^0$
m_b	$m_0, m_{1/2}$	$m_0, m_{1/2}$
m_τ	$\tan\beta$	$\tan\beta$
M_Z	μ	μ
$b \rightarrow s\gamma$	(A_0)	A_0
τ_{Universe}		

Low and High $\tan\beta$ Solutions



Requirements:

- EWSB
- $b\tau$ unification

Low $\tan\beta$ solution

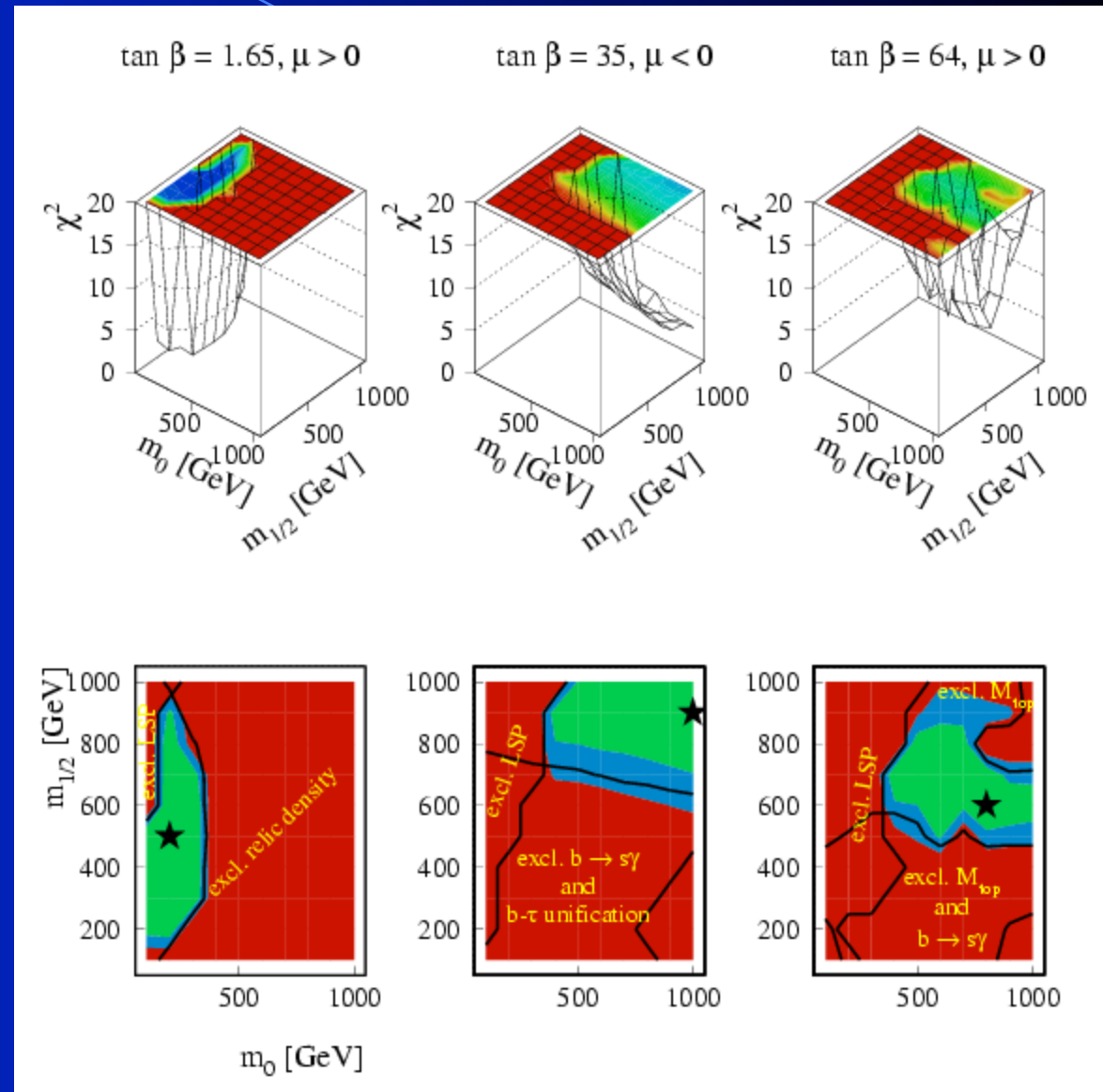
High $\tan\beta$ solution

- $b\tau$ unification is the consequence of GUT
- Non working for the light generations

Allowed Regions in Parameter Space

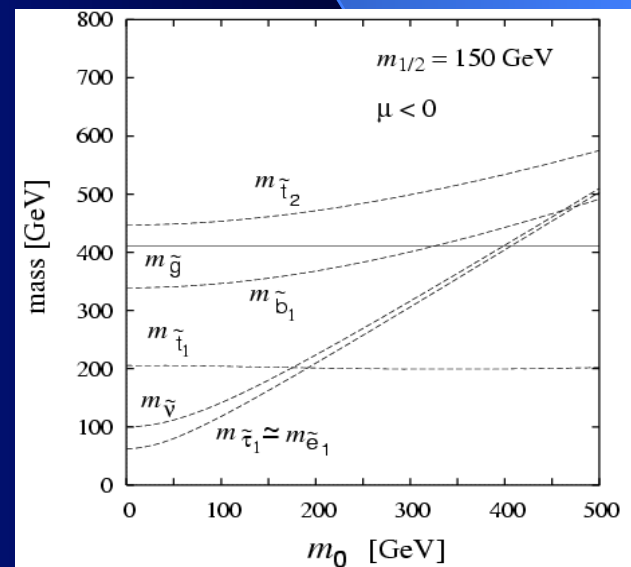
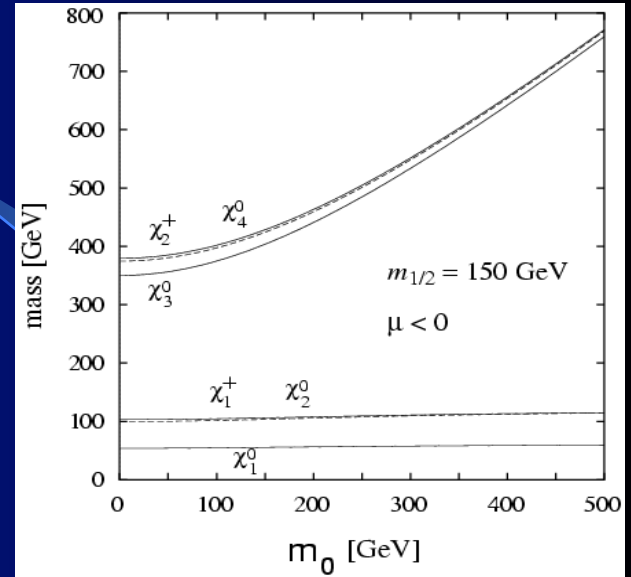
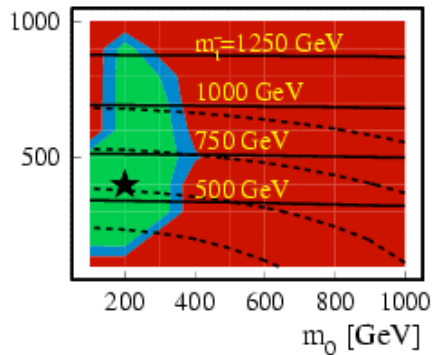
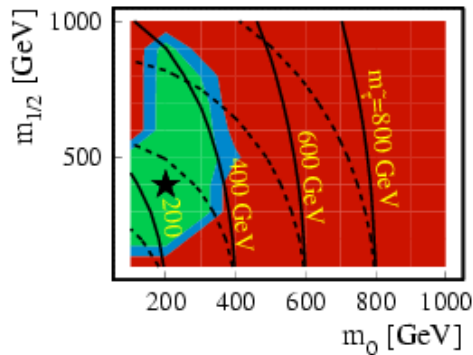
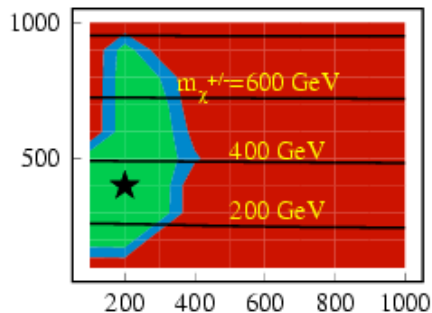
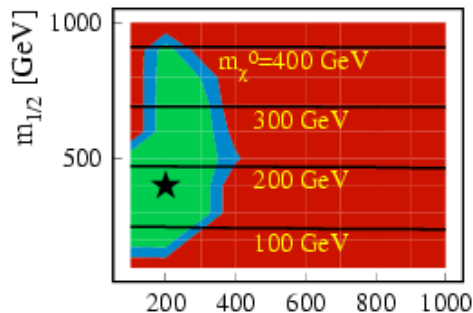
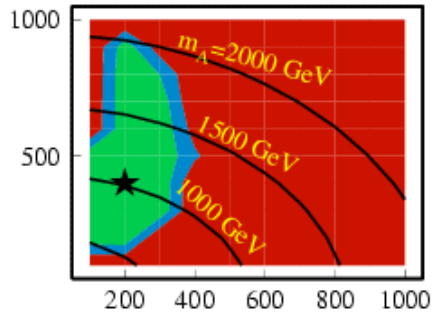
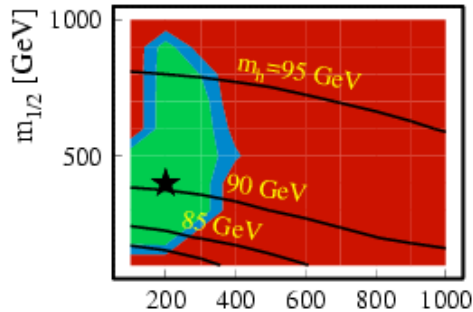
All the requirements are fulfilled simultaneously !

- μ is defined from the EWSB
 - $A_0 = 0$
- ★ - is the best fit value



Masses of Superpartners

$\tan \beta = 1.65, \mu > 0$



Mass Spectrum in CMSSM

SUSY Masses in GeV

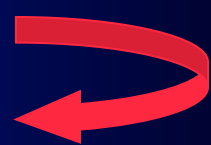
Fitted SUSY Parameters

Symbol	Low tan β	High tan β
Tan β	1.71	35.0
m_0	200	600
$m_{1/2}$	500	400
$\mu(0)$	1084	-558
$A(0)$	0	0
$1/\alpha_{\text{GUT}}$	24.8	24.8
M_{GUT}	$1.6 \cdot 10^{16}$	$1.6 \cdot 10^{16}$

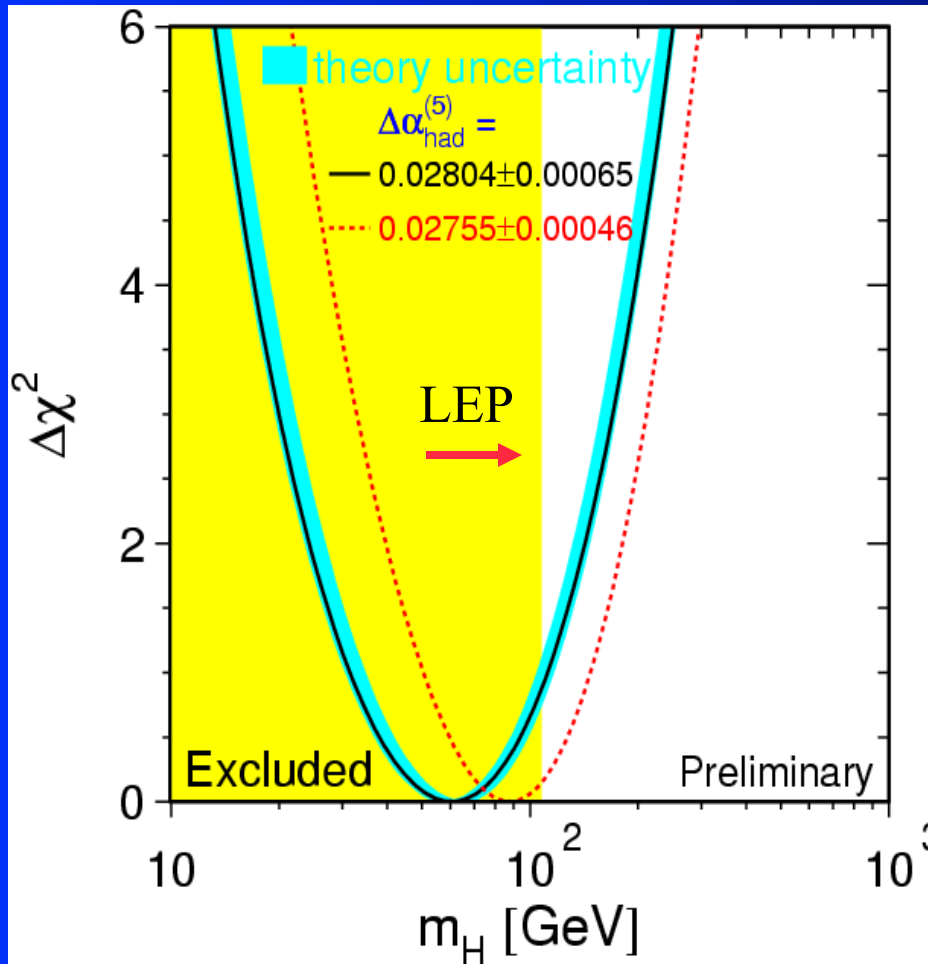
Symbol	Low tan β	High tan β
$\tilde{\chi}_1^0(\tilde{B}), \tilde{\chi}_2^0(\tilde{W}^3)$	214, 413	170, 322
$\tilde{\chi}_3^0(\tilde{H}_1), \tilde{\chi}_4^0(\tilde{H}_2)$	1028, 1016	481, 498
$\tilde{\chi}_1^\pm(\tilde{W}^\pm), \tilde{\chi}_2^\pm(\tilde{H}^\pm)$	413, 1026	322, 499
\tilde{g}	1155	950
\tilde{e}_L, \tilde{e}_R	303, 270	663, 621
$\tilde{\nu}_L$	290	658
\tilde{q}_L, \tilde{q}_R	1028, 936	1040, 1010
$\tilde{\tau}_1, \tilde{\tau}_2$	279, 403	537, 634
\tilde{b}_1, \tilde{b}_2	953, 1010	835, 915
\tilde{t}_1, \tilde{t}_2	727, 1017	735, 906
h, H	95, 1344	119, 565
A, H $^\pm$	1340, 1344	565, 571 ⁵³

The Lightest Superparticle

		<u>property</u>	<u>signature</u>
• <u>Gravity mediation</u>	LSP = $\tilde{\chi}_1^0$	stable	jets/leptons + \cancel{E}_T
• <u>Gauge mediation</u>	LSP = \tilde{G}	stable	\cancel{E}_T
	NLSP =		
	$\left\{ \begin{array}{l} \tilde{\chi}_1^0 \\ \tilde{l}_R \end{array} \right.$	$\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}, h \tilde{G}, Z \tilde{G}$ $\tilde{l}_R \rightarrow \tau \tilde{G}$	photons/jets + \cancel{E}_T lepton + \cancel{E}_T
• <u>Anomaly mediation</u>	LSP =		
	$\left\{ \begin{array}{l} \tilde{\chi}_1^0 \\ \tilde{\nu}_L \end{array} \right.$	stable stable	lepton + \cancel{E}_T
• <u>R-parity violation</u>	LSP is unstable \rightarrow SM particles		
• <u>Modern limit</u>	$M_{LSP} \geq 40 \text{ GeV}$		Rare decays Neutrinoless double β decay



The Higgs Mass Limit

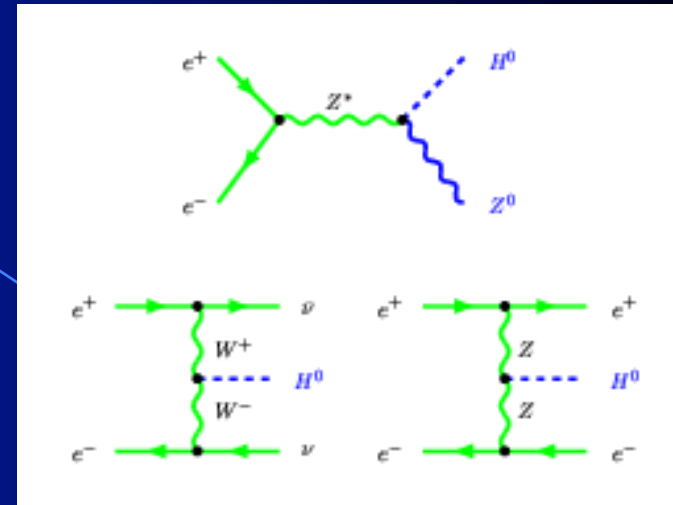
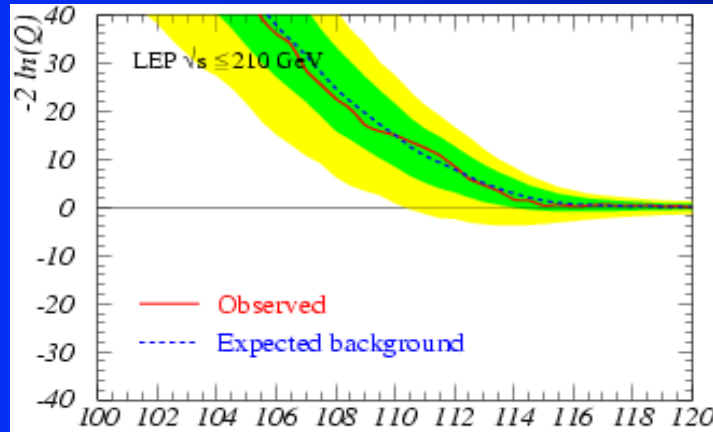


- Indirect limit from radiative corrections
- Direct limit from Higgs non-observation at LEP II (CERN)

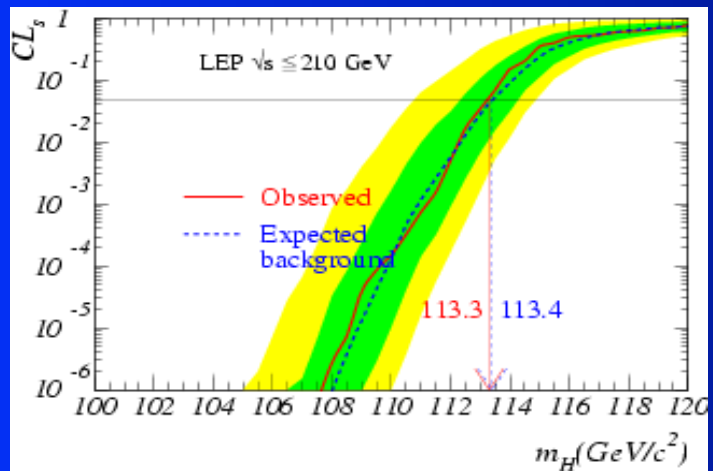
$$113 < m_H < 200 \text{ GeV}$$

At 95 % C.L.

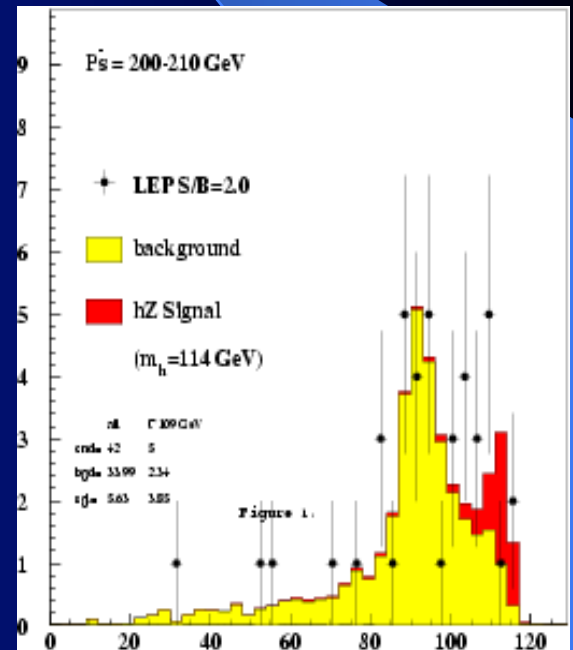
Higgs Searches



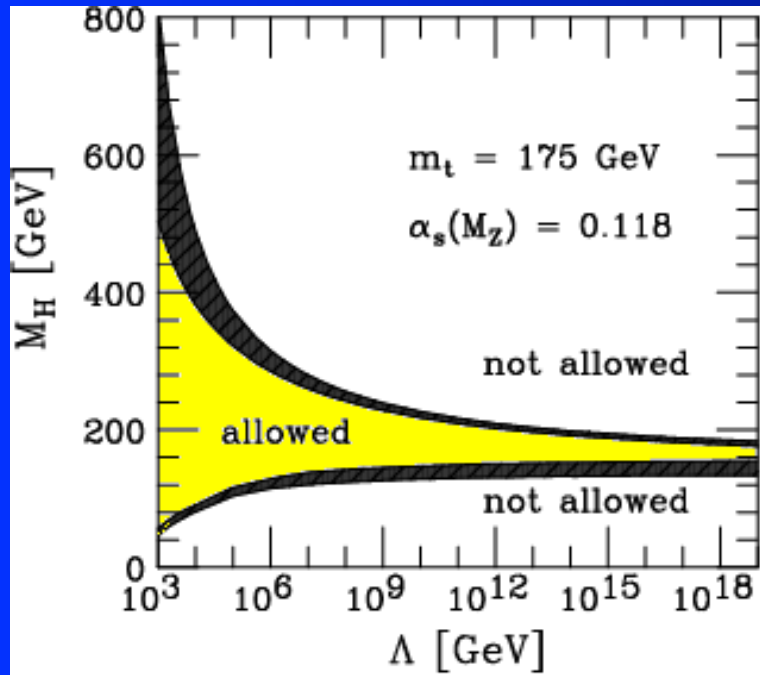
$m_H \geq 113.4$ GeV at 95 % C.L.



114 -115 GeV Event

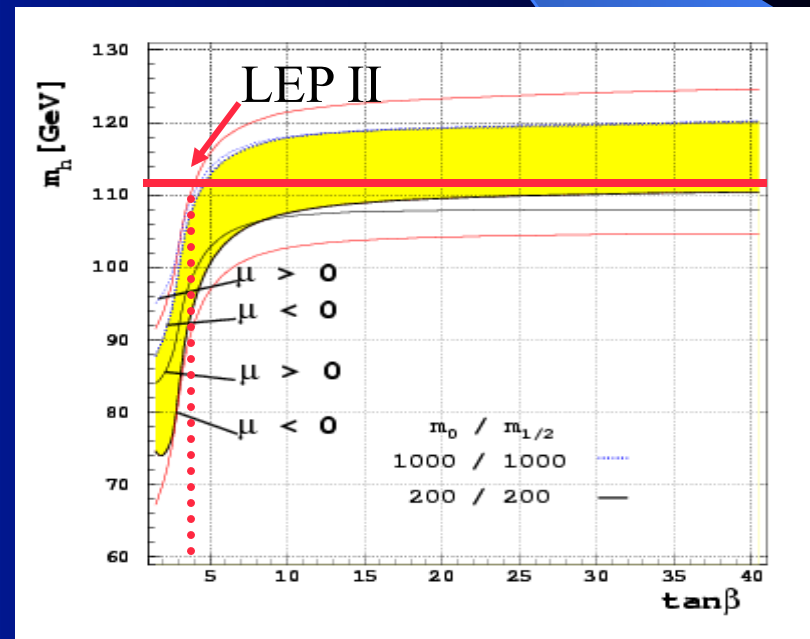
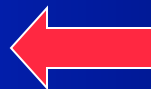


The Higgs Mass Limit (Theory)

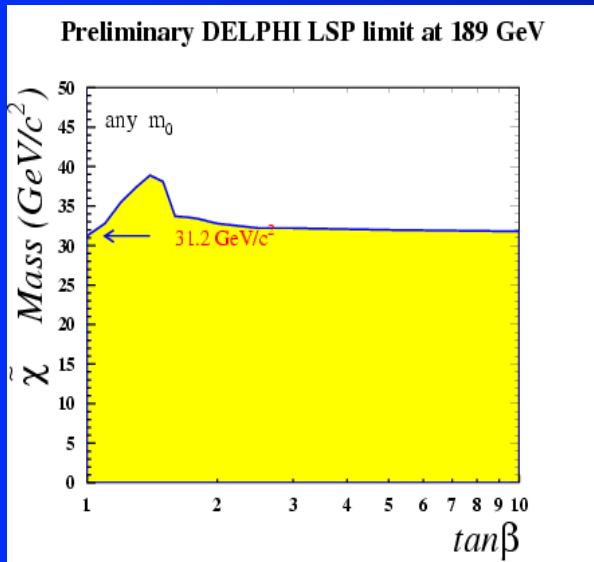


- The SM Higgs $m_H \geq 134 \text{ GeV}$

SUSY Higgs $m_H \leq 130 \text{ GeV}$



SUSY Searches at LEP



neutralinos

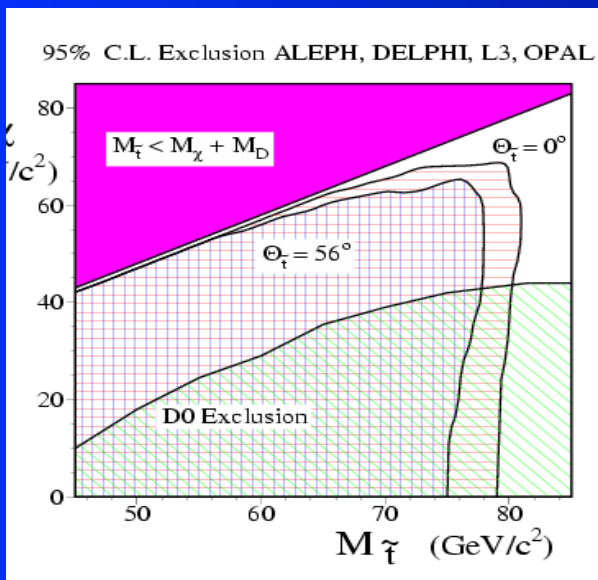
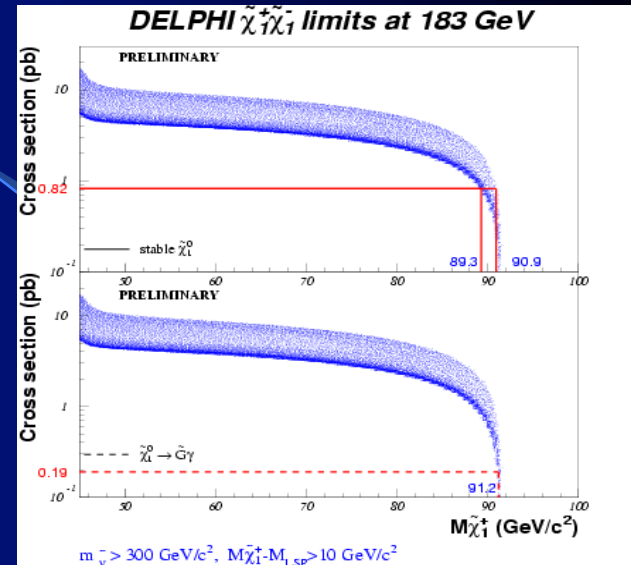


$$\tilde{M}_{\chi_0} \geq 40 \text{ GeV}$$

charginos



$$\tilde{m}_{\chi^+} \geq 100 \text{ GeV}$$



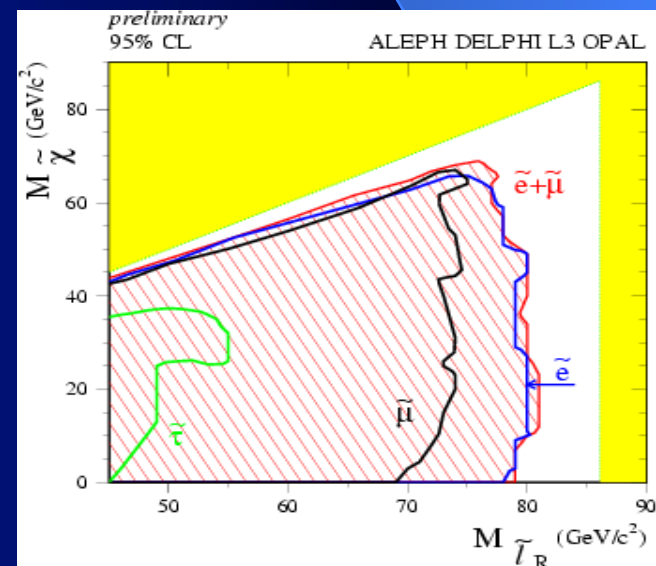
squarks



sleptons

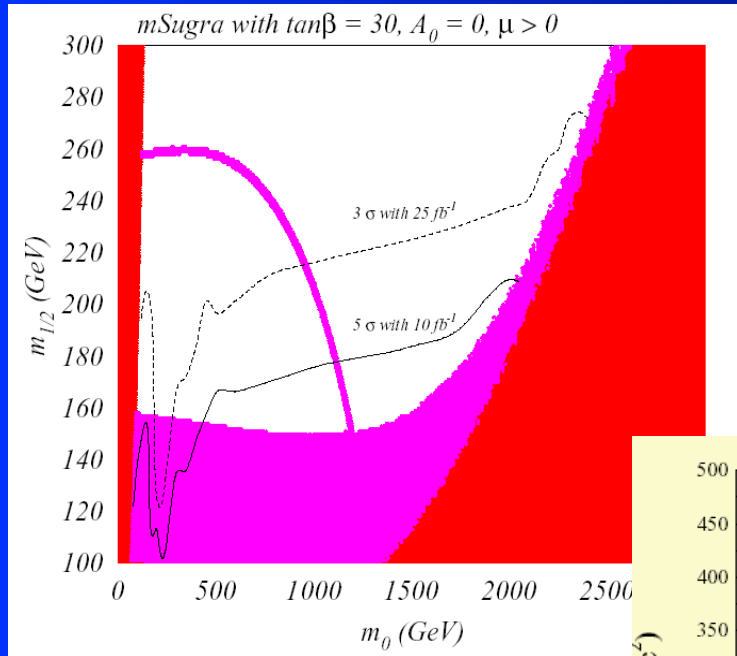


$$\tilde{m}_l \geq 100 \text{ GeV}$$



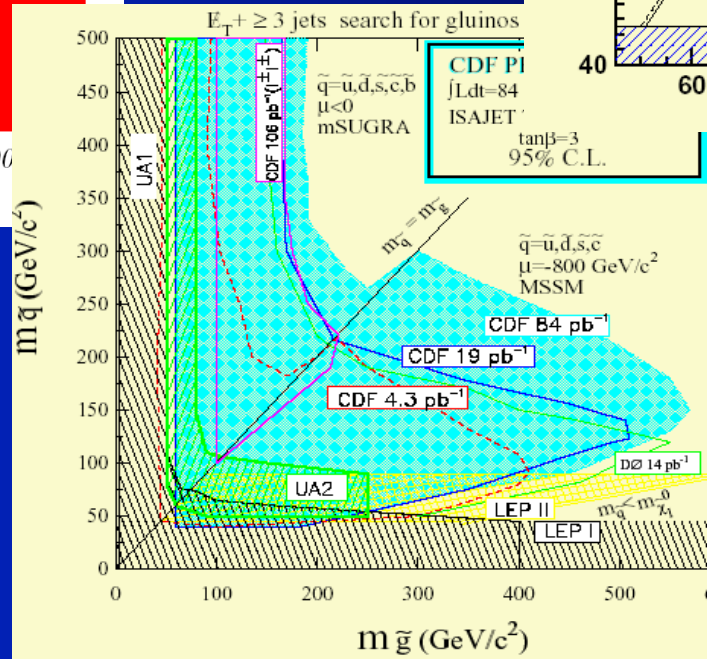
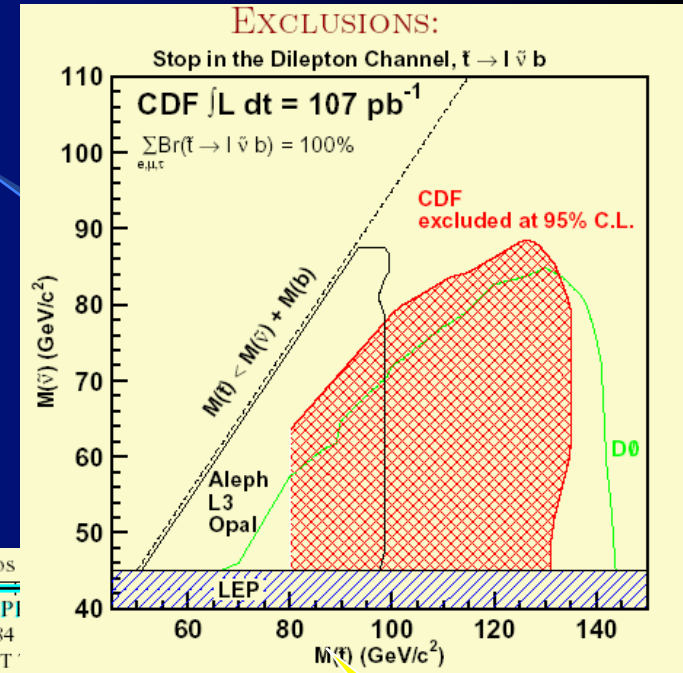
SUSY Searches at Tevatron

The reach of Tevatron in $m_0 / m_{1/2}$ plane



Exclusion:
World's Best Limits

$M_{\tilde{q}} \geq 300 \text{ GeV}$
 $m_{\tilde{g}} \geq 195 \text{ GeV}$



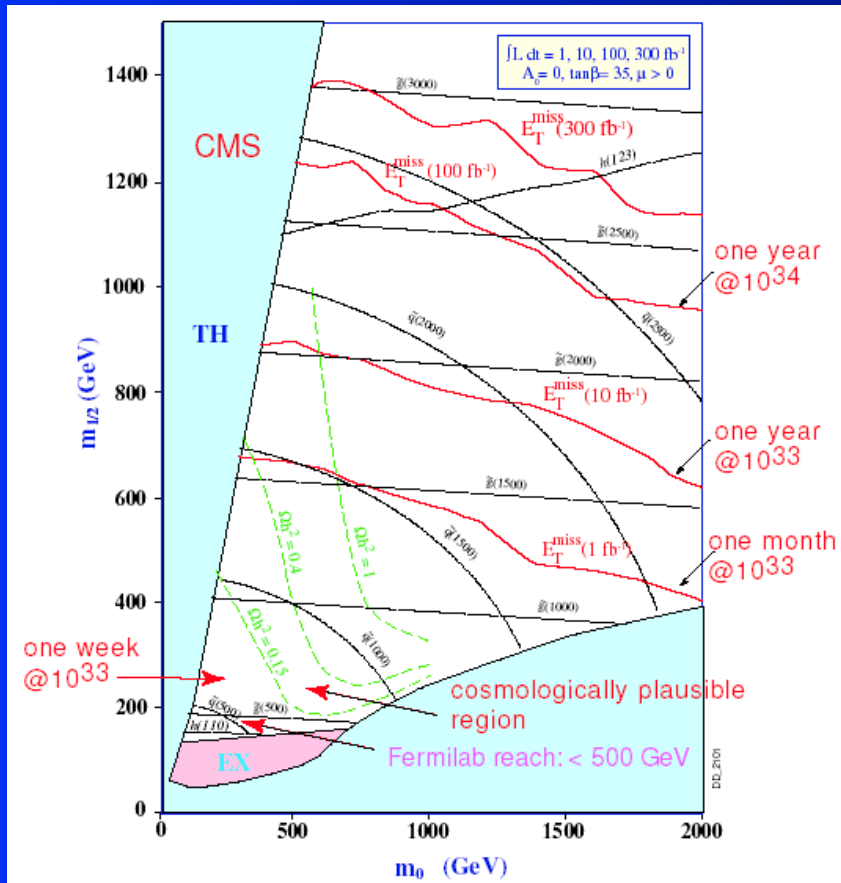
Dilepton
Channel

3 jet channel

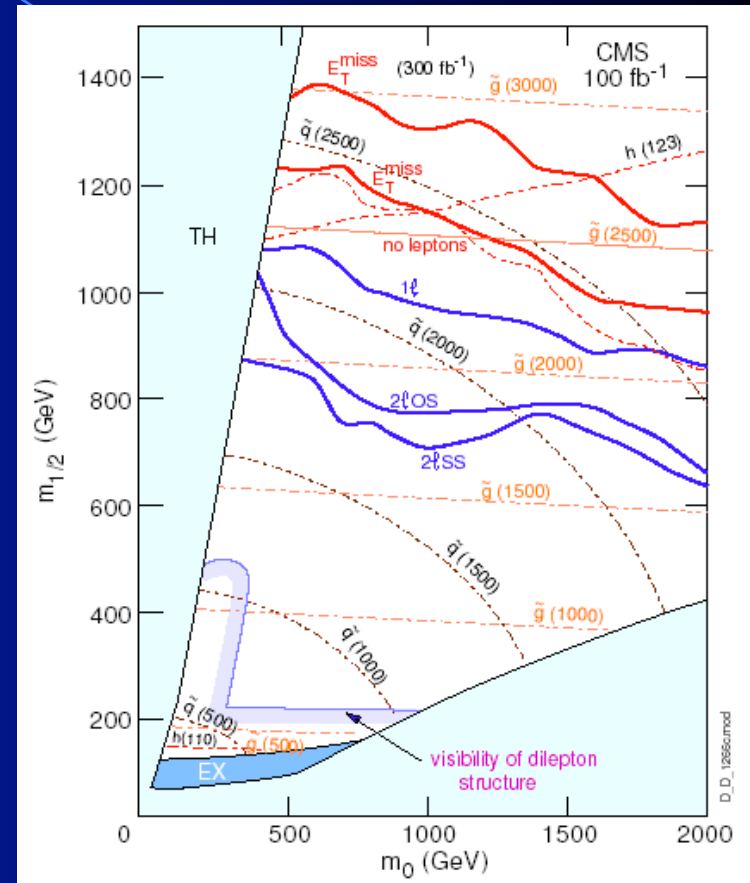
Tevatron Discovery Reach

Decay ($Br = 100\%$)	Subsequent Decay	Final State of $\tilde{b}_1\tilde{\bar{b}}_1$ or $\tilde{t}_1\tilde{\bar{t}}_1$	Discovery Reach in $M_{\tilde{t}_1}$ or $M_{\tilde{b}_1}$ @20 fb $^{-1}$ (Run I)	
$\tilde{b}_1 \rightarrow b\tilde{\chi}_1^0$		$bb\cancel{E}_T$	260 GeV/ c^2 (146 GeV/ c^2 [26])	
$\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$		$cc\cancel{E}_T$	220 GeV/ c^2 (116 GeV/ c^2 [26])	
$\tilde{t}_1 \rightarrow bl\tilde{\nu}$	$\tilde{\nu} \rightarrow \nu\tilde{\chi}_1^0$	$\ell^+\ell^-b\cancel{E}_T$	240 GeV/ c^2 (140 GeV/ c^2 [28])	
$\tilde{t}_1 \rightarrow bl\nu\tilde{\chi}_1^0$		$\ell^+\ell^-b\cancel{E}_T$	- (129 GeV/ c^2 [28])	
$\tilde{t}_1 \rightarrow b\tilde{\chi}_1^\pm$	$\tilde{\chi}_1^\pm \rightarrow W^{(*)}\tilde{\chi}_1^0$	$\ell bj\cancel{E}_T$ and $\ell^+\ell^-j\cancel{E}_T$	210 GeV/ c^2 (-)	
$\tilde{t}_1 \rightarrow bW\tilde{\chi}_1^0$		$\ell bj\cancel{E}_T$	190 GeV/ c^2 (-)	
NLSP	Decay Mode	$c\tau$ Prod.	Key Final State(s)	Discovery Reach @30 fb $^{-1}$
Bino $\tilde{\chi}_1^0$	$\gamma + \tilde{G}$	p all d all	$\gamma\gamma\cancel{E}_T + X$ $\gamma dj\cancel{E}_T$ or $\gamma\gamma\cancel{E}_T + X$	340 GeV/ c^2 ($\tilde{\chi}_1^\pm$) 300 GeV/ c^2 ($\tilde{\chi}_1^\pm$) ($c\tau = 50$ cm)
Higgsino $\tilde{\chi}_1^0$	$(h, Z, \gamma) + \tilde{G}$	p all d all d all	$(hh, h\gamma, hZ, Z\gamma, ZZ, \gamma\gamma)\cancel{E}_T + X$ $\delta_{ip} < 0$ for $h \rightarrow bb, Z \rightarrow \ell^+\ell^-$ $\gamma_d + X$	220 GeV/ c^2 ($\tilde{\chi}_1^\pm$) - -
$\tilde{\tau}$	$\tau + \tilde{G}$	p all ll all ll all ll all	$\ell\ell\ell j\cancel{E}_T, \ell^\pm\ell^\pm jj\cancel{E}_T, \tau_h\tau_h\cancel{E}_T$ $\mu(dE/dx) + \ell\ell(M_{\ell\ell} > 150 \text{ GeV}/c^2)$ $\mu(dE/dx) + X$ $\mu(dE/dx + \text{TOF}) + X$	230 GeV/ c^2 ($\tilde{\chi}_1^\pm$) 120 GeV/ c^2 ($\tilde{\tau}_1$) 420 GeV/ c^2 ($\tilde{\chi}_1^\pm$) 210 GeV/ c^2 ($\tilde{\tau}$) 180 GeV/ c^2 ($\tilde{\tau}$) 210 GeV/ c^2 ($\tilde{\tau}$)
\tilde{t}_1	$(c, bW) + \tilde{G}$	p $\tilde{t}_1\tilde{\bar{t}}_1$	$cc\cancel{E}_T$ or $\ell + jets + \cancel{E}_T$	175 GeV/ c^2 (\tilde{t}_1)

SUSY Searches at LHC



5 σ reach in jets + \cancel{E}_T channel



Reach limits for various channels at 100 fb^{-1}

Superparticles



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[C. Caso et al.](#), The European Physical Journal **C183** (2018) 1 (2018 Authors)

- **2019** [2019 Web update of Reviews, Tables, Plots](#) [New November 2, 2019](#)
- **2019** [2019 Web update of Sparticle Listings](#) [New July 6, 2019](#)
- **2018** [2018 Summary Tables and Conservation Laws](#)
- [2018 Reviews, Tables, Plots \(incl. Intro. Text\)](#) [Superseded by 2019 Web Version](#)
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- [Errata](#) (last changed January 18, 2020)
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Discovery of
the new world
of SUSY

Back to 60's

New
discoveries
every year