Full two-loop electroweak corrections to the pole masses of gauge bosons

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The Loop Verein,

- Pole Mass
- Calculations
- $\overline{\mathrm{MS}}$ renormalization and two-loop results
- Conclusion

Pole mass. General consideration.

The position of the pole s_P and the wave-function renormalization constant Z_2 of the propagator of a massive gauge-boson are defined by the relation

$$\frac{1}{s - m_0^2 - \Pi(p^2, m_0^2, \cdots)} \simeq \frac{Z_2}{s - M^2 + iM\Gamma} \qquad s \simeq s_P$$

where $\Pi(p^2, \cdots)$ is the transversal part of the one-particle irreducible self-energy (depends on all SM parameters) and m_0^2 is the bare mass. Here we have adopted the standard parameterization of the pole $s_{P,V} = M_V^2 - iM_V\Gamma_V$, where Γ_V is the width of the unstable gauge-boson.

The pole s_P is

- gauge invariant
- infrared stable

By iterative solution of pole formula up to 2-loops we have:

$$\boxed{\begin{aligned} s_P &= m^2 \\ &+ \Pi^{(1)}(m^2, m^2) \\ &+ \Pi^{(2)}(m^2, m^2) + \Pi^{(1)}(m^2, m^2)\Pi^{(1)\prime}(m^2, m^2), \\ Z_2^{-1} &= 1 - \Pi^{(1)\prime} - \Pi^{(2)\prime} - \Pi^{(1)}\Pi^{(1)\prime\prime} . \end{aligned}}$$

Pole mass and γ-Z mixing.

In neutral sector because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

$$\boxed{s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.}$$

- ♣ Mixing term $\Pi^2_{\gamma Z}$ starts contributing at two-loop
- ♠ Photon term $\Pi_{\gamma\gamma}$ only contributes beyond the two-loop

Solution (up to 2-loops):

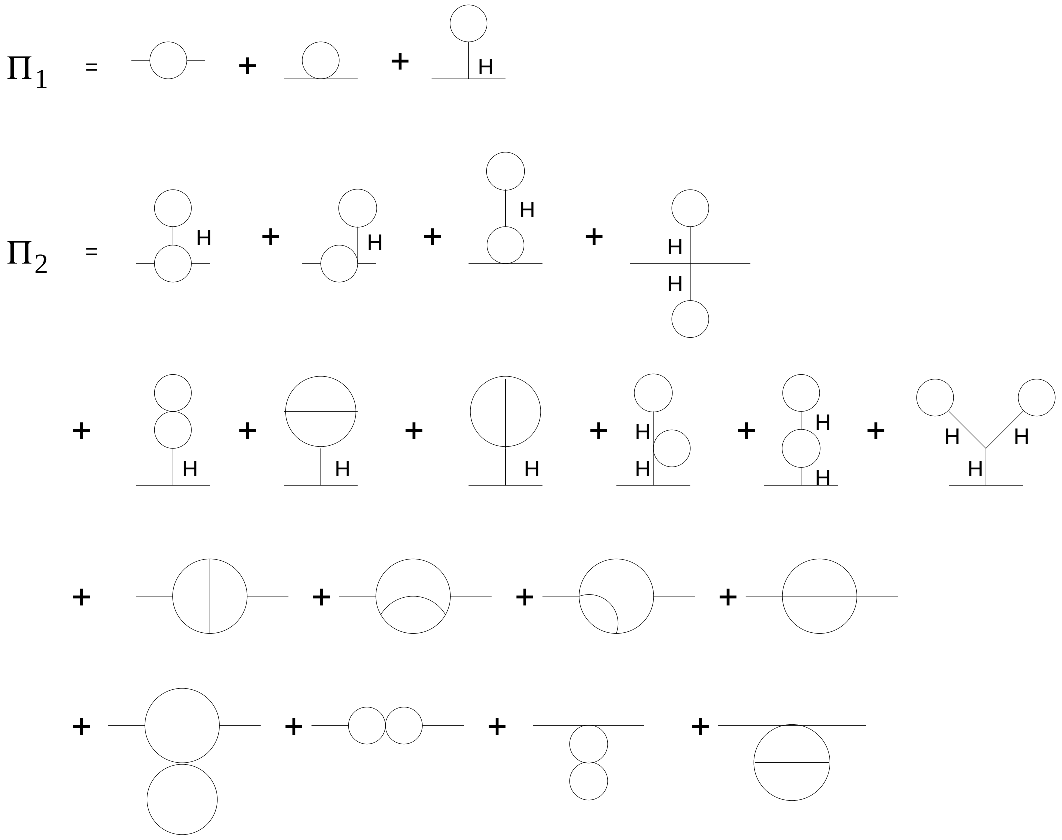
$$\begin{aligned} s_Z &= m_Z^2 \\ &+ \Pi^{(1)}_{ZZ}(m_Z^2) \\ &+ \Pi^{(2)}_{ZZ}(m_Z^2) + \Pi^{(1)}_{ZZ}(m_Z^2)\Pi^{(1)}_{ZZ}{}'(m_Z^2) + \frac{\left[\Pi^{(1)}_{\gamma Z}(m_Z^2)\right]^2}{m_Z^2}. \\ Z^{-1}_{ZZ} &= 1 - \Pi^{(1)}_{ZZ}{}' - \Pi^{(2)}_{ZZ}{}' - \Pi^{(1)}_{ZZ}\Pi^{(1)}_{ZZ}{}'' - \frac{2}{m_Z^2}\Pi^{(1)}_{\gamma Z}\Pi^{(1)}_{\gamma Z}{}' . \end{aligned}$$

The prime denotes the derivative with respect to p^2.

Diagrams and topologies

We need to evaluate propagator type diagrams and their derivatives at $p^2 = m^2$:

$$\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \dots$$



Bosonic contribution				
Number of diagrams	linearR_ξ *gauge*		nonlinearR_ξ *gauge*	
one-loop :	~ 50			
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	616	2348	410	1837
W	792	4084	537	2942
With one massive fermion family				
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	802	4410	550	3631
W	990	7780	669	5604

Gauge

The gauge fixing Lagrangian is

$$L_{g.f.} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z} (\partial_\mu Z^\mu - \xi_Z M_Z h)^2$$

where for the linear R_ξ gauge is

$$F^\pm = \partial_\mu W_\mu^\pm \mp i \xi_W M_W \phi^\pm$$

and the nonlinear R_ξ gauge is defined as

$$F^\pm = \partial_\mu W_\mu^\pm \mp i \xi_W M_W \phi^\pm \mp ie A_\mu W_\mu^\pm \pm ig \frac{\sin^2\theta_W}{\cos\theta_W} Z_\mu W_\mu^\pm$$

K. Fujikawa '73
D.A. Dicus & C. Kao '94

The old vertices:	
$\{A_\mu, Z_\mu\} W_\mu^\pm \phi^\mp$	*absent*
$\{A_\sigma, Z_\sigma, A_\mu A_\nu, A_\mu Z_\nu, Z_\mu Z_\nu\} W_\mu^\pm W_\nu^\mp$	*modified*
$\{A_\mu, Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*modified*
$W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*modified*
The new vertices:	
$\{A_\mu A_\mu, A_\mu Z_\mu, Z_\mu Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*new*
$\{A_\mu, Z_\mu\} W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*new*

Evaluation of 2-loop self-energies

One can choose one of the approaches:

♣ Analytical ♢ Numerical ♠ Semianalytical

Each of them meets with the typical problems:

- Reduction of tensor integrals to set of scalar integrals
- Extraction of UV- and IR-poles
- Analytical/numerical evaluation of scalar integrals for different region of variables (numerical stability)

Let us consider some of the approaches within our task:

- J. Fleischer & O.V. Tarasov '94
  Small momentum expansion and construction of Padé approximants.
  Demand a lot of coefficients of expansion, specially for on-shell diagrams.

- A. Ghinculov & Y.-P. Yao '98; '01
  Each two-loop diagram is reduced to ten scalar integrals and their derivatives which can be evaluated numerically.
  Applicable to diagrams (their combination) which are free of mass (infrared) singularities.

- G. Passarino '01; '02
  Under construction

- V.A. Smirnov '90; '95; '99; '01;
  The asymptotic expansion with respect to small parameter(s) existing in the initial diagram.
  It is applied for special values of kinematical variables.

- Differential Equation Method
  For each diagram the linear system of partial differential equations in masses and momenta can be constructed, which can be used for analytical A. Kotikov '91 or numerical S. Laporta '00 calculation of diagram.

- Difference Equation Method
  A linear system of difference equations is constructed and solved analytically O.V. Tarasov '96; '00 or numerically S. Laporta '00 .
  Main application is the calculations of master integrals. Not yet applied to real multi-loop calculations.

- G. Weiglein, R. Scharf & M. Böhm '94
  S. Bauberger,F.A. Berends, M. Böhm, M. Buza, '95
  S. Bauberger, M. Böhm, '95
  Reduction of two-loop self-energy diagrams to scalar integrals without numerator which can be calculated numerically or analytically.

- O.V. Tarasov '97
  Exact reduction of tensor two-loop propagator type diagrams with arbitrary masses and momentum to set of scalar master integrals.
  Several masses led to the cumbersome expressions, which are difficult to simplify.
  The result is expressible in terms of master-integrals which demand a special consideration

- FORM based package ON-SHELL2
  Single scale propagator-type diagrams with arbitrary power of propagators are reduced to a set of master-integrals which calculated analytically.

Evaluation by expansion

In order to check the gauge invariance we perform all calculations in R_ξ gauge with three independent gauge parameters ξ_W, ξ_Z, ξ_γ. $\rightarrow$ Vector boson propagator reads

$$D_{\mu\nu}(p) = \frac{i}{p^2 - m_Z^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_Z^2}\right)$$

Then there are several scales: m_V, $\sqrt{\xi_V} m_V$, m_H, m_t

We perform expansion in 3 steps:

1. Taylor (naive) expansion in $(\xi_V - 1)$: i.e., propagator of the vector bosons look like

$$D_{\mu\nu}^V(p) \sim -g_{\mu\nu} + (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - m_V^2} - (1-\xi_V)^2 \frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2}$$

2. Expansion in small parameter

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} < 0.25.$$

   After expansion in $\sin^2\theta_W$: the diagrams without Higgs are the two-loop on-shell diagrams can be evaluated analytically by FORM package **ONSHELL2**. J. Fleischer & M.Yu. Kalmykov '00

   Diagrams with massless fermion loop demand special consideration.

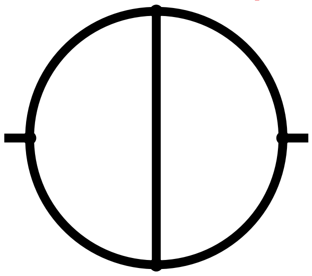
3. Large mass expansion in $m_Z^2/m_t^2 < 0.3; m_Z^2/m_H^2 < 0.64$
   Package **TLAMM** L.V. Avdeev et al., '97

ONSHELL2

www-zeuthen.desy.de/~kalmykov/onshell2/onshell2.html

J. Fleischer, M.Yu. Kalmykov & A. Kotikov '99
A.I. Davydychev & M.Yu. Kalmykov '01
J. Fleischer & M.Yu. Kalmykov '00

All two-loop on-shell self-energy diagrams with one mass, with arbitrary power of propagators are reduced to 13 scalars integrals plus bubble-type integrals and product of the one-loop integrals.



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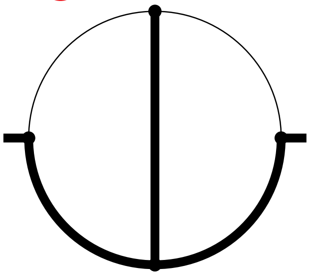

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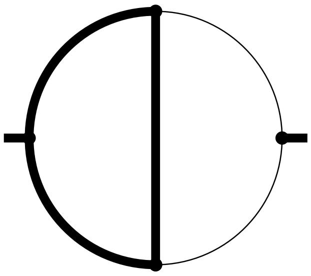

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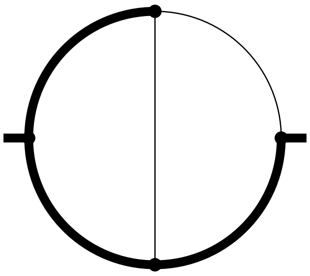

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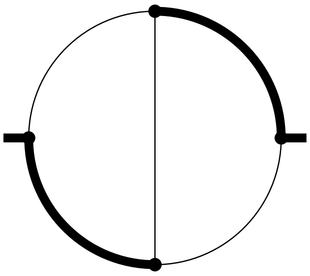

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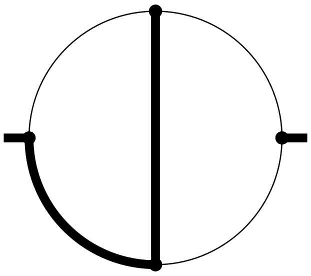

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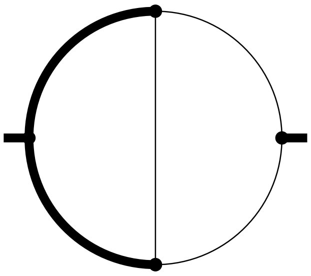

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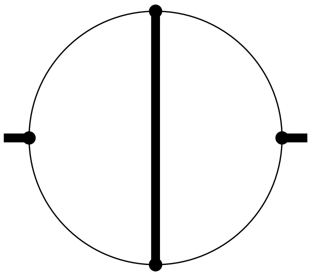

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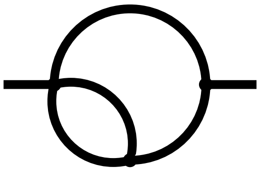

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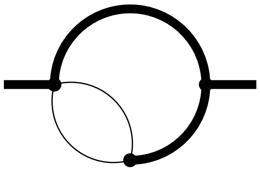

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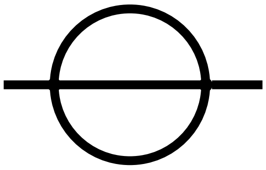

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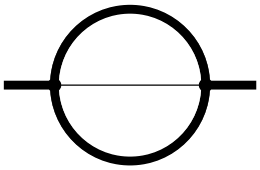

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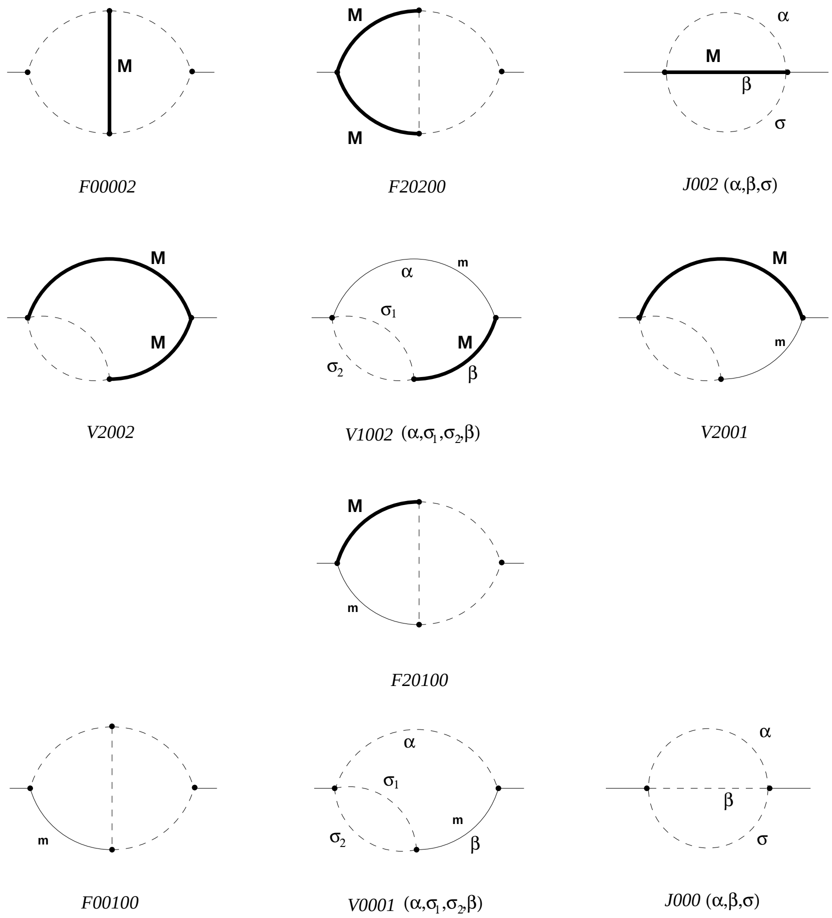
Bold and thin lines correspond to the mass and massless propagators, respectively.

Massless fermion contribution

In contrast to the bosonic corrections where all diagrams can be expanded from the very beginning in the small parameter $\sin^2\theta_W$ the individual diagrams with massless fermion loop develop the thresholds singularity which behave like $\ln^j \sin^2\theta_W$. To control these terms we need the exact analytical result without expansion. Using Tarasov's recurrence relations we reduce all diagrams with massless fermion loop to a minimal set of master-integrals



The analytical results for these diagrams are presented by Scharf, Tausk, '94 We performed an independent analytical calculations of these master integrals.

After summing of all contribution the singular terms, $\ln^j \sin^2\theta_W$ are canceled.

Master-integrals

E.E. Boos & A.I. Davydychev, '91
J. Fleischer, A. Kotikov & O.Veretin '99
T. van Ritbergen & R. G. Stuart '00
A.I. Davydychev & M.Yu. Kalmykov '01

$$
\begin{aligned}
m^2 F_{20100} = \sum_{n=1}^{\infty} \binom{2n}{n} \frac{x^n}{n} \Biggl\{ & -\frac{1}{4}\ln^2 x + \frac{1}{2}\ln(-x)\ln x \\
& + \left[2S_1 - 2\bar{S}_1 + \frac{1}{n} \right] \ln x + \left[-S_1 + \bar{S}_1 - \frac{1}{n} \right] \ln(-x) \\
& - \zeta_2 - \frac{3}{2} V_2 - \frac{3}{2} S_2 + 3\bar{S}_2 - 3S_1^2 + 6S_1\bar{S}_1 - 3\bar{S}_1^2 \\
& - 4\frac{S_1}{n} + 4\frac{\bar{S}_1}{n} - \frac{3}{2}\frac{1}{n^2} - 2\frac{1}{\binom{2n}{n}}\frac{1}{n^2} \Biggr\}
\end{aligned}
$$

with

$$
x = \frac{m^2}{M^2}, \quad S_a = \sum_{j=1}^{n-1} \frac{1}{j^a}, \quad \bar{S}_a = \sum_{j=1}^{2n-1} \frac{1}{j^a},
$$

$$
\begin{aligned}
\sum_{k=1}^{\infty} \binom{2k}{k} \frac{x^k}{k^4} = & -8\mathrm{S}_{1,2}(-\chi)\ln(1+\chi) - \mathrm{Li}_4(-\chi) \\
& + 4\mathrm{Li}_3(-\chi)\ln(1+\chi) - 4\mathrm{Li}_2(-\chi)\ln^2(1+\chi) \\
& - \frac{2}{3}\ln^4(1+\chi) + 4\mathrm{S}_{2,2}(-\chi) - 8\mathrm{S}_{1,3}(-\chi)
\end{aligned}
$$

where

$$
\chi = \frac{1-\sqrt{1-4x}}{1+\sqrt{1-4x}}, \quad x = \frac{\chi}{(1+\chi)^2},
$$

Renormalization.

We have s_P parametrized in terms of *bare* parameters

$$s_P = f(e_0, m_{Z,0}, m_{W,0}, m_{H,0}).$$

Renormalization of charges and masses:

$$\begin{aligned} e_0 &= \mu^\varepsilon e Z_e \\ m_0^2 &= m^2 Z_m \end{aligned}$$

We need:
+ Z_e, Z_H, Z_t up to 1-loop
+ Z_W, Z_Z up to 2-loops

We don't need:
− field renormalization
− renormalization of ghost sector
− gauge parameters renormalization

The charge $\overline{\mathrm{MS}}$ renormalization constant can be calculated in the unbroken theory

$$Z_e = 1 + \frac{e^2}{16\pi^2\varepsilon}\left(-\frac{7}{2} + \frac{16}{9}n_F\right),$$

where e, m_Z, m_W, m_H are $\overline{\mathrm{MS}}$ parameters, n_F is a number of fermion families ($n_F = 3$).

Mass renormalization.

$$m_0^2 = m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right)$$

One-loop results:

$$
\begin{aligned}
Z_H^{(1,1)} &= -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2} + \sum_{\text{fermion}} \frac{1}{2}\frac{m_f^2}{m_W^2} \\
Z_W^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{3}{4}\frac{m_Z^2}{m_W^2} - \frac{17}{3} \\
&\quad + \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right) + \frac{4}{3} n_F \\
Z_Z^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} + \frac{11}{12}\frac{m_Z^2}{m_W^2} \\
&\quad - 7\frac{m_W^2}{m_Z^2} + \frac{7}{6} + \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right) \\
&\quad + \frac{1}{9} n_F \left[20\frac{m_Z^2}{m_W^2} + 32\frac{m_W^2}{m_Z^2} - 40 \right] \\
Z_{top}^{(1,1)} &= \frac{2}{3} - \frac{2}{3}\frac{m_Z^2}{m_W^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} - \frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} \\
&\quad + \frac{3}{4}\frac{m_t^2 - m_b^2}{m_W^2} + 2\sum_{\text{lepton}} \frac{m_l^4}{m_W^2 m_H^2} + 6\sum_{\text{quark}} \frac{m_q^4}{m_W^2 m_H^2} ,
\end{aligned}
$$

2-loop renormalization constants in $\overline{\mathrm{MS}}$.

$$
\begin{aligned}
Z_W^{(2,1)} = {} & \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} - \frac{53}{3} + \frac{109}{18} n_F \\
& - \left[\frac{301}{192} + \frac{25}{36} n_F\right] \frac{m_Z^4}{m_W^4} + \left[\frac{17}{12} + \frac{14}{9} n_F\right] \frac{m_Z^2}{m_W^2} \\
& + \left[\frac{31}{12} - \frac{80}{9} n_F\right] \frac{m_Z^4}{m_H^2 m_W^2} - \left[\frac{17}{2} - \frac{64}{9} n_F\right] \frac{m_Z^2}{m_H^2} \\
& - \left[\frac{176}{3} - \frac{16}{3} n_F\right] \frac{m_W^2}{m_H^2} + \left[\frac{59}{24} + \frac{40}{9} n_F\right] \frac{m_Z^6}{m_H^2 m_W^4} \\
& - \frac{15}{2}\frac{m_t^6}{m_W^4 m_H^2} + \frac{4}{3}\frac{m_t^4 m_Z^2}{m_W^4 m_H^2} - \frac{21}{32}\frac{m_t^4}{m_W^4} - \frac{4}{3}\frac{m_t^4}{m_W^2 m_H^2} \\
& - \frac{85}{96}\frac{m_t^2 m_Z^2}{m_W^4} + \frac{19}{4}\frac{m_t^2 m_Z^4}{m_W^4 m_H^2} + \frac{9}{8}\frac{m_t^2 m_H^2}{m_W^4} \\
& - 20\frac{m_t^2 m_Z^2}{m_W^2 m_H^2} - \frac{43}{48}\frac{m_t^2}{m_W^2} + \frac{35}{2}\frac{m_t^2}{m_H^2}
\end{aligned}
$$

- $Z_Z^{(2,1)}$ has similar structure.

- To restore gauge invariance of renormalization constants the tadpole contribution should be taken into account.

- The terms proportional to m_H^4 are generated by pure bosonic contributions.

- The terms proportional to $m_t^6/(m_H^2 m_V^4)$ are generated by tadpole diagrams.

Renormalization group analysis

We can check two-loop coefficients $Z^{(2,2)}$ and $Z^{(2,1)}$ in mass renormalization formula

$$
\begin{aligned}
m_0^2 &= m^2 \left(1 + \frac{g^2}{16\pi^2}\frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2}\frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2}\frac{Z^{(2,2)}}{\varepsilon^2} \right) \\
&= m^2 \left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \ldots \right)
\end{aligned}
$$

using renormalization group.

Introduce anomalous mass dimension

$$
\gamma = \mu^2 \frac{d}{d\mu^2} \log m^2 .
$$

From

$$
0 = \mu^2 \frac{d}{d\mu^2} \log m_0^2
$$

we have

$$
\begin{aligned}
\gamma &= \frac{1}{2} \sum_{j=g,g_S} g_j \frac{\partial}{\partial g_j} z^{(1)}, \\
\left(\gamma + \sum_i \beta_{g_i} \frac{\partial}{\partial g_i} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) z^{(n)} &= \frac{1}{2} \sum_{j=g,g_S} g_j \frac{\partial}{\partial g_j} z^{(n+1)},
\end{aligned}
$$

where

$$
z^{(n)} = \sum_{k=1} \left(\frac{g^2}{16\pi^2} \right)^k Z^{(n,k)} .
$$

Pole structure

Check of $1/\varepsilon^2$ pole:

From previous formula we can derive

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\frac{16\pi^2}{g^3}\beta_g^{(1)} Z^{(1,1)} + \sum_i Z_{m_i}^{(1,1)} m_i^2 \frac{\partial}{\partial m_i^2} Z^{(1,1)}$$

Check $1/\varepsilon$ pole:

Use relation between charges

$$e^2 = g^2\left(1 - \frac{m_W^2}{m_Z^2}\right) \quad \text{and} \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}.$$

Differentiating the former formulae w.r.t. $\log\mu^2$ we derive

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(Z_Z^{(2,1)} - Z_W^{(2,1)}\right) c^2 = g^2 \frac{\beta_{g'}^{(2)}}{g'^3} s^4 - \frac{\beta_g^{(2)}}{g} s^2 c^2.$$

β-functions for g and g' are calculated from symmetric phase theory

D.R.T. Jones '82

M.E. Machacek & M.T. Vaughn '83

$$\begin{aligned}
\beta_{g'} &= \left(\frac{1}{12} + \frac{10}{9}n_F\right)\frac{g'^3}{16\pi^2} - \frac{17}{24}\frac{m_t^2}{m_W^2}\frac{g'^3 g^2}{(16\pi^2)^2} \\
&+ \left(\frac{1}{4} + \frac{95}{54}n_F\right)\frac{g'^5}{(16\pi^2)^2} + \left(\frac{3}{4} + \frac{1}{2}n_F\right)\frac{g'^3 g^2}{(16\pi^2)^2} \\
\beta_g &= \left(-\frac{43}{12} + \frac{2}{3}n_F\right)\frac{g^3}{16\pi^2} - \frac{3}{8}\frac{m_t^2}{m_W^2}\frac{g^5}{(16\pi^2)^2} \\
&+ \left(-\frac{259}{12} + \frac{49}{6}n_F\right)\frac{g^5}{(16\pi^2)^2} + \left(\frac{1}{4} + \frac{n_F}{6}\right)\frac{g^3 g'^2}{(16\pi^2)^2}
\end{aligned}$$

Connection with RG functions of unbroken phase

In the SM it is interesting to compare the RG equations calculated in broken phase with the ones obtained in the unbroken phase. Let us remind that at the tree-level the vacuum expectation value v^2 is given by $v^2 \equiv \frac{m^2}{\lambda}$, where m^2 and λ are the parameters of the symmetric scalar potential

$$V = -m^2 \phi^+ \phi + \lambda \left(\phi^+ \phi \right)^2$$

$$\gamma_W - 2\frac{\beta_g}{g} = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} ,$$

$$\gamma_Z - \gamma_{m^2} + \frac{\beta_\lambda}{\lambda} = 2 \left(\cos^2 \theta_W \frac{\beta_g}{g} + \sin^2 \theta_W \frac{\beta_{g'}}{g'} \right) ,$$

where the 2-loop RG functions $\beta_g, \beta_{g'}, \beta_\lambda, \gamma_{m^2}$ have been calculated in the unbroken phase

D.R.T. Jones '82
M.E. Machacek & M.T. Vaughn '83
C. Ford, I. Jack and D.R. Jones, '92

We have verified in the $\overline{MS}$ scheme, that these relations are valid up to 2-loop order in the broken phase with the same RG functions. Thus the RG equations for the $\overline{MS}$ masses in the broken theory can be written

$$m_W^2(\mu^2) = \frac{1}{4} \frac{g^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) ,$$

$$m_Z^2(\mu^2) = \frac{1}{4} \frac{g^2(\mu^2) + g'(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) ,$$

$$m_H^2(\mu^2) = 2m^2(\mu^2) ,$$

$$m_t^2(\mu^2) = \frac{1}{2} \frac{y_t^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) .$$

$\overline{\mathrm{MS}}$-Renormalization

The bare relation is

$$s_V = m_{0,V}^2 + \Pi_{0,V}^{(1)} + \Pi_{0,V}^{(1)}{}'$$

$\overline{\mathrm{MS}}$-renormalized relation can be written as

$$s_V = m_V^2(\mu) + \left\{ \Pi_{0,V}^{(1)} \right\}_{MS} + \left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS}$$

and

$$\Pi_{0,V}^{(j)} = m_{0,V}^2 \left(\frac{g_0^2}{16\pi^2} \right)^j X_{0,V}^{(j)}$$

$$\left\{ \Pi_{0,V}^{(1)} \right\}_{MS} = m_V^2(\mu) \frac{e^2}{16\pi^2 \sin^2\theta_W} \lim_{\varepsilon \to 0} \left(\frac{1}{\varepsilon} Z_V^{(1,1)} + X_{0,V}^{(1)} \right)$$

$$\left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS} = \lim_{\varepsilon \to 0} \Bigg(\Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}'$$

$$+ \frac{m_V^2(\mu)}{\varepsilon} \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[Z_V^{(1,1)} + \Delta g^2 + \sum_j Z_{m_j^2}^{(1,1)} \frac{\partial}{\partial m_j^2} \right] X_{0,V}^{(1)}$$

$$+ m_V^2(\mu) \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[\frac{1}{\varepsilon} Z_V^{(2,1)} + \frac{1}{\varepsilon^2} Z_V^{(2,2)} \right] \Bigg)$$

where the sum runs over all species of particles $j = Z,\ W,\ H,\ t$ and

$$\Delta g^2 = \frac{\cos^2\theta_W}{\sin^2\theta_W} \left(Z_W^{(1,1)} - Z_Z^{(1,1)} \right) + \sin^2\theta_W \left(-7 + \frac{32}{9} n_F \right)$$

Results

$$\frac{s_{P,V}}{m_V^2} = 1 + \frac{e^2}{16\pi s^2} X_{1,V} + \left(\frac{e^2}{16\pi s^2} \right)^2 X_{2,V},$$

$$X_{2,V} \quad = \quad \sum_{i=0}^{5} s^{2i} \sum_{j=0}^{5} A^V_{i,j}(m_H^2, m_t^2, \mu^2) \sum_{a+b=j} \left(\frac{m_V^2}{m_H^2} \right)^a \left(\frac{m_V^2}{m_t^2} \right)^b \quad .$$

- All parameters in $\overline{\mathrm{MS}}$- scheme.
- Expansion in powers and $\log$'s (i.e. is asymptotic expansion not naive Taylor expansion).
- www-zeuthen.desy.de/~kalmykov/pole/pole.html

The position of the complex pole s_p of a the gauge-bosons (Z, W) propagator is a gauge invariant quantity up to two-loop after inclusion of the Higgs tadpole contributions.

Stuart '91, Sirlin'91, $\cdots$ Kniehl & Sirlin '98

After exact UV renormalization the pole mass is a finite expression. $\longrightarrow$ infrared finiteness of two-loop EW corrections to the pole mass.

Gambino & Grassi '00

RG invariance of the pole s_p and gauge invariance of the parameter renormalizationen also requires the inclusion of tadpole.

By our calculation we have proven that the $\overline{MS}$ renormalization scheme is self consistent and works properly in case of unstable particles.

Our results for the 2-loop mass renormalization constants in the on-shell and the $\overline{MS}$ scheme can be applied in calculations of physical quantities in both of these schemes at the 2-loop level. Examples of such calculations, where results of our paper first have been used, are the computations of the bosonic 2-loop contributions to the Muon life-time

A. Onishchenko and O. Veretin, '02

M. Awramik et al., '02

Factorization of $\log$

The knowledge of the anomalous dimensions γ_V allow us to write expression for the pole positions $s_{P,V}$ with explicit factorization of the RG logarithms. What we get is

$$
\begin{aligned}
s_{P,V} &= m_V^2 \left(1 - g^2 \gamma_V^{(1)} L_a\right) + g^2 X_{V,1} \\
&+ g^4 m_V^2 \left(C^{(2,2)} L_a^2 - C^{(2,1)} L_a\right) + g^4 X_{V,2} \ ,
\end{aligned}
$$

where $\mu^2 \frac{\partial}{\partial \mu^2} \ln m_a^2 = \gamma_a^{(1)} g^2 + \gamma_a^{(2)} g^4$, $L_a = \ln \frac{\mu^2}{m_a^2}$,

$$
\begin{aligned}
2C_V^{(2,2)} &= \left[\theta + \sum_j \gamma_{m_j}^{(1)} m_j^2 \frac{\partial}{\partial m_j^2}\right] \gamma_V^{(1)} \ , \\
C_V^{(2,1)} &= \gamma_V^{(2)} + \gamma_V^{(1)} \gamma_a^{(1)} \\
&+ \frac{1}{m_V^2} \left[2\beta_g^{(1)} + \sum_j \gamma_{m_j}^{(1)} m_j^2 \frac{\partial}{\partial m_j^2}\right] X_{V,1} \ ,
\end{aligned}
$$

and $\theta = \gamma_V^{(1)} + 2\beta_g^{(1)}$, $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g^{(1)} g^3$. In contrast to QCD, $\gamma_V^{(1)}$ and $C_V^{(i,j)}$ have non-polynomial structure in the massless coupling constants which originated from the tadpole contributions. The the amplitudes $X_{V,j}$ have no explicit μ dependence.

Scheme dependence

Our results reveal terms of unexpectedly high powers of the Higgs and the top-quark masses, arising from 2-loop corrections. In fact the purely bosonic diagrams yield m_H^4/m_V^4 terms and the (t,b) quark-doublet $(m_t \gg m_b)$ contributes $m_t^6/(m_H^2 m_V^4)$ power corrections. At a first glance, such terms contradict Veltman's screening theorem, which states, that the L-loop Higgs dependence of a physical observable is bounded by $(m_H^2)^{L-1} \ln^L m_H^2$ for large Higgs masses. However, this theorem only applies to physical observables like cross sections and asymmetries. If we consider quantities like Δr (which is an observable in the on-shell scheme) in the $\overline{\rm MS}$ scheme the screening theorem does not hold in general. To illustrate this, let us compare the 1-loop EW corrections to the Fermi constant in the on-shell and the $\overline{\rm MS}$ scheme

$$
\begin{aligned}
G_F &= \frac{\pi\alpha}{\sqrt{2}} \frac{1}{M_W^2 \sin^2\Theta_W} \left(1 + \Delta r_{OS}^{(1)}\right) \\
&= \frac{\sqrt{2}\, e^2(\mu^2)}{8 m_W^2(\mu^2) \sin^2\theta_W(\mu^2)} \left(1 + \Delta r_{MS}^{(1)}\right)
\end{aligned}
$$

$$
\Delta r_{MS}^{(1)} = \Delta r_{OS}^{(1)} + \left[\frac{m_W^2}{M_W^2} - 1\right] + \left[\frac{\sin^2\theta_W}{\sin^2\Theta_W} - 1\right] + \Delta\alpha \; .
$$

The 1-loop correction $\Delta r_{OS}^{(1)}$ in the limit of a heavy Higgs boson has a logarithmic Higgs mass dependence only . In contrast, in the $\overline{\rm MS}$scheme, higher powers of the Higgs mass are showing up, because $\frac{m_W^2}{M_W^2} - 1$ contributes the term

$$
\Delta r_{MS}^{(1)} \sim -\frac{\sqrt{2}\hat{G}_F(\mu^2) m_W^2}{16\pi^2} \frac{7}{2} \frac{m_H^2}{m_W^2} + O(m_H^2) \; ,
$$

while all other corrections exhibit logarithmic behavior in the Higgs mass, only.

$\overline{\mathrm{MS}}$ mass in terms of on-shell mass

Inverse of our "master formula" express all $\overline{\mathrm{MS}}$ parameters in terms of on-shell ones:

$$
\begin{aligned}
m_V^2 &= M_V^2 - \left\{ \Pi_V^{(1)} \right\}_{MS} - \left\{ \Pi_V^{(2)} + \Pi_V^{(1)} \Pi_V^{(1)\prime} \right\}_{MS} \\
&- \sum_j (\Delta m_j^2)^{(1)} \frac{\partial}{\partial m_j^2} \left\{ \Pi_V^{(1)} \right\}_{MS} - (\Delta e)^{(1)} \frac{\partial}{\partial e} \left\{ \Pi_V^{(1)} \right\}_{MS} \Bigg|_{m_j^2 = M_j^2}^{e = e_{\mathrm{OS}}} ,
\end{aligned}
$$

where the sum runs over all species of particles $j = Z,\ W,\ H\ t$ and

$$
(\Delta m_j^2)^{(1)} = -\mathrm{Re} \left\{ \Pi_V^{(1)} \right\}_{MS} \Bigg|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}}
$$

stands for the self-energy of the jth particle at $p^2 = m_j^2$ in the $\overline{\mathrm{MS}}$scheme and parameters replaced by the on-shell ones. Note that in the above relation we had to perform a change from the $\overline{\mathrm{MS}}$ to the on-shell scheme also for the electric charge.

Unphysical terms, proportional to m_H^4 and m_t^6 drop out in $\sin^2\theta_W$

$$
\begin{aligned}
\sin^2\theta_W &= 1 - \frac{m_W^2}{m_Z^2} \left(1 - \frac{M_W^2}{M_Z^2} \right) \times \\
&\left[1 - \frac{\cos^2\theta_W}{\sin^2\theta_W} \left((\delta_W^{(1)} - \delta_Z^{(1)})(1 - \delta_Z^{(1)}) + \delta_W^{(2)} - \delta_Z^{(2)} \right) \right]
\end{aligned}
$$

where $m_V^2 / M_V^2 = 1 + \delta_V^{(1)} + \delta_V^{(2)}$