Pole mass of gauge bosons within SM at two-loop order

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The Loop Verein

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- Pole Mass
- Expansion procedure and calculation
- $\overline{\mathrm{MS}}$ renormalization and two-loop results

Pole mass. General consideration.

The position of the pole s_P of the massive boson propagator in a QFT is defined by equation:

$$\boxed{s_P - m^2 - \Pi(s_P, m^2, \cdots) = 0}$$

m is a $\overline{\mathrm{MS}}$ or *bare* mass. For gauge bosons Π is *transversal* part of the self-energy.

The pole s_P is

- gauge invariant
- infrared stable
- generally complex: $\mathrm{Im} s_p \neq 0$

We can define *pole* or *on-shell* mass M and width Γ via

$$s_P = M^2 - \mathrm{i} M \Gamma$$

Solution (up to 2-loops):

$$\begin{aligned} s_P &= m^2 \\ &+ \Pi^{(1)}(m^2, m^2) \\ &+ \Pi^{(2)}(m^2, m^2) + \Pi^{(1)}(m^2, m^2)\Pi^{(1)\prime}(m^2, m^2). \end{aligned}$$

where we use loop expansion for $\Pi(p^2, m^2, \ldots)$:

$$\Pi = \Pi^{(1)} + \Pi^{(2)} + \ldots$$

Pole mass (2). Case of Z-γ mixing.

In neutral sector because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

$$\boxed{s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.}$$

Mixing term $\Pi_{\gamma Z}$ starts to contribute from 2-loops. Term $\Pi_{\gamma\gamma}$ contributes only beyond the two-loop approximation.

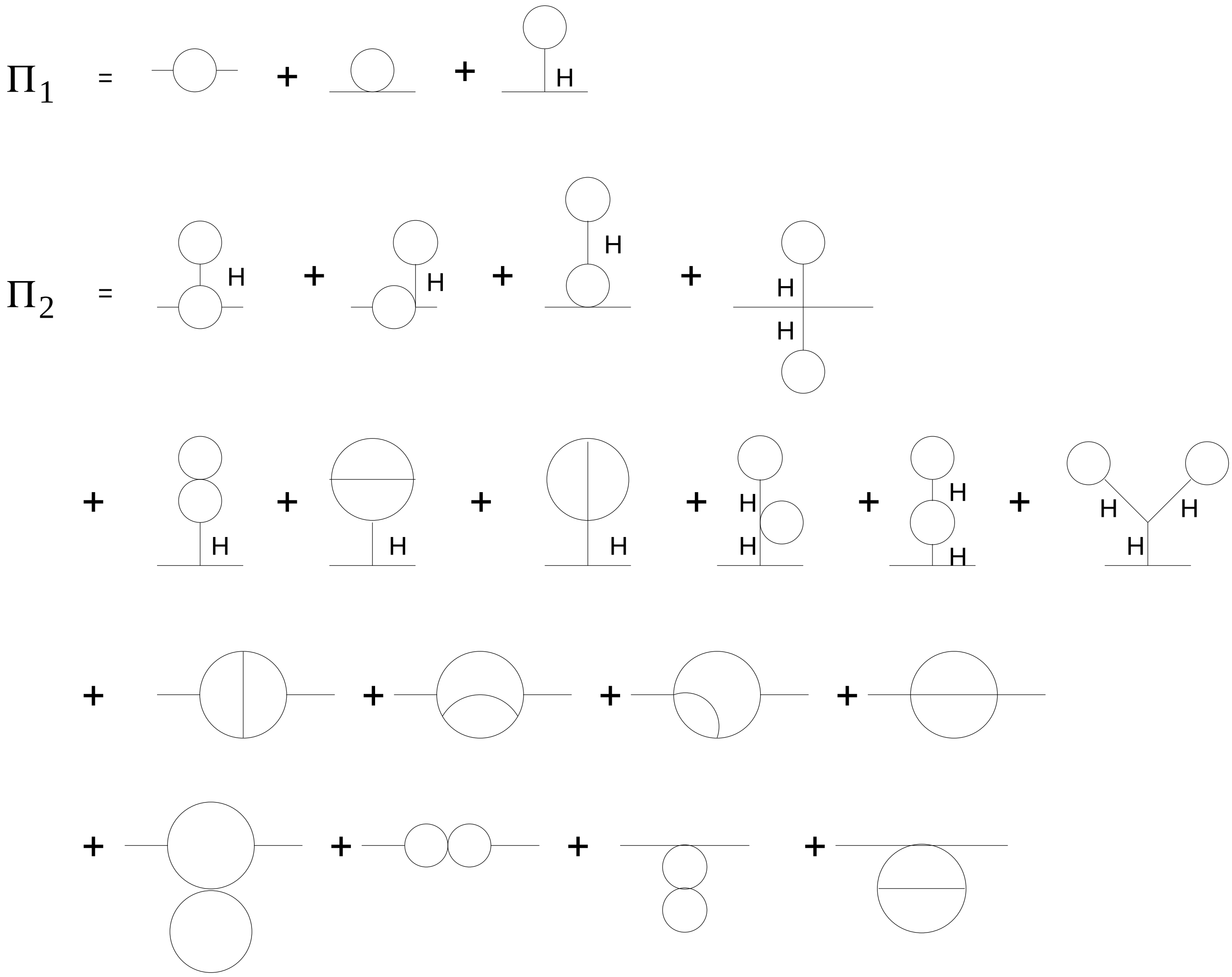
Solution (up to 2-loops):

$$\begin{aligned} s_Z &= m_Z^2 \\ &+ \Pi^{(1)}_{ZZ}(m_Z^2) \\ &+ \Pi^{(2)}_{ZZ}(m_Z^2) + \Pi^{(1)}_{ZZ}(m_Z^2)\Pi^{(1)}_{ZZ}{}'(m_Z^2) + \frac{\left[\Pi^{(1)}_{\gamma Z}(m_Z^2)\right]^2}{m_Z^2}. \end{aligned}$$

Diagrams and topologies

To be computed on-shell ($p^2 = m_Z^2$):

$$\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \ldots$$



Number of diagrams in	linear R_ξ *gauge*		nonlinear R_ξ *gauge*	
At $1 - loop\ level$:	~ 50			
At $2 - loop\ level$:	$1PI$	$Total$	$1PI$	$Total$
Z	616	2348	410	1837
W	792	4084	537	2942

Gauge

The gauge fixing Lagrangian is

$$L_{g.f.} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2 - \frac{1}{2\xi_Z} \left(\partial_\mu Z^\mu - \xi_Z M_Z h\right)^2$$

where for the linear R_ξ gauge is

$$F^\pm = \partial_\mu W^\pm_\mu \mp i \xi_W M_W \phi^\pm$$

and the nonlinear R_ξ gauge is defined as

$$F^\pm = \partial_\mu W^\pm_\mu \mp i \xi_W M_W \phi^\pm \mp ie A_\mu W^\pm_\mu \pm ig \frac{\sin^2\theta_W}{\cos\theta_W} Z_\mu W^\pm_\mu$$

K. Fujikawa '73
D.A. Dicus & C. Kao '94

The old vertices:	
$\{A_\mu, Z_\mu\} W^\pm_\mu \phi^\mp$	*absent*
$\{A_\mu A_\nu, A_\mu Z_\nu, Z_\mu Z_\nu\} W^\pm_\mu W^\mp_\nu$	*modified*
$\{A_\mu, Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*modified*
$W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*modified*
The new vertices:	
$\{A_\mu A_\mu, A_\mu Z_\mu, Z_\mu Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*new*
$\{A_\mu, Z_\mu\} W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*new*

Evaluation

One can choose one of the approaches:
♣ Analytical ♢ Numerical ♠ Semianalytical

Each of them meets with the typical problems:

- Reduction of tensor integrals to set of scalar integrals
- Extraction of UV- and IR-poles
- Analytical/numerical evaluation of scalar integrals for different region of variables (numerical stability)

Let us consider some of the approaches within our task:

- D. Kreimer '91; '92
  The algorithms has realized in package XLOOPS `http://wwwthep.physik.uni-mainz.de/~xloops`
  Does not exist working version applicable for real two-loop calculations.

- J. Fleischer & O.V. Tarasov '94
  Small momentum expansion and construction of Padé approximants.
  Demand a lot of coefficients of expansion, specially for on-shell diagrams.

- A. Ghinculov & Y.-P. Yao '98; '01
  Each two-loop diagram is reduced to ten scalar integrals and their derivatives which can be evaluated numerically.
  Applicable to diagrams (their combination) which are free of mass (infrared) singularities.

- G. Passarino '01
  It is recently described [hep-ph/0108252].

- Differential Equation Method
  For each diagram the linear system of partial differential equations in masses and momenta can be constructed, which can be used for analytical A. Kotikov '91 or numerical S. Laporta '00 calculation of diagram.
- Difference Equation Method
  A linear system of difference equations is constructed and solved analytically O.V. Tarasov '96; '00 or numerically S. Laporta '00 .
  Main application is the calculations of master integrals.
  Not yet applied to real multi-loop calculations.

- V.A. Smirnov '90; '95; '99; '01;
  The asymptotic expansion with respect to small parameter(s) existing in the initial diagram.
  It is applied for special values of kinematical variables.

- G. Weiglein, R. Scharf & M. Böhm '94
  Reduction of two-loop self-energy diagrams to scalar integrals without numerator which can be calculated numerically or analytically.
- O.V. Tarasov '97
  Exact reduction of tensor two-loop propagator type diagrams with arbitrary masses and momentum to set of scalar master integrals.
  Several masses led to the cumbersome expressions, which are difficult to simplify.
  The result is expressible in terms of master-integrals which demand a special consideration

Evaluation (2)

In order to check explicitely the gauge invariance we perform all calculations in R_ξ gauge with three independent gauge parameters ξ_W, ξ_Z, ξ_γ. E.g. Z boson propagator reads

$$D^Z_{\mu\nu}(p) = \frac{i}{p^2 - m_Z^2}\left(-g_{\mu\nu} + (1-\xi_Z)\frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}\right).$$

Then there are several scales: $m_V, \; \sqrt{\xi_V} m_V, \; m_H, \; m_t$

We perform 3 steps:

1. Taylor (naive) expansion in $(\xi_Z - 1)$ and $(\xi_W - 1)$.

$$D^V_{\mu\nu}(p) \sim g_{\mu\nu} + (\xi_V - 1)\frac{p_\mu p_\nu}{p^2 - m_V^2} + (\xi_V - 1)^2 \frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2}$$

2. Expansion in small parameter

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} < 0.25.$$

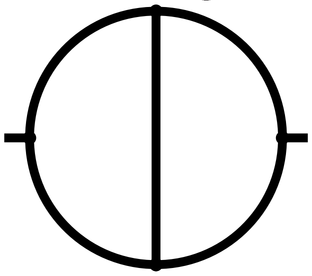
   After expansion in $\sin^2\theta_W$: $m_W^2 = m_Z^2(1 - \sin^2\theta_W)$ the diagrams without Higgs–two-loop on-shell diagrams – can be evaluated analytically by FORM package **ONSHELL2**. J. Fleischer & M.Yu. Kalmykov '00

3. Large mass expansion in $m_Z^2/m_H^2 < 0.64$.
   Package **TLAMM** L.V. Avdeev et al., '97

ONSHELL2

J. Fleischer, M.Yu. Kalmykov & A. Kotikov '99
A.I. Davydychev & M.Yu. Kalmykov '01
J. Fleischer & M.Yu. Kalmykov '00

All two-loop on-shell self-energy diagrams with one mass, with arbitrary power of propagators are reduced to 13 scalars integrals plus bubble-type integrals and product of the one-loop integrals.



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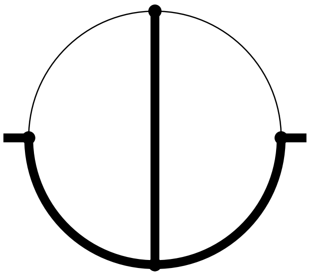

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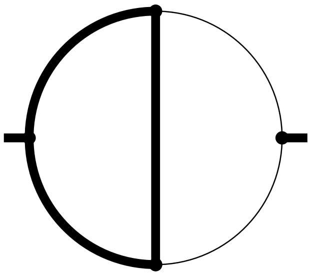

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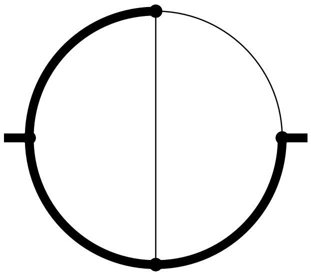

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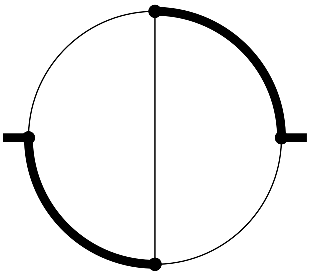

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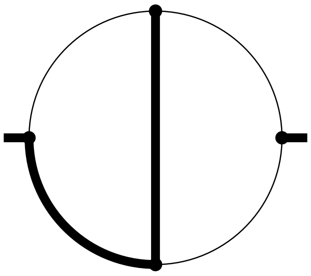

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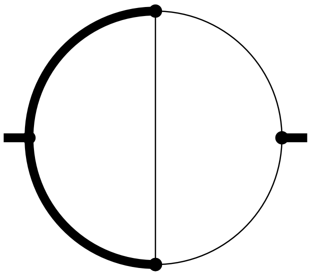

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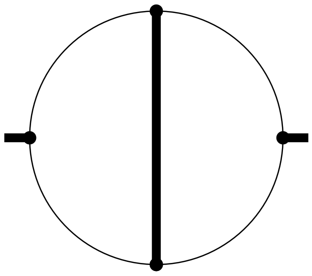

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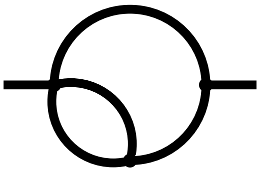

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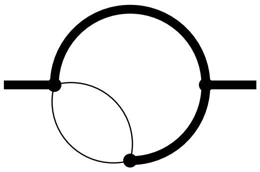

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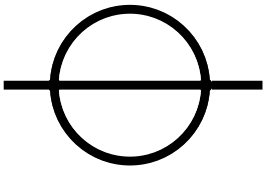

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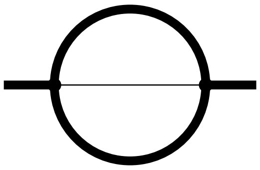

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Bold and thin lines correspond to the mass and massless propagators, respectively.

Renormalization.

We have s_P parametrized in terms of *bare* parameters

$$s_P = f(e_0, m_{Z,0}, m_{W,0}, m_{H,0}).$$

Renormalization of charges and masses:

$$\begin{aligned} e_0 &= \mu^{\varepsilon}\, e\, Z_e \\ m_0^2 &= m^2\, Z_m \end{aligned}$$

<u>We need:</u>

+ Z_e, Z_H, Z_t up to 1-loop
+ Z_W, Z_Z up to 2-loops

<u>We dont need:</u>

- field renormalization
- renormalization of ghost sector
- gauge parameters renormalization

The charge $\overline{\mathrm{MS}}$ renormalization constant can be calculated in the unbroken theory

$$Z_e = 1 - \frac{e^2}{16\pi^2}\frac{7}{2\varepsilon}$$

where e, m_Z, m_W, m_H and s^2 are $\overline{\mathrm{MS}}$ parameters.

Mass renormalization.

$$m_0^2 = m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right)$$

The mass renormalization constants cannot be calculated from the unbroken theory. The calculation of the Feynman diagrams in the Standard Model is required.

One-loop results:

$$\begin{aligned}
Z_H^{(1,1)} &= -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2}, \\
Z_W^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{3}{4}\frac{m_Z^2}{m_W^2} - \frac{17}{3}, \\
Z_Z^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} + \frac{11}{12}\frac{m_Z^2}{m_W^2} \\
&\quad -7\frac{m_W^2}{m_Z^2} + \frac{7}{6}.
\end{aligned}$$

Violation of polynomiality in $\overline{\mathrm{MS}}$ in SM:

$\frac{1}{m_Z^2}$ — $Z - \gamma$ mixing: actual Z boson coupling is $\sqrt{g^2 + g'^2} = \frac{g}{\cos\theta_W}$

$\frac{1}{m_H^2}$ — inverse powers of the Higgs self-coupling (tadpoles)

2-loop constants in $\overline{\mathrm{MS}}$. Exact results:

$$
\begin{aligned}
Z_W^{(2,1)} &= \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} - \frac{301}{192}\frac{m_Z^4}{m_W^4} \\
&+ \frac{17}{12}\frac{m_Z^2}{m_W^2} - \frac{53}{3} + \frac{31}{12}\frac{m_Z^4}{m_H^2 m_W^2} - \frac{17}{2}\frac{m_Z^2}{m_H^2} \\
&- \frac{176}{3}\frac{m_W^2}{m_H^2} + \frac{59}{24}\frac{m_Z^6}{m_H^2 m_W^4}, \\
Z_W^{(2,2)} &= -\frac{9}{32}\frac{m_H^4}{m_W^4} + \frac{43}{8}\frac{m_H^2}{m_W^2} + \frac{65}{32}\frac{m_Z^4}{m_W^4} - \frac{35}{8}\frac{m_Z^2}{m_W^2} + \frac{334}{9} \\
&+ \frac{27}{4}\frac{m_Z^4}{m_H^2 m_W^2} + 6\frac{m_Z^2}{m_H^2} + 34\frac{m_W^2}{m_H^2} - \frac{5}{2}\frac{m_Z^6}{m_H^2 m_W^4} \\
&+ 9\frac{m_W^4}{m_H^4} + 9\frac{m_Z^4}{m_H^4} + \frac{9}{4}\frac{m_Z^8}{m_H^4 m_W^4}, \\
Z_Z^{(2,1)} &= \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} + \frac{11}{3} + \frac{7}{6}\frac{m_Z^2}{m_W^2} \\
&- \frac{64}{3}\frac{m_W^2}{m_Z^2} - \frac{253}{192}\frac{m_Z^4}{m_W^4} - \frac{176}{3}\frac{m_W^2}{m_H^2} - \frac{17}{2}\frac{m_Z^2}{m_H^2} \\
&+ \frac{31}{12}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{59}{24}\frac{m_Z^6}{m_H^2 m_W^4}, \\
Z_Z^{(2,2)} &= -\frac{9}{32}\frac{m_H^4}{m_W^4} + \frac{1}{4}\frac{m_H^2}{m_W^2} - \frac{1}{8}\frac{m_H^2 m_Z^2}{m_W^4} + \frac{21}{4}\frac{m_H^2}{m_Z^2} \\
&- \frac{55}{6} + \frac{11}{12}\frac{m_Z^2}{m_W^2} + \frac{629}{288}\frac{m_Z^4}{m_W^4} + \frac{245}{6}\frac{m_W^2}{m_Z^2} + \frac{27}{2}\frac{m_W^2}{m_H^2} \\
&+ 16\frac{m_Z^2}{m_H^2} + 21\frac{m_W^4}{m_Z^2 m_H^2} - \frac{7}{2}\frac{m_Z^4}{m_H^2 m_W^2} - \frac{11}{4}\frac{m_Z^6}{m_H^2 m_W^4} \\
&+ 9\frac{m_W^4}{m_H^4} + 9\frac{m_Z^4}{m_H^4} + \frac{9}{4}\frac{m_Z^8}{m_H^4 m_W^4}.
\end{aligned}
$$

Cross-checks. RG analysis.

We can check two-loop coefficients $Z^{(2,2)}$ and $Z^{(2,1)}$ in mass renormalization formula

$$
\begin{aligned}
m_0^2 &= m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right) \\
&= m^2 \left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \dots \right)
\end{aligned}
$$

using renormalization group.

Introduce anomalous mass dimension

$$
\gamma = \mu^2 \frac{d}{d\mu^2} \log m^2 .
$$

From

$$
0 = \mu^2 \frac{d}{d\mu^2} \log m_0^2
$$

we have

$$
\begin{aligned}
\gamma &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(1)}, \\
\left(\gamma + \beta_g \frac{\partial}{\partial g} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) z^{(n)} &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(n+1)},
\end{aligned}
$$

where

$$
z^{(n)} = \sum_{k=1} \left(\frac{g^2}{16\pi^2} \right)^k Z^{(n,k)} .
$$

Cross-checks. RG analysis (2)

Check of $1/\varepsilon^2$ pole:

From previous formula we can derive

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\frac{16\pi^2}{g^3}\beta_g^{(1)} Z^{(1,1)} + \sum_i Z_{m_i}^{(1,1)} m_i^2 \frac{\partial}{\partial m_i^2} Z^{(1,1)}$$

Check $1/\varepsilon$ pole:

Use relation between charges

$$e^2 = g^2\left(1 - \frac{m_W^2}{m_Z^2}\right) \quad \text{and} \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}.$$

Differentiating the former formulae w.r.t. $\log\mu^2$ we derive

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(Z_Z^{(2,1)} - Z_W^{(2,1)}\right) c^2 = g^2 \frac{\beta_{g'}^{(2)}}{g'^3} s^4 - \frac{\beta_g^{(2)}}{g} s^2 c^2.$$

We can use 2-loop β-functions for g and g' calculated from symmetric phase theory

D.R.T. Jones '82
M.E. Machacek & M.T. Vaughn '83

$$\beta_{g'} = \frac{1}{12}\frac{g'^3}{16\pi^2} + \frac{1}{4}\frac{g'^5}{(16\pi^2)^2} + \frac{3}{4}\frac{g'^3 g^2}{(16\pi^2)^2},$$

$$\beta_g = -\frac{43}{12}\frac{g^3}{16\pi^2} - \frac{259}{12}\frac{g^5}{(16\pi^2)^2} + \frac{1}{4}\frac{g^3 g'^2}{(16\pi^2)^2}.$$

Two-loop results for the pole masses of Z and W

After UV renormalization the pole mass is a finite expression. This fact confirms the infrared finiteness of the bosonic contribution to the pole mass.

P. Gambino & P.A. Grassi '00

$$\frac{M_V^2}{m_V^2} = 1 + \frac{e^2}{16\pi s^2} X_{1,V} + \left(\frac{e^2}{16\pi s^2}\right)^2 X_{2,V},$$

$$X_{2,V} = \frac{m_H^4}{m_V^4} \sum_{i=0}^{5} A_i s^{2i},$$

$$A_i = \sum_{j=0}^{5} A_j^V \left(\frac{m_V^2}{m_H^2}\right)^j, \qquad V = Z, W$$

This is not a naive Taylor expansion. The general rules for asymptotic expansions allow us to extract also logarithmic dependences.

Coefficients $A_{i,j}^V$ after expansion are expressible in terms of restricted set of transcendental constants, like

$$S_0 = \frac{\pi}{\sqrt{3}}, \qquad S_1 = \frac{\pi}{\sqrt{3}} \ln 3,$$

$$S_2 = \frac{4}{9\sqrt{3}} \mathrm{Cl}_2(\pi/3), \qquad S_3 = \pi \mathrm{Cl}_2(\pi/3),$$

J. Fleischer & M. Kalmykov '99
A. Davydychev & M. Kalmykov '01

Two-loop results for the pole masses of Z and W (2)

Numerically,

$$
\begin{aligned}
A_0^Z = & \left[-1.23825 + 2.0625 \ln m_H^2 - 0.28125 \ln^2 m_H^2\right] \frac{M_Z^4}{M_W^4} \\
& + h_z \big[6.76278 - 5.03125 \ln h_z + 4.3125 \ln h_z \ln m_H^2 \\
& - 4.9959 \ln m_H^2 + 5.375 \ln^2 m_H^2\big] \\
& + h_z^2 \big[-50.9669 - 38.165 \ln h_z + 37.6042 \ln^2 h_z \\
& + 81.2917 \ln h_z \ln m_H^2 - 25.544 \ln m_H^2 + 34.7674 \ln^2 m_H^2 \\
& + h_z^3 \big[-28.4156 - 154.515 \ln h_z + 70.4896 \ln^2 h_z \\
& + 108.938 \ln h_z \ln m_H^2 - 90.9077 \ln m_H^2 + 44.25 \ln^2 m_H^2\big] \\
& + h_z^4 \big[-108.146 - 29.409 \ln h_z + 93.8760 \ln^2 h_z \\
& + 93.5 \ln h_z \ln m_H^2 + 65.5542 \ln m_H^2 + 20.25 \ln^2 m_H^2\big] \\
& + h_z^5 \big[-286.699 + 51.671 \ln h_z + 266.783 \ln^2 h_z \\
& + 171.486 \ln h_z \ln m_H^2 + 209.429 \ln m_H^2\big]
\end{aligned}
$$

where

$$
\ln m_H^2 = \ln \frac{m_H^2}{\mu^2}, \qquad h_V = \frac{m_V^2}{m_H^2}
$$

Two-loop results for the pole masses of Z and W (3)

$$
\begin{aligned}
A_1^W &= 0 \\
&+h_w\left[7.86 + 0.66\ln h_w - 0.56\ln h_w \ln m_H^2 - 4.52\ln m_H^2\right] \\
&+h_w^2\left[-8.0073 - 32.2802\ln h_w - 3.84722\ln^2 h_w \right.\\
&\left. -1.25\ln h_w \ln m_H^2 - 27.9831\ln m_H^2 - 0.3125\ln^2 m_H^2\right] \\
&+h_w^3\left[24.6496 + 19.1329\ln h_w + 19.4514\ln^2 h_w \right.\\
&\left. +24.81\ln h_w \ln m_H^2 + 15.36\ln m_H^2 + 12.0\ln^2 m_H^2\right] \\
&+O(h_w^4)
\end{aligned}
$$

$$
\begin{aligned}
A_1^Z &= 0 \\
&+h_z\left[19.81 + 1.167\ln h_z - 1.\ln h_z \ln m_H^2 - 7.38\ln m_H^2\right] \\
&+h_z^2\left[31.674 - 62.2730\ln h_z - 40.7222\ln^2 h_z \right.\\
&\left. -70.2222\ln h_z \ln m_H^2 - 54.5640\ln m_H^2 - 35.5486\ln^2 m_H^2\right] \\
&+h_z^3\left[111.233 - 6.00536\ln h_z - 79.1319\ln^2 h_z \right.\\
&\left. -122.\ln h_z \ln m_H^2 + 44.6422\ln m_H^2 - 64.5000\ln^2 m_H^2\right] \\
&+O(h_z^4)
\end{aligned}
$$

Auxiliary relations

Inverse relation (up to 2-loops):

$$
\begin{aligned}
m_V^2 &= M_V^2 - \Pi_1 - \Pi_2 - \Pi_1' \Pi_1 - \Pi_1 \Pi_1 + \sum_j \Pi_{j,1} \frac{\partial}{\partial m_j^2} \Pi_1 \\
&- \frac{e^2}{16\pi^2} \left(7 \ln \left(\frac{M_W^2}{\mu^2} \right) - \frac{2}{3} \right) \Pi_1 \Big|_{p^2 = M_V^2,\, m_j^2 = M_j^2,\, e = e_{\rm OS}},
\end{aligned}
$$

In the above relation we had to perform a change from the $\overline{\rm MS}$ to the on-shell scheme also for the electric charge

$$
e(\mu^2) = e_{\rm OS} \left[1 + \frac{e_{\rm OS}^2}{16\pi^2} \left(\frac{7}{2} \ln \left(\frac{M_W^2}{\mu^2} \right) - \frac{1}{3} \right) \right]
$$

with $e_{\rm OS}^2/4\pi = \alpha \sim 1/137$.

$$
\begin{aligned}
\sin^2\theta_W &= 1 - \frac{m_W^2}{m_Z^2} = 1 - \frac{M_W^2}{M_Z^2} \left(\frac{1 + \delta_W^{(1)} + \delta_W^{(2)}}{1 + \delta_Z^{(1)} + \delta_Z^{(2)}} \right) \\
&= \left(1 - \frac{M_W^2}{M_Z^2} \right) \times \\
&\left[1 - \frac{\cos^2\theta_W}{\sin^2\theta_W} \left((\delta_W^{(1)} - \delta_Z^{(1)})(1 - \delta_Z^{(1)}) + \delta_W^{(2)} - \delta_Z^{(2)} \right) \right]
\end{aligned}
$$

where $m_V^2/M_V^2 = 1 + \delta_V^{(1)} + \delta_V^{(2)}$

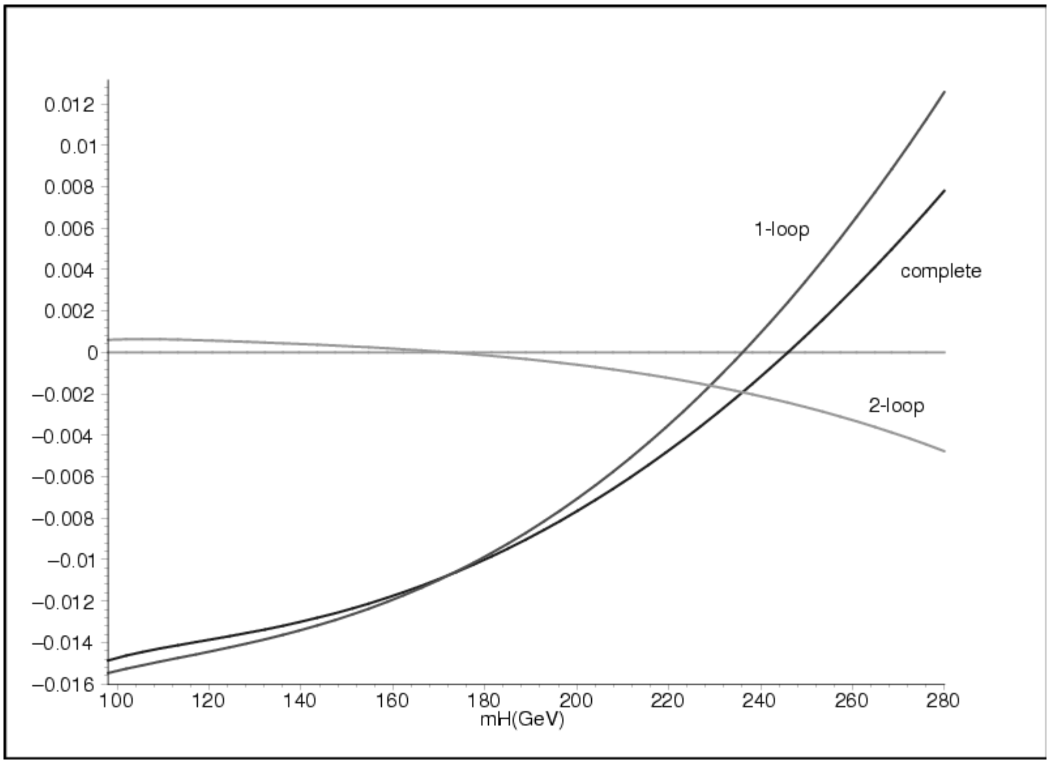


Figure 1:

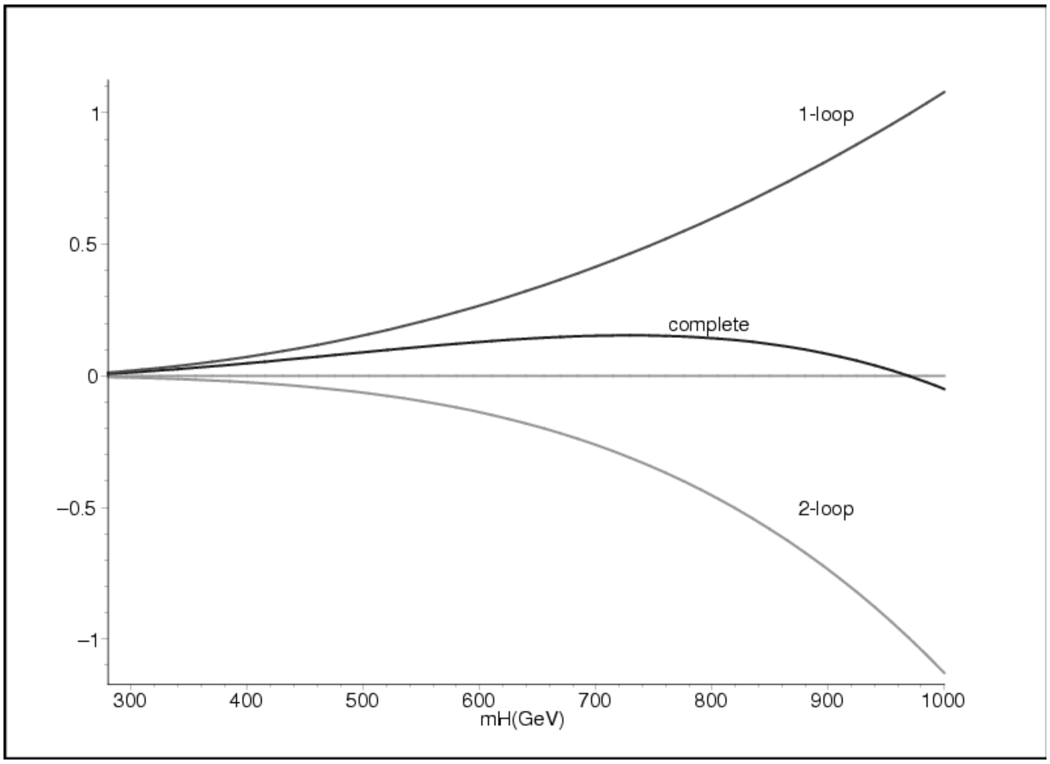


Figure 2: One- and two-loop corrections to the relation $\left[m_Z^2(M_Z)/M_Z^2 - 1\right]$ for a Higgs.

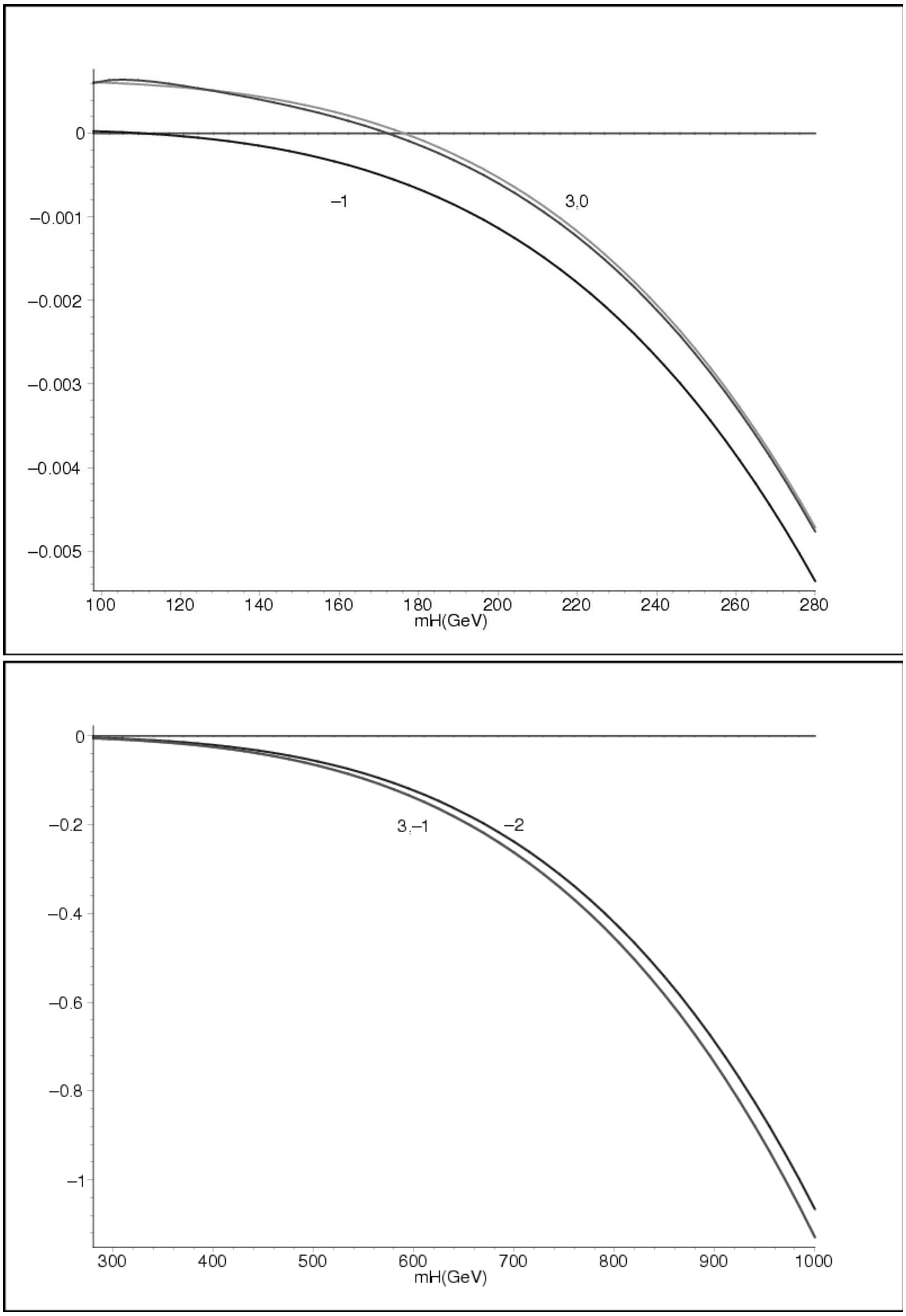
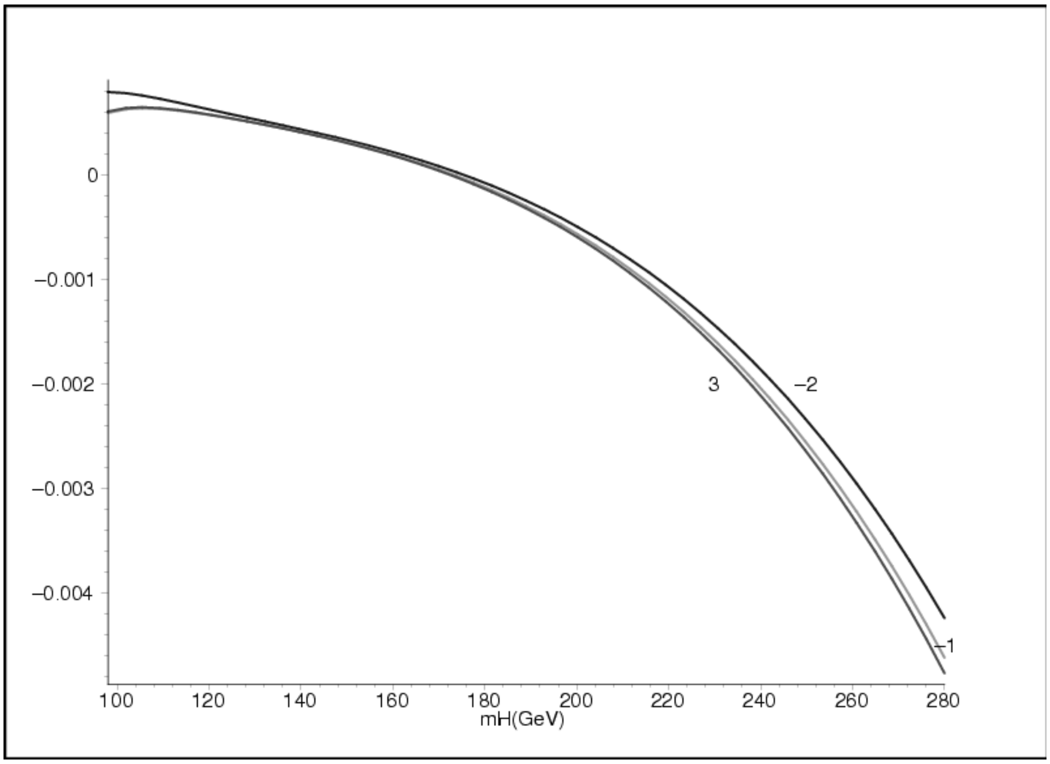


Figure 4: The dependence of the two-loop corrections on the ratio $m_Z^2(M_Z^2)/M_Z^2$ as a function of the Higgs mass and numbers of coefficients of expansion used for their evaluation. "-2" corresponds to leading order, "-1" denotes the value obtained from the first two coefficients (up to next-to-leading order) and "3" is a complete result.



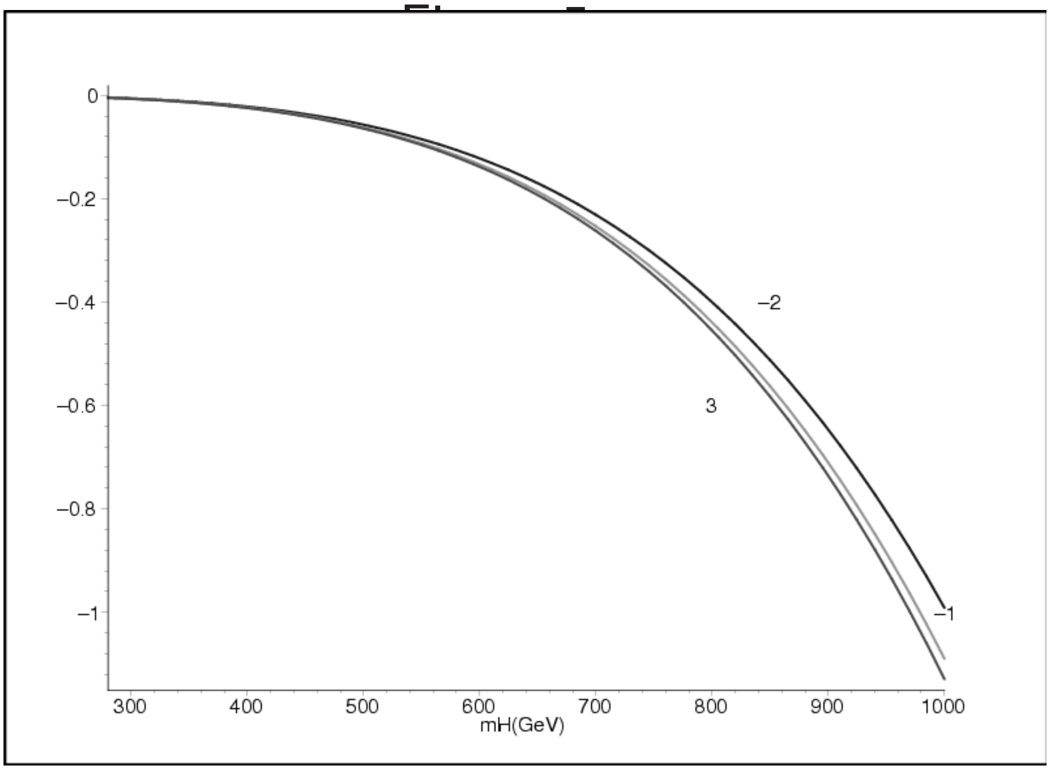


Figure 6: The dependence of the two-loop corrections on the ratio $m_Z^2(M_Z^2)/M_Z^2$ as a function of the Higgs mass and the number of coefficients of expansion with respect to $\sin^2\theta_W$. "-2", "-1" and "3" mean that we take only leading order, next-to-leading order and complete result.

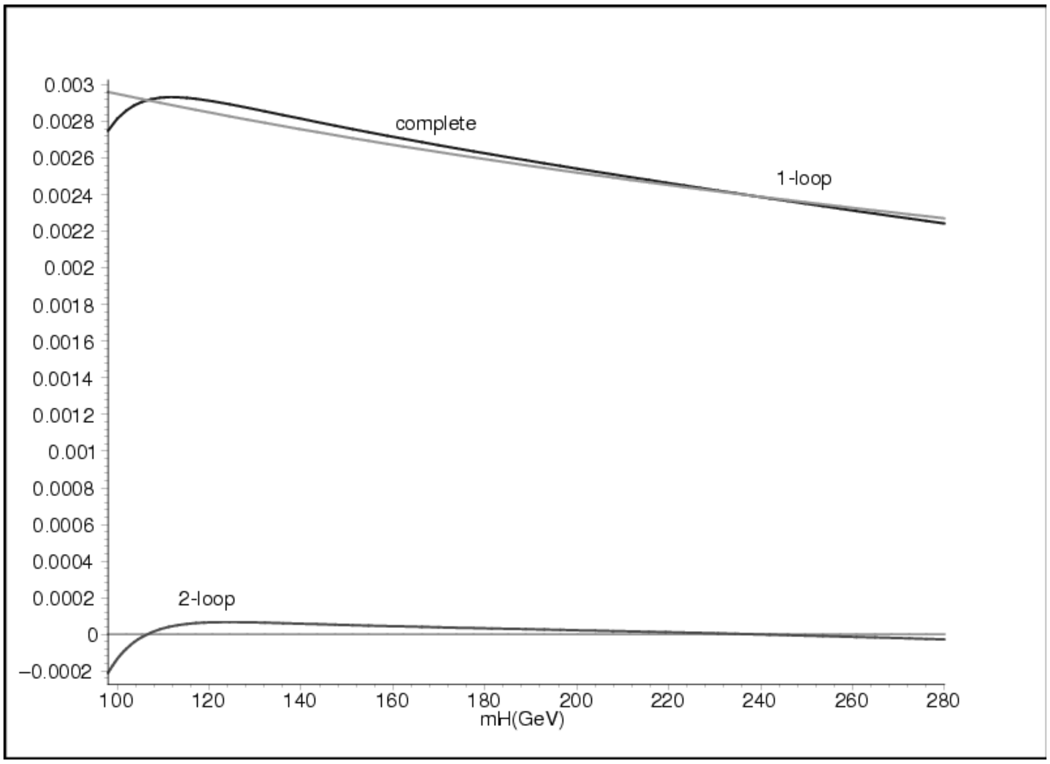


Figure 7: One- and two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ as a function of the (light) Higgs mass.

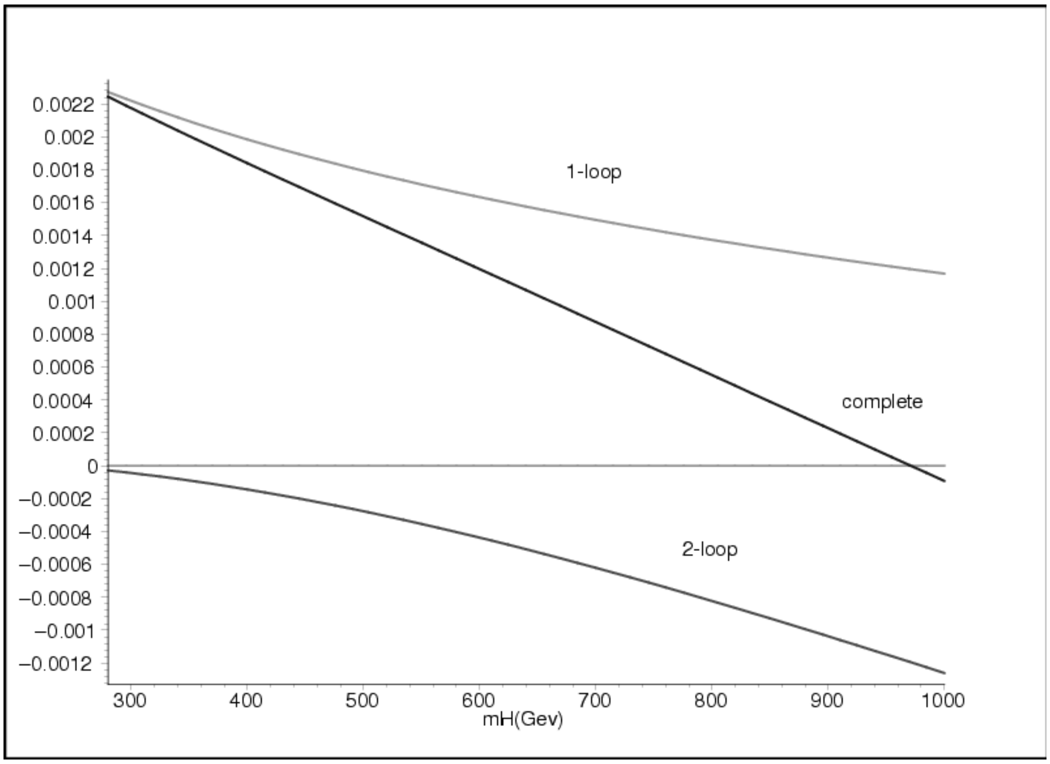


Figure 8: One- and two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ as a function of the (heavy) Higgs mass.

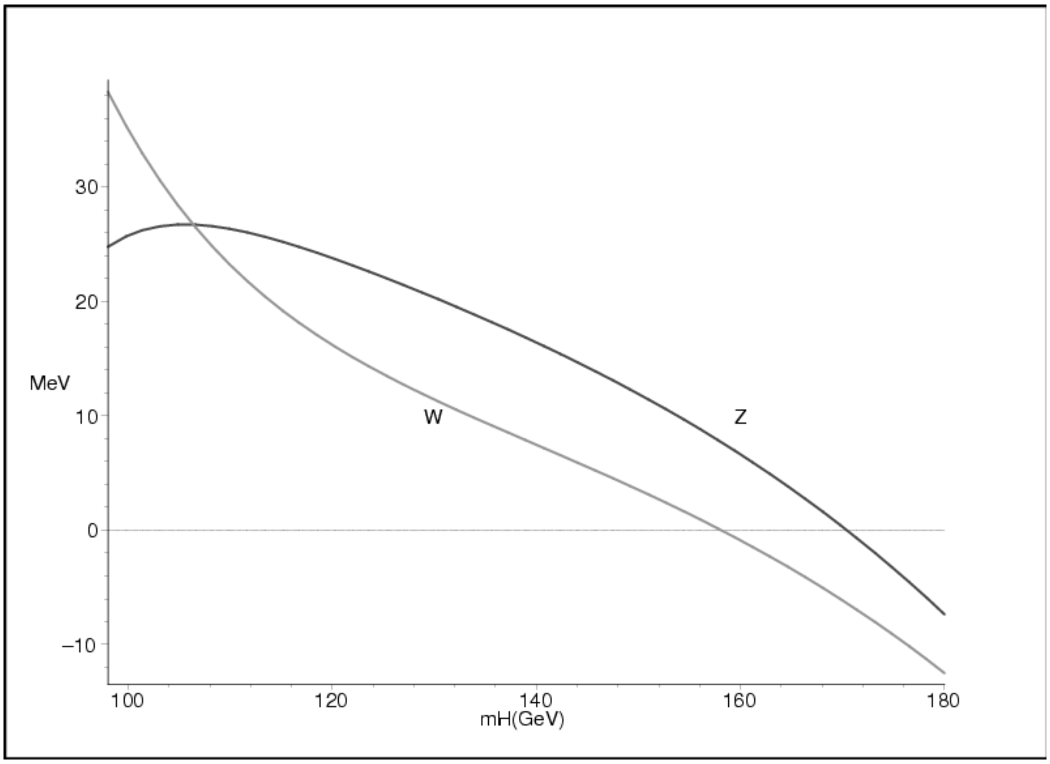


Figure 9: Two-loop corrections to the relation $m_V^2(M_V^2) - M_V^2$ at $\mu^2 = M_V^2$ for the restricted values of Higgs mass

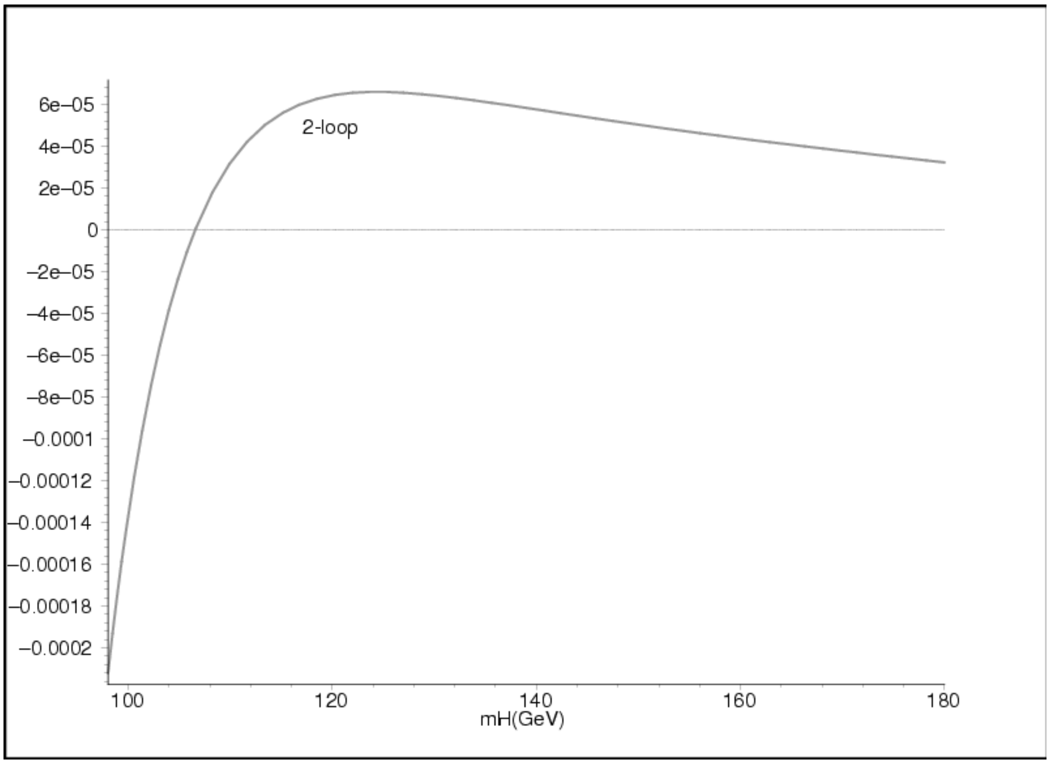


Figure 10: Two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ for the restricted values of Higgs mass