Two-loop electroweak bosonic corrections to the relation between $\overline{\mathrm{MS}}$ and on-shell massses of gauge bosons

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Pole mass. General consideration.

The position of the pole s_P of the massive gauge boson propagator in a QFT is defined by equation:

$$s_P - m^2 - \Pi(s_P, m^2, \cdots) = 0$$

m is a $\overline{\rm MS}$ or *bare* mass. For gauge bosons Π is *transversal* part of the self-energy.

The pole s_P is

- gauge invariant
- infrared stable
- generally complex: $\mathrm{Im} s_p \neq 0$

We can define *pole* or *on-shell* mass M and width Γ via

$$s_P = M^2 - \mathrm{i} M \Gamma$$

Solution (up to 2-loops):

$$\begin{aligned} s_P = \; & m^2 \\ & + \Pi_1(m^2, m^2) \\ & + \Pi_2(m^2, m^2) + \Pi_1(m^2, m^2) \Pi_1'(m^2, m^2). \end{aligned}$$

where we use loop expansion for $\Pi(p^2, m^2, \ldots)$:

$$\Pi = \Pi_1 + \Pi_2 + \ldots$$

Pole mass (2). Case of Z-γ mixing.

In neutral sector because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

Equation:

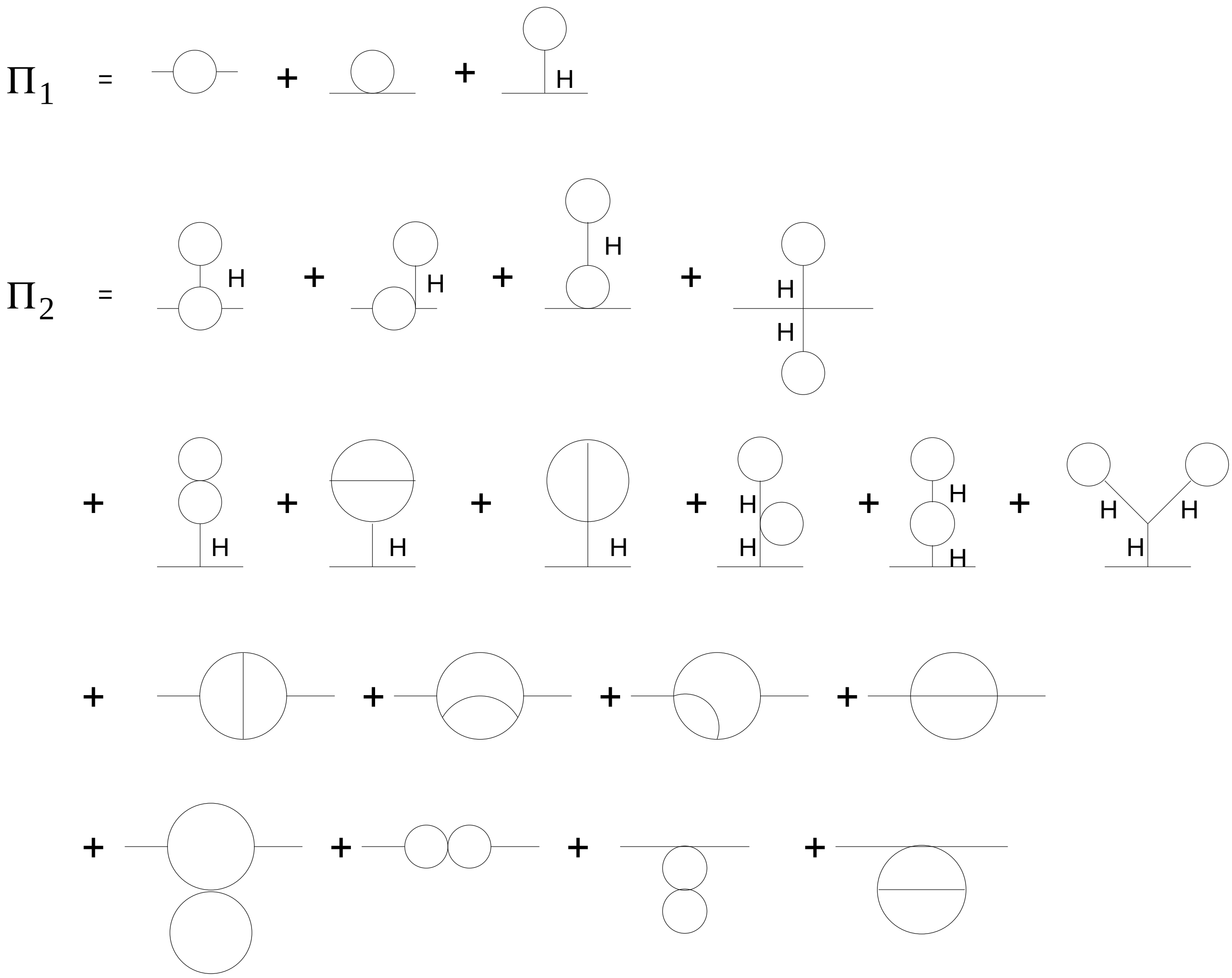
$$s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.$$

Mixing term $\Pi_{\gamma Z}$ starts to contribute from 2-loops. Term $\Pi_{\gamma\gamma}$ contributes only in 3-loop approximation.

Diagrams and topologies

To be computed on-shell ($p^2 = m_Z^2$):

$$\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \ldots$$



Number of diagrams in R_ξ gauge

At 1-loop level:	∼50	
At 2-loop level: (only bosons)	1PI	Total
Z	616	2348
W	792	4084

Evaluation

One can choose one of the following schemes:

Numerical approach:

- Reduction of tensor integrals to few masterintegrals
- Evaluation of masterintegrals

Semianalytical approach:

- Reduce number of scale to one (Expansion)
- Evaluate single scale diagrams (Constants)

Evaluation (2)

In order to check explicitely the gauge invariance we perform all calculations in R_ξ gauge with three independent gauge parameters ξ_W, ξ_Z, ξ_γ. E.g. Z boson propagator reads

$$D^Z_{\mu\nu}(p) = \frac{i}{p^2 - m_Z^2}\left(-g_{\mu\nu} + (1-\xi_Z)\frac{p_\mu p_\nu}{p^2 - \xi_Z m_Z^2}\right).$$

Then

there are the following 5 scales in the problem:

$$m_W, \quad \sqrt{\xi_W} m_W, \quad m_Z, \quad \sqrt{\xi_Z} m_Z, \quad m_H, \quad m_t.$$

We perform 3 steps:

1. Taylor (naive) expansion in $(\xi_Z - 1)$ and $(\xi_W - 1)$.

$$D^V_{\mu\nu}(p) \sim g_{\mu\nu} + (\xi_V - 1)\frac{p_\mu p_\nu}{p^2 - m_V^2} + (\xi_V - 1)^2 \frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2}$$

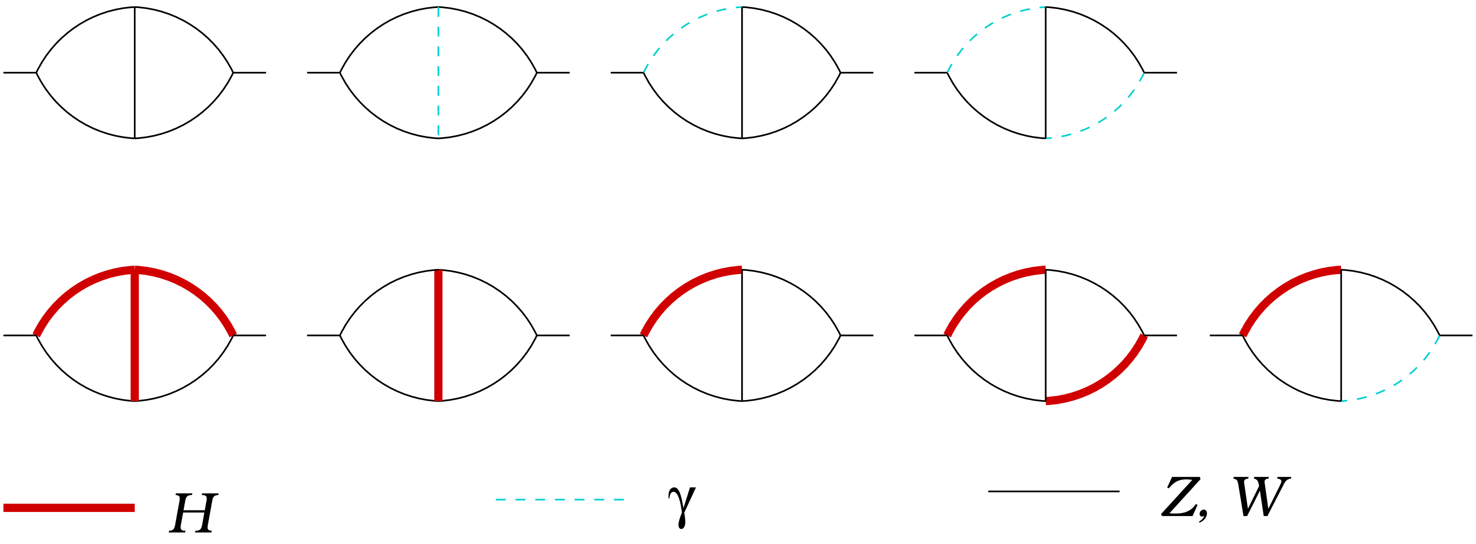
2. Expansion in small parameter

$$s^2 = \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} < 0.25.$$

3. Large mass expansion in $m_Z^2/m_H^2 < 0.64$.

Diagram prototypes

We need to evaluate following on-shell diagrams ($p^2 = m_Z^2$).



♣ Expansion in $\sin^2\theta_W$:

$$m_W^2 = m_Z^2(1 - s^2)$$

Diagrams without Higgs—two-loop on-shell diagrams.
Single scale! Can be evaluated analytically.
FORM package **ONSHELL2**

J. Fleischer & O.V. Tarasov 92
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♠ Diagrams with Higgs:
Heavy Higgs expansion (HHE).

$$\frac{m_Z^2}{m_H^2} \ll 1$$

V.A. Smirnov 90

Renormalization.

We have s_P parametrized in terms of *bare* parameters

$$s_P = f(e_0, m_{Z,0}, m_{W,0}, m_{H,0}).$$

Renormalization of charges and masses:

$$\begin{aligned} e_0 &= \mu^\varepsilon e Z_e \\ m_0^2 &= m^2 Z_m \end{aligned}$$

We need:
+ Z_e, Z_H, Z_t up to 1-loop
+ Z_W, Z_Z up to 2-loops

We dont need:
– field renormalization
– renoramlization of ghost sector
– gauge parameters renormalization

The charge $\overline{\mathrm{MS}}$ renormalization constant can be calculated in the unbroken theory

$$Z_e = 1 - \frac{e^2}{16\pi^2} \frac{7}{2\varepsilon}$$

where e, m_Z, m_W, m_H and s^2 are $\overline{\mathrm{MS}}$ parameters.

Mass renormalization.

$$m_0^2 = m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right)$$

The mass renormalization constants cannot be calculated from the unbroken theory. The calculation of the Feynman diagrams in the Standard Model is required.

One-loop results:

$$\begin{aligned}
Z_H^{(1,1)} &= -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2}, \\
Z_W^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{3}{4}\frac{m_Z^2}{m_W^2} - \frac{17}{3}, \\
Z_Z^{(1,1)} &= -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} + \frac{11}{12}\frac{m_Z^2}{m_W^2} \\
& \quad -7\frac{m_W^2}{m_Z^2} + \frac{7}{6}.
\end{aligned}$$

Violation of polynomiality in $\overline{\mathrm{MS}}$ in SM:

$\dfrac{1}{m_Z^2}$ — $Z - \gamma$ mixing: actual Z boson coupling is $\sqrt{g^2 + g'^2} = \dfrac{g}{\cos\theta_W}$

$\dfrac{1}{m_H^2}$ — inverse powers of the Higgs self-coupling (tadpoles)

2-loop constants in $\overline{\mathrm{MS}}$. Exact results:

$$
\begin{aligned}
Z_W^{(2,1)} &= \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} - \frac{301}{192}\frac{m_Z^4}{m_W^4} \\
&+ \frac{17}{12}\frac{m_Z^2}{m_W^2} - \frac{53}{3} + \frac{31}{12}\frac{m_Z^4}{m_H^2 m_W^2} - \frac{17}{2}\frac{m_Z^2}{m_H^2} \\
&- \frac{176}{3}\frac{m_W^2}{m_H^2} + \frac{59}{24}\frac{m_Z^6}{m_H^2 m_W^4}, \\
Z_W^{(2,2)} &= -\frac{9}{32}\frac{m_H^4}{m_W^4} + \frac{43}{8}\frac{m_H^2}{m_W^2} + \frac{65}{32}\frac{m_Z^4}{m_W^4} - \frac{35}{8}\frac{m_Z^2}{m_W^2} + \frac{334}{9} \\
&+ \frac{27}{4}\frac{m_Z^4}{m_H^2 m_W^2} + 6\frac{m_Z^2}{m_H^2} + 34\frac{m_W^2}{m_H^2} - \frac{5}{2}\frac{m_Z^6}{m_H^2 m_W^4} \\
&+ 9\frac{m_W^4}{m_H^4} + 9\frac{m_Z^4}{m_H^4} + \frac{9}{4}\frac{m_Z^8}{m_H^4 m_W^4}, \\
Z_Z^{(2,1)} &= \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} + \frac{11}{3} + \frac{7}{6}\frac{m_Z^2}{m_W^2} \\
&- \frac{64}{3}\frac{m_W^2}{m_Z^2} - \frac{253}{192}\frac{m_Z^4}{m_W^4} - \frac{176}{3}\frac{m_W^2}{m_H^2} - \frac{17}{2}\frac{m_Z^2}{m_H^2} \\
&+ \frac{31}{12}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{59}{24}\frac{m_Z^6}{m_H^2 m_W^4}, \\
Z_Z^{(2,2)} &= -\frac{9}{32}\frac{m_H^4}{m_W^4} + \frac{1}{4}\frac{m_H^2}{m_W^2} - \frac{1}{8}\frac{m_H^2 m_Z^2}{m_W^4} + \frac{21}{4}\frac{m_H^2}{m_Z^2} \\
&- \frac{55}{6} + \frac{11}{12}\frac{m_Z^2}{m_W^2} + \frac{629}{288}\frac{m_Z^4}{m_W^4} + \frac{245}{6}\frac{m_W^2}{m_Z^2} + \frac{27}{2}\frac{m_W^2}{m_H^2} \\
&+ 16\frac{m_Z^2}{m_H^2} + 21\frac{m_W^4}{m_Z^2 m_H^2} - \frac{7}{2}\frac{m_Z^4}{m_H^2 m_W^2} - \frac{11}{4}\frac{m_Z^6}{m_H^2 m_W^4} \\
&+ 9\frac{m_W^4}{m_H^4} + 9\frac{m_Z^4}{m_H^4} + \frac{9}{4}\frac{m_Z^8}{m_H^4 m_W^4}.
\end{aligned}
$$

Cross-checks. RG analysis.

We can check two-loop coefficients $Z^{(2,2)}$ and $Z^{(2,1)}$ in mass renormalization formula

$$
\begin{aligned}
m_0^2 &= m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right) \\
&= m^2 \left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \ldots \right)
\end{aligned}
$$

using renormalization group.

Introduce anomalous mass dimension

$$\gamma = \mu^2 \frac{d}{d\mu^2} \log m^2 .$$

From

$$0 = \mu^2 \frac{d}{d\mu^2} \log m_0^2$$

we have

$$
\begin{aligned}
\gamma &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(1)}, \\
\left(\gamma + \beta_g \frac{\partial}{\partial g} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) z^{(n)} &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(n+1)},
\end{aligned}
$$

where

$$z^{(n)} = \sum_{k=1} \left(\frac{g^2}{16\pi^2} \right)^k Z^{(n,k)} .$$

Cross-checks. RG analysis (2)

Check of $1/\varepsilon^2$ pole:

From previous formula we can derive

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\frac{16\pi^2}{g^3}\beta_g^{(1)} Z^{(1,1)} + \sum_i Z_{m_i}^{(1,1)} m_i^2 \frac{\partial}{\partial m_i^2} Z^{(1,1)}$$

Check $1/\varepsilon$ pole:

Use relation between charges

$$e^2 = g^2\left(1 - \frac{m_W^2}{m_Z^2}\right) \quad \text{and} \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}.$$

Differentiating the former formule w.r.t. $\log\mu^2$ we derive

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(Z_Z^{(2,1)} - Z_W^{(2,1)}\right) c^2 = g^2 \frac{\beta_{g'}^{(2)}}{g'^3} s^4 - \frac{\beta_g^{(2)}}{g} s^2 c^2.$$

We can use 2-loop β-functions for g and g' calculated from symmetric phase theory

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$$\begin{aligned}
\beta_{g'} &= \frac{1}{12}\frac{g'^3}{16\pi^2} + \frac{1}{4}\frac{g'^5}{(16\pi^2)^2} + \frac{3}{4}\frac{g'^3 g^2}{(16\pi^2)^2}, \\
\beta_g &= -\frac{43}{12}\frac{g^3}{16\pi^2} - \frac{259}{12}\frac{g^5}{(16\pi^2)^2} + \frac{1}{4}\frac{g^3 g'^2}{(16\pi^2)^2}.
\end{aligned}$$

Two-loop results for the pole masses of Z and W

After UV renormalization the pole mass is a finite expression. This fact confirms the infrared finiteness of the bosonic contribution to the pole mass.

$$\frac{M_V^2}{m_V^2} = 1 + \frac{e^2}{16\pi s^2} X_{1,V} + \left(\frac{e^2}{16\pi s^2}\right)^2 X_{2,V},$$

$$\begin{aligned} X_{2,V} &= \frac{m_H^4}{m_V^4} \sum_{i,j=0}^{5} A_{i,j}^V s^{2i} \left(\frac{m_V^2}{m_H^2}\right)^j \\ &\sim a_0 + s^2 a_V + O(s^4), \qquad V = Z, W \end{aligned}$$

This is not a naive Taylor expansion. The general rules for asymptotic expansions allow us to extract also logarithmic dependences.

Coefficient a_0 is universal for Z and W bosons to be

$$\begin{aligned} a_0 &= \frac{m_H^4}{m_W^4} \left(-\frac{359}{128} + \frac{243}{32} S_2 - \frac{1}{24}\pi^2 + \frac{33}{16} L_h - \frac{9}{32} L_h^2 \right) \\ &+ \frac{m_H^2}{m_V^2} \Big(\frac{2483}{192} - \frac{231}{32} S_0 + \frac{243}{32} S_2 + \frac{433}{864}\pi^2 - \frac{161}{32} \ln \frac{m_V^2}{m_H^2} \\ &\quad + \frac{69}{16} \ln \frac{m_V^2}{m_H^2} L_h - \frac{519}{32} L_h + \frac{99}{16} L_h S_0 + \frac{43}{8} L_h^2 \Big) \end{aligned}$$

where

$$L_h = \log \frac{m_H^2}{\mu^2}, \qquad S_0 = \frac{\pi}{\sqrt{3}}, \qquad S_2 = \frac{4}{9\sqrt{3}} \mathrm{Cl}_2(\pi/3),$$

Two-loop results for the pole masses of Z and W (2)

$$
\begin{aligned}
a_W \;=\; & \frac{m_H^2}{m_W^2}\Big(\frac{79}{48} + \frac{77}{32}S_0 + \frac{3}{16}\pi^2 + \frac{21}{32}\ln\frac{m_W^2}{m_H^2} - \frac{9}{16}\ln\frac{m_W^2}{m_H^2}L_h \\
& -\frac{25}{32}L_h^2 - \frac{33}{16}L_h S_0\Big) \\
+ & \Big(-\tfrac{15491}{576} + \tfrac{65}{6}S_3 - \tfrac{11183}{48}S_0 + 66 S_1 - \tfrac{4239}{16}S_2 \\
& + \tfrac{8837}{216}\pi^2 - \tfrac{22}{3}\pi^2\ln 3 + \tfrac{67}{4}\zeta_3 - \tfrac{149}{216}\ln\frac{m_W^2}{m_H^2} \\
& - \tfrac{209}{12}\ln\frac{m_W^2}{m_H^2}S_0 - \tfrac{277}{72}\ln^2\frac{m_W^2}{m_H^2} - \tfrac{5}{4}\ln\frac{m_W^2}{m_H^2}L_h \\
& + \tfrac{559}{72}L_h - \tfrac{473}{24}L_h S_0 - \tfrac{5}{16}L_h^2\Big)
\end{aligned}
$$

$$
\begin{aligned}
a_Z \;=\; & \frac{m_H^2}{m_Z^2}\Big(-\frac{2243}{288} + \frac{77}{8}S_0 + \frac{243}{16}S_2 + \frac{271}{432}\pi^2 + \frac{7}{6}\ln\frac{m_Z^2}{m_H^2} \\
& - \ln\frac{m_Z^2}{m_H^2}L_h + \frac{91}{12}L_h - \frac{33}{4}L_h^2 S_0\Big) \\
+ & \Big(-\tfrac{4637683}{10368} + \tfrac{589}{12}S_3 - \tfrac{12295}{24}S_0 + 66 S_1 + \tfrac{157725}{64}S_2 \\
& + \tfrac{99227}{1728}\pi^2 + \tfrac{11}{3}\pi^2\ln 3 - \tfrac{853}{8}\zeta_3 + \tfrac{225601}{864}\ln\frac{m_Z^2}{m_H^2} \\
& - \tfrac{4279}{24}\ln\frac{m_Z^2}{m_H^2}S_0 - \tfrac{733}{18}\ln^2\frac{m_Z^2}{m_H^2} - \tfrac{632}{9}\ln\frac{m_Z^2}{m_H^2}L_h \\
& + \tfrac{203531}{864}L_h - \tfrac{3839}{24}L_h S_0 - \tfrac{5119}{144}L_h^2\Big)
\end{aligned}
$$

Auxiliary relations

Inverse relation (up to 2-loops):

$$
\begin{aligned}
m_V^2 &= M_V^2 - \Pi_1 - \Pi_2 - \Pi_1' \Pi_1 - \Pi_1 \Pi_1 + \sum_j \Pi_{j,1} \frac{\partial}{\partial m_j^2} \Pi_1 \\
&- \frac{e^2}{16\pi^2} \left(7 \ln \left(\frac{M_W^2}{\mu^2} \right) - \frac{2}{3} \right) \Pi_1 \Big|_{p^2 = M_V^2,\, m_j^2 = M_j^2,\, e = e_{\rm OS}},
\end{aligned}
$$

In the above relation we had to perform a change from the $\overline{\rm MS}$ to the on-shell scheme also for the electric charge

$$
e(\mu^2) = e_{\rm OS} \left[1 + \frac{e_{\rm OS}^2}{16\pi^2} \left(\frac{7}{2} \ln \left(\frac{M_W^2}{\mu^2} \right) - \frac{1}{3} \right) \right]
$$

with $e_{\rm OS}^2/4\pi = \alpha \sim 1/137$.

$$
\begin{aligned}
\sin^2\theta_W &= 1 - \frac{m_W^2}{m_Z^2} = 1 - \frac{M_W^2}{M_Z^2} \left(\frac{1 + \delta_W^{(1)} + \delta_W^{(2)}}{1 + \delta_Z^{(1)} + \delta_Z^{(2)}} \right) \\
&= \left(1 - \frac{M_W^2}{M_Z^2} \right) \times \\
& \left[1 - \frac{\cos^2\theta_W}{\sin^2\theta_W} \left((\delta_W^{(1)} - \delta_Z^{(1)})(1 - \delta_Z^{(1)}) + \delta_W^{(2)} - \delta_Z^{(2)} \right) \right]
\end{aligned}
$$

where $m_V^2/M_V^2 = 1 + \delta_V^{(1)} + \delta_V^{(2)}$

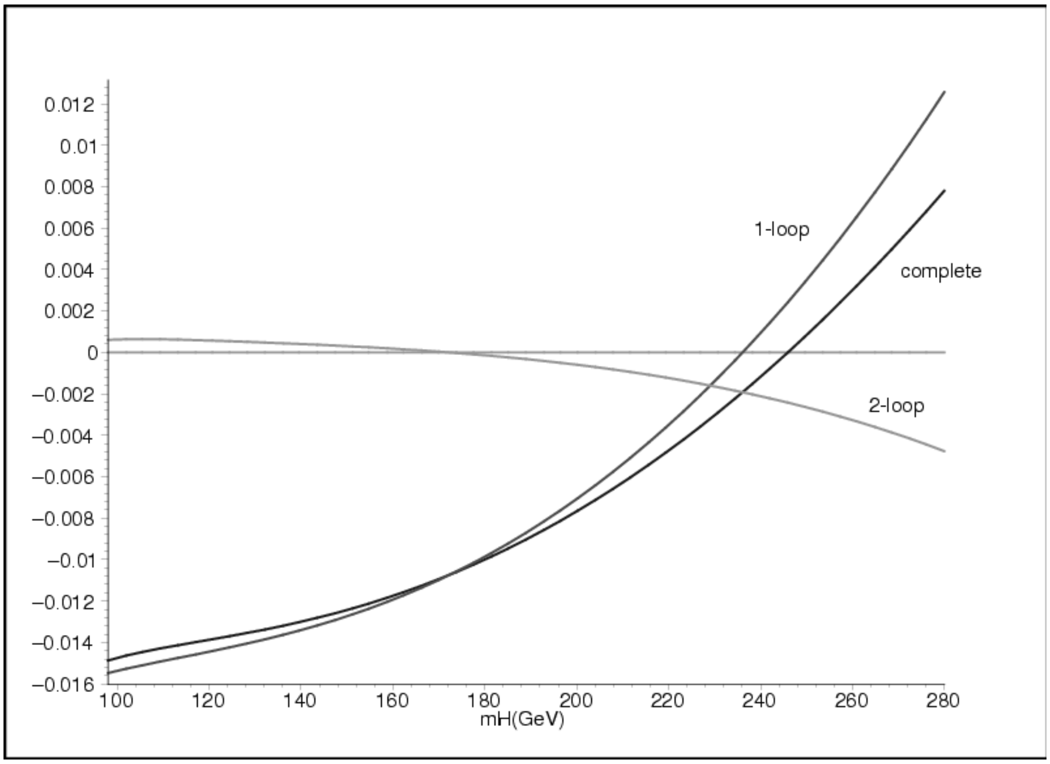


Figure 1:

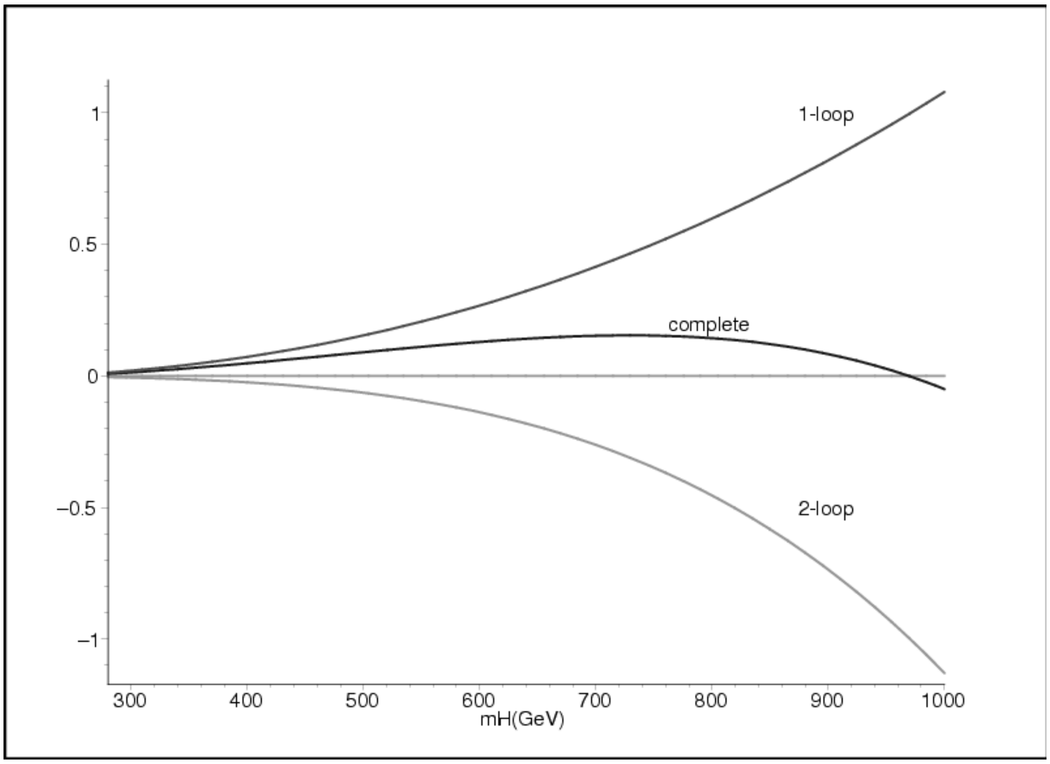


Figure 2: One- and two-loop corrections to the relation $\left[m_Z^2(M_Z)/M_Z^2 - 1\right]$ for a Higgs.

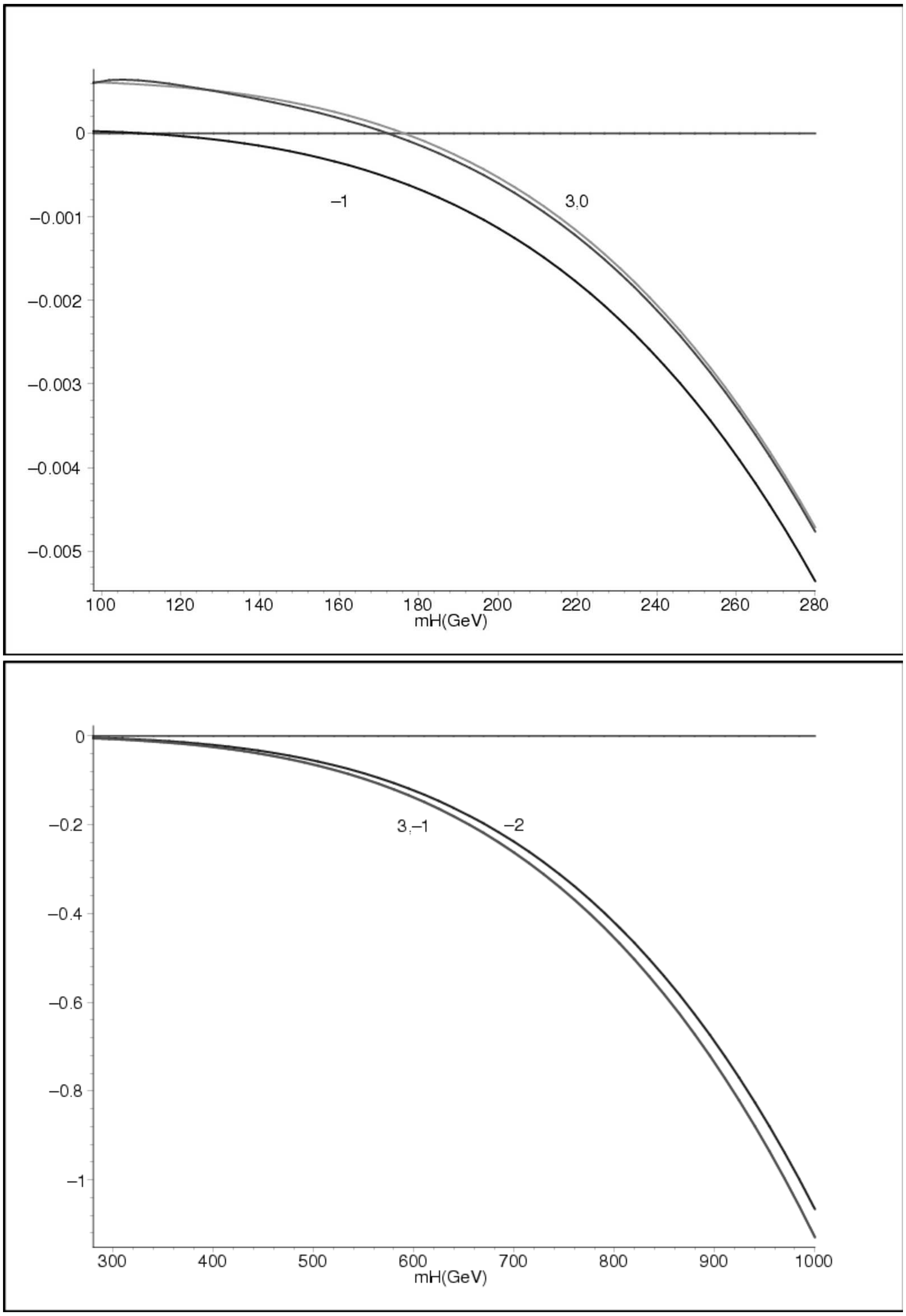
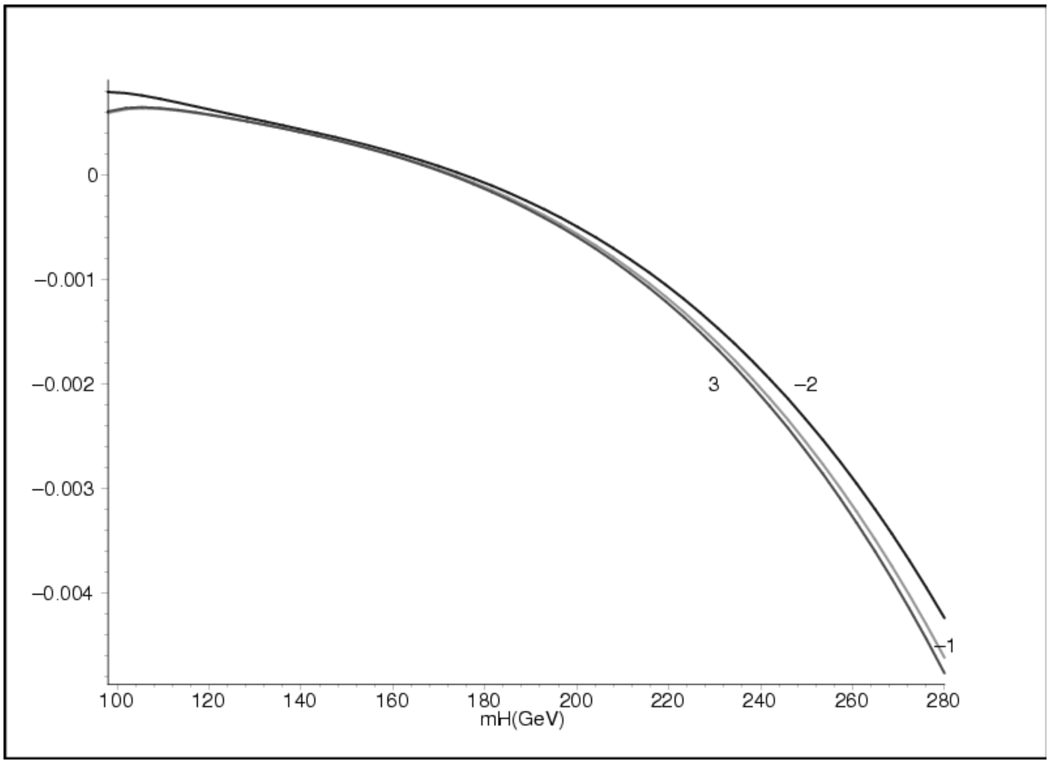


Figure 4: The dependence of the two-loop corrections on the ratio $m_Z^2(M_Z^2)/M_Z^2$ as a function of the Higgs mass and numbers of coefficients of expansion used for their evaluation. "-2" corresponds to leading order, "-1" denotes the value obtained from the first two coefficients (up to next-to-leading order) and "3" is a complete result.



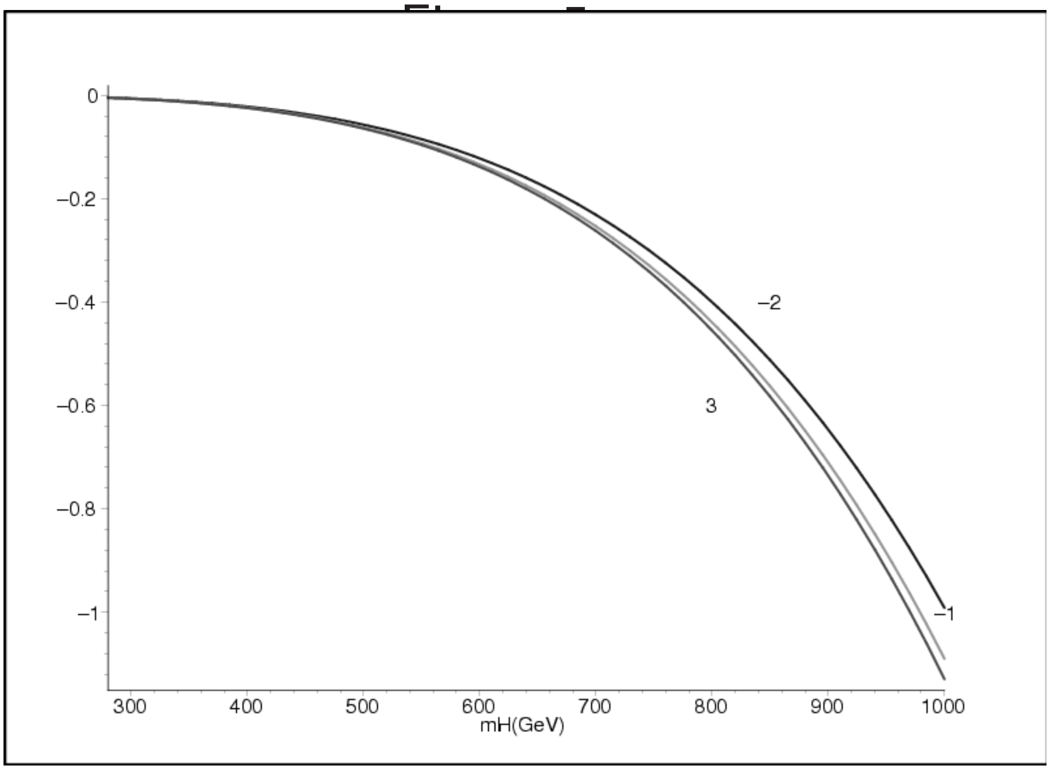


Figure 6: The dependence of the two-loop corrections on the ratio $m_Z^2(M_Z^2)/M_Z^2$ as a function of the Higgs mass and the number of coefficients of expansion with respect to $\sin^2\theta_W$. "-2", "-1" and "3" mean that we take only leading order, next-to-leading order and complete result.

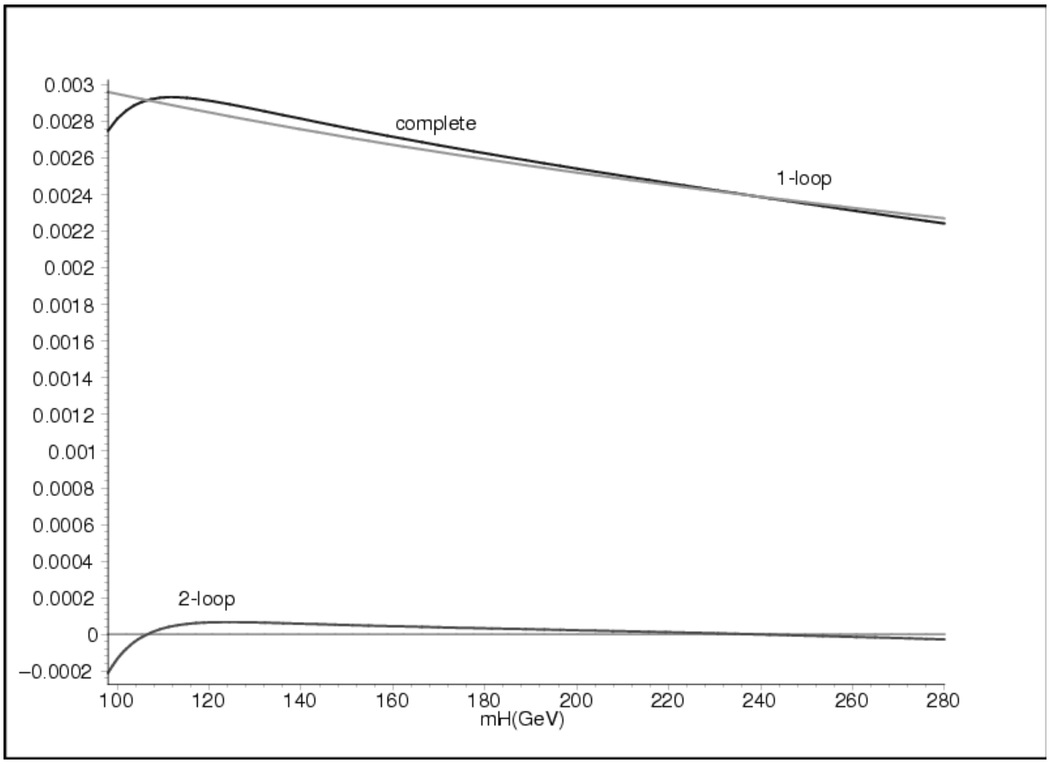


Figure 7: One- and two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ as a function of the (light) Higgs mass.

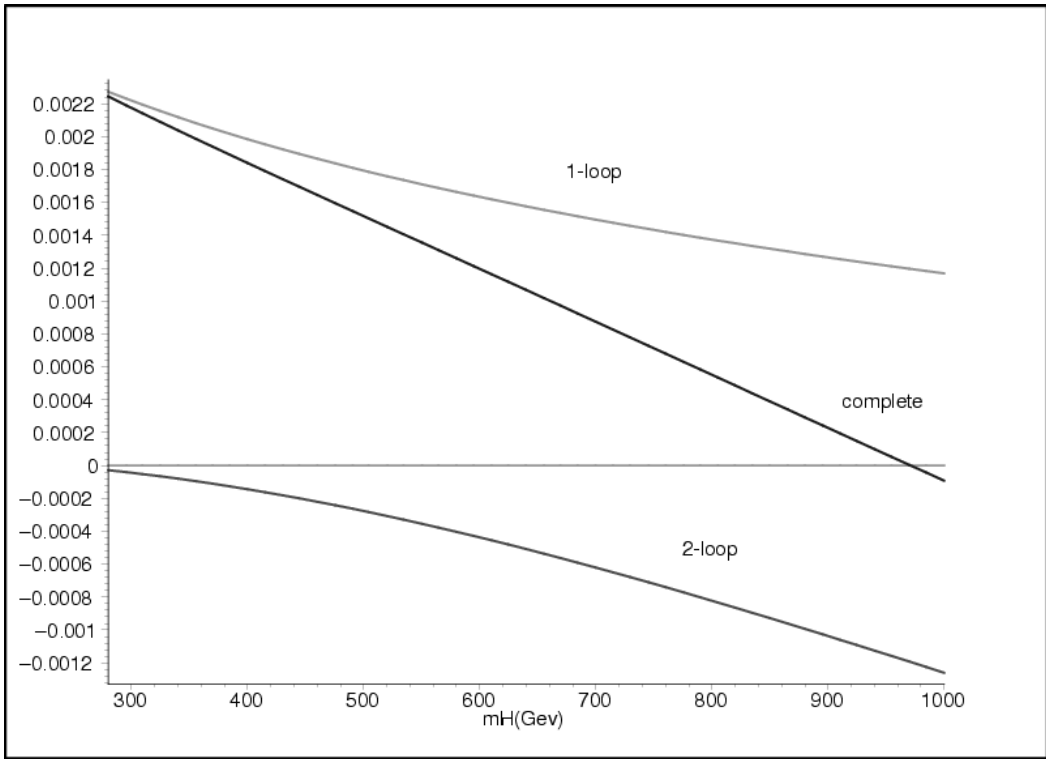


Figure 8: One- and two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ as a function of the (heavy) Higgs mass.

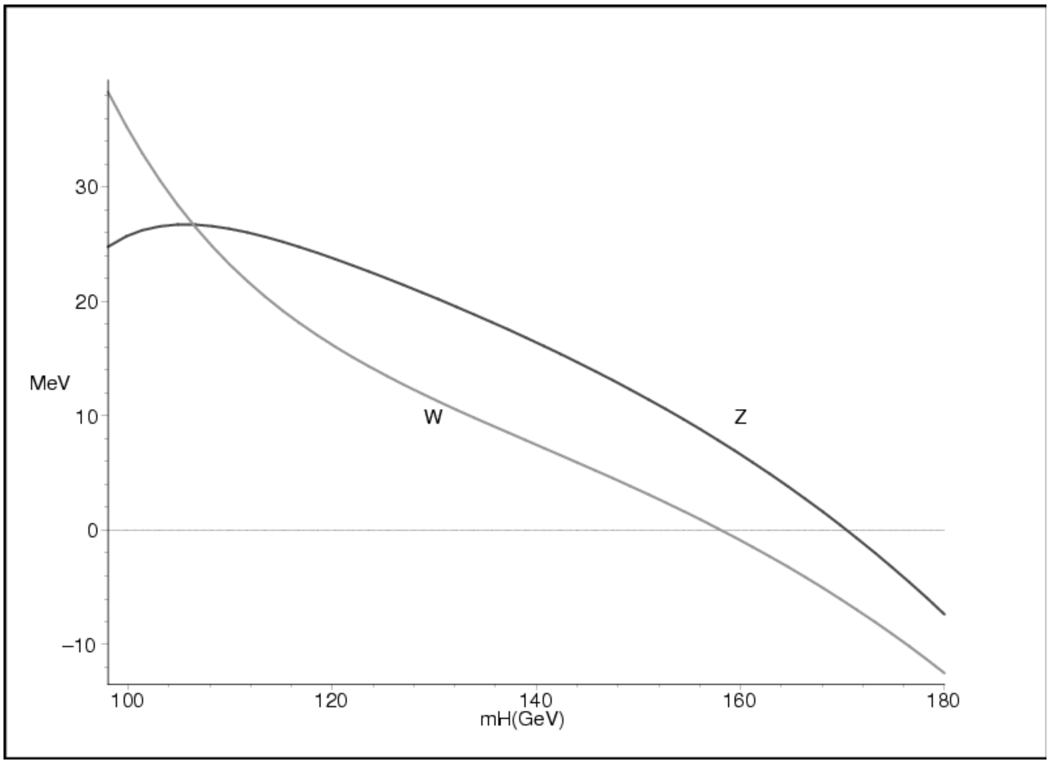


Figure 9: Two-loop corrections to the relation $m_V^2(M_V^2) - M_V^2$ at $\mu^2 = M_V^2$ for the restricted values of Higgs mass

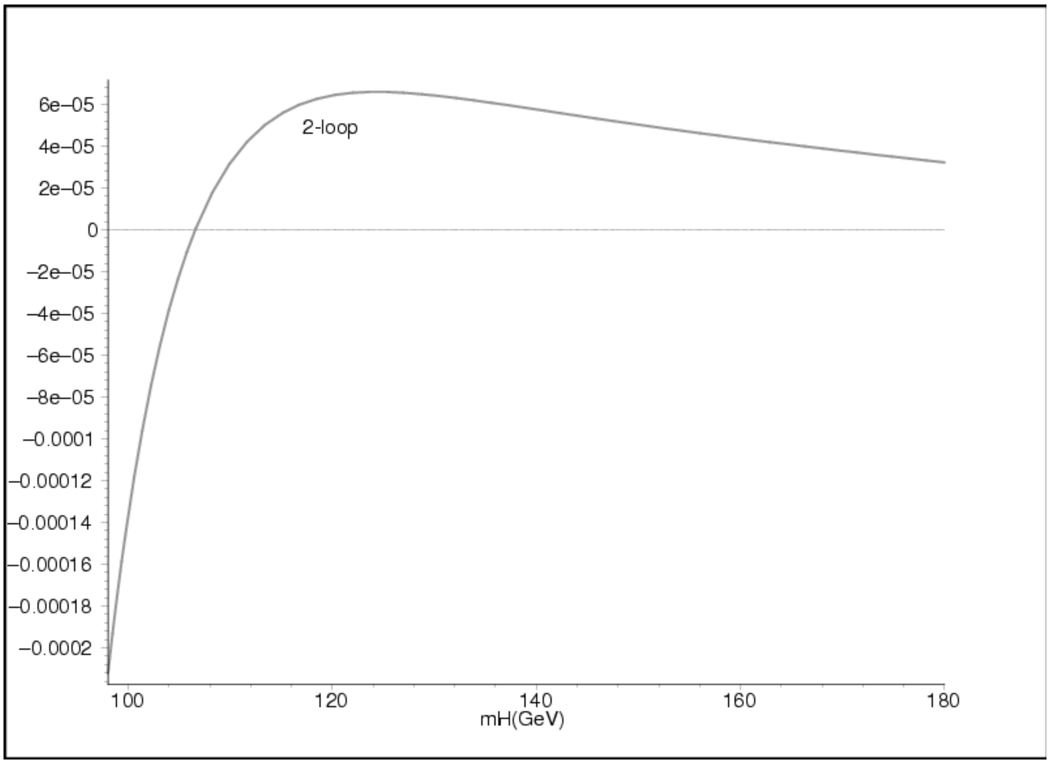


Figure 10: Two-loop corrections to the $\sin^2\theta_W$ at $\mu^2 = M_Z^2$ for the restricted values of Higgs mass