Pole masses of gauge bosons

F.Jegerlehner, M.Yu.Kalmykov, O.Veretin.

DESY Zeuthen
JINR Dubna
Karlsruhe U.

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- Pole Mass
- Expansion procedure and calculation
- $\overline{\mathrm{MS}}$ renormalization and two-loop results

Pole mass. General consideration.

The position of the pole s_P of the massive (gauge) boson propagator in a QFT is defined by equation:

$$\boxed{s_P - m^2 - \Pi(s_P, m^2, \cdots) = 0}$$

For gauge bosons Π is transversal part of the one-particle irreducible self-energy (depends on all SM parameters).

The pole s_P is

- gauge invariant
- infrared stable
- complex in general: $\mathrm{Im} s_p \neq 0$

We can define pole (on-shell) mass M and width Γ via

$$s_P = M^2 - \mathrm{i} M \Gamma$$

By iterative solution of pole formula up to 2-loops we have:

$$\boxed{\begin{aligned} s_P = & \; m^2 \\ + & \; \Pi^{(1)}(m^2, m^2) \\ + & \; \Pi^{(2)}(m^2, m^2) + \Pi^{(1)}(m^2, m^2) \Pi^{(1)\prime}(m^2, m^2). \end{aligned}}$$

$\Pi^{(L)}$ is the bare $(m = m_0)$ or $\overline{\mathrm{MS}}$- renormalized (m the $\overline{\mathrm{MS}}$-mass) L-loop contribution to Π, and the prime denotes the derivative with respect to p^2. In this way we need to evaluate propagator type diagrams and their derivatives at $p^2 = m^2$.

Pole mass and γ-Z mixing.

In neutral sector because of mixing two neutral bosons Z and γ form a 2×2 matrix propagator

$$D^{-1}(p^2) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{\gamma\gamma}(p^2) \end{pmatrix}$$

The equation for the pole is now modified

$$\boxed{s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.}$$

♣ Mixing term $\Pi^2_{\gamma Z}$ starts contributing at two-loop
♠ Photon term $\Pi_{\gamma\gamma}$ only contributes beyond the two-loop

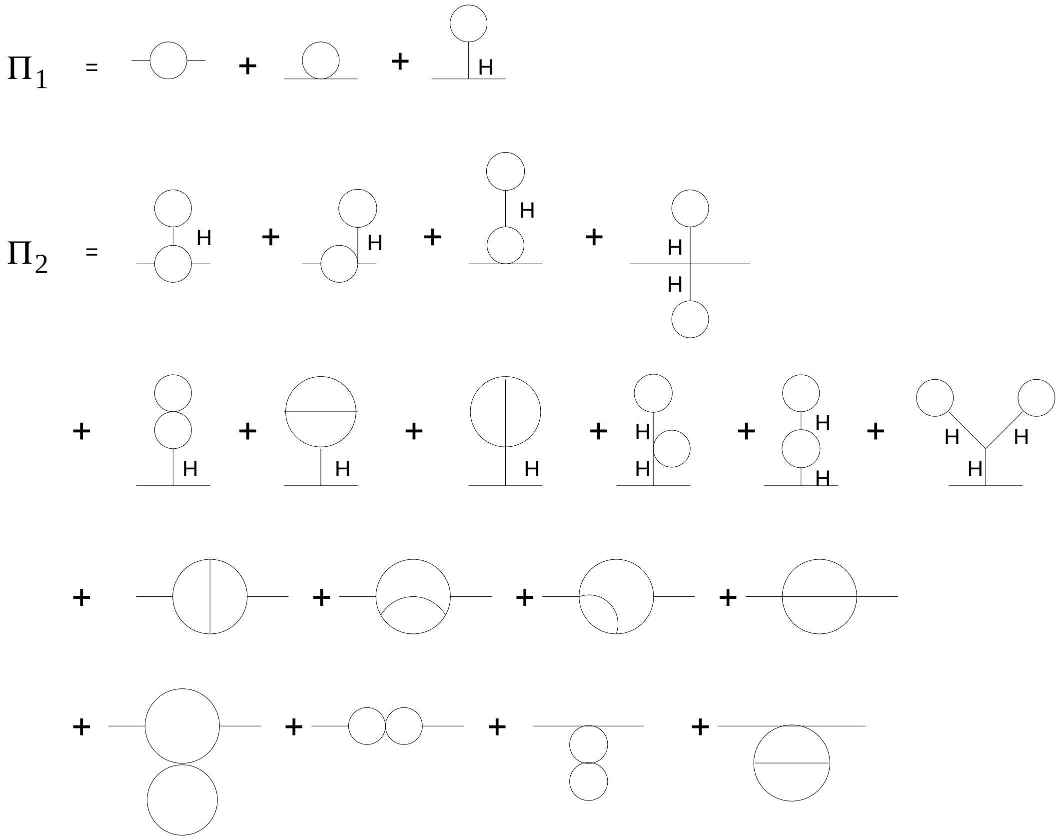
Solution (up to 2-loops):

$$\begin{aligned} s_Z &= m_Z^2 \\ &+ \Pi^{(1)}_{ZZ}(m_Z^2) \\ &+ \Pi^{(2)}_{ZZ}(m_Z^2) + \Pi^{(1)}_{ZZ}(m_Z^2)\Pi^{(1)}_{ZZ}{}'(m_Z^2) + \frac{\left[\Pi^{(1)}_{\gamma Z}(m_Z^2)\right]^2}{m_Z^2}. \end{aligned}$$

Diagrams and topologies

To be computed on-shell ($p^2 = m_Z^2$):

$$\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \ldots$$



Bosonic contribution				
Number of diagrams	linear R_ξ *gauge*		nonlinear R_ξ *gauge*	
one-loop :	~ 50			
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	616	2348	410	1837
W	792	4084	537	2942
With one fermion family				
two-loop :	$1PI$	$Total$	$1PI$	$Total$
Z	802	4410	550	3631
W	990	7780	669	5604

Gauge

The gauge fixing Lagrangian is

$$L_{g.f.} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi} \left(\partial_\mu A^\mu\right)^2 - \frac{1}{2\xi_Z} \left(\partial_\mu Z^\mu - \xi_Z M_Z h\right)^2$$

where for the linear R_ξ gauge is

$$F^\pm = \partial_\mu W^\pm_\mu \mp i \xi_W M_W \phi^\pm$$

and the nonlinear R_ξ gauge is defined as

$$F^\pm = \partial_\mu W^\pm_\mu \mp i \xi_W M_W \phi^\pm \mp ie A_\mu W^\pm_\mu \pm ig \frac{\sin^2\theta_W}{\cos\theta_W} Z_\mu W^\pm_\mu$$

K. Fujikawa '73
D.A. Dicus & C. Kao '94

The old vertices:	
$\{A_\mu, Z_\mu\} W^\pm_\mu \phi^\mp$	*absent*
$\{A_\sigma, Z_\sigma, A_\mu A_\nu, A_\mu Z_\nu, Z_\mu Z_\nu\} W^\pm_\mu W^\mp_\nu$	*modified*
$\{A_\mu, Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*modified*
$W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*modified*
The new vertices:	
$\{A_\mu A_\mu, A_\mu Z_\mu, Z_\mu Z_\mu\} \overline{\eta}^\pm \eta^\pm$	*new*
$\{A_\mu, Z_\mu\} W^\pm \overline{\eta}^\pm \{c_\gamma, \Delta_Z\}$	*new*

Evaluation of 2-loop self-energies

One can choose one of the approaches:

♣ Analytical ♢ Numerical ♠ Semianalytical

Each of them meets with the typical problems:

- Reduction of tensor integrals to set of scalar integrals
- Extraction of UV- and IR-poles
- Analytical/numerical evaluation of scalar integrals for different region of variables (numerical stability)

Let us consider some of the approaches within our task:

- J. Fleischer & O.V. Tarasov '94
  Small momentum expansion and construction of Padé approximants.
  Demand a lot of coefficients of expansion, specially for on-shell diagrams.

- A. Ghinculov & Y.-P. Yao '98; '01
  Each two-loop diagram is reduced to ten scalar integrals and their derivatives which can be evaluated numerically.
  Applicable to diagrams (their combination) which are free of mass (infrared) singularities.

- G. Passarino '01; '02
  Present talk

- V.A. Smirnov '90; '95; '99; '01;
  The asymptotic expansion with respect to small parameter(s) existing in the initial diagram.
  It is applied for special values of kinematical variables.

- Differential Equation Method
  For each diagram the linear system of partial differential equations in masses and momenta can be constructed, which can be used for analytical A. Kotikov '91 or numerical S. Laporta '00 calculation of diagram.
- Difference Equation Method
  A linear system of difference equations is constructed and solved analytically O.V. Tarasov '96; '00 or numerically S. Laporta '00 .
  Main application is the calculations of master integrals. Not yet applied to real multi-loop calculations.
- G. Weiglein, R. Scharf & M. Böhm '94
  S. Bauberger,F.A. Berends, M. Böhm, M. Buza, '95
  S. Bauberger, M. Böhm, '95
  Reduction of two-loop self-energy diagrams to scalar integrals without numerator which can be calculated numerically or analytically.
- O.V. Tarasov '97
  Exact reduction of tensor two-loop propagator type diagrams with arbitrary masses and momentum to set of scalar master integrals.
  Several masses led to the cumbersome expressions, which are difficult to simplify.
  The result is expressible in terms of master-integrals which demand a special consideration
- FORM based package ON-SHELL2
  Single scale propagator-type diagrams with arbitrary power of propagators are reduced to a set of master-integrals which calculated analytically.

Evaluation by expansion

In order to check the gauge invariance we perform all calculations in R_ξ gauge with three independent gauge parameters ξ_W, ξ_Z, ξ_γ. $\rightarrow$ Vector boson propagator reads

$$D_{\mu\nu}(p) = \frac{i}{p^2 - m_Z^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - m_Z^2}\right)$$

Then there are several scales: $m_V, \sqrt{\xi_V} m_V, m_H, m_t$

We perform expansion in 3 steps:

1. Taylor (naive) expansion in $(\xi_V - 1)$: i.e., propagator of the vector bosons look like

$$D_{\mu\nu}^V(p) \sim -g_{\mu\nu} + (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - m_V^2} - (1-\xi_V)^2 \frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2}$$

2. Expansion in small parameter

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} < 0.25.$$

   After expansion in $\sin^2\theta_W$: the diagrams without Higgs are the two-loop on-shell diagrams can be evaluated analytically by FORM package **ONSHELL2**.
   J. Fleischer & M.Yu. Kalmykov '00
   Diagrams with massless fermion loop demand special consideration.

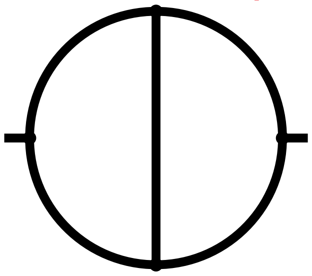
3. Large mass expansion in $m_Z^2/m_H^2 < 0.64$
   Package **TLAMM** L.V. Avdeev et al., '97

ONSHELL2

www-zeuthen.desy.de/~kalmykov/onshell2/onshell2.html

J. Fleischer, M.Yu. Kalmykov & A. Kotikov '99
A.I. Davydychev & M.Yu. Kalmykov '01
J. Fleischer & M.Yu. Kalmykov '00

All two-loop on-shell self-energy diagrams with one mass, with arbitrary power of propagators are reduced to 13 scalars integrals plus bubble-type integrals and product of the one-loop integrals.



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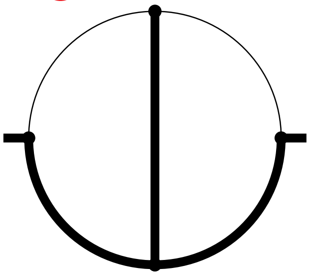

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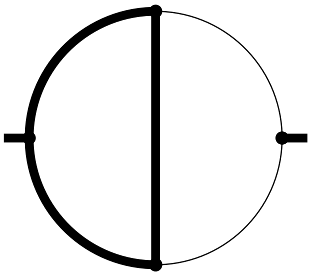

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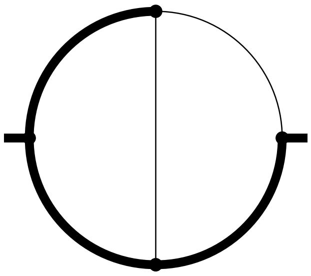

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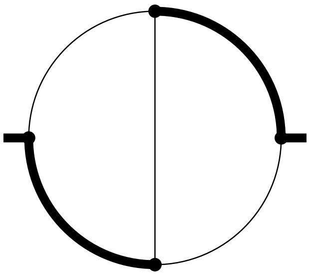

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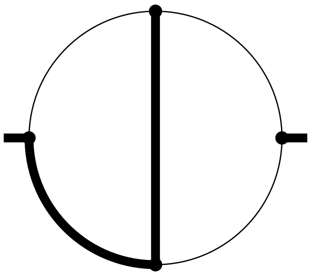

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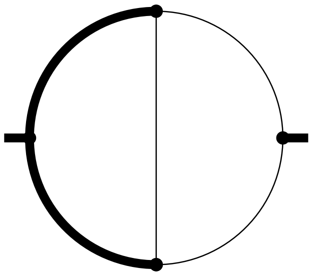

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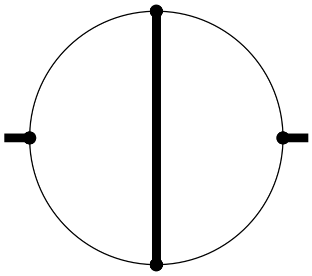

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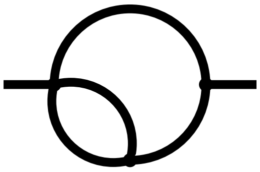

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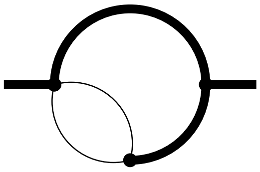
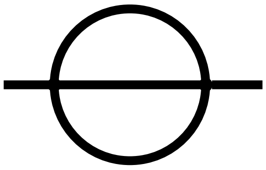
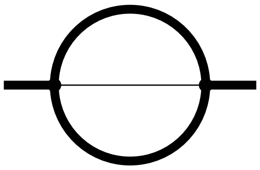


V1001



J111

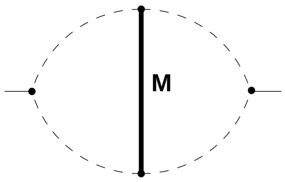


J011

Bold and thin lines correspond to the mass and massless propagators, respectively.

Massless fermion contribution

In contrast to the bosonic corrections where all diagrams can be expanded from very beginning in the small parameter $\sin^2\theta_W$ the individual diagrams with massless fermion loop develop the thresholds singularity which look like $\ln^j \sin^2\theta_W$. To control these terms it is desirable to have the exact analytical result without expansion. Using Tarasov's recurrence relations we reduce all diagrams with massless fermion loop to following set of master-integrals



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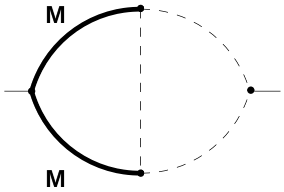

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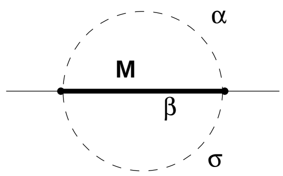
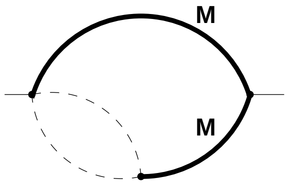
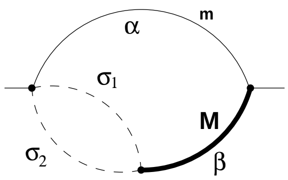


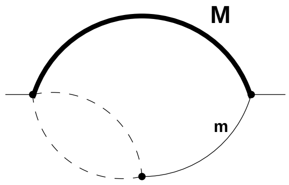
J002 (α,β,σ)



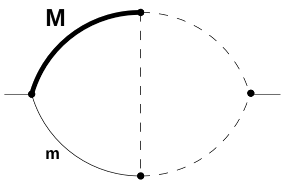
V2002



V1002 (α,σ1,σ2,β)



V2001



F20100

The analytical results for these diagrams are presented by Scharf, Tausk, '94 We performed an independent analytical calculations of these master integrals.

After summing of all contribution the singular terms, $\ln^j \sin^2\theta_W$ are canceled.

Renormalization.

We have s_P parametrized in terms of *bare* parameters

$$s_P = f(e_0, m_{Z,0}, m_{W,0}, m_{H,0}).$$

Renormalization of charges and masses:

$$\begin{aligned} e_0 &= \mu^{\varepsilon}\, e\, Z_e \\ m_0^2 &= m^2\, Z_m \end{aligned}$$

We need:
+ Z_e, Z_H, Z_t up to 1-loop
+ Z_W, Z_Z up to 2-loops

We don't need:
- field renormalization
- renormalization of ghost sector
- gauge parameters renormalization

The charge $\overline{\mathrm{MS}}$ renormalization constant can be calculated in the unbroken theory

$$Z_e = 1 + \frac{e^2}{16\pi^2 \varepsilon}\left(-\frac{7}{2} + \frac{1}{3} n_G \left[\frac{10}{9} N_c + 2\right]\right) ,$$

where e, m_Z, m_W, m_H are $\overline{\mathrm{MS}}$ parameters. N_c is color factor ($N_c = 3$) and n_G is a number of fermion families ($n_G = 3$).

Mass renormalization.

$$m_0^2 = m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right)$$

The mass renormalization constants cannot be calculated from the unbroken theory. The calculation of the Feynman diagrams in the Standard Model is required.

One-loop results:

$$Z_H^{(1,1)} = -\frac{3}{2} - \frac{3}{4}\frac{m_Z^2}{m_W^2} + \frac{3}{4}\frac{m_H^2}{m_W^2} + \sum_{\text{fermion}} \frac{1}{2}\frac{m_f^2}{m_W^2}$$

$$Z_W^{(1,1)} = -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_H^2 m_W^2} + \frac{3}{4}\frac{m_Z^2}{m_W^2} - \frac{17}{3}$$
$$+ \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right) + \frac{1}{3} n_G (N_c + 1)$$

$$Z_Z^{(1,1)} = -\frac{3}{4}\frac{m_H^2}{m_W^2} - 3\frac{m_W^2}{m_H^2} - \frac{3}{2}\frac{m_Z^4}{m_W^2 m_H^2} + \frac{11}{12}\frac{m_Z^2}{m_W^2}$$
$$-7\frac{m_W^2}{m_Z^2} + \frac{7}{6} + \sum_{\text{fermion}} \left(2\frac{m_f^4}{m_W^2 m_H^2} - \frac{1}{2}\frac{m_f^2}{m_W^2} \right)$$
$$+\frac{1}{3} n_G \left[\frac{m_Z^2}{m_W^2} \left(\frac{11}{9} N_c + 3 \right) + \frac{m_W^2}{m_Z^2} \left(\frac{20}{9} N_c + 4 \right) \right]$$
$$-\frac{1}{3} n_G \left[\frac{22}{9} N_c + 6 \right]$$

2-loop constants in $\overline{\mathrm{MS}}$. Exact results:

$$
\begin{aligned}
Z_W^{(2,1)} = \; & \frac{63}{64}\frac{m_H^4}{m_W^4} - \frac{3}{4}\frac{m_H^2}{m_W^2} - \frac{3}{8}\frac{m_H^2 m_Z^2}{m_W^4} \\
& - \left[\frac{301}{192} + \left(\frac{55}{432}N_c + \frac{5}{16}\right) n_G\right]\frac{m_Z^4}{m_W^4} \\
& + \left[\frac{17}{12} + \left(\frac{29}{108}N_c + \frac{3}{4}\right) n_G\right]\frac{m_Z^2}{m_W^2} \\
& + \left[\frac{31}{12} - \left(\frac{44}{27}N_c + 4\right) n_G\right]\frac{m_Z^4}{m_H^2 m_W^2} \\
& - \left[\frac{17}{2} - \left(\frac{40}{27}N_c + \frac{8}{3}\right) n_G\right]\frac{m_Z^2}{m_H^2} \\
& - \left[\frac{176}{3} - \frac{4}{3}(N_c + 1) n_G\right]\frac{m_W^2}{m_H^2} \\
& + \left[\frac{59}{24} + \left(\frac{22}{27}N_c + 2\right) n_G\right]\frac{m_Z^6}{m_H^2 m_W^4} \\
& - \frac{53}{3} + \left(\frac{343}{216}N_c + \frac{31}{24}\right) n_G
\end{aligned}
$$

- $Z_W^{(2,2)}, Z_Z^{(2,1)}, Z_Z^{(2,2)}$ have similar structure.
- Massive top corrections m_t generate terms proportional to powers of m_t.
- The terms proportional to m_H^4, m_H^2 are generated by pure bosonic contributions.

Renormalization group analysis

We can check two-loop coefficients $Z^{(2,2)}$ and $Z^{(2,1)}$ in mass renormalization formula

$$
\begin{aligned}
m_0^2 &= m^2 \left(1 + \frac{g^2}{16\pi^2} \frac{Z^{(1,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,1)}}{\varepsilon} + \frac{g^4}{(16\pi^2)^2} \frac{Z^{(2,2)}}{\varepsilon^2} \right) \\
&= m^2 \left(1 + \frac{z^{(1)}}{\varepsilon} + \frac{z^{(2)}}{\varepsilon^2} + \dots \right)
\end{aligned}
$$

using renormalization group.

Introduce anomalous mass dimension

$$
\gamma = \mu^2 \frac{d}{d\mu^2} \log m^2 .
$$

From

$$
0 = \mu^2 \frac{d}{d\mu^2} \log m_0^2
$$

we have

$$
\begin{aligned}
\gamma &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(1)}, \\
\left(\gamma + \sum_i \beta_{g_i} \frac{\partial}{\partial g_i} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2} \right) z^{(n)} &= \frac{1}{2} g \frac{\partial}{\partial g} z^{(n+1)},
\end{aligned}
$$

where

$$
z^{(n)} = \sum_{k=1} \left(\frac{g^2}{16\pi^2} \right)^k Z^{(n,k)} .
$$

Pole structure

Check of $1/\varepsilon^2$ pole:

From previous formula we can derive

$$2Z^{(2,2)} = \left(Z^{(1,1)}\right)^2 + 2\frac{16\pi^2}{g^3}\beta_g^{(1)} Z^{(1,1)} + \sum_i Z_{m_i}^{(1,1)} m_i^2 \frac{\partial}{\partial m_i^2} Z^{(1,1)}$$

Check $1/\varepsilon$ pole:

Use relation between charges

$$e^2 = g^2\left(1 - \frac{m_W^2}{m_Z^2}\right) \quad \text{and} \quad \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}.$$

Differentiating the former formulae w.r.t. $\log\mu^2$ we derive

$$\left(\frac{g^2}{16\pi^2}\right)^2 \left(Z_Z^{(2,1)} - Z_W^{(2,1)}\right) c^2 = g^2 \frac{\beta_{g'}^{(2)}}{g'^3} s^4 - \frac{\beta_g^{(2)}}{g} s^2 c^2.$$

β-functions for g and g' are calculated from symmetric phase theory

D.R.T. Jones '82
M.E. Machacek & M.T. Vaughn '83

$$\begin{aligned}
\beta_{g'}\,|_{N_c=3} &= \left(\frac{1}{12} + \frac{10}{9} n_G\right) \frac{g'^3}{16\pi^2} \\
&+ \left(\frac{1}{4} + \frac{95}{54} n_G\right) \frac{g'^5}{(16\pi^2)^2} + \left(\frac{3}{4} + \frac{1}{2} n_G\right) \frac{g'^3 g^2}{(16\pi^2)^2} \\
\beta_g\,|_{N_c=3} &= \left(-\frac{43}{12} + \frac{2}{3} n_G\right) \frac{g^3}{16\pi^2} \\
&+ \left(-\frac{259}{12} + \frac{49}{6} n_G\right) \frac{g^5}{(16\pi^2)^2} + \left(\frac{1}{4} + \frac{n_G}{6}\right) \frac{g^3 g'^2}{(16\pi^2)^2}
\end{aligned}$$

$\overline{\mathrm{MS}}$Renormalization

$\overline{\mathrm{MS}}$renormalized relation can be written as

$$s_Z = m_Z^2 + \left\{ \Pi_{0,V}^{(1)} \right\}_{MS} + \left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS}$$

where

$$\left\{ \Pi_{0,V}^{(1)} \right\}_{MS} = m_V^2(\mu) \frac{e^2}{16\pi^2 \sin^2\theta_W} \lim_{\varepsilon \to 0} \left(\frac{1}{\varepsilon} Z_V^{(1,1)} + X_{0,V}^{(1)} \right)$$

where we have introduced new function

$$\Pi_{0,V}^{(1)} = m_{0,V}^2 \frac{g_0^2}{16\pi^2} X_{0,V}^{(1)}$$

$$\begin{aligned}
\left\{ \Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \right\}_{MS} &= \lim_{\varepsilon \to 0} \Bigg(\Pi_{0,V}^{(2)} + \Pi_{0,V}^{(1)} \Pi_{0,V}^{(1)}{}' \\
&+ m_V^2(\mu) \frac{1}{\varepsilon} \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[Z_V^{(1,1)} + \left[\frac{\Delta g^2}{g^2} \right] + \sum_j Z_{m_j^2}^{(1,1)} \frac{\partial}{\partial m_j^2} \right] X \\
&+ m_V^2(\mu) \left(\frac{e^2}{16\pi^2 \sin^2\theta_W} \right)^2 \left[\frac{1}{\varepsilon} Z_V^{(2,1)} + \frac{1}{\varepsilon^2} Z_V^{(2,2)} \right] \Bigg)
\end{aligned}$$

where the sum runs over all species of particles $j = Z,\ W,\ H,\ t$ and

$$\begin{aligned}
\left[\frac{\Delta g^2}{g^2} \right] &= \frac{\cos^2\theta_W}{\sin^2\theta_W} \left(Z_W^{(1,1)} - Z_Z^{(1,1)} \right) \\
&+ \sin^2\theta_W \left(-7 + \frac{2}{3} n_G \left[\frac{10}{9} N_c + 2 \right] \right)
\end{aligned}$$

Results

$$\frac{M_V^2}{m_V^2} = 1 + \frac{e^2}{16\pi s^2} X_{1,V} + \left(\frac{e^2}{16\pi s^2}\right)^2 X_{2,V},$$

$$X_{2,V} = \frac{m_H^4}{m_V^4} \sum_{i=0}^{5} A_i s^{2i},$$

$$A_i = \sum_{j=0}^{5} A_j^V \left(\frac{m_V^2}{m_H^2}\right)^j, \qquad V = Z, W$$

All parameters in $\overline{\mathrm{MS}}$scheme.
Six coefficients calculated analytically.
Expansion in powers and $\log$'s (i.e. is asymptotic expansion not naive Taylor expansion).
Expansion coefficients $A_{i,j}^V$ given by a small set of transcendental constants, like

$$S_0 = \frac{\pi}{\sqrt{3}}, \qquad S_1 = \frac{\pi}{\sqrt{3}} \ln 3,$$

$$S_2 = \frac{4}{9\sqrt{3}} \mathrm{Cl}_2(\pi/3), \qquad S_3 = \pi \mathrm{Cl}_2(\pi/3),$$

After summing all diagrams with massless fermion loop all singularites of type $\ln^j \sin^2\theta_W$ canseled $\longrightarrow$ infrared finiteness of the massless fermion contribution to the pole mass.
After UV renormalization the pole mass is a finite expression. $\longrightarrow$ infrared finiteness of the bosonic contribution to the pole mass.

$$\boxed{\text{Pole mass of gauge boson is infrared finite quantity}}$$

To restore gauge invariance of pole s_p the tadpole shall be taking into account.
RG invariance also requires the tadpole - $\alpha\alpha_s$ contribution

$\overline{\mathrm{MS}}$ mass in terms of on-shell mass

Inverse of our "master formula" express all $\overline{\mathrm{MS}}$ parameters in terms of on-shell ones:

$$
\begin{aligned}
m_V^2 \quad = \quad & M_V^2 - \hat{\Pi}_V^{(1)} - \left\{ \Pi_V^{(2)} + \Pi_V^{(1)} \Pi_V^{(1)\prime} \right\}_{MS} \\
& - \sum_j (\Delta m_j^2)^{(1)} \frac{\partial}{\partial m_j^2} \hat{\Pi}_V^{(1)} - (\Delta e)^{(1)} \frac{\partial}{\partial e} \hat{\Pi}_V^{(1)} \Bigg|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}} ,
\end{aligned}
$$

where the sum runs over all species of particles $j = Z,\ W,\ H$ and

$$
(\Delta m_j^2)^{(1)} = -\mathrm{Re} \hat{\Pi}_V^{(1)} \Bigg|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}}
$$

stands for the self-energy of the jth particle at $p^2 = m_j^2$ in the $\overline{\mathrm{MS}}$scheme and parameters replaced by the on-shell ones. Note that in the above relation we had to perform a change from the $\overline{\mathrm{MS}}$ to the on-shell scheme also for the electric charge.

Unphysical terms, propartional to m_H^4 drop out in $\sin^2 \theta_W$

$$
\begin{aligned}
\sin^2 \theta_W \quad = \quad & 1 - \frac{m_W^2}{m_Z^2} \left(1 - \frac{M_W^2}{M_Z^2} \right) \times \\
& \left[1 - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left((\delta_W^{(1)} - \delta_Z^{(1)})(1 - \delta_Z^{(1)}) + \delta_W^{(2)} - \delta_Z^{(2)} \right) \right]
\end{aligned}
$$

where $m_V^2 / M_V^2 = 1 + \delta_V^{(1)} + \delta_V^{(2)}$