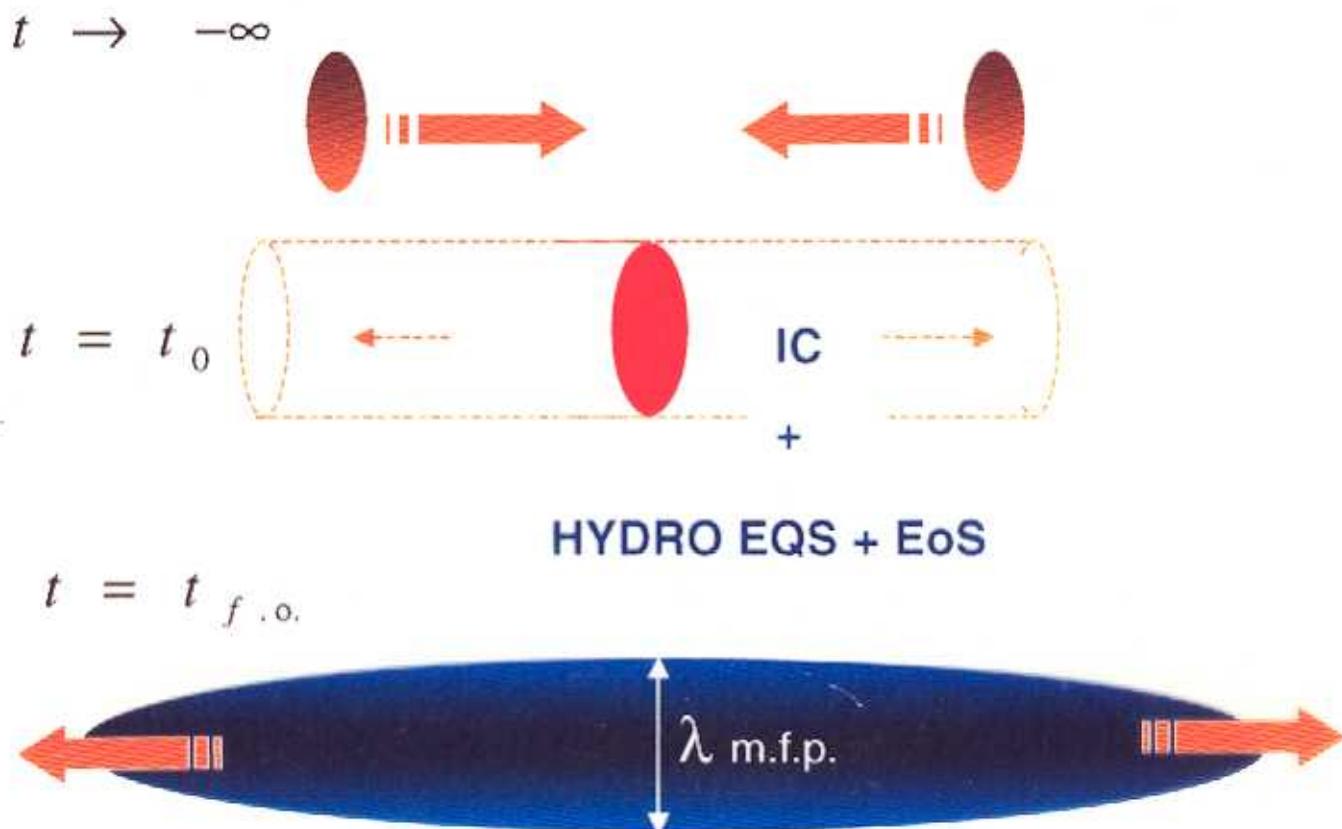


On a problem of spectra formation in hydro-kinetic approach to $A + A$ collisions

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Hydrodynamic approach to multiparticle production:

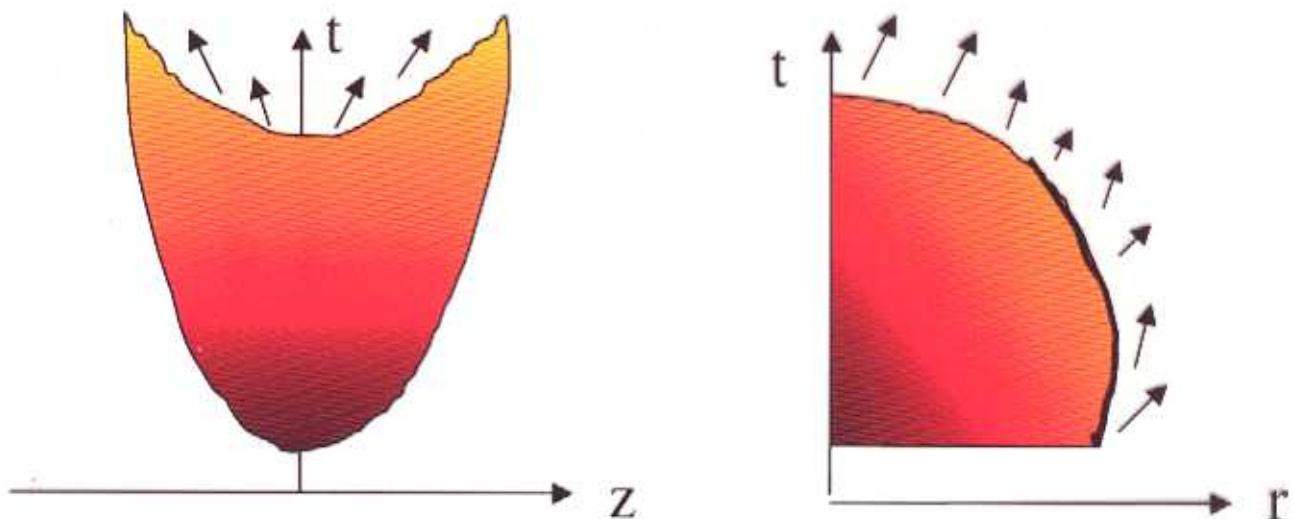
L.D. Landau, 1953



Studying of (one- and multi- particle) spectra versus IC and
can get, in principle information about earlier partonic stage of
possible formation of QGP or even type of the phase transition

**But: whether do the predicted spectra with given IC and
EoS in Hydro are unambiguous?**

Cooper-Frye prescription (CFp)



$$\sigma_{f.o.} : \tau \equiv \sqrt{t^2 - z^2} = const$$

at $r = const$

$$\sigma_{f.o.} : \tau(r) \text{ at } z = 0$$

CFp gets serious problems:

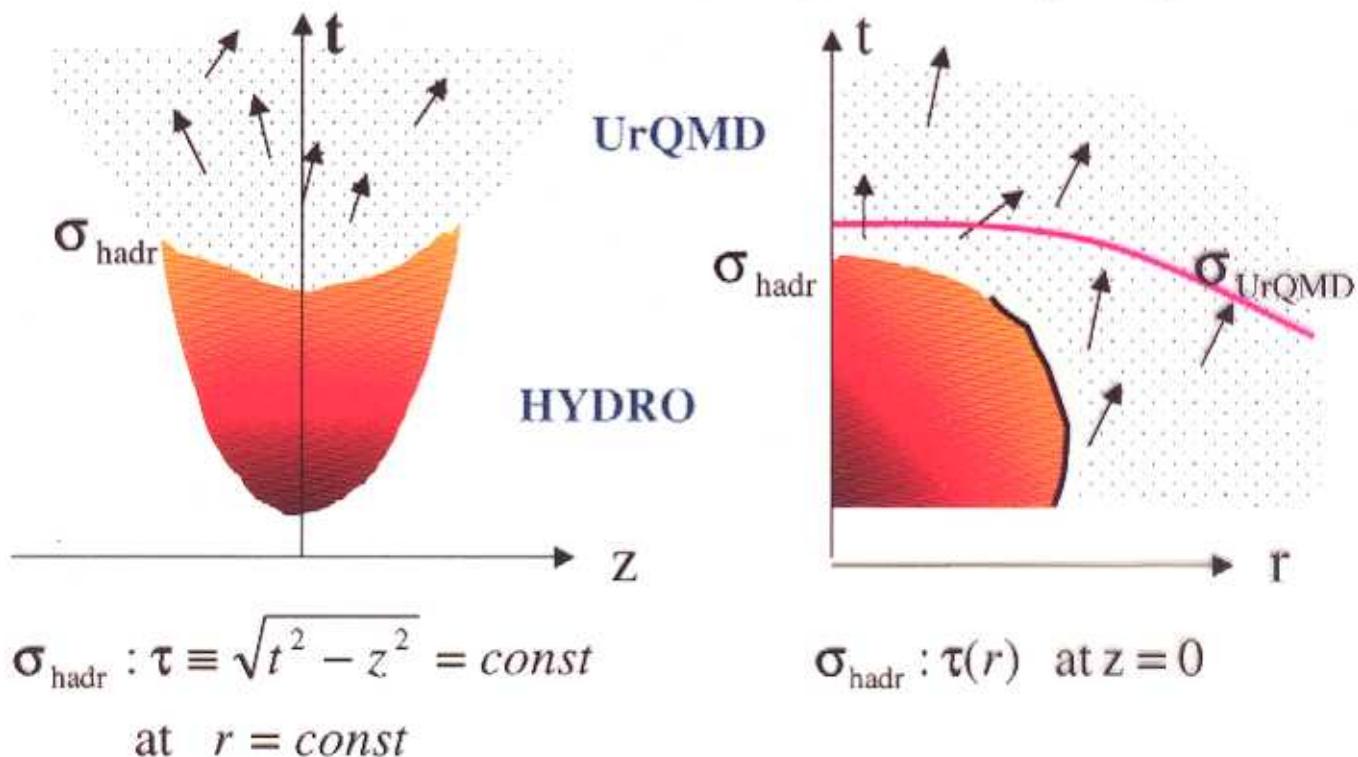
- Freeze-out hypersurface $\tau(r)$ contains non-space-like sectors →
- artificial discontinuities appears across $\sigma_{f.o.}$
Sinyukov (1989), Bugaev (1996), Andrelak et al (1999);
- cascade models show that particles escape from the system about whole time of its evolution.



The method of continuous emission (Grassi, Hama, Kodama (1995), Magas et al (1999)) fails as the particle escape probability becomes large.

Hybrid models: HYDRO + UrQMD

Bass, Dumitru (2000)



The problems:

- the system just after hadronization is not so dilute to apply hadronic cascade models;
- hadronization hypersurface $\tau(r)$ contains non-space-like sectors;
- hadronization happens in fairly wide 4D-region, not just at hypersurface σ_{hadr} , especially in crossover scenario.



The initial conditions for hadronic cascade models should be based on non-local equilibrium distributions

The most important focal-points of hydro-kinetic approach, based on the method of escape probability

■ Boltzmann Equation: $\frac{p^\mu}{p^0} \frac{\partial f(x, p)}{\partial x^\mu} = F^{gain}(x, p) - F^{loss}(x, p)$

$$F^{loss}(x, p) = R(x, p)f(x, p) \quad \text{where } R(x, p) = \langle \sigma v_{rel} \rangle n(x)$$

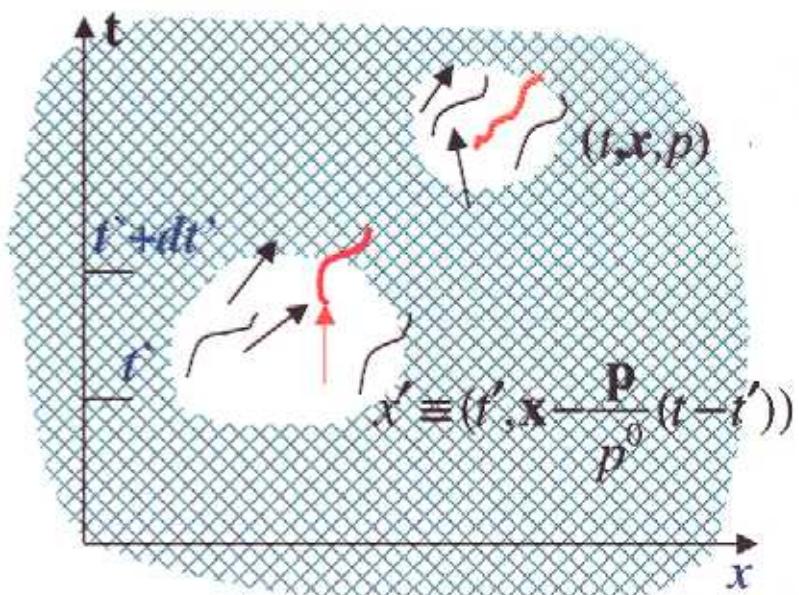
$$F^{gain}(x, p) = \mathfrak{R}(d\sigma, f(x, p))$$

↑
rate of collisions

■ Escape function: $f(x, p) = f_{int}(x, p) + f_{esc}(x, p)$

The *additional*
portion of escaped
particles is

$$P(x', p) F^{gain}(x', p) dt'$$



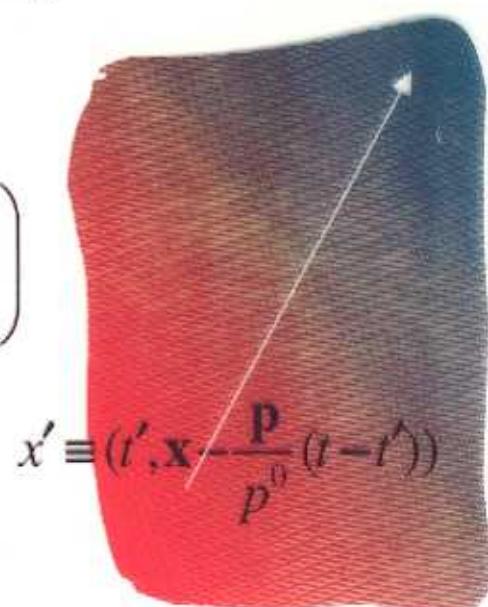
Escape function:

$$f_{\text{esc}}(x, p) = f_{\text{esc}}(x_0, p) + \int_{t_0}^t dt' P(x', p) F^{\text{gain}}(x', p)$$

where $f_{\text{esc}}(x_0, p)$ corresponds to the portion of the particles, which are already free at t_0

Escape probability:

$$P(x, p) = \exp \left(- \int_t^\infty dt' R(x', p) \right)$$



$$x' \equiv (t', \mathbf{x}' - \frac{\mathbf{p}}{p^0}(t - t'))$$

Probability definition:

$$P(x, p) = \frac{f_{\text{esc}}(x, p)}{f(x, p)} \quad \longleftrightarrow \quad f(x, p) = \frac{f_{\text{esc}}(x, p)}{P(x, p)}$$



Boltzmann equation

(integral form)

The essence of the method of escape probability

$$f_{\text{esc}}(x, p) = P(x, p) f(x, p)$$

If system is initially finite, it becomes free at large enough times t_{out}

$$P(x, p) \rightarrow 1, \quad f_{\text{esc}}(x, p) \rightarrow f(x, p)$$



The main point is to utilize, for the description of particle spectra in A+A collisions, the escape function with $P(x, p)$ and $F^{\text{gain}}(x, p)$ evaluated just in local equilibrium (l.eq.) approximation for $f(x, p)$

It corresponds to kinetic equation in relaxation time approximation:

$$\frac{p^\mu}{p^0} \frac{\partial f(x, p)}{\partial x^\mu} = -\frac{f(x, p) - f_{\text{l.eq.}}(x, p)}{\tau(x, p)}, \quad \text{where } \tau = 1/R_{\text{l.eq.}}$$

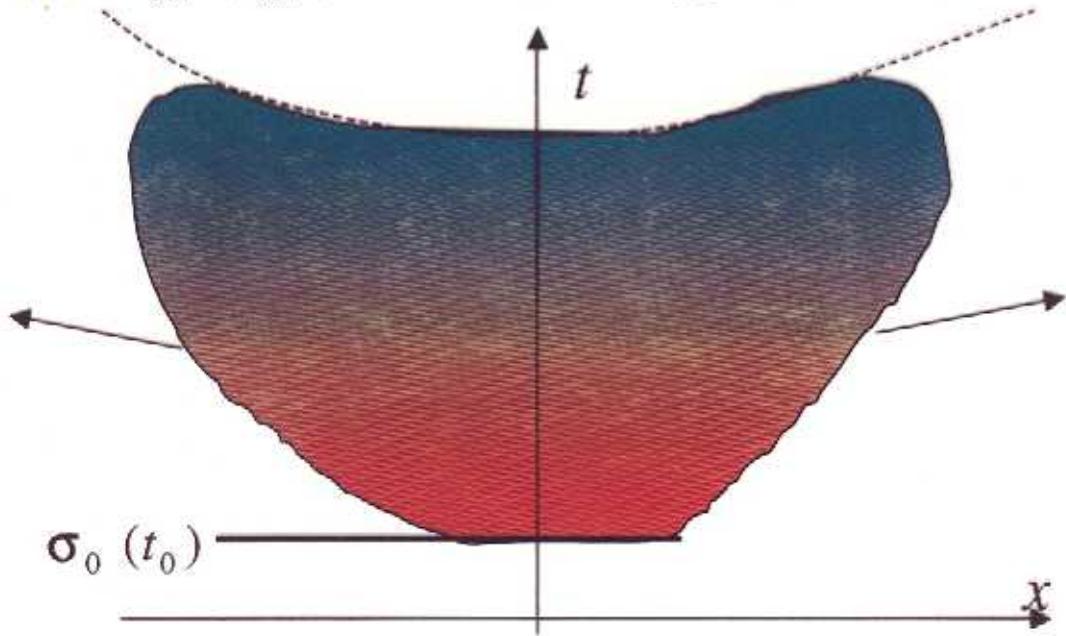
Particle spectra and correlations

$$p^0 \frac{dN}{d\mathbf{p}} = \langle a_p^+ a_p \rangle, \quad p_1^0 p_2^0 \frac{dN}{d\mathbf{p}_1 d\mathbf{p}_2} = \langle a_{p_1}^+ a_{p_2}^+ a_{p_1} a_{p_2} \rangle$$

Irreducible operator averages:

$$\langle a_{p_1}^+ a_{p_2} \rangle = \int_{\sigma_{out}} d\sigma_\mu p^\mu \exp(iqx) f(x, p); \quad p = (p_1 + p_2)/2, q = p_1 - p_2$$

At $\sigma_{out}(t_{out})$: $f(x, p) \rightarrow f_{esc}(x, p)$



$$\frac{p^\mu}{p^0} \frac{\partial f_{esc}(x, p)}{\partial x^\mu} = P(x, p) F^{gain}(x, p)$$

$$\partial_\mu [p^\mu \exp(iqx)] = 0$$

$$\frac{p^\mu}{p^0} \frac{\partial f(x, p)}{\partial x^\mu} = F^{gain}(x, p) - F^{loss}(x, p)$$

Formation of spectra and correlations

$$\langle a_{p_1}^+ a_{p_2} \rangle = p^\mu \int_{\sigma_0} d\sigma_\mu f_{\text{esc}}(x, p) e^{iqx} + p^0 \int_{\sigma_0} d^4x P(x, p) F^{\text{gain}}(x, p) e^{iqx}$$

Emission function $P F^{\text{gain}}$

Source function = +

Direct emission $f_{\text{esc}}(x_0, p)$

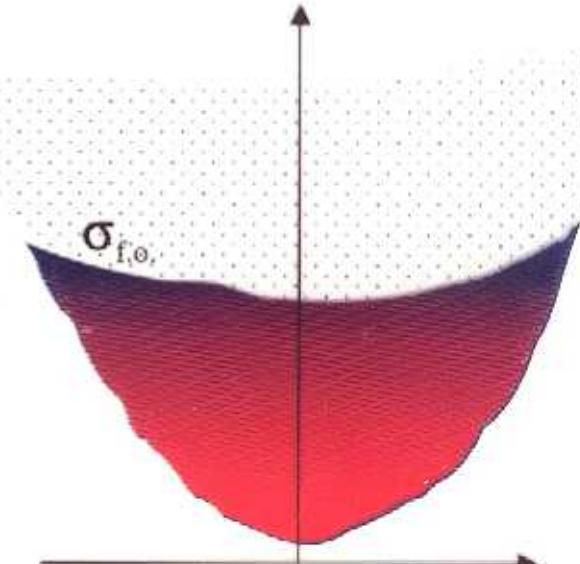
Cooper-Frye prescription

$\sigma_{\text{tot}} \rightarrow \infty$ at $t < t_{\sigma_{f.o.}}$

$\sigma_{\text{tot}} \rightarrow 0$ at $t > t_{\sigma_{f.o.}}$

$$P(t, x) = \Theta(t - t_{\sigma_{f.o.}}(x))$$

$$S = P F^{\text{gain}} = \delta(t - t_{\sigma_{f.o.}}(x))$$



The role of dissipative effects in spectra formation

$$\langle a_{p_1}^+ a_{p_2} \rangle = p^\mu \int_{\sigma_0} d\sigma_\mu f(x, p) e^{iqx} + p^0 \int_{\sigma_0} d^4 x (F^{gain}(x, p) - F^{loss}) e^{iqx}$$



Spectra and correlations at initial state

Dissipative term,
 $D(x, p)$, (r.h.s. of Boltzmann equation)

■ The difference between observed spectra (at σ_{out})

and initial one (at σ_0) is due to dissipative effects – deviation from local equilibrium (l.eq.).

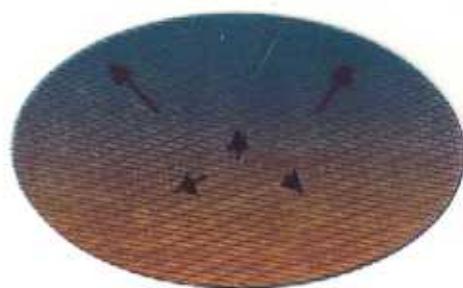
■ The effects exists even for high densities and/or cross-sections, when the conditions of applicability of ideal HYDRO is fairly good, $f(x, p) \approx f_{l.eq.}(x, p)$. In this case, according to Chapman-Enskog method,

$$D(x, p) = p^\mu \partial_\mu f(x, p) \approx p^\mu \partial_\mu f_{l.eq.}(x, p) \propto \kappa f_{l.eq.}(x, p)$$

Simple analytical models

Akkelin, Csorgo, Lukacs, Sinyukov (2001)

Ideal HYDRO solutions with initial conditions at $t = t_0 = 0$:
the n.-r. ideal gas has ellipsoidally symmetric Gaussian density distribution and a self-similar velocity profile $u(x)$.



$$f(t, \mathbf{x}, \mathbf{v}) = \frac{N}{V} \left(\frac{m}{(2\pi)^2 T} \right)^{\frac{3}{2}} \exp \left(-\frac{m(\mathbf{v} - \mathbf{u}(x))^2}{2T} - \sum_i^3 \frac{x_i^2}{2X_i^2} \right)$$

where

$$\mathbf{v} = \mathbf{p}/m, \quad V = X_1 X_2 X_3, \quad X_i X_i = \frac{T}{m}, \quad T = T_0 \left(\frac{V_0}{V} \right)^{\frac{2}{3}}, \quad u_i = \frac{\dot{X}_i}{X_i} x_i$$

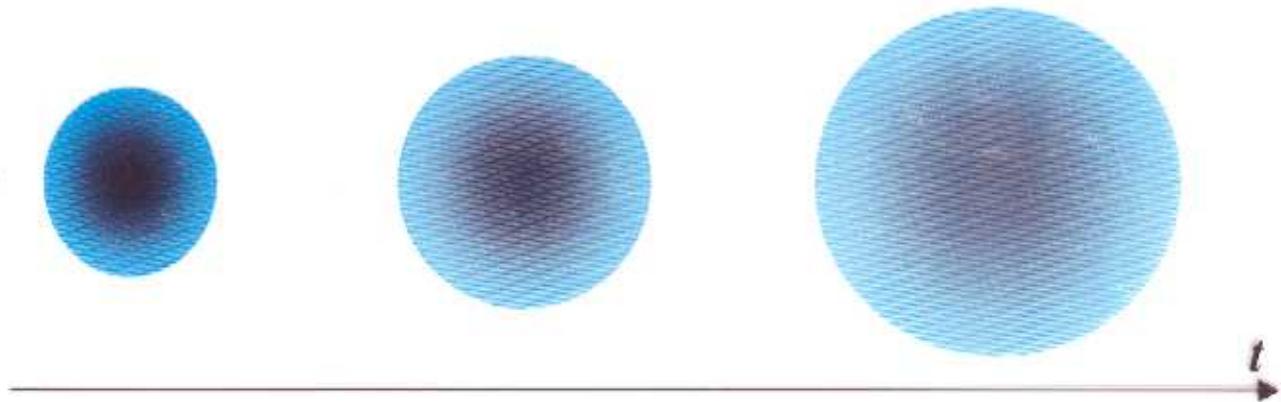
Spherically symmetric solution:

$$X_1 = X_2 = X_3 = R$$

Csizmadia, Csorgo, Lukacs (1998)

Solution of Boltzmann equation for *locally equilibrium* expanding fireball

$$f(t, \mathbf{x}, \mathbf{v}) = \frac{N}{(2\pi R_0)^3} \left(\frac{m}{T_0}\right)^{\frac{3}{2}} \exp\left(-\frac{m\mathbf{v}^2}{2T_0} - \frac{(\mathbf{x} - \mathbf{vt})^2}{2R_0^2}\right)$$



The spectra and interferometry radii do not change:

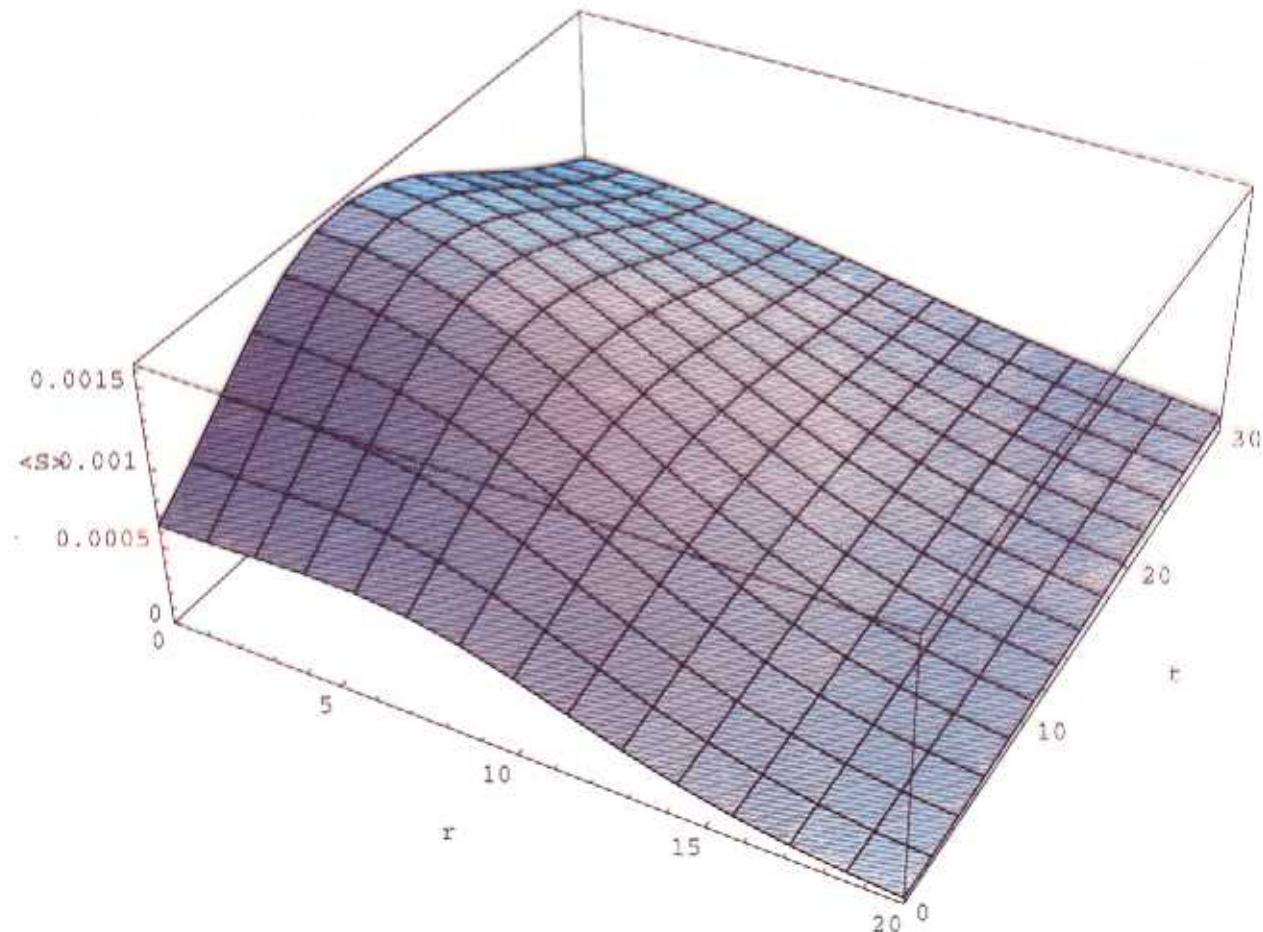
- One particle velocity (momentum) spectrum

$$f(t, \mathbf{v}) = N \left(\frac{m}{2\pi T_0}\right)^{\frac{3}{2}} \exp\left(-\frac{m\mathbf{v}^2}{2T_0}\right) = \underline{f(t=0, \mathbf{v})}$$

- Two particle correlation function

$$C(t, q) = 1 + \frac{\langle a_{p_1}^+ a_{p_2} \rangle^2}{\langle a_{p_1}^+ a_{p_1} \rangle \langle a_{p_2}^+ a_{p_2} \rangle} = 1 + \exp(-q^2 R_0^2) = \underline{C(t=0, q)}$$

Emission function for expanding fireball

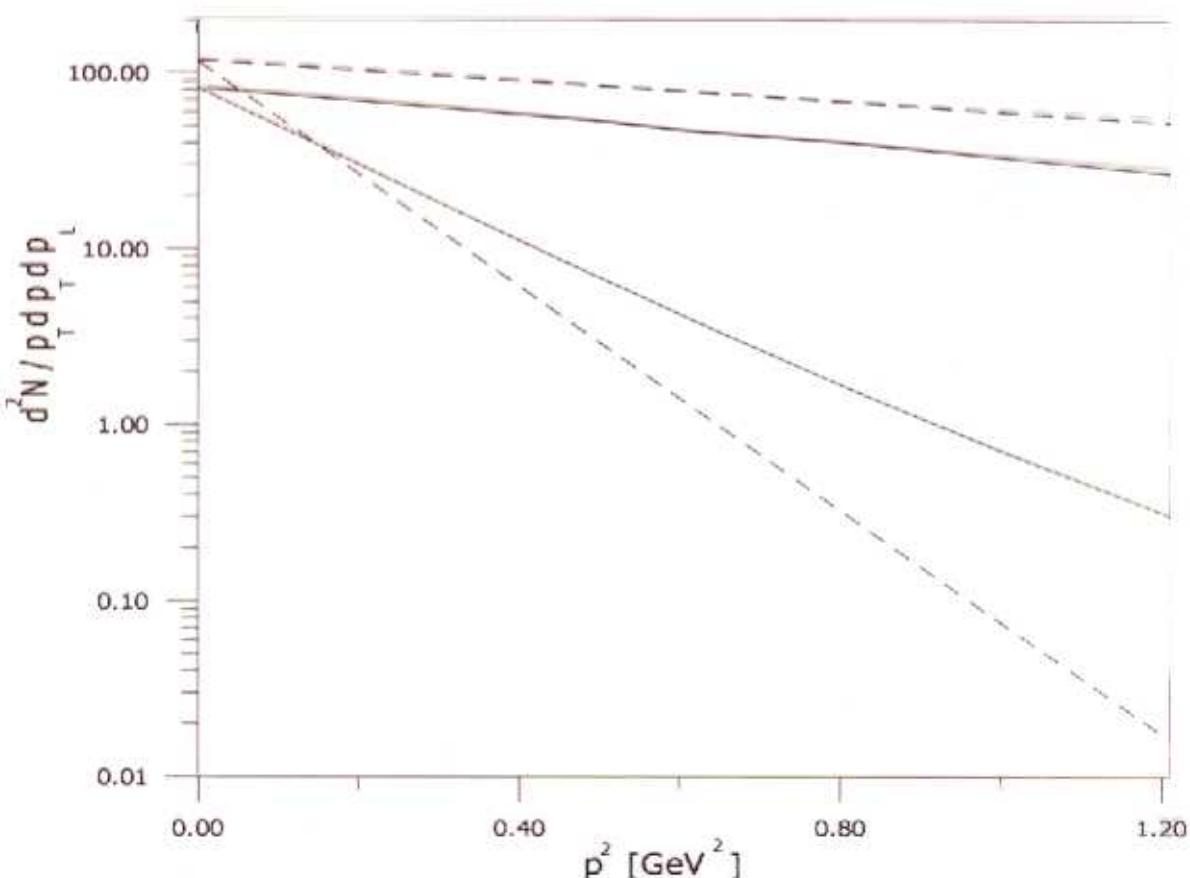


The space-time (t, r) dependence of the emission function $S(x, p)$, averaged over momenta, for an expanding spherically symmetric fireball containing 400 particles with mass $m=1$ GeV and with cross section $\sigma = 40$ mb, initially at rest and localized with Gaussian radius parameter $R = 7$ fm and temperature $T = 0.130$ GeV.

What does exact solution of BE demonstrate?

- The difference between observed (at σ_{out}) spectra and correlations and the initial ones (at σ_0) is defined by dissipative effects – deviations from local equilibrium.
- The smoothness in time of the emission function does not mean automatically an inapplicability of CFp: the latter can formally give true results at some $\sigma_{f.o.}$, if the dissipative effects beyond of the $\sigma_{f.o.}$ are fairly small, despite collisions still continue.
- Symmetry properties of the dynamics give influence to a value of the dissipation effects $D(x, p) \propto \kappa f_{l.eq}(x, p)$.
For spherical symmetry of the field of hydro-velocity – in our example - $\kappa=0$.
Typically, however, $\kappa \neq 0$ and hadron system is not at local equilibrium.

Application of the escape probability method



The transverse (bottom, at $p_L = 0$) and longitudinal (top, at $p_T = 0$) spectra of particles, escaped until $t = 8 \text{ fm/c}$ from the expanding ellipsoidally symmetric fireball of the particles with mass $m = 1 \text{ GeV}$ and with cross section $\sigma = 40 \text{ mb}$, initially at rest and localized with Gaussian radius parameters $X_1 = X_2 = 7 \text{ fm}$, $X_3 = 0.7 \text{ fm}$ and temperature $T_0 = 0.3 \text{ GeV}$. Dashed line correspond to spectra calculated according to CFp applied to I.eq. distribution function at $T(t) = 0.063 \text{ GeV}$

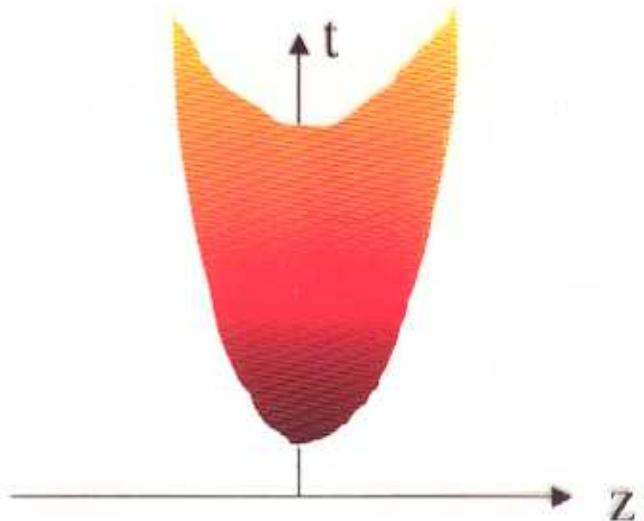
Conclusion

- The proposed method allows to describe, in hydrodynamic approach for A+A collisions, the evolution of the matter from I.eq. to free streaming.
- The method is differ from the continuous emission where the central object is the interacting component which, as we found, is neither isotropic nor thermal in its local rest frame.
- The method, being applied to the evolution of rarefied hadron gas, overlaps with transport models like RQMD. In this aspect, exact solution of BE can be used as a test of numerical cascade algorithms.
- The advantage of the method reveals when the system is not dilute, has more complicated interactions and display a collective behavior.

HYDRO + ESCAPE FUNCTION

OUTLOOK

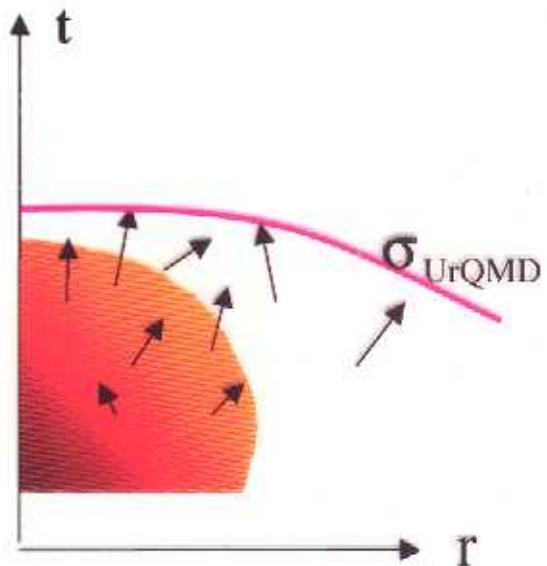
I. HYDRO of QGP + HG
with corresponding EoS.



II. Chemistry QGP – HG
composition from hydro-solution
 $T(t,x)$, $\mu(t,x)$

III. Kinetic description of
hadronization + escaping from QGP
+ HG hydro-system. It is based on
relaxation time $\tau(x)$ appr.

IV. The resulting hadronic distr.
function is non-l.eq. The initial
conditions for hadronic cascade
models should be based on non-local
equilibrium distributions.



V. Account for back reaction of non-l.eq. hadron emission
on hydro-evolution. The hadron anisotropy (due to
opacity) will result to faster transverse expansion as for
standard HYDRO (I).

(IV-V) \longrightarrow reduction of R_i and ratio R_{out} / R_{side}