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Topical section: S. VII
Low-x & Very High Multiplicity Physics

Study of charged multiplicity of hadronic three-jet events at LEP

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The DELPHI experiment

$$e^+e^- \rightarrow Z \rightarrow q\,\bar{q}\,g$$

The data have been collected in the years 1992 – 1995

<u>CONTENTS</u>

DELPHI (1992–95)

$$e^+e^- \to Z \to q\,\bar{q}\,g$$

Particle and Event Selection

Track selection for charged particles:

∘ Momentum	$p \geq 0.4$ GeV/c
∘ Polar angle	$20^\circ \leq \theta \leq 160^\circ$
∘ Impact parameter	$\epsilon_{R\phi} \leq 5.0$ cm
∘ Impact parameter	$\epsilon_z \leq 10.0$ cm
∘ Track length	$L_{track} \geq 30$ cm
∘ Momentum error	$\Delta p/p \leq 100\%$

Energy cuts for neutral particles:

∘ High Density Projection Chamber	0.5 GeV $\leq E_{HPC} \leq 50$ GeV
∘ Forward ElectroMagnetic Calorimeter	0.5 GeV $\leq E_{FEMC} \leq 30$ GeV
∘ Hadron Calorimeter	1.0 GeV $\leq E_{HAC} \leq 50$ GeV

m (charged particles) $= m\,(\pi^\pm)$　　　**m (neutral particles) $= 0$**

Hadronic event selection:

∘ Charged multiplicity	$N_{ch} \geq 5$
∘ Visible charged energy	$E_{ch}^{tot} \geq 0.15\,\sqrt{s}$
∘ Sphericity angle	$30^\circ \leq \theta_{Sphericity} \leq 150^\circ$
∘ Visible charged energy in one hemisphere	$E_{ch}^{hemi} \geq 0.03\,\sqrt{s}$
∘ Momentum cut on the fastest charged particle	$p_{fast} \leq 45$ GeV/c

Charged and neutral particles are grouped into jets by means of the jet-finding algorithms: the Cambridge, the angular-ordered Durham and the Durham algorithms.

All events are clustered to a configuration of three jets with no cut on y_{ij}.

Cambridge algorithm

Step 0. If only one object remains in the table, then store this as a jet and stop.

Step 1. Select the pair of objects (ij) having the minimal value of $v_{ij} = 2(1-\cos\theta_{ij})$. Order the pair such that $E_i \leq E_j$.

Step 2. Inspect the value of $y_{ij} = v_{ij} \cdot E_i^2 \,/\, E_{vis}^2$.

- If $y_{ij} < y_{cut}$, then update the table by deleting i and j, introducing a new particle (ij) with 4-momentum $p_{ij} = p_i + p_j$, and recomputing the relevant values of v_{ij}.
- If $y_{ij} \geq y_{cut}$, then store i as a jet and delete it from the table.

Step 3. Go to Step 0.

Angular-ordered Durham algorithm

Step 1. Select the pair of objects (ij) with the minimal value of $v_{ij} = 2(1-\cos\theta_{ij})$.

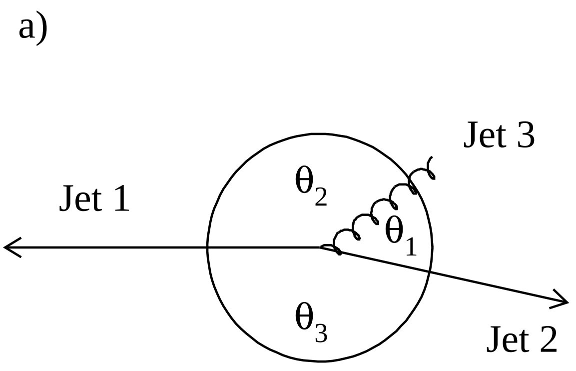
Step 2. Inspect the value of $y_{ij} = v_{ij} \cdot \min\{E_i, E_j\}^2 \,/\, E_{vis}^2$.

- If $y_{ij} < y_{cut}$, then update the table by deleting i and j, introducing a new particle (ij) with 4-momentum $p_{ij} = p_i + p_j$, and recomputing the relevant values of v_{ij}; then go to Step 1.
- If $y_{ij} \geq y_{cut}$, then consider the pair (if any) with the next smallest value of v_{ij} and repeat Step 2. If no such pair exists, then clustering is finished and all remaining objects are jets.

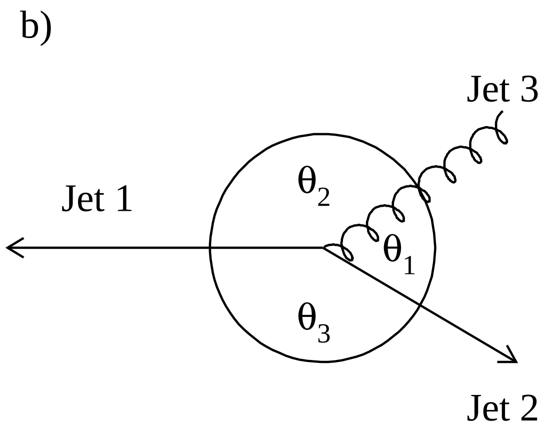
$k_\perp$ Durham algorithm

$$v_{ij} = y_{ij} = 2\,\min\{E_i, E_j\}^2\,(1-\cos\theta_{ij})\,/\,E_{vis}^2$$

with the same Steps as for the *angular-ordered Durham algorithm*.



Asymmetric events



Symmetric (Y) events

$$\theta_1 + \theta_2 + \theta_3 = 360^\circ$$

$$\theta_1 < \theta_2 < \theta_3$$

The jet k is opposite to the inter-jet angle θ_k:

$$E_1 > E_2 > E_3$$

Event topology is given by the two largest angles (θ_2, θ_3).

To choose the (θ_2, θ_3)–range the following considerations have been made:

- For each studied topology class a minimal mean value of $\log_{10}(y_2/y_3)$ of 1 is demanded to avoid artificial clustering of genuine two-jet events into a three-jet configuration. Here y_i is the minimal distance between two jets in an event at a stage of clustering where $i+1$ jets are still present.

- A multiplicative acceptance correction for detector-inefficiencies is calculated from Monte-Carlo simulations and applied to the data. The correction factor ranges from 1.25 to 1.35 depending on the event topology. It is highest for small θ_2 and θ_3 and decreases with larger angles. Angular configurations at the edge of the parameter space where the correction factor starts do deviate from a smooth behaviour have been excluded from this analysis.

- The constraint of a reasonable χ^2/ndf of the performed fit of at least one of the theoretical descriptions, to ensure the applicability of the used theoretical prediction, restricts the (θ_2, θ_3)–range further.

The final (θ_2, θ_3)–intervals are the following:

θ_3	θ_2
$120^\circ \ldots 130^\circ$	$115^\circ \ldots 128^\circ$
$130^\circ \ldots 140^\circ$	$111^\circ \ldots 138^\circ$
$140^\circ \ldots 145^\circ$	$108^\circ \ldots 143^\circ$
$145^\circ \ldots 150^\circ$	$108^\circ \ldots 148^\circ$
$150^\circ \ldots 155^\circ$	$111^\circ \ldots 155^\circ$

The colour dipole model

recoil effects add corrections
to the modified leading log approximation (MLLA)

P. Edén, G. Gustafson, *JHEP* **09** (1998) 015;
P. Edén, G. Gustafson, V. Khoze, *Eur. Phys. J.* **C11** (1999) 345.

$$\frac{d}{dL}N_{gg}(L + c_g - c_q) \;=\; \frac{C_A}{C_F}\cdot\frac{d}{dL}N^h_{q\bar{q}}(L)\cdot(1 - \frac{\alpha_0\cdot c_r}{L})$$

N_{gg} – multiplicity of a two-gluon colour-singlet system
$N^h_{q\bar{q}}$ – multiplicity of inclusive $e^+e^- \to hadrons$ events
C_A and C_F are the Casimir operators of the $SU(3)_C$ representation (the colour charges of gluons and quarks)
$L = \ln(s/\Lambda^2)$ is the energy scale
$\Lambda = 250$ MeV is the QCD scale parameter

$$c_g = \frac{11}{6}\;,\; c_q = \frac{3}{2}\;,\; c_r = \frac{10}{27}\pi^2 - \frac{3}{2}\;,\; \alpha_0 = \frac{6C_A}{11C_A - 2N_f}\;,\; N_f = 5$$

The solution of this differential equation implies a constant of integration. This constant has been fixed by a measurement of the multiplicity in $\chi_b'(J{=}2) \to gg$ decays:

CLEO Collab., M.S. Alam et al., *Phys. Rev.* **D46** (1992) 4822

$$N_{gg}(E_{cm} = m_{\chi_b'(J=2)} = 9.9132\text{ GeV}) \;=\; 9.339 \pm 0.090 \pm 0.045$$

Two alternative expressions for the multiplicity of e^+e^- three-jet events

$$N_{q\bar{q}g} = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{\mathrm{Lu}}) + \tfrac{1}{2}N_{gg}(\kappa_{\mathrm{Le}}) \qquad \textbf{a)}$$

$$N_{q\bar{q}g} = N_{q\bar{q}}(L, \kappa_{\mathrm{Lu}}) \quad + \tfrac{1}{2}N_{gg}(\kappa_{\mathrm{Lu}}) \qquad \textbf{b)}$$

$$s_{q\bar{q}} = p_q \cdot p_{\bar{q}} \qquad s_{ig} = p_i \cdot p_g \quad (i = q,\ \bar{q})$$

$$\kappa_{\mathrm{Le}} = \ln(p^2_{\perp\,\mathrm{Le}} \,/\, \Lambda^2) \qquad p^2_{\perp\,\mathrm{Le}} = s_{qg} \cdot s_{\bar{q}g} \,/\, s_{q\bar{q}}$$

$$\kappa_{\mathrm{Lu}} = \ln(p^2_{\perp\,\mathrm{Lu}} \,/\, \Lambda^2) \qquad p^2_{\perp\,\mathrm{Lu}} = s_{qg} \cdot s_{\bar{q}g} \,/\, s$$

$$L = \ln(s \,/\, \Lambda^2) \qquad L_{q\bar{q}} = \ln(s_{q\bar{q}} \,/\, \Lambda^2)$$

The multiplicity of a two-jet $q\bar{q}$–system $N_{q\bar{q}}(L, \kappa_{\mathrm{cut}})$ takes into account that the resolution of a gluon jet at a given cutoff κ_{cut} implies restrictions on the phase space of the $q\bar{q}$–system. This restricted multiplicity is linked to the multiplicity of inclusive $e^+e^- \to$ *hadrons* events $N^h_{q\bar{q}}$ via:

$$N_{q\bar{q}}(L, \kappa_{\mathrm{cut}}) = N^h_{q\bar{q}}(L') + (L - L')\frac{d}{dL'}N^h_{q\bar{q}}(L') \ , \ \ \textit{where} \ \ L' = \kappa_{\mathrm{cut}} + c_q$$

The above expressions are valid for jet-finding algorithms (Cambridge, angular-ordered Durham and Durham) which employ a transverse momentum cutoff $k_\perp$ to resolve "two-jet" $q\bar{q}$ from "three-jet" $q\bar{q}g$ events.

The DELPHI measurement of the charged multiplicity in e^+e^-–annihilations into $q\bar{q}$ at $\sqrt{s} = 206$ GeV

Previous DELPHI measurements at center-of-mass energies:

- $\sqrt{s} = 70 - 78$ GeV   [$q\bar{q}(\gamma)$ events at the Z]
- $\sqrt{s} = M_W$
- $\sqrt{s} = M_Z$
- $\sqrt{s} = 130$ GeV
- $\sqrt{s} = 161$ and 172 GeV
- $\sqrt{s} = 183$, 189 and 200 GeV

From the data collected in the year 2000 with the DELPHI detector at $\sqrt{s} = 206$ GeV:

$$N^h_{q\bar{q}} = 28.03 \pm 0.22(stat) \pm 0.27(syst)$$

Comparison with lower energy points:

- JADE
- PLUTO
- TASSO
- MARK II
- HRS
- AMY
- DELPHI for $q\bar{q}(\gamma)$ events at the Z
- world averages at $\sqrt{s} = M_Z$ and $\sqrt{s} = M_W$
- LEP at high energies ($\sqrt{s} \geq 130$ GeV)

A parameterization of the function $N^h_{q\bar{q}}(\sqrt{s})$

Including the resummation of leading (LLA) and next-to-leading (NLLA) corrections, the QCD predicts:

$$N^h_{q\bar{q}}(\sqrt{s}) \;=\; A \cdot [\alpha_s(\sqrt{s})]^{\,b} \cdot \exp[c/\sqrt{\alpha_s(\sqrt{s})}]$$

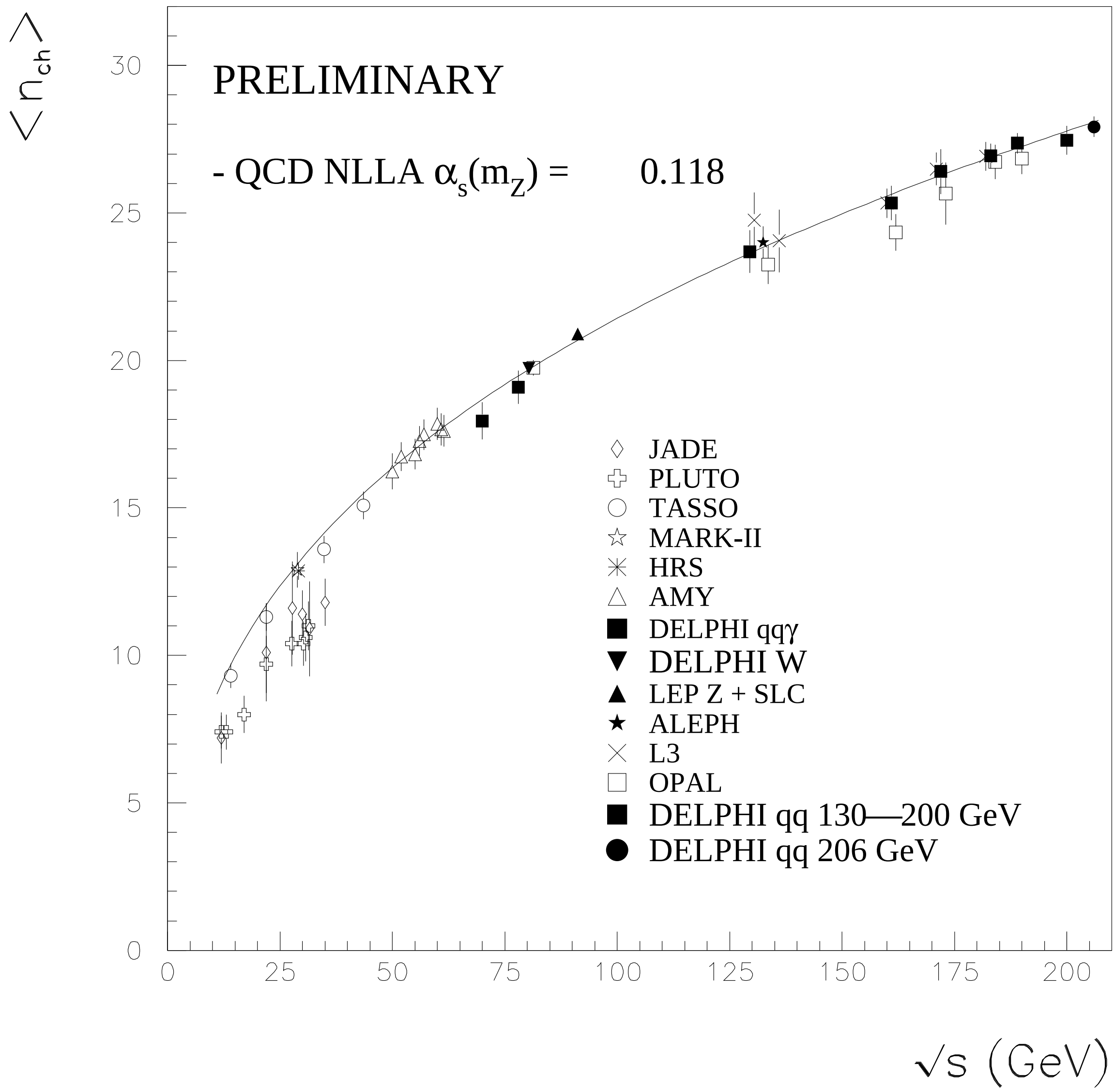
$$b = 0.49 \;\; , \;\; c = 2.27 \;\; , \;\; \alpha_s(M_Z) = 0.118 \pm 0.002$$

- The points from JADE, PLUTO and MARK II do not include the decay products of short lived K^0 and Λ. They are excluded from the fit of the QCD–NLLA prediction.

- The points at $\sqrt{s} \geq 70$ GeV are corrected to account for the different proportion of $b\bar{b}$ and $c\bar{c}$ events at the Z and W with respect to the e^+e^- continuum with pure photon contribution.

- The fit of the QCD–NLLA prediction to the data in the energy range 14 GeV $\leq \sqrt{s} \leq$ 206 GeV gives:

$$A = 0.043 \pm 0.001 \text{ and } \chi^2/ndf = 0.6.$$



PRELIMINARY
- QCD NLLA $\alpha_s(m_Z)$ = 0.118
$\langle n_{ch} \rangle$
$\sqrt{s}$ (GeV)
JADE
PLUTO
TASSO
MARK-II
HRS
AMY
DELPHI qqγ
DELPHI W
LEP Z + SLC
ALEPH
L3
OPAL
DELPHI qq 130—200 GeV
DELPHI qq 206 GeV

Fit of $C_A \,/\, C_F$

• Assuming massless kinematics of the jets, the dynamical scales

$$s_{q\bar{q}} \quad , \quad p_{\perp\,\mathrm{Lu}} \quad \text{and} \quad p_{\perp\,\mathrm{Le}}$$

can be calculated from the inter-jet angles only leading to a function

$$N_{q\bar{q}g} \;=\; N_{q\bar{q}g}\,(\theta_1, \theta_3).$$

• The function $N^h_{q\bar{q}}(\sqrt{s})$ is a parameterization of the measurements of the charged multiplicity $N^{ch}_{e^+e^-}$ obtained in e^+e^-–annihilations at several c.m. energies. The values of the charged multiplicity $N^{ch}_{e^+e^-}$ are corrected not to include contributions of initial b-quarks.

• In this analysis no b-tagging procedure is applied to the data. Instead, an additive offset parameter N_0 is introduced to the fitting procedure

$$N_{q\bar{q}g} \;=\; N^{exp}_{q\bar{q}g} - N_0$$

to account for the additional multiplicity due to the decay of hadrons containing the b-quark. The value of this offset is expected to be of order $N_0 \simeq 0.62$ and to be constant over a wide range of energy.

• The offset N_0 also accounts for small differences in the multiplicity normalization of this measurement and the overall charged hadron multiplicity measurements in e^+e^-–annihilations. As the colour factor ratio C_A/C_F enters the prediction only via a derivative dN/dL, the introduction of N_0 also assures that the value obtained for C_A/C_F is not affected by the overall normalization, but only by the slope of the measured multiplicity.

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- The gluon jet is not identified explicitly in this analysis.
- The predictions for the multiplicity at the topologies with the gluon jet being *jet* 1, *jet* 2 or *jet* 3 have to be added with proper weights for each angular region.
- These weights are obtained from the QCD three-jet matrix element to the first order. A comparison with probabilities calculated from Monte-Carlo events shows a good agreement with this method in the used angular region.

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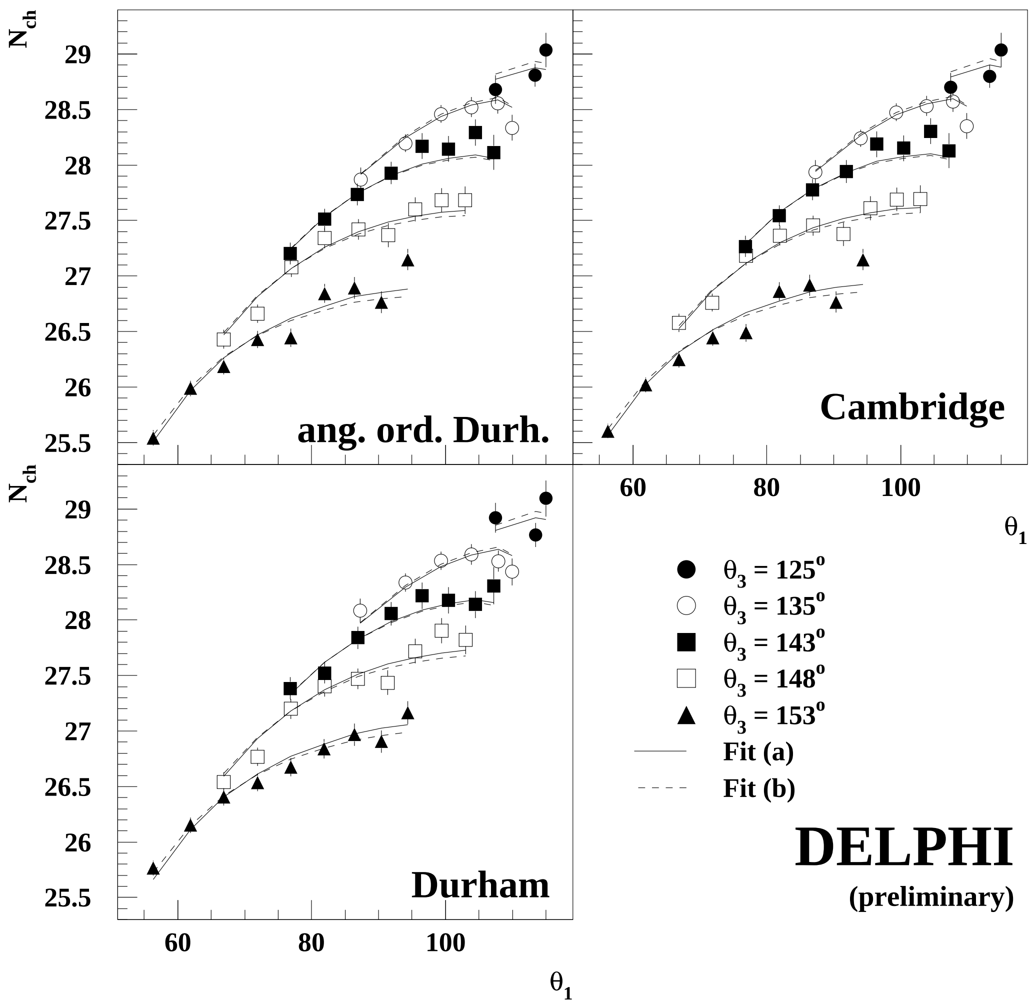
- The hadronization process affects the angles between the jets by pulling close jets even closer together.
- To take this effect into account, a hadronization correction is calculated by comparing the multiplicities evaluated for the topology taken from the *partonic level* with the multiplicities measured when the angles are taken from the *hadronic level* in events generated with the ARIADNE Monte-Carlo simulation.
- This correction is well below 3% all over the used angular region.



N_ch
29
28.5
28
27.5
27
26.5
26
25.5
ang. ord. Durh.
Cambridge
Durham
60
80
100
θ_1
θ_3 = 125°
θ_3 = 135°
θ_3 = 143°
θ_3 = 148°
θ_3 = 153°
Fit (a)
Fit (b)
DELPHI
(preliminary)

	Pred. **a)**		
	C_A/C_F	N_0	χ^2/ndf
Cambridge	2.300±0.01	0.69±0.03	0.92
ang. ord. Durham	2.329±0.03	0.60±0.08	1.03
Durham	2.228±0.02	0.97±0.03	0.89
	Pred. **b)**		
	C_A/C_F	N_0	χ^2/ndf
Cambridge	2.11 ±0.06	0.36±0.08	1.17
ang. ord. Durham	2.151±0.04	0.27±0.04	1.33
Durham	2.000±0.05	0.60±0.06	0.96

A weighted average over the values, obtained for the different clustering algorithms, results in a mean value of

$$C_A/C_F = 2.277 \pm 0.02 \pm 0.05 \qquad \textbf{for Pred. a)}$$

$$C_A/C_F = 2.093 \pm 0.05 \pm 0.08 \qquad \textbf{for Pred. b)}$$

where the first error is the mean statistical error and the second one is given by half the spread of the values obtained with a given prediction.

Extraction of N_{gg}

The charged multiplicity of a two-gluon colour-singlet system is measured for effective energy scale values from 16 to 52 GeV.

Pred. **a)**

$$N_{gg}(\kappa_{\rm Le}) \;=\; 2 \cdot [\, N_{q\bar{q}g}(\theta_1,\theta_3) - N_{q\bar{q}}(L_{q\bar{q}},\kappa_{\rm Lu}) - N_0 \,]$$

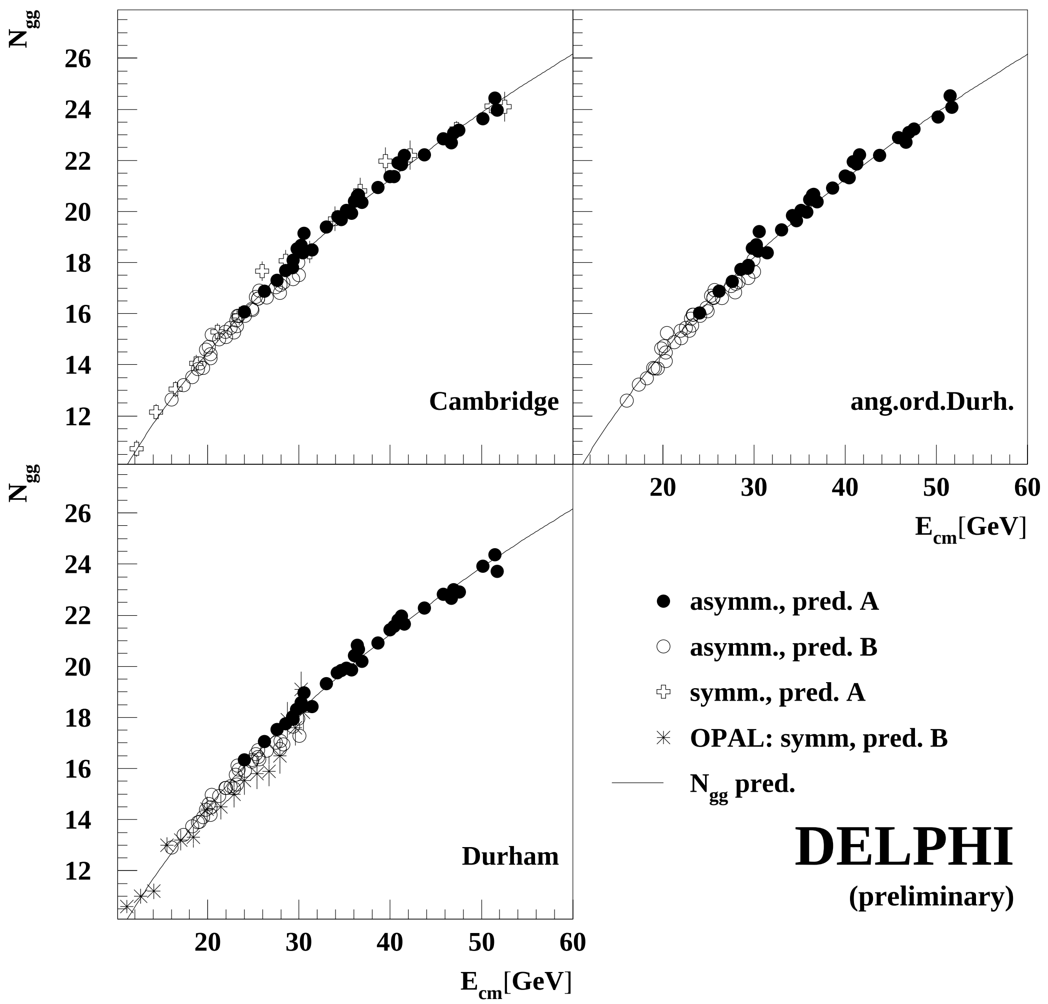
Pred. **b)**

$$N_{gg}(\kappa_{\rm Lu}) \;=\; 2 \cdot [\, N_{q\bar{q}g}(\theta_1,\theta_3) - N_{q\bar{q}}(L,\kappa_{\rm Lu}) \;\; - N_0 \,]$$

C_A/C_F is kept at the QCD value of

$$\frac{C_A}{C_F} \;=\; \frac{2\,N_c^2}{N_c^2-1} \;=\; \frac{9}{4}$$

	Pred. **a)**		Pred. **b)**	
	N_0	χ^2/ndf	N_0	χ^2/ndf
Cambridge	0.812±0.006	0.99	0.175±0.015	1.42
ang. ord. Durham	0.779±0.016	1.17	0.140±0.016	1.46
Durham	0.913±0.014	0.90	0.275±0.016	1.70



N_gg
26
24
22
20
18
16
14
12
Cambridge
ang.ord.Durh.
Durham
20
30
40
50
60
E_cm[GeV]
asymm., pred. A
asymm., pred. B
symm., pred. A
OPAL: symm, pred. B
N_gg pred.
DELPHI
(preliminary)

COMPARISON

- **This analysis** *(DELPHI preliminary)*

  $C_A/C_F = 2.277 \pm 0.054$ **Pred. a)**
  $C_A/C_F = 2.093 \pm 0.094$ **Pred. b)**

- **DELPHI value for symmetric (Y) three-jet events**
  (contributed paper 640 to ICHEP 2000, Osaka, Japan)

  $C_A/C_F = 2.22 \pm 0.11$

- **OPAL value for symmetric (Y) three-jet events**
  (G. Abbiendi et al., Eur. Phys. J. C23 (2002) 597)

  $C_A/C_F = 2.23 \pm 0.14$

- **QCD prediction for** $N_c = 3$

  $C_A/C_F = \frac{2\,N_c^2}{N_c^2-1} = \frac{9}{4} = 2.25$

SUMMARY

- **The charged multiplicity of hadronic three-jet events has been measured with the DELPHI detector at LEP at $\sqrt{s} = M_Z$ in dependence on the event topology covering the angular range:**

$$120^\circ < \theta_3 < 155^\circ \quad , \quad 108^\circ < \theta_2 < 155^\circ.$$

- **A fit of the theoretical prediction to the data yields the following values of the colour factor ratio C_A/C_F**

$$\textbf{a)} \quad 2.277 \pm 0.02 \pm 0.05 \quad \textbf{and} \quad \textbf{b)} \quad 2.093 \pm 0.05 \pm 0.08$$

for two different expressions of the theoretical prediction, in good agreement with the results obtained by DELPHI and OPAL for symmetric three-jet events.

- **Although the less satisfactory χ^2/ndf and the low values for the offset N_0, found by applying the expression b) of the prediction, disfavour this expression, no compelling evidence for the preference of the expression a) is given.**

- **The multiplicity of a two-gluon colour-singlet system at effective center-of-mass energies from 16 to 52 GeV is extracted from the measured multiplicity of three-jet events and found in good agreement with previous measurements and the theoretical prediction.**