Energy-energy and transversal energy-energy correlations in e^+e^- and pp collisions

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Talk Plan

1. Introduction
2. A. Subprocess $Q_1Q_2 \rightarrow Q_1Q_{2g}, QQ \rightarrow QQ_g, q\bar{q} \rightarrow ggg$
3. B. Subprocess $qg \rightarrow qgg$ and $gg \rightarrow q\bar{q}g, gg \rightarrow ggg$
4. Appendix

Motivation

The energy-energy correlations for sub-processes $2 \to 3$ for high energy proton-proton correlations are calculated in Born approximation. The sub-processes $\gamma^* \to q\bar{q}g$ and $qq' \to qq'g$, $qq \to qqg$, $gg \to ggg$,$gq \to ggq$, $gg \to q\bar{q}g$, $qg \to q'\bar{q}'q$, $gq \to qq\bar{q}$ are taken into account explicitly. These processes can be measured in electron-positron colliders and hadron-hadron ones. The transverse energy azimuthal correlations and the rapidity distributions for processes with two jet production are considered. The application for LHC conditions is discussed.

Introduction

In experiments of 1981 year with measuring the energy-energy correlation of jets the energy-weighted cross section was measured (Pluto Collaboration, C. Berger et al. Phys. Lett.99B(1981),292)

$$\frac{d\Sigma}{d\theta} = 2\sum_{a,b}\int \frac{d^3\sigma}{dz_a dz_b d\theta} z_a z_b dz_a dz_b, \tag{1}$$

with $z_{a,b} = E_{a,b}/E$, θ is the angle between the directions of two jets (the center of mass frame is implied).

Such kind of the measurable are free as from infrared as well as from collinear (mass) singularities, was shown (Yu.L. Dokshitzer,D.I. Djakonov and S.I. Troyan, Phys.Lett.B78(1978)290, C.L. Basham,L.S. Brown,S.D. Ellis and S.T. Love, Phys.Rev.Lett. 41 (1978),1585; Phys.Rev.D 19(7)(1979),2018).

In electron-positron colliders the simplest QCD process annihilation e^+e^- to the the pair of quark and anti-quark, accompanied by emission of a hard gluon with the subsequent conversion to three jets can be studied

$$e^+(p_+) + e^-(p_-) \to \bar{Q}(q_+) + Q(q_-) + g(k). \tag{2}$$

Introduction

The energy-weighted cross section was suggested to measure (C.L. Basham,L.S. Brown,S.D. Ellis and S.T. Love, Phys.Rev.Lett. 41 (1978),1585; Phys.Rev.D 19(7)(1979)2018, A. Ali,F. Barreiro, Phys.Lett.118B,(1982),155; Nucl.Phys. B236(1984),269)

$$\frac{d\Sigma}{d\cos\eta} = \frac{1}{8s}\int\sum|M^{e^+e^-\to q\bar{q}g}|^2[z_+z_-\delta(\cos\eta - c_{+-}) + z_+z\delta(\cos\eta - c_{+0}) + z_-z\delta(\cos\eta - c_{-0})]d\Gamma_3, \quad z_\pm = \frac{q_{\pm 0}}{E}, z = \frac{k_0}{E}, s = 4E^2, \quad (3)$$

with $M^{e^+e^-\to q\bar{q}g}$ is the matrix element of process and the phase volume of final particles is

$$\int d\Gamma_3 = \int \frac{d^3q_+}{2E_+}\frac{d^3q_-}{2E_-}\frac{d^3k}{2k_0}\frac{(2\pi)^4}{(2\pi)^9}\delta^4(p_+ + p_- - q_+ - q_- - k). \quad (4)$$

It was obtained (see Appendix,part 1.)

$$\frac{d\Sigma}{d\cos\eta} = \frac{\alpha_s}{\pi}\sigma_0 F(\xi),$$

$$F(\xi) = \frac{3-2\xi}{\xi^5\bar{\xi}}[2(3-6\xi+2\xi^2)\ln(1-\xi) + 3\xi(2-3\xi)], \xi = \frac{1}{2}(1-\cos\eta). \quad (5)$$

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We suppose here that real photon emission is switched out.
Some information can be extracted measuring two jet production processes in hadron collisions. The relevant sub-process is

$$a(p_1) + b(p_2) \to c(p_3) + d(p_4). \quad (6)$$

Here the polar angle $\theta = (\vec{p}_- \vec{q}_-)$ distribution of one of jet (or the rapidity distribution) can be measured

$$\frac{d\Sigma}{dc} = \sum_{abcd} \int\limits_0^1 dx_1 f_a(x_1) \int\limits_0^1 dx_2 f_b(x_2) z_c z_d \frac{d\sigma_{ab}^{cd}}{dc}, c = \cos\theta. \quad (7)$$

Definite expressions for the cross sections of relevant partonic cross sections processes $a + b \to c + d$ are given in Appendix(part 3).
Some experimental interest have a transversal energies correlation (K. Nakamura et al.,(PDG), J.Phys.G v37,075021 (2010)) defined for process with creation of three jets in the high energy hadron-hadron collisions.

Introduction

The transversal energy correlations of two jets produced in electron-positron annihilation to quark anti-quark and gluon can be measured studying the dependence on the azimuthal angle between the pair of final particles of process $a(p_1) + b(p_2) \to c(q_1) + d(q_2) + f(q_3)$.
In center of mass frame of initial particles we define for partons momenta

$$\frac{1}{E}p_1 = x_1(1,1,0,0); \frac{1}{E}p_2 = x_2(1,-1,0,0); \frac{1}{E}q_i = y_i(1,c_i,s_i\vec{n}_i), \vec{n}_i^2 = 1, i = 1,2,3.$$

with $c_i, s_i = \cos\theta_i, \sin\theta_i$ and θ_i is the polar angle between the beam axes and 3-momentum of one of final particles. We have for the transversal energy azimuthal correlation

$$E^2\frac{d\Sigma}{d\phi} = \frac{\alpha^2\alpha_s}{4\pi}\sum_{abcdf}\int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}f_{p/a}(x_1)f_{p/b}(x_2)R_{ab}^{cdf}\delta(x_1+x_2-y_1-y_2-y_3)$$
$$y_1y_2y_3dy_1dy_2dy_3dc_1dc_2dc_3\delta(x_1-x_2-y_1c_1-y_2c_2-y_3c_3)$$
$$\left[s_1s_2\delta_{12}+s_1s_3\delta_{13}+s_3s_2\delta_{32}\right],$$

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with $R = (E^2/(16e^4g^2))\sum|M^{ab\to cdf}|^2$ and

$$\delta_{12} = \delta(2x_1x_2 - y_1\Delta_1 - y_2\Delta_2 + y_1y_2(1 - c_1c_2 - s_1s_2\cos\phi));$$
$$\delta_{13} = \delta(2x_1x_2 - y_1\Delta_1 - y_3\Delta_3 + y_1y_3(1 - c_1c_3 - s_1s_3\cos\phi));$$
$$\delta_{23} = \delta(2x_1x_2 - y_3\Delta_3 - y_2\Delta_2 + y_3y_2(1 - c_3c_2 - s_3s_2\cos\phi));$$
$$\Delta_i = x_1(1 - c_i) + x_2(1 + c_i). \quad (10)$$

The expression for the azimuthal correlations for QED process $e^+e^- \to Q\bar{Q}g$ is considered in Appendix(part2).
Compared with electron-positron colliding beams where the initial state is virtual photon, decaying to quark-anti-quark pair and gluons, in proton-proton collisions initial state can be constructed from the constituents of protons-quarks of two kinds and gluons. To provide the similar test for LHC facility the sub-processes of type $2 \to 3$ with the initial particles: two sorts of quarks and gluons- must be considered: $qg, gg, qq', qq \to ggg, qqg, qq'g, q\bar{q}g$. Matrix elements squared, summed on polarization states of these sub-processes where calculated in series of papers of CALCUL collaboration (F.A. Berends,R. Kleiss,P.De Gausmaecker, R. Gastmans and T.T. Wu, Phys.Lett. 103B,(1981),124).

Below we will consider each of this sub-processes separately. Keeping in mind the content of each proton as $2u+d+g$ the schematic picture of pp collision is

$$(2u+d+g)_1(2u+d+g)_2 \to 4u_1u_2 + d_1d_2 + 2(u_1d_2+u_2d_1) + g_1g_2 + g_1(2u_2+d_2) + g_2(2u_1+d_1) \qquad (11)$$

with final state

$$uug, ddg, udg, ggg, ugg, dgg, gu\bar{u}, gd\bar{d}. \qquad (12)$$

Using the definition of 4-vectors of process $a(p_1)+b(p_2) \to c(q_1)+d(q_2)+f(q_3)$ we obtain for the dimensionless quantities

$$s = \frac{2p_1p_2}{E^2} = 4x_1x_2;\ s' = \frac{2p_1p_2}{E^2} = 4x_1x_2 - y_3\Delta_3;$$
$$t_1 = -\frac{2p_2q_2}{E^2} = -2x_2y_2(1+c_2);\ t' = -\frac{2p_1q_1}{E^2} = -2x_1y_1(1-c_1);$$
$$u = -\frac{2p_2q_1}{E^2} = -2x_2y_1(1+c_1);\ u' = -\frac{2p_1q_2}{E^2} = -2x_1y_2(1-c_2);$$
$$a_3 = \frac{q_3p_1}{E^2} = x_1y_3(1-c_3);\ b_3 = \frac{p_2q_3}{E^2} = x_2y_3(1+c_3); \qquad (13)$$

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$$c_{13} = \frac{q_1 q_3}{E^2} = 2x_1 x_2 - y_2 \Delta_2; \;\; c_{23} = \frac{q_2 q_3}{E^2} = 2x_1 x_2 - y_1 \Delta_1,$$
$$c_{12} = \frac{q_1 q_2}{E^2} = 2x_1 x_2 - y_3 \Delta_3 \quad , \quad . \tag{14}$$

The relevant squares of matrix elements summed on spin and color states of partons was obtained, using the chiral amplitudes method by the CALCUL collaboration (F.A. Berends,R. Kleiss,P.De Gausmaecker, R. Gastmans and T.T. Wu, Phys.Lett. 103B,(1981),124). They are listed in section two. The problem of taking into account the contributions of higher orders of PQCD are discussed in section 3.

For quark-quark scattering with different flavors,

$$Q_1(p_1) + Q_2(p_2) \to Q_1(q_1) + Q_2(q_2) + g(q_3), \tag{15}$$

we have (here one imply $|M|^2 = (1/(4N^2)) \sum_{\lambda_i, color} |M^{\lambda_i, color}|^2$

$$|M^{Q_1Q_2 \to Q_1Q_2g}|^2 = \frac{2g^6}{a_1b_1c_{13}c_{23}}[C_1[(u+u_1)A + C] - C_2[(s+s_1)B + D]]F,$$
$$a_1 = (p_1q_3), b_1 = (p_2q_3), c_{13} = (q_1q_3), c_{23} = (q_2q_3), \tag{16}$$

with

$$C_1 = \frac{(N^2-1)^2}{4N^3} = \frac{16}{27}, C_2 = \frac{N^2-1}{4N^3} = \frac{2}{27}, \tag{17}$$

and

$$F = \frac{s^2 + s'^2 + u^2 + u'^2}{tt'};$$
$$A = ss' + tt' - uu', B = ss' - tt' - uu';$$
$$C = u(st + s't') + u'(st' + s't); D = 2tt'(u+u') + 2uu'(t+t'). \tag{18}$$

subprocess $Q_1Q_2 \to Q_1Q_2g, QQ \to QQg, q\bar{q} \to ggg$

The result is somewhat more complicated for the case quark-quark scattering with identical flavors:

$$|M^{QQ\to QQg}|^2 = \frac{2g^6}{a_1b_1c_{13}c_{23}}(1 + P(t \leftrightarrow u; t' \leftrightarrow u'))$$

$$\Bigg[[C_1[(u+u')A + C] - C_2[(s+s')B + D]]F +$$

$$\left[C_3[(s+s')B + D] + C_4[(s+s')B - D - C]\right]\frac{B(s^2+s'^2)}{2tt'uu'}\Bigg], \tag{19}$$

with operation P interchanging the invariants t, u and t', u'

$$C_3 = (N^4-1)/(8N^4) = \frac{10}{81}, \;\; C_4 = (N^2-1)/(8N^4) = \frac{8}{81}. \tag{20}$$

The averaged by spin and color states of initial particles matrix element square of annihilation sub-process $\bar{q}(p_1) + q(p_2) \to g(q_1) + g(q_2) + g(q_3)$ have a form

$$|M|^2 = g^6\frac{N^2-1}{4N^4}G(a_i, b_i, c_{ij}, \sigma), \tag{21}$$

with

$$G = \frac{1}{\Pi_1^3 a_i b_i} \sum_1^3 a_i b_i (a_i^2 + b_i^2)[\frac{\sigma}{2}(1 + N^2) - N^2[\frac{a_1b_2 + a_2b_1}{c_{12}} + \frac{a_2b_3 + a_3b_2}{c_{23}} + \frac{a_1b_3 + a_3b_1}{c_{13}}] + \frac{2N^4}{s}[\frac{a_3b_3(a_1b_2 + a_2b_1)}{c_{23}c_{13}} + \frac{a_1b_1(a_2b_3 + a_3b_2)}{c_{13}c_{12}} + \frac{a_2b_2(a_1b_3 + a_3b_1)}{c_{12}c_{23}}]]. \quad (22)$$

Here

$$a_i = (p_1 q_i), b_i = (p_2 q_i), i = 1, 2, 3; c_{ij} = (q_i q_j), \sigma = 2(p_1 p_2). \quad (23)$$

sub-processes $qg \to qgg$ and $gg \to q\bar{q}g, gg \to ggg$

Using the crossing symmetry we can obtain the expression for $|M|^2$ starting from the known expression for the process $\bar{q}q \to ggg$ (F.A. Berends,R. Kleiss,P.De Gausmaecker, R. Gastmans and T.T. Wu, Phys.Lett. 103B,(1981),124) given above. Using the substitutions $p_1 \to -q_1, q_1 \to -p_1$

$$q(p_1) + g(p_2) \to q(q_1) + g(k_2) + g(k_3) \tag{24}$$

we have

$$|M^{qg \to qgg}|^2 = g^6 \frac{\sqrt{N^2-1}}{4N^3} G(\bar{a}_i, \bar{b}_i, \bar{\sigma}, \bar{c}_{ij}), \tag{25}$$

with

$$\bar{\sigma} = -2b_1, \bar{a}_2 = -c_{12}, \bar{a}_3 = -c_{13}, \bar{b}_1 = -\sigma/2, \bar{c}_{12} = -a_2, \bar{c}_{13} = -a_3, \\ \bar{a}_1 = a_1, \bar{b}_2 = b_2, \bar{b}_3 = b_3, \bar{c}_{23} = c_{23}. \tag{26}$$

sub-processes $qg \to qgg$ and $gg \to q\bar{q}g, gg \to ggg$

For the subprocess $gg \to q\bar{q}g$ we apply the double crossing transformation. It results in additional color factor $N^2/(N^2-1)$ and the replacement of momenta $p_1 \to -q_1, p_2 \to -q_2, q_1 \to -p_1, q_2 \to -q_2$. and

$$|M^{gg\to q\bar{q}g}|^2 = g^6 \frac{\sqrt{1}}{4N^2} G(a'_i, b'_i, \sigma', c'_{ij}), \quad (27)$$

with

$$\begin{aligned} \sigma \to \sigma' = c_{12}, a_1 \to a'_1 = a_1, b_1 \to b'_1 = a_2, a_2 \to a'_2 = b_1, \\ a_3 \to a'_3 = -c_{13}, b_2 \to b'_2 = b_2, \\ b_3 \to b'_3 = -c_{23}, c_{13} \to c'_{13} = -a_3, c_{23} \to c'_{23} = -b_3. \end{aligned} \quad (28)$$

It was obtained (F.A. Berends,R. Kleiss,P.De Gausmaecker, R. Gastmans and T.T. Wu, Phys.Lett. 103B,(1981),124) for process

$$g(k_1) + g(k_2) \to g(k_3) + g(k_4) + g(k_5) \quad (29)$$

sub-processes $qg \to qgg$ and $gg \to q\bar{q}g, gg \to ggg$

the expression

$$|M^{gg\to ggg}|^2 = \frac{g^6 N^2}{2(N^2-1)E^2} R_1 R_2, \quad (30)$$

with

$$R_1 = (12345) + (12354) + (12435) + (12453) + (12534) + (12543) + (13245) +$$
$$(13254) + (13425) + (13524) + (14235) + (14325$$

$$R_2 = \frac{(12)^4 + (13)^4 + (14)^4 + (15)^4 + (23)^4 + (24)^4 + (25)^4 + (34)^4 + (35)^4 + (45)}{(12)(13)(14)(15)(23)(24)(25)(34)(35)(45)}$$

and

$$(ij) = \frac{k_i k_j}{E^2}; (ijnml) = (ij)(jn)(nm)(ml)(li). \quad (32)$$

In terms of universal notations we have

$$(12) = \sigma/2, (13) = a_1, (14) = a_2, (15) = a_3; (23) = b_1, (24) = b_2, (25) = b_3;$$
$$(34) = c_{12}, (35) = c_{13}, (45) = c_{23}. \quad (33)$$

Appendix

1) The analytic expression for the process $e^+e^- \to Q\bar{Q}g$ (see (3))

$$e_+(p_+) + e_-(p_-) \to q(q_-) + \bar{q}(q_-) + g(q), \tag{34}$$

can be obtained. Really, using the known spectral distribution (V.N. Baier, E.A. Kuraev and V.S. Fadin, Sov.J.Nucl.Phys. 31(3),364(1980).)

$$\frac{d\sigma}{d\nu_+ d\nu_-} = \frac{2\alpha_s}{\pi}\sigma_0(\frac{z_+^2 + z_-^2}{(1-z_+)(1-z_-)} + O(\frac{m^2}{s})),$$
$$\sigma_0 = \frac{4\pi\alpha^2}{3s}\sum Q_q^2, \tag{35}$$

we construct the energy-energy correlation

$$\frac{d\Sigma}{d\cos\eta} = \sigma_0 \int dz_+ dz_- dz \delta(2 - z_+ - z_- - z)\frac{z_+^2 + z_-^2}{(1-z_+)(1-z_-)}$$
$$\left[z_+ z_- \delta(\cos\eta - \cos(\vec{q}_+\vec{q}_-)) + z_+ z \delta(\cos\eta - \cos(\vec{q}_+\vec{q})) + z z_- \delta(\cos\eta - \cos(\vec{q}\vec{q}_-))\right]. \; (3$$

Appendix

Performing the one fold integral we obtain for the right hand part

$$\frac{2\alpha_s}{\pi}\sigma_0 F(\xi), F(\xi) = f_1(\xi) + 2f_2(\xi),$$
$$\xi = \frac{1}{2}(1-\cos\eta), \quad (37)$$

with

$$f_1(\xi) = \frac{1}{2\xi^5\bar{\xi}}[2(4-3\xi)\ln\frac{1}{\bar{\xi}} - 8\xi + 2\xi^2 + \frac{1}{3}\xi^3];$$
$$f_2(\xi) = \frac{1}{2\xi^5\bar{\xi}}[(-13+27\xi-18\xi^2+4\xi^3)\ln\frac{1}{\bar{\xi}} + 13\xi - \frac{41}{2}\xi^2 + \frac{53}{6}\xi^3]. \quad (38)$$

As a result we obtain (C.L. Basham,L.S. Brown,S.D. Ellis and S.T. Love, Phys.Rev.Lett. 41 (1978),1585; Phys.Rev.D 19(7)(1979)2018, A. Ali,F. Barreiro, Phys.Lett.118B,(1982),155; Nucl.Phys. B236(1984),269)

$$\frac{d\Sigma}{d\cos\eta} = \frac{\alpha_s}{\pi}\sigma_0 F(\xi),$$
$$F(\xi) = \frac{3-2\xi}{\xi^5\bar{\xi}}[2(3-6\xi+2\xi^2)\ln(1-\xi) + 3\xi(2-3\xi)]. \quad (39)$$

Appendix

2) For process of creation quark anti-quark gluon state in electron-positron annihilation we must use the QED non-singlet Structure function $D(x,\beta)$ (L.N. Lipatov, Yad.Fiz. v 20,(1984),181; E.A. Kuraev and V.S. Fadin, Yad.Fiz.v 41(1985),733) takes into account the radiative corrections in leading order of perturbation theory $D(x,\beta) \approx (1/\beta)(1-x)^{\beta-1}, \beta = (\alpha/\pi)\ln(4E^2/m_2^2)$. In parton language it is the probability to find electron (positron) inside the initial electron (positron). The dimensionless quantity $R^{e^+e^-\to Q\bar{Q}g}$ is

$$R = \frac{8x_1x_2}{c_{13}c_{23}}[-a_1(u+t') - b_1(u'+t) + uu' + tt'] + \frac{1}{c_{13}}[-2a_1t_1 - 2b_1u' + uu' + tt' - 2ut] + \frac{1}{c_{23}}[-2a_1u - 2b_1t + uu' + tt' - 2u't']. \tag{40}$$

Appendix

3)Differential cross sections of processes of type $2 \to 2$

$$a(p_1)+b(p_2)\to c(p_3)+d(p_4),$$
$$p_i^2=0, s=2p_1p_2; t=-2p_1p_3, u=-2p_1p_4, s+t+u=0, \quad (41)$$

have a form (Our result for process $q\bar{q}\to gg$ differs from the one obtained in (J.P. Owens,E. Reya,M. Gluk, Phys.Rev.D v 18,(1978),1501))

$$\frac{d\sigma}{d\tau}=\frac{\pi\alpha_s^2}{s}S_{ab}^{cd}, \quad (42)$$

with $\tau=-t/s, \sigma=-u/s$ and

$$S_{gg}^{gg}=\frac{9}{2}[3-\tau\sigma+\frac{\tau}{\sigma^2}+\frac{\sigma}{\tau^2}], \quad S_{q_1\bar{q}_2}^{q_1\bar{q}_2}=\frac{4}{9}[\sigma^2+\tau^2],$$
$$S_{q\bar{q}}^{gg}=\frac{1}{36}[\frac{4(\sigma^2+\tau^2)}{3\tau\sigma}+3(2\tau\sigma+1+\frac{1}{2\tau\sigma}], \quad S_{q_1q_2}^{q_1q_2}=\frac{4}{9}\frac{1+\sigma^2}{\tau^2},$$
$$S_{qq}^{qq}=\frac{4}{9}[\frac{1+\sigma^2}{\tau^2}+\frac{1+\tau^2}{\sigma^2}]-\frac{8}{27}\frac{1}{\sigma\tau},$$
$$S_{q\bar{q}}^{q\bar{q}}=\frac{4}{9}[\frac{1+\sigma^2}{\tau^2}+\frac{1+\tau^2}{\sigma^2}]-\frac{8}{27}\frac{\sigma^2}{\tau}. \quad (43)$$