Helicity amplitudes and angular decay distributions

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Contents

Introductory remarks

- **In these lectures I want to discuss some examples of sequential cascade decays and their corresponding angular decay distributions.**
- **Angular decay distributions follow from a reasonably simple master formula involving bilinear forms of helicity amplitudes and Wigner's d–functions.**

Some sample cascade decay processes are

- **Top quark decay**
  $t(\uparrow) \to b + W^+(\to \ell^+ \nu_\ell)$
  (the symbol "$(\uparrow)$" denotes polarization)
- **Rare $\Lambda_b(\uparrow)$ decays**
  $\Lambda_b(\uparrow) \to \Lambda_s(\to p\pi^-) + j_{eff}(\to \ell^+ \ell^-)$
- **Higgs decay to gauge bosons**
  $H \to W^+(\to \ell^+ \nu_\ell) + W^{-*}(\ell^- \bar{\nu}_\ell)$
  $H \to Z(\to \ell^+ \ell^-) + Z^*(\to \ell^+ \ell^-)$
  (the symbol "$*$" denotes off-shellness)

- **Rare B decays**
  $B \to D + j_{eff}(\to \ell^+\ell^-)$
  $B \to D^*(\to D\pi) + j_{eff}(\to \ell^+\ell^-)$
- **Semileptonic B decays**
  $B \to D + W_{off-shell}(\to \ell\nu)$
  $B \to D^*(\to D\pi) + W_{off-shell}(\to \ell\nu)$
- **Nonleptonic Λ_b decays**
  $\Lambda_b \to \Lambda(\to p\pi^-) + J/\psi(\to \ell^+\ell^-)$

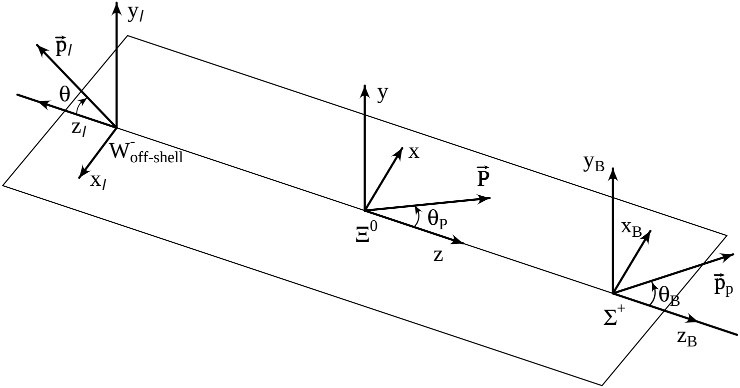
- **Semileptonic hyperon decays**
  $\Xi^0(\uparrow) \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l \;\; (l^- = e^-, \mu^-)$

Interest in angular decay distributions is twofold

1. **Facilitates theoretical analysis. Behaviour under P, CP, ..**
2. **Allows for a Monte Carlo programs to generate experimental decay distributions**

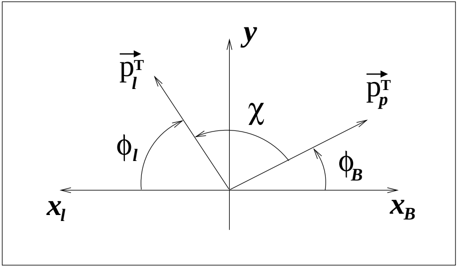
- **Take as an example the semileptonic hyperon decay**
  $\Xi^0(\uparrow) \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l \;\; (l^- = e^-, \mu^-)$
  One needs the three polar angles θ, θ_B and θ_P and the two azimuthal angles ϕ_B and ϕ_ℓ to describe the cascade decay process.

Definition of the three polar angles θ, θ_B and θ_P



Definition of the three polar angles θ, θ_B and θ_P in the semileptonic decay of a polarized Ξ^0 into $\Sigma^+ + l^- + \bar{\nu}_l$ followed by the nonleptonic decay $\Sigma^+ \to p + \pi^0$. The polarization vector of the parent baryon $\vec{P}$ lies in the (x, z)–plane with positive P_x component.

Definition of azimuthal angles ϕ_l and ϕ_B



Definition of the three azimuthal angles ϕ_l, ϕ_B and χ ($\phi_l + \phi_B + \chi = \pi$) in the semileptonic decay of a polarized Ξ^0. Present figure is a view of the last figure from the right along the negative z–direction. $\vec{p}_l^{\,T}$ and $\vec{p}_p^{\,T}$ denote the transverse components of the momentum of the lepton and proton, respectively.

As we shall learn in this lecture the angular decay distribution can be derived from the master formula

$$\mathbf{W(\theta, \theta_P, \theta_B, \phi_B, \phi_\ell) \propto}$$

$$\sum_{\lambda_\ell,\lambda_W,\lambda'_W,J,J',\lambda_2,\lambda'_2,\lambda_3} (-1)^{J+J'} |h^{V-A}_{\lambda_l\lambda_{\nu=\pm1/2}}|^2 e^{i(\lambda_W-\lambda'_W)\phi_\ell}$$
$$\rho_{\lambda_2-\lambda_W,\lambda'_2-\lambda'_W}(\theta_P) d^J_{\lambda_W,\lambda_\ell-\lambda_\nu}(\theta) d^{J'}_{\lambda'_W,\lambda_\ell-\lambda_\nu}(\theta) H_{\lambda_2\lambda_W} H^*_{\lambda'_2\lambda'_W}$$
$$e^{i(\lambda_2-\lambda'_2)\phi_B} d^{\frac{1}{2}}_{\lambda_2\lambda_3}(\theta_B) d^{\frac{1}{2}}_{\lambda'_2\lambda_3}(\theta_B) |h^B_{\lambda_3 0}|^2$$

where

$h^{V-A}_{\lambda_\ell\lambda_{\nu=\pm1/2}}$	:	**helicity amplitudes for the transition** $W_{off-shell} \to \ell + \nu_\ell$
		$+1/2$ for $(\ell^- \bar{\nu}_\ell)$; $-1/2$ for $(\ell^+ \nu_\ell)$
$\rho_{\lambda_1\lambda'_1}$	:	**density matrix for the polarized parent baryon** B_1
$H_{\lambda_2\lambda_W}$	:	**helicity amplitudes for the transition** $B_1 \to B_2 + W_{off-shell}$
$h^B_{\lambda_3 0}$	:	**helicity amplitudes for the transition** $B_2 \to B_3 + \pi$
$d^J_{mm'}$	:	**Wigner's d-functions**

- **The λ_ℓ, λ_W, ... are helicity labels of the baryons, leptons and the $W_{off-shell}$ that participate in the process. They take the values**

$$\begin{aligned} \lambda_\ell,\, \lambda_1,\, \lambda_2,\, \lambda_3 &= \pm 1/2 \\ \lambda_W &= 1,\, 0,\, -1\,(J=1); \quad t\,(J=0) \\ \lambda_{\bar{\nu}} &= +1/2;\, \lambda_\nu = -1/2 \end{aligned}$$

- **We shall see in these lectures that the off-shell gauge boson W has a spin 1 and a spin 0 part. Thus we have to sum over $J = 0, 1$. There is a factor $(-1)^{(J+J')} = \pm 1$ associated with the Minkowski metric of our world.**
- **The angular decay distribution holds for both final lepton states $(\ell^- \bar{\nu}_\ell)$ and $(\ell^+ \nu_\ell)$ which are distinguished through the labelling $\lambda_\nu = \pm 1/2$. This covers the charge conjugated process or also the semileptonic decay $\Sigma^+ \to \Lambda + e^+ \nu_e$.**
- **The master formula is quite general. After appropiate angular integration over θ_B and ϕ_B the master formula also applies to polarized top decay $t \to b + \ell^+ \nu_\ell$, etc., etc..**
- **The summation over helicities can be quite elaborate if done by hand. However, the summation can be done by computer. A FORM package doing the summation automatically is available from M.A. Ivanov.**

Polarization of lepton

In the master formula we have summed over the helicities of the lepton. To obtain the polarization of the lepton leave the lepton helicity unsummed.

$$\sum_{\lambda_\ell,\dots} \quad \rightarrow \quad \sum_{,\dots}$$

- **The longitudinal polarization of the lepton is given by**

$$P^z(\ell) = \frac{W_{\lambda_\ell=+1/2} - W_{\lambda_\ell=-1/2}}{W_{\lambda_\ell=+1/2} + W_{\lambda_\ell=-1/2}}$$

- **similar for $P^x(\ell)$ and $P^y(\ell)$**

Polar angle distribution in $t \to b + W^+$

- **Traditional approach**
  - **Calculate the lepton and hadron tensors. Set $m_b = m_\ell = m_\nu = 0$.**

$$
\begin{aligned}
\mathbf{L}^{\mu\nu} &= \frac{1}{8}\mathrm{Tr}\Big\{ \not{p}_\ell \gamma^\mu (1-\gamma_5) \not{p}_\nu \gamma^\nu (1-\gamma_5) \Big\} \\
&= p_\ell^\mu p_\nu^\nu + p_\ell^\nu p_\nu^\mu - \frac{1}{2} q^2 g^{\mu\nu} - i\varepsilon^{\mu\nu\alpha\beta} p_{\mu\alpha} p_{\nu\beta}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}^{\mu\nu} &= \frac{1}{8}\mathrm{Tr}\Big\{ (\not{p}_t + m_t) \gamma^\mu (1-\gamma_5) \not{p}_b \gamma^\nu (1-\gamma_5) \Big\} \\
&= p_t^\mu p_b^\nu + p_t^\nu p_b^\mu - p_t p_b g^{\mu\nu} - i\varepsilon^{\mu\nu\alpha\beta} p_{t\alpha} p_{b\beta}
\end{aligned}
$$

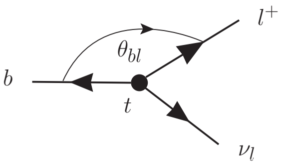
  - **Evaluate $L^{\mu\nu} H_{\mu\nu}$ and use $q = p_\ell + p_\nu$**

$$
\mathbf{L}^{\mu\nu}\mathbf{H}_{\mu\nu} = 4\,(p_t p_\ell)\,(p_b p_\nu) = 4\,(p_t p_\ell)\, p_b (q - p_\ell)
$$

Polar angle in the top quark rest frame

- **Dependence on the angle** $\cos\theta_{b\ell}$ **in the top quark rest system**

$$L_{\mu\nu}H^{\mu\nu} = (m_t E_\ell)E_b\Big(q_0 + |\vec{q}| - E_\ell(1+\cos\theta_{b\ell})\Big)$$

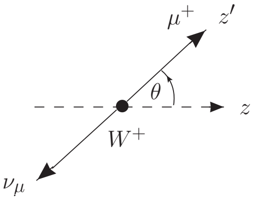


- **This is how one explores angular correlations in low energy applications such as in neutron** β**–decay.**
- **disadvantage: opening angle is restricted to**

$$\boldsymbol{\theta_{b\ell}(\mathrm{min}) \quad \leq \quad \theta_{b\ell} \quad \leq \quad \theta_{b\ell}(\mathrm{max})}$$

Polar angle distribution in $\cos\theta$

- **Definition of angle θ in the W^+ rest frame. z–axis is original flight direction of W^+.**



- **Dependence on** $\cos\theta$**. The lepton momentum in the W^+ rest frame is given by**

$$p_\ell^\mu = \frac{m_W}{2}(1; \sin\theta, 0, \cos\theta)$$

Polar angle in the top quark rest frame

- **Boost the lepton momentum to the top quark rest system ($y^2 = q^2/m_t^2$)**

$$
\begin{aligned}
p_\ell(t_{rf}) &= \begin{pmatrix} q_0/m_W & 0 & 0 & |\vec{q}|/m_W \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ |\vec{q}|/m_W & 0 & 0 & q_0/m_W \end{pmatrix} \frac{m_W}{2} \begin{pmatrix} 1 \\ \sin\theta \\ 0 \\ \cos\theta \end{pmatrix} \\
&= \frac{m_t}{4} \begin{pmatrix} (1+y^2) + (1-y^2)\cos\theta \\ 2y\sin\theta \\ 0 \\ (1-y^2) + (1+y^2)\cos\theta \end{pmatrix}
\end{aligned}
$$

- $L_{\mu\nu}H^{\mu\nu}$ **can now be evaluated in the top quark rest frame**

$$
\begin{aligned}
L_{\mu\nu}H^{\mu\nu} &= 4(p_t p_\ell)\Big(p_b(q - p_\ell)\Big) \\
&= \frac{m_t^4}{4}(1-y^2)\Big(\sin^2\theta + y^2(1-\cos\theta)^2\Big)
\end{aligned}
$$

- **The same angular decay distribution in** $\cos\theta$ **can be obtained much faster using helicity methods**

Off-shell effects and scalar degrees of freedom

When the gauge boson is off its mass shell $q^2 \neq m^2_{W,Z}$ one has to take into account the scalar degrees of freedom of the gauge bosons. Take the unitary gauge and write out the gauge boson propagator

$$H_{\mu\nu} L^{\mu\nu} \quad \Rightarrow \quad H_{\mu\nu} \left(g^{\mu\mu'} - \frac{q^\mu q^{\mu'}}{m_W^2} \right) \left(g^{\nu\nu'} - \frac{q^\nu q^{\nu'}}{m_W^2} \right) L_{\mu'\nu'}$$

Comment: In low energy applications one drops the terms $q^\mu q^{\mu'}/m_W^2$. Split the propagator numerator into a spin 1 and a spin 0 piece

$$\Big(\underbrace{-g^{\mu\mu'} + \frac{q^\mu q^{\mu'}}{m^2 W}}_{\text{spin 1}} - \underbrace{\frac{q^\mu q^{\mu'}}{q^2}(1 - \frac{q^2}{m_W^2})}_{\text{spin 0}} \Big) \Big(\underbrace{-g^{\nu\nu'} + \frac{q^\nu q^{\nu'}}{m^2 W}}_{\text{spin 1}} - \underbrace{\frac{q^\nu q^{\nu'}}{q^2}(1 - \frac{q^2}{m_W^2})}_{\text{spin 0}} \Big)$$

There are three contributions

$$\begin{array}{ll} \mathbf{A:} & \text{spin}\,\mathbf{1} \otimes \text{spin}\,\mathbf{1} \\ \mathbf{B:} & -\,(\,\text{spin}\,\mathbf{1} \otimes \text{spin}\,\mathbf{0} + \text{spin}\,\mathbf{0} \otimes \text{spin}\,\mathbf{1}\,) \\ \mathbf{C:} & \text{spin}\,\mathbf{0} \otimes \text{spin}\,\mathbf{0} \end{array}$$

Note the minus sign in case B. This minus sign has the same origin as was encountered before in the factor $(-1)^{(J+J')}$.

- **Scalar contributions are $O(m_\ell^2)$ since $q^\mu L_{\mu\nu} \sim \mathbf{O}(\mathbf{m}_\ell)$**
- **However, q^2 can be small since the range of off-shellness is**

$$(m_{\ell\,1} + m_{\ell\,2})^2 \le q^2 \le (M_1 - M_2)^2$$

for $\quad B_1(M_1) \to B_2(M_2) + \ell_1(m_{\ell_1}) + \ell_2(m_{\ell_2})$

- **The issue of gauge invariance**
  An identical result is obtained in the general 't Hooft-Feynman R_ξ gauge (for $\xi \to \infty$) where one has to take into account also Goldstone boson exchange. The coupling to the final state fermion-pair is crucial in the R_ξ gauge.
- **The term q^2/M_W^2 in the spin 0 propagator term is usually dropped in low energy applications and also in the charm and bottom sector.**

Off-shell effects in the decay $t \to b + W^+$

- **Zero width approximation**

$$\frac{d\,\Gamma}{d\,q^2} \sim H_{\mu\nu}\left(g^{\mu\mu'} - \frac{q^\mu q^{\mu'}}{m_W^2}\right)\left(g^{\nu\nu'} - \frac{q^\nu q^{\nu'}}{m_W^2}\right) L_{\mu'\nu'} \quad \delta(q^2 - m_W^2)$$

- **Finite width approximation**
Smear the finite-width formula using

$$\delta(q^2 - m_W^2) \quad \longrightarrow \quad \frac{m_W \Gamma_W}{\pi} \frac{1}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2}$$

and integate in the limits

$$m_\ell^2 \leq q^2 \leq (m_t - m_b)^2$$

Numerically the finite width corrections amount to -1.55% in $\Gamma_{t\to b+W^+}$. Couriously, the negative finite width corrections are almost completely cancelled by the positive first order electroweak corrections.

Scalar contributions in some sample decay processes

Scalar contributions are of $O(m_\ell^2)$. They are therefore important for decay processes where the lepton mass is comparable to the scale of the decay process. Sample decay processes are

- **Decays involving the τ**

$$\begin{aligned}
B \to D + \tau\nu_\tau{}^{*} \quad &: \quad \Gamma_S/\Gamma \approx 58\,\% \\
B \to D^* + \tau\nu_\tau{}^{*} \quad &: \quad \Gamma_S/\Gamma \approx 7\,\% \\
B \to \pi + \tau\nu_\tau{}^{*} \quad &: \quad \Gamma_S/\Gamma \approx (30-50)\,\% \\
H \to W^+W^{-*}(\to \tau^-\nu_\tau) \quad &: \quad \Gamma_S/\Gamma = 0.73\,\% \\
H \to ZZ^*(\to \tau^+\tau^-) \quad &: \quad \Gamma_S/\Gamma = 1.19\,\%
\end{aligned}$$

  * **These decays have been widely discussed in the literature because the scalar contribution can be augmented by charged Higgs exchange.**

- **Hadronic semi-inclusive decays $H \to ZZ^*(\to b\bar{b})$**

$$H \to ZZ^*(\to b\bar{b}) : \quad \Gamma_S/\Gamma = 7.9\,\%$$

- **Neutron β-decay.**
  The electron mass is comparable to the scale $(m_n - m_p)$.

$$n \to p + e^- \bar{\nu}_e : \qquad \Gamma_S/\Gamma = 19\,\%$$

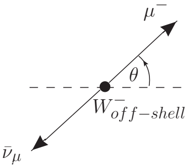
- **Semileptonic hyperon decays involving the μ**
  The muon mass is comparable to the scale $(m_{\Xi^0} - m_{\Sigma^+})$

$$\Xi^0 \to \Sigma^+ + \mu^- \bar{\nu}_\mu$$

Forward-Backward Asymmetry A_{FB} of lepton pair

- **An important observation: There are parity-conserving Forward-Backward asymmetries arising from scalar-vector interference effects.**
- **Forward-Backward Asymmetry**

$$A_{FB} = \frac{\Gamma_F - \Gamma_B}{\Gamma_F + \Gamma_B}$$



- $A_{FB} \neq 0$: **parity-odd effect**
- J^P **content of currents:** $V(1^-, 0^+)$ **and** $A(1^+, 0^-)$
- **Two sources of parity-odd effects**

$$\begin{array}{lll} 1) & \text{parity} - \text{violating from} & V(1^-)A(1^+),\ V(0^+)A(0^-) \\ 2) & \text{parity} - \text{conserving from} & V(0^+)V(1^-),\ A(0^-)A(1^+) \end{array}$$

Rotation of density matrices

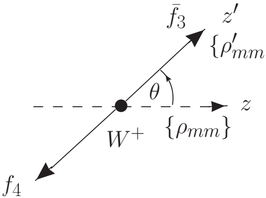
For concreteness we discus the decay of a W^+ into a fermion pair, i.e. $W^+ \to \bar{f}_3 f_4$ described by the helicity amplitudes $h_{\lambda_3 \lambda_4}$ ($\lambda_3, \lambda_4 = \pm 1/2$). In the rest frame of the W^+ the antifermion $\bar{f}_3$ moves in the z'–direction.

- **Consider first the decay of an unpolarized W^+ into a fermion pair. The decay rate is given by**

$$\Gamma \quad \sim \quad \sum_{\text{helicities}} |h_{\lambda_3 \lambda_4}|^2$$

- **Consider next the decay of a polarized W^+ into a fermion pair. The polarization is given in terms of the spin density matrix ρ'_{mm} with $m = \lambda_3 - \lambda_4$. One then has**

$$\Gamma \quad \sim \quad \sum_{\text{helicities}} \rho'_{mm} |h_{\lambda_3 \lambda_4}|^2$$



Rotation of density matrices

- **Now assume that the W^+ was polarized in a production process characterized by a z–axis as e.g. in the decay $t \to b + W^+$ discussed before. The spin density matrix of the W^+ in the z–system is given in terms of the helicity amplitudes for the decay $t \to b + W^+$, i.e. $H_{\lambda_1;\lambda_2\lambda_W}$. One has**

$$\rho_{m=\lambda_W,\, m=\lambda_W} = |H_{\lambda_2\lambda_W}|^2$$

Then "rotate" the density matrix. Rotation is from (x, y, z) to (x', y, z') by angle θ around the y–axis.

$$\Gamma(\theta) \sim \sum_{\text{helicities}} |h_{\lambda_3\lambda_4}|^2 \underbrace{d^1_{\lambda_W,\lambda_3-\lambda_4}(\theta)\rho_{\lambda_W,\lambda'_W} d^1_{\lambda'_W,\lambda_3-\lambda_4}(\theta)}_{\text{rotated density matrix } \rho'}$$

General polarized two-body decay

- Take the two particle decay $a \to b + c$ of a spin J_a particle where the polarization of particle a in the frame (x, y, z) is given by $\rho_{\lambda_a \lambda'_a}$.
- Consider a second frame (x', y', z') obtained from (x, y, z) by the rotation $R(\theta, \phi, 0)$ and whose z–axis is defined by particle b. The polarization density matrix ρ' in the frame (x', y', z') is obtained by a "rotation" of the density matrix ρ from the frame (x, y, z) to the frame (x', y', z').
- The rate for $a \to b + c$ is then given by the the sum of the decay probabilities $|h_{\lambda_b \lambda_c}|^2$ (with $\lambda_a = \lambda_b - \lambda_c$) weighted by the diagonal terms of the density matrix ρ' of particle a in the frame (x', y', z'). One has

$$\Gamma_{a\to b+c}(\theta,\phi) \propto \sum_{\lambda_a,\lambda'_a,\lambda_b,\lambda_c} |h_{\lambda_b\lambda_c}|^2 \underbrace{D^{J*}_{\lambda_a,\lambda_b-\lambda_c}(\theta,\phi)\rho_{\lambda_a,\lambda'_a} D^{J}_{\lambda'_a,\lambda_b-\lambda_c}(\theta,\phi)}_{\text{rotated density matrix } \rho'}$$

  where

$$D^J_{m,m'}(\theta,\phi) = e^{-im\phi} d^J_{m\,m'}(\theta)$$

- All master formulas discussed in this lecture can be obtained by a repeated application of the basic two-body formula.

Wigner's d^J–functions for J=1/2 and J=1

- **Spin** $1/2$

$$d^{1/2}_{mm'}(\theta) = \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix}$$

- **Spin 1**

$$d^{1}_{mm'}(\theta) = \begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{\sqrt{2}}\sin\theta & \cos\theta & -\frac{1}{\sqrt{2}}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{pmatrix}$$

The rows and columns are labeled in the order $(1/2, -1/2)$ **and** $(1, 0, -1)$**, respectively.**

T-odd contributions

- **Take again the cascade decay** $\Xi^0 \to \Sigma^+(\to p\pi^0) + W^-_{off-shell}(\to \ell^- \nu_\ell)$
- **Consider the helicity configuration**

$$(\lambda_\Sigma = -1/2, \lambda_W = 0; \lambda'_\Sigma = 1/2, \lambda'_W = 1)$$
$$(\lambda_S igma = 1/2, \lambda_W = 1; \lambda'_\Sigma = -1/2, \lambda'_W = 0)$$

- **This will lead to the bilinear combinations**

$$\begin{aligned} H_{\frac{1}{2}1} H^*_{-\frac{1}{2}0}\, e^{i(\pi-\chi)} \quad &+ \quad H_{-\frac{1}{2}0} H^*_{\frac{1}{2}1}\, e^{-i(\pi-\chi)} \\ &= \quad -2\cos\chi \mathrm{Re} \mathrm{H}_{\frac{1}{2}1} \mathrm{H}^*_{-\frac{1}{2}0} - 2\sin\chi \mathrm{Im} \mathrm{H}_{\frac{1}{2}1} \mathrm{H}^*_{-\frac{1}{2}0}\ . \end{aligned}$$

- **Take the imaginary part contributions and put in the remaining θ and θ_B dependent trigonometric factors. One has the two terms**

$$\sin\theta \sin\chi \sin\theta_B \mathrm{Im} \mathrm{H}_{\frac{1}{2}1} \mathrm{H}^*_{-\frac{1}{2}0}$$

  and

$$\cos\theta \sin\theta \sin\chi \sin\theta_B\, \mathrm{Im} \mathrm{H}_{\frac{1}{2}1} \mathrm{H}^*_{-\frac{1}{2}0}$$

  proportional to $\sin\chi$**.**
- **Rewrite the product of angular factors in terms of scalar and pseudoscalar products using the momentum representations in the (x, y, z)–system.**

T-odd contributions

- **For the normalized momenta one has ($\hat{p}^2 = 1$)**

$$
\begin{aligned}
\hat{p}_{l^-} &= (\sin\theta\cos\chi, \sin\theta\sin\chi, -\cos\theta) \\
\hat{p}_W &= (0, 0, -1) \\
\hat{p}_{\Sigma^+} &= (0, 0, 1) \\
\hat{p}_p &= (\sin\theta_B, 0, \cos\theta_B)\ ,
\end{aligned}
$$

where the momenta have unit length indicated by a hat notation.

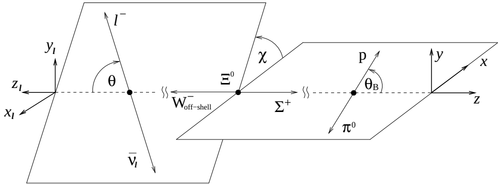


Figure: Definition of the polar angles θ and θ_B, and the azimuthal angle χ in the joint angular decay distribution of an unpolarized Ξ^0 in the cascade decay $\Xi^0 \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l$. The coordinate system (x_l, y_l, z_l) is obtained from the coordinate system (x, y, z) by a 180° rotation around the y–axis.

T-odd contributions

- **The angular factors can be rewritten as**

$$\begin{aligned} \sin\theta \sin\chi \sin\theta_B &= \hat{p}_W \cdot (\hat{p}_{l^-} \times \hat{p}_p) \\ \cos\theta \sin\theta \sin\chi \sin\theta_B &= (\hat{p}_{l^-} \cdot \hat{p}_W)\,[\,\hat{p}_W \cdot (\,\hat{p}_{l^-} \times \hat{p}_p)] \end{aligned}$$

- **Under time reversal ($t \to -t$) one has ($p \to -p$). The above invariants involve an odd number of momenta, i.e. they change sign under time reversal.**
- **This has led to the notion of the so–called T–odd obsevables. Observables that multiply T–odd momentum invariants are called T–odd obsevables.**

T-odd contributions

- **They can be contributed to by true CP–violating effects or by final state interaction effects (imaginary parts of loop contributions)**
- **One can distinguish between the two sources of T–odd effects by comparing with the corresponding antihyperon decays since phases from CP–violating effects change sign whereas phases from final state interaction effects do not change sign when going from hyperon to antihyperon decays.**

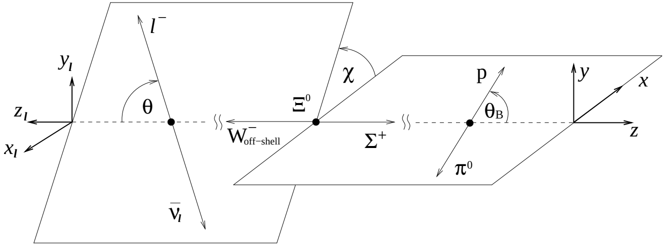
Joint angular decay distribution $\Xi^0 \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l$

Consider the cascade decay $\Xi^0 \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l$ for an unpolarized Ξ^0. The angular dependence can be obtained by setting $\vec{P}^0_\Xi = 0$ in the five-fold angular decay distribution.

- **The relevant master formula reads**

$$W(\theta, \chi, \theta_B) \propto \sum_{\lambda_l, \lambda_W, \lambda'_W, J, J', \lambda_2, \lambda'_2, \lambda_3} (-1)^{J+J'} |h^{V-A}_{\lambda_\ell \lambda_\nu = \pm 1/2}|^2 e^{i(\lambda_W - \lambda'_W)(\pi - \chi)}$$
$$\delta_{\lambda_2 - \lambda_W, \lambda'_2 - \lambda'_W} d^J_{\lambda_W, \lambda_l - \lambda_\nu}(\theta) d^{J'}_{\lambda'_W, \lambda_l - \lambda_\nu}(\theta) H_{\lambda_2 \lambda_W} H^*_{\lambda'_2 \lambda'_W}$$
$$d^{\frac{1}{2}}_{\lambda_2 \lambda_3}(\theta_B) d^{\frac{1}{2}}_{\lambda'_2 \lambda_3}(\theta_B) |h^B_{\lambda_3 0}|^2 \ .$$

Joint angular decay distribution $\Xi^0 \to \Sigma^+(\to p + \pi^0) + \ell^- + \bar{\nu}_\ell$



Definition of the polar angles θ and θ_B, and the azimuthal angle χ in the joint angular decay distribution of an unpolarized Ξ^0 in the cascade decay $\Xi^0 \to \Sigma^+(\to p + \pi^0) + l^- + \bar{\nu}_l$. The coordinate system (x_l, y_l, z_l) is obtained from the coordinate system (x, y, z) by a 180° rotation around the y–axis.

Helicity non-flip term

- **Write out the corresponding normalized decay distribution. Helicity amplitudes are are assumed relatively real. One obtains**

$$\frac{d\Gamma}{dq^2 d\cos\theta d\chi d\cos\theta_B} = B(B_2 \to B_3 + \pi)\frac{1}{6}\frac{G^2}{(2\pi)^4}|V_{us}|^2\frac{(q^2 - m_l^2)^2 p}{8M_1^2 q^2} \quad (6.1)$$

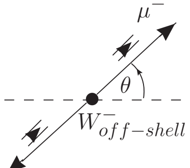
$$\Big[\frac{3}{8}(1 \mp \cos\theta)^2|H_{\frac{1}{2}1}|^2(1 + \alpha_B\cos\theta_B)$$

$$+\frac{3}{8}(1 \pm \cos\theta)^2|H_{-\frac{1}{2}-1}|^2(1 - \alpha_B\cos\theta_B)$$

$$+\frac{3}{4}\sin^2\theta(|H_{\frac{1}{2}0}|^2(1 + \alpha_B\cos\theta_B) + |H_{-\frac{1}{2}0}|^2(1 - \alpha_B\cos\theta_B))$$

$$\pm\frac{3}{2\sqrt{2}}\alpha_B\sin\theta\cos\chi\sin\theta_B((1 \mp \cos\theta)H_{-\frac{1}{2}0}H_{\frac{1}{2}1}$$

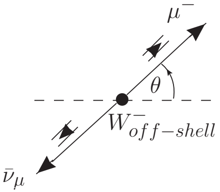
$$+(1 \pm \cos\theta)H_{\frac{1}{2}0}H_{-\frac{1}{2}-1}) + \textbf{lepton helicity flip terms}\Big]$$



Helicity flip term

- **We have to add the helicity flip terms in the square bracket $\Big[\ldots\Big]$**

$$
\begin{aligned}
&+\frac{m_l^2}{2q^2}\Big\{\frac{3}{2}|H_{\frac{1}{2}t}|^2(1+\alpha_B\cos\theta_B)+\frac{3}{2}|H_{-\frac{1}{2}t}|^2(1-\alpha_B\cos\theta_B)\\
&-3\cos\theta\,(H_{\frac{1}{2}t}H_{\frac{1}{2}0}(1+\alpha_B\cos\theta_B)+H_{-\frac{1}{2}t}H_{-\frac{1}{2}0}(1-\alpha_B\cos\theta_B))\\
&+\frac{3}{2}\cos^2\theta(|H_{\frac{1}{2}0}|^2(1+\alpha_B\cos\theta_B)+|H_{-\frac{1}{2}0}|^2(1-\alpha_B\cos\theta_B))\\
&+\frac{3}{4}\sin^2\theta(|H_{\frac{1}{2}1}|^2(1+\alpha_B\cos\theta_B)+|H_{-\frac{1}{2}-1}|^2(1-\alpha_B\cos\theta_B))\\
&-\frac{3}{\sqrt{2}}\alpha_B\sin\theta\cos\chi\sin\theta_B(H_{-\frac{1}{2}t}H_{\frac{1}{2}1}-H_{\frac{1}{2}t}H_{-\frac{1}{2}-1})\\
&+\frac{3}{\sqrt{2}}\alpha_B\sin\theta\cos\theta\cos\chi\sin\theta_B(H_{-\frac{1}{2}0}H_{\frac{1}{2}1}-H_{\frac{1}{2}0}H_{-\frac{1}{2}-1})\Big\}
\end{aligned}
$$



Helicity and invariant amplitudes

System 1

- **System 1 B_1 at rest. The effective current j_{eff} with momentum q^μ moves in the positive z–direction while B_2 moves in the negative z–direction.**

$$p_1 = (M_1; 0, 0, 0) \qquad q^\mu = (q_0; 0, 0, |\vec{q}|) \qquad p_2^\mu = (E_2; 0, 0, -|\vec{q}|)$$

$$\lambda_1 = -\lambda_2 + \lambda_j$$

Convenient relations in system 1 are

$$\begin{aligned} 2M_1(E_2 + M_2) &= Q_+ \\ 2M_1|\vec{q}| &= \sqrt{Q_+ Q_-} \end{aligned}$$

where

$$Q_\pm = (M_1 \pm M_2)^2 - q^2$$

System 1

spinors are given by

$$\bar{u}_2(\pm\frac{1}{2}, p_2) = \sqrt{E_2 + M_2}\left(\chi^\dagger_\mp, \frac{\mp|\vec{q}\,|}{E_2 + M_2}\chi^\dagger_\mp\right)$$

$$u_1(\pm\frac{1}{2}, p_1) = \sqrt{2M_1}\begin{pmatrix} \chi_\pm \\ 0 \end{pmatrix},$$

where $\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **and** $\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **are the usual Pauli two-spinors.For the four polarization four-vectors of the effective current we have**

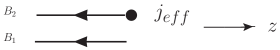
$$\epsilon^\mu(t) = \frac{1}{\sqrt{q^2}}(q_0; 0, 0, |\vec{q}\,|)$$

$$\epsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}}(0; \mp 1, -i, 0)$$

$$\epsilon^\mu(0) = \frac{1}{\sqrt{q^2}}(|\vec{q}\,|; 0, 0, q_0)$$

System 2

- **System 2 The effective current j_{eff} is at rest. Both B_1 and B_2 move in the negative z-direction.**



$$p_1^\mu = (E_1'; 0, 0, -|\vec{p}|\,') \qquad q^\mu = (\sqrt{q^2}; 0, 0, 0) \qquad p_2^\mu = (E_2'; 0, 0, -|\vec{p}\,'|)$$

$$\lambda_1 = -\lambda_2 + \lambda_j$$

Convenient relations in system II are

$$\begin{aligned} |\vec{p}\,'| &= \frac{\sqrt{Q_+ Q_-}}{2\sqrt{q^2}} \\ (E_1' + M_1)(E_2' + M_2) &= \frac{Q_+}{4q^2}(M_1 - M_2 + \sqrt{q^2})^2 \end{aligned}$$

System 2

- **The spinors in system 2 are given by**

$$\bar{u}_2(\pm\frac{1}{2}, p_2) = \sqrt{E'_2 + M_2}\left(\chi^\dagger_\mp, \frac{\mp|\vec{p}\,'|}{E'_2 + M_2}\chi^\dagger_\mp\right)$$

$$u_1(\pm\frac{1}{2}, p_1) = \sqrt{E'_1 + M_1}\begin{pmatrix} \chi_\mp \\ \frac{\pm|\vec{p}\,'|}{E'_1 + M_1}\chi_\mp \end{pmatrix}$$

- **For the four polarization four-vectors of the effective current we have**

$$\epsilon^\mu(t) = (1;0,0,0)$$

$$\epsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}}(0;\mp 1, -i, 0)$$

$$\epsilon^\mu(0) = (0;0,0,1)$$

Evaluation of helicity amplitudes in system 1

- **Definition of helicity amplitudes**

$$H^{V,A}_{\lambda_2 \lambda_W} = M^{V,A}_{\mu}(\lambda_2)\epsilon^{*\mu}(\lambda_j) \ .$$

- **The current matrix elements are expanded in terms of invariants**

$$M^V_\mu =< B_2|J^V_\mu|B_1 >= \bar{u}_2(p_2)\left[F^V_1(q^2)\gamma_\mu - \frac{F^V_2(q^2)}{M_1} i\sigma_{\mu\nu}q^\nu + \frac{F^V_3(q^2)}{M_1} q_\mu\right] u_1(p_1) \ ,$$

$$M^A_\mu =< B_2|J^A_\mu|B_1 >= \bar{u}_2(p_2)\left[F^A_1(q^2)\gamma_\mu - \frac{F^A_2(q^2)}{M_1} i\sigma_{\mu\nu}q^\nu + \frac{F^A_3(q^2)}{M_1} q_\mu\right] \gamma_5 u_1(p_1)$$

(we define $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$.
- **We do not explicitly annote the helicity of the parent baryon B_1 in the helicity amplitudes since (in system 1) λ_1 is fixed by the relation $\lambda_1 = -\lambda_2 + \lambda_W$.**
- **Possible helicity configurations** $(\lambda_1 = -\lambda_2 + \lambda_W)$**:**

λ_1	λ_2	λ_W
1/2	-1/2	0
-1/2	1/2	0
1/2	1/2	1
-1/2	-1/2	-1

Results for helicity amplitudes in system 1

- **The helicity amplitudes are given by**

$$
\begin{aligned}
H^V_{\frac{1}{2}t} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\Big((M_1 - M_2)F_1^V + q^2/M_1 F_3^V\Big) \\
H^V_{\frac{1}{2}1} &= \sqrt{2Q_-}\Big(+F_1^V + (M_1 + M_2)/M_1 F_2^V\Big) \\
H^V_{\frac{1}{2}0} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\Big((M_1 + M_2)F_1^V + q^2/M_1 F_2^V\Big)
\end{aligned}
$$

$$
\begin{aligned}
H^A_{\frac{1}{2}t} &= \frac{\sqrt{Q_-}}{\sqrt{q^2}}\Big((M_1 + M_2)F_1^A - q^2/M_1 F_3^A\Big) \\
H^A_{\frac{1}{2}1} &= \sqrt{2Q_+}\Big(F_1^A - (M_1 - M_2)/M_1 F_2^A\Big) \\
H^A_{\frac{1}{2}0} &= \frac{\sqrt{Q_+}}{\sqrt{q^2}}\Big((M_1 - M_2)F_1^A - q^2/M_1 F_2^A\Big)
\end{aligned}
$$

Parity relations

- **From parity or from an explicit calculation one has**

$$
\begin{aligned}
H^V_{-\lambda_2,-\lambda_W} &= H^V_{\lambda_2,\lambda_W} \\
H^A_{-\lambda_2,-\lambda_W} &= -H^A_{\lambda_2,\lambda_W}
\end{aligned}
$$

With a little bit of work one can show that

$$\mathbf{H}^{\mathbf{V,A}}_{\lambda_2,\lambda_W}(\text{system2}) = \mathbf{H}^{\mathbf{V,A}}_{\lambda_2,\lambda_W}(\text{system1})$$

- **One can conclude: Helicity amplitudes are boost invariant. Thus λ_W can be interpeted as the m quantum number of the W in the W rest system!**

Polar angle distribution using helicity methods

- **We are finally ready to derive the polar angle distribution $W(\theta)$ using helicity methods**

$$L_{\mu\nu}H^{\mu\nu} = \frac{1}{8}\sum_{\lambda_j=0,-1} |H^{V-A}_{-\frac{1}{2}\,\lambda_j}|^2 \, d^1_{\lambda_j 1}(\theta)\, d^1_{\lambda_j 1}(\theta)\, |h^{V-A}_{\frac{1}{2},-\frac{1}{2}}|^2$$

$$= \frac{m_t^4}{4}(1-y^2)\left(\sin^2\theta + y^2(1-\cos\theta)^2\right)$$

where we have used $(y^2 = q^2/m_t^2)$

$$|H^{V-A}_{-\frac{1}{2}\,0}|^2 = 4m_t^2\frac{1-y^2}{y^2} \qquad |H^{V-A}_{-\frac{1}{2}\,-1}|^2 = 8m_t^2(1-y^2)$$

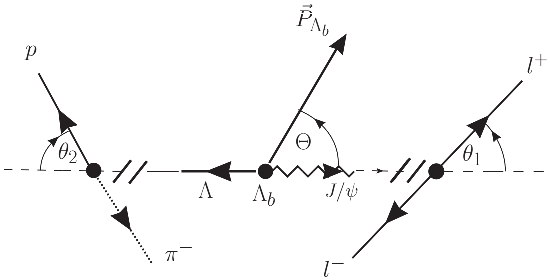
$$d^1_{01}(\theta) = \frac{1}{\sqrt{2}}\sin\theta \qquad d^1_{-11}(\theta) = \frac{1}{2}(1-\cos\theta)$$

$$|h^{V-A}_{\frac{1}{2},-\frac{1}{2}}|^2 = 8q^2$$

- **Same result as before**

The decay $\Lambda_b(\uparrow) \to \Lambda + J/\psi$

- **There has been a longstanding interest to measure the polarization of hadronically produced hyperons, and charm and bottom baryons (R. Lednicky 1986,1995). Recently the *LHCb* Collaboration has attempted to measure the polarization of hadronically produced Λ_b's. At the same time they measured ratios of squared helicity amplitudes in the decay $\Lambda_b(\uparrow) \to \Lambda + J/\psi$ through an analysis of polar correlations in the cascade decay process.**
- **Consider the the three polar angles that characterize the cascade decay $\Lambda_b(\uparrow) \to \Lambda(\to p + \pi^-) + J/\psi(\to \ell^+\ell^-)$**



The master formula for the polar angle distribution

- **By now we know how to write down the master formula for this three-fold polar angle distribution.**

$$W(\theta,\theta_1,\theta_2) \propto \frac{1}{2}\sum_{helicities} |h^V_{\lambda_1\lambda_2}|^2 \left[d^1_{\lambda_V,\lambda_1-\lambda_2}(\theta_2)\right]^2 \rho_{\lambda_b,\lambda_b}(\theta)$$

$$\delta_{\lambda_b.\lambda_V-\Lambda_\Lambda}|H_{\lambda_\Lambda\lambda_V}|^2 \left[d^{1/2}_{\lambda_\Lambda\lambda_p}(\theta_1)\right]^2 |h^B_{\lambda_p,0}|^2$$

The lepton helicity amplitudes are given by

$$h^V_{-\frac{1}{2},-\frac{1}{2}} = h^V_{+\frac{1}{2},+\frac{1}{2}} = 2m_l\,, \qquad h^V_{-\frac{1}{2},+\frac{1}{2}} = h^V_{+\frac{1}{2},-\frac{1}{2}} = \sqrt{2q^2}\,.$$

- **We also know how to rotate the density matrix of the Λ_b from its production direction (perpendicular to the production plane). In fact, only the diagonal matrix elements are needed.**

$$\rho_{\lambda_\mathrm{b}\lambda'_\mathrm{b}}(\theta_\mathrm{P}) = \frac{1}{2}\begin{pmatrix} 1+\mathrm{P}_b\cos\theta_\mathrm{P} & \mathrm{P}_b\sin\theta_\mathrm{P} \\ \mathrm{P}_b\sin\theta_\mathrm{P} & 1-\mathrm{P}_b\cos\theta_\mathrm{P} \end{pmatrix}$$

The polar angle distribution

- **One can do the helicity sums by hand or use the program by M.A. Ivanov.**

$$
\begin{aligned}
& W(\theta,\theta_1,\theta_2) \propto \\
& \frac{1}{2}|H_{+\frac{1}{2}1}|^2 \left[q^2(1+\cos^2\theta_2)+4m_l^2\sin^2\theta_2\right](1-P_b\cos\theta)(1+\alpha_\Lambda\cos\theta_1) \\
+ \quad & \frac{1}{2}|H_{-\frac{1}{2}-1}|^2 \left[q^2(1+\cos^2\theta_2)+4m_l^2\sin^2\theta_2\right](1+P_b\cos\theta)(1-\alpha_\Lambda\cos\theta_1) \\
+ \quad & |H_{+\frac{1}{2}0}|^2 \left[q^2\sin^2\theta_2+4m_l^2\cos^2\theta_2\right](1+P_b\cos\theta)(1+\alpha_\Lambda\cos\theta_1) \\
+ \quad & |H_{-\frac{1}{2}0}|^2 \left(q^2\sin^2\theta_2+4m_l^2\cos^2\theta_2\right)(1-P_b\cos\theta)(1-\alpha_\Lambda\cos\theta_1)
\end{aligned}
$$

Normalize the helicity amplitudes to the sum of the squared helicity amplitudes $|\widehat{H}_{\lambda_\lambda\lambda_j}|^2 = |H_{\lambda_\lambda\lambda_j}|^2/N$ **with**

$$
N = |H_{+\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}-1}|^2 + |H_{+\frac{1}{2}+1}|^2
$$

.

The polar angle distribution

- **Introduce the linear combinations**

$$
\begin{aligned}
\alpha_b &= |\widehat{H}_{+\frac{1}{2}0}|^2 - |\widehat{H}_{-\frac{1}{2}0}|^2 + |\widehat{H}_{-\frac{1}{2}-1}|^2 - |\widehat{H}_{+\frac{1}{2}+1}|^2 \,, \\
r_0 &= |\widehat{H}_{+\frac{1}{2}0}|^2 + |\widehat{H}_{-\frac{1}{2}0}|^2 \,, \\
r_1 &= |\widehat{H}_{+\frac{1}{2}0}|^2 - |\widehat{H}_{-\frac{1}{2}0}|^2 \,,
\end{aligned}
$$

and write The angular decay distribution can be rearranged into the form ($\varepsilon = m_l^2/q^2$)

$$
\begin{aligned}
\widetilde{W}(\theta, \theta_1, \theta_2) &= \sum_{i=0}^{7} f_i(\alpha_b, r_0, r_1)\, g_i(P_b, \alpha_\Lambda)\, h_i(\cos\theta, \cos\theta_1, \cos\theta_2)\, \ell_i(\varepsilon) \\
&= 1 + \sum_{i=1}^{7} f_i(\alpha_b, r_0, r_1)\, g_i(P_b, \alpha_\Lambda)\, h_i(\cos\theta, \cos\theta_1, \cos\theta_2)\, \ell_i(\varepsilon)
\end{aligned}
$$

The terms $i = 1, ..7$ integrate to zero.

The symbols are

P_b	:	**polarization of** Λ_b
α_b	:	**asymmetry parameter in the decay** $\Lambda \to p + \pi^-$
$v = \sqrt{1 - 4m_\ell^2/m_{J/\psi}^2}$	:	**velocity of the lepton**

i	$f_i(\alpha_b, r_0, r_1)$	$g_i(P_b, \alpha_\Lambda)$	$h_i(\cos\theta, \cos\theta_1, \cos\theta_2)$	$\ell_i(\varepsilon)$
0	1	1	1	$\mathbf{v} \cdot (1 + 2\varepsilon)$
1	α_b	P_b	$\cos\theta$	$\mathbf{v} \cdot (1 + 2\varepsilon)$
2	$2r_1 - \alpha_b$	α_Λ	$\cos\theta_1$	$\mathbf{v} \cdot (1 + 2\varepsilon)$
3	$2r_0 - 1$	$P_b\alpha_\Lambda$	$\cos\theta\cos\theta_1$	$\mathbf{v} \cdot (1 + 2\varepsilon)$
4	$\frac{1}{2}(1 - 3r_0)$	1	$\frac{1}{2}(3\cos^2\theta_2 - 1)$	$\mathbf{v} \cdot \mathbf{v}^2$
5	$\frac{1}{2}(\alpha_b - 3r_1)$	P_b	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta$	$\mathbf{v} \cdot \mathbf{v}^2$
6	$-\frac{1}{2}(\alpha_b + r_1)$	α_Λ	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta_1$	$\mathbf{v} \cdot \mathbf{v}^2$
7	$-\frac{1}{2}(1 + r_0)$	$P_b\alpha_\Lambda$	$\frac{1}{2}(3\cos^2\theta_2 - 1)\cos\theta\cos\theta_1$	$\mathbf{v} \cdot \mathbf{v}^2$

The polar angle distribution

- **The overall factor v in the fifth column is the phase space factor for $J/\psi \to \ell^+\ell^-$. The factors $(1 + 2\epsilon)$ (S-wave dominance) and v^2 (($S - D$)-wave interference) were calculated by us for the first time: T. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij and P. Santorelli**
- **The LHCb Collaboration finds a very small polarization of the Λ_b**

$$P_b = 0.05 \pm 0.07 \pm 0.02$$

- **Our results on helicity amplitudes for the transitions $\Lambda_b \to \Lambda$ agree with experiment. Our calculation is based on the confined covariant quark model developed by us.**

The confined covariant quark model in a nutshell

The confined covariant quark model provides a fieldtheoretic frame work for the constituent quark model.
Its main features are:

- **Particle transitions are calculated from Feynman diagrams involving quark loops**
  Example: $\Lambda_b \to \Lambda$ transition is described by a genuine two-loop diagram
- **High energy behaviour of quark loops are tempered by nonlocal Gaussian-type vertex functions. Particle-quark vertices have interpolating current structure.**
- **Use free local quark propagators**
- **Normalization of vertices provided by the compositeness condition $\to$ correct charge normalization**
- **Universal infrared cut-off provides for effective confinement of quarks $\to$ no quark poles in Feynman diagrams**
- **HQET relations are recovered by using a static propagator for the heavy quark (k_1 is loop momentum)**

$$\frac{1}{m_b - \not{k}_1 - \not{p}_1} \quad \to \quad \frac{1 + \not{v}_1}{-2k_1 v_1 - 2\bar{\Lambda}}$$

Summary

- **The helicity method provides an easy and simple access to angular decay distributions**
- **Polarization and mass effects are readily incorporated**