Heavy quark physics
in the covariant quark model

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HQP'13

Contents

Collaboration

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- **Bratislava (S. Dubnicka, A.Z. Dubnickova, A. Liptaj)**
- **Dubna (M. A. Ivanov)**
- **Mainz (J. G. Körner)**
- **Napoli (P. Santorelli)**
- **Tübingen (T. Gutsche, V. E. Lyubovitskij)**
- **Valparaiso (S. G. Kovalenko)**

Covariant quark model of hadrons

- **Main assumption: hadrons interact via quark exchange only**
- **Interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

Covariant quark model of hadrons

- **Main assumption: hadrons interact via quark exchange only**
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$$\mathcal{L}_{int} = g_H \cdot H(x) \cdot J_H(x)$$

- **Quark currents**

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_M \, q^a_{f_2}(x_2) \qquad \text{Meson}$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \qquad \text{Baryon}$$
$$\times \, \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left[\varepsilon^{a_1 a_2 a_3} q^{T\, a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3) \right]$$

$$J_T(x) = \int dx_1 \ldots \int dx_4 \, F_T(x; x_1, \ldots, x_4) \qquad \text{Tetraquark}$$
$$\times \left[\varepsilon^{a_1 a_2 c} q^{T\, a_1}_{f_1}(x_1) \, C\Gamma_1 \, q^{a_2}_{f_2}(x_2) \right] \cdot \left[\varepsilon^{a_3 a_4 c} \bar{q}^{T\, a_3}_{f_3}(x_3) \, \Gamma_2 C \, \bar{q}^{a_4}_{f_4}(x_4) \right]$$

Compositeness condition $Z_H = 0$

Salam 1962; Weinberg 1963

- A composite field and its constituents are introduced as elementary particles
- The transition of a composite field to its constituents is provided by the interaction Lagrangian
- The renormalization constant $Z^{1/2}$ is the matrix element between a physical state and the corresponding bare state. If there is a stable bound state which we wish to represent by introducing a quasi-particle H, then elementary particle must have renormalization factor Z equal to zero

$$Z_H^{1/2} = < H_{\rm bare} | H_{\rm dressed} > = 0$$

We use the compositeness condition to determine the hadron-quark coupling constant, e.g. in the case of mesons

$$Z_M = 1 - \tilde{\Pi}'(m_M^2) = 0$$

where $\tilde{\Pi}(p^2)$ is the meson mass operator.

The vertex functions and quark propagators

- Translational invariance for the vertex function

$$F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)\,, \qquad \forall a\,.$$

- Our choice:

$$F_B(x, x_1, \ldots, x_n) \;=\; \delta^{(4)}\Big(x - \sum_{i=1}^{n} w_i x_i\Big)\;\Phi_H\Big(\sum_{i<j}(x_i - x_j)^2\Big)$$

where $w_i = m_i / \sum_i m_i$.

- The quark propagators

$$S_q(x_1 - x_2) \;=\; \int \frac{d^4k}{(2\pi)^4 i}\,\frac{e^{-ik(x_1-x_2)}}{m_q - \not{k}}$$

The matrix elements

- The matrix elements are described by a set of the Feynman diagrams which are convolution of the quark propagators and vertex functions.
- Let Π be the matrix element corresponding to the Feynman diagram:

j external momenta;
n quark propagators;
ℓ loop integrations;
m vertices.
In the momentum space it will be represented as

$$\Pi(p_1, \dots, p_j) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

$\tilde{k}_i$ are linear combinations of the loop momenta k_i
$\tilde{p}_i$ are linear combinations of the external momenta p_i

Infrared confinement

- Use the Schwinger representation of the propagator:

$$\frac{m+\not{k}}{m^2-k^2} = (m+\not{k}) \int_0^\infty d\alpha \, \exp[-\alpha(m^2-k^2)]$$

- Choose a simple Gaissian form for the vertex function

$$\Phi(-K^2) = \exp\left(K^2/\Lambda^2\right)$$

where the parameter Λ characterizes the hadron size.

- We imply that the loop integration k proceed over Euclidean space:

$$k^0 \to e^{i\frac{\pi}{2}} k_4 = ik_4 \,, \qquad k^2 = (k^0)^2 - \vec{k}^2 \to -k_E^2 \le 0 \,.$$

- We also put all external momenta p to Euclidean space:

$$p^0 \to e^{i\frac{\pi}{2}} p_4 = ip_4 \,, \qquad p^2 = (p^0)^2 - \vec{p}^2 \to -p_E^2 \le 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

Infrared confinement

- Convert the loop momemta in the numerator into derivatives over external momenta:

$$k_i^{\mu} e^{2kr} = \frac{1}{2} \frac{\partial}{\partial r_{i\,\mu}} e^{2kr},$$

- Move the derivatives outside of the loop integrals.
- Calculate the scalar loop integral:

$$\prod_{i=1}^{n} \int \frac{d^4 k_i}{i\pi^2} e^{kAk+2kr} = \prod_{i=1}^{n} \int \frac{d^4 k_i^E}{\pi^2} e^{-k_E A k_E - 2k_E r_E} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

where a symmetric $n \times n$ real matrix A is positive-definite.

- Use the identity

$$P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1}r\right)$$

to move the exponent to the left.

Infrared confinement

- **Employ the commutator**

$$[\frac{\partial}{\partial r_{i\,\mu}}, r_{j\,\nu}] = \delta_{ij}\, g_{\mu\nu}$$

to make differentiation in

$$P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1} r\right)$$

for any polynomial P. The necessary commutations of the differential operators are done by a FORM program.

- **One obtains**

$$\Pi = \int_0^\infty d^n\alpha\, F(\alpha_1, \ldots, \alpha_n)\,,$$

where F stands for the whole structure of a given diagram.

Infrared confinement

The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt\, \delta(t - \sum_{i=1}^n \alpha_i)$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha\, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big)\, F(t\alpha_1, \ldots, t\alpha_n)\,.$$

Infrared confinement

- **Cut off the upper integration at $1/\lambda^2$**

$$\Pi^c = \int\limits_0^{1/\lambda^2} dt t^{n-1} \int\limits_0^1 d^n\alpha \, \delta\Big(1 - \sum_{i=1}^n \alpha_i\Big) \, F(t\alpha_1, \ldots, t\alpha_n)$$

- **The infrared cut-off has removed all possible thresholds in the quark loop diagram.**

- **We take the cut-off parameter λ to be the same in all physical processes.**

T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij,
Phys. Rev. D81, 034010 (2010)

Infrared confinement

- An example of a scalar one–loop two–point function:

$$\Pi_2(p^2) = \int \frac{d^4k_E}{\pi^2} \frac{e^{-s\,k_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

where the numerator factor $e^{-s\,k_E^2}$ comes from the product of nonlocal vertex form factors of Gaussian form. k_E, p_E are Euclidean momenta ($p_E^2 = -\,p^2$).

- Doing the loop integration one obtains

$$\Pi_2(p^2) = \int_0^\infty dt \frac{t}{(s+t)^2} \int_0^1 d\alpha \, \exp\left\{ -\,t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2 \right\}$$

A branch point at $p^2 = 4m^2$.

Infrared confinement

- By introducing a cut-off in the t–integration one obtains

$$\Pi_2^c(p^2) = \int_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int_0^1 d\alpha \, \exp\left\{ -t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2 \right\}$$

where the one–loop two–point function $\Pi_2^c(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

- The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.

Subtleties: gauging

In order to guarantee local invariance of the strong interaction Lagrangian one multiplies each quark field $q(x_i)$ in nonlocal quark current $J_H(x)$ with a gauge field exponential:

$$q_i(x_i) \to e^{-ie_{q_1} I(x_i,x,P)} q_i(x_i) \quad \text{where} \quad I(x_i,x,P) = \int_x^{x_i} dz_\mu A^\mu(z).$$

The path P connects the end-points of the path integral.
We use the path-independent definition of the derivative of $I(x,y,P)$:

Mandelstam,1962,Terning,1991

$$\lim_{dx^\mu \to 0} dx^\mu \frac{\partial}{\partial x^\mu} I(x,y,P) = \lim_{dx^\mu \to 0} [I(x+dx,y,P') - I(x,y,P)]$$

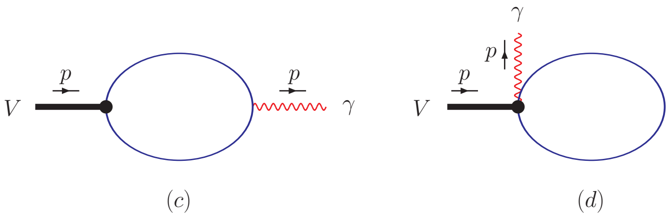
where the path P' is obtained from P by shifting the end-point x by dx.
The definition leads to the key rule

$$\frac{\partial}{\partial x^\mu} I(x,y,P) = A_\mu(x)$$

which in turn states that the derivative of the path integral $I(x,y,P)$ does not depend on the path P originally used in the definition.

Subtleties: gauging

Diagrams describing $V \to \gamma$ transition:



$$M_c^{\mu\nu}(p) = \int \frac{d^4k}{4\pi^2 i} \Phi_V(-k^2)\, \mathrm{tr}\Big(\gamma^\mu S(k + \tfrac{1}{2}p)\gamma^\nu S(k - \tfrac{1}{2}p)\Big)$$

$$M_d^{\mu\nu}(p) = -\int \frac{d^4k}{4\pi^2 i} (2k + \tfrac{1}{2}p)^\mu \int_0^1 d\alpha \Phi_V'\Big(-\alpha(k+\tfrac{1}{2}p)^2 - (1-\alpha)k^2\Big)$$

$$\times \quad \mathrm{tr}\Big(\gamma^\nu S(k)\Big)$$

Subtleties: gauging

If $p = 0$ then the second diagram maybe transfered to the first one by using integration by parts

$$\int \frac{d^4k}{4\pi^2 i} \frac{\partial}{\partial k^\mu}\Big\{\Phi_V\Big(-k^2\Big)\mathrm{tr}\Big(\gamma^\nu S(k)\Big)\Big\} =$$

$$= \int \frac{d^4k}{4\pi^2 i}\Big\{-2k^\mu \Phi_V'\Big(-k^2\Big)\mathrm{tr}\Big(\gamma^\nu S(k)\Big)$$

$$+\Phi_V\Big(-k^2\Big)\mathrm{tr}\Big(\gamma^\mu S(k)\gamma^\nu S(k)\Big)\Big\} = 0.$$

Model parameters

M. A. I., J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D85, 034004 (2012)

Input values for the leptonic decay constants f_H (in MeV) and our least-squares fit values.

	Fit Values	PDG/LAT		This work	PDG/LAT
f_π	128.7	130.4 ± 0.2	f_ω	198.5	198 ± 2
f_K	156.1	156.1 ± 0.8	f_ϕ	228.2	227 ± 2
f_D	205.9	206.7 ± 8.9	$f_{J/\psi}$	415.0	415 ± 7
f_{D_s}	257.5	257.5 ± 6.1	f_{K^*}	213.7	217 ± 7
f_B	191.1	192.8 ± 9.9	f_{D^*}	243.3	245 ± 20
f_{B_s}	234.9	238.8 ± 9.5	$f_{D_s^*}$	272.0	272 ± 26
f_{B_c}	489.0	489 ± 5	f_{B^*}	196.0	196 ± 44
f_ρ	221.1	221 ± 1	$f_{B_s^*}$	229.0	229 ± 46

Model parameters

Input values for some basic electromagnetic decay widths and our least-squares fit values (in keV).

Process	Fit Values	PDG
$\pi^0 \to \gamma\gamma$	5.06×10^{-3}	$(7.7 \pm 0.4) \times 10^{-3}$
$\eta_c \to \gamma\gamma$	1.61	1.8 ± 0.8
$\rho^\pm \to \pi^\pm\gamma$	76.0	67 ± 7
$\omega \to \pi^0\gamma$	672	703 ± 25
$K^{*\pm} \to K^\pm\gamma$	55.1	50 ± 5
$K^{*0} \to K^0\gamma$	116	116 ± 10
$D^{*\pm} \to D^\pm\gamma$	1.22	1.5 ± 0.5
$J/\psi \to \eta_c\gamma$	1.43	1.58 ± 0.37

Model parameters

The results of the fit for the values of quark masses m_{q_i}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} (all in GeV).

m_u	m_s	m_c	m_b	λ	
0.235	0.424	2.16	5.09	0.181	GeV

Λ_π	Λ_K	Λ_D	Λ_{D_s}	Λ_B	Λ_{B_s}	Λ_{B_c}	Λ_ρ
0.87	1.04	1.47	1.57	1.88	1.95	2.42	0.61

Λ_ω	Λ_ϕ	$\Lambda_{J/\psi}$	Λ_{K^*}	Λ_{D^*}	$\Lambda_{D_s^*}$	Λ_{B^*}	$\Lambda_{B_s^*}$
0.47	0.88	1.48	0.72	1.16	1.17	1.72	1.71

The study of heavy flavor physics: motivation

- **To determine the Cabibbo-Kobayashi-Maskawa matrix elements.**
- **To provide insights into the origin of flavor and CP-violation.**
- **To look for new physics beyond the standard model.**
- **The subject to study are heavy hadrons containing a b– or a c–quark and their weak decays.**

The study of heavy flavor physics: motivation

- **To determine the Cabibbo-Kobayashi-Maskawa matrix elements.**
- **To provide insights into the origin of flavor and CP-violation.**
- **To look for new physics beyond the standard model.**
- **The subject to study are heavy hadrons containing a b– or a c–quark and their weak decays.**
- **The main idea in the theoretical studies of heavy-flavor decays is to separate short-distance (perturbative) QCD dynamics from long-distance (nonperturbative) hadronic effects.**
- **One uses the so-called** *naive* **factorization approach which is based on the weak effective Hamiltonian describing quark and lepton transitions in terms of local operators that are multiplied by Wilson coefficients.**
- **The Wilson coefficients characterize the short-distance dynamics and may be reliably evaluated by perturbative methods.**

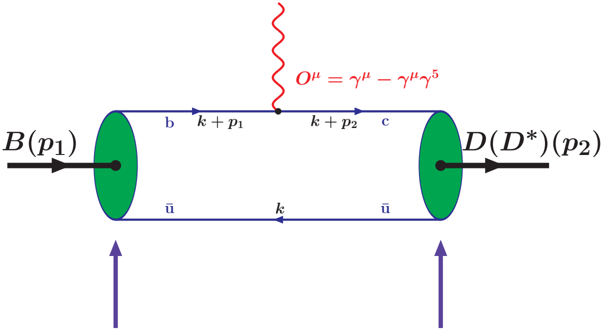
The study of heavy flavor physics: motivation

- **The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.**
- **Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.**
- **A variety of theoretical approaches have been used to evaluate the hadronic form factors:**

The study of heavy flavor physics: motivation

- **The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks.**
- **Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.**
- **A variety of theoretical approaches have been used to evaluate the hadronic form factors:**
  - **The light-cone sum rule (LCSR) approach (Braun, Ball, Khodjamirian et al.)**
  - **Dyson-Schwinger equations in QCD (C.D. Roberts et al.)**
  - **A relativistic quark model (Faustov, Galkin et al.)**
  - **The constituent quark model with dispersion relations (Melikhov et al.)**
  - **A QCD relativistic potential model (Ladisa, et al.)**
  - **A QCD sum rule analysis (P. Colangelo et al.)**

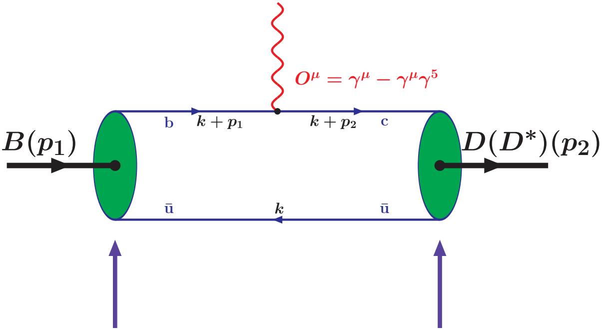
Semileptonic B − D transition



$$\phi_B\left(-\left(k + w_u\, p_1\right)^2\right) \quad \phi_{D(D^*)}\left(-\left(k + w'_u\, p_2\right)^2\right)$$

$$w_u = \frac{m_u}{m_u + m_b} \qquad w'_u = \frac{m_u}{m_u + m_c}$$

Semileptonic $\mathbf{B - D}$ transition



$$\phi_B\left(-(k + w_u\,p_1)^2\right) \quad \phi_{D(D^*)}\left(-(k + w'_u\,p_2)^2\right)$$

$$w_u = \frac{m_u}{m_u+m_b} \qquad w'_u = \frac{m_u}{m_u+m_c}$$

Heavy quark limit: $m_H = m_Q + E, \quad m_Q \to \infty; \quad \Lambda_B = \Lambda_D = \Lambda_{D^*}.$

Isgur-Wise function

$$\frac{1}{m_i - \not{k} - \not{p}_i} \quad \to \quad -\frac{1+\not{v}_i}{2} \cdot \frac{1}{k v_i + E}, \qquad v = \frac{p}{m}$$

Isgur-Wise function

$$\frac{1}{m_i - \not{k} - \not{p}_i} \to -\frac{1+\not{v}_i}{2} \cdot \frac{1}{kv_i + E}, \qquad v = \frac{p}{m}$$

$$M^{\mu}_{BD}(p_1, p_2) = f_+(q^2)(p_1 + p_2)^{\mu} + f_-(q^2)(p_1 - p_2)^{\mu},$$

$$f_{\pm} = \frac{M_c \pm M_b}{2\sqrt{M_b M_c}} \cdot \xi(w),$$

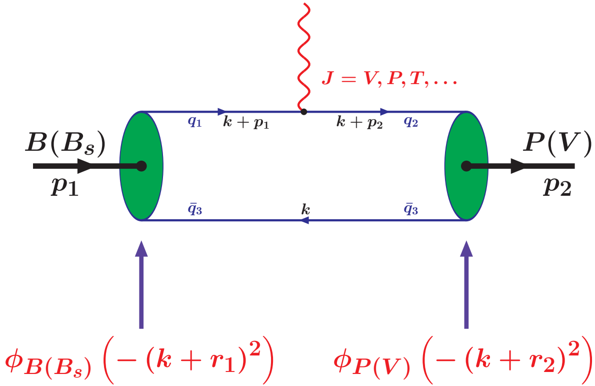
The compositeness condition $Z_H = 0$ provides the correct normalization

$$\xi(w = 1) = 1$$

.

Form factors for semileptonic, nonleptonic and rare **B (B$_s$)** meson decays

M. A. I., J. G. Körner, S. G. Kovalenko, P. Santorelli and G. G. Saidullaeva, Phys. Rev. D85, 034004 (2012)



$$r_i = \frac{m_{q_3}}{m_{q_i}+m_{q_3}}\,p_i$$

The definition of the form factors

$$\langle P'_{[\bar q_3 q_2]}(p_2)\,|\,\bar q_2\, O^{\mu}\, q_1\,|P_{[\bar q_3 q_1]}(p_1)\rangle \;=\; F_+(q^2)\,P^{\mu} + F_-(q^2)\,q^{\mu}$$

$$\langle P'_{[\bar q_3 q_2]}(p_2)\,|\,\bar q_2\,(\sigma^{\mu\nu} q_\nu)\,q_1\,|P_{[\bar q_3 q_1]}(p_1)\rangle \;=\; \frac{i}{m_1+m_2}\left(q^2\,P^{\mu} - q\cdot P\,q^{\mu}\right) F_T(q^2)$$

$$\langle V(p_2,\epsilon_2)_{[\bar q_3 q_2]}\,|\,\bar q_2\, O^{\mu}\, q_1\,|\,P_{[\bar q_3 q_1]}(p_1)\rangle \;=$$

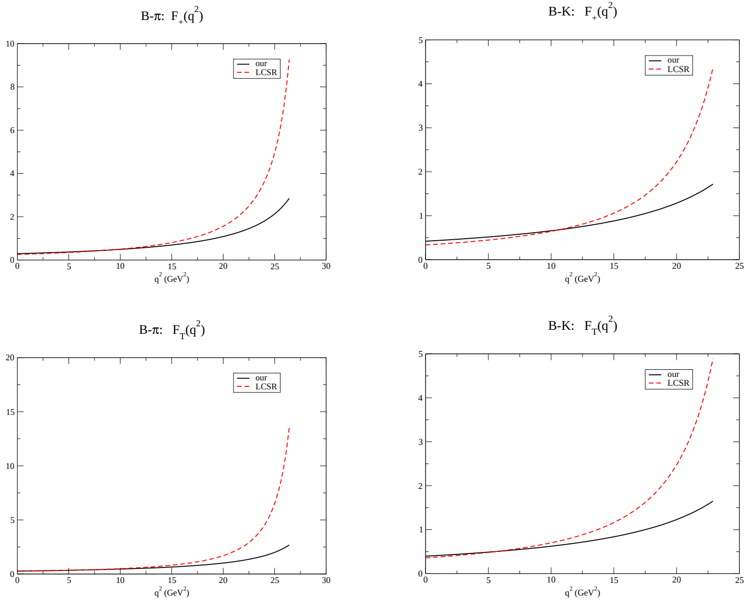
$$= \frac{\epsilon^{\dagger}_{\nu}}{m_1+m_2}\Big(-g^{\mu\nu}\,P\cdot q\,A_0(q^2) + P^{\mu}P^{\nu}A_+(q^2) + q^{\mu}P^{\nu}A_-(q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}P_\alpha\, q_\beta\, V(q^2)\Big)$$

$$\langle V(p_2,\epsilon_2)_{[\bar q_3 q_2]}\,|\,\bar q_2\,(\sigma^{\mu\nu} q_\nu(1+\gamma^5))\,q_1\,|\,P_{[\bar q_3 q_1]}(p_1)\rangle \;=$$

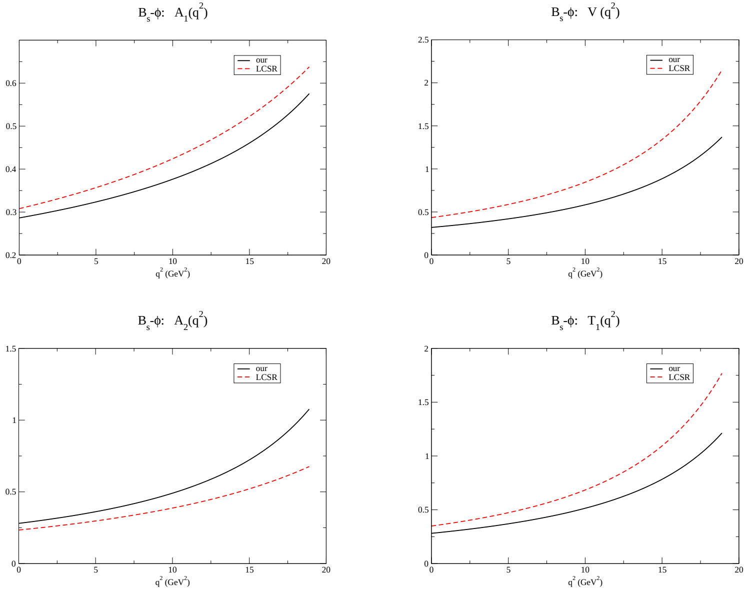
$$= \epsilon^{\dagger}_{\nu}\Big(-(g^{\mu\nu} - q^{\mu}q^{\nu}/q^2)\,P\cdot q\,a_0(q^2) + (P^{\mu}P^{\nu} - q^{\mu}P^{\nu}\,P\cdot q/q^2)\,a_+(q^2) + i\,\varepsilon^{\mu\nu\alpha\beta}P_\alpha\, q_\beta\, g(q^2)\Big)$$

$$P = p_1 + p_2\,, \qquad q = p_1 - p_2\,, \qquad \epsilon^{\dagger}_2\cdot p_2 = 0\,.$$

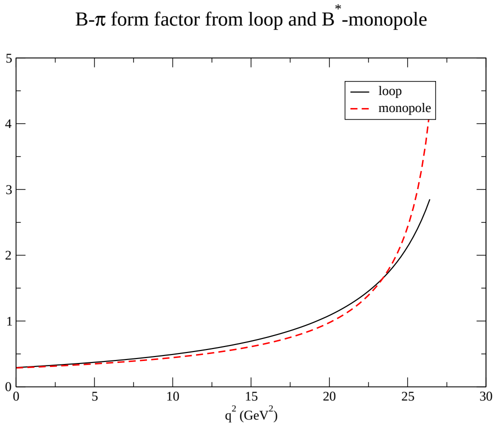
Form factors



Form factors



Form factors



$$F^{B\pi}_{\mathrm{VDM}}(q^2) = \frac{F^{B\pi}_{+}(0)}{m^2_{B*} - q^2}.$$

Nonleptonic $\mathbf{B_s}$ decays

- **The modes $B_s \to D_s^- D_s^+,\ D_s^{*-} D_s^+ + D_s^- D_s^{*+},\ D_s^{*-} D_s^{*+}$ give the largest contribution to $\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H$ for the $B_s - \bar{B}_s$ system.**

- **The mode $B_s \to J/\psi\phi$ is color–suppressed but it is interesting for the search of possible CP-violating new physics effects in $B_s - \bar{B}_s$ mixing.**

- **Nonleptonic $B_s^0 \to J/\psi\eta(\eta')$ decays were observed by Belle Coll.:**

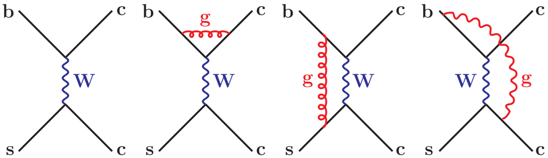
  J. Li *et al.* [Belle Collaboration], Phys. Rev. Lett. 108, 181808 (2012)

- **Their decay widths were calculated in our approach by**

  S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and A. Liptaj Phys. Rev. D 87, 074201 (2013)

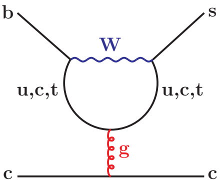
The effective Hamiltonian

Current-current diagrams



tree

QCD one-loop



QCD penguin

The effective Hamiltonian

- **The effective Hamiltonian describing the B_s nonleptonic decays:**

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^{\dagger} \sum_{i=1}^{6} C_i\, Q_i$$

- **Current-current diagrams:**

$$Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} \qquad Q_2 = (\bar{c}_{a_1}\, b_{a_1})_{V-A}, (\bar{s}_{a_2}\, c_{a_2})_{V-A}$$

- **QCD penguin diagram:**

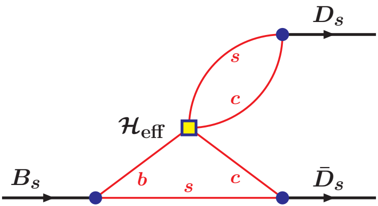
$$Q_3 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A} \qquad Q_4 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A}$$

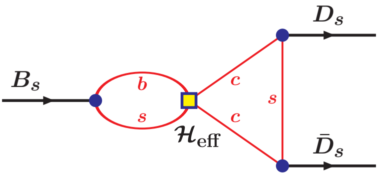
$$Q_5 = (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A} \qquad Q_6 = (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A}$$

$$(\bar{q}q)_{V-A} = \bar{q}\gamma^{\mu}(1-\gamma^5)q \qquad \text{left–chiral current}$$

$$(\bar{q}q)_{V+A} = \bar{q}\gamma^{\mu}(1+\gamma^5)q \qquad \text{right–chiral current}$$

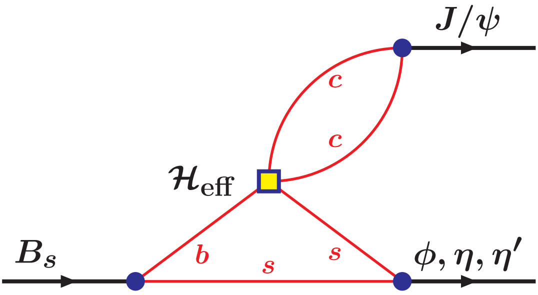
Nonleptonic B_s decays





Annihilation diagram

Nonleptonic B_s decays



Nonleptonic B_s decays

Calculated branching ratios (%) of the B_s nonleptonic decays.

Process	This work	PDG
$B_s \to D_s^- D_s^+$	1.65	$1.04^{+0.29}_{-0.26}$
$B_s \to D_s^- D_s^{*+} + D_s^{*-} D_s^+$	2.40	2.8 ± 1.0
$B_s \to D_s^{*-} D_s^{*+}$	3.18	3.1 ± 1.4
$B_s \to J/\psi \phi$	0.16	0.14 ± 0.05

Nucleon as three-quark state: Lagrangian

T. Gutsche, M. A. I., J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D 86, 074013 (2012))

Lagrangian describing the interaction of proton (antiproton) with its constituents:

$$\mathcal{L}^{p}_{int}(x) = g_N \, \bar{p}(x) \cdot J_p(x) + h.c.$$

The interpolating three-quark current:

$$\begin{aligned} J_p(x) &= \int dx_1 \int dx_2 \int dx_3 \, F_N(x; x_1, x_2, x_3) \, J^{(p)}_{3q}(x_1, x_2, x_3) \\ J^{(p)}_{3q}(x_1, x_2, x_3) &= \Gamma^A \gamma^5 \, d^{a_1}(x_1) \cdot [\epsilon^{a_1 a_2 a_3} \, u^{a_2}(x_2) \, C \, \Gamma_A \, u^{a_3}(x_3)] \, . \end{aligned}$$

There are two kinds of three-quark currents:

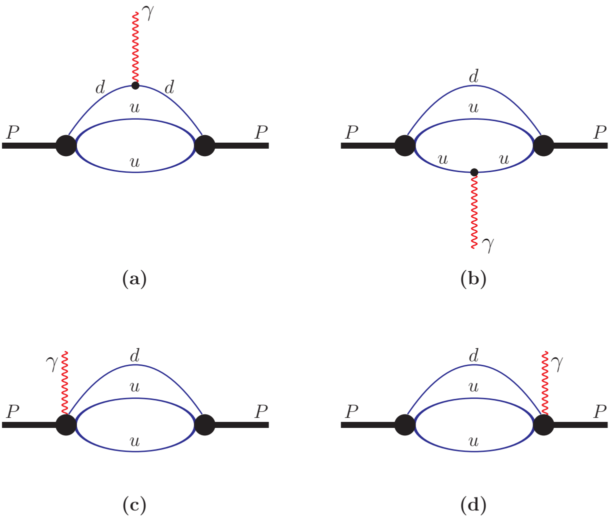
$$\Gamma^A \otimes \Gamma_A = \gamma^\alpha \otimes \gamma_\alpha \quad \text{(vector)} \quad \Gamma^A \otimes \Gamma_A = \frac{1}{2} \sigma^{\alpha\beta} \otimes \sigma_{\alpha\beta} \quad \text{(tensor)}$$

We consider a general linear superposition:

$$J_N = x J_N^T + (1 - x) J_N^V \, , \quad N = p, n$$

with a mixing parameter x $(0 \leq x \leq 1)$.

Electromagnetic vertex function of proton



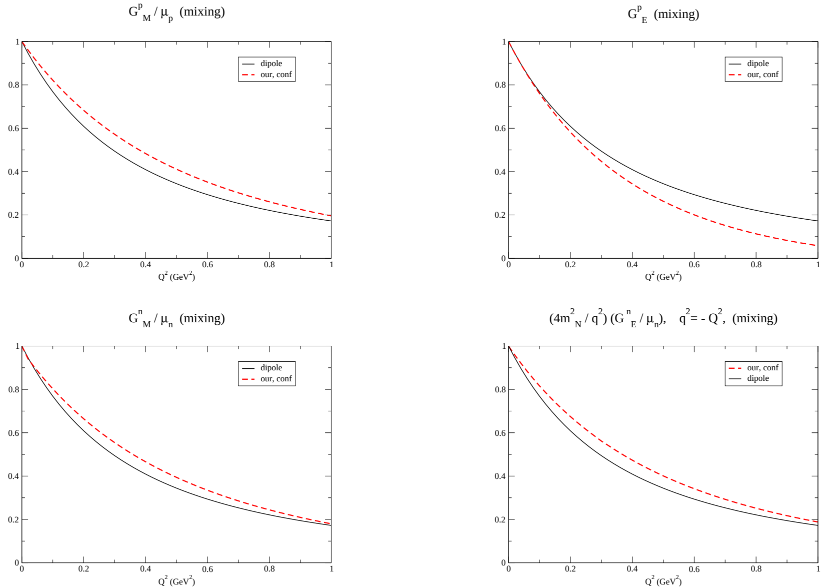
Static properties of nucleons

Parameters:

- a superposition of the V– and T–currents of nucleons with $x = 0.8$
- the size parameter of the nucleon we take $\Lambda_N = 0.5$ GeV.

Quantity	Our results	PDG
μ_p (in n.m.)	2.96	2.793
μ_n (in n.m.)	-1.83	-1.913
r_E^p (fm)	0.805	0.8768 ± 0.0069
$< r_E^2 >^n$ (fm^2)	-0.121	-0.1161 ± 0.0022
r_M^p (fm)	0.688	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.685	$0.862^{+0.009}_{-0.008}$

Electromagnetic form factors



Rare baryon decays $\Lambda_b \to \Lambda \ell^+ \ell^-$: motivation

- The decay $\Lambda_b \to \Lambda\, \ell^+ \ell^-$ is complement to the well–analyzed rare meson decays $B \to K^{(*)}\, \ell^+ \ell^-$ etc. to study the short– and long–distance dynamics of rare decays induced by the transition $b \to s\, \ell^+ \ell^-$.

- The experimental measurements:

$$\mathcal{B}(\Lambda_b \to \Lambda \mu^+ \mu^-) = (1.73 \pm 0.42(\mathrm{stat}) \pm 0.55(\mathrm{syst})) \cdot 10^{-6}$$

T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. 107, 201802 (2011)

$$\mathcal{B}(\Lambda_b \to \Lambda \mu^+ \mu^-) = (0.96 \pm 0.16(\mathrm{stat}) \pm 0.13(\mathrm{syst}) \pm 0.21(\mathrm{norm})) \cdot 10^{-6}$$

RAaij *et al.* [LHCb Collaboration], arXiv:1306.2577 [hep-ex].

- A number of theoretical papers use the SM (penguin) operators and their non–Standard Model extensions to describe the short distance dynamics.

- Nonperturbative approaches to calculate the transition matrix element $\langle \Lambda |\, O_i \,| \Lambda_b \rangle$.

Lagrangian and 3-quark currents

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87, 074031 (2013)

$$\mathcal{L}^{\Lambda}_{\rm int}(x) = g_\Lambda \, \bar{\Lambda}(x) \cdot J_\Lambda(x) + g_\Lambda \, \bar{J}_\Lambda(x) \cdot \Lambda(x)$$

$$J_\Lambda(x) = \int dx_1 \int dx_2 \int dx_3 \, F_\Lambda(x; x_1, x_2, x_3) \, J^{(\Lambda)}_{3q}(x_1, x_2, x_3)$$

$$J^{(\Lambda)}_{3q}(x_1, x_2, x_3) = Q^{a_1}(x_1) \cdot \epsilon^{a_1 a_2 a_3} \, u^{T\, a_2}(x_2) \, C \, \gamma^5 \, d^{a_3}(x_3)$$

$$Q = s, c, b$$

The vertex function is chosen in the form

$$F_\Lambda(x; x_1, x_2, x_3) = \delta^{(4)}\Big(x - \sum_{i=1}^{3} w_i x_i\Big) \, \Phi_\Lambda\Big(\sum_{i<j} (x_i - x_j)^2\Big) \qquad w_i = \frac{m_i}{m_1 + m_2 + m_3}$$

The rare baryon decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$

- The effective Hamiltonian leads to the quark decay amplitudes $b \to s\ell^+\ell^-$:

$$
\begin{aligned}
M(b \to s\ell^+\ell^-) &= \frac{G_F}{\sqrt{2}}\frac{\alpha\lambda_t}{2\pi}\Big\{C_9^{\rm eff}\,(\bar{s}O^\mu b)\,(\bar{\ell}\gamma_\mu\ell) + C_{10}\,(\bar{s}O^\mu b)\,(\bar{\ell}\gamma_\mu\gamma_5\ell) \\
&- \frac{2}{q^2}C_7^{\rm eff}\left[m_b\left(\bar{s}\,i\sigma^{\mu q}\,(1+\gamma^5)\,b\right) + O(m_s)\right](\bar{\ell}\gamma_\mu\ell)\Big\}.
\end{aligned}
$$

- and $b \to s\gamma$:

$$
M(b \to s\gamma) = -\frac{G_F}{\sqrt{2}}\frac{e\lambda_t}{4\pi^2}C_7^{\rm eff}\left[m_b\left(\bar{s}\,i\sigma^{\mu q}\,(1+\gamma^5)\,b\right) + O(m_s)\right]\epsilon_\mu\,,
$$

where $\lambda_t \equiv V_{ts}^\dagger V_{tb}$.

- The Wilson coefficient $C_9^{\rm eff}$ effectively takes into account, first, the contributions from the four-quark operators $Q_i (i = 1, \cdots, 6)$ and, second, the nonperturbative effects (long–distance contributions) coming from the $c\bar{c}$-resonance contributions what are, as usual, parametrized by a Breit-Wigner ansatz.

The rare baryon decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$

The hadronic matrix elements are expanded in terms of dimensionless form factors:

$$\langle B_2 \,|\, \bar{s}\,\gamma^\mu\, b \,|B_1\rangle = \bar{u}_2(p_2)\Big[f_1^V(q^2)\gamma^\mu - f_2^V(q^2) i\sigma^{\mu q_1} + f_3^V(q^2) q_1^\mu\Big] u_1(p_1)$$

$$\langle B_2 \,|\, \bar{s}\,\gamma^\mu\gamma^5\, b \,|B_1\rangle = \bar{u}_2(p_2)\Big[f_1^A(q^2)\gamma^\mu - f_2^A(q^2) i\sigma^{\mu q_1} + f_3^A(q^2) q_1^\mu\Big]\gamma^5 u_1(p_1)$$

$$\langle B_2 \,|\, \bar{s}\, i\sigma^{\mu q}\, b \,|B_1\rangle = \bar{u}_2(p_2)\Big[f_1^{TV}(q^2)(\gamma^\mu q_1^2 - q_1^\mu \not{q}_1) - f_2^{TV}(q^2) i\sigma^{\mu q_1}\Big] u_1(p_1)$$

$$\langle B_2 \,|\, \bar{s}\, i\sigma^{\mu q_1}\gamma^5\, b \,|B_1\rangle = \bar{u}_2(p_2)\Big[f_1^{TA}(q^2)(\gamma^\mu q_1^2 - q_1^\mu \not{q}_1) - f_2^{TA}(q^2) i\sigma^{\mu q_1}\Big]\gamma^5 u_1(p_1)$$

where $q = p_1 - p_2$ **and** $q_1 = q/M_1$.

The fit of the size parameters

- **We use the same values of the quark masses and the infrared cut-off as in meson sector.**
- **We determine the set of size parameters Λ_{Λ_s}, Λ_{Λ_c} and Λ_{Λ_b} by fitting data on the magnetic moment of the Λ-hyperon and the branching ratios of the semileptonic decays $\Lambda_c \to \Lambda \ell^+ \nu_\ell$ and $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$ by a one-parameter fit to these values.**
- **With the choice of dimensional parameters in GeV**

$$\Lambda_{\Lambda_s} = 0.490 \qquad \Lambda_{\Lambda_c} = 0.864 \qquad \Lambda_{\Lambda_b} = 0.569$$

we get:

$$\mu_{\Lambda_s} = -0.73 \qquad \mu_{\Lambda_s}^{\rm expt} = -0.613 \pm 0.004$$

$$\mu_{\Lambda_c} = +0.39$$

$$\mu_{\Lambda_b} = -0.06$$

The fit of the size parameters

Branching ratios of semileptonic decays of heavy baryons in %.

Mode	Our results	Data
$\Lambda_c \to \Lambda e^+ \nu_e$	2.0	2.1 ± 0.6
$\Lambda_c \to \Lambda \mu^+ \nu_\mu$	2.0	2.0 ± 0.7
$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	6.6	$6.5^{+3.2}_{-2.5}$
$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$	6.6	
$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$	1.8	

Asymmetry parameter α in the semileptonic decays of heavy baryons.

Mode	Our results	Data
$\Lambda_c \to \Lambda e^+ \nu_e$	0.828	0.86 ± 0.04
$\Lambda_c \to \Lambda \mu^+ \nu_\mu$	0.825	
$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	0.831	
$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$	0.831	
$\Lambda_b \to \Lambda_c \tau^- \bar{\nu}_\tau$	0.731	

The rare baryon decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$

Our results:

$$\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-) = 1.0 \cdot 10^{-6}$$

T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87 074031 (2013)

to be compared with the recent LHCb data:

$$\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-) = (0.96 \pm 0.16(\text{stat}) \pm 0.13(\text{syst}) \pm 0.21(\text{norm})) \cdot 10^{-6}$$

RAaij *et al.* [LHCb Collaboration], arXiv:1306.2577 [hep-ex].

$$\mathcal{B}(\Lambda_b \to \Lambda\gamma) = 0.4 \cdot 10^{-5} \qquad (\text{experimental upper bound} < 130 \cdot 10^{-5})$$

The angular decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\gamma$

$$\frac{d\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\gamma)}{d\cos\theta_B} = \mathrm{Br}(\Lambda \to p\pi^-)\,\frac{1}{2}\,\Gamma(\Lambda_b \to \Lambda\gamma)(1 + \alpha_B \tilde{P}_z^{\Lambda}\cos\theta_B)$$

where α_B is the asymmetry parameter in the decay $\Lambda \to p + \pi^-$ for which we take the experimental value $\alpha_B = 0.642 \pm 0.013$.

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{\alpha}{2}\left(\frac{G_F m_b\,|\lambda_t|\,C_7^{\mathrm{eff}}}{4\pi^2\sqrt{2}}\right)^2 \frac{(M_1^2 - M_2^2)^3}{M_1^3}\left[\left(f_2^{TV}(0)\right)^2 + \left(f_2^{TA}(0)\right)^2\right]$$

The z–component of the polarization of the Λ is given by

$$\tilde{P}_z^{\Lambda} = -2\frac{f_2^{TV}(0)f_2^{TA}(0)}{(f_2^{TV}(0))^2 + (f_2^{TA}(0))^2}$$

One can show that $f_2^{TV}(0) \equiv f_2^{TA}(0)$. Therefore, $\tilde{P}_z^{\Lambda} \equiv -1$ and finally

$$\frac{1}{\Gamma_{tot}}\frac{d\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\gamma)}{d\cos\theta_B} = \mathrm{Br}(\Lambda \to p\pi^-)\,\frac{1}{2}\,\mathrm{Br}(\Lambda_b \to \Lambda\gamma)(1 - \alpha_B\cos\theta_B)$$

The angular decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

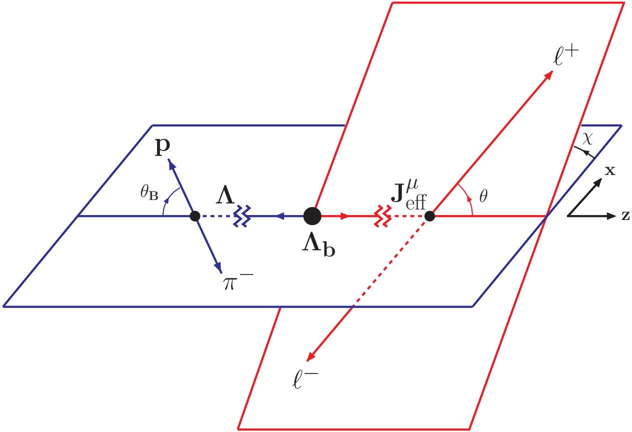


Figure: Definition of angles θ, θ_B and χ in the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-) + J_{\rm eff}(\to \ell^+\ell^-)$.

The differential rate of the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2} = \frac{v^2}{2} \cdot \left(U^{11+22} + L^{11+22}\right) + \frac{2m_\ell^2}{q^2} \cdot \frac{3}{2} \cdot \left(U^{11} + L^{11} + S^{22}\right)$$

The total rate is obtained by q^2–integration in the range

$$4m_\ell^2 \;\leq\; q^2 \;\leq\; (M_1 - M_2)^2$$

The short notations:

$$X^{mm'} = \frac{1}{2}\,\frac{G_F^2}{(2\pi)^3}\left(\frac{\alpha|\lambda_t|}{2\pi}\right)^2 \frac{|\vec{p}_2|\,q^2\,v}{12\,M_1^2}\,H_X^{mm'}\,,$$

where $\lambda_t = V_{ts}^\dagger V_{tb} = 0.041$ and $v = \sqrt{1 - 4m_\ell^2/q^2}$ is the lepton velocity in the $(\ell^+\ell^-)$ CM frame.

Lepton–side decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2 d\cos\theta} = v^2 \cdot \left[\frac{3}{8}(1+\cos^2\theta)\cdot\frac{1}{2}U^{11+22} + \frac{3}{4}\sin^2\theta\cdot\frac{1}{2}L^{11+22}\right]$$
$$- \; v\cdot\frac{3}{4}\cos\theta\cdot P^{12} + \frac{2m_\ell^2}{q^2}\cdot\frac{3}{4}\cdot\left[U^{11}+L^{11}+S^{22}\right]$$

One can define a lepton–side forward–backward asymmetry A_{FB}^{ℓ} by $A_{FB}^{\ell} = (F-B)/(F+B)$ where F and B denote the rates in the forward and backward hemispheres.

$$A_{FB}^{\ell}(q^2) = -\frac{3}{2}\,\frac{v\cdot P^{12}}{v^2\cdot\left(U^{11+22}+L^{11+22}\right)+\frac{2m_\ell^2}{q^2}\cdot 3\cdot\left(U^{11}+L^{11}+S^{22}\right)}\,.$$

The integrated forward–backward asymmetry is defined as the ratio of the integrals of the numerator and denominator over q^2 in the full kinematical region.

Λ–polarization and hadron–side decay distribution for the cascade decay $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$

$$\frac{d\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-)}{dq^2\, d\cos\theta_B} = \mathrm{Br}(\Lambda \to p\pi^-)\cdot\frac{1}{2}\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2} \times \left(1 + \alpha_B P_z^{\Lambda}\cos\theta_B\right)$$

Thez–component of the polarization of the daughter baryon Λ:

$$P_z^{\Lambda} = \frac{v^2\cdot\left(P^{11+22} + L_P^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot 3\cdot\left(P^{11} + L_P^{11} + S_P^{22}\right)}{v^2\cdot\left(U^{11+22} + L^{11+22}\right) + \frac{2m_\ell^2}{q^2}\cdot 3\cdot\left(U^{11} + L^{11} + S^{22}\right)}$$

The forward–backward asymmetry is simply related to the polarization P_z^{Λ} via

$$A_{FB}^{h}(q^2) = \frac{\alpha_B}{2}\cdot P_z^{\Lambda}(q^2)$$

Asymmetries A^l_{FB} and A^h_{FB} with (without) long–distance contributions

Mode	A^l_{FB}	A^h_{FB}
$\Lambda_b \to \Lambda e^+ e^-$	3.2×10^{-10} (1.2×10^{-8})	-0.321 (-0.321)
$\Lambda_b \to \Lambda \mu^+ \mu^-$	1.7×10^{-4} (8.0×10^{-4})	-0.300 (-0.294)
$\Lambda_b \to \Lambda \tau^+ \tau^-$	5.9×10^{-4} (9.6×10^{-4})	-0.265 (-0.259)

X(3872)-meson: short introduction

- A narrow charmonium-like state **X(3872)** was observed in the exclusive decay process:

$$B^+ \to K^+\pi^+\pi^- J/\psi$$

S. K. Choi *et al.* [Belle Collaboration] Phys. Rev. Lett. 91, 262001 (2003)

- X-mass is close to $D^0 - D^{*0}$ mass threshold:

$$M_X = 3871.68 \pm 0.17\,\mathrm{MeV}, \qquad \mathrm{PDG'12}$$

$$M_{D^0} + M_{D^{*0}} = 3871.81 \pm 0.25\,\mathrm{MeV}$$

- Its width $\Gamma_X \leq 1.2$ MeV at 90% CL.

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- **Its width $\Gamma_X \leq 1.2$ MeV at 90% CL.**

- **The state was confirmed in B-decays by BaBar experiment**

B. Aubert *et al.* Phys. Rev. Lett. 93, 041801 (2004)

and in $p\bar{p}$ production by Tevatron experiments CDF and DØ.

D. E. Acosta *et al.* [CDF Collaboration] Phys. Rev. Lett. 93, 072001 (2004);

V. M. Abazov *et al.* [D0 Collaboration] Phys. Rev. Lett. 93, 162002 (2004)

X(3872)-meson: short introduction

- From the observation of decays $X(3872) \to J/\psi\gamma$ reported by both Belle and BaBar collaborations and from the angular analysis performed by CDF experiment it was shown that the only quantum numbers $J^{PC} = 1^{++}$ or 2^{-+} are compatible with data.

  K. Abe *et al.*, [Belle Collaboration], arXiv:hep-ex/0505037; hep-ex/0505038
  B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D 74, 071101 (2006)

  A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. 98, 132002 (2007)

- The observation of decays into $D^0\overline{D}^0\pi^0$ by Belle and BaBar collaborations allows one to exclude the choice 2^{-+} because the near-threshold decay $X \to D^0\overline{D}^0\pi^0$ is expected to be strongly suppressed for $J = 2$.

  G. Gokhroo *et al.* [Belle Collaboration], Phys. Rev. Lett. 97, 162002 (2006)

  B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D 77, 011102 (2008).

- The quantum numbers of the $X(3872)$ meson were determined from the analysis of angular correlations in $B^+ \to X(3872)K^+$ decays, where $X(3872) \to \pi^+\pi^-J/\psi$ and $J/\psi \to \mu^+\mu^-$.

  RAaij *et al.* [LHCb Coll.], Phys. Rev. Lett. 110, 222001 (2013) [arXiv:1302.6269 [hep-ex]].

  The quantum numbers of the $X(3872)$ are

$$J^{PC} = 1^{++}$$

X(3872)-meson: short introduction

- **Belle collaboration has reported evidence for the decay mode $X \to \pi^+\pi^-\pi^0 J/\psi$ dominated by the sub-threshold decay $X \to \omega J/\psi$.**

K. Abe *et al.*, [Belle Collaboration], arXiv:hep-ex/0505037,hep-ex/0505038

- **It was found that the branching ratio of this mode is almost the same as of $X \to \pi^+\pi^- J/\psi$ decay:**

$$\frac{\mathcal{B}(X \to J/\psi \pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi \pi^+\pi^-)} = 1.0 \pm 0.4\,(\mathrm{stat}) \pm 0.3\,(\mathrm{syst}).$$

- **It implies strong isospin violation because the three-pion decay proceeds via intermediate ω-meson with isospin 0 whereas the two-pion decay proceeds via intermediate ρ-meson with isospin 1.**

X(3872)-meson: short introduction

- **The two-pion decay via intermediate ρ-meson is very difficult to explain by using an interpretation of the X(3872) as simple $c\bar{c}$ charmonium state with isospin 0.**

- **The possible candidate from $\bar{c}c$-spectroscopy:**

$$\chi_{c_1}(2^3P_1) - \text{state with } J^{PC} = 1^{++}$$

BUT the value of its mass varies from 3925 up to 3953 MeV. Also the decay width calculated in variuos models is too large.

- **The X(3872) IS NOT the pure $\bar{c}c$-state**

X(3872)-meson: short introduction

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BUT the value of its mass varies from 3925 up to 3953 MeV. Also the decay width calculated in variuos models is too large.

- **The X(3872) IS NOT the pure $\bar{c}c$-state**

- **a molecule bound state $D^0\bar{D}^{*0}$ with small binding energy**

- **a tetraquark state composed from a diquark and antidiquark**

- **threshold cusps**

- **hybrids and glueballs**

X(3872)-meson: short introduction

- **An intepretation of the X(3872) as a tetraquark was suggested in**

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71, 014028 (2005)

$$\mathbf{X_q \Longrightarrow [cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1}, \qquad (q = u, d)}$$

- **Isospin breaking: the state $\mathbf{X_u}$ breaks isospin symmetry maximally:**

$$\mathbf{X_u = \frac{1}{\sqrt{2}}\left\{ \underbrace{\frac{X_u + X_d}{\sqrt{2}}}_{I=0} + \underbrace{\frac{X_u - X_d}{\sqrt{2}}}_{I=1} \right\}.}$$

X(3872)-meson: short introduction

- **The physical states are the mixing of $\mathbf{X_u}$ and $\mathbf{X_d}$**

$$\mathbf{X_l} \equiv \mathbf{X_{low}} = \mathbf{X_u}\ \cos\theta + \mathbf{X_d}\ \sin\theta,$$
$$\mathbf{X_h} \equiv \mathbf{X_{high}} = -\mathbf{X_u}\ \sin\theta + \mathbf{X_d}\ \cos\theta.$$

- **The mixing angle θ is supposed to be found from the known ratio of the two-pion (via ρ) and three-pion (via ω) decay widths.**

X(3872)-meson: short introduction

- The physical states are the mixing of X_u and X_d

$$
\begin{aligned}
X_l \equiv X_{\rm low} &= \quad X_u \cos\theta + X_d \sin\theta, \\
X_h \equiv X_{\rm high} &= -X_u \sin\theta + X_d \cos\theta.
\end{aligned}
$$

- The mixing angle θ is supposed to be found from the known ratio of the two-pion (via ρ) and three-pion (via ω) decay widths.

- We have performed independent analysis of the X(3872)-meson considered as a tetraquark state in the framework of the covariant quark model with infrared confinement.

X(3872)-meson as a tetraquark state: Lagrangian

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010)

- **An effective interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_X\, X_{q\,\mu}(x) \cdot J^{\mu}_{X_q}(x), \qquad (q = u, d).$$

- **The nonlocal version of the four-quark interpolating current**

$$J^{\mu}_{X_q}(x) = \int dx_1 \ldots \int dx_4\, \delta\Big(x - \sum_{i=1}^{4} w_i x_i\Big)\, \Phi_X\Big(\sum_{i<j}(x_i - x_j)^2\Big)\, J^{\mu}_{4q}(x_1, \ldots, x_4)$$

$$J^{\mu}_{4q} = \frac{1}{\sqrt{2}}\, \varepsilon_{abc}\, [q_a(x_4) C\gamma^5 c_b(x_1)]\, \varepsilon_{dec}\, [\bar{q}_d(x_3)\gamma^{\mu} C\bar{c}_e(x_2)] + (\gamma^5 \leftrightarrow \gamma^{\mu}),$$

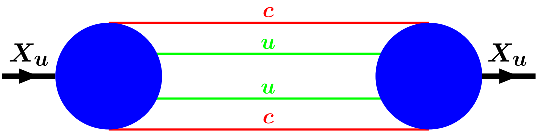
$$w_1 = w_2 = \frac{m_c}{2(m_q + m_c)} \equiv \frac{w_c}{2}, \qquad w_3 = w_4 = \frac{m_q}{2(m_q + m_c)} \equiv \frac{w_q}{2}.$$

Compositeness condition

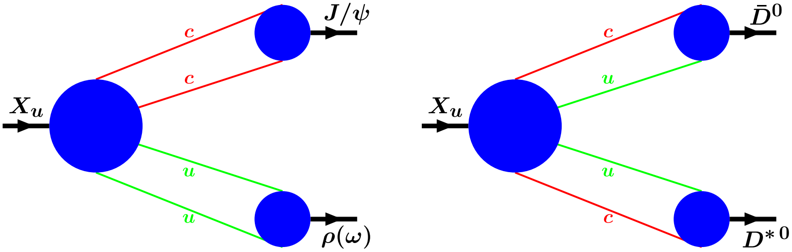
The coupling constant g_X is determined from the compositeness condition

$$Z_X = 1 - \Pi'_X(M_X^2) = 0$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator.



Strong off-shell decays



Since the X(3872) lies nearly the respective thresholds in both cases,

$$\begin{aligned} m_X - (m_{J/\psi} + m_\rho) &= -0.90 \pm 0.41\,\mathrm{MeV}, \\ m_X - (m_{D^0} + m_{D^{*\,0}}) &= -0.30 \pm 0.34\,\mathrm{MeV} \end{aligned}$$

the intermediate $\rho(\omega)$ and D^* mesons should be taken off-shell.

The narrow width approximation

$$\frac{d\Gamma(X \to J/\psi + n\pi)}{dq^2} = \frac{1}{8\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X \to J/\psi + v^0)|^2$$
$$\times \frac{\Gamma_{v^0}\, m_{v^0}}{\pi} \frac{p^*(q^2)}{(m_{v^0}^2 - q^2)^2 + \Gamma_{v^0}^2\, m_{v^0}^2} \mathrm{Br}(v^0 \to n\pi),$$

$$\frac{d\Gamma(X_u \to \bar{D}^0 D^0 \pi^0)}{dq^2} = \frac{1}{2\, m_X^2\, \pi} \cdot \frac{1}{3} |M(X_u \to \bar{D}^0 D^{*\,0})|^2$$
$$\times \frac{\Gamma_{D^{*\,0}}\, m_{D^{*\,0}}}{\pi} \frac{p^*(q^2)\, \mathcal{B}(D^{*\,0} \to D^0 \pi^0)}{(m_{D^{*\,0}}^2 - q^2)^2 + \Gamma_{D^{*\,0}}^2\, m_{D^{*\,0}}^2},$$

Strong decay widths

- **Two new adjustable parameters:** θ **and** Λ_X.
- **The ratio**

$$\frac{\Gamma(X_u \to J/\psi + 3\,\pi)}{\Gamma(X_u \to J/\psi + 2\,\pi)} \approx 0.25$$

is very stable under variation of Λ_X.

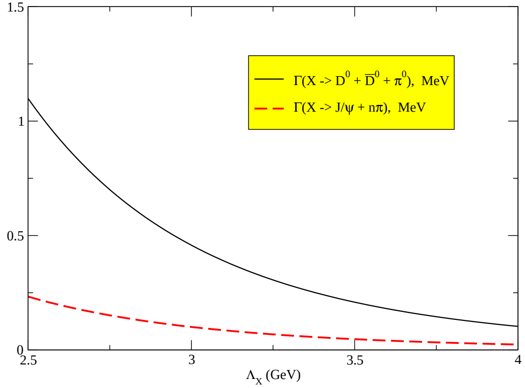
- **Using this result and the central value of the experimental data**

$$\frac{\Gamma(X_{l,h} \to J/\psi + 3\,\pi)}{\Gamma(X_{l,h} \to J/\psi + 2\,\pi)} \approx 0.25 \cdot \left(\frac{1 \pm \tan\theta}{1 \mp \tan\theta}\right)^2 \approx 1$$

gives $\theta \approx \pm 18.4^o$ **for** X_l ("+") **and** X_h ("−")**, respectively.**

- **This is in agreement with the results obtained by both Maiani:** $\theta \approx \pm 20^o$ **and Nielsen:** $\theta \approx \pm 23.5^o$.

Strong decay widths

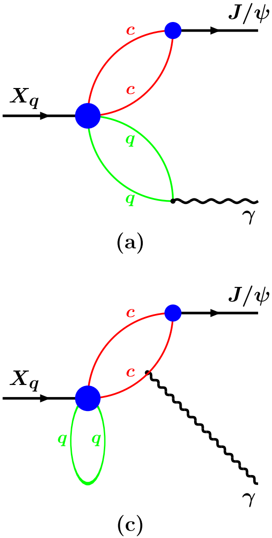


$$\frac{\Gamma(X\to D^0\bar{D}^0\pi^0)}{\Gamma(X\to J/\psi\pi^+\pi^-)} = \begin{cases} 4.5 \pm 0.2 & \text{theor} \\ 10.5\pm 4.7 & \text{expt} \end{cases}$$

Radiative X-decay

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Koerner, P. Santorelli and G. G. Saidullaeva,

Phys. Rev. D 84, 014006 (2011)



Radiative X-decay

The on-mass shell conditions

$$\varepsilon_X^\mu p_\mu = 0, \qquad \varepsilon_{J/\psi}^\nu q_{1\nu} = 0, \qquad \varepsilon_\gamma^\rho q_{2\rho} = 0$$

leave us five Lorentz structures:

$$\begin{aligned} T_{\mu\rho\nu}(q_1, q_2) &= \varepsilon_{q_2\mu\nu\rho}(q_1 \cdot q_2)\, W_1 + \varepsilon_{q_1 q_2 \nu\rho} q_{1\mu}\, W_2 + \varepsilon_{q_1 q_2 \mu\rho} q_{2\nu}\, W_3 \\ &+ \varepsilon_{q_1 q_2 \mu\nu} q_{1\rho}\, W_4 + \varepsilon_{q_1 \mu\nu\rho}(q_1 \cdot q_2)\, W_5 \,. \end{aligned}$$

Using the gauge invariance condition

$$q_2^\rho T_{\mu\rho\nu} = (q_1 \cdot q_2)\varepsilon_{q_1 q_2 \mu\nu}(W_4 + W_5) = 0$$

one has $W_4 = -W_5$ which reduces the set of independent covariants to four. However, there are two nontrivial relations among the four covariants which can be derived by noting that the tensor

$$T_{\mu[\nu_1\nu_2\nu_3\nu_4\nu_5]} = g_{\mu\nu_1}\varepsilon_{\nu_2\nu_3\nu_4\nu_5} + \text{cycl.}(\nu_1\nu_2\nu_3\nu_4\nu_5)$$

vanishes in four dimensions since it is totally antisymmetric in the five indices $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5)$.

Radiative X-decay

The two conditions reduce the set of independent covariants to two. This is the appropriate number of independent covariants since the photon transition is described by two independent amplitudes as e.g. by the E1 and M2 transition amplitudes. One has

$$\Gamma(X \to \gamma\, J/\psi) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}\left(|H_L|^2 + |H_T|^2\right) = \frac{1}{12\pi}\frac{|\vec{q}_2|}{m_X^2}\left(|A_{E1}|^2 + |A_{M2}|^2\right),$$

where the helicity amplitudes H_L and H_T are expressed in terms of the Lorentz amplitudes as

$$H_L = i\frac{m_X^2}{m_{J/\psi}}|\vec{q}_2|^2\left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}W_4\right],$$

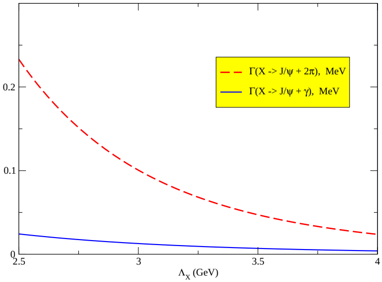
$$H_T = -im_X|\vec{q}_2|^2\left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X|\vec{q}_2|}\right)W_4\right],$$

$$|\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

The E1 and M2 multipole amplitudes are obtained via

$$A_{E1/M2} = (H_L \mp H_T)/\sqrt{2}.$$

Radiative X-decay



If one takes $\Lambda_X \in (3,4)$ GeV with the central value $\Lambda_X = 3.5$ GeV then our prediction for the ratio of widths reads

$$\left.\frac{\Gamma(X_l \to \gamma + J/\psi)}{\Gamma(X_l \to J/\psi + 2\pi)}\right|_{\rm theor} = 0.15 \pm 0.03$$

which fits very well the experimental data from the Belle Collaboration

$$\frac{\Gamma(X \to \gamma + J/\psi)}{\Gamma(X \to J/\psi\, 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \textbf{Belle} \\ 0.22 \pm 0.06 & \textbf{BaBar} \end{cases}$$

Summary and outlook

- **We have presented a refined covariant quark model which includes infrared confinement of quarks.**

- **We have calculated the transition form factors of the heavy B and B_s mesons to light pseudoscalar and vector mesons, which are needed as ingredients for the calculation of the semileptonic, nonleptonic, and rare decays of the B and B_s mesons. Our form factor results hold in the full kinematical range of momentum transfer.**

- **We have made use of the calculated form factors to calculate the nonleptonic decays $B_s \to D_s\bar{D}_s, \ldots$ and $B_s \to J/\psi\phi$, which have been widely discussed recently in the context of $B_s - \bar{B}_s$–mixing and CP violation.**

- **We have applied our approach to baryon physics by using the same values of the constituent quark masses and infrared cutoff as in meson sector.**

- **We have calculated the nucleon magnetic moments and charge radii and also electromagnetic form factors at low energies.**

Summary and outlook

- We have explored the rare baryon decays $\Lambda_b \to \Lambda + \ell^+\ell^-$ and $\Lambda_b \to \Lambda + \gamma$.
- We have studied the properties of the X(3872) as a tetraquark.
- We have calculated the strong decays $X \to J/\psi + \rho(\to 2\pi)$, $X \to J/\psi + \omega(\to 3\pi)$, $X \to D + \bar{D}^*(\to D\pi)$ and electromagnetic decay $X \to \gamma + J/\psi$.
- The comparison with available experimental data allows one to conclude that the X(3872) can be a tetraquark state.