Effective weak Lagrangians in the Standard Model and B decays

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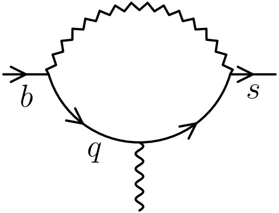
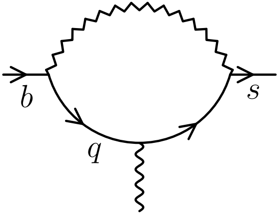
$$b \to s$$

Dipole operator

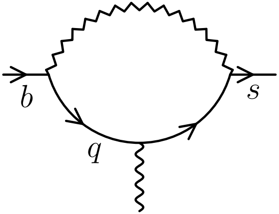


Dipole operator



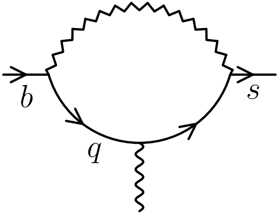
$$\bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b$$

Dipole operator



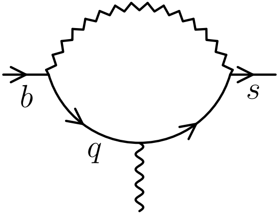
$$\bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

Dipole operator



$$O_g = m_b \bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

Dipole operator

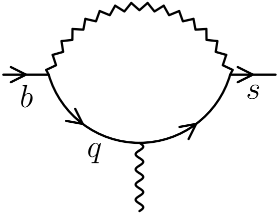


$$O_g = m_b \bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

$$\frac{g_2^2 g_s}{M_W^2} \sum_{q=u,c,t} V_{qb} V^*_{qs} E(x_q)$$

$$x_q = \frac{m_q^2}{M_W^2}$$

Dipole operator



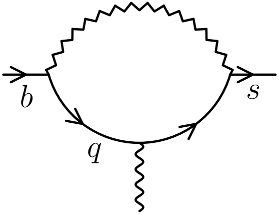
$$O_g = m_b \bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

$$\frac{g_2^2 g_s}{M_W^2} \sum_{q=u,c,t} V_{qb} V_{qs}^* \left(E(x_q) - E(0) \right)$$

$$x_q = \frac{m_q^2}{M_W^2}$$

$$\sum_{q=u,c,t} V_{qb} V_{qs}^* = 0$$

Dipole operator

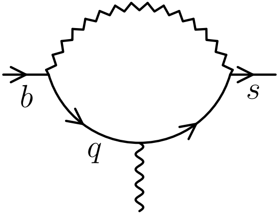


$$O_g = m_b \bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

$$\frac{g_2^2 g_s}{M_W^2} V_{tb} V_{ts}^* \left(E(x_t) - E(0)\right)$$

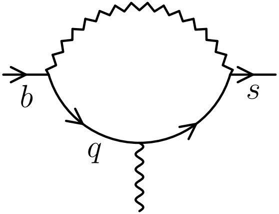
$$x_q = \frac{m_q^2}{M_W^2}$$

Penguin operator



$$\bar{s}_L D^\nu G^a_{\mu\nu} t^a \gamma^\mu b_L$$

Penguin operator



$$\bar{s}_L D^\nu G^a_{\mu\nu} t^a \gamma^\mu b_L$$

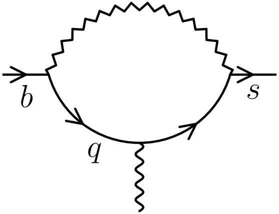
EOM

$$D^\nu G^a_{\mu\nu} = g_s \sum_q \bar{q} t^a \gamma_\mu q$$

Up to an EOM-vanishing operator

$$(\bar{s}_L t^a \gamma^\alpha b_L) \sum_q (\bar{q} t^a \gamma_\alpha q)$$

Penguin operator



$$\bar{s}_L D^\nu G^a_{\mu\nu} t^a \gamma^\mu b_L$$

EOM

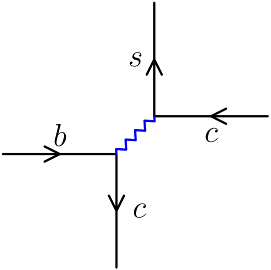
$$D^\nu G^a_{\mu\nu} = g_s \sum_q \bar{q} t^a \gamma_\mu q$$

Up to an EOM-vanishing operator

$$(\bar{s}_L t^a \gamma^\alpha b_L) \sum_q (\bar{q} t^a \gamma_\alpha q)$$

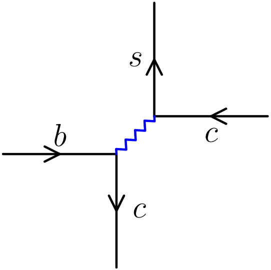
GIM mechanism

Full set of operators



$$O_{1c} = \left(\bar{c}_{Li}\gamma^{\alpha}b_L^i\right)\left(\bar{s}_{Lj}\gamma_{\alpha}c_L^j\right)$$
$$O_{2c} = \left(\bar{c}_{Li}\gamma^{\alpha}b_L^j\right)\left(\bar{s}_{Lj}\gamma_{\alpha}c_L^i\right)$$

Full set of operators



$$O_{1c} = \left(\bar{c}_{Li}\gamma^{\alpha}b_L^i\right)\left(\bar{s}_{Lj}\gamma_{\alpha}c_L^j\right)$$
$$O_{2c} = \left(\bar{c}_{Li}\gamma^{\alpha}b_L^j\right)\left(\bar{s}_{Lj}\gamma_{\alpha}c_L^i\right)$$

$$O_{1u} = \left(\bar{u}_{Li}\gamma^{\alpha}b_L^i\right)\left(\bar{s}_{Lj}\gamma_{\alpha}u_L^j\right)$$
$$O_{2u} = \left(\bar{u}_{Li}\gamma^{\alpha}b_L^j\right)\left(\bar{s}_{Lj}\gamma_{\alpha}u_L^i\right)$$

$$O_1 = (\bar{c}_{Li}\gamma^\alpha b_L^i)\,(\bar{s}_{Lj}\gamma_\alpha c_L^j)$$
$$O_2 = (\bar{c}_{Li}\gamma^\alpha b_L^j)\,(\bar{s}_{Lj}\gamma_\alpha c_L^i)$$

$$O_1 = (\bar{c}_{Li}\gamma^\alpha b_L^i)\,(\bar{s}_{Lj}\gamma_\alpha c_L^j)$$
$$O_2 = (\bar{c}_{Li}\gamma^\alpha b_L^j)\,(\bar{s}_{Lj}\gamma_\alpha c_L^i)$$

Penguin operators

$$O_3 = (\bar{s}_{Li}\gamma^\alpha b_L^i)\,\sum_q (\bar{q}_j\gamma_\alpha q^j)$$
$$O_4 = (\bar{s}_{Li}\gamma^\alpha b_L^j)\,\sum_q (\bar{q}_j\gamma_\alpha q^i)$$
$$O_5 = (\bar{s}_{Li}\gamma^\alpha\gamma^\beta\gamma^\gamma b_L^i)\,\sum_q (\bar{q}_j\gamma_\gamma\gamma_\beta\gamma_\alpha q^j)$$
$$O_6 = (\bar{s}_{Li}\gamma^\alpha\gamma^\beta\gamma^\gamma b_L^j)\,\sum_q (\bar{q}_j\gamma_\gamma\gamma_\beta\gamma_\alpha q^i)$$

$$O_1 = (\bar{c}_{Li}\gamma^\alpha b_L^i)\,(\bar{s}_{Lj}\gamma_\alpha c_L^j)$$
$$O_2 = (\bar{c}_{Li}\gamma^\alpha b_L^j)\,(\bar{s}_{Lj}\gamma_\alpha c_L^i)$$

Penguin operators

$$O_3 = (\bar{s}_{Li}\gamma^\alpha b_L^i)\ \sum_q (\bar{q}_j\gamma_\alpha q^j)$$
$$O_4 = (\bar{s}_{Li}\gamma^\alpha b_L^j)\ \sum_q (\bar{q}_j\gamma_\alpha q^i)$$
$$O_5 = (\bar{s}_{Li}\gamma^\alpha\gamma^\beta\gamma^\gamma b_L^i)\ \sum_q (\bar{q}_j\gamma_\gamma\gamma_\beta\gamma_\alpha q^j)$$
$$O_6 = (\bar{s}_{Li}\gamma^\alpha\gamma^\beta\gamma^\gamma b_L^j)\ \sum_q (\bar{q}_j\gamma_\gamma\gamma_\beta\gamma_\alpha q^i)$$

Dipole operator

$$O_g = m_b \bar{s}_L G^a_{\mu\nu} t^a \sigma^{\mu\nu} b_R$$

Lagrangian

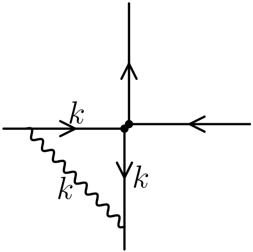
$$L = \frac{g_2^2}{2M_W^2}\Big[V_{us}^* V_{ub}\left(c_{1u}O_{1u} + c_{2u}O_{2u}\right)$$
$$+ V_{cs}^* V_{cb}\left(c_{1c}O_{1c} + c_{2c}O_{2c}\right)$$
$$+ V_{ts}^* V_{tb}\sum_{i=3}^{6} c_i O_i\Big]$$

$<O_1^0>$

$$\left[1 - 2C_F\frac{\alpha_s}{4\pi\varepsilon}\right] <O_1>$$

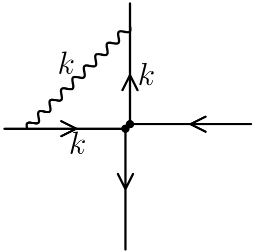
$<O_1^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_1>$$



$$C_F T_1 \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\mu \gamma^\lambda \gamma^\alpha \gamma_\lambda \gamma_\mu \otimes \gamma_\alpha$$

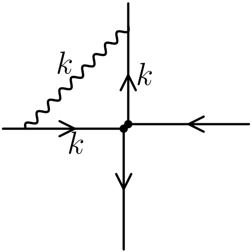
$<O_1^0>$



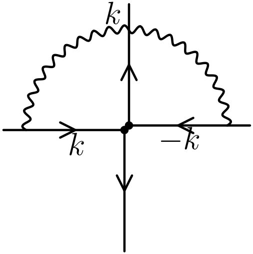
$$T_F\left(T_2 - \frac{T_1}{N_c}\right)$$

$$\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\alpha$$

$<O_1^0>$



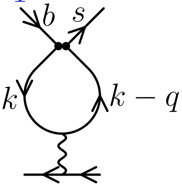
$$T_F \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\mu \gamma_\lambda \gamma_\alpha$$



$$-T_F \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\alpha \gamma_\lambda \gamma_\mu$$

$$\gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\alpha \gamma_\lambda \gamma_\mu = -\gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\mu \gamma_\lambda \gamma_\alpha + 20 \gamma^\alpha \otimes \gamma_\alpha$$

$<O_1^0>$



$$I^{\rho\sigma} = \frac{1}{i\pi^{d/2}} \int d^d k \frac{k^\rho (k-q)^\sigma}{k^2 (k-q)^2}$$
$$= -\frac{G_{11}(-q^2)^{-\varepsilon}}{4(d-1)} \left[q^2 g^{\rho\sigma} + (d-2) q^\rho q^\sigma\right]$$

$$- T_F \left(T_2 - \frac{T_1}{N_c}\right) \frac{\alpha_s}{4\pi\varepsilon} \frac{q^2 g^{\rho\sigma} + 2 q^\rho q^\sigma}{12 q^2} \gamma_\alpha \gamma_\rho \gamma_\mu \gamma_\sigma \gamma^\alpha \otimes \gamma^\mu$$
$$= -T_F \left(T_2 - \frac{T_1}{N_c}\right) \frac{\alpha_s}{4\pi\varepsilon} \frac{2}{3} \gamma^\mu \otimes \gamma_\mu$$

$<O_1^0>$ result

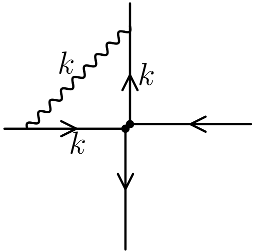
$$
\begin{aligned}
<O_1^0> = {} & <O_1> \\
& - 6T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_2> - \frac{<O_1>}{N_c} \right) \\
& - \frac{2}{3} T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_4> - \frac{<O_3>}{N_c} \right)
\end{aligned}
$$

$<O_2^0>$

Penguin diagram has 0 color factor

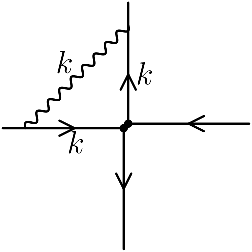
$$<O_2^0> = <O_2> - 6T_F \frac{\alpha_s}{4\pi\varepsilon}\left(<O_1> - \frac{<O_2>}{N_c}\right)$$

$<O_3^0>$

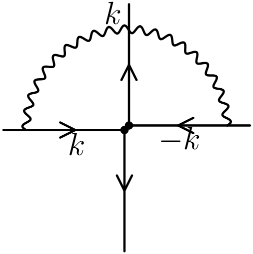


$$T_F\left(T_2 - \frac{T_1}{N_c}\right)$$
$$\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\alpha$$

$<O_3^0>$



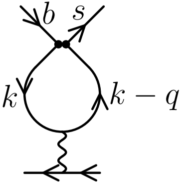
$$T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\alpha$$



$$-T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\lambda\gamma^\mu\otimes\gamma_\alpha\gamma_\lambda\gamma_\mu$$

$$\gamma^\alpha\gamma^\lambda\gamma^\mu\otimes\gamma_\alpha\gamma_\lambda\gamma_\mu=-\gamma^\alpha\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\alpha+20\gamma^\alpha\otimes\gamma_\alpha$$

$<O_3^0>$



$$-T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{q^2g^{\rho\sigma}+2q^\rho q^\sigma}{12q^2}\gamma_\alpha\gamma_\rho\gamma_\mu\gamma_\sigma\gamma^\alpha\otimes\gamma^\mu$$

$$=-T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{2}{3}\gamma^\mu\otimes\gamma_\mu$$

$<O_3^0>$ result

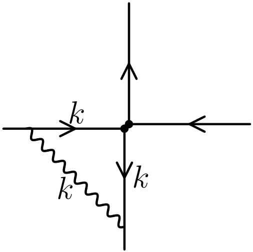
$$<O_3^0> = <O_3> + T_F \frac{\alpha_s}{4\pi\varepsilon}\left(<O_6> - \frac{<O_5>}{N_c}\right) - \frac{32}{3} T_F \frac{\alpha_s}{4\pi\varepsilon}\left(<O_4> - \frac{<O_3>}{N_c}\right)$$

$<O_4^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_4>$$

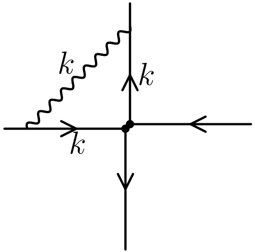
$<O_4^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_4>$$



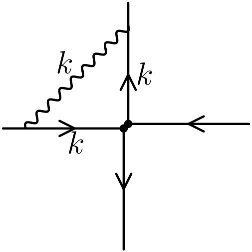
$$T_F \left(T_1 - \frac{T_2}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\mu \gamma^\lambda \gamma^\alpha \gamma_\lambda \gamma_\mu \otimes \gamma_\alpha$$

$<O_4^0>$

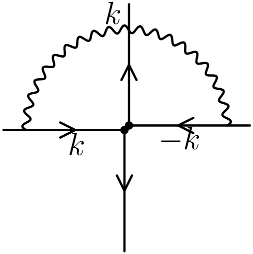


$$C_F T_2 \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\mu \gamma_\lambda \gamma_\alpha$$

$<O_4^0>$



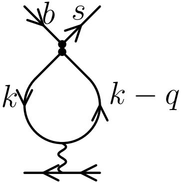
$$C_F T_2 \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\mu \gamma_\lambda \gamma_\alpha$$



$$-T_F \left(T_1 - \frac{T_2}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\alpha \gamma_\lambda \gamma_\mu$$

$$\gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\alpha \gamma_\lambda \gamma_\mu = -\gamma^\alpha \gamma^\lambda \gamma^\mu \otimes \gamma_\mu \gamma_\lambda \gamma_\alpha + 20 \gamma^\alpha \otimes \gamma_\alpha$$

$<O_4^0>$



$$T_F n_f \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{q^2 g^{\rho\sigma} + 2q^\rho q^\sigma}{12q^2} \gamma_\alpha \gamma_\rho \gamma_\mu \gamma_\sigma \gamma^\alpha \otimes \gamma^\mu$$

$$= T_F n_f \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{2}{3} \gamma^\mu \otimes \gamma_\mu$$

$<O_4^0>$ result

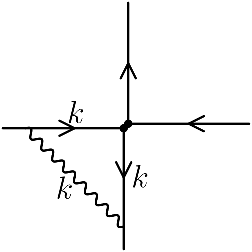
$$
\begin{aligned}
<O_4^0> = & \left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_4> \\
& + 8T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_3> - \frac{<O_4>}{N_c} \right) \\
& + \frac{1}{2} C_F \frac{\alpha_s}{4\pi\varepsilon} <O_6> + \frac{1}{2} T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_5> - \frac{<O_6>}{N_c} \right) \\
& + \frac{2}{3} T_F n_f \frac{\alpha_s}{4\pi\varepsilon} \left(<O_4> - \frac{<O_3>}{N_c} \right)
\end{aligned}
$$

$<O_5^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_5>$$

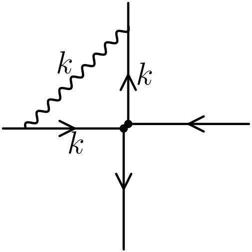
$<O_5^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_5>$$



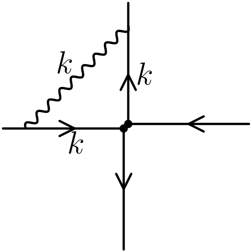
$$C_F T_1 \frac{\alpha_s}{4\pi\varepsilon} \frac{1}{4} \gamma^\mu \gamma^\lambda \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma_\lambda \gamma_\mu \otimes \gamma_\gamma \gamma_\beta \gamma_\alpha$$

cancels



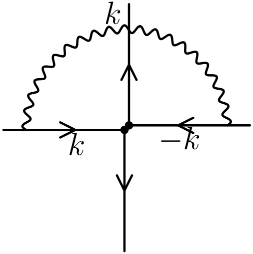
$$T_F\left(T_2-\frac{T_1}{N_c}\right)$$
$$\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha$$

$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha=20\gamma^\alpha\gamma^\beta\gamma^\gamma\otimes\gamma_\gamma\gamma_\beta\gamma_\alpha-64\gamma^\alpha\otimes\gamma_\alpha$$



$$T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha$$

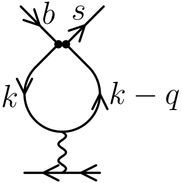
$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha=20\gamma^\alpha\gamma^\beta\gamma^\gamma\otimes\gamma_\gamma\gamma_\beta\gamma_\alpha-64\gamma^\alpha\otimes\gamma_\alpha$$



$$-T_F\left(T_2-\frac{T_1}{N_c}\right)\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\gamma\gamma_\beta\gamma_\alpha\gamma_\lambda\gamma_\mu$$

$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu\otimes\gamma_\gamma\gamma_\beta\gamma_\alpha\gamma_\lambda\gamma_\mu=64\gamma^\alpha\otimes\gamma_\alpha$$

$<O_5^0>$



$$- T_F \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{q^2 g^{\rho\sigma} + 2q^\rho q^\sigma}{12 q^2} \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\rho \gamma_\mu \gamma_\sigma \gamma^\gamma \gamma^\beta \gamma^\alpha \otimes \gamma^\mu$$

$$= -T_F \left(T_2 - \frac{T_1}{N_c} \right) \frac{\alpha_s}{4\pi\varepsilon} \frac{8}{3} \gamma^\mu \otimes \gamma_\mu$$

$<O_5^0>$ result

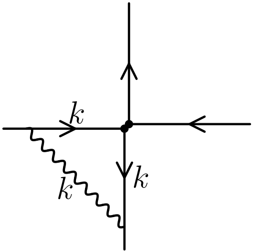
$$
\begin{aligned}
<O_5^0> = {} & <O_5> \\
& + 10 T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_6> - \frac{<O_5>}{N_c} \right) \\
& - \frac{200}{3} T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_4> - \frac{<O_3>}{N_c} \right)
\end{aligned}
$$

$<O_6^0>$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_6>$$

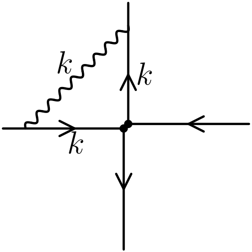
$\langle O_6^0 \rangle$

$$\left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] \langle O_6 \rangle$$



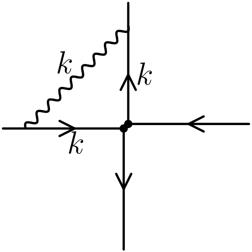
$$T_F \left(T_1 - \frac{T_2}{N_c}\right) \frac{\alpha_s}{4\pi\varepsilon}$$

$$\frac{1}{4}\gamma^\mu\gamma^\lambda\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma_\lambda\gamma_\mu \otimes \gamma_\gamma\gamma_\beta\gamma_\alpha$$



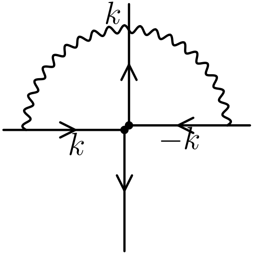
$$C_F T_2 \frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha$$

$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha = 20\gamma^\alpha\gamma^\beta\gamma^\gamma \otimes \gamma_\gamma\gamma_\beta\gamma_\alpha - 64\gamma^\alpha \otimes \gamma_\alpha$$



$$C_F T_2 \frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha$$

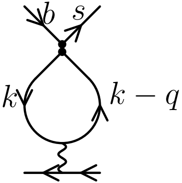
$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\mu\gamma_\lambda\gamma_\gamma\gamma_\beta\gamma_\alpha = 20\gamma^\alpha\gamma^\beta\gamma^\gamma \otimes \gamma_\gamma\gamma_\beta\gamma_\alpha - 64\gamma^\alpha \otimes \gamma_\alpha$$



$$-T_F\left(T_1 - \frac{T_2}{N_c}\right)$$
$$\frac{\alpha_s}{4\pi\varepsilon}\frac{1}{4}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\gamma\gamma_\beta\gamma_\alpha\gamma_\lambda\gamma_\mu$$

$$\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda\gamma^\mu \otimes \gamma_\gamma\gamma_\beta\gamma_\alpha\gamma_\lambda\gamma_\mu = 64\gamma^\alpha \otimes \gamma_\alpha$$

$<O_6^0>$



$$T_F n_f \left(T_2 - \frac{T_1}{N_c}\right) \frac{\alpha_s}{4\pi\varepsilon} \frac{q^2 g^{\rho\sigma} + 2q^\rho q^\sigma}{12q^2} \gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\rho\gamma_\mu\gamma_\sigma\gamma^\gamma\gamma^\beta\gamma^\alpha \otimes \gamma^\mu$$

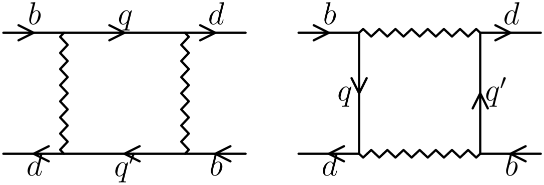
$$= T_F n_f \left(T_2 - \frac{T_1}{N_c}\right) \frac{\alpha_s}{4\pi\varepsilon} \frac{8}{3} \gamma^\mu \otimes \gamma_\mu$$

$<O_6^0>$ result

$$
\begin{aligned}
<O_6^0> = & \left[1 - 2C_F \frac{\alpha_s}{4\pi\varepsilon}\right] <O_6> \\
& + 2T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_5> - \frac{<O_6>}{N_c} \right) \\
& + \frac{1}{2} C_F \frac{\alpha_s}{4\pi\varepsilon} (20 <O_6> - 64 <O_4>) \\
& - 32 T_F \frac{\alpha_s}{4\pi\varepsilon} \left(<O_3> - \frac{<O_4>}{N_c} \right) \\
& + \frac{8}{3} T_F n_f \frac{\alpha_s}{4\pi\varepsilon} \left(<O_4> - \frac{<O_3>}{N_c} \right)
\end{aligned}
$$

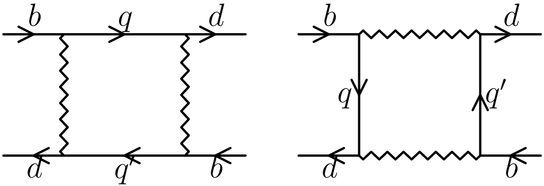
1-loop anomalous dimensions

$$B^0 \leftrightarrow \bar{B}^0$$



$$O = \left(\bar{d}_L \gamma^\alpha b_L\right)\left(\bar{d}_L \gamma^\alpha b_L\right)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

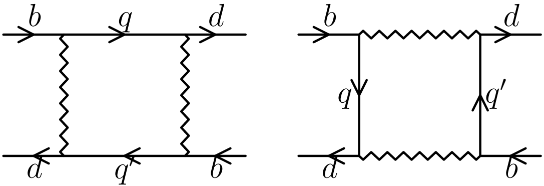


$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = \sum_{q,q'=u,c,t} V_{qb}^* V_{qd} V_{q'b}^* V_{q'd} S(x_q, x_{q'})$$

$$x_q = \frac{m_q^2}{M_W^2}$$

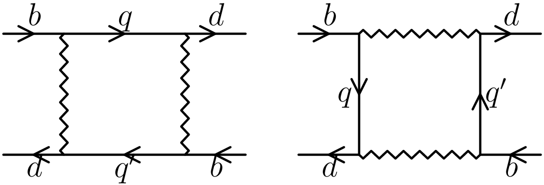


$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = \sum_{q,q'=u,c,t} V_{qb}^* V_{qd} V_{q'b}^* V_{q'd}\,[S(x_q, x_{q'}) - S(x_q, 0)]$$

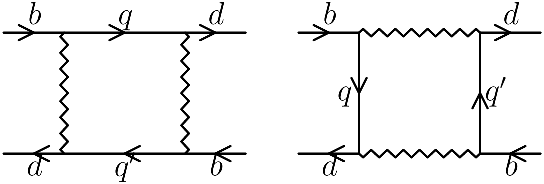
$$\sum_{q'=u,c,t} V_{q'b}^* V_{q'd} = 0$$



$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = V_{tb}^* V_{td} \sum_{q=u,c,t} V_{qb}^* V_{qd} \left[S(x_q, x_t) - S(x_q, 0)\right]$$

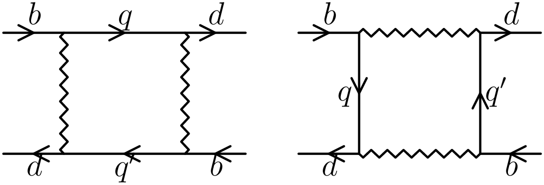


$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = V_{tb}^* V_{td} \sum_{q=u,c,t} V_{qb}^* V_{qd} \left[S(x_q, x_t) - S(x_q, 0) - S(0, x_t) + S(0,0)\right]$$

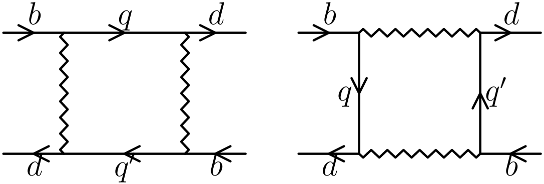
$$\sum_{q=u,c,t} V_{qb}^* V_{qd} = 0$$



$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = (V_{tb}^* V_{td})^2 \left[S(x_t, x_t) - S(x_t, 0) - S(0, x_t) + S(0, 0)\right]$$



$$O = (\bar{d}_L \gamma^\alpha b_L)\,(\bar{d}_L \gamma^\alpha b_L)$$

$$L = \frac{g_2^4}{512\pi^2 M_W^2} C O$$

$$C = (V_{tb}^* V_{td})^2 \left[S(x_t, x_t) - 2S(x_t, 0) + S(0, 0)\right]$$

1-loop anomalous dimension

$$\gamma_0 = \lambda_+ = 12T_F\left(1 - \frac{1}{N_c}\right)$$