New physics in $B \to D^{(*)} \tau \nu_\tau$ decay

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Lecture I

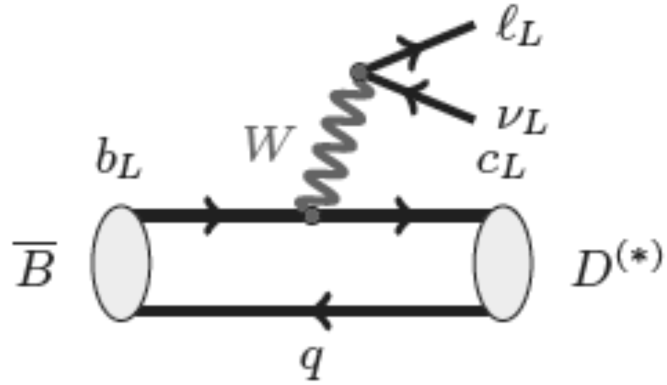
$B \to D^{(*)} l \nu_l$ decay in SM

Lecture II

$B \to D^{(*)} \tau \nu_\tau$ decay and New Physics

Outline

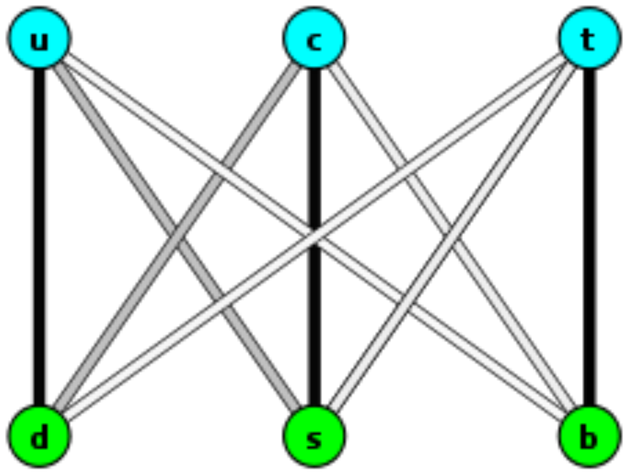
- Motivation;
- Exclusive decay modes $\bar{B} \to D l \nu_l$ and $\bar{B} \to D^* l \nu_l$
- Hadronic matrix elements (symmetries);
- Form factors and heavy-quark symmetry (Isgur-Wise function)
- Helicity amplitudes;
- Branching ratios in SM.



Motivation

We have to know CKM matrix elements as precise as possible!

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & \boxed{V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$



HFAG: 1207.1158

$$|V_{cb}| = (39.54 \pm 0.50_{\rm exp} \pm 0.74_{\rm th}) \times 10^{-3}$$ B$\rightarrow$ $D l \nu_l$

$$|V_{cb}| = (39.70 \pm 1.42_{\rm exp} \pm 0.89_{\rm th}) \times 10^{-3}$$ B$\rightarrow$ $D^* l \nu_l$

SM in $b \to c(u)\tau\nu_\tau$

$$\mathcal{H}_{\rm eff}^{b\to q} = \frac{G_F}{\sqrt{2}} V_{qb} \sum_{l=e,\mu,\tau} [(\bar{q}\gamma_\mu(1-\gamma_5)b)(\bar{l}\gamma^\mu(1-\gamma_5)\nu)]$$

/c

Why important?

$$\begin{array}{lcl} B & \to & \tau\nu_\tau \\ B & \to & X_c\tau\nu_\tau \\ B & \to & \pi(\rho)\tau\nu_\tau \\ B & \to & D(D^*)\tau\nu_\tau \end{array}$$

i) Precise knowledge of V_{cb} and V_{ub} CKM

ii) learn about SM – decay constants & form-factors;

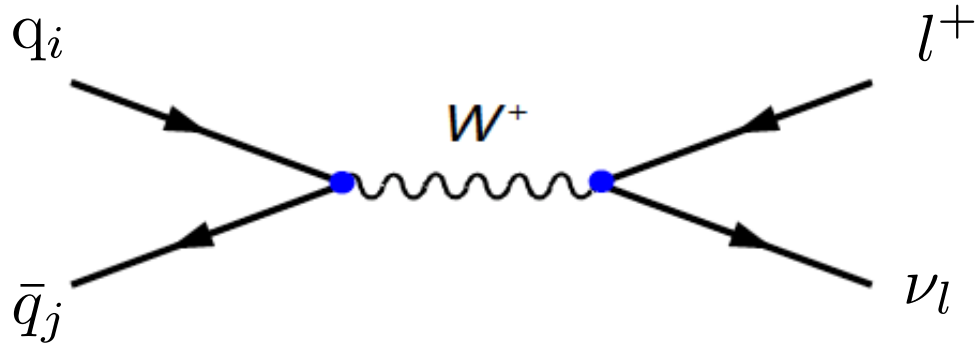
iii) form-factors which cannot be accessed in other semileptonic decays;

iv) NP at tree level;

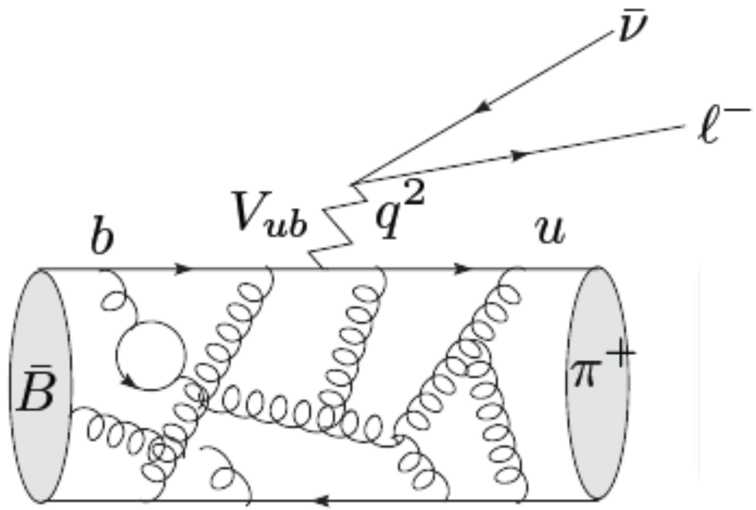
v) possible tests of lepton universality.

Meson semileptonic decays

quarks interaction



QCD @ work!



$$\bar{B} \to \pi^+ l^- \bar{\nu}_l$$

No information on the weak interactions without knowing QCD effects!

For semileptonic decays the main issue is how to determine hadronic matrix element

$$< M|j_\mu|P >$$

Methods:

- Symmetries (Lorentz symmetry, symmetries of strong interactions);
- Lattice gauge theories: simple hadronic matrix elements are calculable
- or to use Effective theories:
  - different scales;
  - QCD effects on EW scales are calculable perturbatively;

QCD sum rules.

Parametrization of hadronic amplitudes

General decomposition using particles degrees of freedom

e.g. $$\langle P_1(p_1)P_2(p_2)|V(p_V,\epsilon)\rangle = G_{P_1P_2V}\,\epsilon\cdot p_1$$

$$\epsilon\cdot p_V = 0$$

$$\begin{aligned}\langle 0|\, J_\mu \,|P(p)\rangle &= i f_P p_\mu, \\ \langle 0|\, J_\mu \,|V(p,\epsilon)\rangle &= f_V m_V \epsilon_\mu\end{aligned}$$

For semileptonic decay of pseudoscalar meson P_i to P_f:

$$\langle P_f(p_f)|\, J_V^\mu \,|P_i(p_i)\rangle = F_+(s)(p_i+p_f)^\mu + F_-(s)(p_i-p_f)^\mu$$

$$m_l^2 \le s \le (m_{Pi} - m_{Pf})^2$$

Let's start with:

$$\langle 0|\, J_V^\mu \,|P_i(p_i)P_f(p_f)\rangle = F_+(t)(p_i - p_f)^\mu + F_-(t)(p_i + p_f)^\mu$$

(crossing symmetry $p_f \to -p_f$)

$$t = (p_i + p_f)^2$$

c.m. system of $P_i P_f$

$$|P_i(\mathbf{p_i})P_f(-\mathbf{p_i})\rangle = \sum_{L,m} Y_L^m(\mathbf{p}/|\mathbf{p}|)\,|P_i P_f(\mathbf{p}, L, m)\rangle$$

p is a center of mass three momentum $J^P = 1^-$ or 0^+

$$\langle 0|\, J_V^\mu \,|P_i(p_i)P_f(p_f)\rangle = \langle 0|\, J_V^\mu \,|P_i P_f(\mathbf{p}, 0, 0)\rangle + \sum_m \langle 0|\, J_V^\mu \,|P_i P_f(\mathbf{p}, 1, m)\rangle\, Y_1^m(\mathbf{p}/|\mathbf{p}|)$$

nonvanishing for μ=0 nonvanishing for μ=1,2,3

$$t = (p_i^0 + p_f^0)^2 \qquad p_i^0 - p_f^0 = (m_{P_i}^2 - m_{P_f}^2)/\sqrt{t}$$

$\mu = 0$ Scalar form factor ⟶ spin 0 particles

$$\langle 0|\, J_V^0 \,|P_i P_f(p,0,0)\rangle = \sqrt{t}\left[\frac{m_{P_i}^2 - m_{P_f}^2}{t} F_+(t) + F_-(t)\right] = \sqrt{t} F_0(t)$$

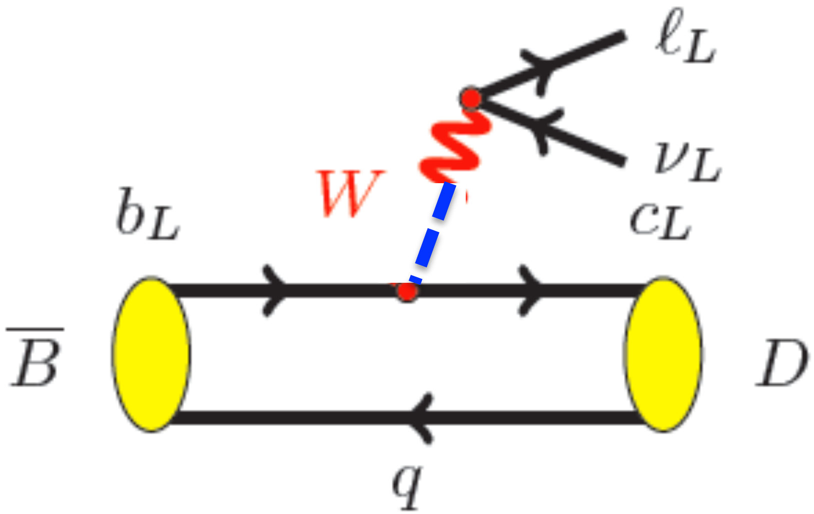
$\mu = 1,\ 2,\ 3$ “Vector form factor ⟶ spin 1 particles

$$\sum_m \langle 0|\, \mathbf{J}_V \,|P_i P_f(p,1,m)\rangle\, Y_1^m(\mathbf{p}/|\mathbf{p}|) = 2\mathbf{p_i} F_+(t)$$

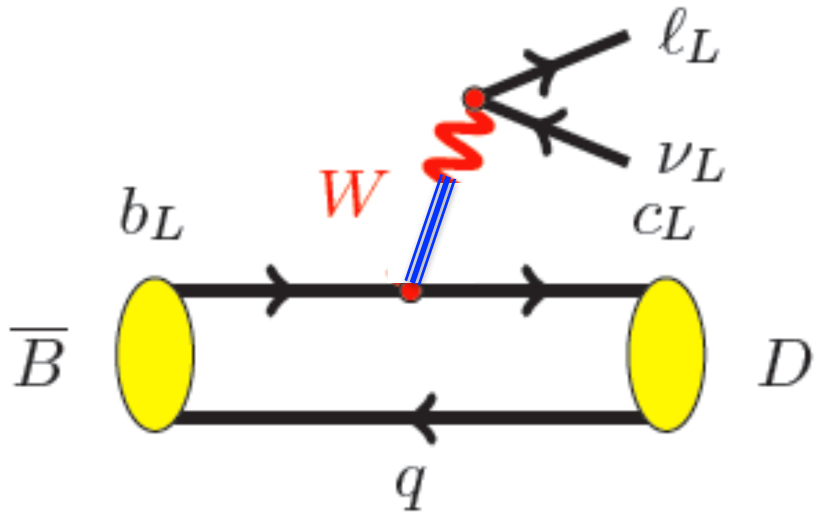
$$\begin{aligned}\langle P_f(p_f)|\, J_{V-A}^\mu \,|P_i(p_i)\rangle \quad &= \quad F_+(s)\left((p_i + p_f)^\mu - \frac{m_{P_i}^2 - m_{P_f}^2}{s}(p_i - p_f)^\mu\right) \\ &+ F_0(s)\frac{m_{P_i}^2 - m_{P_f}^2}{s}(p_i - p_f)^\mu,\end{aligned}$$

$$F_+(0) = F_0(0)$$

Vector form factor, accounts spin 1 particles



F_0: scalar particle



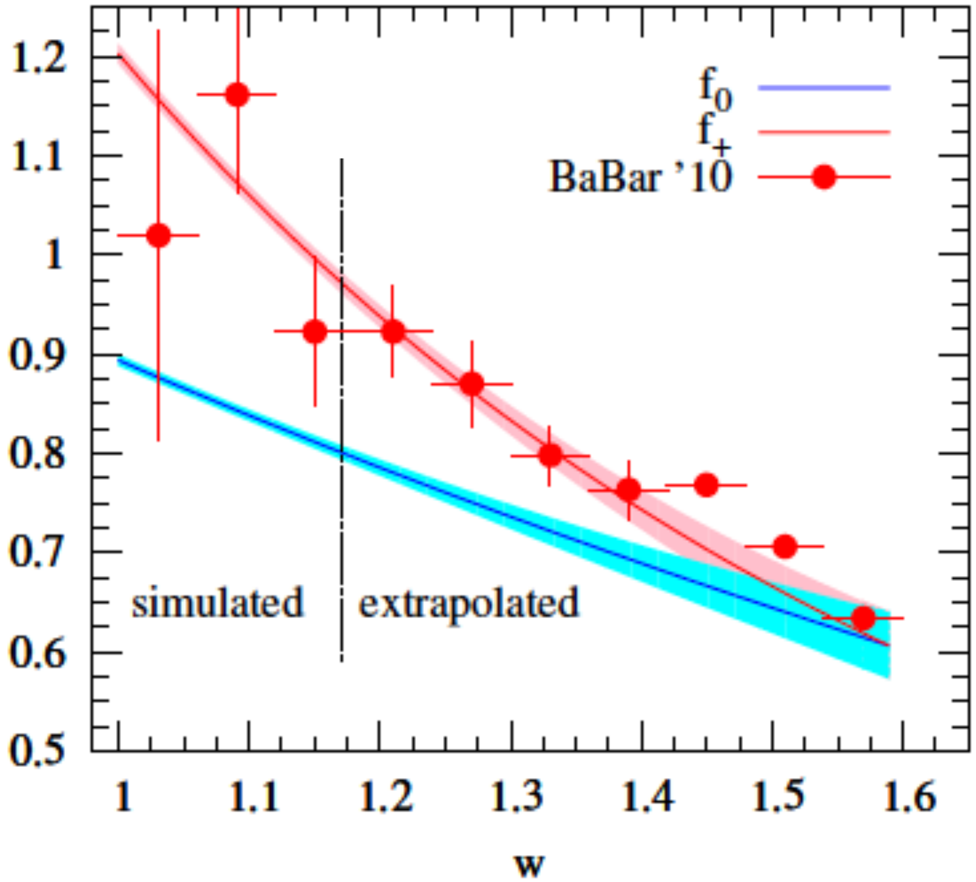
F_+: vector particle

The basic problem: knowledge of all for factors!

- In the case of $\mathrm{B}\rightarrow\ D l \nu_l$ decay the lattice QCD has already found the form of form factors!

- in the case $\mathrm{B}\rightarrow\ D^* l \nu_l$ one has to rely on other methods, e.g. use heavy quark symmetry!

Recent Lattice QCD results on $B \rightarrow D\,\tau\,\nu_\tau$



$$w = (M_B^2 + M_D^2 - q^2)/(2M_D M_B)$$

MILC collaboration:
1206.4992, first SM
"ab intio full QCD"
calculation (unquenched)

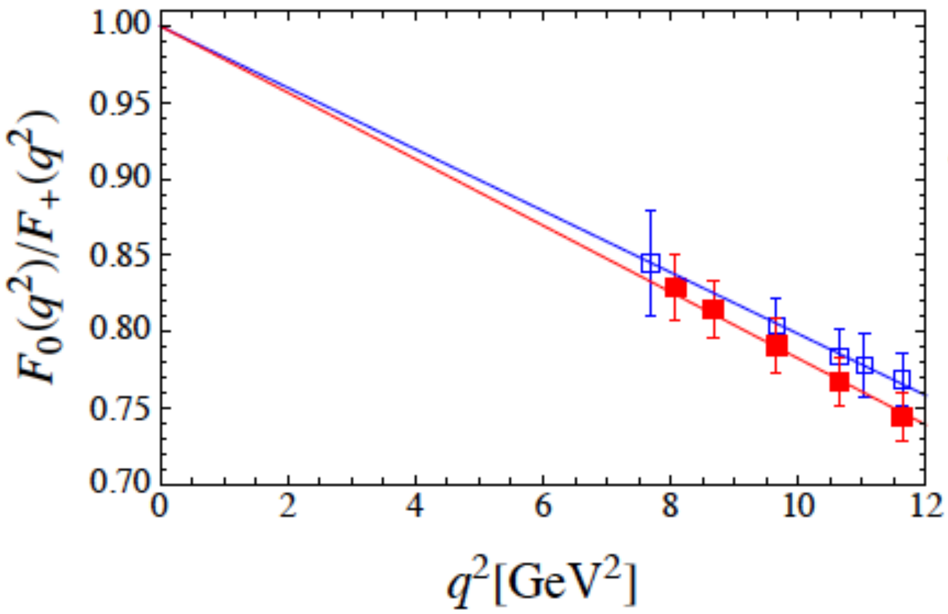
$$R(D) = 0.316(12)(7)$$

D. Becirevic Kosnik, Tayduganov , (1206.4977) proposal, using lattice data:

$$\frac{F_0(q^2)}{F_+(q^2)} = 1 - \alpha\, q^2$$

$$\alpha = 0.020(1)\ \mathrm{GeV}^{-2}$$

lattice result, in agreement with pole model, QCD sum rule study, etc



Proposal: experimenters should make cut at $q^2 \approx 8$ GeV^2, then full shape of the vector form factor could be reconstructed from the rate

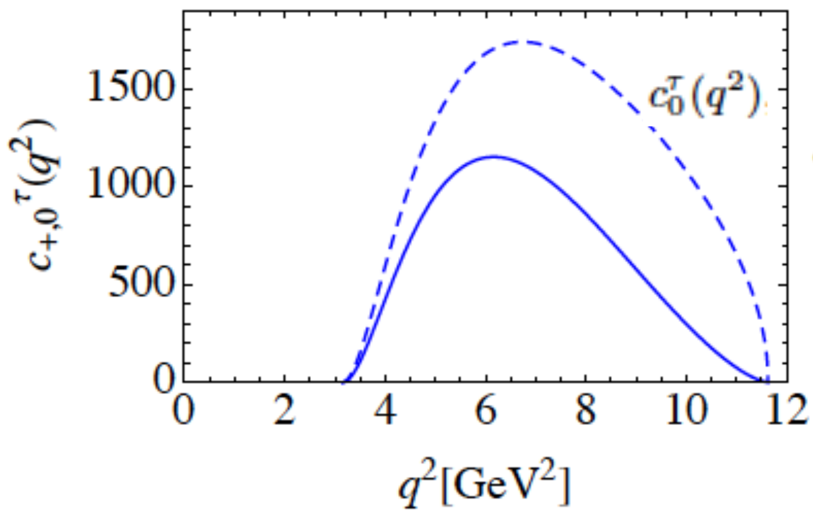
$$\left.\frac{\mathcal{B}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D\mu\bar{\nu}_\mu)}\right|_{q^2 \leq 8\ \mathrm{GeV}^2}$$

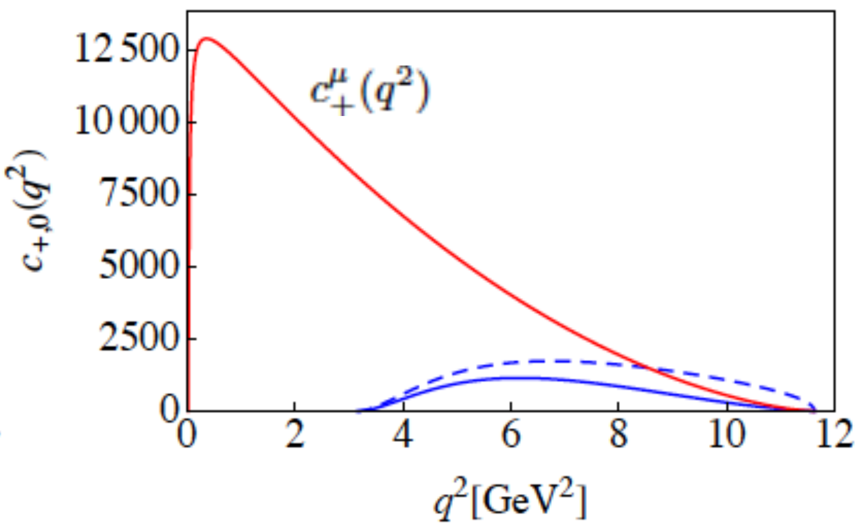
Ratio. scalar and vector form factor known from lattice

$$\frac{d\mathcal{B}(\bar{B} \to D\ell\bar{\nu}_\ell)}{dq^2} = \tau_{B^0} \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left[c_+^\ell(q^2)|F_+(q^2)|^2 + c_0^\ell(q^2)|F_0(q^2)|^2 \right]$$
$$= |V_{cb}|^2 \mathcal{B}_0 |F_+(q^2)|^2 \left[c_+^\ell(q^2) + c_0^\ell(q^2) \left| \frac{F_0(q^2)}{F_+(q^2)} \right|^2 \right],$$

D.Becirevic et al, 1206.4977

The difference in phase space contributions for τ and μ leptonic pair





Parametrization of a weak current in $B \to D^* l \nu_l$:

1 vector form factor and 3 axial vectors

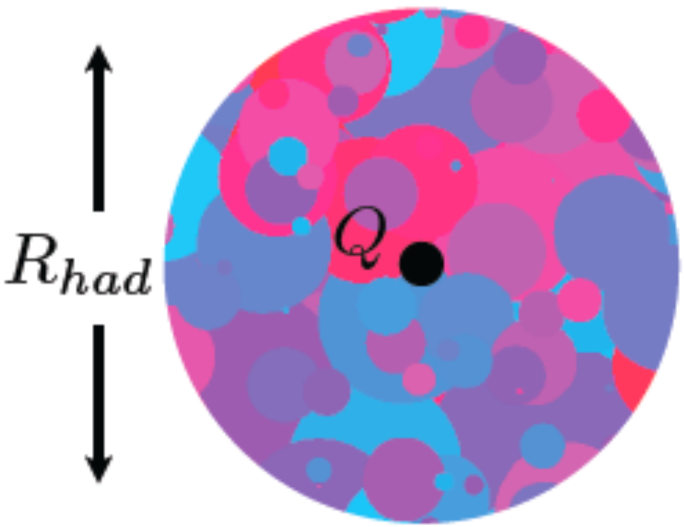
$$\langle \underbrace{D(p', \epsilon)}_{D^*} \mid \bar{c}\gamma^{\mu}(1-\gamma_5)b \mid B(p)\rangle = \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B + m_{D*}} \epsilon^*_{\nu} p'_{\alpha} p_{\beta} V(q^2)$$
$$- \left[(m_B + m_{D*})\epsilon^{*\mu} A_1(q^2) - \frac{\epsilon^* \cdot q}{m_B + m^*_D}(p' + p)^{\mu} A_2(q^2) \right]$$
$$- 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q^{\mu} A_0(q^2).$$

$A_3(0) = A_0(q^2)$ in order to cancel unphysical pole at $q^2 = 0$

$$A_3(q^2) = \frac{m_B + m_D}{2m_{D^*}} A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(q^2)$$

For all form factors mostly used result based on Heavy quark symmetry

Heavy quark symmetry



• Light quarks do not feel mass of the heavy quark $m_Q \to \infty$;

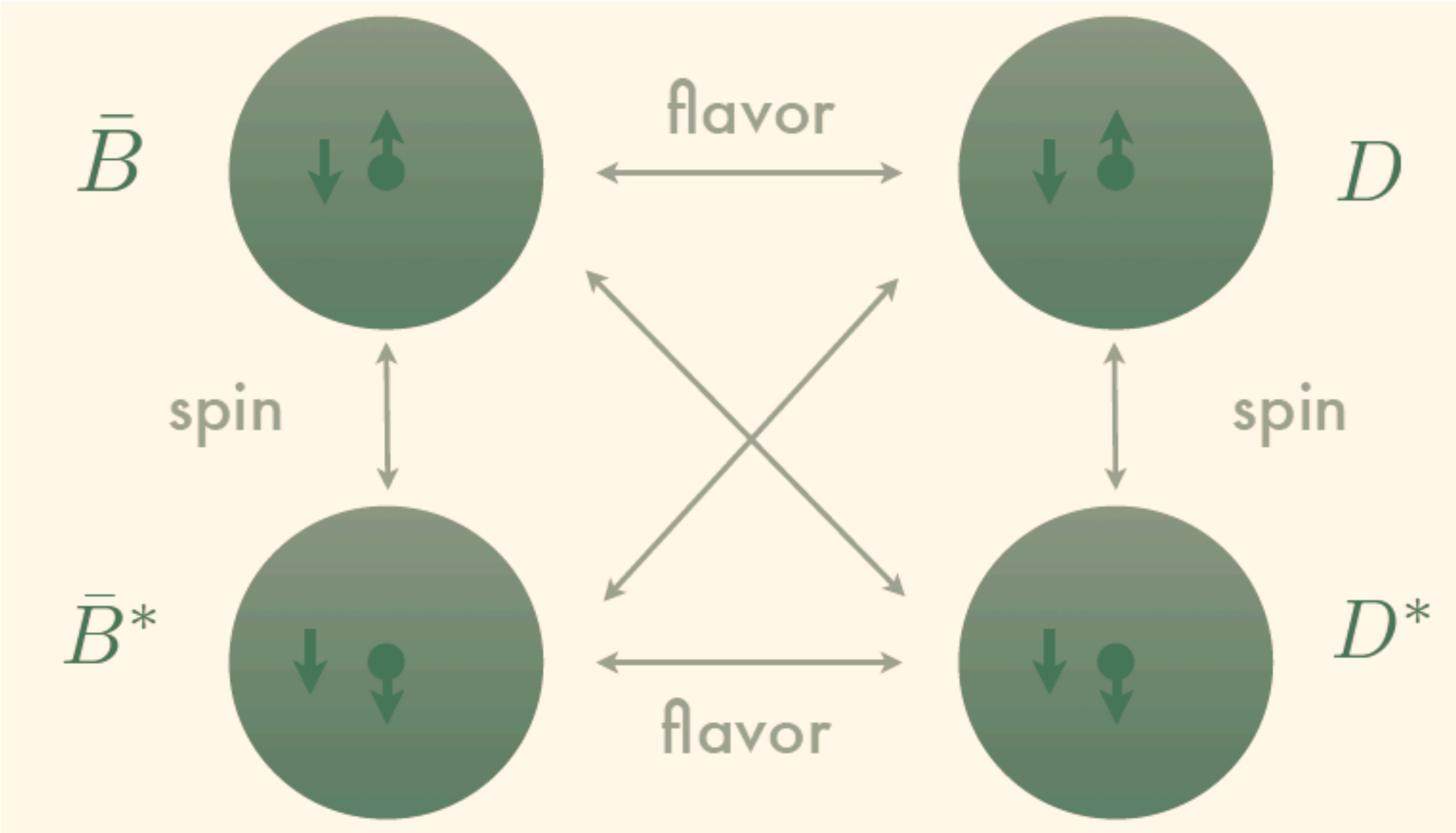
• Flavor symmetry;

• Spin symmetry (magnetic moments decouple $\mu \sim 1/m_Q$).

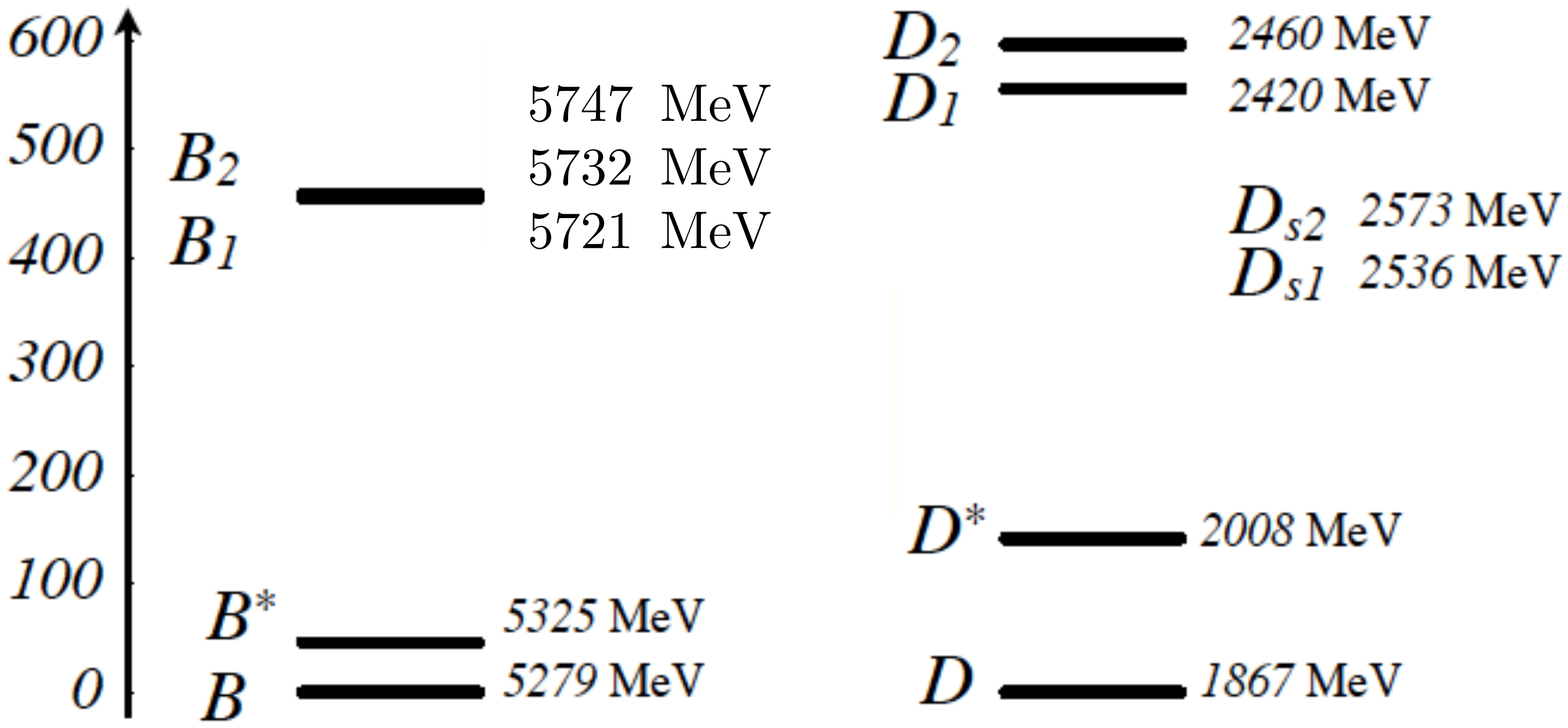
$$R_{had} \sim \frac{1}{\Lambda_{\rm QCD}} \gg \frac{1}{m_Q} = \lambda_Q$$

$$\alpha_s(m_Q) \ll 1$$

Spin-flavor symmetry



Heavy meson mass spectrum



Spin doublets almost degenerate $M_{B1} - M_B \simeq M_{D1} - M_D$

Spin doublets almost degenerate $M_{B*} - M_B = 46\ \text{MeV}$

Heavy quark effective Lagrangian

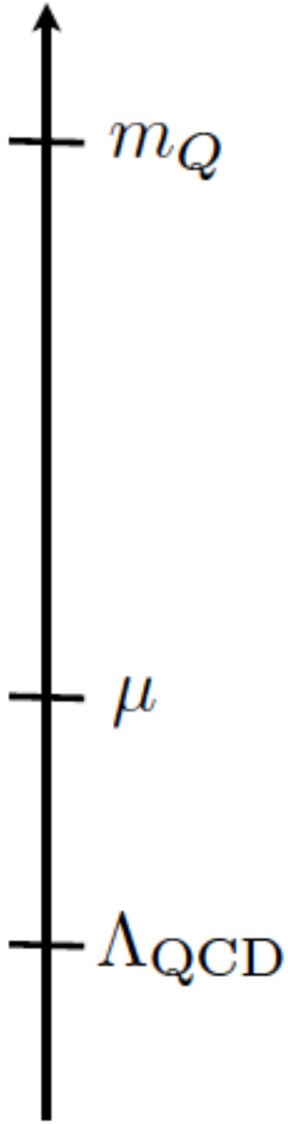
$$\mathcal{L}_{eff} = \mathcal{L}_{LO} + \mathcal{L}_{NLO} + \ldots$$

$$m_Q \to \infty \qquad 1/m_Q$$

Perturbative expansion in $1/m_Q$

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_Q(i\not{D} - m_Q)\psi_Q$$

$$\gamma_\mu(i\partial_\mu + g_s G^i_\mu T_i)$$



$$\psi_Q(x) = e^{-im_Q t}\,\psi_Q(0) \longrightarrow m_Q(\gamma_0 - 1)\psi_Q(0) = 0$$

In an arbitrary frame: v_μ 4-vector of velocity,
leads to projection operators

$$P_+ = \frac{1+\not{v}}{2} \qquad P_- = \frac{1-\not{v}}{2}$$

for v_μ=(1,0,0,0) $P_+ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$ $P_- = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$$(\not{v}\,\not{v} = v^2 = 1) \qquad P_\pm^2 = P_\pm\,, \qquad P_+\,P_- = P_-\,P_+ = 0$$

$$\begin{aligned} \psi_Q(x) &= e^{-im_Q v\cdot x}\ \tilde{\psi}_Q(x) \\ &= e^{-im_Q v\cdot x}\left[P_+\,\tilde{\psi}_Q(x) + P_-\,\tilde{\psi}_Q(x)\right] \\ &= e^{-im_Q v\cdot x}\left[h_v(x) + H_v(x)\right] \end{aligned}$$

Quark field
redefinition

Dirac equation becomes

$$\left\{m_Q \not{v} + i\not{D} - m_Q\right\}[h_v(x) + H_v(x)] = 0$$

$$\leftrightarrow \; i\not{D} h_v(x) + (i\not{D} - 2m_Q)H_v(x) = 0$$

$$\not{v} h_v = h_v \,, \; \not{v} H_v = -H_v$$

Multiplying equations by $P_-(P_+)$ and with $a_\perp^\mu = a^\mu - v \cdot a\, v^\mu$ using

$$P_+ \not{a} \quad = \not{a}_\perp P_- + v \cdot a P_+$$

$$P_- \not{a} \quad = \not{a}_\perp P_+ - v \cdot a P_-$$

$$iv \cdot D\, h_v(x) + i\not{D}_\perp\, H_v \quad = 0$$

$$i\not{D}_\perp\, h_v(x) - (iv \cdot D + 2m_Q)H_v(x) \quad = 0$$

$$H_v(x) \approx \frac{1}{2m_Q} i\not{D}_\perp\, h_v(x)$$

$$
\begin{aligned}
i\not{D}_\perp\, i\not{D}_\perp \quad &= iD_\perp^\mu\, iD_\perp^\nu \,\left(\frac{1}{2}\{\gamma_\mu, \gamma_\nu\} + \frac{1}{2}[\gamma_\mu, \gamma_\nu]\right) \\
&= iD_\perp^\mu\, iD_\perp^\nu \,(g_{\mu\nu} - i\sigma_{\mu\nu}) \\
&= (iD_\perp)^2 + \frac{i}{2}[D_\perp^\mu, D_\perp^\nu]\sigma_{\mu\nu} \\
&= (iD_\perp)^2 + \frac{g_s}{2}\sigma_{\mu\nu}G_\perp^{\mu\nu}
\end{aligned}
$$

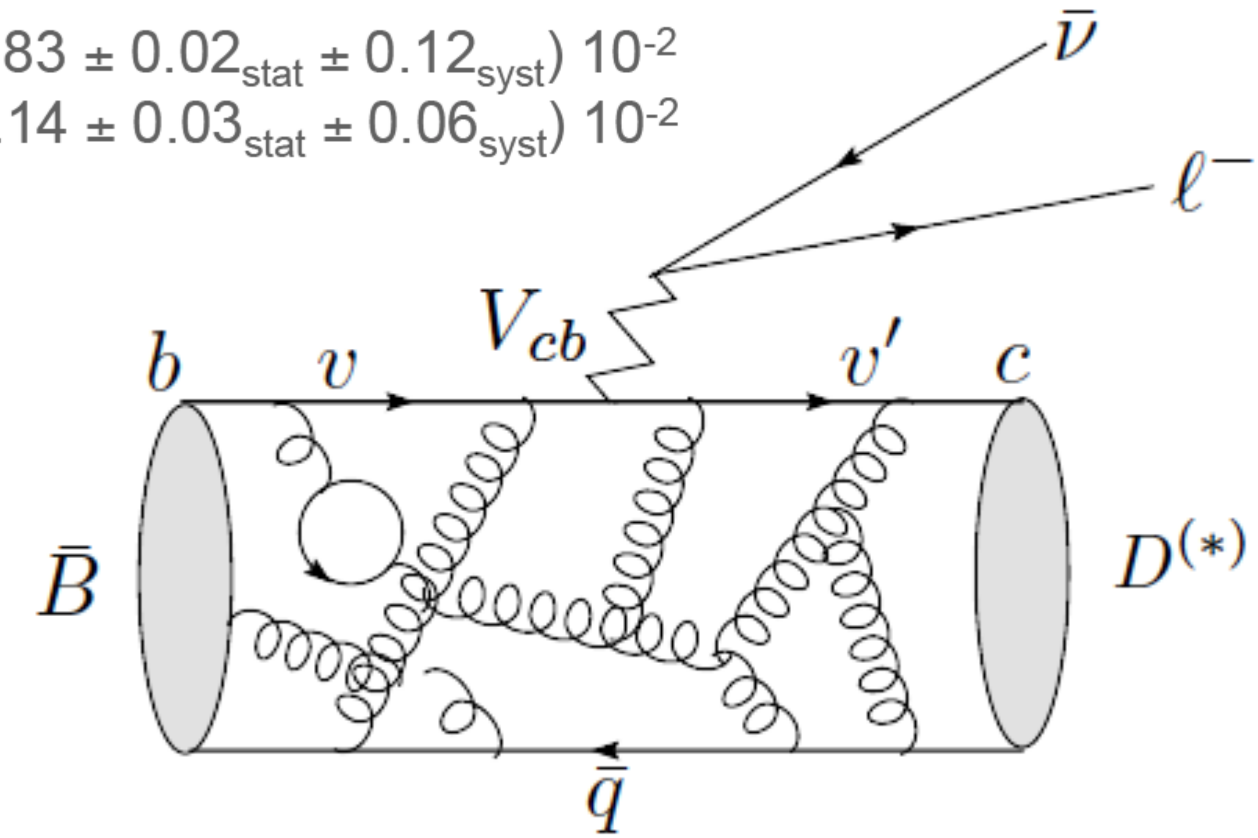
$$
\mathcal{L}_{\rm eff} = \bar{h}_v\, iv \cdot D\, h_v + \frac{1}{2m_Q}\bar{h}_v(iD_\perp)^2 h_v + \frac{g_s}{4m_Q}\bar{h}_v\sigma_{\mu\nu}G_\perp^{\mu\nu}\, h_v + \mathcal{O}(\Lambda/m_Q^2)
$$

flavor and spin symmetric

$1/m_Q$ corrections

$B \to D^{(*)} l \nu_l$ and heavy quark symmetry

$BF_{D^{*+}l^-\nu} = (4.83 \pm 0.02_{stat} \pm 0.12_{syst})\ 10^{-2}$
$BF_{D^+l^-\nu} = (2.14 \pm 0.03_{stat} \pm 0.06_{syst})\ 10^{-2}$



Heavy quark symmetry: for v=v' "zero recoil point", corresponds to maximum of lepton pair momentum transfer.

Why this result is interesting?

The goal precise determination of the V_{cb} CKM matrix element.

In the case of $B \rightarrow D^* l \nu_l$ Luke found that first order in $1/m_Q$ corrections vanish at zero-recoil point (Luke's theorem).

Procedure:

- measure branching rate as a function of w, and then extrapolate to the zero recoil point w=1;

- calculate second order power correction using lattice QCD;
- one single parameter ρ is sufficient to parametrize shape of the form-factor.

HQS

$$< \bar{B}(p')|\bar{b}\gamma_\mu b|\bar{B}(p) >= F(q^2)(p+p')_\mu$$ QCD

$$p' \simeq m_b v' \qquad p \simeq m_b v$$

normalization

$$< \bar{B}(p')|\bar{B}(p) >= 2m_B v^0 (2\pi)^3 \delta^3(\vec{p}-\vec{p}')$$

Heavy quark effective theory

$$\frac{1}{m_B} < \bar{B}(v')|\bar{b}\gamma_\mu b|\bar{B}(v) >= \xi(v\cdot v')(v+v')_\mu + \mathcal{O}(\frac{1}{m_B})$$

$$F(q^2) = \xi(v\cdot v')$$

$$q^2 = -2m_B^2(v\cdot v' - 1)$$

$$\boxed{F(0) = 1 \leftrightarrow \xi(1) = 1}$$

This means nothing happens with heavy meson

$$w = v \cdot v' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

For $B \to D^{(*)} e \bar{\nu}_e$ w is variable in the range

$$1 < w \lesssim 1.6$$

$$\frac{\langle D(p')|V^\mu|B(p)\rangle}{\sqrt{m_B m_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$\frac{\langle D^*(p',\varepsilon)|V^\mu|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = i h_V(w) \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* v'_\alpha v_\beta$$

$$\frac{\langle D^*(p',\varepsilon)|A^\mu|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w)(w+1)\varepsilon^{*\mu} - h_{A_2}(\varepsilon^* \cdot v)v^\mu$$

$$- h_{A_3}(w)(\varepsilon^* \cdot v)v^\mu.$$

heavy quark symmetry allows to replace b quark by c quark:

$$\frac{1}{m_B} < \bar{B}(v')|\bar{b}\gamma_\mu b|\bar{B}(v) >= \xi(v \cdot v')(v + v')_\mu$$

$$\frac{1}{\sqrt{m_B m_D}} < D(v')|\bar{b}\gamma_\mu b|\bar{B}(v) >= \xi(v \cdot v')(v + v')_\mu$$

Without heavy quark symmetry

$$< D(p')|\bar{b}\gamma_\mu b|\bar{B}(p) >= f_+(q^2)(p + p')_\mu + f_-(q^2)(p - p')_\mu$$

$$f_\pm(q^2) = \frac{m_B \pm m_D}{2\sqrt{m_B m_D}}\xi(v \cdot v')$$

$$q^2 = m_B^2 + m_D^2 - 2m_B m_D v \cdot v'$$

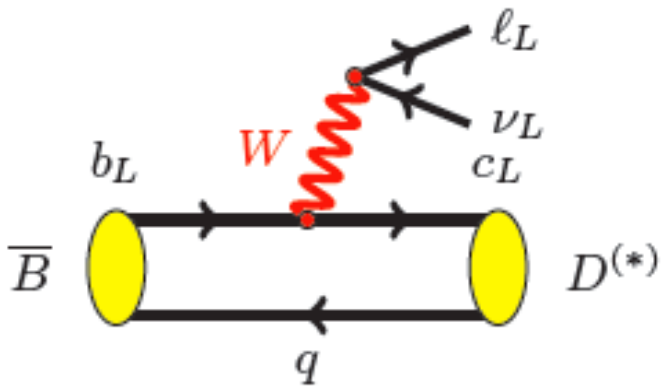
It is obvious that in the limit

$$m_B, m_D \to \infty$$

$$f_- \to 0$$

and only f_+ survives

Effective weak Lagrangian is



$$\mathcal{L}_{eff} = -\frac{G}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1-\gamma_5) b \;\; \bar{l} \gamma^\mu (1-\gamma_5) \nu_l$$

The amplitude is

$$\mathcal{M} = \frac{G}{\sqrt{2}} V_{cb} < D^{(*)}(v') | \bar{c} \gamma_\mu (1-\gamma_5) b | B(v) > \bar{u}_l(p_l) \gamma^\mu (1-\gamma_5) v_{\nu_l}(p_\nu)$$

To obtain branching ratio: square of the amplitude, sum over spins and integrate over phase space.

By neglecting corrections: $\alpha_s(m_{c,b})$ and $\Lambda_{QCD}/m_{c,b}$
one can match full theory to HQET

$$\bar{c}\gamma_\mu b = \bar{c}_{v'}\gamma_\mu b_v$$

$$\bar{c}\gamma_\mu\gamma_5 b = \bar{c}_{v'}\gamma_\mu\gamma_5 b_v$$

Covariant representation of fields

Meson consisting of heavy quark Q and light anti-quark q̄ can be described by the object, which has all transformation properties of this structure

$$H_v^{(Q)} = \frac{(1+\not{v})}{2}[\gamma_\mu P_v^{*\mu(Q)} - \gamma_5 P_v^{(Q)}]$$

$$\not{v} H_v^{(Q)} = H_v^{(Q)}$$

$$v^\mu P_{\mu v}^{*(Q)} = 0.$$

$$H_v^{(Q)} \rightarrow D(R)_Q H_v^{(Q)}$$

$$H_v^{(Q)}\not{v} = -H_v^{(Q)}$$

Dirac four component spinor representation under parity $$\gamma^0 D^\dagger(R)_Q \gamma^0 = D(R)_Q^{-1}$$

$$x_P = (x^0, -\mathbf{x}), \qquad v_P = (v^0, -\mathbf{v})$$

$$\bar{H}_v^{(Q)} = [\gamma^\mu P_{v\mu}^{*(Q)\dagger} + \gamma_5 P_v^{(Q)\dagger}] \frac{(1+\not{v})}{2}$$

$$Tr \bar{H}_v^{(Q)} H_v^{(Q)} = -2 P_v^{(Q)\dagger} P_v^{(Q)} + 2 P_{v\mu}^{*(Q)\dagger} P_v^{*(Q)\mu}$$

Transformation properties of $\bar{c}_{v'} \Gamma b_v$

We want that this operator transforms the same way as a quark operator

$$\bar{c}_{v'} \Gamma b_v = Tr \bar{H}_{v'}^{(c)} \Gamma H_v^{(b)} X$$

$$X = X_0 + X_1 \not{v} + X_2 \not{v}' + X_3 \not{v} \not{v}'$$

$$\not{v} H_v^{(b)} = H_v^{(b)} \qquad \bar{H}_{v'}^{(c)} \not{v}' = -\bar{H}_{v'}^{(c)}$$

Due to these properties only one function remains:

$$\boxed{X = -\xi(w)}$$

Isgur-Wise function

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)$$
$$h_-(w) = h_{A_2}(w) = 0.$$

Matrix elements of the vector and axial current with HQS

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(v',\varepsilon)|\, \bar{c}_{v'} \gamma^\mu b_v \,|\bar{B}(v)\rangle = i\epsilon^{\mu\nu\alpha\beta}\, \varepsilon^*_\nu\, v'_\alpha v_\beta\; \xi(v\cdot v'),$$

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(v',\varepsilon)|\, \bar{c}_{v'} \gamma^\mu \gamma_5\, b_v \,|\bar{B}(v)\rangle = \Big[\varepsilon^{*\mu}\,(v\cdot v'+1) - v'^\mu\, \varepsilon^* \cdot v\Big]\xi(v\cdot v')$$

Heavy quark symmetry reduces 4 form factors to only one:

$$\frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}}\,\xi(v\cdot v') = V(q^2) = A_0(q^2) = A_2(q^2)$$

$$= \left[1 - \frac{q^2}{(m_B + m_D)^2}\right]^{-1} A_1(q^2)$$

$$q^2 = m_B^2 + m_{D^*}^2 - 2 m_B m_{D^*}\, v\cdot v'$$

$$\frac{d\Gamma(\bar{B} \to D\,\ell\,\bar{\nu})}{dw} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\,(m_B + m_D)^2\,m_D^3\,(w^2-1)^{3/2}\,\boxed{\xi^2(w)}\,,$$

$$\frac{d\Gamma(\bar{B} \to D^*\ell\,\bar{\nu})}{dw} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\,(m_B - m_{D^*})^2\,m_{D^*}^3\,\sqrt{w^2-1}\,(w+1)^2$$

$$\times\left[1 + \frac{4w}{w+1}\,\frac{m_B^2 - 2w\,m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right]\boxed{\xi^2(w)}\,.$$

In both rates there is only one form factor $\xi(w), \;\; w = v \cdot v'$
with normalization $\xi(1) = 1$

What are the corrections to this rates?
- QCD $\alpha_s(m_Q)$ known 2 loops ;
- Λ/m_Q should come from lattice QCD.

Here lepton mass negligible!

For $B \to D^* l\nu$ there is no lattice calculations of the form factors yet!

Caprini, Lellouch and Neubert, NPB 530 (1998) 153:
Dispersive bound on shape of $\bar{B} \to D^{(*)} \ell \bar{\nu}$ form factors

• Ingridents: heavy quark, spin symmetry short distance and $1/m_Q$ corrections provide relations between form factors near zero recoil.

• Dispersive bounds are derived using dispersion relations and complex analysis techniques combing QCD calculations of current-current correlation functions in the Euclidean domain with the spectral representation of these functions in terms of sums over intermediate hadronic states.
The contributions of BD, BD*, B*D, B*D* states are considered in the unitarity sum, leading short distance contribution and $1/m_Q$ to the heavy quark Limit.

$$h_{A_1}(w) = A_1(q^2)\frac{1}{R_{D*}}\frac{2}{w+1}$$

$$A_0(q^2) = \frac{R_0(w)}{R_{D*}}h_{A_1}(w)$$

$$A_2(q^2) = \frac{R_2(w)}{R_{D*}}h_{A_1}(w)$$

$$V(q^2) = \frac{R_1(w)}{R_{D*}}h_{A_1}(w)$$

$$\begin{aligned}
h_{A_1}(w) &= h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right] , \\
R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \\
R_2(w) &= R_2(1) + 0.11(w-1) - 0.06(w-1)^2, \\
R_0(w) &= R_0(1) - 0.11(w-1) + 0.01(w-1)^2,
\end{aligned}$$

$$z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2}) \qquad R_{D*} = 2\sqrt{m_B m_{D*}}/(m_B + m_{D*}).$$

$R_2(1) = 0.80$ $R_0(1) = 1.22$ Caprini et al,

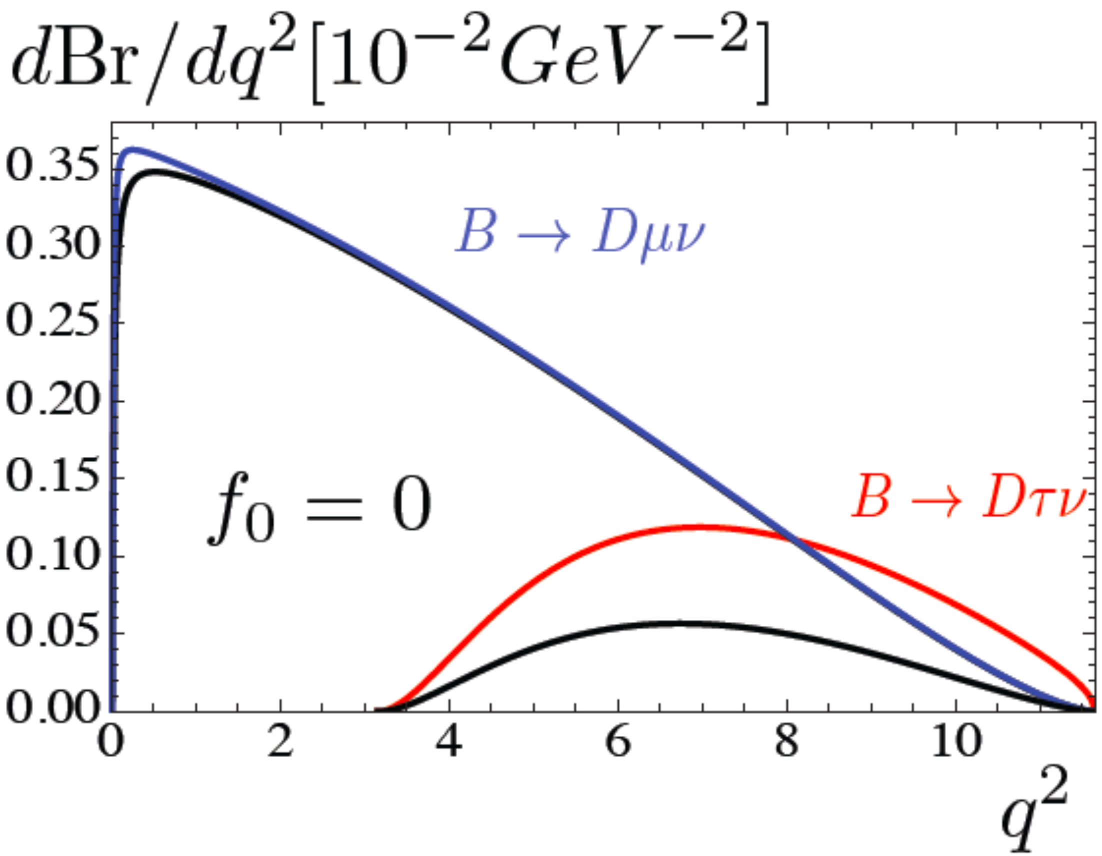
$$R_3(1) \equiv \frac{R_2(1)(1-r) + r[R_0(1)(1+r) - 2]}{(1-r)^2} = 0.97$$

Improvement using Belle collaboration result, PRD 82 (2010) 112007

$R_2(1) = 0.864(25)$ $R_0(1) = 1.14.$

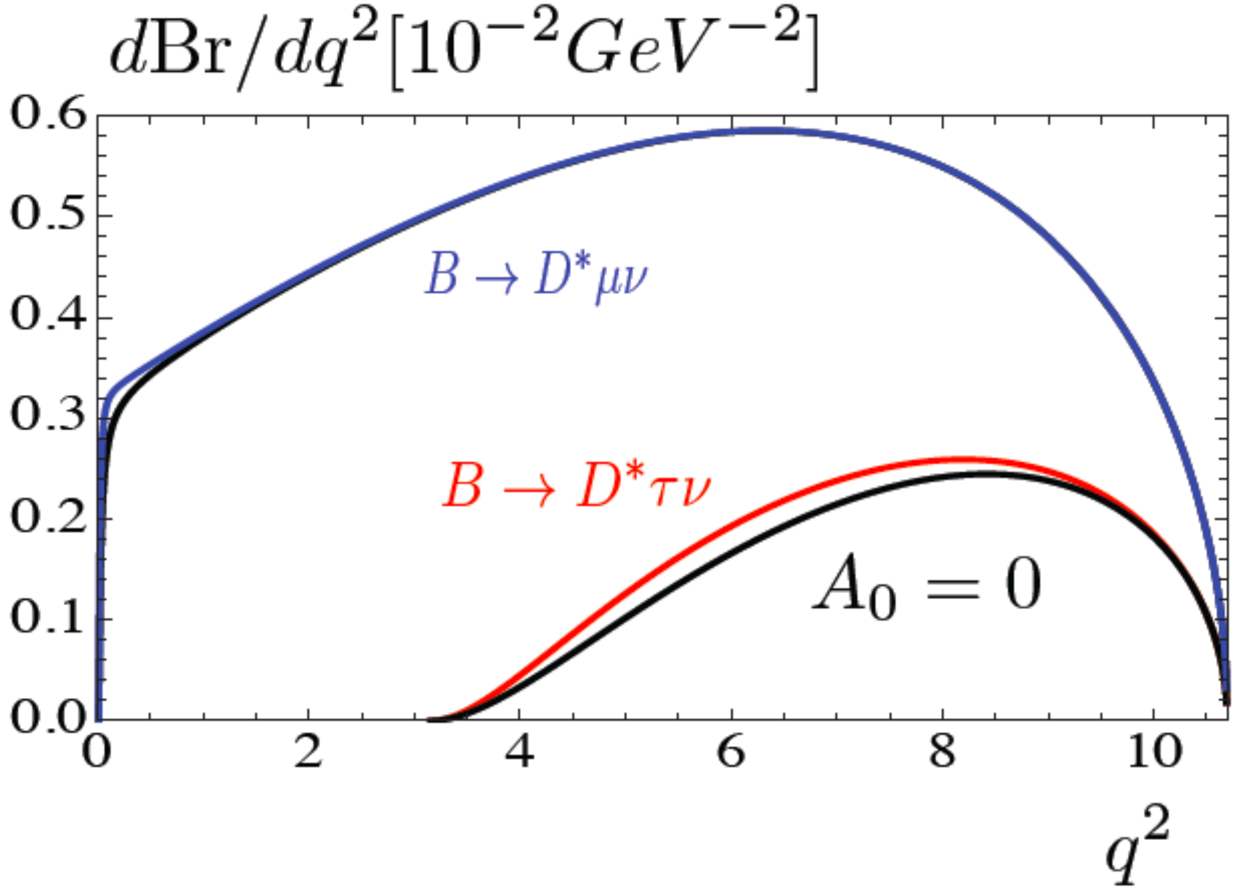
Comparison of differential momentum transfer distribution

$B \to D\mu\nu_\mu$ and $B \to D\tau\nu_\tau$



Comparison of differential momentum transfer distribution

$B \to D^* \mu \nu_\mu$ and $B \to D^* \tau \nu_\tau$



Experimental approach

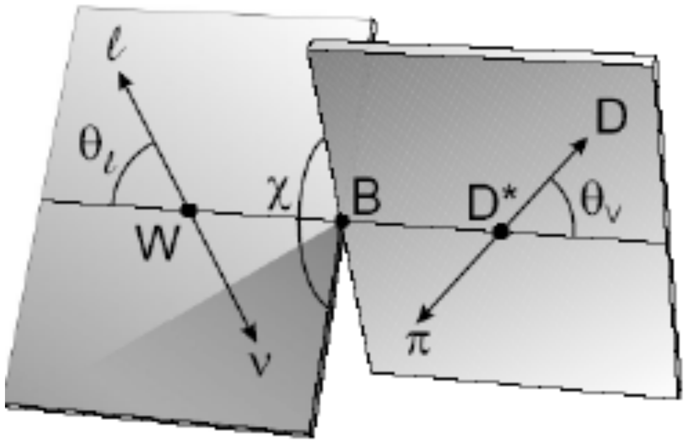
Extraction of form factors by fits to differential decay rates:

$\bar{B} \to D l \nu_l$

Fit to one decay distribution G(w)
Determination of $|V_{cb}|$ G(1) and slope ρ^2

$\bar{B} \to D^{(*)} l \nu_l$

Fit to 4-dim. decay distribution $G(w,\theta_\lambda,\theta_v,\chi)$
determination of $|V_{cb}|$ F(1) and slope ρ^2,
$R_1(w=1)$ and $R_2(w=1)$



Fit gives ρ and G/F and $|V_{cb}|$

Lattice QCD results give

$$G(1) = 1.074 \pm 0.018_{stat} \pm 0.016_{syst}$$
$$F(1) = 0.908 \pm 0.005_{stat} \pm 0.016_{syst}$$

NP Suppl. 140, 461 (2005)
FNAL/MILC 2011

BaBar and Belle measurements give

$D \ell \nu$: $|V_{cb}| = (39.46 \pm 1.54_{exp} \pm 0.88_{LQCD})\ x10^{-3}$

$D^* \ell \nu$: $|V_{cb}| = (39.04 \pm 0.55_{exp} \pm 0.73_{LQCD})\ x10^{-3}$

HQS based calculations

$G(1)=1.02 \pm 0.04$ $|V_{cb}| = (41.84 \pm 1.64_{exp} \pm 1.63_{SR})\ x10^{-3}$

$F(1) =0.86 \pm 0.02$ $|V_{cb}| = (41.22 \pm 0.58_{exp} \pm 0.95_{SR})\ x10^{-3}$

PLB585, 53(2004)
PRD 81, 113002 (2010)

BaBar result

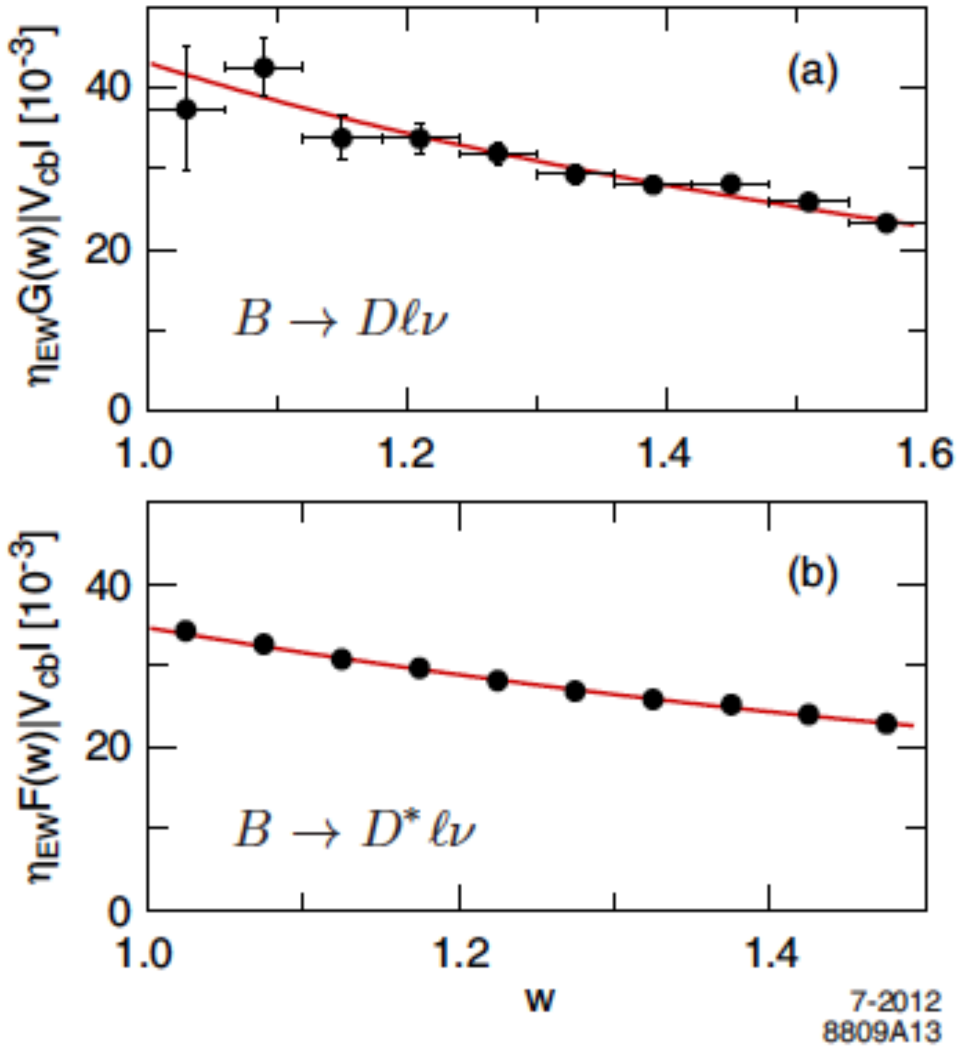
$$\frac{d\Gamma(B \to D\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2\mathcal{K}_D(w)\,\eta_{\rm EW}^2\,\mathcal{G}^2(w)$$

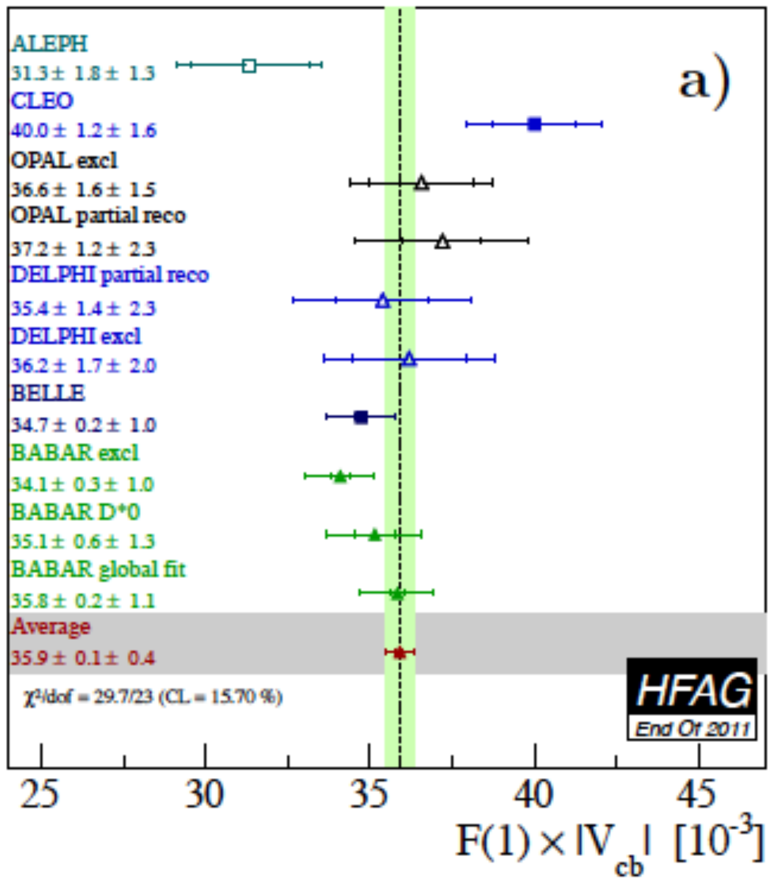
$B \to D^* \ell\nu$

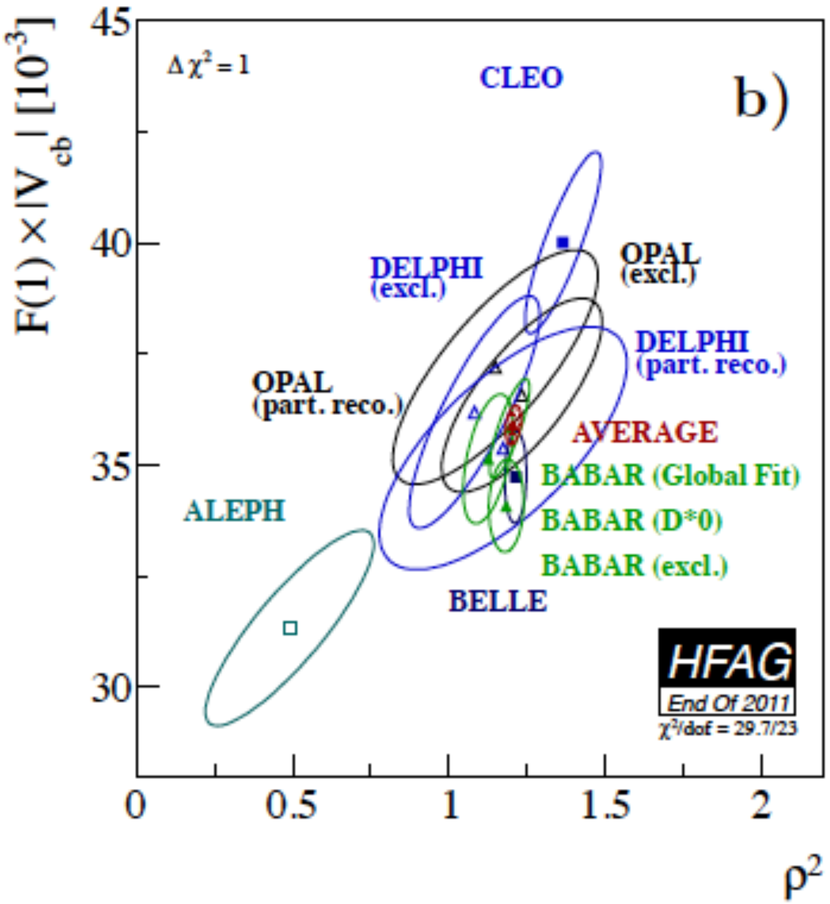
$A_1(w)$ form factor dominate as $w \to 1$

$$\eta_{EW} = 1.0066$$

(one-loop electroweak correction)





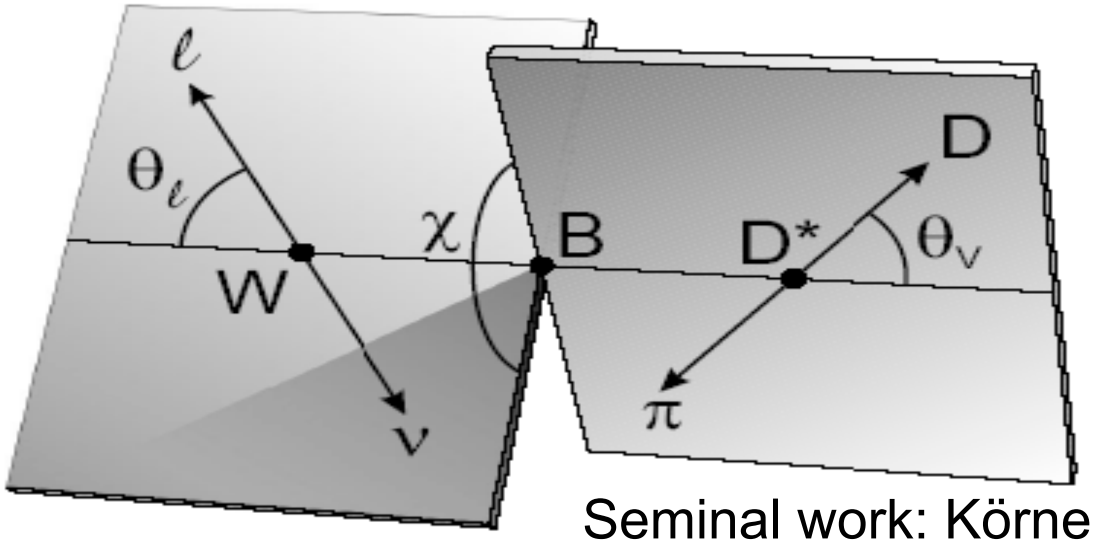


LQCD, arXive:1011.2166

$$|V_{cb}| = (39.70 \pm 1.42_{\text{exp}} \pm 0.89_{\text{th}}) \times 10^{-3}$$ B$\rightarrow$ $Dl\nu_l$

$$|V_{cb}| = (39.54 \pm 0.50_{\text{exp}} \pm 0.74_{\text{th}}) \times 10^{-3}$$ B$\rightarrow$ $D^*l\nu_l$

Helicity amplitudes in $\bar{B} \to D^* l \nu_l$:



helicity amplitudes

Seminal work: Körner& Schuller, ZPC 38 (1988) 511;
S.F., Nisanndzic, J.F.Kamenik, 1203.2654

$$\tilde{\varepsilon}_\mu(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0)\,,$$

$$\tilde{\varepsilon}_\mu(0) = \frac{1}{\sqrt{q^2}}(|\mathbf{p}\,|, 0, 0, -q_0)\,,$$

$$\tilde{\varepsilon}_\mu(t) = \frac{1}{\sqrt{q^2}}(q_0, 0, 0, -|\mathbf{p}\,|),$$

W polarization in the B meson rest frame

$$q_0 = (m_B^2 - m_{D^*}^2 + q^2)/2m_B$$

$$|\mathbf{p}\,| = \frac{\lambda^{1/2}(m_B^2, m_{D^*}^2, q^2)}{2m_B} \qquad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$$

$$\tilde{\varepsilon}^*_\mu(m)\tilde{\varepsilon}^\mu(m') = g_{mm'}, \quad \text{for} \quad (m, m' = t, \pm, 0) \quad \sum_{m,m'} \tilde{\varepsilon}_\mu(m)\tilde{\varepsilon}^*_\nu(m')g_{mm'} = g_{\mu\nu}\,.$$

$$\varepsilon_\alpha(\pm) = \mp\frac{1}{\sqrt{2}}(0, 1, \pm i, 0)\,,$$

$$\varepsilon_\alpha(0) = \frac{1}{m_{D^*}}(|\mathbf{p}\,|, 0, 0, E_{D^*}),$$

polarizations of D*

$$\varepsilon^*_\alpha(m)\varepsilon^\alpha(m') = -\delta_{mm'}$$

$$\sum_{mm'} \varepsilon_\alpha(m)\varepsilon_\beta(m')\delta_{mm'} = -g_{\alpha\beta} + \frac{p_{D^*\alpha}p_{D^*\beta}}{m_{D^*}^2}$$

$$H_{mm}(q^2) = \tilde{\varepsilon}(m)^{\mu *} H_\mu(m), \quad \text{for} \quad m = 0, \pm\,,$$
$$H_{0t}(q^2) = \tilde{\varepsilon}(m = t)^{\mu *} H_\mu(n = 0).$$

time-like component

$$\frac{d^2\Gamma_\ell}{dq^2 d\cos\theta d\chi} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^4} \frac{|\mathbf{p}|}{2m_B^2} \left(1 - \frac{m_\ell^2}{q^2}\right) L_{\mu\nu} H^{\mu\nu}$$

$$L_{\mu\nu} H^{\mu\nu} = L_{\mu'\nu'} g^{\mu'\mu} g^{\nu'\nu} H_{\mu\nu} = \sum_{mm', nn'} \left(L_{\mu'\nu'} \tilde{\varepsilon}^{\mu'}(m) \tilde{\varepsilon}^{\nu'}(n) g_{mm'} g_{nn'}\right) \left(\tilde{\varepsilon}^{\mu *}(m') \tilde{\varepsilon}^{\nu}(n') H_{\mu\nu}\right)$$

$$L_{\mu\nu} H^{\mu\nu} = \frac{1}{8} \sum_{\lambda_\ell, \lambda_{D*}, \lambda_{\ell\nu}, \lambda'_{\ell\nu}, J, J'} (-1)^{J+J'} |h_{(\lambda_\ell, \lambda_\nu)}|^2 \, \delta_{\lambda_{D*}\lambda_{\ell\nu}} \delta_{\lambda_{D*}\lambda'_{\ell\nu}}$$
$$\times d^J_{\lambda_{\ell\nu}, \lambda_\ell - 1/2}(\theta) d^{J}_{\lambda'_{\ell\nu}, \lambda_\ell - 1/2}(\theta) H_{\lambda_{D*}\lambda_{\ell\nu}} H^*_{\lambda_{D*}\lambda'_{\ell\nu}}\,,$$

$$H_{\pm\pm}^{\mathrm{SM}}(q^2) = (m_B + m_{D^*})A_1(q^2) \mp \frac{2m_B}{m_B + m_{D^*}}|\mathbf{p}|V(q^2)\,,$$

$$H_{00}^{\mathrm{SM}}(q^2) = \frac{1}{2m_{D^*}\sqrt{q^2}}\left[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*})A_1(q^2) - \frac{4m_B^2|\mathbf{p}|^2}{m_B + m_{D^*}}A_2(q^2)\right]\,,$$

$$H_{0t}^{\mathrm{SM}}(q^2) = \frac{2m_B|\mathbf{p}|}{\sqrt{q^2}}A_0(q^2)\,.$$

$$h_{(\lambda_\ell,\lambda_\nu)} = \frac{1}{2}\bar{u}_\ell(\lambda_\ell)\gamma^\mu(1-\gamma^5)v_\nu(\lambda_\nu)\tilde{\epsilon}_\mu(\lambda_{\ell\nu})$$

right-handed anti-neutrino $\quad \lambda_\nu = 1/2$

$$\lambda_{\ell\nu} = \lambda_\ell - \lambda_\nu$$

$$|h_{-1/2,1/2}|^2 = 2(q^2 - m_\ell^2) \qquad |h_{1/2,1/2}|^2 = 2\frac{m_\ell^2}{2q^2}(q^2 - m_\ell^2)$$

Differential distribution for two leptonic polarizations

$$\frac{d^2\Gamma_\ell}{dq^2 d\cos\theta}(\lambda_\ell = -1/2) = \frac{G_F^2 |V_{cb}|^2 |\mathbf{p}| q^2}{256\pi^3 m_B^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times$$

$$[(1-\cos\theta)^2 H_{++}^2 + (1+\cos\theta)^2 H_{--}^2 + 2\sin^2\theta H_{00}^2]$$

$$\frac{d^2\Gamma_\ell}{dq^2 d\cos\theta}(\lambda_\ell = 1/2) = \frac{G_F^2 |V_{cb}|^2 |\mathbf{p}| q^2}{256\pi^3 m_B^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \frac{m_\ell^2}{q^2} \quad \times$$

$$[(\sin^2\theta (H_{++}^2 + H_{--}^2) + 2(H_{0t} - H_{00}\cos\theta)^2]$$

$$\frac{d^2\Gamma_\tau}{dq^2 d\cos\theta} = \frac{G_F^2 |V_{cb}|^2 |\mathbf{p}| q^2}{256\pi^3 m_B^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times$$

$$\left[(1-\cos\theta)^2 |H_{++}|^2 + (1+\cos\theta)^2 |H_{--}|^2 + 2\sin^2\theta |H_{00}|^2 + \right.$$

$$\left. \underbrace{\frac{m_\tau^2}{q^2}\left((\sin^2\theta(|H_{++}|^2 + |H_{--}|^2) + 2|H_{0t} - H_{00}\cos\theta|^2\right)} \right],$$

If instead of τ is a light lepton then second part is negligible!

$$H_{\pm\pm}^{\rm SM}(q^2) = (m_B + m_{D^*})A_1(q^2) \mp \frac{2m_B}{m_B + m_{D^*}}|\mathbf{p}|V(q^2)\,,$$

$$H_{00}^{\rm SM}(q^2) = \frac{1}{2m_{D^*}\sqrt{q^2}}\left[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*})A_1(q^2) - \frac{4m_B^2|\mathbf{p}|^2}{m_B + m_{D^*}}A_2(q^2)\right]$$

$$H_{0t}^{\rm SM}(q^2) = \frac{2m_B|\mathbf{p}|}{\sqrt{q^2}}A_0(q^2)\,.$$

Heavy Quark limit for b and c quarks ⟶ only one form-factor!

$$A_0(q^2) = \frac{R_0(w)}{R_{D^*}}h_{A_1}(w)$$

$$A_2(q^2) = \frac{R_2(w)}{R_{D^*}}h_{A_1}(w)$$

$$V(q^2) = \frac{R_1(w)}{R_{D^*}}h_{A_1}(w)$$

$$h_{A_1}(w) = A_1(q^2)\frac{1}{R_{D^*}}\frac{2}{w+1}$$

$$w \equiv v_B \cdot v_{D^*} = \frac{m_B^2 + m_{D*}^2 - q^2}{2m_B m_{D*}}$$

Caprini et al., hep-ph/9712417

recent work form-factors: Gambino et al., **1206.2296**

Conclusions

- HQS very useful tool in simplifying number of form factors;
- Lattice QCD progress in scalar and vector form factors for $B \to D l \nu_l$;
- Results of Lattice QCD for $B \to D^* l \nu_l$ would help in the analysis;
- More experimental data on the existing observable in both $B \to D^{(*)} l \nu_l$ and new ones will shed more light on the current problem.

Literature HQET

- A.J.Buras, Weak Hamiltonian, CP violation and rare decays”, hep-ph/9806471;
- M.Neubert, “Heavy quark symmetry”, Phys. Rept. 245, 259 (1994);
- A.V. Manohar and M.B. Wise, “Heavy quark physics”, Camb. Part. Phys. 10, 1 (2000);
- M.B. Wise, Heavy quark physics”, hep-ph/9805468.

Properties of axial current

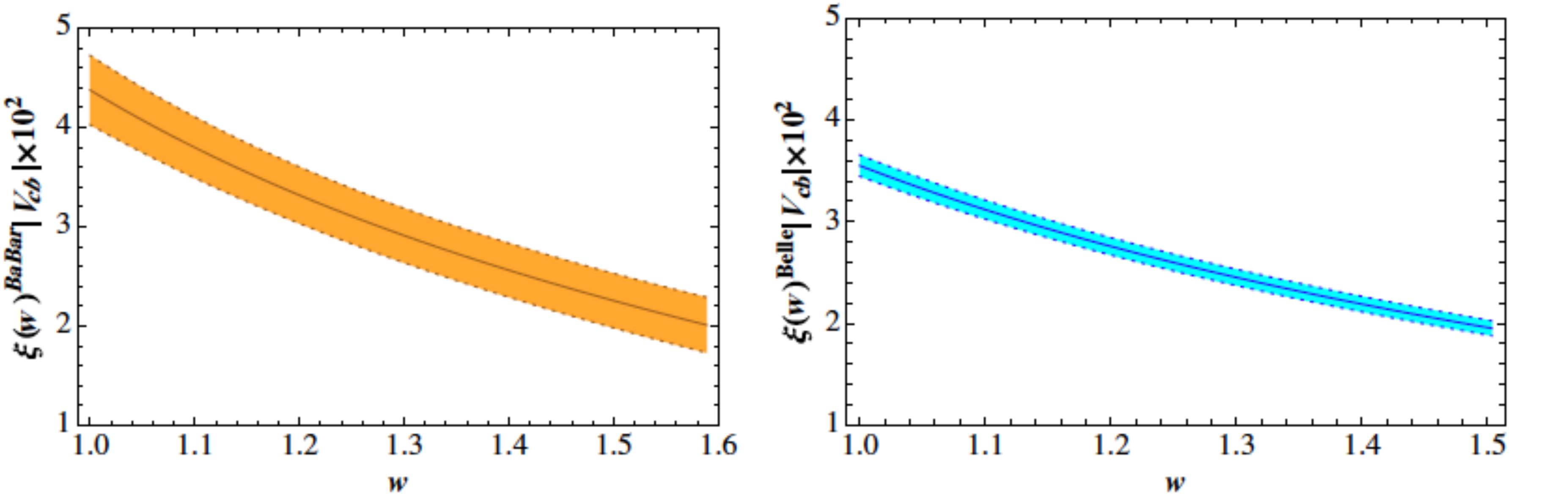
$$P^{-1}\boldsymbol{A}P = \boldsymbol{A},$$

$$P^{-1}A^0P = -A^0$$

$$P|D(p)\rangle = -|D(p_P)\rangle \qquad p_P = (p^0, -\boldsymbol{p})$$

$$\langle D(p'_P)|A^\mu|B(p_P)\rangle = h_{A_+}(w)(v_P + v'_P)^\mu + h_{A_-}(w)(v_P - v'_P)^\mu$$

$$= \langle PD(p')|A^\mu|PB(p)\rangle = \langle D(p')|P^{-1}A^\mu P|B(p)\rangle.$$



From Biancofiore, P. Colangelo, F. DeFazio et al.
1302.1042,

Reparametrization invariance

$$p_Q = m_Q v + k \qquad k \sim O(\Lambda_{QCD})$$

Λ_{QCD}/m_Q

If velocity is changed by Λ_{QCD}/m_Q then one can modify small momentum k by

$$\text{v}\to v + \epsilon/m_Q,$$

$$\text{k}\to k - \epsilon$$

Since $v^2 = 1$ then $v \cdot \epsilon = 0$

$$\not{v} Q_v = Q_v \qquad \left(\not{v} + \frac{\not{\epsilon}}{m_Q}\right)(Q_v + \delta Q_v) = Q_v + \delta Q_v$$

$$(1 - \not{v})\delta Q_v = \frac{\not{\epsilon}}{m_Q} Q_v$$

It means that any change in velocity implies that theory is invariant under

$$v \to v + \varepsilon / m_Q$$

$$Q_v \to e^{i\varepsilon \cdot x} \left(1 + \frac{\not{\varepsilon}}{2m_Q}\right) Q_v$$

$$\mathcal{L}_0 = \bar{Q}_v iv \cdot D Q_v$$

$$\mathcal{L}_0 \to \bar{Q}_v \left(1 + \frac{\not{\varepsilon}}{2m_Q}\right) \left(iv \cdot D + \frac{i\varepsilon \cdot D}{m_Q}\right) \left(1 + \frac{\not{\varepsilon}}{2m_Q}\right) Q_v$$

$$\delta \mathcal{L}_0 = \frac{1}{m_Q} \bar{Q}_v (i\varepsilon \cdot D_\perp) Q_v$$

$$\delta \mathcal{L}_1 \to -\bar{Q}_v i2 \frac{\varepsilon \cdot D_\perp}{2m_Q} Q_v = -\frac{1}{m_Q} \bar{Q}_v i\varepsilon \cdot D_\perp Q_v.$$

$\mathcal{L}_0 + \mathcal{L}_1$ the theory is invariant under the small change in velocity