Rare B Decays in the SM & Current Experiments

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Interest in Rare B Decays

- Rare B Decays $(b \to s\gamma, b \to s\ell^+\ell^-, ...)$ are Flavour-Changing-Neutral-Current (FCNC) processes $(|\Delta B| = 1, |\Delta Q| = 0)$; not allowed at the Tree level in the SM
- FCNC processes are governed by the GIM mechanism, which imparts them sensitivity to higher scales (m_t, m_W) and the CKM matrix elements, in particular, $V_{ti}; i = d, s, b$
- FCNC processes are sensitive to physics beyond the SM, such as supersymmetry, and the BSM amplitudes can be comparable to the (tW)-part of the GIM amplitudes
- Last, but not least, Rare B-decays enjoy great interest in the ongoing and planned experimental programme in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

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- Basic Formalism for QCD Effects in Weak decays
- Operator Product Expansion in Weak Decays
- The Standard Candle in Rare B-Decays: $\mathbf{B} \to X_s \gamma$
- Electroweak Penguins: $\mathbf{B} \to X_s \ell^+ \ell^-$
- Exclusive Radiative Decays $\mathbf{B} \to K^* \gamma$ & $\mathbf{B} \to (\rho, \omega)\gamma$
- Exclusive Decays $\mathbf{B} \to (K, K^*, \pi)\ell^+ \ell^-$
- Current Frontier of Rare B Decays: $\mathbf{B}_s \to \mu^+ \mu^-$ & $\mathbf{B}_d \to \mu^+ \mu^-$
- Outlook & Summary

Standard Model Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm GSW} + \mathcal{L}_{\rm QCD}$$

QCD [SU(3)]

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i \sum_q \bar{\psi}^\alpha_q \gamma^\mu (D_\mu)_{\alpha\beta} \psi^\beta_q$$

with $F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu - g_s f_{abc} A^{(b)}_\mu A^{(c)}_\nu; \quad a, b, c = 1, ..., 8$

and $(D_\mu)_{\alpha\beta} = \delta_{\alpha\beta}\partial_\mu + i g_s \sum_a \frac{1}{2} \lambda^{(a)}_{\alpha\beta} A^{(a)}_\mu$

Electroweak $[SU(2)_I \times U(1)_Y]$

$$\mathcal{L}_{\rm GSW} = \mathcal{L}_{\rm gauge}(W_i, B, \psi_j) + \mathcal{L}_{\rm Higgs}(\phi_k, W_i, B, \psi_j)$$

$$\mathcal{L}_{\rm gauge}(W_i, B, \psi_j) = -\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{\psi_L} \overline{\psi_L} i D_\mu \gamma^\mu \psi_L + \sum_{\psi_R} \overline{\psi_R} i D_\mu \gamma^\mu \psi_R$$

$$\mathcal{L}_{\rm Higgs}(\phi_k, W_i, B, \psi_j) = \mathcal{L}_{\rm Higgs}(\rm gauge) + \mathcal{L}_{\rm Higgs}(\rm fermions)$$

$$\mathcal{L}_{\rm Higgs}(\rm gauge) = (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi)$$

$$D_\mu\Phi = (\mathrm{I}(\partial_\mu + i\frac{g_1}{2}B_\mu) + ig_2\frac{\tau}{2}\cdot \mathrm{W}_\mu)\Phi;\ V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$\mathcal{L}_{\text{Higgs}}(\text{fermions}) = Y_u^{ij}\bar{Q}_{L,i}\tilde{\Phi}u_{R,j} + Y_d^{ij}\bar{Q}_{L,i}\Phi d_{R,j} + \text{h.c.} + \ldots$$

- 3 Quark families: $Q_{L_j} = (u_L,\ d_L); (c_L,\ s_L); (t_L;\ b_L); \bar{u}_R,\ \bar{d}_R; \ldots$
- Flavour mixings in the SM reside in the Higgs-Yukawa sector of the theory
- Flavour symmetry broken by Yukawa interactions

$$Q_i Y_d^{ij} d_j \phi \longrightarrow Q_i M_d^{ij} d_j$$

$$Q_i Y_u^{ij} u_j \phi^c \longrightarrow Q_i M_u^{ij} u_j$$

$$M_d = \text{diag}(m_d,\ m_s,\ m_b);\ \ M_u^\dagger = \text{diag}(m_u,\ m_c,\ m_t) \times V_{\text{CKM}}$$

- V_{CKM} a (3×3) unitary matrix is the only source of Flavour Violation, as all gauge interactions (involving $\gamma,\ Z^0,\ g$) are Flavour diagonal
- All observed phenomena involving flavour changes in the hadrons are consistently described by the CKM framework; i.e., in terms of 10 fundamental parameters: 6 quark masses, 3 mixing angles and 1 phase

The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

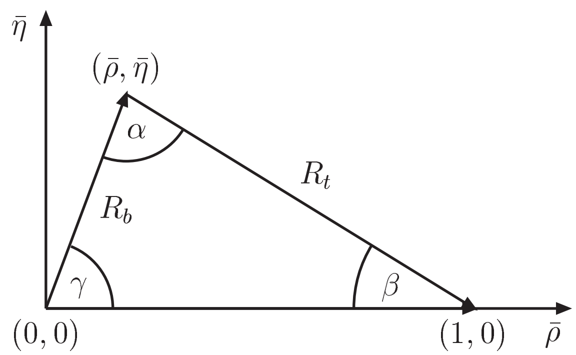
- Customary to use the handy Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2(1+i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters: $A, \ \lambda, \ \rho, \ \eta$
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2), \ \ \bar{\eta} = \eta(1-\lambda^2/2)$$

- The CKM-Unitarity triangle $[\phi_1 = \beta; \ \ \phi_2 = \alpha; \ \ \phi_3 = \gamma]$



Phases and sides of the UT

$$\alpha \equiv \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right)$$

- β and γ have simple interpretation

$$V_{td} = |V_{td}| e^{-i\beta}, \qquad V_{ub} = |V_{ub}| e^{-i\gamma}$$

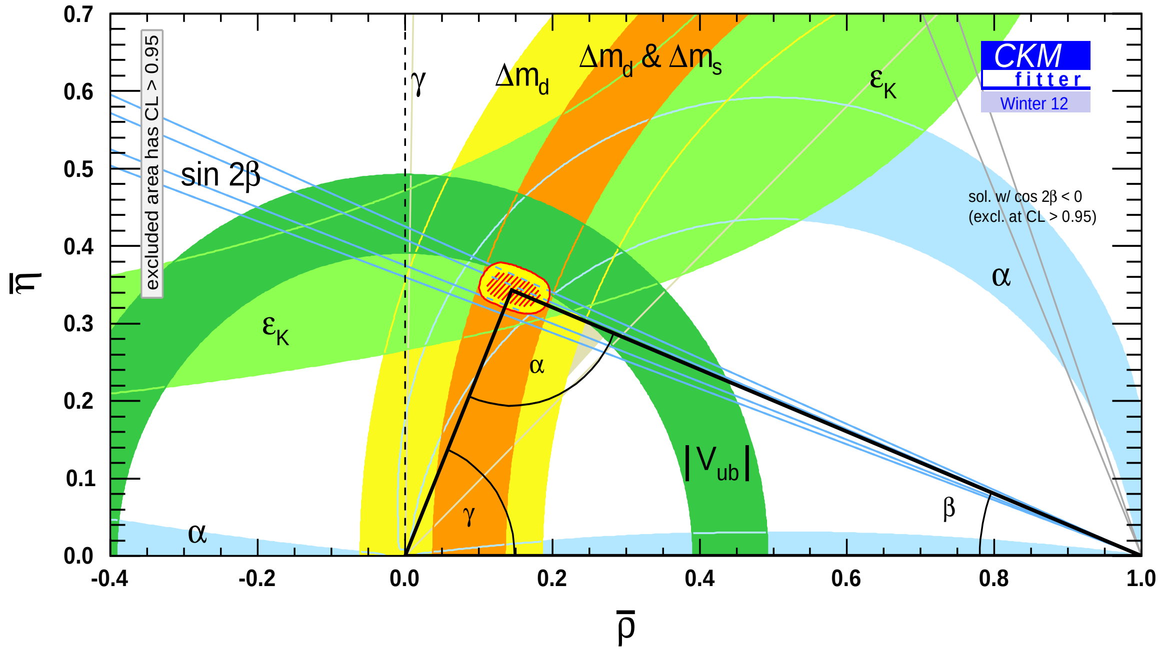
- α defined by the relation: $\alpha = \pi - \beta - \gamma$
- The Unitarity Triangle (UT) is defined by:

$$R_b \mathrm{e}^{i\gamma} + R_t \mathrm{e}^{-i\beta} = 1$$

$$R_b \equiv \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|$$

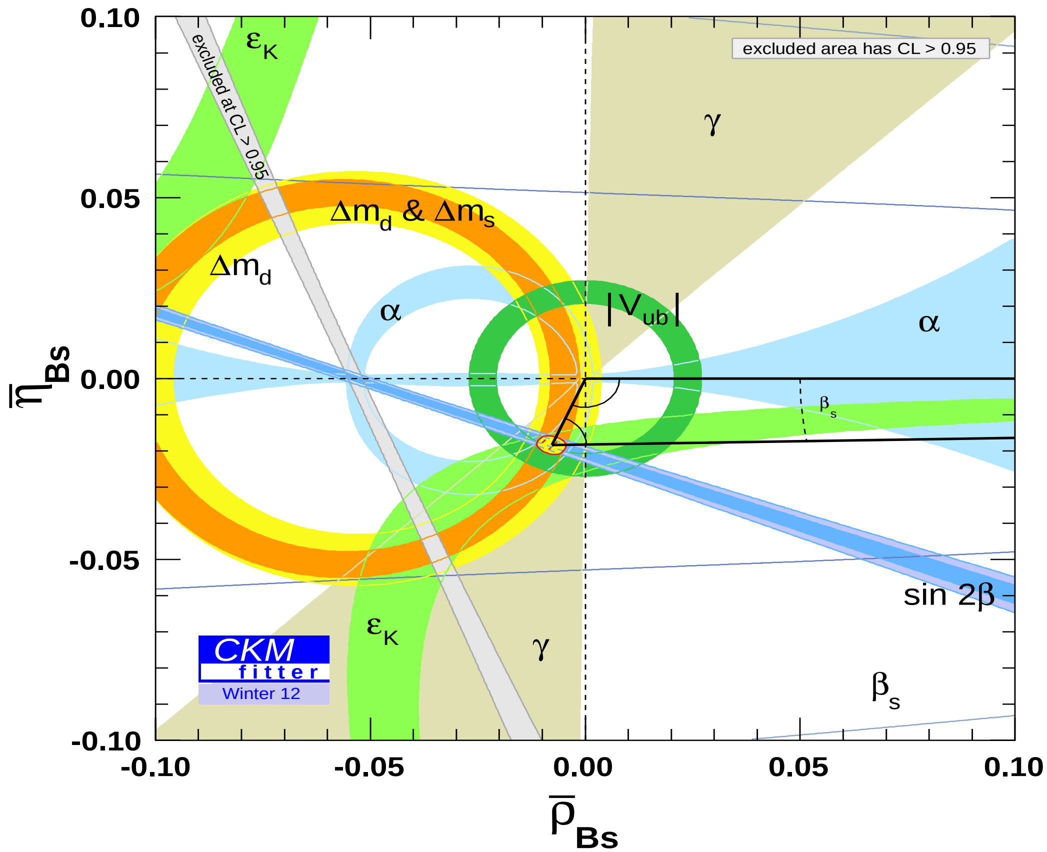
$$R_t \equiv \frac{|V_{tb}^* V_{td}|}{|V_{cb}^* V_{cd}|} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left|\frac{V_{td}}{V_{cb}}\right|$$

Current Status of the CKM-Unitarity Triangle [CKMfitter]



- $\sin 2\beta = 0.820^{+0.024}_{-0.028}$ [Fit-value]
  ($= 0.691 \pm 0.020$ [Direct Measurement]
- $\alpha = [95.9^{+2.2}_{-5.6}]^\circ$ [Fit-value]
  $\alpha = [88.7^{+4.6}_{-4.2}]^\circ$ [Direct Measurement]
- $\gamma = [67.1 \pm 4.3]^\circ$[Fit-value]
  $\gamma = [66 \pm 12]^\circ$ [Direct Measurement]
- Direct and indirect measurements of angles agree well; largest Pull is on $\sin 2\beta$ ($= 2.6\ \sigma$)

Current Status of the Squashed UT_s Triangle [CKMfitter]



- $\bar{\rho}_{B_s} = -0.0078 \pm 0.0015$ [Fit-value]
- $\bar{\eta}_{B_s} = -0.01837^{+0.00080}_{-0.00082}$ [Fit-value]
- $\sin 2\beta_s = 0.0364 \pm 0.0016$[Fit-value]
  where $\beta_s = -\arg(-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*)$]

Basic Formalism for including QCD Effects in Weak decays

- Recall the renormalization procedure in QCD

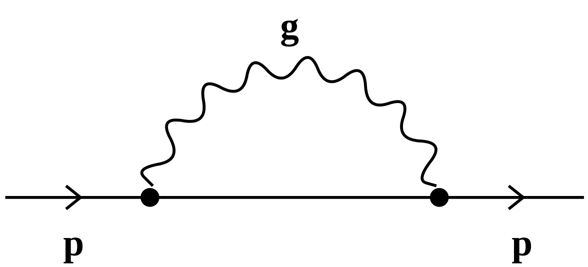
$$A^a_{0\mu} = Z_3^{1/2} A^a_\mu \qquad q_0 = Z_q^{1/2} q$$
$$g_{0,s} = Z_g g_s \mu^\varepsilon \qquad m_0 = Z_m m$$

- The index "0" indicates unrenormalized quantities. A^a_μ and q are renormalized fields, g_s is the renormalized QCD coupling and m the renormalized quark mass
- Dimensional Regularization is used in which Feynman diagrams are evaluated in $D = 4 - 2\varepsilon$ space-time dimensions and the singularities are extracted as $1/\varepsilon$ poles
- The simplest renormalization scheme is the *Minimal Subtraction Scheme* MS in which only divergences ($1/\varepsilon$ poles) are subtracted

$$Z_i = \frac{\alpha_s}{4\pi}\frac{a_{1i}}{\varepsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{a_{2i}}{\varepsilon^2} + \frac{b_{2i}}{\varepsilon}\right) + \mathcal{O}(\alpha_s^3)$$

a_{ji} and b_{ji} are μ-independent constants.

- Example: Quark Self-Energy Correction in the MS scheme:



$$i\Sigma_{\alpha\beta} = i \not{p}\, C_F\, \delta_{\alpha\beta}\frac{\alpha_s}{4\pi}[\frac{1}{\varepsilon} + \ln 4\pi - \gamma_E + \ln\frac{\mu^2}{-p^2} + 1]$$

where $C_F = 4/3$ and γ_E is the Euler constant $\gamma_E = 0.5772...$

- The so-called $\overline{\mathrm{MS}}$-scheme is defined by: $\mu_{\overline{MS}} = \mu e^{\gamma_E/2}(4\pi)^{-1/2}$

$$(i\Sigma_{\alpha\beta})_{div} = iC_F\delta_{\alpha\beta}\frac{\alpha_s}{4\pi}(\not{p} - 4m)\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

- Adding the counter-term $i\delta_{\alpha\beta}[(Z_q - 1)\not{p} - (Z_q Z_m - 1)m]$
  and requiring the final result to be zero yields the Renormalization constants

$$Z_q = 1 - \frac{\alpha_s}{4\pi}C_F\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

$$Z_m = 1 - \frac{\alpha_s}{4\pi}3C_F\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

- Z_3 and Z_g calculated likewise from the gluon propagator and the $g\bar{q}q$ vertex

$$Z_3 = 1 - \frac{\alpha_s}{4\pi}\left[\frac{2}{3}f - \frac{5}{3}N\right]\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

$$Z_g = 1 - \frac{\alpha_s}{4\pi}\left[\frac{11}{6}N - \frac{2}{6}f\right]\frac{1}{\varepsilon} + \mathcal{O}(\alpha_s^2)$$

Basic RG Equations in QCD & their Solutions

- Scale (μ)-dependence of renormalized coupling $g_s(\mu)$ ($g \equiv g_s$) and quark mass $m(\mu)$:

$$\frac{dg(\mu)}{d\ln\mu} = \beta(g(\mu), \varepsilon)$$

$$\frac{dm(\mu)}{d\ln\mu} = -\gamma_m(g(\mu))m(\mu)$$

where

$$\beta(g, \varepsilon) = -\varepsilon g + \beta(g),$$

$$\beta(g) = -g\frac{1}{Z_g}\frac{dZ_g}{d\ln\mu}, \qquad \gamma_m(g) = \frac{1}{Z_m}\frac{dZ_m}{d\ln\mu}$$

- Series expansion in $1/\varepsilon^k$: $Z_i = 1 + \sum_{k=1}^{\infty} \frac{1}{\varepsilon^k} Z_{i,k}(g)$ one finds

$$\beta(g) = 2g^3\frac{dZ_{g,1}(g)}{dg^2},$$

$$\gamma_m(g) = -2g^2\frac{dZ_{m,1}(g)}{dg^2}$$

- $\beta(g)$ and $\gamma_m(g)$ can be obtained from the $1/\varepsilon$-pole parts of Z_g and Z_m

Compendium of Useful Results

- $\beta(g)$, $\gamma(\alpha_s)$ and $Z_{q,1}(\alpha_s)$ up to two-loops are

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

$$\gamma_m(\alpha_s) = \gamma_m^{(0)} \frac{\alpha_s}{4\pi} + \gamma_m^{(1)} \left(\frac{\alpha_s}{4\pi}\right)^2$$

$$Z_{q,1}(\alpha_s) = a_1 \frac{\alpha_s}{4\pi} + a_2 \left(\frac{\alpha_s}{4\pi}\right)^2$$

where

$$\beta_0 = \frac{11N - 2f}{3} \qquad \beta_1 = \frac{34}{3}N^2 - \frac{10}{3}Nf - 2C_F f$$

$$\gamma_m^{(0)} = 6C_F \qquad \gamma_m^{(1)} = C_F \left(3C_F + \frac{97}{3}N - \frac{10}{3}f\right)$$

$$a_1 = -C_F \qquad a_2 = C_F \left(\frac{3}{4}C_F - \frac{17}{4}N + \frac{1}{2}f\right)$$

$$C_F = \frac{N^2 - 1}{2N}$$

Running Coupling Constant

- The RG equation for $g(\mu)$ can be written as:

$$\frac{d\alpha_s}{d\ln\mu} = -2\beta_0\frac{\alpha_s^2}{4\pi} - 2\beta_1\frac{\alpha_s^3}{(4\pi)^2}$$

- The solution is:

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0\ln(\mu^2/\Lambda_{\overline{MS}}^2)} - \frac{\beta_1}{\beta_0^3}\frac{\ln\ln(\mu^2/\Lambda_{\overline{MS}}^2)}{\ln^2(\mu^2/\Lambda_{\overline{MS}}^2)}$$

- $\Lambda_{\overline{MS}}$ is a QCD scale characteristic for the $\overline{\mathrm{MS}}$ scheme.

- $\Lambda_{\overline{MS}}$ and $\alpha_s(\mu)$ depend on f, the number of "effective" flavours present in β_0 and β_1, and depends on the scale μ. As a working procedure $f = 6$ for $\mu > m_t$, $f = 5$ for $m_b \leq \mu \leq m_t$ etc.

- Denoting by $\alpha_s^{(f)}(\mu)$ the effective coupling constant for a theory with f effective flavours, the current world average is

$$\alpha_s^{(5)}(M_Z) = 0.1184 \pm 0.0007$$

Running Quark Masses

- The RG equation for $m(\mu)$ can be written as:

$$\frac{dm(\mu)}{d\ln\mu} = -\gamma_m(g)m(\mu)$$

- With $dg/d\ln\mu = \beta(g)$ the solution is:

$$m(\mu) = m(\mu_0)\exp\left[-\int_{g(\mu_0)}^{g(\mu)} dg' \frac{\gamma_m(g')}{\beta(g')}\right]$$

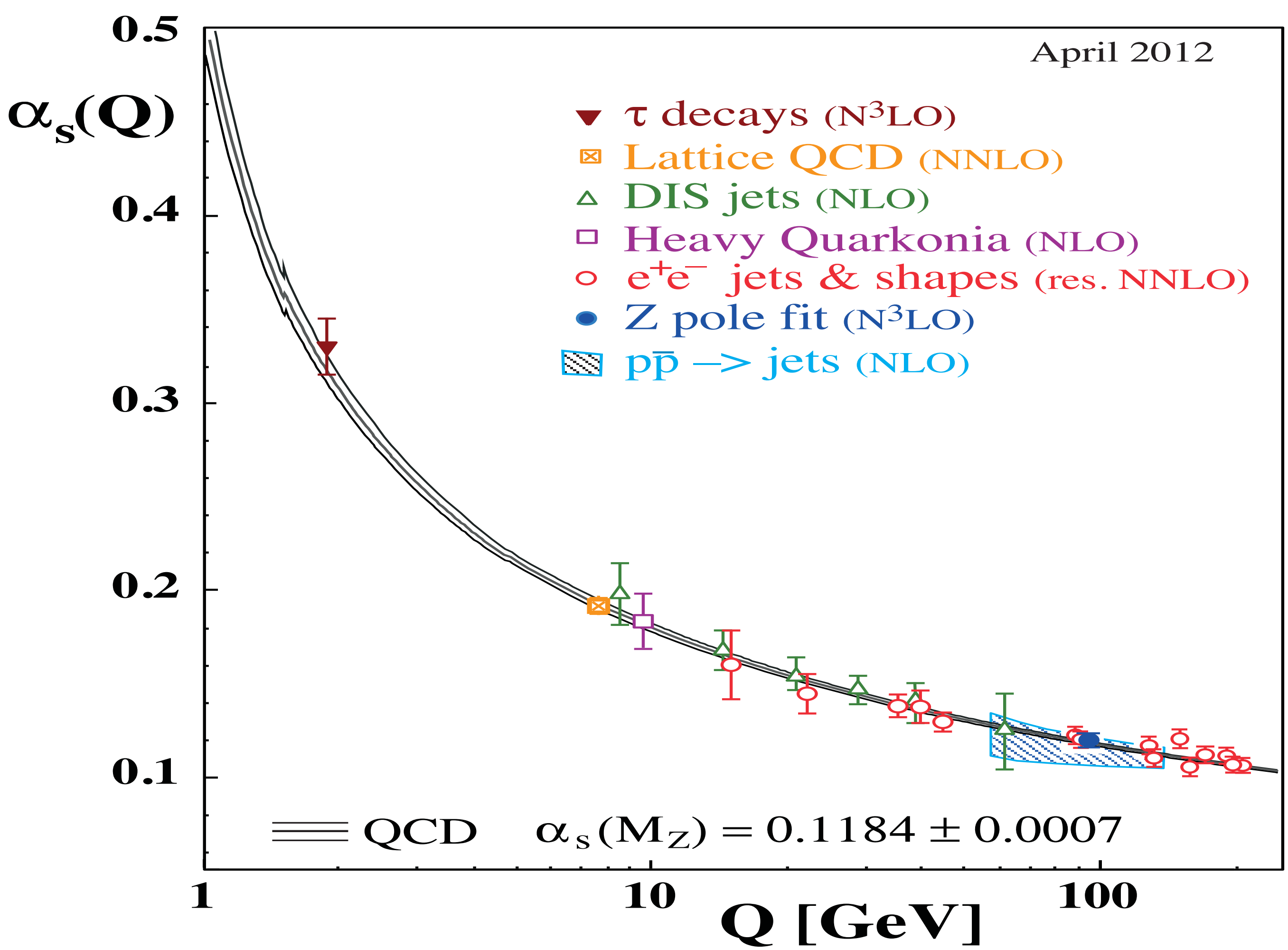
- $m(\mu_0)$ is the value of the running quark mass at the scale μ_0. Inserting the expansions for $\gamma_m(g)$ and $\beta(g)$ and expanding in α_s gives:

$$m(\mu) = m(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_m^{(0)}}{2\beta_0}}\left[1 + \left(\frac{\gamma_m^{(1)}}{2\beta_0} - \frac{\beta_1\gamma_m^{(0)}}{2\beta_0^2}\right)\frac{\alpha_s(\mu)-\alpha_s(\mu_0)}{4\pi}\right]$$

- Since $\frac{\gamma_m^{(0)}}{2\beta_0}$ is a positive number, quark masses $m(\mu)$ decrease as μ increases, and they require a scheme and a scale to be quantified much like $\alpha_s(\mu)$

Experimental Evidence of Asymptotic Freedom

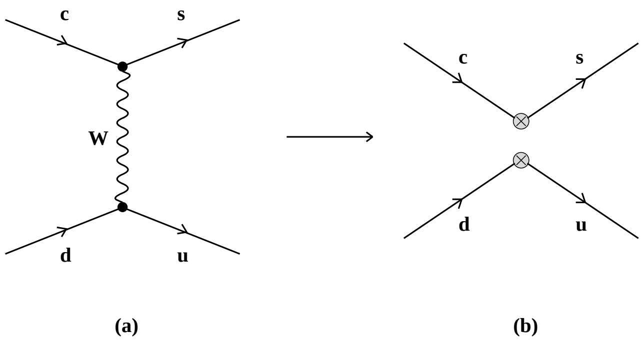
PDG, 2012



Operator Product Expansion in Weak decays

Basic Idea

- Consider the quark level transition $c \to s u \bar{d}$



- The tree-level W-exchange amplitude is:

$$A = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{M_W^2}{k^2 - M_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A}$$
$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \mathcal{O}(\frac{k^2}{M_W^2})$$

where $(\bar{s}c)_{V-A} \equiv \bar{s}\gamma_\mu(1-\gamma_5)c$

- Ignoring $\mathcal{O}(k^2/M_W^2)$ terms, the amplitude A may also be obtained from

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \textbf{High D Operators}$$

- Basic idea of Operator Product Expansion (OPE): the product of two current operators is expanded into a series of local operators, weighted by the effective coupling constants, the Wilson Coefficients
  OPE & Short-distance QCD Effects
- Rewriting the $c \to su\bar{d}$ transition to make the quark color-indices explicit

$$\mathcal{H}_{eff}^{(0)} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

- With QCD effects $\mathcal{H}_{eff}^{(0)}$ is generalized to

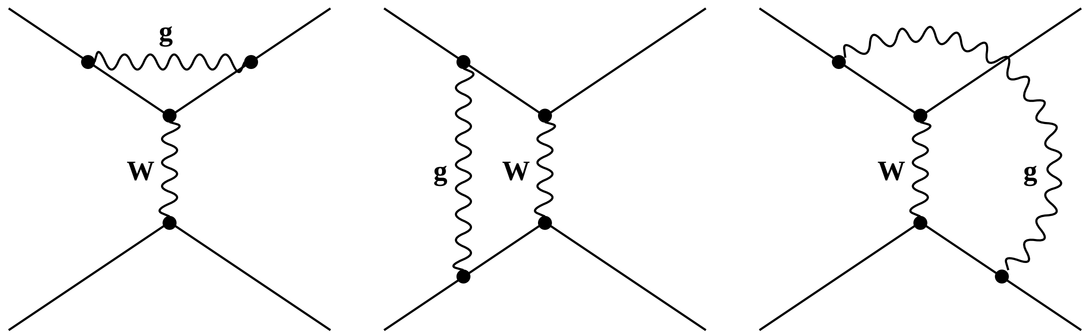
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1(\mu) Q_1 + C_2(\mu) Q_2)$$

  where

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$
$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

- In addition to the original operator Q_2 , a new operator Q_1 with the *same flavour* form but *different colour structure* is generated, as is evident from the colour structure

$$T^a_{\alpha\beta} T^a_{\gamma\rho} = -\frac{1}{2N} \delta_{\alpha\beta} \delta_{\gamma\delta} + \frac{1}{2} \delta_{\alpha\delta} \delta_{\gamma\beta}$$



(a) (b) (c)

- The Wilson coefficients C_1 and C_2, become calculable nontrivial functions of α_s, M_W and the renormalization scale μ.

Calculation of Wilson Coefficients

- They are determined by the requirement that the amplitude A_{full} in the SM is reproduced by the amplitude in the effective theory A_{eff}

$$A_{full} = A_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1 \langle Q_1 \rangle + C_2 \langle Q_2 \rangle)$$

- There are three steps involved in this procedure, outlined below

Step 1: Calculation of A_{full}

- In the SM, A_{full} to $\mathcal{O}(\alpha_s)$ ($m_i = 0$, $p^2 < 0$):

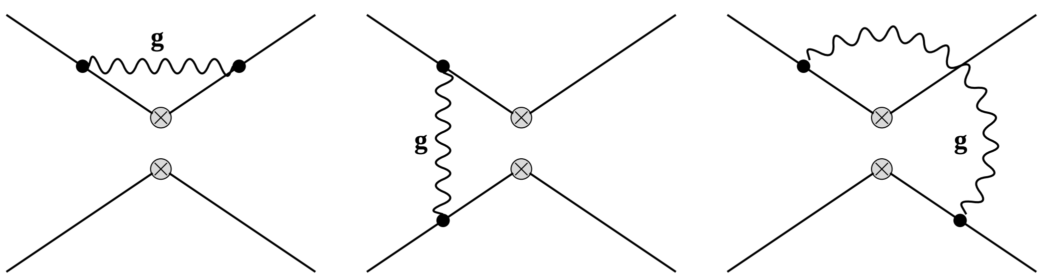
$$A_{full} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Big[\Big(1 + 2 C_F \frac{\alpha_s}{4\pi} \big(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \big) \Big) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_2 - 3 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} S_1 \Big]$$

Here S_1 and S_2 are the tree level matrix elements of Q_1 and Q_2

$$S_1 \equiv \langle Q_1 \rangle_{tree} = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$

$$S_2 \equiv \langle Q_2 \rangle_{tree} = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

- The singularity $1/\varepsilon$ can be removed by quark field renormalization



(a) (b) (c)

Step 2: Calculation of Matrix Elements $\langle Q_i \rangle$

- The unrenormalized matrix elements of Q_1 and Q_2 are found at $\mathcal{O}(\alpha_s)$ by calculating the diagrams in the effective theory

$$\begin{aligned}
\langle Q_1 \rangle^{(0)} &= \left(1 + 2C_F \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right)\right) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right) S_1 \\
&\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right) S_2
\end{aligned}$$

$$\begin{aligned}
\langle Q_2 \rangle^{(0)} &= \left(1 + 2C_F \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right)\right) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right) S_2 \\
&\quad -3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{-p^2}\right) S_1
\end{aligned}$$

- The divergence in the first terms can again be removed by quark field renormalization. However, the resulting expressions are still divergent. To remove them, *Operator renormalization* is necessary

$$Q_i^{(0)} = Z_{ij} Q_j$$

- The renormalization constant is a 2×2 matrix $\hat{Z}$. the relation between the unrenormalized ($\langle Q_i \rangle^{(0)}$) and the renormalized amputated Green functions ($\langle Q_i \rangle$) is:

$$\langle Q_i \rangle^{(0)} = Z_q^{-2} Z_{ij} \langle Q_j \rangle$$

- Z_q^{-2} removes the $1/\varepsilon$ divergences in the first terms discussed above. Z_{ij} remove the remaining divergences. In the $\overline{\mathrm{MS}}$-scheme:

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

- The renormalized matrix elements $\langle Q_i \rangle$ are given by

$$\langle Q_1 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1 - 3\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2$$

$$\langle Q_2 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_2 - 3\frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1$$

Step 3: Extraction of C_i

- Inserting $\langle Q_i \rangle$ in A_{eff} and comparing with A_{full} yields the Wilson coefficients C_1 and C_2

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2}\,, \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{\mu^2}$$

The Standard Candle: $B \to X_s \gamma$

Interest in the rare decay $B \to X_s \gamma$ transcends B Physics!

- First measurements by CLEO (1995); well measured at the B-factories by Belle and BaBar; more precise measurements anticipated at SuperB-factories

Theoretical Interest:

- A monumental theoretical effort has gone in improving the perturbative precision; $B \to X_s \gamma$ in NNLO completed in 2006

  - First estimate of $\mathcal{B}(B \to X_s \gamma)$: M. Misiak et al., Phys. Rev. Lett. 98:022002 (2007)

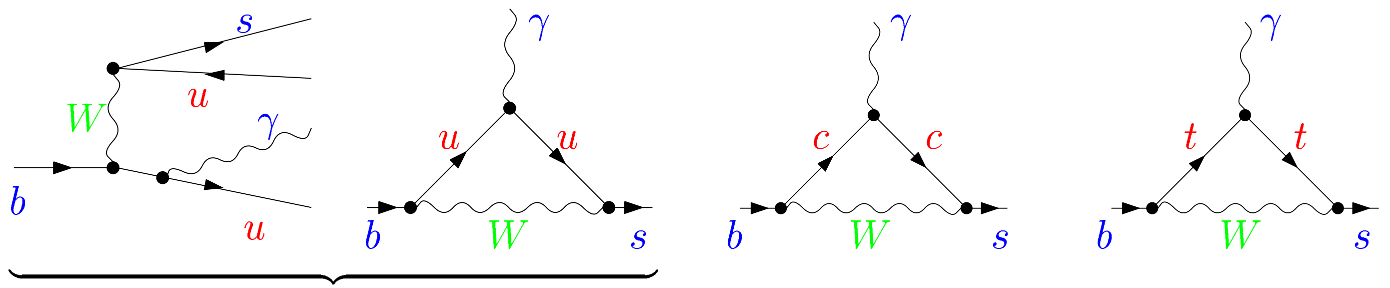
  - Analysis of $\mathcal{B}(B \to X_s \gamma)$ at NNLO with a cut on the Photon energy, T. Becher and M. Neubert, Phys. Rev. Lett. 98:022003 (2007)

- Non-perturbative effects under control thanks to HQET

- Sensitivity to new physics; hence constrains parameters of the BSM models such as the 2HDMs and Supersymmetry

- A crucial input in a large number of precision tests of the SM in $b \to s$ processes, such as $B \to X_s \ell^+ \ell^-$

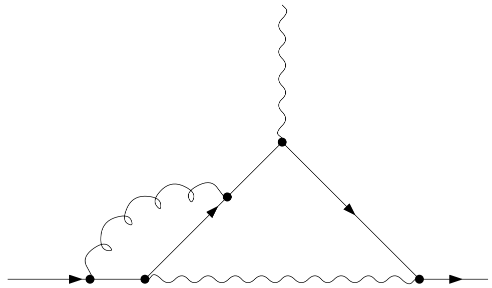
Examples of the leading electroweak diagrams for $B \to X_s \gamma$



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

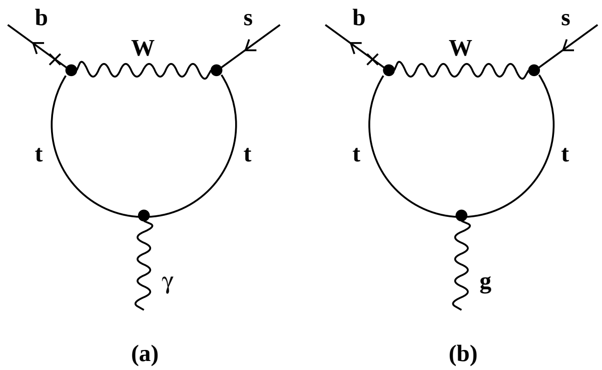
$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.

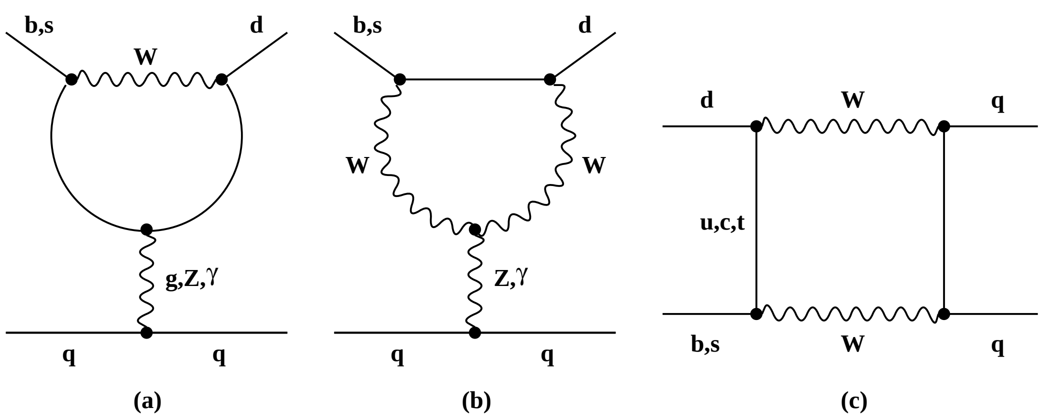


- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($B \to X_s\gamma$) more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

QCD Penguin and Box Diagrams in SM for $b \to sg$ & $b \to sq\bar{q}$

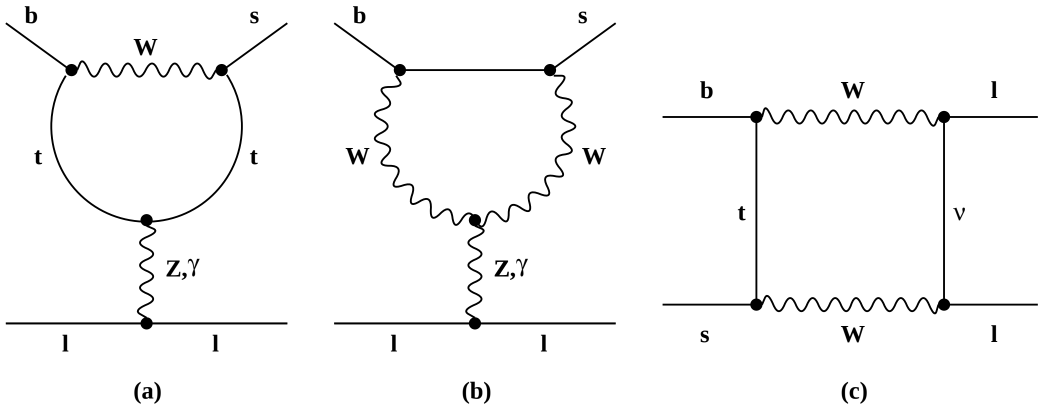


QED & QCD Penguin Diagrams in the SM

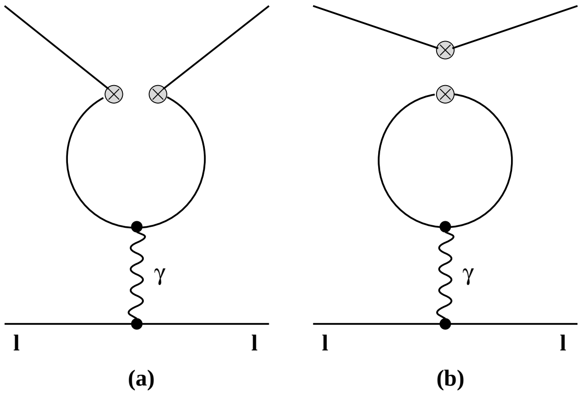


Penguins and Box Diagrams in the SM

Some representative diagrams in $b \to s\ell^+\ell^-$



Diagrams in the full theory



Diagrams in the effective theory

The effective Lagrangian for $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD \times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$$(q = u, d, s, c, b, \; l = e, \mu, \tau)$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1,2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3,4,5,6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Structure of the SM calculations for $\bar{B} \to X_s \gamma$ & $\bar{B} \to X_s \ell^+ \ell^-$

$$\mathcal{H}_{\text{eff}} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

• $\mathcal{H}_{\text{eff}}$ independent of the scale μ, while $C_i(\mu)$ and $O_i(\mu)$ depend on μ
$\Longrightarrow$ Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^{\text{T}} C_j(\mu)$$

• γ_{ij}: anomalous dimension matrix

• Matching usually done at high scale $(\mu_0 \sim M_W, m_t)$

• Full theory and the matrix elements of the effective operators have the same large logarithms

$\mu_0 \sim O(M_W)$
$\downarrow$ RGE
$\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$

• Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Renormalization Group Evolution for $\bar{B} \to X_s \gamma$

- $C_i(\mu_b)$ are calculated by using

$$\vec{C}(\mu_b) = \hat{U}_5(\mu_b, \mu_W)\vec{C}(\mu_W)$$

$\hat{U}_5(\mu_b, \mu_W)$ is the 8×8 evolution matrix

- In LO: $\hat{U}_5(\mu_b, \mu_W)$ is to be replaced by $\hat{U}_5^{(0)}(\mu_b, \mu_W)$,

$$\hat{U}^{(0)}(\mu, \mu_W) = \hat{V} \left(\left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu)} \right]^{\frac{\vec{\gamma}^{(0)}}{2\beta_0}} \right)_D \hat{V}^{-1}$$

where $\hat{V}$ diagonalizes $\hat{\gamma}^{(0)T}$

$$\hat{\gamma}_D^{(0)} = \hat{V}^{-1} \gamma^{(0)T} \hat{V}$$

and $\vec{\gamma}^{(0)}$ is the vector containing the diagonal elements of the diagonal matrix $\hat{\gamma}_D^{(0)}$,

- Initial conditions $\vec{C}^{(0)}(\mu_W)$ are (other coefficients are set to zero at $\mu = \mu_W$): :

$$C_2^{(0)}(\mu_W) = 1$$

$$C_{7\gamma}^{(0)}(\mu_W) = \frac{3x_t^3 - 2x_t^2}{4(x_t - 1)^4} \ln x_t + \frac{-8x_t^3 - 5x_t^2 + 7x_t}{24(x_t - 1)^3}$$

$$C_{8G}^{(0)}(\mu_W) = \frac{-3x_t^2}{4(x_t - 1)^4} \ln x_t + \frac{-x_t^3 + 5x_t^2 + 2x_t}{8(x_t - 1)^3}$$

- Using RG, one obtains the following LO results for the Wilson coefficients

$$C_j^{(0)}(\mu_b) = \sum_{i=1}^{8} k_{ji}\eta^{a_i} \qquad (j=1,\ldots,6)$$

$$C_{7\gamma}^{(0)eff}(\mu_b) = \eta^{\frac{16}{23}} C_{7\gamma}^{(0)}(\mu_W) + \frac{8}{3}\left(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}\right) C_{8G}^{(0)}(\mu_W) + C_2^{(0)}(\mu_W)\sum_{i=1}^{8} h_i \eta^{a_i},$$

$$C_{8G}^{(0)eff}(\mu_b) = \eta^{\frac{14}{23}} C_{8G}^{(0)}(\mu_W) + C_2^{(0)}(\mu_W)\sum_{i=1}^{8} \bar{h}_i \eta^{a_i},$$

with $\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}$

Table 1: Magic Numbers

i	1	2	3	4	5	6	7	8
a_i	$\frac{14}{23}$	$\frac{16}{23}$	$\frac{6}{23}$	$-\frac{12}{23}$	0.4086	-0.4230	-0.8994	0.1456
h_i	2.2996	-1.0880	$-\frac{3}{7}$	$-\frac{1}{14}$	-0.6494	-0.0380	-0.0185	-0.0057
$\bar{h}_i$	0.8623	0	0	0	-0.9135	0.0873	-0.0571	0.0209

For $m_t = 170$GeV, $\mu_b = 5$GeV and $\alpha_s^{(5)}(M_Z) = 0.118$ one obtains

$$C_{7\gamma}^{(0)\mathrm{eff}}(\mu_b) = -0.300; \quad C_{8G}^{(0)\mathrm{eff}}(\mu_b) = -0.144$$

Status of the SM calculations for $\bar{B} \to X_s \gamma$

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$:	tree	1-loop	2-loop	[Bobeth, Misiak, Urban, NPB 574 (2000) 291]
$i = 7, 8$:	1-loop	2-loop	3-loop	[Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR$(\bar{B} \to X_s \gamma)$

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1\text{L} & 2\text{L} \\ 0 & 1\text{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2\text{L} & 3\text{L} \\ 0 & 2\text{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3\text{L} & 4\text{L} \\ 0 & 3\text{L} \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

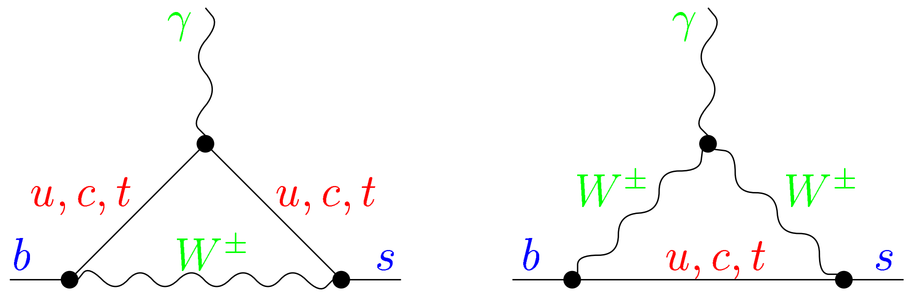
Matrix elements $(\mu_b \sim m_b)$:

$$\langle O_i \rangle(\mu_b) = \langle O_i \rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$$

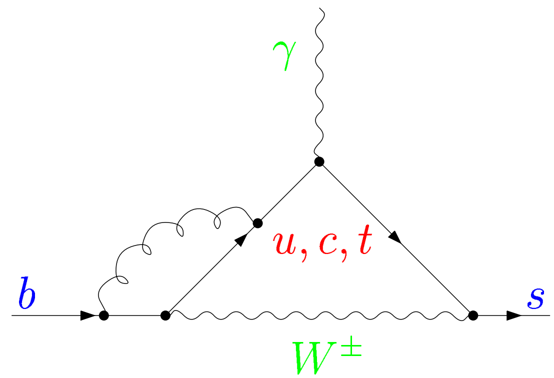
$i = 1, ..., 6$:	1-loop	2-loop	3-loop	[Bieri, Greub, Steinhauser, hep-ph/0302051] $\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak
$i = 7, 8$:	tree	1-loop		2-loop [Greub, Hurth, Asatrian]

Examples of SM diagrams for the matching of $C_7(\mu_0)$:

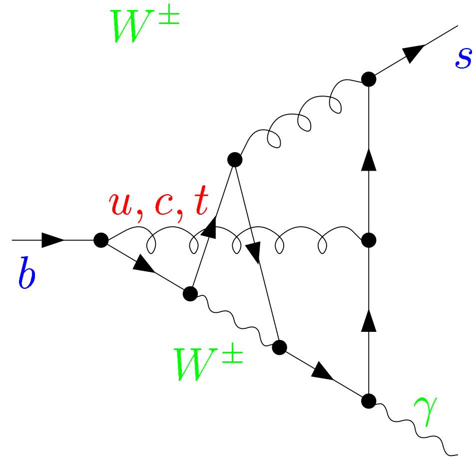
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, Misiak, 2004]



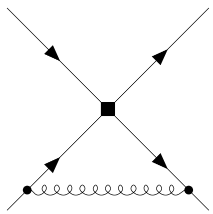
Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \to s\gamma$ amplitude

RGE for the Wilson coefficients $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

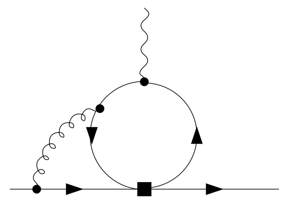
- Renormalization constants $\Longrightarrow$ γ_{ij}: $C_j(\mu)$ known in the meanwhile to NLL accuracy

Z_{22} Z_{27} Z_{87}

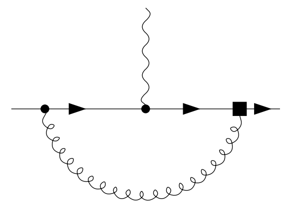
LO



[Gaillard, Lee, 1974]
[Altarelli, Maiani, 1974]

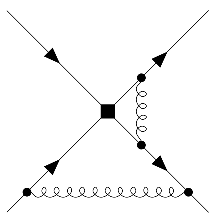


[Grinstein *et al.*, 1990]

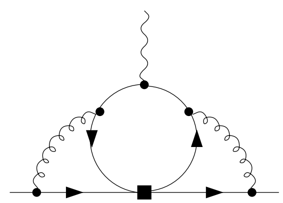


[Shifman *et al.*, 1978]
[Grigjanis *et al.*, 1988]

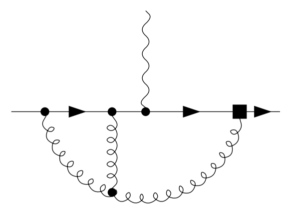
NLO



[Altarelli *et al.*, 1981]
[Buras, Weisz, 1990]

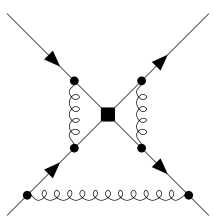


[Chetyrkin *et al.*, 1997]

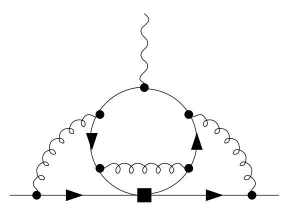


[Misiak, Münz, 1995]

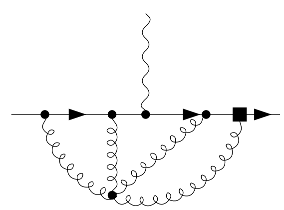
NNLO



[Gambino *et al.*, 2004]
(unpublished yet)



in progress: Czakon, Haisch, Misiak, Schröder.

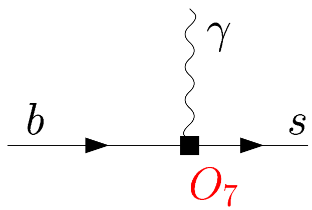


in progress: Gambino, Gorbahn, Haisch.

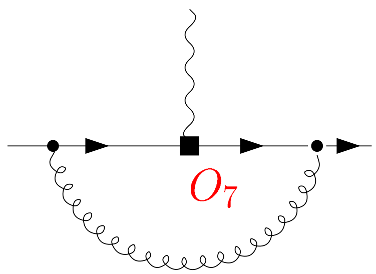
The $b \to s\gamma$ matrix elements

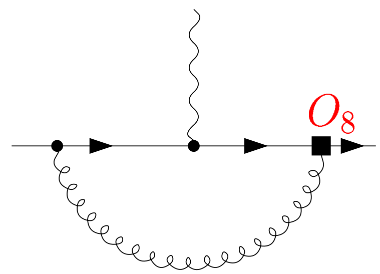
Perturbative on-shell amplitudes

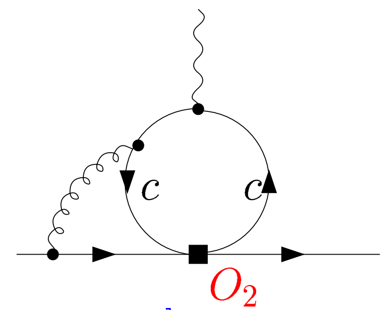
LO



NLO



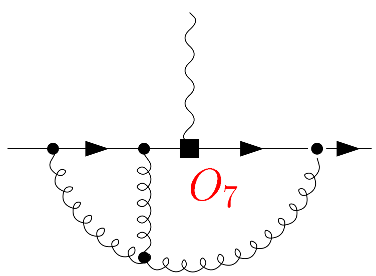


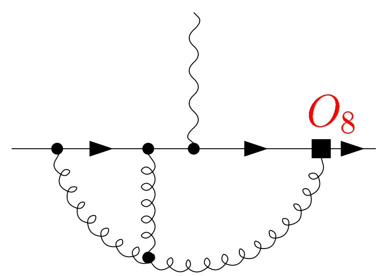


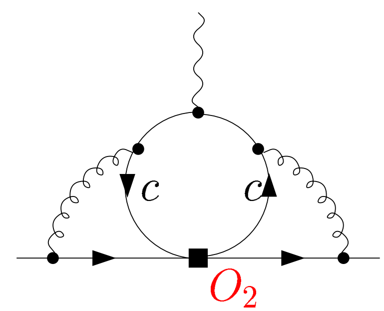
[Ali, Greub, 1991]

[Greub, Hurth, Wyler, 1996]

NNLO







in progress: Asatrian, Greub, Hurth

[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, Misiak
(extrapolation in m_c)

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

- Three-loop running is used for α_s coupling with $\Lambda^{(5)}_{\overline{\text{MS}}} = 220$ MeV

Non-perturbative effects in $\bar{B} \to X_s\gamma$

We need to sum the matrix elements of the effective Hamiltonian:

$$\Sigma_{X_s} \left| C_7\langle X_s\gamma|O_7|\bar{B}\rangle + C_2\langle X_s\gamma|O_2|\bar{B}\rangle + ... \right|^2$$

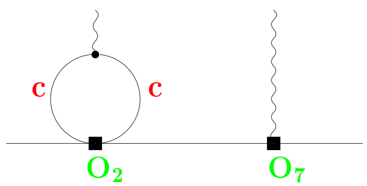
- The "77" term in the above sum can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude; HQET gives us a double expansion

$$\Sigma_{X_s} \mathrm{BR}(\bar{B} \to X_s\gamma) = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B}\right)^2 + ... \right] + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B}\right)^2 + ... \right]$$

$$+ \mathcal{O}\left[\left(\frac{\alpha_s(m_b)}{\pi}\right)^2\right] + \text{[Contributions other than the "77" term]}$$

- $\frac{\Delta\Gamma^{O_7,O_7}_{B\to X_s\gamma}}{\Gamma^{\mathrm{LL}}_{b\to s\gamma}} = 1 + \frac{\delta^{\mathrm{NP}}_{\mathrm{rad}}}{m_b^2}; \quad \delta^{\mathrm{NP}}_{\mathrm{rad}} = \frac{1}{2}\lambda_1 - \frac{9}{2}\lambda_2$

- Contributions from Operators containing the charm quark at the leading order in α_s :

$\langle\bar{B}|$  $|\bar{B}\rangle = \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b\Lambda}{m_c^2}\right)^n,$

- $\frac{\Delta\Gamma^{O_2,O_7}_{B\to X_s\gamma}}{\Gamma^{\mathrm{LL}}_{b\to s\gamma}} = \frac{1}{9}\frac{C_2}{C_7}\frac{\lambda_2}{m_c^2} \simeq +0.03$

E_γ-Spectrum in $B \to X_s \gamma$ in $O(\alpha_s^2)$

Melnikov and Mitov; hep-ph/0505097

- Assuming that the decay is dominated by $\mathcal{O}_7$; calculate normalized E_γ-spectrum in $O(\alpha_s^2)$ $[z = 2E_\gamma/m_b]$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz} = \delta(1-z) + (\frac{\alpha_s}{\pi})C_F F^{(1)}(z) + (\frac{\alpha_s}{\pi})^2 C_F F^{(2)}(z)$$

- Normalization $\int_0^1 \frac{1}{\Gamma}\frac{d\Gamma}{dz}dz = 1$ allows to fix the $\delta(1-z)$ term

- $O(\alpha_s)$ contribution [Greub, AA; Z. Phys. C49 ('91) 431]

$$F^{(1)}(z) = -\frac{31}{12}\delta(1-z) - \left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{7}{4}\left[\frac{1}{1-z}\right]_+ - \frac{z+1}{2}\ln(1-z) + \frac{7+z-2z^2}{4}$$

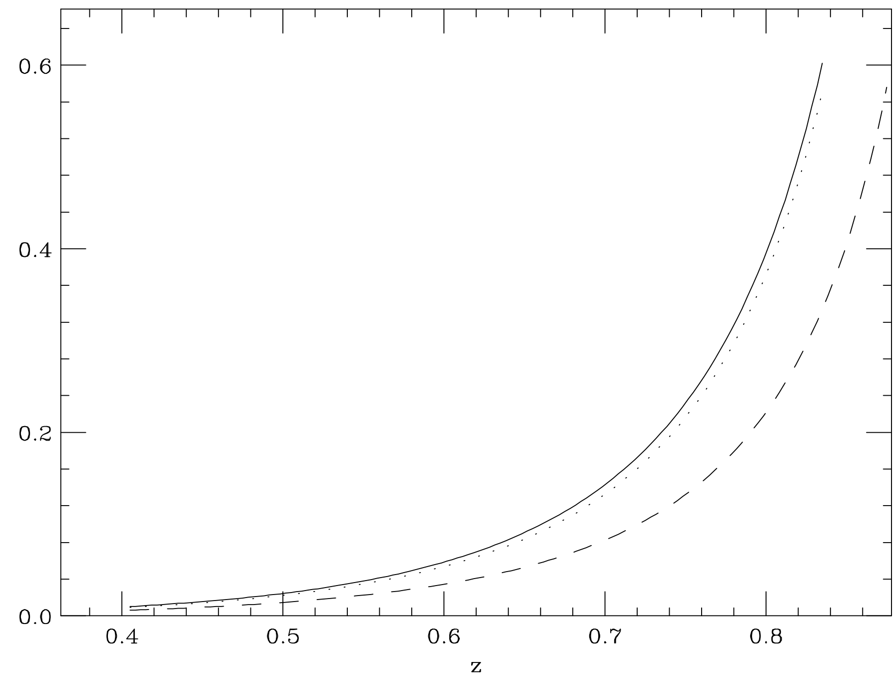
- BLM [Brodsky-Lepage-Mackenzie] corrections to $O(\alpha_s)^2\beta_0$ obtained by calculating the $O(\alpha_s)^2 n_f$ piece and making the identification $-2n_f/3 \to \beta_0$ [Ligeti, Luke, Manohar, Wise; hep-ph/9903305]

- BLM corrections summed to all orders in α_s[Benson, Bigi, Uraltsev; hep-ph/0410080]

E_γ-Spectrum in $B \to X_s \gamma$ in $O(\alpha_s^2)$ (Contd.)

Melnikov and Mitov; hep-ph/0505097

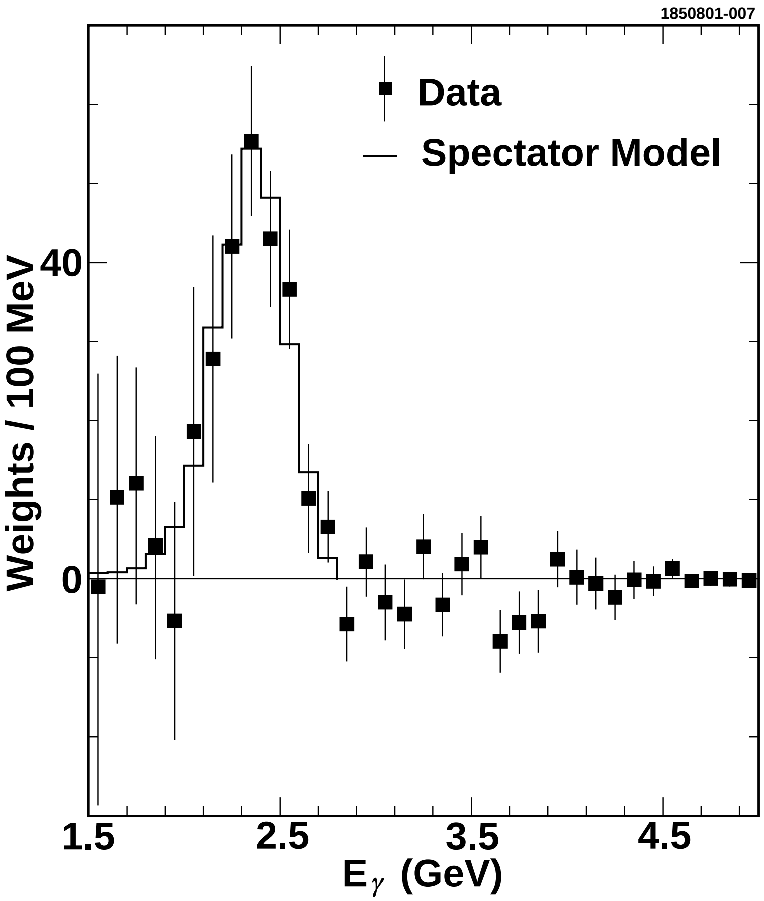
- E_γ-spectrum in $\mathcal{O}(\alpha_s^2)$ (solid), BLM (dots) and $\mathcal{O}(\alpha_s)$ (dashed)



- Effect of the non-BLM terms is about 1% for $\mu = m_b$

Measurement of $\bar{B} \to X_s \gamma$

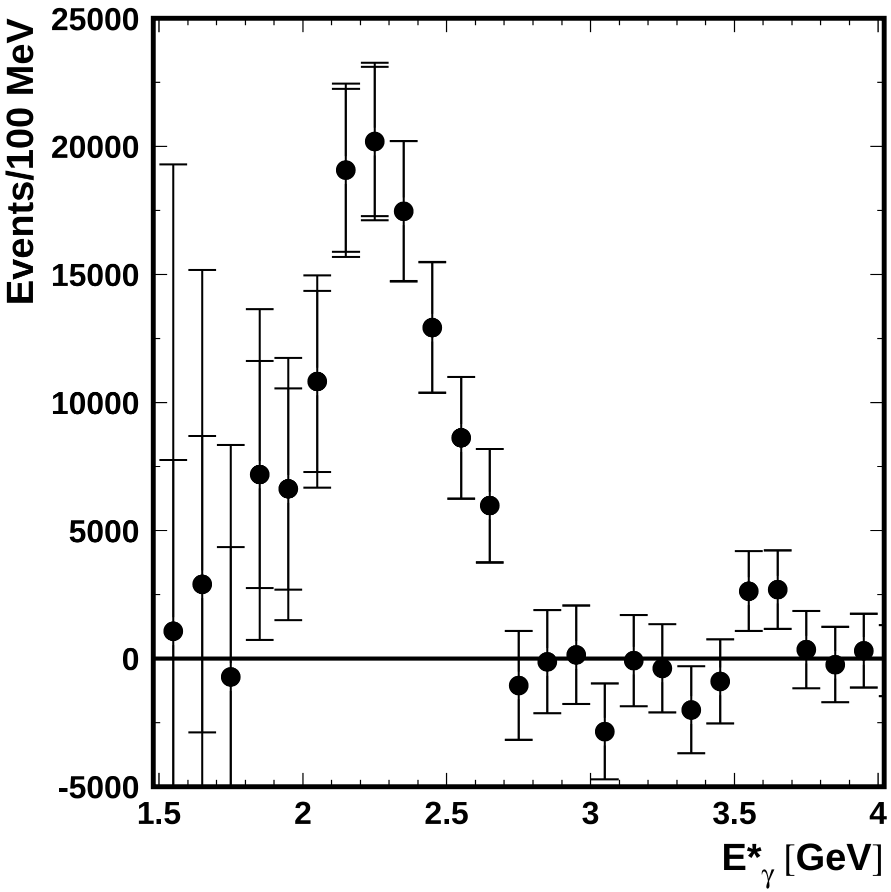
Spectator Model: Greub, AA; PLB 259, 182 (1991)



CLEO

hep-ex/0108032

PRL 87 (2001) 251807



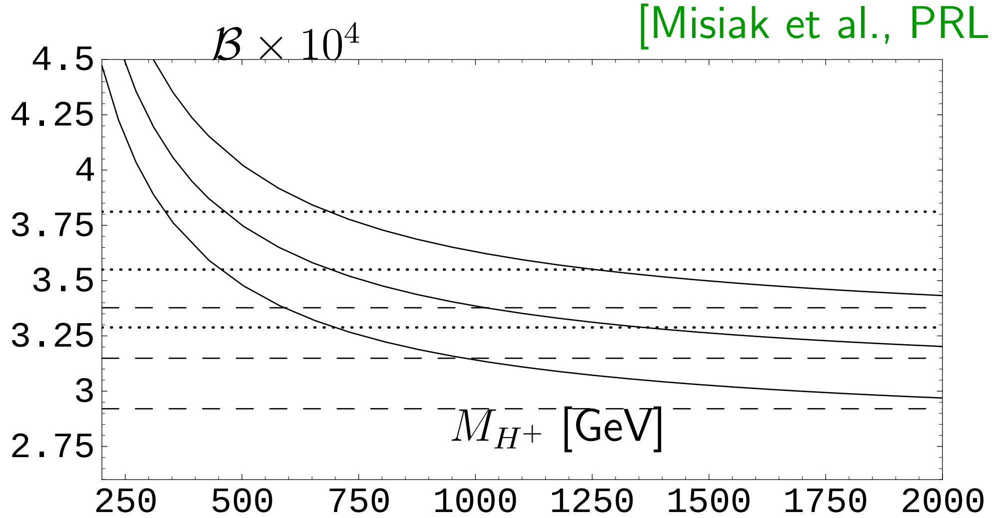
BELLE

hep-ex/0403004

PRL 93 (2004) 061803

$\mathcal{B}(\bar{B} \to X_s \gamma)$: Experiment vs. SM & 2HDM

[Misiak et al., PRL 98:022002 (2007)]



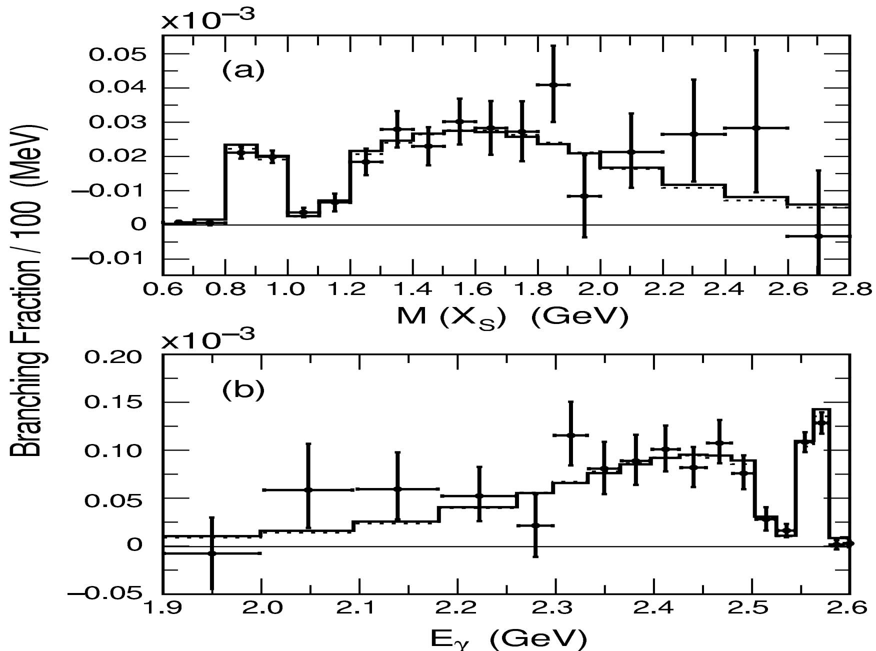
[$\cdots$ (exp); $-\,-\,-\,-$ (SM); solid (2HDM)]

- Expt. [ICHEP 2012]: $(E_\gamma > 1.6$ GeV): $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.37 \pm 0.23) \times 10^{-4}$
- NNLO SM: $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$
- Ratio=Expt./SM $= 1.07 \pm 0.10$, Limits most NP models
- In 2HDM, $\mathcal{B}(B \to X_s \gamma)$ bounds M_{H^+}

Photon Energy Spectrum from Sum of Exclusive Final States

BABAR Collaboration, PR D72:052004 (2005)

- Theory: Shape function [Kagan, Neubert; Neubert et al.]
  Kinetic quark mass scheme: [Benson, Bigi, Uraltsev]



- $\mathcal{B}(B \to X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.35 \pm 0.19^{+0.56+0.04}_{-0.41-0.09}) \times 10^{-4}$
- Isospin-asymmetry:

$$\Delta_{0-} = \frac{\Gamma(\bar{B}^0 \to X_{s\bar{d}}\gamma) - \Gamma(B^- \to X_{s\bar{u}}\gamma)}{\Gamma(\bar{B}^0 \to X_{s\bar{d}}\gamma) + \Gamma(B^- \to X_{s\bar{u}}\gamma)} = -0.01 \pm 0.066$$

- Consistent with SM, where Δ_{0-} power (Λ/m_b) suppressed; typically a few %

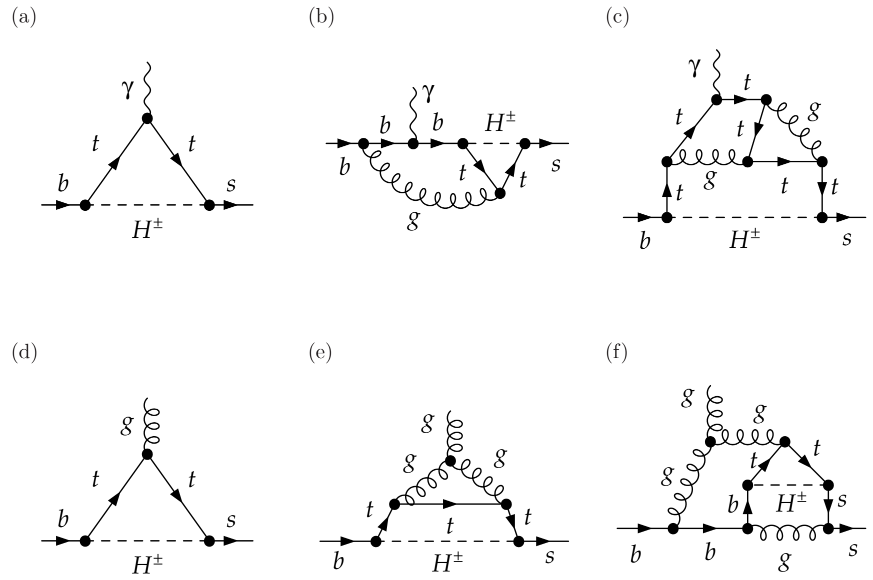
$B \to X_s\gamma$ in 2HDM

- NNLO in 2HDM calculated recently [Hermann, Misiak, Steinhauser; arxiv:1208.2788]

$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2}\Sigma_{i,j=1}^{3}\bar{u}_i(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

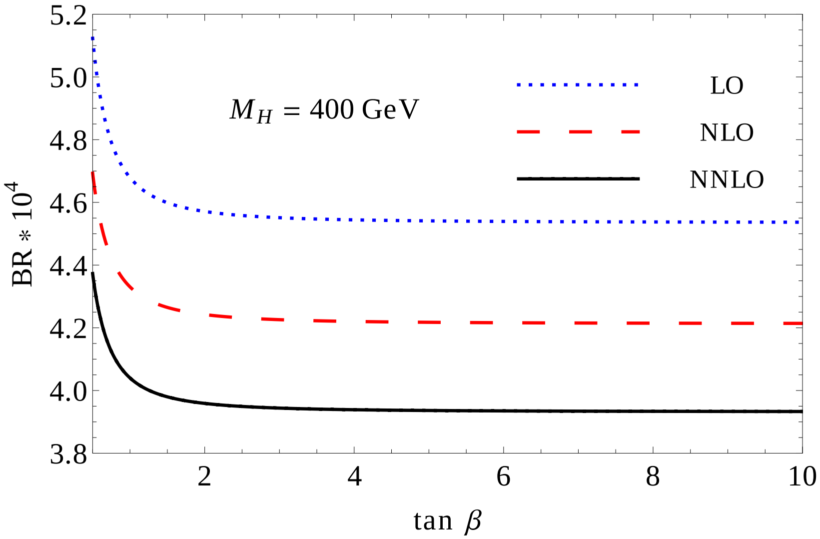
with $P_{L/R} = (1 \mp \gamma_5)/2$

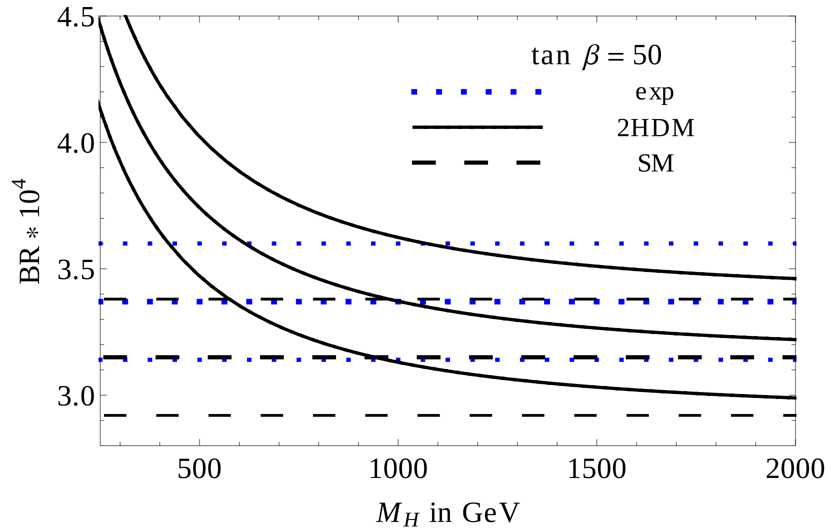
- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
  - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan\beta}$
  - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan\beta}$



$\bar{B} \to X_s \gamma$ in Type-II 2HDM

[Hermann, Misiak, Steinhauser; arxiv:1208.2788]





- $M_{H^+} > 380$ GeV (at 95% C.L.)
- $M_{H^+} > 289$ GeV (at 99% C.L.)

$\bar{B} \to X_s l^+ l^-$

• The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

• Wilson Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, 10$$

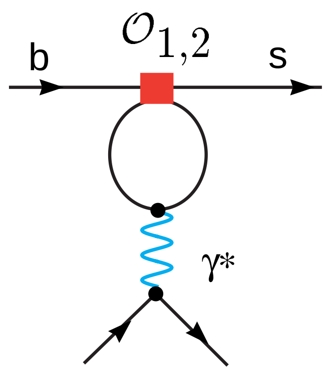
have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

• After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electroweak logarithm that originates from photonic penguins with charm quark loops:

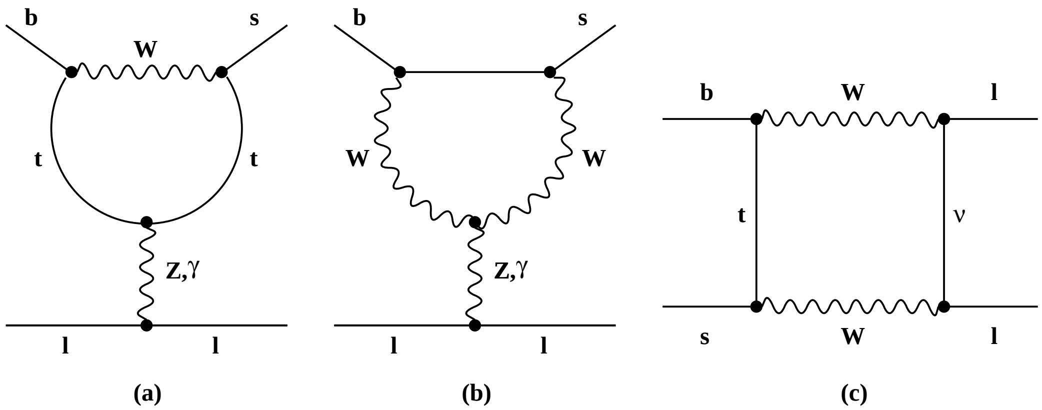
$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



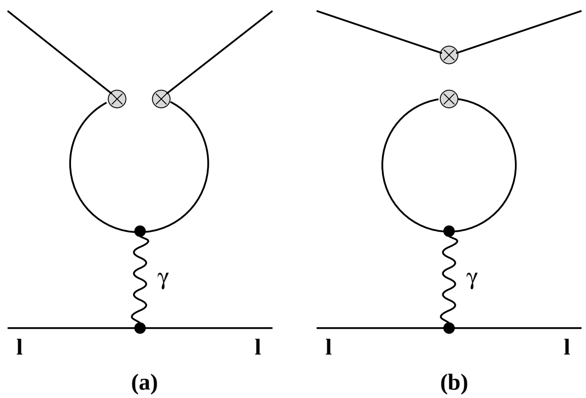
$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

On the other hand: $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO

Some representative diagrams in $b \to s\ell^+\ell^-$



Diagrams in the full theory



Diagrams in the effective theory

NNLO Calculations of $\mathrm{BR}(\bar{B} \to X_s \ell^+ \ell^-)$

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:
    [Asatryan, Asatrian, Greub, Walker;
    Asatrian, Bieri, Greub, Hovhannissyan;
    Ghinculov, Hurth, Isidori, Yao;
    Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays [AA, Greub, Hiller, Lunghi]
  - $\mathrm{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathrm{BR}(\bar{B} \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Inclusive $B \to X_s \ell^+ \ell^-$ in NNLO in SM

Dilepton Invariant Mass

$$\frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} (1-\hat{s})^2$$
$$\times \left((1+2\hat{s}) \left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\tilde{C}_{10}^{\text{eff}}\right|^2 \right) + 4\,(1+2/\hat{s}) \left|\tilde{C}_7^{\text{eff}}\right|^2 + 12\text{Re}\left(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}\right) \right)$$

$$\begin{aligned}
\tilde{C}_7^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_7(\hat{s})\right) A_7 \\
&\quad - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(7)}(\hat{s}) + C_2^{(0)} F_2^{(7)}(\hat{s}) + A_8^{(0)} F_8^{(7)}(\hat{s})\right) \\
\tilde{C}_9^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) \left(A_9 + T_9\, h(\hat{m}_c^2, \hat{s}) + U_9\, h(1, \hat{s}) + W_9\, h(0, \hat{s})\right) \\
&\quad - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(9)}(\hat{s}) + C_2^{(0)} F_2^{(9)}(\hat{s}) + A_8^{(0)} F_8^{(9)}(\hat{s})\right) \\
\tilde{C}_{10}^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) A_{10}
\end{aligned}$$

- A_7, A_8, A_9, A_{10}, T_9, U_9, W_9 are linear combinations of the Wilson coefficients

Comparison of $B \to X_s \ell^+ \ell^-$ with Data

[AA,Greub, Hiller, Lunghi 2001 (AGHL); Ghinculov, Hurth, Isidori, Yao 2004 (GHIY); Huber, Lunghi, Misiak, Wyler 2005 (HLMW); Bobeth, Gambino, Gorbahn, Haisch 2003]

- Inclusive $B \to X_s \ell^+ \ell^-$ BRs

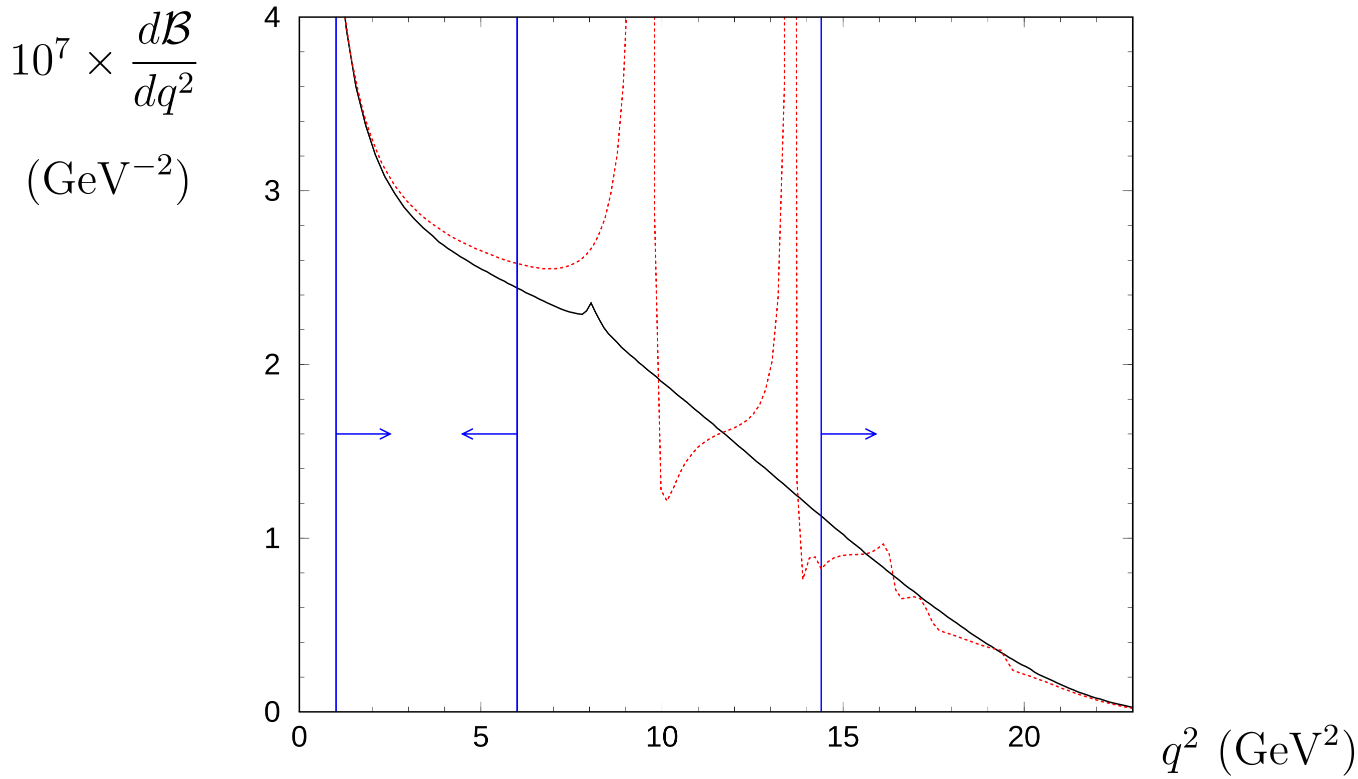
$$\mathcal{B}(B \to X_s \ell^+ \ell^-)(M_{\ell\ell} > 0.2 \text{ GeV}) = (3.66^{+0.76}_{-0.77}) \times 10^{-6} \ \ [\text{HFAG}'12]$$

$$SM : (4.2 \pm 0.7) \times 10^{-6} \ \ [\text{AGHL}]; \ \ (4.6 \pm 0.8) \times 10^{-6} \ \ [\text{GHIY}]$$

- Partial BRs (integrated over lower range of q^2)
  - $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 \in \ [1, 6] \text{ GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$ [GHIY]
  - $\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 \in \ [1, 6] \text{ GeV}^2 = (1.59 \pm 0.11) \times 10^{-6}$ [HLMW]
  - $\mathcal{B}(\bar{B} \to X_s e^+ e^-); \quad q^2 \in \ [1, 6] \text{ GeV}^2 = (1.63 \pm 0.11) \times 10^{-6}$ [HLMW]
  - Experiment: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \quad q^2 \in \ [1, 6] \text{ GeV}^2 = (1.60 \pm 0.51) \times 10^{-6}$
- Partial BRs (integrated over higher range of q^2)
  - $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 > 14 \text{ GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$ [GHIY]
  - $\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 14.4 \text{ GeV}^2 = 2.40(1^{+0.29}_{-0.26}) \times 10^{-7}$ [HLMW]
  - $\mathcal{B}(\bar{B} \to X_s e^+ e^-); \quad q^2 > 14.4 \text{ GeV}^2 = 2.09(1^{+0.32}_{-0.30}) \times 10^{-7}$ [HLMW]
  - Experiment: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \quad q^2 > 14.4 \text{ GeV}^2 = (4.4 \pm 1.2) \times 10^{-7}$

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]

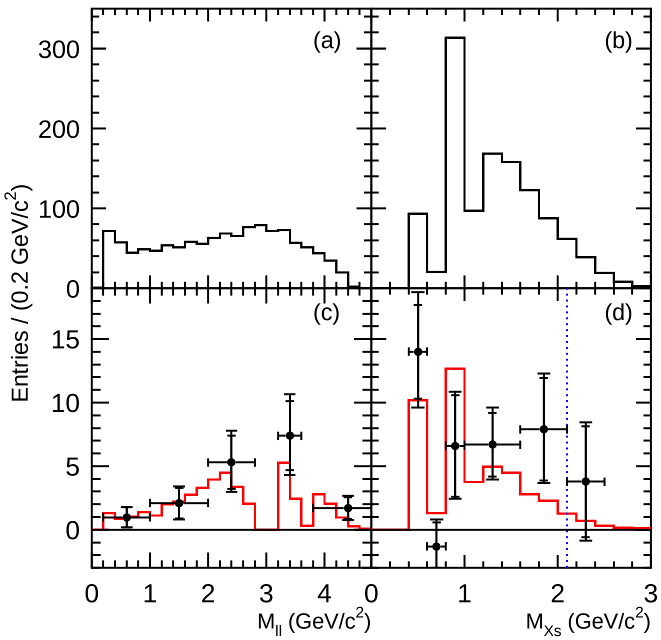


- $\mathrm{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $q^2 \in [1, 6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathrm{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathrm{BR}(\bar{B} \to X_s \ell^+ \ell^-)$; $q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$,

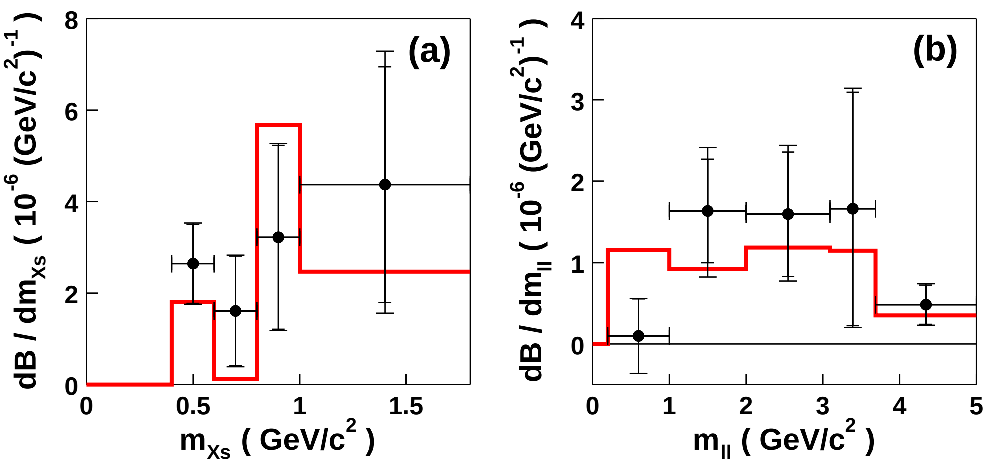
Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



[BABAR]



- In agreement with the NNLO SM calculations

Forward-Backward Asymmetry in $B \to X_s \ell^+\ell^-$

[Proposed in AA, Mannel, Morozumi, PLB 273, 505 (1991)]
[NNLL: Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\overline{A}_{\rm FB}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,\mathrm{sgn}(z)\,dz}{\int_{-1}^{1} \frac{d^2\Gamma(b\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,dz}$$

Unnormalized FB Asymmetry

$$A_{\rm FB}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,\mathrm{sgn}(z)\,dz}{\Gamma(B\to X_c e\bar{\nu}_e)}\,\mathrm{BR}_{\rm sl}$$

$$\int_{-1}^{1} \frac{d^2\Gamma(b\to X_s\,\ell^+\ell^-)}{d\hat{s}\,dz}\,\mathrm{sgn}(z)\,dz = \left(\frac{\alpha_{\rm em}}{4\,\pi}\right)^2 \frac{G_F^2\, m_{b,\rm pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\,\pi^3}(1-\hat{s})^2$$
$$\times \left[-3\,\hat{s}\,\mathrm{Re}(\widetilde{C}_9^{\rm eff}\widetilde{C}_{10}^{\rm eff*})\left(1+\frac{2\alpha_s}{\pi}\,f_{910}(\hat{s})\right) - 6\,\mathrm{Re}(\widetilde{C}_7^{\rm eff}\widetilde{C}_{10}^{\rm eff*})\left(1+\frac{2\alpha_s}{\pi}\,f_{710}(\hat{s})\right)\right]$$

- NNLL Contributions stabilize the scale ($=\mu$) dependence of the FB Asymmetry

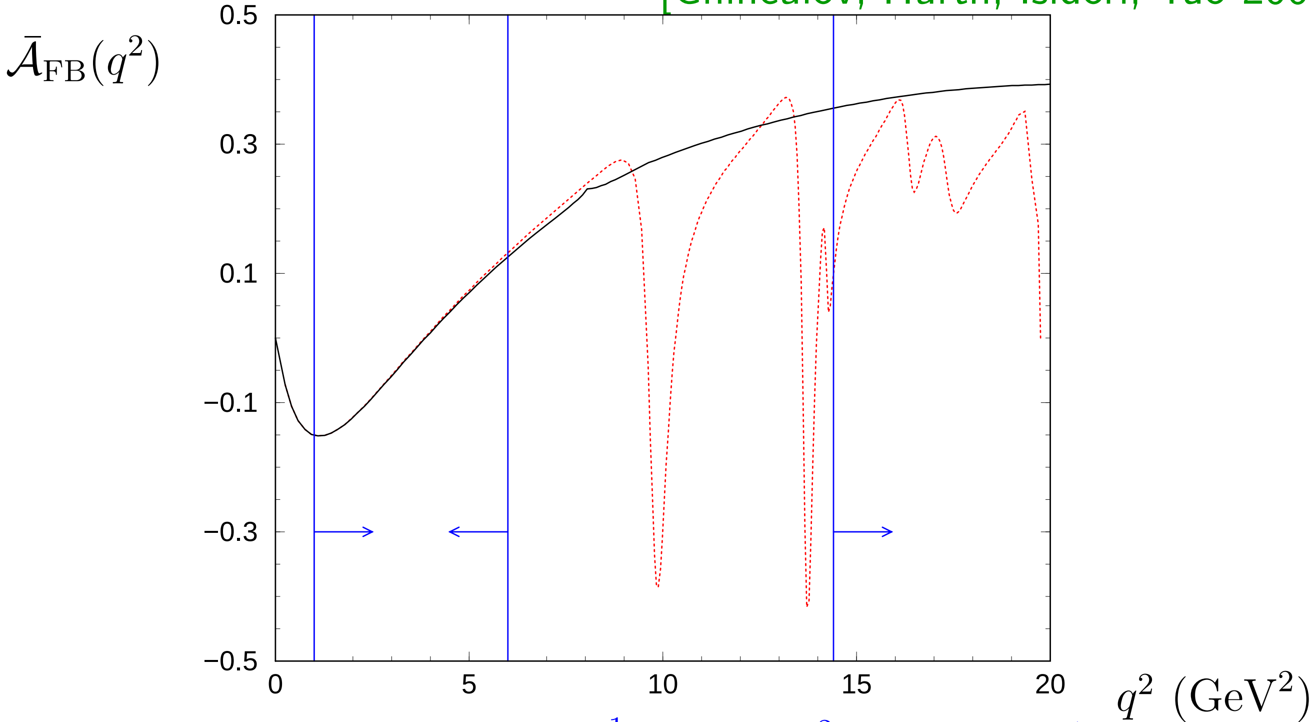
$$A_{\rm FB}^{\rm NLL}(0) = -(2.51\pm 0.28)\times 10^{-6}; \quad A_{\rm FB}^{\rm NNLL}(0) = -(2.30\pm 0.10)\times 10^{-6}$$

- Zero of the FB Asymmetry is a precise test of the SM, correlating $\widetilde{C}_7^{\rm eff}$ and $\widetilde{C}_9^{\rm eff}$

$$\hat{s}_0^{\rm NLL} = 0.144\pm 0.020; \quad \hat{s}_0^{\rm NNLL} = 0.162\pm 0.008$$

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \, d\cos\theta_\ell} \text{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$q_0^2 = (3.90 \pm 0.25)$ GeV^2 [Ghinculov, Hurth, Isidori, Yao 2004]

$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})$ GeV^2 [Bobeth, Gambino, Gorbahn, Haisch 2003]

Experimental data

Experimental Data on $B \to V\gamma$ Decays

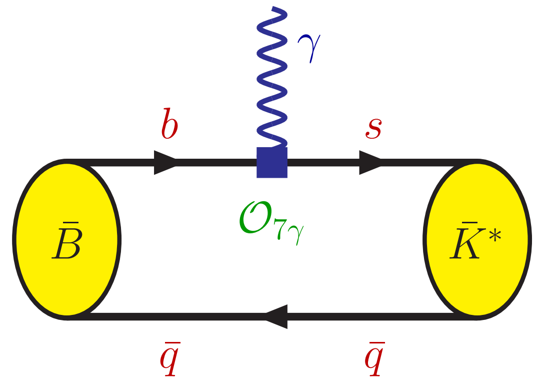
Branching ratios (in units of 10^{-6}) [HFAG, Summer 2012]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B \to X_s\gamma$	$331 \pm 35 \pm 34$	$347 \pm 15 \pm 40$	$327 \pm 44 \pm 28$	337 ± 23 ‡
$B^+ \to K^*(892)^+\gamma$	$42.1 \pm 1.4 \pm 1.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	42.1 ± 1.8
$B^0 \to K^*(892)^0\gamma$	$44.7 \pm 1.0 \pm 1.6$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	43.3 ± 1.5
$B^+ \to K_1(1270)^+\gamma$		$43 \pm 9 \pm 9$		43 ± 12
$B^+ \to K_2^*(1430)^+\gamma$	$14.5 \pm 4.0 \pm 1.5$			14.5 ± 4.3
$B^0 \to K_2^*(1430)^0\gamma$	$12.2 \pm 2.5 \pm 1.0$	$13.0 \pm 5.0 \pm 1.0$		12.4 ± 2.4
$B^+ \to \rho^+\gamma$	$1.20^{+0.42}_{-0.37} \pm 0.20$	$0.87^{+0.29+0.09}_{-0.27-0.11}$	< 13.0	$0.98^{+0.25}_{-0.24}$
$B^0 \to \rho^0\gamma$	$0.97^{+0.24}_{-0.22} \pm 0.06$	$0.78 \pm 0.17 \pm 0.09$	< 17.0	0.86 ± 0.14
$B^0 \to \omega\gamma$	$0.50^{+0.27}_{-0.23} \pm 0.09$	$0.40^{+0.19}_{-0.17} \pm 0.11$	< 9.2	$0.44^{+0.18}_{-0.16}$
$B \to (\rho,\omega)\gamma$	$1.63 \pm 0.29 \pm 0.16$	$1.14 \pm 0.20 \pm 0.11$	< 14.0	1.30 ± 0.18
$B^0 \to \phi\gamma$	< 0.85		< 3.3	< 0.85
$B^0 \to J/\psi\,\gamma$	< 1.6			< 1.6

‡ Calculated for the photon energy range $E_\gamma > 1.6$ GeV

$B \to K^*\gamma$ Branching Fraction in LO

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \to K^*\gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\rm LO} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* C_7^{(0)\rm eff} \frac{e\bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^*\varepsilon^*) - (e^*P)(\varepsilon^*q) + i\,{\rm eps}(e^*,\varepsilon^*,P,q)\right]$$

Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- Branching ratio:

$$\mathcal{B}^{\rm LO}(B \to K^*\gamma) = \tau_B \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b)\, |C_7^{(0)\rm eff}(\mu_b)|^2\, |T_1^{(K^*)}(0)|^2$$

$B \to K^* \gamma$ decay rates in NLO

- Perturbative improvements undertaken in three approaches (QCD-F; PQCD; SCET)

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

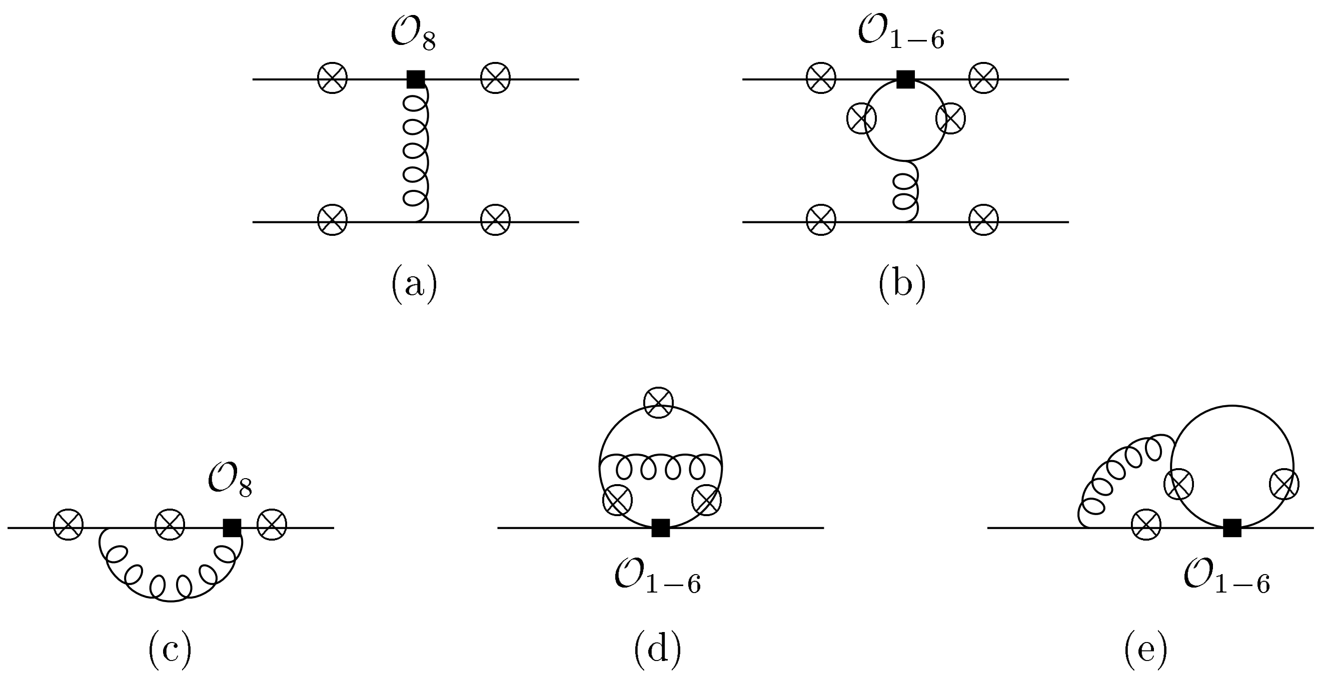
$$\langle V\gamma|Q_i|\bar{B}\rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})$$

- $\zeta_{V_\perp}$ (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + ..., \quad t^{II} = \mathcal{O}(\alpha_s) + ...$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$ for some time; include Hard-scattering and Vertex corrections [Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

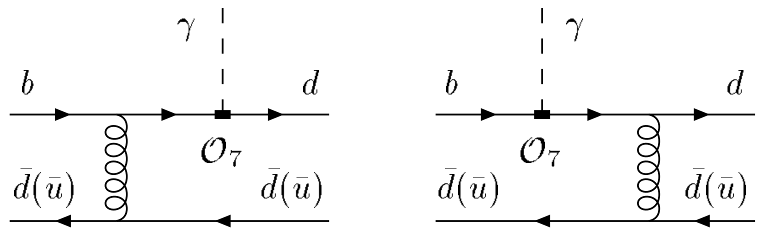
Nonfactorizable α_s Corrections



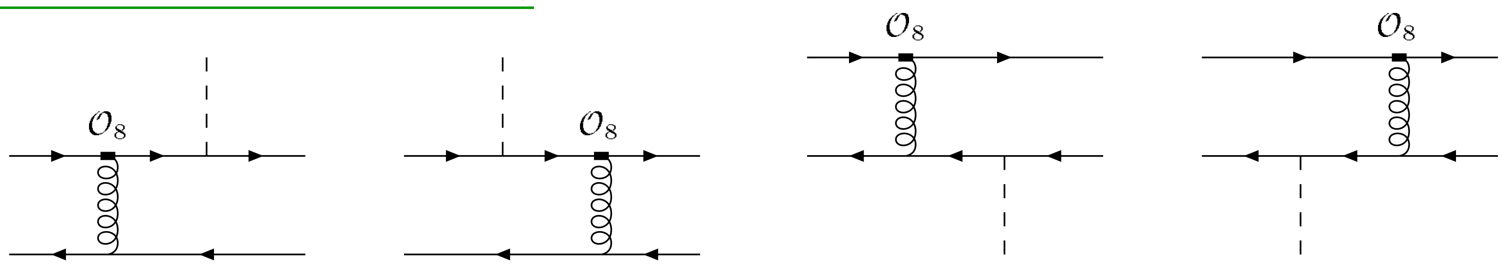
- First line: hard-spectator corrections
- Second line: $b \to s\gamma$ vertex corrections

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

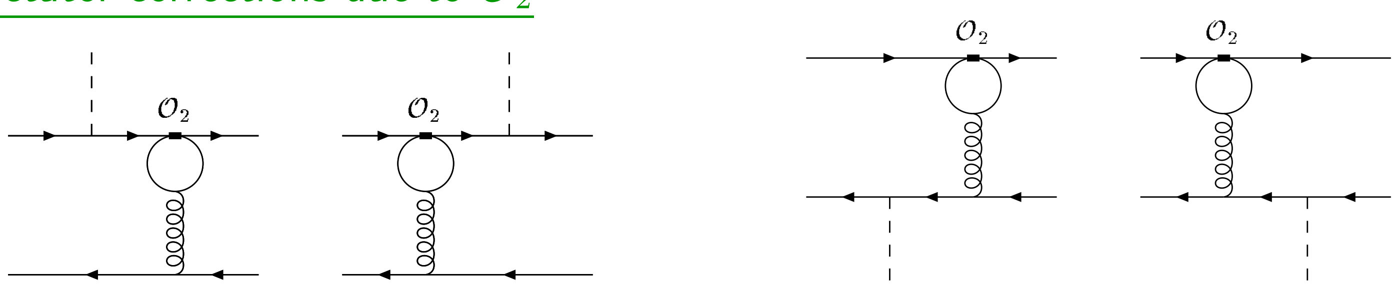
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Estimates of BR($B \to K^* \gamma$) in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.2 ± 0.18 (HFAG 2012)];

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.33 ± 0.15 (HFAG 2012)];

$$\mathcal{B}(B_s \to \phi\gamma) = 4.3 \pm 1.1[\zeta_{\phi}] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+2.1}_{-1.8}$ (BELLE); 3.9 ± 0.5 (LHCb)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm exp}} = 1.10 \pm 0.35[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*0}\gamma)_{\rm exp}} = 1.00 \pm 0.32[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B_s \to \phi\gamma)_{\rm NNLO}}{\mathcal{B}(B_s \to \phi\gamma)_{\rm exp}} = 1.1 \pm 0.3[\text{theory}] \pm 0.1[\text{exp}]$
- Theory error is about 30%; dominantly from $\zeta_{V_\perp}$, m_c and λ_B; SM decay rates in good agreement with the data

$B \to \rho\gamma$ decay

Penguin amplitude $\mathcal{M}_{\rm P}(B \to \rho\gamma)$

$$-\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta - i \left[g^{\mu\nu}(q.p) - p^\mu q^\nu \right] \right) T_1^{(\rho)}(0)$$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$

$$e \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_A^{(\rho);\rm p.v.} - i \left[g^{\mu\nu}(q.p) - p^\mu q^\nu \right] F_A^{(\rho);\rm p.c.} \right)$$

- $F_A^{(\rho);\rm p.v.}(0) \simeq F_A^{(\rho);\rm p.c.}(0) = F_A^{(\rho)}(0)$ [e.g., Byer, Melikhov, Stech]

$$\epsilon_{\rm A}(\rho^\pm\gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

- Holds in factorization approximation
- $O(\alpha_s)$ corrections to annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$: Leading-twist contribution vanishes in the chiral limit [Grinstein, Pirjol]; non-factorizing annihilation contribution likely small; testable in $B^\pm \to \ell^\pm \nu_\ell \gamma$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^0 \to \rho^0\gamma)$

• Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\rm A}(\rho^0\gamma) \simeq 0.05$$

$B \to (\rho,\ \omega)\gamma$ decay rates

[Parkhomenko, A.A.; Bosch, Buchalla; Lunghi, Parkhomenko, AA; Beneke, Feldmann, Seidel]

$$R(\rho\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \rho\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \, [1 + \Delta R(\rho/K^*)]$$

$$R(\omega\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \omega\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = 1/2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\omega^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \, [1 + \Delta R(\omega/K^*)]$$

- $S_\rho = 1$ for $B^\pm \to \rho^\pm\gamma$; $= 1/2$ for $B^0 \to \rho^0\gamma$
- $\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.85 \pm 0.10$; $T_1^\omega(0) \simeq T_1^{(\rho)}(0)$ [SRs, Lattice Average]

  $\zeta = \simeq 0.85 \pm 0.06$; $T_1^\omega(0) \simeq T_1^{(\rho)}(0)$ [Ball, Zwicky, 2006]
- $\Delta R(\rho^\pm/K^{*\pm}) = 0.12 \pm 0.10$
- $\Delta R(\rho^0/K^{*0}) \simeq \Delta R(\omega/K^{*0}) = 0.1 \pm 0.07$

Branching Ratios (SM) vs. Expt.

$\mathrm{BR}(\mathrm{B}^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}$ (SM)$= (0.98 \pm 0.24) \times 10^{-6}$ (Expt.)
$\mathrm{BR}(\mathrm{B}^0 \to \rho^0\gamma) \simeq \mathrm{BR}(\mathrm{B}^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}$
$\mathrm{BR}(\mathrm{B}^0 \to \rho^0\gamma)$ (Expt.) $= (0.86 \pm 0.14) \times 10^{-6}$
$\mathrm{BR}(\mathrm{B}^0 \to \omega\gamma)$ (Expt.) $= (1.30 \pm 0.18) \times 10^{-6}$

Experiment vs. SM $(b \to d\gamma)$

SM Estimates [Lunghi, Parkhomenko, AA; PLB 595 (2004) 323]

$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\} = (1.38 \pm 0.42) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] \equiv \frac{\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma]}{\bar{\mathcal{B}}[B \to K^*\,\gamma]} = 0.033 \pm 0.010$$

Expt. HFAG-2012

$$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] = (1.30^{+0.18}_{-0.19}) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] = 0.030 \pm 0.005\ ({\rm stat})^{+0.003}_{-0.002}\ ({\rm syst})$$

$$|V_{td}/V_{ts}| = 0.20 \pm 0.02\ ({\rm exp}) \pm 0.04\ ({\rm theo})$$

- In good agreement with the determination from the ratio $\Delta M_s/\Delta M_d \Longrightarrow$ $|V_{td}|/|V_{ts}| = 0.211 \pm 0.001({\rm exp}) \pm 0.006\ ({\rm theo})$ in the SM, but less precise
- A correlated study of $R[(\rho,\omega)/K^*]$ and $\Delta M_s/\Delta M_d$ provides valuable constraints on the parameters of the underlying theory

D. Mohapatra (BELLE)[EPS 2005]

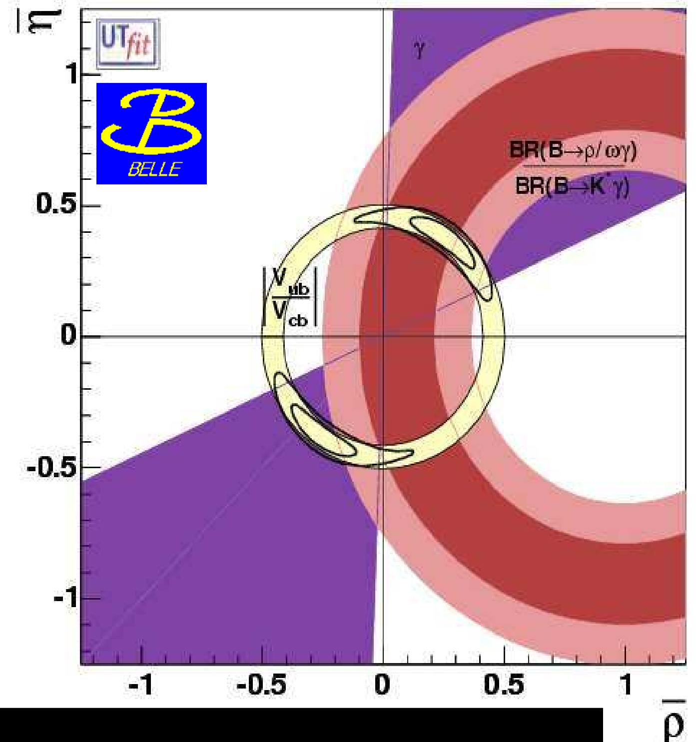
Extraction of $|V_{td}/V_{ts}|$

$$\frac{B(\overline{B}\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{1-M_\rho^2/M_B^2}{1-M_{K^*}^2/M_B^2}\right)\zeta^2[1+\Delta R]$$

Form factor ratio $\zeta = 0.85 \pm 0.10$

SU(3)-breaking effect $\Delta R = 0.1 \pm 0.1$

$$\frac{B(B\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)} = 0.032 \pm 0.008^{+0.003}_{-0.002}$$



$$0.143 < \left|\frac{V_{td}}{V_{ts}}\right| < 0.260$$

(95 % C.L. interval)

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.200^{+0.026}_{-0.025}(\text{expt.})^{+0.038}_{-0.029}(\text{theo.})$$

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Isospin violation in $B \to \rho\gamma$ decays

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \qquad \Delta^{\pm 0} \equiv \frac{\Gamma(B^{\pm} \to \rho^{\pm}\gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0\gamma)} - 1$$

$$\Delta_{\rm LO} = 2\epsilon_A \left[F_1 + \frac{\epsilon_A}{2}(F_1^2 + F_2^2)\right] = 2\epsilon_A F\cos\alpha + O(\epsilon_A^2)$$

$$\Delta_{\rm NLO} \simeq \Delta_{\rm LO} - \frac{2\epsilon_A}{C_7^{(0)\rm eff}} F\cos\alpha \left[A_R^{(1)t} + A_R^u F\cos 2\alpha\right] + O(\epsilon_A^2)$$

$$F_1 = F\cos\alpha; \quad F_2 = F\sin\alpha; \quad F = \frac{R_b}{R_t} \simeq 0.5$$

$$\Delta^{\rm SM}(\rho\gamma) = (1.1 \pm 3.9)\% \ \text{ for } \ \alpha = (92 \pm 11)^\circ; \quad \Delta^{\rm expt}(\rho\gamma) = -0.46^{+0.17}_{-0.16}$$

$$\Delta^{(\rho/\omega)} \equiv \tfrac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right]$$

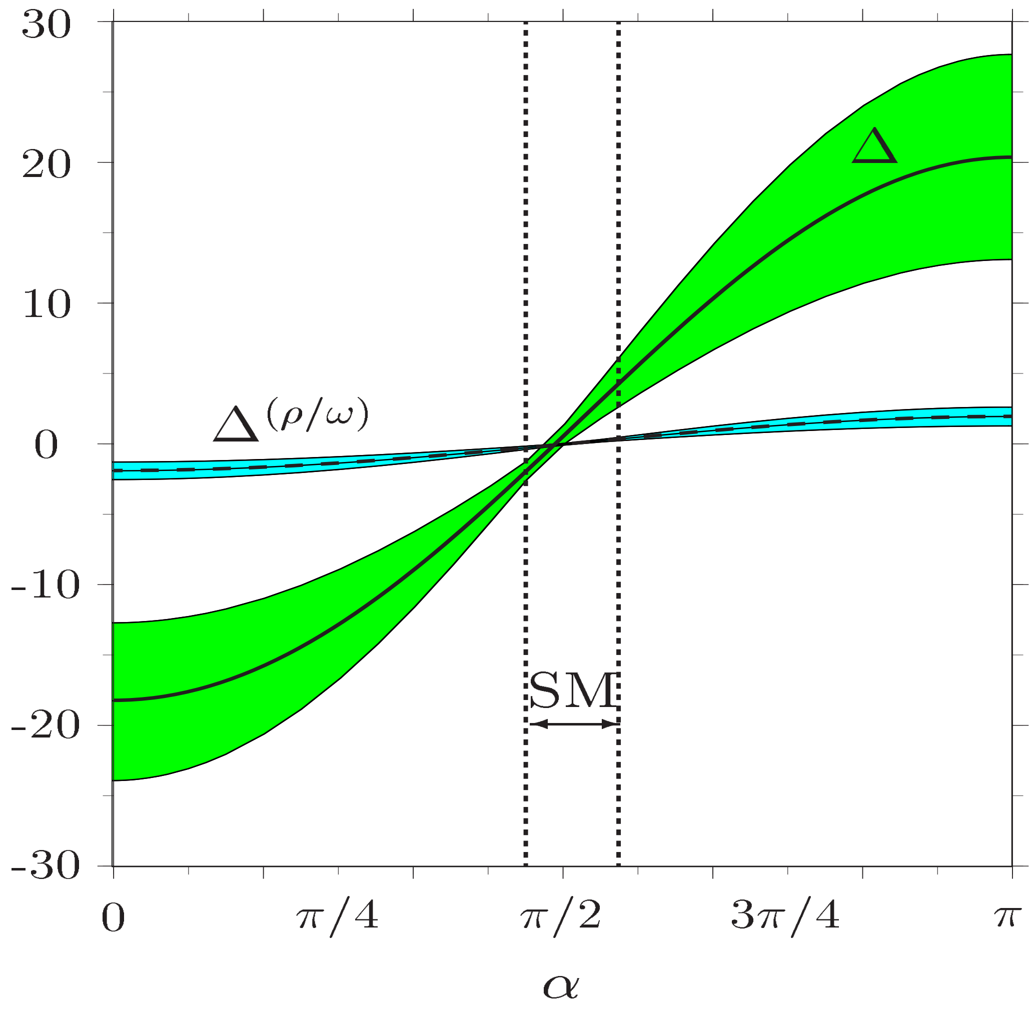
$$\Delta_B^{(\rho/\omega)} \equiv \frac{(M_B^2 - m_\omega^2)^3 \mathcal{B}(B^0 \to \rho\gamma) - (M_B^2 - m_\rho^2)^3 \mathcal{B}(B^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3 \mathcal{B}(B^0 \to \rho\gamma) + (M_B^2 - m_\rho^2)^3 \mathcal{B}(B^0 \to \omega\gamma)}$$

with $\Delta_{\bar{B}}^{(\rho/\omega)} = \Delta_B^{(\rho/\omega)}(B^0 \to \bar{B}^0)$

$$\Delta_B^{(\rho/\omega)} = (0.3 \pm 3.9) \times 10^{-3} \ \text{ for } \ \alpha = (92 \pm 11)^\circ$$

Isospin-violating ratio Δ in $B \to \rho\gamma$ decays

[AA, Lunghi, Parkhomenko; PLB 595 (2004) 323]



CP Asymmetry in $B \to (\rho, \omega)\gamma$ decays

Direct CP Asymmetry in $B \to (\rho, \omega)\gamma$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\rho^\pm\gamma) \equiv \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)} = -0.11 \pm 0.032 \pm 0.09 \text{ [Expt]}.$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) \equiv \frac{\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) - \mathcal{B}(B_d^0 \to \rho^0\gamma)}{\mathcal{B}(\bar{B}_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \rho^0\gamma)} = -0.44 \pm 0.49 \pm 0.14 \text{ [Expt]}$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(\omega\gamma) \equiv \frac{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) - \mathcal{B}(B_d^0 \to \omega\gamma)}{\mathcal{B}(\bar{B}_d^0 \to \omega\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)}$$

$$\mathcal{A}_{\rm CP}(\rho/\omega\gamma) = \frac{2F \sin\alpha (A_I^u - \epsilon_A A_I^{(1)t})}{C_7^{(0)\rm eff}(1 + \Delta_{\rm LO})}$$

Mixing-induced CP Asymmetery in $B^0 \to (\rho, \omega)\gamma$

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma}\cos(\Delta M_d t) + S_{\rho\gamma}\sin(\Delta M_d t)$$

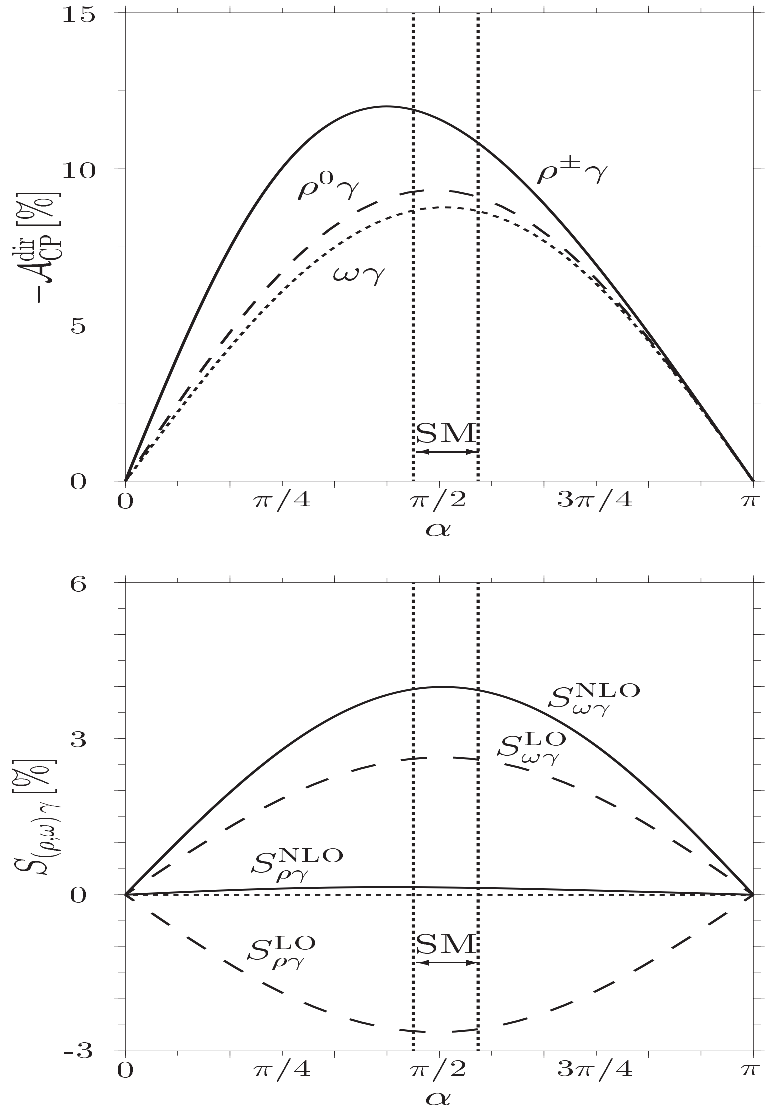
$$\lambda_{\rho\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}_d^0 \to \rho^0\gamma)}{A(B_d^0 \to \rho^0\gamma)} = \frac{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,\mathrm{e}^{+i\alpha}}{C_7^{(0)\rm eff} + A^{(1)t} - [C_7^{(0)\rm eff}\varepsilon_A^{(0)} + A^u]\,F\,\mathrm{e}^{-i\alpha}}$$

where $p/q \simeq \exp(2i\beta)$ and $F = R_b/R_t$

$$C_{\rho\gamma} = -\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma) = \frac{1 - |\lambda_{\rho\gamma}|^2}{1 + |\lambda_{\rho\gamma}|^2}, \qquad S_{\rho\gamma} = \frac{2\,\mathrm{Im}(\lambda_{\rho\gamma})}{1 + |\lambda_{\rho\gamma}|^2} = -0.83 \pm 0.65 \pm 0.18 \text{ [Expt]}$$

CP-violating Asymmetries in $B \to (\rho, \omega)\gamma$ decays

[AA, Lunghi, Parkhomenko; PLB 595 (2004) 323]



Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ (pseudoscalar P); $B \to K^*$ (Vector V) Transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \quad \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

$$\langle P|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle P|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle V|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle V|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- 10 non-perturbative q^2-dependent functions (Form factors) $\Longrightarrow$ model-dependence
- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-Approach allows to reduce the number of independent form factors from 10 to 3; perturbative symmetry-breaking corrections [Beneke, Feldmann, Seidel; Beneke, Feldmann]
- HQET & $SU(3)_F$ relate $B \to (\pi, \rho)\ell\nu_\ell$ and $B \to (K, K^*)\ell^+\ell^-$ to determine the remaining FF's

Experimental data vs. SM in $B \to (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6}) [HFAG: 2012]

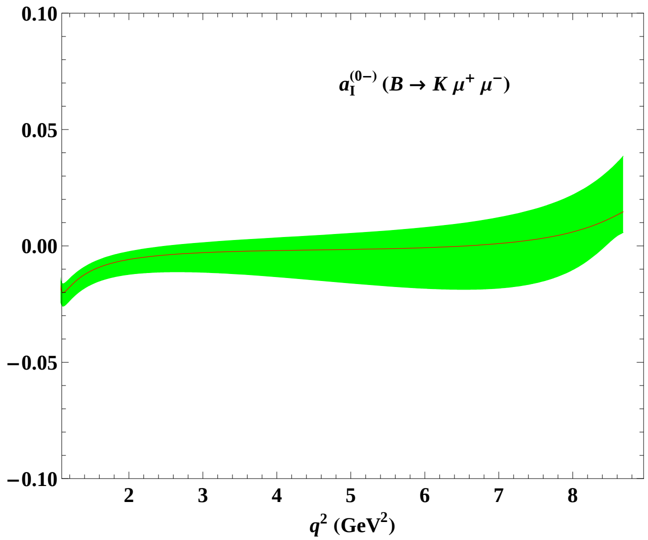
SM: [A.A., Greub, Hiller, Lunghi PR D66 (2002) 034002]

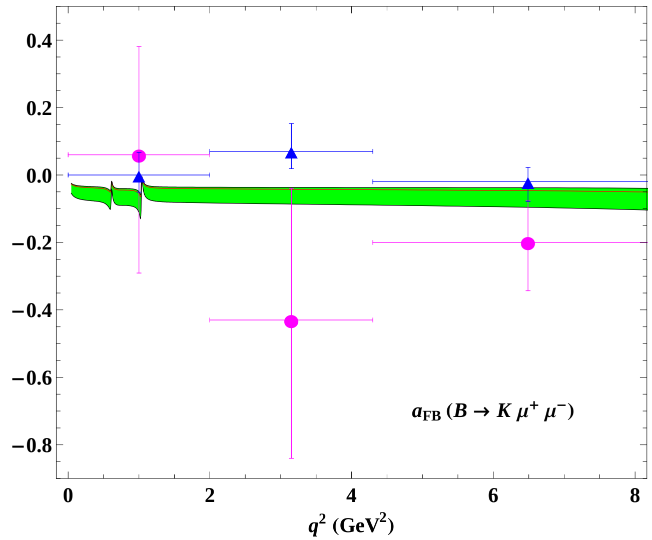
Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \to K\ell^+\ell^-$	0.45 ± 0.04	0.35 ± 0.12
$B \to K^* e^+ e^-$	$1.19^{+0.17}_{-0.16}$	1.58 ± 0.49
$B \to K^* \mu^+\mu^-$	$1.15^{+0.16}_{-0.15}$	1.19 ± 0.39
$B \to X_s \mu^+\mu^-$	$2.23^{+0.97}_{-0.98}$	4.2 ± 0.7
$B \to X_s e^+ e^-$	$4.91^{+1.04}_{-1.06}$	4.2 ± 0.7
$B \to X_s \ell^+\ell^-$	$3.66^{+0.76}_{-0.77}$	4.2 ± 0.7

- Inclusive measurements and the SM rates include the cut $M_{\ell^+\ell^-} > 0.2$ GeV
- SM & Data agree within $\mathbf{25\%}$

Analysis at Large Recoil of $B \to K\ell^+\ell^-$

Khodjamirian, Mannel, Wang [1211.0234]





- Includes an estimate of the non-local contributions based on QCD sum rules and dispersion relations at large hadronic recoil; find these effects are modest
- Re-evaluated $d\Gamma(B \to K\ell^+\ell^-)/dq^2$, CP-averaged isospin asymmetry $a_I^{(0-)}(q^2)$, and forward-backward asymmetry $a_{\rm FB}(q^2) \Longrightarrow$ improved theoretical estimates at large recoil
- Both $a_I^{(0-)}(q^2)$ and $a_{\rm FB}(q^2)$ are small in the SM
- $a_{\rm FB}(q^2)$ in agreement with SM, but current data hints at significantly larger isospin asymmetry $a_I^{(0-)}(q^2)$

Isospin Asymmetries (Current Experimental Summary)

[HFAG 2012]

- $\mathbf{\Delta_{0-}(K^*\gamma) = 0.052 \pm 0.026}$
- $\mathbf{\Delta_{0-}(X_s\gamma) = -0.01 \pm 0.06}$
- $\mathbf{\Delta_{0-}(\rho\gamma) = -0.46^{+0.17}_{-0.16}}$
- $\mathbf{\Delta_{0-}(K\ell\ell) = -0.40^{+0.16}_{-0.15}}$
- $\mathbf{\Delta_{0-}(K^*\ell\ell) = -0.44^{+0.13}_{-0.12}}$

- Currently, there is no measurement of $\mathbf{\Delta_{0-}(X_d\gamma)}$
- Others remain to be well measured; all will be undertaken at Belle II & LHCb
- More theoretical work needed to reduce the parametric uncertainties

Forward-Backward Asymmetry in $B \to K^* \ell^+ \ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u} d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u} \frac{d\Gamma}{d\hat{u} d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff}) V A_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{eff} (V T_2 (1 - \hat{m}_V) + A_1 T_1 (1 + \hat{m}_V))]$$

- $\boldsymbol{T_1, T_2, V, A_1}$ form factors
  - Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m^2_{J/\psi}$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^* \ell^+ \ell^-$

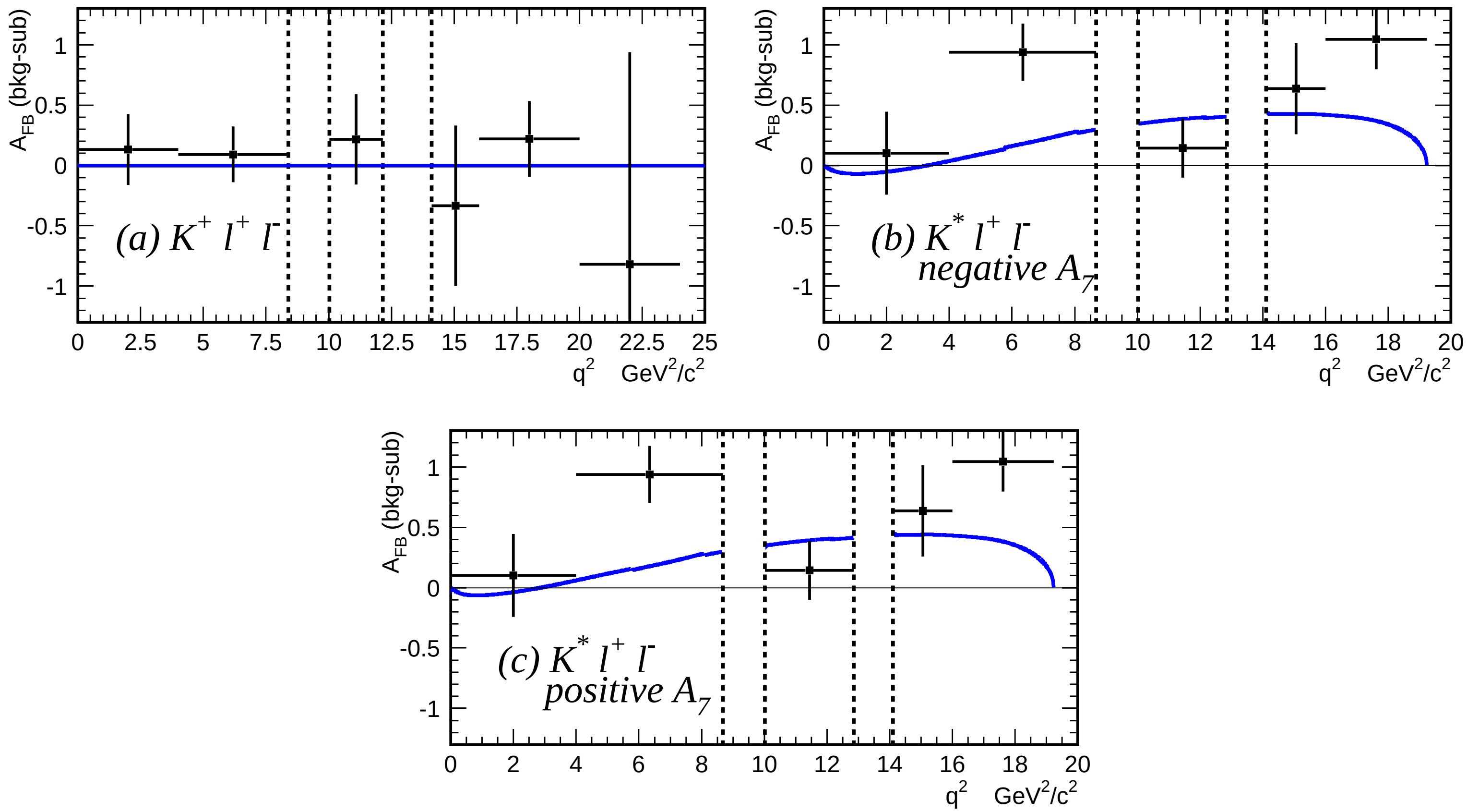
$$\mathrm{Re}(\mathrm{C_9^{eff}}(\hat{\mathrm{s}}_0)) = -\frac{\hat{\mathrm{m}}_\mathrm{b}}{\hat{\mathrm{s}}_0} \mathrm{C_7^{eff}} (\frac{\mathrm{T_2}(\hat{\mathrm{s}}_0)}{\mathrm{A_1}(\hat{\mathrm{s}}_0)}(1 - \hat{\mathrm{m}}_\mathrm{V}) + \frac{\mathrm{T_1}(\hat{\mathrm{s}}_0)}{\mathrm{V}(\hat{\mathrm{s}}_0)}(1 + \hat{\mathrm{m}}_\mathrm{V}))$$

- Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
- HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ ($E = \frac{m_B^2 + m_{K^*}^2 - q^2}{2 m_B}$) and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1 + \hat{m}_V}{1 + \hat{m}_V^2 - \hat{s}}(1 - \frac{\hat{s}}{1 - \hat{m}_V^2}); \quad \frac{T_1}{V} = \frac{1}{1 + \hat{m}_V}$$

- No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]:
  $C_9^{eff}(\hat{s}_0) = -\frac{2 m_b M_B}{s_0} C_7^{eff}$

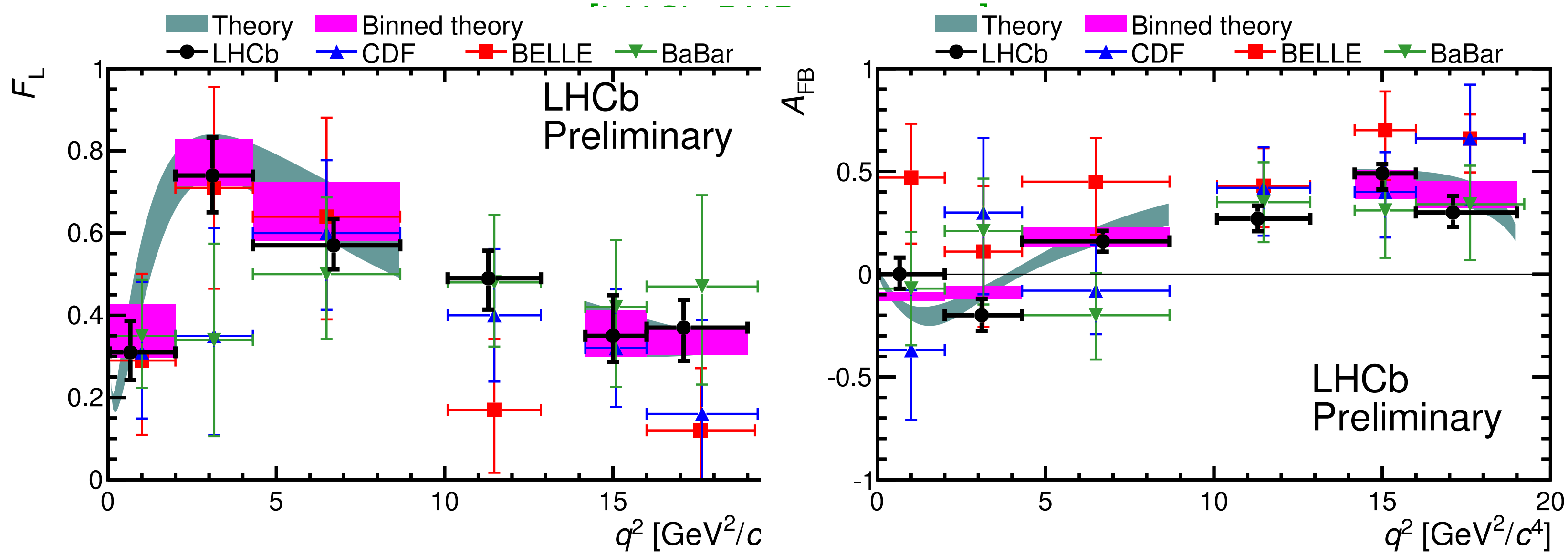
Belle FB Asymmetry Distributions (EPS 2005)



Best Fits

- $A_7 = -0.33$: $A_9/A_7 = -15.3^{+3.4}_{-4.8}$; $A_{10}/A_7 = 10.3^{+5.2}_{-3.5}$
- $A_7 = +0.33$: $A_9/A_7 = -16.3^{+3.7}_{-5.7}$; $A_{10}/A_7 = 11.1^{+6.0}_{-3.9}$
- SM: $A_7 = -0.33$; $A_9/A_7 = -12.3$; $A_{10}/A_7 = 12.8$

Recent Measurements of Angular Observables in $B \to K^* \mu^+ \mu^-$



- Angular variables F_L and A_{FB} have been extracted from the decays $B \to K^*(\to K\pi)\mu^+\mu^-$ from the following expressions

$$\frac{d\Gamma'}{d\theta_K} = \frac{3\Gamma'}{4} \sin\theta_K \left(2F_L \cos^2\theta_K + (1 - F_L)\sin^2\theta_K\right)$$

$$\frac{d\Gamma'}{d\theta_\ell} = \Gamma' \left(\frac{3}{4}F_L \sin^2_{\theta_\ell} + \frac{3}{8}(1 - F_L)(1 + \cos^2_{\theta_\ell}) + A_{FB}\cos\theta_\ell\right) \sin\theta_\ell$$

with $\Gamma' = \Gamma + \bar{\Gamma}$

- Their dependence on the Wilson Coeffs. and the FFs has been worked out in great detail and the measurements are in agreement with the SM

$B_s \to \mu^+\mu^-$ in the SM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}{}^* V_{tb} \sum_i \left[C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu)\right]$$

$$\mathcal{O}_{10} = (\bar{s}_\alpha \gamma^\mu P_L b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l)\,, \qquad \mathcal{O}_{10}' = (\bar{s}_\alpha \gamma^\mu P_R b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l)$$

$$\mathcal{O}_S = m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}l)\,, \qquad \mathcal{O}_S' = m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}l)$$

$$\mathcal{O}_P = m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}\gamma_5 l)\,, \qquad \mathcal{O}_P' = m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}\gamma_5 l)$$

$$\mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right) = \frac{G_F^2\alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3}|V_{ts}{}^* V_{tb}|^2\sqrt{1-4\hat{m}_\mu^2} \times \left[\left(1-4\hat{m}_\mu^2\right)|F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2\right]$$

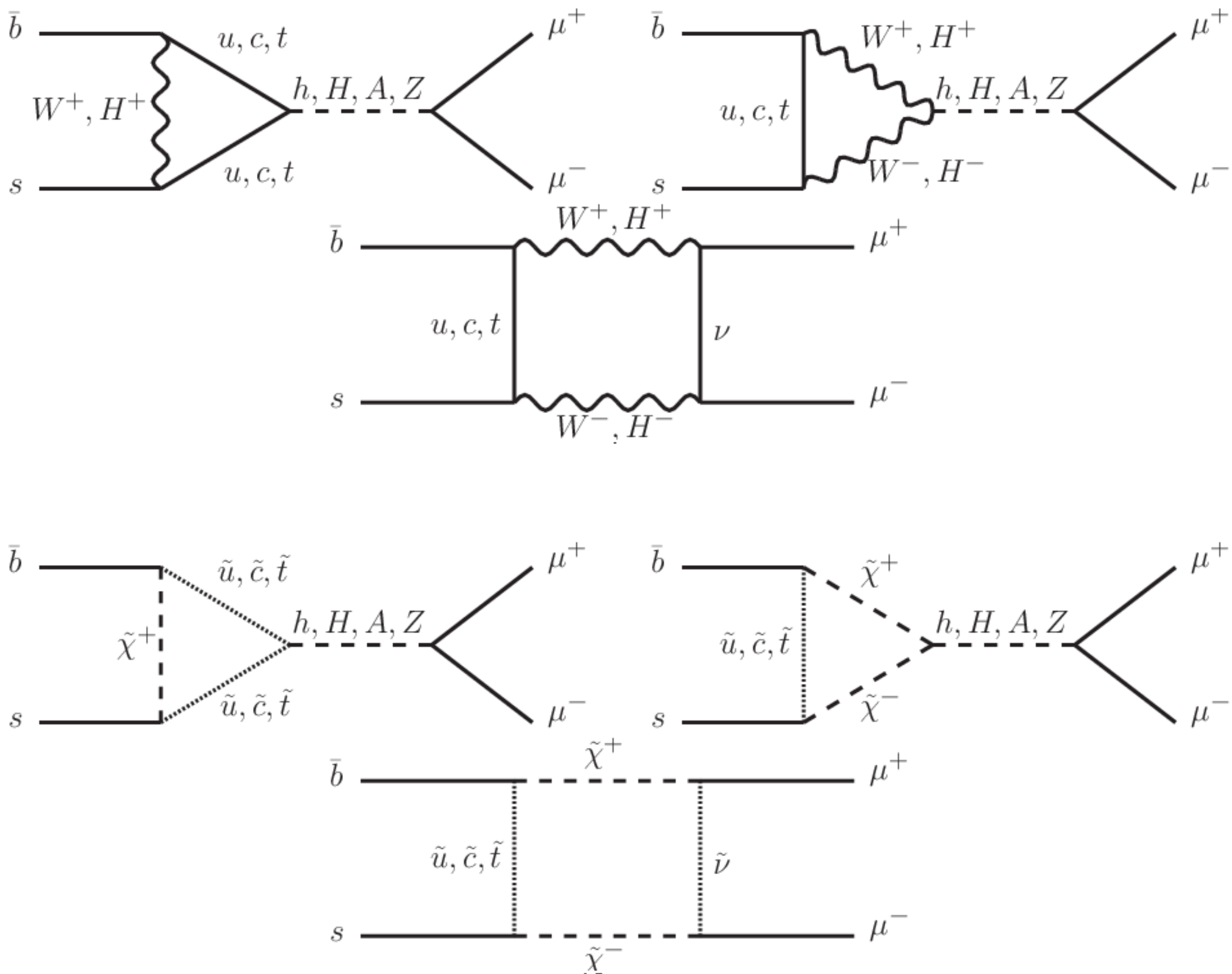
where $\hat{m}_\mu = m_\mu/m_{B_s}$ and

$$F_{S,P} = m_{B_s}\left[\frac{C_{S,P}m_b - C_{S,P}'m_s}{m_b+m_s}\right], \qquad F_{10} = C_{10} - C_{10}'$$

$$\mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right)_{\mathrm{SM}} = (3.23 \pm 0.27) \times 10^{-9}$$ [Buras et al.; arxiv:1208.09344]

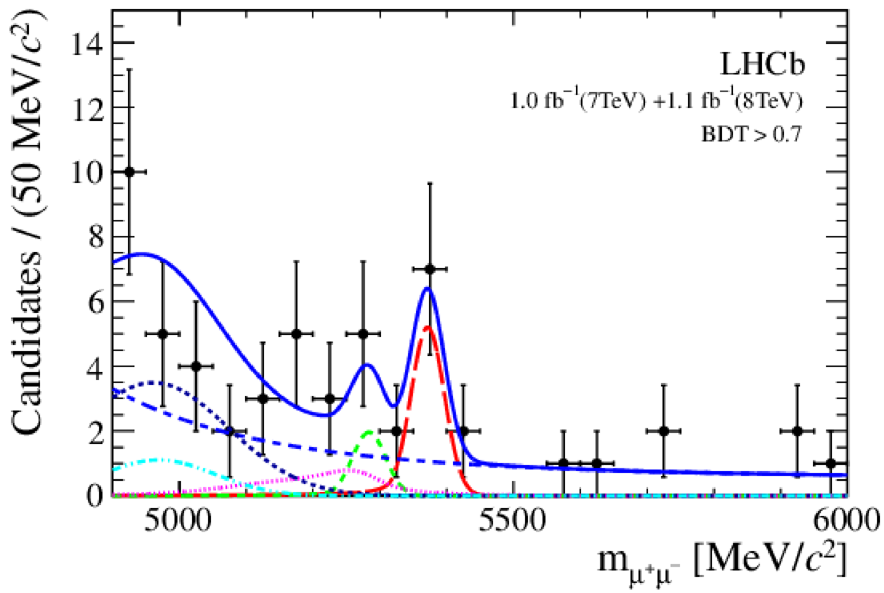
- Experimentally, the measured BR is time-averaged (TA), which differs from this value beacause of $y_S^{\mathrm{SM}} = \Delta\Gamma_s/\Gamma_s = 0.088 \pm 0.014$
  $BR(B_s^0 \to \mu^+\mu^-)_{\mathrm{TA}}^{\mathrm{SM}} = (3.54 \pm 0.30) \times 10^{-9}$; $= (3.2^{+1.5}_{-1.2}) \times 10^{-9}$ (LHCb: PRL 110, 021801 (2013))

Leading diagrams for $B_s \to \mu^+\mu^-$ in SM, 2HDM & MSSM



First Evidence for the Decays $\mathcal{B}(B_s \to \mu^+\mu^-)$

[R. Aaij et al. (LHCb), PRL 110, 021801 (2013)]]



$B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ decays in SCET

- Soft Collinear Effective Theory (SCET): Applicable to any QCD processes which contain collinear meson or jet, i.e. $P^2 \ll Q^2$, in the final states
  [Bauer, Feming, Luke, Pirjol, Stewart (2001, 2002); Beneke, Chapovsky, Diehl, Feldmann (2003)]

- SCET allows for the separation of scales in a multiscale problem, allowing for an operator definition of objects in the factorization formulae derived in the $1/m_b$-expansion

- $B \to K^*\gamma$ in the SCET approach is worked out in the RG-improved perturbation theory (NLO). The main result is a Factorization formula, written as
  [Chay & Kim (2003); Grinstein, Grossman, Ligeti, Pirjol (2005); Bechher, Hill, Neubert (2005)]

$$\langle V\gamma\,|Q_i|\,\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + \frac{\sqrt{m_B} F f_{V_\perp}}{4}\int d\omega\, du\, \phi_+^B(\omega)\, \phi_\perp^V(u)\, t_i^{\rm II}(\omega, u)$$

- F and $f_{V_\perp}$ are meson decay constants; The SCET form factor $\zeta_{V_\perp}$ is related to the QCD form factor $T_1^{K^*}(0)$ through perturbative and power corrections

- In SCET, the perturbative hard-scattering kernels are the matching coefficients $\Delta_i C^A$ and $t_i^{\rm II}$

- For a given operator O_i, $t_i^{\rm II}$ is sub-factorized into the convolution of a hard-coefficient function with a universal jet function

$$t_i^{\rm II}(u,\omega) = \int_0^1 d\tau \Delta_i C^{B1}(\tau) j_\perp(\tau,u,\omega) \equiv \Delta_i C^{B1} \star j_\perp$$

- The coefficients $\Delta_i C^{B1}$ contain physics at the hard scale m_b, while the jet function $j_\perp$ contains physics at the hard-collinear scale $\sqrt{m_b \Lambda}$
- The hard coefficient is identified in a first step of matching $\mathbf{QCD} \to \mathbf{SCET_I}$, and the jet function in a second step of matching $\mathbf{SCET_I} \to \mathbf{SCET_{II}}$.
- For $B \to K^* \ell^+ \ell^-$ decay, in the region 1 GeV$^2 \leq q^2 \leq 8$ GeV^2, a factorization formula, valid in leading power in $1/m_b$, is also derived in SCET [AA, Kramer, Zhu (2006)]

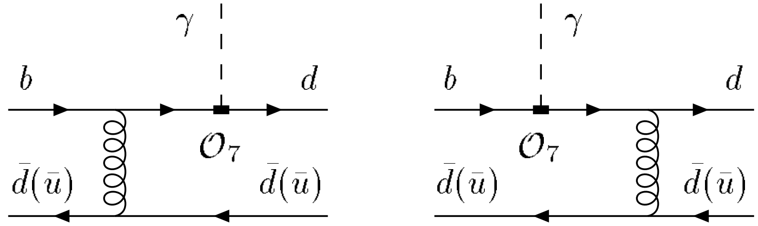
$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2)\, \xi_a(q^2) + \\ + \sum_\pm \int_0^\infty \frac{d\omega}{\omega} \phi_\pm^B(\omega) \int_0^1 du\, \phi_{K^*}^B(u)\, T_{a,\pm}^{II}(\omega,u,q^2)$$

where $a = \parallel, \perp$ denotes the polarization of the K^* meson.

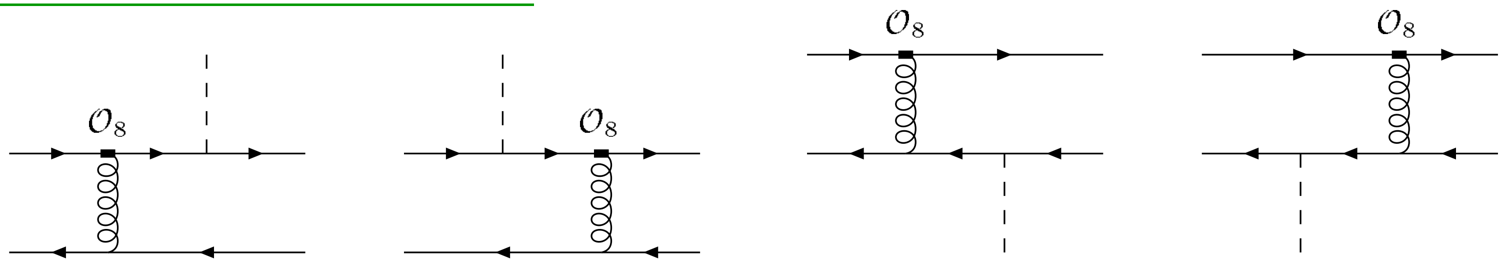
- SCET approach to these processes is the aim of this lecture

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

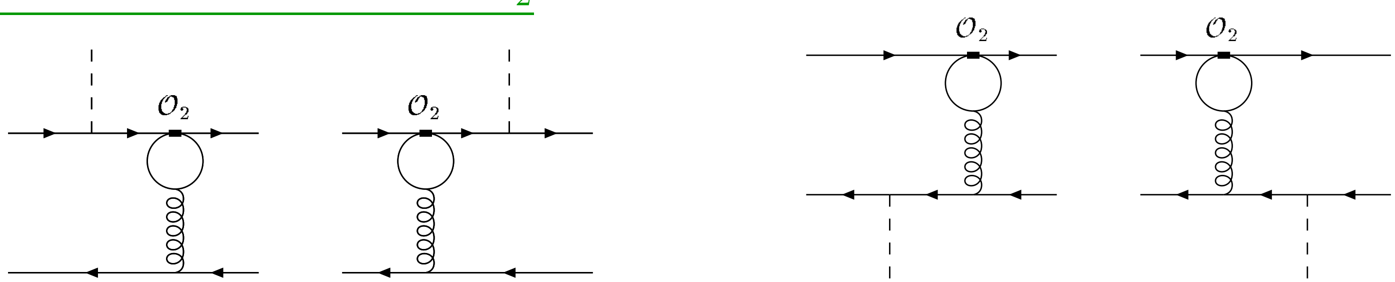
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Momentum regions in $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$ decays

- The connection between SCET and perturbative QCD is provided by the method of regions [Smirnov; Beneke, Smirnov]

- A number of different momentum regions appear in the analysis. To identify these, introduce two light-like vectors $n_\pm$

  $n^\mu = (1,0,0,1)$, $\bar{n}^\mu = (1,0,0,-1)$, satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$

- The outgoing K^* is assumed to be along the n_- direction, and define n_+ such that the velocity of the b quark is given by

$$v^\mu = n_-^\mu \frac{n_+ v}{2} + n_+^\mu \frac{n_- v}{2}$$

- To perform the expansion in $1/m_b$, we define the parameter $\Lambda^2 = (p_B - m_b v)^2$ and the dimensionless parameter $\lambda = \Lambda/m_b \ll 1$

- The regions are classified according to the scaling of their light-cone components with the expansion parameter λ

- Denoting the light-cone components of a generic four-vector p by $(n_+p, p_\perp, n_-p)$, the relevant momentum regions are

Perturbative

hard	$m_b(1,1,1)$
hard-collinear	$m_b(1,\sqrt{\lambda},\lambda)$

Non-perturbative

soft	$m_b(\lambda,\lambda,\lambda)$
collinear	$m_b(1,\lambda,\lambda^2)$
soft-collinear	$m_b(\lambda,\lambda^{3/2},\lambda^2)$

- In the effective theory, contributions from the perturbative regions are encoded in Wilson coefficients of operators built from fields representing the regions of lower virtuality
- Factorize the two perturbative scales m_b^2 and $m_b\Lambda$ using a two-step matching procedure $\mathrm{QCD} \to \mathrm{SCET_I} \to \mathrm{SCET_{II}}$

Effective Fields of SCET

- Hard mode (h)
  $P \sim E(1,1,1)$, integrated out in QCD $\to$ SCET$_I$

- Hard-collinear mode (hc)
  $P \sim E(\lambda, 1, \sqrt{\lambda})$, integrated out in SCET$_I$ $\to$ SCET$_{II}$,

$$\xi_{hc} \sim \sqrt{\lambda} \qquad A_{hc} \sim (\lambda, 1, \sqrt{\lambda})$$

- Power counting
  $\xi_{hc} = \frac{\not{n}\not{\bar{n}}}{4}\psi_{hc} \Longrightarrow$
  $\int d^4x\, e^{ip\cdot x} \langle 0|\, T\, \{\xi_{hc}(x)\bar{\xi}_{hc}(0)\}\, |0\rangle = \frac{\not{n}\not{\bar{n}}}{4}(\frac{i\not{p}}{p^2})\frac{\not{\bar{n}}\not{n}}{4} = \frac{i\bar{n}\cdot p}{p^2}\frac{\not{n}}{2}$
  $d^4x \sim 1/d^4p \sim \lambda^{-2}$, $p^2 \sim \lambda$, $\bar{n}\cdot p \sim 1 \Longrightarrow \xi_{hc} \sim \sqrt{\lambda}$

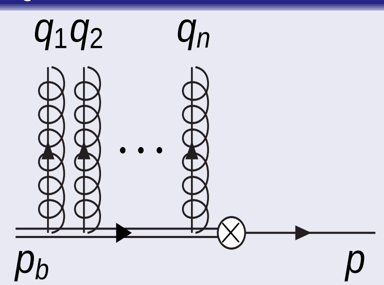
- Collinear mode (c)
  $P \sim E(\lambda^2, 1, \lambda)$, long-distance mode, $\quad \xi_c \sim \lambda \quad A_c \sim (\lambda^2, 1, \lambda)$

- Soft mode (s)
  $P \sim E(\lambda, \lambda, \lambda)$, long-distance mode, $\quad q_s \sim \lambda^{3/2} \quad A_s \sim (\lambda, \lambda, \lambda)$

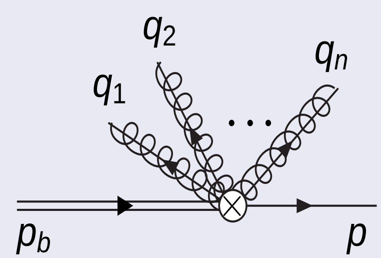
Defining $\chi_{\rm hc}$

Wilson lines

Full QCD



SCET$_I$



Hard-collinear Wilson line

$$W_{hc} = \mathrm{P}\,\exp\left(ig\int_{-\infty}^{y} ds\ \bar{n}\cdot A_{hc}(s\bar{n})\right)$$

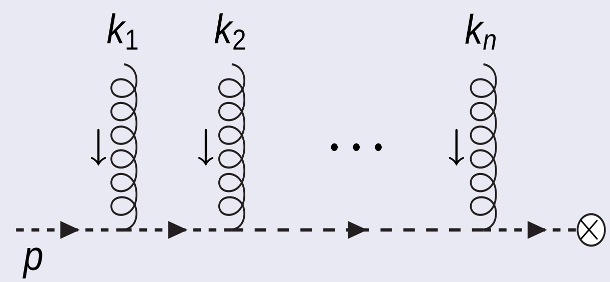
$$\bar{q}\Gamma b \quad \Longrightarrow \quad (\bar{\xi}_{hc} W_{hc})\Gamma' h_v$$

- $\mathcal{X}_{hc} = W_{hc}^{\dagger}\xi_{hc}$: collinear gauge invariance

Defining Y_s

Soft Wilson line

Soft-collinear interaction



Soft Wilson line

- The interaction between soft gluon and hard-collinear quarks or gluons can be resummed into

$$Y_s = \mathrm{P} \exp \left(ig \int_{-\infty}^{x} \mathrm{d}s \; n \cdot A_s(sn) \right)$$

- This property is crucial to prove the soft-collinear factorization
- $\mathcal{Q}_s = Y_s^\dagger q_s$: soft gauge invariance

SCET approach to $B \to K^*\gamma$ decay

- The objects of interest are the hadronic matrix elements $\langle K^*\gamma|Q_i|B\rangle$
- First matching step: the hard scale m_b^2 is integrated out by matching the operators Q_i onto a set of operators in $\mathrm{SCET_I}$
- For $B \to V\gamma$, the matching takes the form

$$Q_i \to \Delta_i C^A J^A + \Delta_i C^{B1} \star J^{B1} + \Delta_i C^{B2} \star J^{B2}$$

- The momentum-space Wilson coefficients depend only on quantities at the hard scale m_b^2. The exact form of the operators $J^{(i)}$ is:

$$\begin{aligned} J^A &= \left(\bar{\xi} W_{hc}\right) \not{\epsilon}_\perp (1-\gamma_5) h_v, \\ J^{B1} &= \left(\bar{\xi} W_{hc}\right) \not{\epsilon}_\perp \not{\mathcal{A}}_{hc\perp} (1+\gamma_5) h_v, \\ J^{B2} &= \left(\bar{\xi} W_{hc}\right) \not{\mathcal{A}}_{hc\perp} \not{\epsilon}_\perp (1+\gamma_5) h_v \end{aligned}$$

- The operators contain a hard-collinear quark field ξ, a composite object $\mathcal{A}_{hc}$, which in light-cone gauge is the hard-collinear gluon field, and W_{hc}, a Wilson line
- In SCET the b-quark field is treated as in HQET
- The B-type operators are power suppressed in $\mathrm{SCET_I}$, but contribute at the same order as the A-type operator upon the transition to $\mathrm{SCET_{II}}$

SCET factorization formula for $B \to K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma|Q_i|\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients

$$t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda}) \quad \text{(subfactorization)}$$

- Derivation of factorization in SCET
  1) QCD $\to$ SCET$_I$: Integrate out m_b; Defines vertex corrections $\Delta_i C^A = t_i^I$

$$Q_i \to \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + \ldots$$

  2) SCET$_I$ $\to$ SCET$_{II}$: Integrate out $\sqrt{m_b \Lambda_{\rm QCD}}$; Defines spectator corrections

$$J^{B1} \to j_\perp(\sqrt{m_b \Lambda_{\rm QCD}}) \otimes O^{B1,\text{SCET}_{II}}(\Lambda_{\rm QCD})$$

  3) Large logs in t_i^{II} resummed by solving RG equations

$$[\Delta_i C^{B1} \otimes j_\perp] \to [\Delta_i C^{B1}(\mu_h) \otimes U(\mu_h, \mu_{hc}) \otimes j_\perp(\mu_{hc})]$$

$B \to K^* \gamma$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contributions from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contribution from O_2 exact at NLO $O(\alpha_s)$ but only large-β_0 limit at $O(\alpha_s^2)$

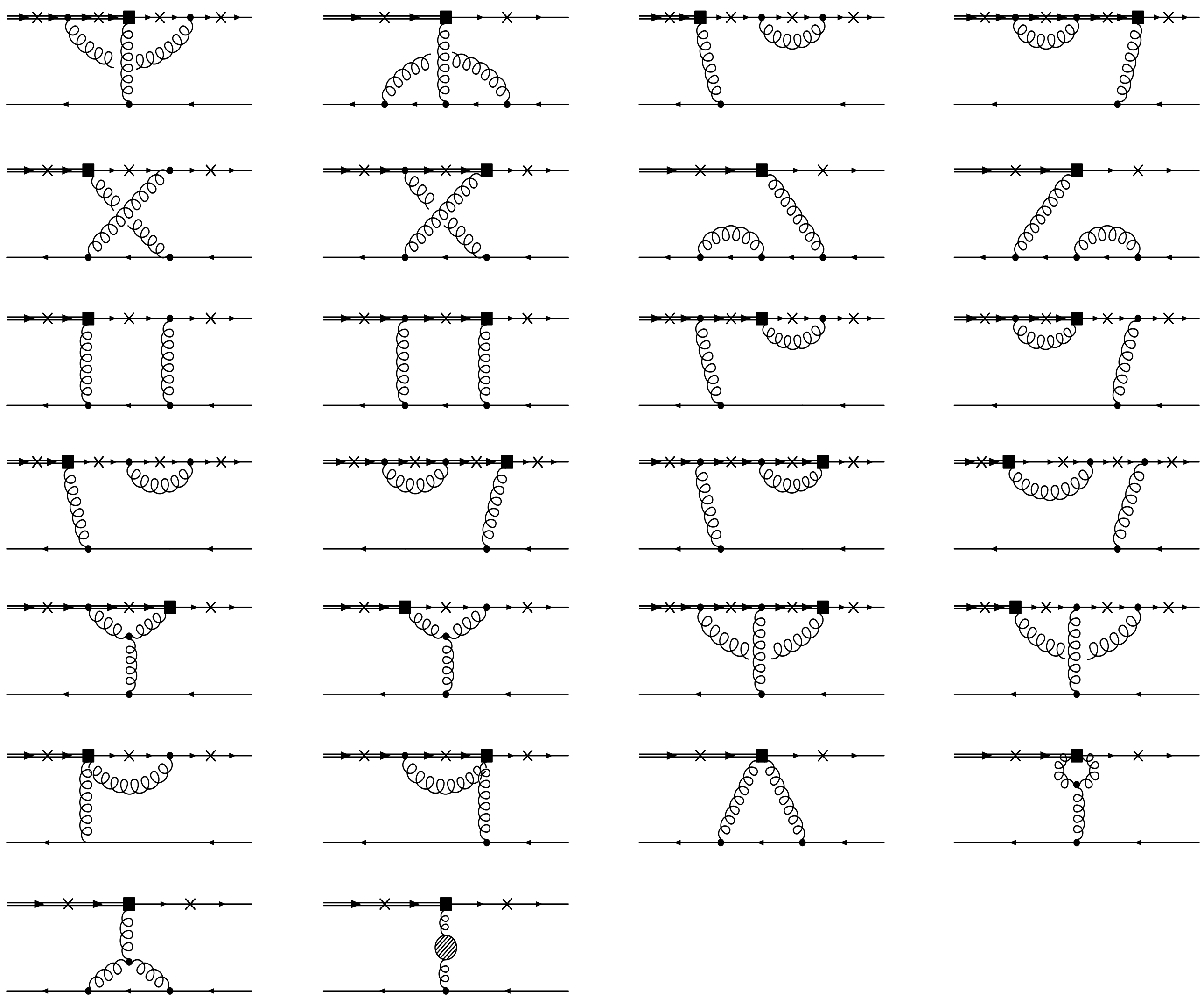
Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u, \omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- Status of $O(\alpha_s^2)$ Calculations
  - The one-loop jet-function $j_\perp^{(1)}$ known
    [Becher and Hill '04; Beneke and Yang '05]
  - The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known
    [Beneke, Kiyo, Yang '04; Becher and Hill '04]
  - The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known
    [Pecjak, Greub, AA '07]
  - $\Delta_i C^{B1(1)}$ $(i = 1, ..., 6)$ remain unknown (require two loops)

One-loop corrections to spectator scattering with O_8

Greub, Pecjak, AA (2008)



Estimates of BR($B \to K^* \gamma$) in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.2 ± 0.18 (HFAG 2012)];

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.33 ± 0.15 (HFAG 2012)];

$$\mathcal{B}(B_s \to \phi\gamma) = 4.3 \pm 1.1[\zeta_{\phi}] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+2.1}_{-1.8}$ (BELLE); 3.9 ± 0.5 (LHCb)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm exp}} = 1.10 \pm 0.35[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*0}\gamma)_{\rm exp}} = 1.00 \pm 0.32[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B_s \to \phi\gamma)_{\rm NNLO}}{\mathcal{B}(B_s \to \phi\gamma)_{\rm exp}} = 1.1 \pm 0.3[\text{theory}] \pm 0.1[\text{exp}]$
- Theory error is about 30%; dominantly from $\zeta_{V_\perp}$, m_c and λ_B; SM decay rates in good agreement with the data

Operators for $B \to K^*\ell^+\ell^-$ in SCET

QCD $\to$ SCET$_I$ matching

$$\sum_{i=1}^{10} C_i(\mu) Q_i(\mu) \to \sum_{i=1}^{4} \widetilde{C}_i^A J_i^A + \sum_{j=1}^{4} \widetilde{C}_j^B J_j^B + \widetilde{C}^C J^C$$

$$Q_7 = -\frac{g_{em}\overline{m}_b}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}\,, \qquad Q_{9,10} = \frac{\alpha_{em}}{2\pi}(\bar{s}b)_{V-A}(\bar{\ell}\gamma^\mu\gamma_5\ell)\,,$$

$$Q_{1,2} = (\bar{s}T^A c)_{V-A}(\bar{c}T^A b)_{V-A}\,, \qquad Q_{3,4} = 2\,(\bar{s}T^A b)_{V-A}\sum_q(\bar{q}\gamma^\mu T^A q)\,,$$

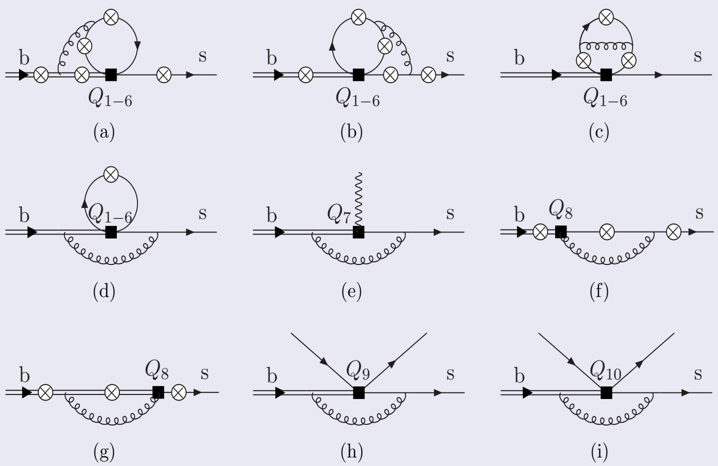
$$Q_{5,6} = 2\,\bar{s}\gamma_\mu\gamma_\nu\gamma_\rho(1-\gamma_5)T^A b\,\sum_q(\bar{q}\gamma^\mu\gamma^\nu\gamma^\rho T^A q)\,,$$

$$Q_8 = -\frac{g_s\overline{m}_b}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)T^A b\,G^A_{\mu\nu}\,,$$

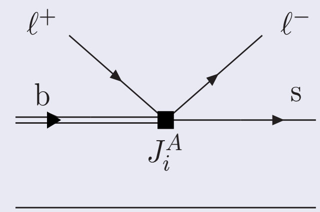
Full QCD → J_i^A in SCET

SCET operators J_i^A

Full QCD



SCET$_I$



$\propto \zeta_{\perp,\parallel}(q^2)$

Crossed circles: locations where the virtual photon is emitted

$$J_{1,3}^A = \bar{\mathfrak{X}}_{hc}(s\bar{n})(1+\gamma_5)\gamma_\perp^\mu h(0)\,\bar{\ell}\gamma_\mu\gamma_5\ell\,,\quad J_{2,4}^A = \bar{\mathfrak{X}}_{hc}(s\bar{n})(1+\gamma_5)\frac{n^\mu}{n\cdot v}h(0)\,\bar{\ell}\gamma_\mu\gamma_5\ell$$

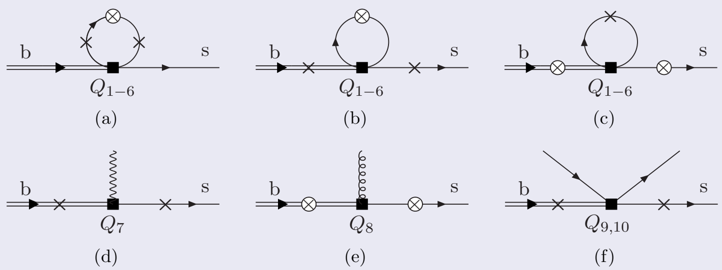
(a)-(d): calculated in $B \to X_s\ell^+\ell^-$ [Asatryan/Asatryan/Greub/Walker,2001]

(e)-(i): Form factor analysis [Bauer/Fleming/Pirjol/Stewart,2001, Beneke/Kiyo/Yang,2004]

Full QCD $\to J_i^B$ in SCET

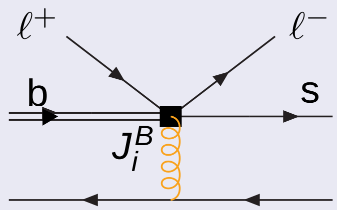
SCET operators J_i^B

Full QCD

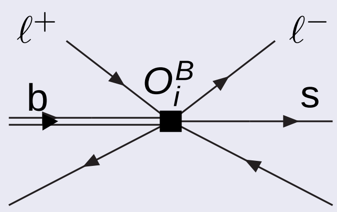


Crossed circles: locations where the virtual photon is emitted. The crosses mark the possible places where a gluon line may be attached.

SCET$_I$



SCET$_{II}$



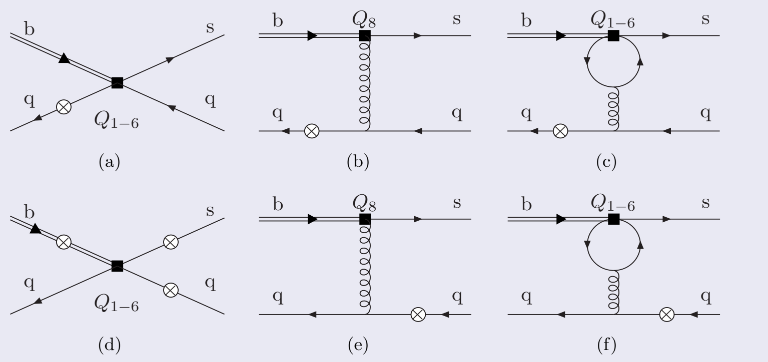
$$J^B_{1,3} = \bar{\chi}_{hc}(s\bar{n})(1+\gamma_5)\gamma_\perp^\mu \mathcal{A}_{hc\perp}(r\bar{n})h(0)\ \bar{\ell}\gamma_\mu\gamma_5\ell\ ,$$

$$J^B_{2,4} = \bar{\chi}_{hc}(s\bar{n})(1+\gamma_5)\mathcal{A}_{hc\perp}(r\bar{n})\frac{n^\mu}{n\cdot v}h(0)\ \bar{\ell}\gamma_\mu\gamma_5\ell$$

Full QCD $\rightarrow J_i^C$ in SCET

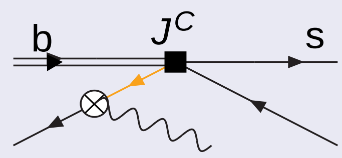
SCET operators J^C

Full QCD: photon from spectator quark

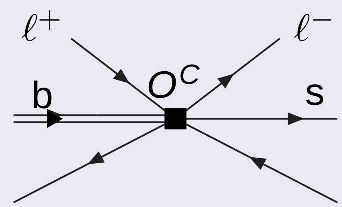


Crossed circles: locations where the virtual photon is emitted. The last 3 diagrams are $1/m_b$ suppressed.

SCET$_I$



SCET$_{II}$



$$J^C = \bar{\chi}_{hc}(s\bar{n})(1+\gamma_5)\frac{\not{\bar{n}}}{2}\chi_{hc}(r\bar{n})\ \bar{\chi}_{\overline{hc}}(an)(1+\gamma_5)\frac{\not{n}}{2}h(0)$$

Matrix elements of SCET Operators

[AA, Kramer, Zhu; EPJ (2006) 625]

- We define the matrix elements of the A-type operators in SCET$_1$

$$\langle M(p)|\bar{\mathfrak{X}}_{hc}\Gamma h|B(v)\rangle = -2E\zeta_M(E)tr[\overline{\mathcal{M}}_M(n)\Gamma\mathcal{M}_B(v)]$$

where the projection operators are

$$\mathcal{M}_B(v) = -\frac{1+\not{v}}{2}\gamma_5\ , \qquad \overline{\mathcal{M}}_{K^*_\perp}(n) = \not{\varepsilon}^*_\perp\frac{\not{\bar{n}}\not{n}}{4}\ , \qquad \overline{\mathcal{M}}_{K^*_\parallel}(n) = -\frac{\not{\bar{n}}\not{n}}{4}$$

with $\varepsilon^\mu_\perp$ being the polarization vector of the $K^*_\perp$ meson

- With this, the matrix elements of the SCET$_I$ currents J_i^A are

$$\langle K^*\ell^+\ell^-|J_1^A|B\rangle = -2E\zeta_\perp(g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu})\varepsilon^*_{\perp\nu}\bar{\ell}\gamma_\mu\ell\ , \ \langle K^*\ell^+\ell^-|J_2^A|B\rangle = -2E\zeta_\parallel\frac{n^\mu}{n\cdot v}\bar{\ell}\gamma_\mu\ell\ ,$$

$$\langle K^*\ell^+\ell^-|J_3^A|B\rangle = -2E\zeta_\perp(g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu})\varepsilon^*_{\perp\nu}\bar{\ell}\gamma_\mu\gamma_5\ell\ , \ \langle K^*\ell^+\ell^-|J_4^A|B\rangle = -2E\zeta_\parallel\frac{n^\mu}{n\cdot v}\bar{\ell}\gamma_\mu\gamma_5\ell$$

where $g_\perp^{\mu\nu} \equiv g^{\mu\nu} - (n^\mu\bar{n}^\nu + \bar{n}^\mu n^\nu)/2$ and $\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}v_\rho n_\sigma/(n\cdot v)$.

Matrix elements of SCET Operators

- For the matrix elements of the B-type operators in SCET$_{I1}$, we need to define first the LCDAs of the mesons:

$$\langle 0|\bar{\mathfrak{Q}}_s(tn)\Gamma\mathcal{H}_s(0)|B(v)\rangle = \frac{iF(\mu)}{2}\sqrt{m_B}\int_0^\infty d\omega\ e^{-i\omega n\cdot vt}$$
$$tr\left[\left(\phi_+^B(\omega,\mu) - \frac{\not{n}}{2n\cdot v}(\phi_-^B(\omega,\mu) - \phi_+^B(\omega,\mu))\right)\Gamma\mathcal{M}_B(v)\right] ,$$
$$\langle K^*(p)|\bar{\mathfrak{X}}_c(s\bar{n})\Gamma\frac{\not{\bar{n}}}{2}\mathfrak{X}_c(0)|0\rangle = \frac{if_{K^*}(\mu)}{4}\bar{n}\cdot p\ tr[\overline{\mathcal{M}}_{K^*}\Gamma]\int_0^1 du\ e^{ius\bar{n}\cdot p}\phi_{K^*}(u,\mu)$$

- Here two different K^*-distribution amplitudes ($\phi^{\parallel}_{K^*}(u,\mu)$ for $\Gamma = 1$ and $\phi^{\perp}_{K^*}(u,\mu)$ for $\Gamma = \gamma_\perp$) with their corresponding decay constants $f^{\parallel}_{K^*}$ and $f^{\perp}_{K^*}(\mu)$, respectively, are involved

- $F(\mu)$ is related to the B meson decay constant f_B up to higher orders in $1/m_b$

$$f_B\sqrt{m_B} = F(\mu)\left(1 + \frac{C_F\alpha_s(\mu)}{4\pi}\left(3\ln\frac{m_b}{\mu} - 2\right)\right)$$

- With the above LCDAs, the matrix elements of the operators O_i^B can be written as

$$\langle K^*\ell^+\ell^-|C_1^B O_1^B|B\rangle = -\frac{F(\mu)m_B^{3/2}}{4}(1-\hat{s})(g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu})\varepsilon^*_{\perp\nu}\bar{\ell}\gamma_\mu\ell \int_0^\infty \frac{d\omega}{\omega}\phi_+^B(\omega,\mu)$$

$$\times \int_0^1 du\ f_{K_\perp^*}(\mu)\phi_{K_\perp^*}(u,\mu)\int_0^1 dv \mathcal{J}_\perp(u,v,\ln\frac{m_b\omega(1-\hat{s})}{\mu^2},\mu)C_1^B(v,\mu)$$

$$\equiv -\frac{F(\mu)m_B^{3/2}}{4}(1-\hat{s})(g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu})\varepsilon^*_{\perp\nu}\bar{\ell}\gamma_\mu\ell\ \phi_+^B \otimes f_{K_\perp^*}\phi_{K_\perp^*} \otimes \mathcal{J}_\perp \otimes C_1^B\ ,$$

$$\langle K^*\ell^+\ell^-|C_2^B O_2^B|B\rangle = -\frac{F(\mu)m_B^{3/2}}{4}(1-\hat{s})\frac{n^\mu}{n\cdot v}\bar{\ell}\gamma_\mu\ell\ \phi_+^B \otimes f_{K_\parallel^*}\phi_{K_\parallel^*} \otimes \mathcal{J}_\parallel \otimes C_2^B$$

- Matrix element of $C_3^B O_3^B (C_4^B O_4^B)$ is be obtained by replacing the lepton current $\bar{\ell}\gamma_\mu\ell$ by $\bar{\ell}\gamma_\mu\gamma_5\ell$, and also replacing $C_1^B \to C_3^B$ ($C_2^B \to C_4^B$).
- Matrix element of O^C is obtained likewise, with the result

$$\langle K^*\ell^+\ell^-|D^C O^C|B\rangle = -\frac{F(\mu)m_B^{3/2}}{4}(1-\hat{s})\frac{\bar{n}^\mu}{\bar{n}\cdot v}\bar{\ell}\gamma_\mu\ell\ \frac{\omega\phi_-^B}{\omega - q^2/m_b - i\epsilon} \otimes f_{K_\parallel^*}\phi_{K_\parallel^*} \otimes \widehat{D}^C$$

- Since $\phi_-^B(\omega)$ does not vanish as ω approaches zero, the integral $\int d\omega\ \phi_-^B(\omega)/(\omega - q^2/m_b)$ would be divergent if $q^2 \to 0$. We restrict the invariant mass of the lepton pair, say $q^2 \geq 1$ GeV^2.

Leading order in $1/m_b$ and all orders in α_s

[AA, Kramer, Zhu; EPJ (2006) 625]

The factorization formula in SCET

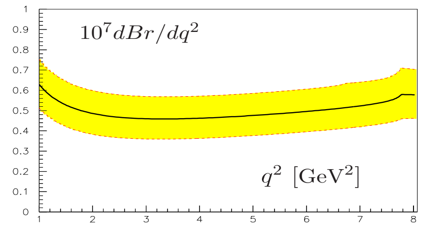
$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2)\, \xi_a(q^2) + + \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega}\, \phi_\pm^B(\omega) \int_0^1 du\, \phi_{K^*}^B(u)\, T_{a,\pm}^{II}(\omega, u, q^2)$$

where $a = \parallel, \perp$ denotes the polarization of the K^* meson.

- formally coincides with the formula in QCD Factorization [Beneke/Feldmann/Seidel 2001], but valid to all orders of α_S,
- for T^{II}, the logarithms are summed from $\mu = m_b$ to $\sqrt{m_b \Lambda_h}$,
- compared with BFS, the definition of $\xi_{\parallel,\perp}$ is also different here.

Comparison with Data

Numerical results

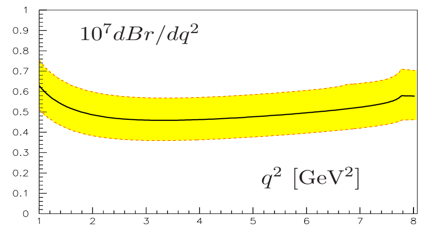


Theor. vs. Belle

$$Br|_{q^2 \in [4,8]\ \text{GeV}^2} = (1.94^{+0.44}_{-0.40}) \times 10^{-7}$$
$$= (4.8^{+1.4}_{-1.2}|_{\text{stat}} \pm 0.3|_{\text{syst}} \pm 0.3|_{\text{model}}) \times 10^{-7}$$

Comparison with experiments

Numerical results



Form factor determination

LCSRs $\zeta_{\|}(0) = 0.40 \pm 0.05$, $\zeta_{\perp}(0) = 0.40 \pm 0.04$, their q^2 dependencies

LCSRs + $B \to K^* \gamma$ $\zeta_{\perp}(0) = 0.32 \pm 0.02$

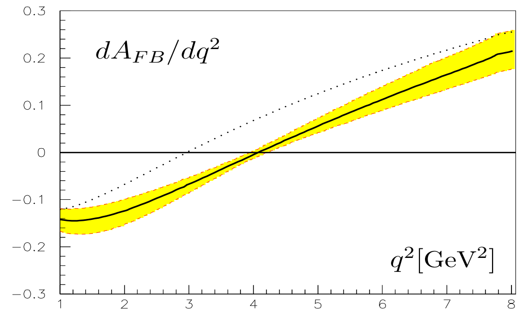
Theor. vs. BaBar

$$Br|_{q^2 \in [1,7]\ \mathrm{GeV}^2} = (2.92^{+0.57}_{-0.50}|_{\zeta_{\|}}\ {}^{+0.30}_{-0.28}|_{\mathrm{CKM}}\ {}^{+0.18}_{-0.20}) \times 10^{-7}$$

$$Br|_{q^2 \in [0.1,8.4]\ \mathrm{GeV}^2} = (2.7^{+1.2}_{-1.0}|_{\mathrm{stat}} \pm 0.5|_{\mathrm{syst}}) \times 10^{-7}$$

Reduction of Scale Uncertainty in SCET

Forward-backward asymmetry



$A_{FB}(q_0^2) = 0$ free of hadronic uncertainties [Burdman1998, Ali et al., 2000]

$q_0^2 = (4.07^{+0.16}_{-0.13})$ GeV^2 with $\Delta(q_0^2)_{\text{scale}} = ^{+0.08}_{-0.05}$ GeV^2

QCD-F [Beneke/Feldmann/Seidel 2001]

$q_0^2 = (4.39^{+0.38}_{-0.35})$ GeV^2 with $\Delta(q_0^2)_{\text{scale}} = \pm 0.25$ GeV^2

Experiment: $q_0^2(B^0 \to K^{*0}\mu^+\mu^-) = 4.9^{+1.3}_{-1.1}$ GeV^2

- In agreement with the SM; however, current precision on q_0^2 is only 25%. Will improve at the upgraded LHCb and Super-B factories

Summary

- Thanks to dedicated experiments and progress in theoretical techniques (Pert. QCD, Lattice-QCD, QCD Sum Rules, Heavy quark Expansion, SCET) Rare B-Decays are under quantitative control, but the precision varies between (10 - 30)%
- From the CKM Phenomenology, there is added value in precisely measuring Rare B-Decays and in improving the SM theoretical accuracy, as this would overconstrain $|V_{ts}|$ and $|V_{td}|$
- Rare B-Decays provide invaluable constraints on Beyond-the-SM Physics; theoretical interest in their dedicated studies remains high and they may turn out to be the harbinger of BSM physics, as they probe very high mass scales
- A new chapter on precision B_s-meson physics has opened at the LHC, in particular, by the LHCb, resolving some open issues and testing SM at an unprecedented rate, of which $B_s^0 \to \mu^+\mu^-$ is a shining example
- We look forward to new data from the ongoing and planned experiments at the LHC and the Super-B factories