Rare Meson Decays in theories beyond the Standard Model

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1. Rare Meson Decays mediated by massive Majorana neutrinos

The width of the rare decay:

$$\Gamma_{\ell\ell'} = \left(1 - \frac{1}{2}\delta_{\ell\ell'}\right) \int (2\pi)^4 \delta^{(4)}\left(P' + p + p' - P\right) \frac{|A_t + A_b|^2}{2m_M} \frac{d^3P' d^3p\, d^3p'}{2^3 (2\pi)^9 P'^0 p^0 p'^0} \,, \tag{1}$$

here A_t (A_b) is tree (box)– diagram.

$$A_i = \frac{1}{(2\pi)^8} \int d^4q d^4q' H^{(i)}_{\mu\nu} L_i^{\mu\nu} \;, i = t,\ b; \tag{2}$$

$L_i^{\mu\nu}$ — lepton tensor, $H^{(i)}_{\mu\nu}$ — hadron tensor.

The lepton tensor

$$L_i^{\mu\nu} = \frac{g^4}{4} \frac{g^{\mu\alpha}}{p_i^2 - m_W^2} \frac{g^{\nu\beta}}{p_i'^2 - m_W^2} \sum_N U_{\ell N} U_{\ell' N} \eta_N m_N$$
$$\times \left(\overline{v^c}(p) \left[\frac{\gamma_\alpha\gamma_\beta}{(p_i - p)^2 - m_N^2} + \frac{\gamma_\beta\gamma_\alpha}{(p_i - p')^2 - m_N^2}\right] \frac{1+\gamma^5}{2} v(p')\right), \tag{3}$$

$\eta_N = \pm 1$ are the relative CP-phases,

$$p_t = P,\ p_t' = P';\ p_b = \frac{1}{2}(P - P') + q' - q,\ p_b' = \frac{1}{2}(P - P') - q' + q. \tag{4}$$

The hadron tensor

$$H^{(t)}_{\mu\nu} = \mathrm{Tr}\left\{\chi_P(q) V_{12}\gamma_\mu \frac{1+\gamma^5}{2}\right\} \mathrm{Tr}\left\{\overline{\chi}_{P'}(q') V_{43}\gamma_\nu \frac{1+\gamma^5}{2}\right\},$$
$$H^{(b)}_{\mu\nu} = \mathrm{Tr}\left\{\chi_P(q) V_{13}\gamma_\mu \frac{1+\gamma^5}{2} \overline{\chi}_{P'}(q') V_{42}\gamma_\nu \frac{1+\gamma^5}{2}\right\}. \tag{5}$$

We take into account $m_M \ll m_W$.

$$A_t = -\frac{1}{4} f_M f_{M'} V_{12} V_{43} P_\mu P'_\nu L_t^{\mu\nu}. \tag{6}$$

$$A_b = 2V_{13}V_{42}\delta_M\delta_{M'}(P_\mu P'_\nu + P_\nu P'_\mu - g_{\mu\nu} P\cdot P' + i\varepsilon_{\mu\nu\alpha\beta}P^\alpha P'^\beta) \times$$
$$\times \int \tfrac{d^4q}{(2\pi)^4}\tfrac{d^4q'}{(2\pi)^4}\varphi_P(q)\,\varphi_{P'}(q')\,L_b^{\mu\nu}. \tag{7}$$

The Bethe-Salpeter formalism

Bethe-Salpeter amplitudes are used to describe mesons as bound states of a quark and an antiquark.

$$\chi_P(x_1, x_2) = -\tfrac{i}{\sqrt{N_c}}\left\langle 0\left|T\{q_1^a(x_1)\bar{q}_{2a}(x_2)\}\right| M(P)\right\rangle = e^{-iP\cdot X}\chi_P(x),$$

here X and $x = x_1 - x_2$ are respectively the centre-of-mass and the relative coordinates; $a = 1,\ 2,\ 3$ — color index ($N_c = 3$).

$$\chi_P(q) = \int d^4x\, e^{iq\cdot x}\chi_P(x) = \gamma^5(1-\delta_M\hat{P})\varphi_P(q)\,\phi_G\ . \tag{8}$$

Here

$$\delta_M = (m_1 + m_2)/m_M^2, \tag{9}$$

m_M is the mass of the meson M having a quark q_1 and an antiquark $\bar{q}_2$ with masses m_1 , m_2 and relative 4-momentum $q = (p_1 - p_2)/2$, $P = p_1 + p_2$ is the total 4-momentum of the meson, $\hat{P} = \gamma^\mu P_\mu$; $\varphi_P(q)$ is model dependent function; ϕ_G — $SU(N_f)\times SU(N_c)$-group factor.

$$\varphi_P(q) = \frac{4\pi}{\alpha^2}\left[1-(m_M\delta_M)^2\right]^{-1/2}\exp\left\{-\frac{1}{2\alpha^2}\left[2\left(\frac{P\cdot q}{m_M}\right)^2 - q^2\right]\right\}, \tag{10}$$

here α is a parameter of the model.

From the definition of the decay constant of the meson:

$$if_M P^\mu = \left\langle 0\left|\overline{q}_{2a}(0)\,\gamma^\mu\gamma^5 q_1^a(0)\right| M(P)\right\rangle = -i\sqrt{N_c}\,\mathrm{Tr}[\gamma^\mu\gamma^5\chi_P(x=0)].$$

Then

$$f_M = 4\sqrt{N_c}\,\delta_M\int\frac{d^4q}{(2\pi)^4}\,\varphi_P(q)\,. \tag{11}$$

Heavy neutrinos $m_N \gg m_M$

The total amplitude in this case is MODEL INDEPENDENT:

$$\begin{aligned} A = A_t + A_b = 4K_V f_M f_{M'}(P\cdot P')L(p,p')\ , \\ K_V = V_{12}V_{43} + \tfrac{1}{N_c}V_{13}V_{42}\ . \end{aligned} \tag{12}$$

The total decay widths:

$$\Gamma_{\ell\ell'} = \frac{G_F^4 m_M^7}{128\pi^3} f_M^2 f_{M'}^2 \left|K_V\right|^2 \left\langle m_{\ell\ell'}^{-1}\right\rangle^2 \Phi_{\ell\ell'}. \tag{13}$$

Here $\Phi_{\ell\ell'}$ is reduced phase space integral and *the effective inverse Majorana masses*:

$$\left\langle m_{\ell\ell'}^{-1}\right\rangle = \left|\sum_N U_{\ell N}U_{\ell' N}\eta_N\frac{1}{m_N}\right|. \tag{14}$$

For identical leptons ($\ell' = \ell$)

$$\Phi_{\ell\ell} = \int_{4z_0}^{z_1} dz \, (z - 2z_0) \left[\left(1 - \frac{4z_0}{z} \right) (z_1 - z) \, (z_2 - z) \right]^{1/2} (1 + z_3 - z)^2 \, . \tag{15}$$

For the case of distinct leptons, assuming $m_{\ell'}/m_\ell \ll 1$:

$$\Phi_{\ell\ell'} = 2 \int_{z_0}^{z_1} \frac{dz}{z} \, (z - z_0)^2 \left[(z_1 - z) \, (z_2 - z) \right]^{1/2} (1 + z_3 - z)^2 \, , \tag{16}$$

here the parameters are defined as

$$z_0 = \left(\frac{m_\ell}{m_M} \right)^2 , \; z_1 = \left(1 - \frac{m_{M'}}{m_M} \right)^2 , \; z_2 = \left(1 + \frac{m_{M'}}{m_M} \right)^2 ,$$
$$z_3 = \tfrac{1}{2} (z_1 + z_2 - 2) = \left(\frac{m_{M'}}{m_M} \right)^2 , \tag{17}$$

$$z = (P - P')^2 / m_M^2 .$$

The branching ratios:

$$\mathrm{B}_{\ell\ell'} = \frac{\Gamma \left(M^+ \to M'^- \ell^+ \ell'^+ \right)}{\Gamma \left(M^+ \to all \right)} \, . \tag{18}$$

$$C_{\ell\ell'} = \frac{\mathrm{B}_{\ell\ell'}}{\left\langle m_{\ell\ell'}^{-1} \right\rangle^2} \, . \tag{19}$$

Using bounds on matrix elements (14) from the others processes one can obtain indirect bounds on the branching ratios.

The bound from $0\nu\beta\beta$-decay of ^{76}Ge nucleus:

$$\left\langle m_{ee}^{-1} \right\rangle < (1.2 \cdot 10^8 \text{ GeV})^{-1} \, . \tag{20}$$

Also there are bounds on other mixing parameters:

$$\sum_N |U_{eN}|^2 < 6.6 \cdot 10^{-3}, \;\; \sum_N |U_{\mu N}|^2 < 6.0 \cdot 10^{-3} \, . \tag{21}$$

Таблица 1: Experimental and indirect bounds on the branching ratios $\mathrm{B}_{\ell\ell'}$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by Majorana neutrinos ($m_N \gg m_M$)

Rare decay	Exp. upper bounds on $\mathrm{B}_{\ell\ell'}$	$C_{\ell\ell'}$ [Mev^2]	Ind. bounds on $\mathrm{B}_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	$6.4 \cdot 10^{-10}$	$8.5 \cdot 10^{-10}$	$5.9 \cdot 10^{-32}$
$K^+ \to \pi^- \mu^+ \mu^+$	$3.0 \cdot 10^{-9}$	$2.4 \cdot 10^{-10}$	$1.1 \cdot 10^{-24}$
$K^+ \to \pi^- e^+ \mu^+$	$5.0 \cdot 10^{-10}$	$1.0 \cdot 10^{-9}$	$5.1 \cdot 10^{-24}$
$D^+ \to K^- e^+ e^+$	$1.2 \cdot 10^{-4}$	$2.2 \cdot 10^{-9}$	$1.5 \cdot 10^{-31}$
$D^+ \to K^- \mu^+ \mu^+$	$1.3 \cdot 10^{-5}$	$2.0 \cdot 10^{-9}$	$8.9 \cdot 10^{-24}$
$D^+ \to K^- e^+ \mu^+$	$1.3 \cdot 10^{-4}$	$4.2 \cdot 10^{-9}$	$2.1 \cdot 10^{-23}$

From the heavy neutral Majorana lepton detecting experiments:

$$m_N > 90.7 \text{ GeV}. \tag{22}$$

Assuming one heavy neutrino scenario:

$$\left\langle m_{\mu\mu}^{-1} \right\rangle < \left(1.5 \cdot 10^4 \text{ GeV}\right)^{-1}, \quad \left\langle m_{e\mu}^{-1} \right\rangle < \left(1.4 \cdot 10^4 \text{ GeV}\right)^{-1}. \tag{23}$$

Light neutrinos $m_N \ll m_\ell,\ m_{\ell'}$

The total decay widths:

$$\Gamma = \Gamma_t + \Gamma_b + \Gamma_{tb}. \tag{24}$$

The effective Majorana masses:

$$\langle m_{\ell\ell'} \rangle = \left| \sum_N U_{\ell N} U_{\ell' N} \eta_N m_N \right|. \tag{25}$$

$$\Gamma_t = \frac{G_F^4 m_M^3}{16\pi^3} f_M^2 f_{M'}^2 \left|V_{12}V_{43}\right|^2 \left\langle m_{\ell\ell'}\right\rangle^2 \phi_{\ell\ell'}, \tag{26}$$

$\phi_{\ell\ell'}$ — reduced phase space integral ($m_\ell/m_M \to\ 0$ and $m_{\ell'}/m_M \to 0$)

$$z_3 = \left(m_{M'}/m_M\right)^2$$

$$\phi_{\ell\ell'} \simeq \left(1 - \tfrac{1}{2}\delta_{\ell\ell'}\right)\varphi\left(z_3\right); \quad \varphi\left(z_3\right) = \int_0^{z_1} dz z \left[\left(z_1 - z\right)\left(z_2 - z\right)\right]^{1/2} =$$
$$= \left(1 - z_3\right)\left[2z_3 + \tfrac{1}{6}\left(1 - z_3\right)^2\right] + z_3\left(1 + z_3\right)\ln z_3. \tag{27}$$

As in the case of heavy neutrinos :

$$c_{\ell\ell'} = \frac{\mathrm{B}_{\ell\ell'}}{\left\langle m_{\ell\ell'}\right\rangle^2} = c_t + c_b + c_{tb}. \tag{28}$$

In this case the b–amplitude is MODEL DEPENDENT:

$$\Gamma_b = \frac{G_F^4}{2\pi^8}\left(V_{13}V_{24}\right)^2 m_M^3 m_{M'}^4 \left(\delta_M \delta_{M'}\right)^2 \left(\frac{a\alpha^2}{1+a}\right)^2 \left\langle m_{\ell\ell}\right\rangle^2 \phi_b. \tag{29}$$

Here reduced phase space integral ($\ell = \ell'$):

$$\phi_b = \int\limits_0^\pi d\theta \int\limits_0^{y_0} dy \int\limits_{-1}^1 dx \int\limits_{-1}^1 dt \int\limits_0^1 du \int\limits_0^1 du' \left[(1+y^2)\tfrac{vv'}{uu'}\left(1 - \tfrac{4z_0}{z}\right)\right]^{1/2} \times$$
$$\times y^2(z - 2z_0)[(1 + ss')\cos\psi - (s - s')\sin\psi] \times$$
$$\times \exp[-A(z - 4z_0)(F + F')]\ , \tag{30}$$

$$s = \left(\tfrac{u}{1-u}\right)^{1/2},\ v = \left(1 + ab^2uy^2\right),\ z = 1 + z_3 - 2\left[z_3(1 + y^2)\right]^{1/2};$$

$$F = bu + (1-bu)(1-v)\left[xt + \sqrt{(1-x^2)(1-t^2)}\cos\theta\right]^2,$$
$$\psi = A(z-4z_0)(H-H'),$$
$$H = b\sqrt{u(1-u)}\left[1-b^{-2}(1-v)\right], \tag{31}$$

$\mathrm{A} = \left(\frac{m_M}{4\alpha}\right)^2,\ a = \left(\frac{\alpha'}{\alpha}\right)^2,\ b = \frac{2}{1+a},\ y_0 = \left[\frac{(1+z_3-4z_0)^2}{4z_3} - 1\right]^{1/2}$.(31)
Dotted values correspond to the final meson.

Interference ($\ell = \ell'$):

$$\Gamma_{tb} = -\frac{G_F^4}{4\pi^6} V_{12}V_{43}V_{13}V_{24} m_M^2 m_{M'}^3 f_M f_{M'} \delta_M \delta_{M'} \frac{a\alpha^2}{1+a} \langle m_{\ell\ell}\rangle^2 \phi_{tb}\ , \tag{32}$$

here

$$\phi_{tb} = \int\limits_0^\pi d\theta \int\limits_0^{y_0} dy \int\limits_{-1}^1 dx \int\limits_{-1}^1 dt \int\limits_0^1 du \left[\frac{v}{u}\left(1-\frac{4z_0}{z}\right)\right]^{1/2} y^2 \left(s\cos\psi + \sin\psi\right) \times$$
$$\times \left[z + \frac{w(\bar z(z-2z_0)-zw)}{w^2 - z_3(z-4z_0)y^2T^2}\right] \exp\left[-A(z-4z_0)F\right]\ , \tag{33}$$

$$\bar z = \frac{1}{2}(1+z_3-z),\ w = z_3 + z_0 + \bar z\sqrt{z},$$
$$T = xt + \sqrt{(1-x^2)(1-t^2)}\cos\theta\ .$$

The numerical calculations of multidimensional phase integrals were carried out with using the program VEGAS based on Monte–Carlo mechanism.

Bounds on the branching ratios $\mathrm{B}_{\ell\ell'}$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$ mediated by Majorana neutrinos ($m_N \ll m_\ell$)

Rare decay	c_t [MeV^{-2}]	c_b [Mev^{-2}]	c_{tb} [Mev^{-2}]
$K^+ \to \pi^- e^+ e^+$	$5.3 \cdot 10^{-20}$	$3.6 \cdot 10^{-22}$	$-8.8 \cdot 10^{-21}$
$K^+ \to \pi^- \mu^+ \mu^+$	$1.4 \cdot 10^{-20}$	$1.2 \cdot 10^{-22}$	$-2.1 \cdot 10^{-21}$
$K^+ \to \pi^- e^+ \mu^+$	$1.1 \cdot 10^{-19}$	$7.2 \cdot 10^{-22}$	$-1.8 \cdot 10^{-20}$
$D^+ \to K^- e^+ e^+$	$4.1 \cdot 10^{-23}$	$3.6 \cdot 10^{-21}$	$8.5 \cdot 10^{-22}$
$D^+ \to K^- \mu^+ \mu^+$	$4.0 \cdot 10^{-23}$	$3.3 \cdot 10^{-21}$	$7.6 \cdot 10^{-22}$
$D^+ \to K^- e^+ \mu^+$	$8.2 \cdot 10^{-23}$	$7.3 \cdot 10^{-21}$	$1.7 \cdot 10^{-21}$

Rare decay	$c_{\ell\ell'}$ [MeV^{-2}]	Ind. bounds on$\mathrm{B}_{\ell\ell'}$
$K^+ \to \pi^- e^+ e^+$	$4.4 \cdot 10^{-20}$	$2.3 \cdot 10^{-33}$
$K^+ \to \pi^- \mu^+ \mu^+$	$1.2 \cdot 10^{-20}$	$6.2 \cdot 10^{-34}$
$K^+ \to \pi^- e^+ \mu^+$	$8.8 \cdot 10^{-20}$	$2.0 \cdot 10^{-33}$
$D^+ \to K^- e^+ e^+$	$4.5 \cdot 10^{-21}$	$2.4 \cdot 10^{-34}$
$D^+ \to K^- \mu^+ \mu^+$	$4.1 \cdot 10^{-21}$	$2.2 \cdot 10^{-34}$
$D^+ \to K^- e^+ \mu^+$	$9.1 \cdot 10^{-21}$	$2.0 \cdot 10^{-34}$

From the cosmological data and neutrino mixing experiments ($m_1 < m_2 < m_3$):

$$m_3 < 0.23 \text{ eV}. \tag{34}$$

Taking into account $\sum_i |U_{\ell i}|^2 = 1$:

$$\langle m_{\ell\ell} \rangle < 0.23 \text{ eV}. \tag{35}$$

The best fit for mixing matrix from neutrino mixing data:

$$U_{\rm bf} = \begin{pmatrix} 0.84 & 0.55 & 0.00 \\ -0.39 & 0.59 & 0.71 \\ 0.39 & -0.59 & 0.71 \end{pmatrix},$$

one can obtain bounds on non-diagonal elements:

$$\langle m_{e\mu} \rangle < 0.15 \text{ eV}. \tag{36}$$

2. Rare Meson Decays in R-parity-violating SUSY theories

R-parity (R_p) is a discrete, multiplicative symmetry defined as $R_p = (-1)^{3B+L+2S}$, where S, B and L are the spin, the baryon and the lepton quantum number.

The SM fields, including additional Higgs boson fields appearing in the extended gauge models, have $R_p = +1$ while their superpartners have $R_p = -1$. This symmetry has been imposed on the minimal supersymmetric standard model to ensure B and L number conservation. However, SUSY doesn't require R_p conservation.

The most general gauge invariant form of the superpotential in minimal SUSY SM is

$$W = W_{R_p} + W_{RPV}.$$

The R_p violating part of the superpotential can be written

$$W_{RPV} = \lambda_{ijk} L_i L_j \bar{L}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{Q}_j \bar{D}_k,$$

here indices i, j, k denote generations, L, Q are lepton and quark doublet superfields and $\bar{E}, \bar{U}, \bar{D}$ are lepton and up, down quark singlet superfields. All λ are the coupling constants.

The second term yields several contributions to $K^+ \longrightarrow \pi^- \ell^+ \ell'^+$.

$$\mathcal{L} = \mathcal{L}_{RPV} + \mathcal{L}_{\tilde{g}} + \mathcal{L}_{\chi}.$$

The lepton number violating part of the Lagrangian for one generation has the form:

$$\mathcal{L}_{RPV} = -\lambda'_{111}\Bigg[\,(\bar{u}_L \;\; \bar{d}_R)\;\cdot\begin{pmatrix} e^c_R \\ -\nu^c_R \end{pmatrix}\tilde{d}_R + \;(\bar{e}_L \;\; \bar{\nu}_L)\, d_R \cdot \begin{pmatrix} \tilde{u}^*_L \\ -\tilde{d}^*_L \end{pmatrix} +$$

$$+ \,(\bar{u}_L \;\; \bar{d}_L)\, d_R \cdot \begin{pmatrix} \tilde{e}^*_L \\ -\tilde{\nu}^*_L \end{pmatrix} + h.c.\Bigg]$$

The Lagrangian terms corresponding to gluino $\mathcal{L}_{\tilde{g}}$ and neutralino $\mathcal{L}_\chi$ interactions with fermions $\psi = \{u, d, e\}, q = \{u, d\}$ and their superpartners $\tilde{\psi} = \{\tilde{u}, \tilde{d}, \tilde{e}\}, \tilde{q} = \{\tilde{u}, \tilde{d}\}$ are

$$\mathcal{L}_{\tilde{g}} = -\sqrt{2} g_3 \frac{\lambda^{(a)}_{\alpha\beta}}{2}(\bar{q}^a_L \tilde{g} \tilde{q}^\beta_L - \bar{q}^a_R \tilde{g} \tilde{q}^\beta_R) + h.c.,$$

$$\mathcal{L}_\chi = \sqrt{2} g_2 \sum_{i=1}^{4}(\epsilon_{Li}(\psi)\bar{\psi}_L \chi_i \tilde{\psi}_L + \epsilon_{Ri}(\psi)\bar{\psi}_R \chi_i \tilde{\psi}_R) + h.c.$$

Here $\lambda^{(a)}$ are 3×3 Gell-Mann matrices $(a = 1, ..., 8)$.

$$\Gamma_t = (1 - \frac{1}{2}\delta_{\ell\ell'})\frac{f_M^2 f_{M'}^2}{2^8\pi^3 M\delta_M^2\delta_{M'}^2}(\lambda'_{111}\lambda'_{211}g_2^2\sum_{i=1}^{4}\frac{\epsilon_{Li}(\ell)\epsilon_{Li}(\ell')}{m^2_{\tilde{\ell}_L}m^2_{\tilde{\ell}'_L}m_{\chi_i}})^2\Phi_{\ell\ell'}, \tag{37}$$

$$\Gamma_b = \frac{\Gamma_t}{4N_c^2},$$

$$\Gamma_{tb} = -\frac{\Gamma_t}{N_c}.$$

Rare decay	$\Phi_{\ell\ell'}$ MeV^2
$K^+ \to \pi^- e^+ e^+$	$0.872 \cdot 10^9$
$K^+ \to \pi^- \mu^+ \mu^+$	$0.306 \cdot 10^9$
$K^+ \to \pi^- e^+ \mu^+$	$0.562 \cdot 10^9$
$D^+ \to K^- e^+ e^+$	$0.199 \cdot 10^{12}$
$D^+ \to K^- \mu^+ \mu^+$	$0.187 \cdot 10^{12}$
$D^+ \to K^- e^+ \mu^+$	$0.193 \cdot 10^{12}$

From Laurence S. Littenberg and Robert Shrock paper (hep-ph/0005285) the estimate of the branching ratio:

$$B(K^+ \to \pi^-\mu^+\mu^+)_{RPV} \leq 10^{-16}(\lambda'_{212}\lambda'_{211})^2(\frac{200GeV}{m_{SUSY}})^{10}.$$

Bounds on RPV coupling are model–dependent, but typical current upper bounds on $\lambda'_{211}, \lambda'_{212}, \lambda'_{111}, \lambda'_{211} \leq O(0.1)$ and $m_{SUSY} \simeq 250$ GeV. Using these inputs, we find that these R-parity violating contributions could be much larger then those from massive neutrinos.

$$B(K^+ \to \pi^-\mu^+\mu^+)_{RPV} \leq 10^{-21}.$$