Two-Body Nonleptonic B-Meson Decays – II (Analysis of Selected Modes)

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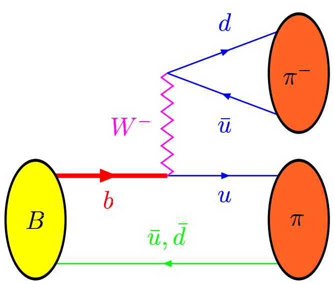
$B \to \pi\pi$ Amplitude Topologies

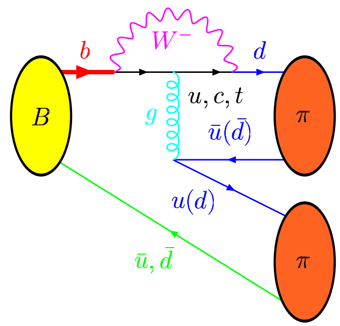
- Dominant topologies contributed within the Standard Model

Tree ($\mathcal{T}$)



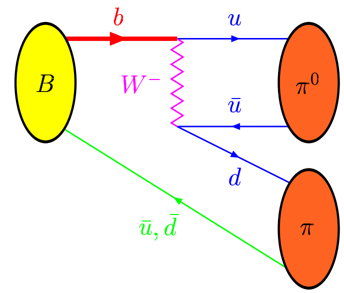
$B^- \to \pi^-\pi^0$
$\bar{B}^0 \to \pi^+\pi^-$

Penguin ($\mathcal{P}$)



$\bar{B}^0 \to \pi^+\pi^-$
$\bar{B}^0 \to \pi^0\pi^0$

Color-suppressed ($\mathcal{C}$)



$B^- \to \pi^-\pi^0$
$\bar{B}^0 \to \pi^0\pi^0$

- Subdominant topologies:
  - exchange ($\mathcal{E}$) $\Longrightarrow$ $\bar{B}^0 \to \pi^+\pi^-,\ \pi^0\pi^0$
  - annihilation ($\mathcal{A}$) $\Longrightarrow$ $B^- \to \pi^-\pi^0$
  - penguin-annihilation ($\mathcal{PA}$) $\Longrightarrow$ $\bar{B}^0 \to \pi^+\pi^-,\ \pi^0\pi^0$
  - electroweak-penguin ($\mathcal{P}_{\rm EW}$) $\Longrightarrow$ $B^- \to \pi^-\pi^0$, $\bar{B}^0 \to \pi^0\pi^0$
  - color-suppressed electroweak-penguin ($\mathcal{P}_{\rm EW}^C$) $\Longrightarrow$ $\bar{B}^0 \to \pi^+\pi^-,\ \pi^0\pi^0$

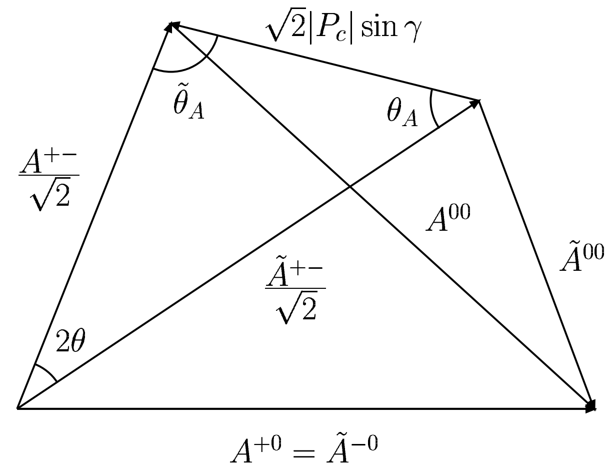
Isospin Relations

- Effective Hamiltonian for $b \to d$ transitions ($\lambda_p^{(d)} = V_{pb} V_{pd}^*$)

$$\mathcal{H}_W^{b\to d} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left(C_1\, \mathcal{O}_1^{(p)} + C_2\, \mathcal{O}_2^{(p)} + \sum_{i=3}^{10} C_i\, \mathcal{O}_i + C_{7\gamma}\, \mathcal{O}_{7\gamma} + C_{8g}\, \mathcal{O}_{8g} \right)$$

- $B \to \pi\pi$ decay amplitudes (subdominant topologies are neglected):

$$\begin{aligned} A^{+0} &= A(B^+ \to \pi^+\pi^0) = -\frac{1}{\sqrt{2}}\,(\mathcal{T} + \mathcal{C}) \\ A^{+-} &= A(B^0 \to \pi^+\pi^-) = -\,(\mathcal{T} + \mathcal{P}) \\ A^{00} &= A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}}\,(\mathcal{P} - \mathcal{C}) \end{aligned}$$

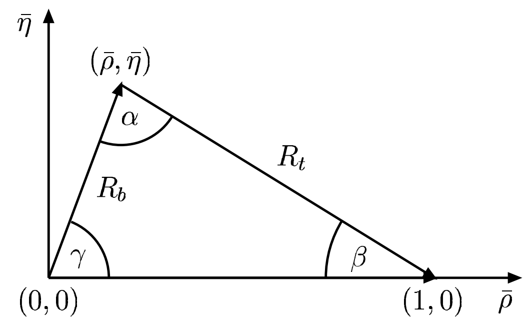


- The amplitudes A^{ij} and their charged-conjugate ones $\bar{A}^{ij}$ are satisfied the isospin relations

$$A^{+0} = \frac{1}{\sqrt{2}}\, A^{+-} + A^{00}, \qquad \bar{A}^{-0} = \frac{1}{\sqrt{2}}\, \bar{A}^{+-} + \bar{A}^{00}$$

Parameterization

- Effective Hamiltonian results two independent CKM factors (improved Wolfenstein parameterization is used)



$$\lambda_c^{(d)} = V_{cb} V_{cd}^* \simeq -A\lambda^3$$
$$\lambda_u^{(d)} = V_{ub} V_{ud}^* \simeq A\lambda^3 (\bar\rho - i\bar\eta) = A\lambda^3 R_b \, \mathrm{e}^{-i\gamma}$$

- In this convention, amplitudes are

$$\begin{aligned}
\sqrt{2}\, A^{+0} &= -(T_c + C_c) = -|T_c|\, \mathrm{e}^{i\delta_T}\, \mathrm{e}^{i\gamma} \left[1 + |C_c/T_c|\, \mathrm{e}^{i\Delta_c}\right] \\
A^{+-} &= -(T_c + P_c) = -|T_c|\, \mathrm{e}^{i\delta_T} \left[\mathrm{e}^{i\gamma} + |P_c/T_c|\, \mathrm{e}^{i\delta_c}\right] \\
\sqrt{2}\, A^{00} &= -(C_c - P_c) = -|T_c|\, \mathrm{e}^{i\delta_T} \left[|C_c/T_c|\, \mathrm{e}^{i\Delta_c}\, \mathrm{e}^{i\gamma} - |P_c/T_c|\, \mathrm{e}^{i\delta_c}\right]
\end{aligned}$$

- Charged-conjugate amplitudes $\bar{A}^{ij}$ differ by the replacement $\gamma \to -\gamma$
- 5 strong parameters [$|T_c|$, $r_c \equiv |P_c/T_c|$, δ_c, $|C_c/T_c|$, Δ_c] (the choice $\delta_T = 0$ for the overall phase can be adopted) and the weak phase γ can be phenomenologically extracted if a complete set of experimental data on $B \to \pi\pi$ decays exist

CP Asymmetry in $B^0 \to \pi\pi$ Decays

- Both final states: $\pi^+\pi^-$ and $\pi^0\pi^0$, are CP conjugate
- The phenomenon of $B^0 - \bar{B}^0$ mixing should be taken into account
- The mixing parameter $q/p \simeq V_{td}/V_{td}^* = \mathrm{e}^{-2i\beta}$ is the pure phase factor with a good accuracy
- Results in the time-dependent CP asymmetry ($\Delta\Gamma_{B^0} = 0$)

$$a_{\pi\pi}^{+-}(t) \equiv \frac{\Gamma[\bar{B}^0(t) \to \pi^+\pi^-] - \Gamma[B^0(t) \to \pi^+\pi^-]}{\Gamma[\bar{B}^0(t) \to \pi^+\pi^-] + \Gamma[B^0(t) \to \pi^+\pi^-]} = S_{\pi\pi}^{+-} \sin(\Delta M_B t) - C_{\pi\pi}^{+-} \cos(\Delta M_B t)$$

- Direct $C_{\pi\pi}^{+-}$ and mixing-induced $S_{\pi\pi}^{+-}$ CP asymmetries can be expressed

$$C_{\pi\pi}^{+-} = \frac{2r_c \sin\delta_c \sin\gamma}{1 + 2r_c \cos\delta_c \cos\gamma + r_c^2}$$

$$S_{\pi\pi}^{+-} = -\frac{\sin(2\beta + 2\gamma) + 2r_c \cos\delta_c \sin(2\beta + \gamma) + r_c^2 \sin(2\beta)}{1 + 2r_c \cos\delta_c \cos\gamma + r_c^2}$$

- Taking weak phases β and γ from elsewhere, for example from the SM fit, the ratio $r_c = |P_c/T_c|$ and strong phase δ_c can be determined

Analysis of $B \to \pi\pi$ Decays

Experimental Data

Branching fractions (in units of 10^{-6})

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B^+ \to \pi^+\pi^0$	$5.8 \pm 0.6 \pm 0.4$	$5.0 \pm 1.2 \pm 0.5$	$4.6^{+1.8+0.6}_{-1.6-0.7}$	5.5 ± 0.6
$B^0 \to \pi^+\pi^-$	$4.7 \pm 0.6 \pm 0.2$	$4.4 \pm 0.6 \pm 0.3$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	$4.5 \pm 0.4^{\ddagger}$
$B^0 \to \pi^0\pi^0$	$1.17 \pm 0.32 \pm 0.10$	$2.3^{+0.4+0.2}_{-0.5-0.3}$	< 4.4	1.45 ± 0.29

‡ The average also includes the recent CDF measurement $\mathcal{B}(\pi^+\pi^-) = 4.4 \pm 1.3$

CP Asymmetry

	BABAR	BELLE	Average [HFAG]
$\mathcal{A}_{\rm CP}(\pi^+\pi^0)$	$-0.01 \pm 0.10 \pm 0.02$	$-0.02 \pm 0.10 \pm 0.01$	-0.02 ± 0.07
$C_{\pi\pi}^{+-} = -A_{\pi\pi}^{+-}$	$-0.09 \pm 0.15 \pm 0.04$	$-0.56 \pm 0.12 \pm 0.06$	-0.37 ± 0.10
$S_{\pi\pi}^{+-}$	$-0.30 \pm 0.17 \pm 0.03$	$-0.67 \pm 0.16 \pm 0.06$	-0.50 ± 0.12
$\mathcal{A}_{\rm CP}(\pi^0\pi^0)$	$0.12 \pm 0.56 \pm 0.06$	$0.44^{+0.53}_{-0.52} \pm 0.17$	$0.28^{+0.40}_{-0.39}$

Life-time ratio $\quad \tau_{B^+}/\tau_{B^0} = 1.081 \pm 0.015$

CP asymmetry in the $B \to J/\psi K_S$ and related decays

$$\sin(2\beta) = \sin(2\phi_1) = 0.725 \pm 0.037, \qquad \beta = \phi_1 = (23.3 \pm 1.5)^\circ$$

Numerical Analysis of $C_{\pi\pi}^{+-}$ and $S_{\pi\pi}^{+-}$

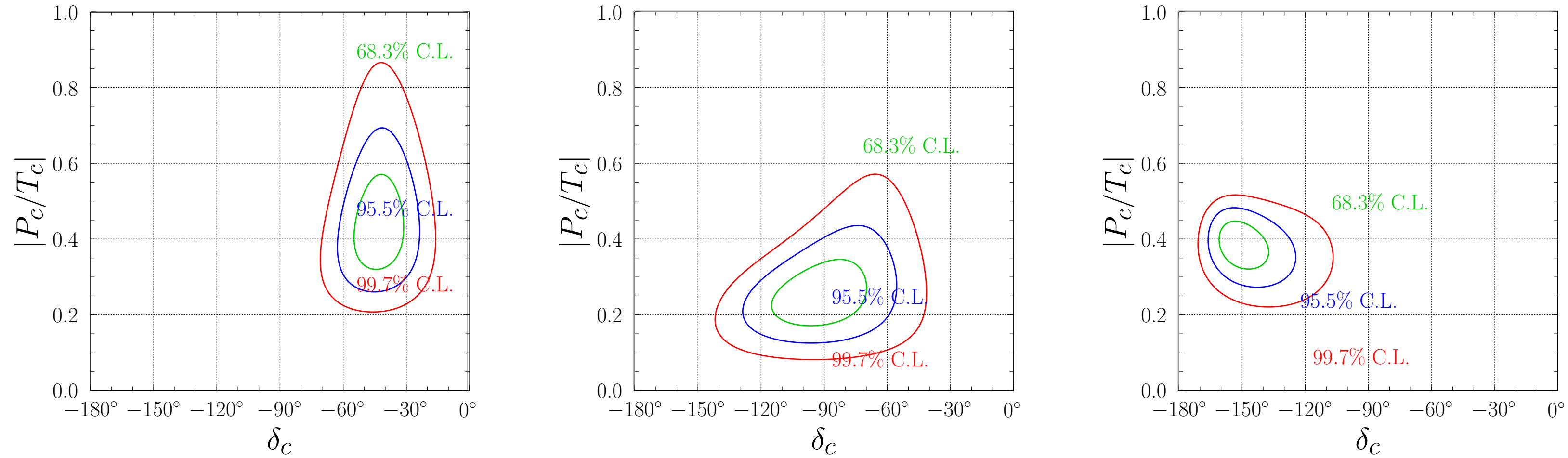
- For the construction of C.L. contours, the χ^2-function is introduced

$$\chi^2\left(\beta, \alpha, |P_c/T_c|, \delta_c\right) = \left[\frac{C_{\pi\pi}^{+-} - (C_{\pi\pi}^{+-})_{\rm exp}}{\Delta C_{\pi\pi}^{+-}}\right]^2 + \left[\frac{S_{\pi\pi}^{+-} - (S_{\pi\pi}^{+-})_{\rm exp}}{\Delta S_{\pi\pi}^{+-}}\right]^2$$

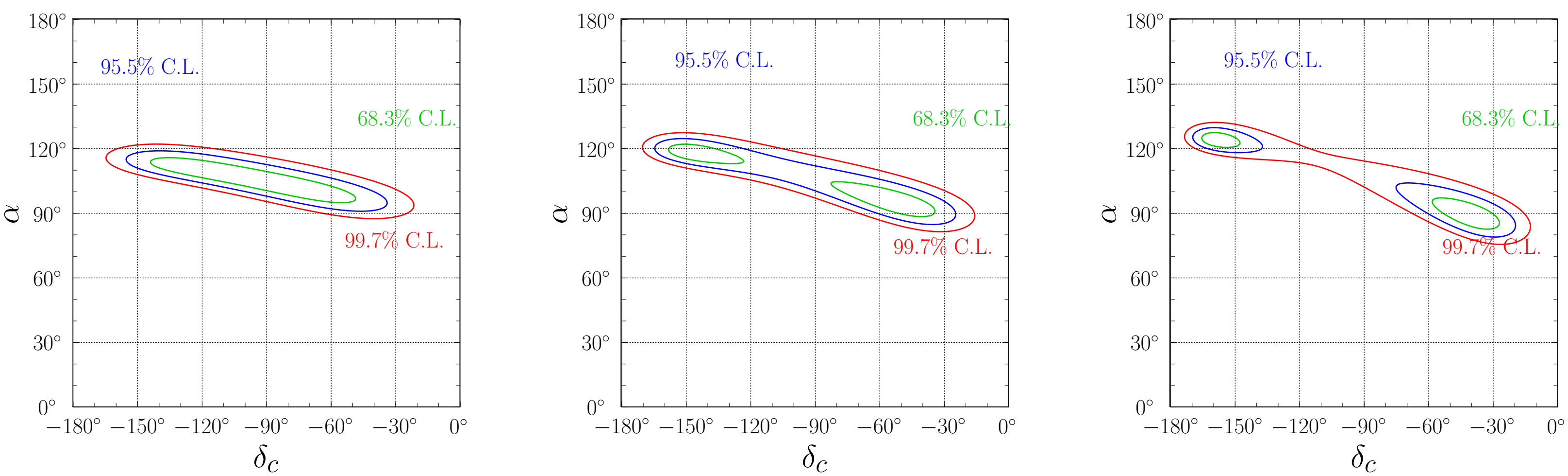
- SM relation $\alpha + \beta + \gamma = \pi$ between the unitarity-triangle angles was used to eliminate γ
- In getting the C.L. contours, χ^2-function was equated to 2.30, 6.18. and 11.83, corresponding to 68.3%, 95.5%, and 99.7%, respectively, for two degrees of freedom
- The correlations $|P_c/T_c| - \delta_c$ at fixed values of α, β and $\alpha - \delta_c$ at fixed values of $|P_c/T_c|$, β are presented further

Analysis of $B \to \pi\pi$ Decays

Correlation $|P_c/T_c| - \delta_c$



Correlation $\alpha - \delta_c$



Results of the SM Fit (Spring 2004)

- Combining the above χ^2-function with the one used for the SM fit results the following 68% C.L. ranges ($C_{\pi\pi}^{+-} = -0.46 \pm 0.13$, $S_{\pi\pi}^{+-} = -0.74 \pm 0.16$)

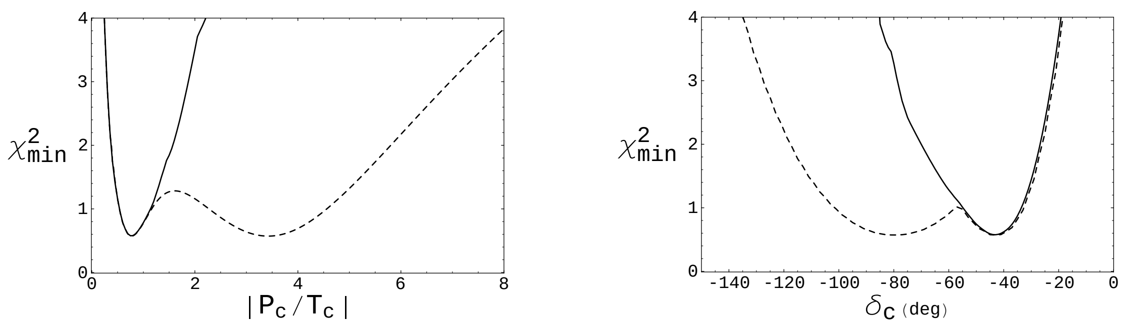
$A = 0.79 \div 0.86$	$\alpha = (81 \div 103)^\circ$	$\Delta M_{B_s} = (16.6 \div 20.3)$ ps^{-1}
$\bar\rho = 0.10 \div 0.24$	$\beta = (21.9 \div 25.5)^\circ$	$\lvert P_c/T_c \rvert = 0.43 \div 5.30$
$\bar\eta = 0.32 \div 0.40$	$\gamma = (54 \div 75)^\circ$	$\delta_c = -(29 \div 112)^\circ$

- The ranges of $|P_c/T_c|$ and δ_c can be reduced if restriction on the penguin "pollution" $\cos(2\theta)$ is taken into account [Gronau et al.]

$$\cos(2\theta) \geq \frac{(B_{\pi\pi}^{+-} + 2B_{\pi\pi}^{+0} - 2B_{\pi\pi}^{00})^2 - 4B_{\pi\pi}^{+-}B_{\pi\pi}^{+0}}{4B_{\pi\pi}^{+-}B_{\pi\pi}^{+0}\sqrt{1-(C_{\pi\pi}^{+-})^2}} \simeq 0.27$$

Here, $B_{\pi\pi}^{ij} = (|A^{ij}|^2 + |\bar A^{ij}|^2)/2$

- $|P_c/T_c| = 0.77^{+0.58}_{-0.34}$ and $\delta_c = (-43^{+14}_{-21})^\circ$ at 68% C.L.



Bounds from $B \to \pi\pi$ Decays

Based on the Quantities

- $B^{ij} \equiv \left(|A^{ij}|^2 + |\bar{A}^{ij}|^2\right)/2$
- $C \equiv \left(|A^{+-}|^2 - |\bar{A}^{+-}|^2\right) / \left(|A^{+-}|^2 + |\bar{A}^{+-}|^2\right)$
- $Y \equiv 2|A^{+-}||\bar{A}^{+-}| / \left(|A^{+-}|^2 + |\bar{A}^{+-}|^2\right)$
- $S \equiv Y \sin(2\alpha_{\rm eff}) \equiv Y \sin(2\alpha + 2\theta)$
- $C^2 + S^2 = 1 - Y^2 \cos^2(2\alpha_{\rm eff}) \le 1$

Bounds on Penguin Pollution

- Grossman and Quinn $\quad \cos(2\theta) \ge 1 - 2B^{00}/B^{+0}$
- Charles $\quad \cos(2\theta) \ge \left[1 - 2B^{00}/B^{+0}\right]/Y$
  $\cos(2\theta) \ge \left[1 - 4B^{00}/B^{+-}\right]/Y$
- Gronau, London, Sinha, Sinha (GLSS)

$$\cos(2\theta) \ge \left[(B^{+-}/2 + B^{+0} - B^{00})^2/(B^{+-}B^{+0}) - 1\right]/Y$$

Bound	$\cos(2\theta)^{\rm cons}_{\rm min}$	$\lvert\theta^{\rm cons}_{\rm max}\rvert$
GQ	0.30	36.4°
Ch-I	0.32	35.7°
GLSS	0.35	34.8°

Bounds from $B \to \pi\pi$ Decays

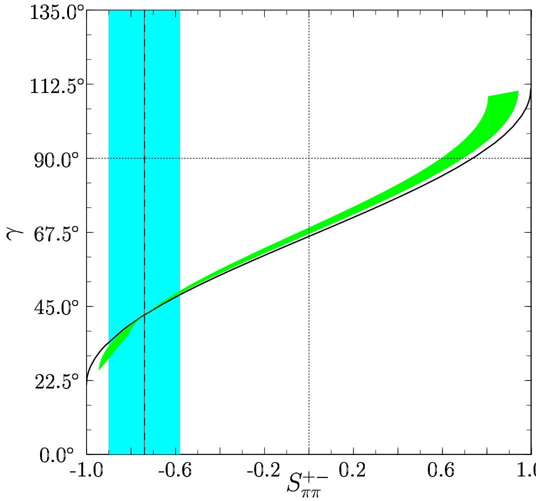
Bounds on the angle γ

- Buchalla and Safir (2004)

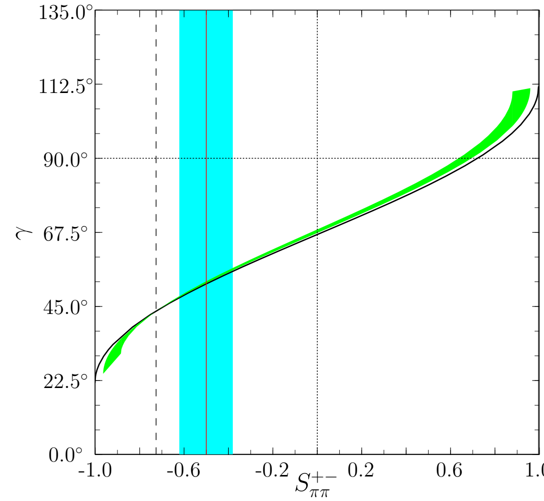
$$\tan\gamma \geq \frac{\cos(2\beta) + \sqrt{1 - S^2}}{\sin(2\beta) - S}$$

- Botella and Silva (2004)

$$\tan\gamma \geq \frac{1 + \sqrt{Y^2 - S^2}\cos(2\beta) + S\sin(2\beta)}{\sqrt{Y^2 - S^2}\sin(2\beta) - S\cos(2\beta)}$$



Summer 2004



Spring 2005

Analysis of $B \to \pi\pi$ Decays

Isospin Analysis

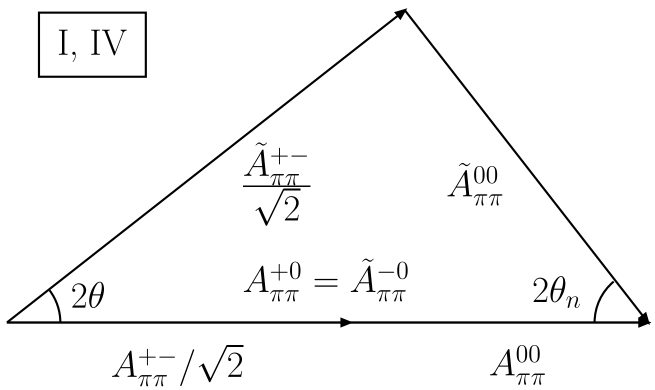
- $\mathcal{B}(B^+ \to \pi^+\pi^0)$, $\mathcal{B}(B^0 \to \pi^+\pi^-)$, $\mathcal{B}(B^0 \to \pi^0\pi^0)$, $S_{\pi\pi}^{+-}$, $C_{\pi\pi}^{+-}$, $C_{\pi\pi}^{00}$ (6 measurements) are included in the χ^2-function; depends on 6 variables ($|T_c|$, r_c, δ_c, x_c, Δ_c, γ); isospin analysis can be done
- The best-fit solutions and 1σ ranges [T_c, P_c, C_c are in units of $3.0 \times 10^{-8}\,\mathrm{GeV}$]

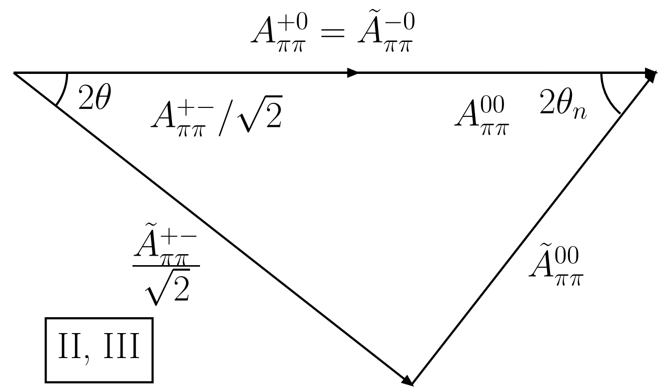
	I	II	III	IV
γ	$(68.9^{+4.1}_{-4.4})^\circ$	$(32.0^{+2.7}_{-2.8})^\circ$	$(11.4^{+2.6}_{-2.6})^\circ$	$(154.6^{+5.0}_{-4.7})^\circ$
r_c	$0.466^{+0.091}_{-0.079}$	$0.466^{+0.031}_{-0.034}$	$1.619^{+0.047}_{-0.041}$	$1.619^{+0.110}_{-0.094}$
δ_c	$(-38.6^{+9.4}_{-9.2})^\circ$	$(-159.2^{+6.6}_{-5.9})^\circ$	$(-152.6^{+4.0}_{-3.4})^\circ$	$(-93.3^{+6.9}_{-8.8})^\circ$
x_c	$1.040^{+0.088}_{-0.096}$	$0.173^{+0.079}_{-0.098}$	$2.213^{+0.064}_{-0.069}$	$2.835^{+0.140}_{-0.151}$
Δ_c	$(-53.5^{+14.3}_{-13.0})^\circ$	$(82.8^{+19.2}_{-21.2})^\circ$	$(-157.8^{+5.4}_{-5.0})^\circ$	$(95.4^{+8.6}_{-9.0})^\circ$
$\|T_c\|$	$0.618^{+0.032}_{-0.032}$	$1.086^{+0.032}_{-0.032}$	$0.838^{+0.060}_{-0.055}$	$0.386^{+0.047}_{-0.049}$
$\|P_c\|$	$0.288^{+0.038}_{-0.037}$	$0.506^{+0.031}_{-0.032}$	$1.358^{+0.027}_{-0.026}$	$0.625^{+0.028}_{-0.027}$
$\|C_c\|$	$0.642^{+0.055}_{-0.059}$	$0.188^{+0.085}_{-0.106}$	$1.856^{+0.054}_{-0.058}$	$1.094^{+0.054}_{-0.058}$
$S_{\pi\pi}^{00}$	$0.801^{+0.125}_{-0.180}$	$-0.835^{+0.078}_{-0.052}$	$-0.835^{+0.165}_{-0.119}$	$0.801^{+0.145}_{-0.271}$

- Larger but consistent with the standard CKM fit $\gamma_{\rm CKM} = (58^{+7}_{-5})^\circ$

Isospin Analysis

- Fourfold ambiguity in determination of γ instead of eightfold
- One of the amplitude triangles collapsed





- "Pollution" angle $\theta = (18.4^{+3.4}_{-3.3})^\circ$; much smaller then the GLSS bound $\theta < 35^\circ$
- The other angle $\theta_n = (-25.6^{+5.7}_{-6.0})^\circ$

Comparison with Theoretical Predictions

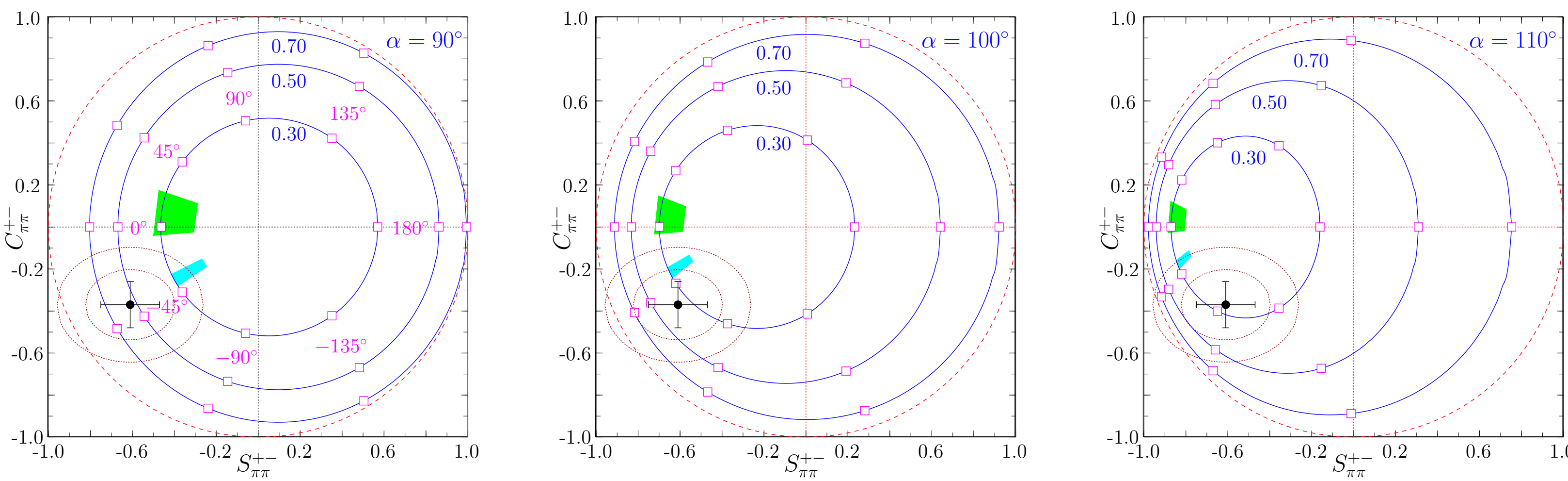
- Allowed values of the dynamical quantities at 68% C.L.

$$|P_c/T_c| = 0.47^{+0.09}_{-0.08} \qquad \delta_c = (-38.6^{+9.4}_{-9.2})^\circ$$

- This should be compared with predictions from the dynamical model

$$|P_c/T_c| = 0.29 \pm 0.09 \quad \delta_c = (9 \pm 15)^\circ \quad \text{QCDF} \quad \text{[Buchalla \& Safir]}$$
$$|P_c/T_c| = 0.23^{+0.07}_{-0.05} \quad \delta_c = (-37 \pm 5)^\circ \quad \text{pQCD} \quad \text{[Keum \& Sanda]}$$

- Phenomenology supports enhanced penguin contribution which is not the case of QCD-F but can be explained within pQCD approach



Similar Phenomenological Analysis of $B \to \pi\pi$ Decays

- Isospin analysis [AP, this School]

$$|P_c/T_c| = 0.47^{+0.09}_{-0.08} \qquad \delta_c = (-38.6^{+9.4}_{-9.2})^\circ$$

- Isospin analysis [A. Höcker, this School]

$$|P_c/T_c| = 0.37 \pm 0.17 \qquad \delta_c = (-36^{+10}_{-12})^\circ$$

- Analysis accounts for the flavor $SU(3)$ symmetry [Buras, Fleischer, Reckseigel, Schwab, hep-ph/0410407; hep-ph/0411373]

$$|P_c/T_c| = 0.51^{+0.26}_{-0.20} \qquad \delta_c = \theta - \pi = (-40^{+14}_{-18})^\circ$$

- Analysis is performed within SCET [Pirjol, hep-ph/0502141]

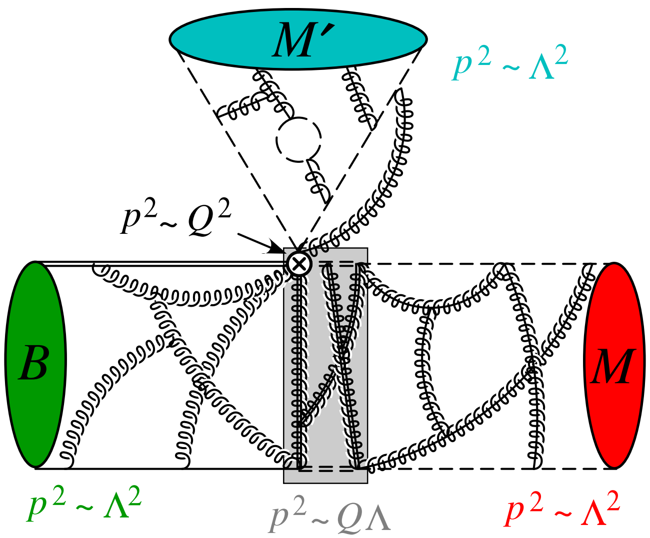
$$|P_c/T_c| = 0.49 \pm 0.14 \qquad \delta_c = -(41 \pm 15)^\circ$$

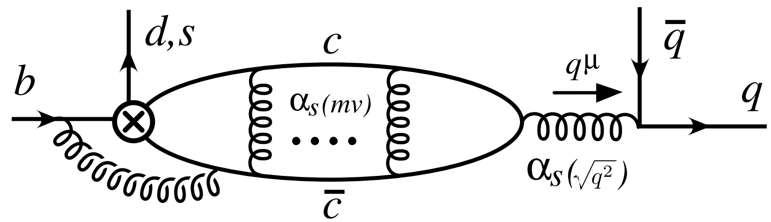
- Isospin analysis of $B^0 \to \pi^+\pi^-$ time-dependent CP asymmetry [Ali, Lunghi and AP, hep-ph/0403275]

$$|P_c/T_c| = 0.77^{+0.58}_{-0.34} \qquad \delta_c = (-43^{+14}_{-21})^\circ$$

$B \to M_1 M_2$ Factorization in SCET

[Chey, Kim, hep-ph/0301262]
[Bauer, Pirjol, Rothstein, Stewart, hep-ph/0401188]





- form-factor and hard-spectator terms are formally the same as in QCD Factorization Approach
- long-distance charming-penguin contribution appears in LO

$$\Lambda^2 \ll Q\lambda \ll Q^2, \quad Q = \{m_b, E, m_c\}$$

$B \to \pi\pi$ Decay Amplitudes in SCET

[Bauer et al., hep-ph/0401188]

In terms of SCET Wilson coefficients $c_i^{(d)}(u)$ and $b_i^{(d)}(u, z)$

$$A(B^- \to \pi^- \pi^0) = \frac{G_F}{2} m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_1^{(d)}(u) + c_2^{(d)}(u) - c_3^{(d)}(u) \right]$$

$$+ \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_1^{(d)}(u, z) + b_2^{(d)}(u, z) - b_3^{(d)}(u, z) \right] \Big\}$$

$$A(\bar{B}^0 \to \pi^+ \pi^-) = \frac{G_F}{\sqrt{2}} m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_1^{(d)}(u) + c_4^{(d)}(u) \right]$$

$$+ \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_1^{(d)}(u, z) + b_4^{(d)}(u, z) \right] \Big\} + \lambda_c^{(d)} A_{\bar{c}c}^{B\pi}$$

$$A(\bar{B}^0 \to \pi^0 \pi^0) = \frac{G_F}{\sqrt{2}} m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_2^{(d)}(u) - c_3^{(d)}(u) - c_4^{(d)}(u) \right]$$

$$+ \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_2^{(d)}(u, z) - b_3^{(d)}(u, z) - b_4^{(d)}(u, z) \right] \Big\} - \lambda_c^{(d)} A_{\bar{c}c}^{B\pi}$$

In agreement with isospin relation

$$\sqrt{2}\, A(B^- \to \pi^- \pi^0) = A(\bar{B}^0 \to \pi^+ \pi^-) + A(\bar{B}^0 \to \pi^0 \pi^0)$$

$B \to \pi\pi$ Phenomenology

[Bauer et al., hep-ph/0401188]

- World averages [HFAG] for branching ratios and CP asymmetries

$$\bar{\mathcal{B}}(B^\pm \to \pi^\pm \pi^0) = (5.5 \pm 0.6) \times 10^{-6} \qquad A_{\rm CP}(\pi^\pm \pi^0) = -0.02 \pm 0.07$$
$$\bar{\mathcal{B}}(B^0 \to \pi^+ \pi^-) = (4.5 \pm 0.4) \times 10^{-6} \qquad C_{\pi\pi}^{+-} = -0.37 \pm 0.10$$
$$\bar{\mathcal{B}}(B^0 \to \pi^0 \pi^0) = (1.45 \pm 0.29) \times 10^{-6} \qquad S_{\pi\pi}^{+-} = -0.50 \pm 0.12$$

blue $S_{\pi\pi}^{00} = -0.28 \pm 0.40$

- Matching full theory onto SCET and neglecting electroweak penguins
- To leading order in $\alpha_s(m_b)$ and assuming $\langle u^{-1} \rangle_\pi \simeq 3$

$$A(B^- \to \pi^- \pi^0) \simeq \frac{G_F}{\sqrt{2}} \frac{f_\pi m_B^2}{3\sqrt{2}} \lambda_u^{(d)} (C_1 + C_2) \left[4\zeta^{B\pi} + 7\zeta_J^{B\pi} \right]$$

$$A(\bar{B}^0 \to \pi^+ \pi^-) = \frac{G_F}{\sqrt{2}} \frac{f_\pi m_B^2}{3} \Big\{ \lambda_u^{(d)} (3C_1 + C_2 + C_3 + 3C_4) \zeta^{B\pi} + \lambda_c^{(d)} (C_3 + 3C_4) \zeta^{B\pi}$$
$$+ \lambda_u^{(d)} (3C_1 + 4C_2 + 4C_3 + 3C_4) \zeta_J^{B\pi} + \lambda_c^{(d)} (4C_3 + 3C_4) \zeta_J^{B\pi} \Big\} + A_{\bar{c}c}^{B\pi}$$

- 4 unknowns: real $\zeta^{B\pi}$ and $\zeta_J^{B\pi} = \int_0^1 dz\, \zeta_J^{B\pi}(z)$; complex $A_{\bar{c}c}^{B\pi}$

Pure Isospin Analysis

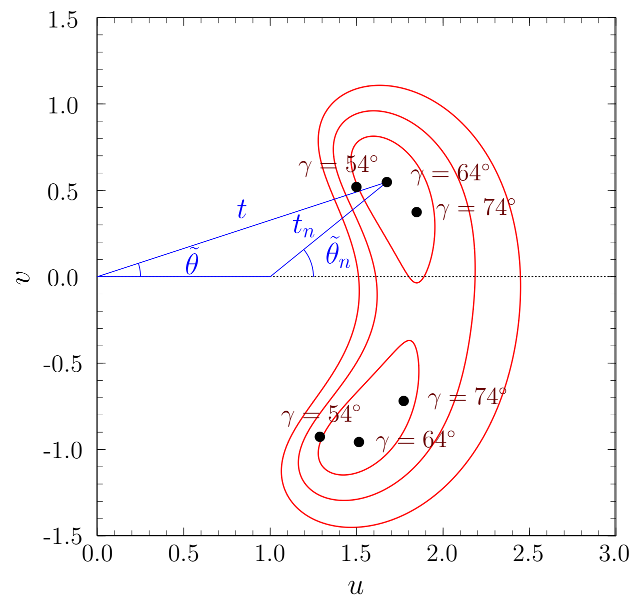
[D.Pirjol, hep-ph/0502141]

$$A = \lambda_u^{(d)} T + \lambda_c^{(d)} P$$

$$\sqrt{2} A(B^- \to \pi^- \pi^o) = \lambda_u^{(d)}\, T$$
$$A(\bar{B}^0 \to \pi^+ \pi^-) = \lambda_u^{(d)}\, T_c\, \left(1 + r_c\, e^{+i\delta_c} e^{+i\gamma}\right)$$
$$A(\bar{B}^0 \to \pi^0 \pi^0) = \lambda_u^{(d)}\, T_n\, \left(1 + r_n\, e^{+i\delta_n} e^{+i\gamma}\right)$$

10 hadronic parameters
- 4 for isospin relation
- an overall phase

5 parameters $[|T|,\, r_c,\, \delta_c,\, u,\, v]$



With $\gamma = 64°$ $\quad |T| = \dfrac{G_F}{\sqrt{2}}\, m_B^2\, f_\pi\, (0.290 \pm 0.016) \left[\dfrac{3.9 \times 10^{-3}}{|V_{ub}|}\right]$

$$r_c = 0.38 \pm 0.08, \quad \delta_c = (-48 \pm 11)°, \quad (u, v) = \begin{cases} (1.51 \pm 0.16, & -0.90 \pm 0.15) \\ (1.68 \pm 0.15, & 0.55 \pm 0.19) \end{cases}$$

Penguin contribution enhanced and color-suppressed contribution is large

At this order the "tree" isospin triangle is predicted to be flat $t - t_n = 1$

Results for Non-Perturbative Parameters

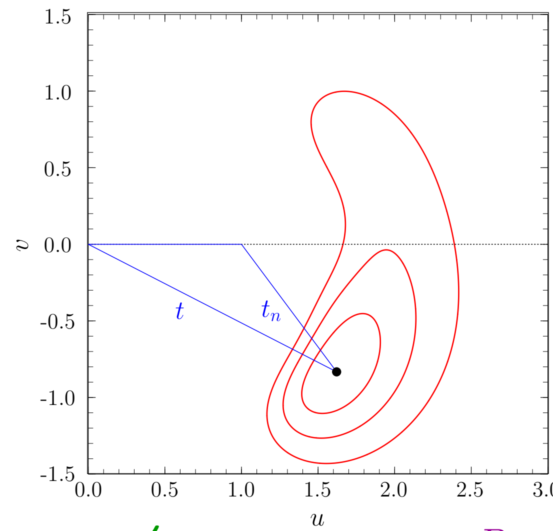
[D. Pirjol, hep-ph/0502141]

Best-fit values and 1σ ranges

$$\gamma = (68.8^{+4.4}_{-4.7})^\circ$$

$$r_c = 0.46 \pm 0.09 \qquad \delta_c = (-38.7^{+9.4}_{-9.2})^\circ$$

$$u = 1.62^{+0.15}_{-0.13} \qquad v = -0.83^{+0.18}_{-0.17}$$



SCET amplitudes in LO in Λ/Q and $\alpha_s(m_b)$

$[\zeta^{B\pi},\ \zeta_J^{B\pi},\ P_c$ (or $A_{\bar{c}c}^{B\pi}$)] fitted from $[|T|,\ r_c,\ \delta_c,\ u,\ v]$

$r_c,\ \delta_c$ $$P_c\Big|_{\gamma=64^\circ} = \frac{G_F}{\sqrt{2}}\, m_B^2 f_\pi\, (0.032 \pm 0.010)\, e^{i(141\pm 11)^\circ}$$ $\Longleftarrow$ large $A_{\bar{c}c}^{B\pi}$

$|T|,\ u,\ v$

$$\zeta^{B\pi}\Big|_{\gamma=64^\circ} = (0.08 \pm 0.03)\left[\frac{3.9\times 10^{-3}}{|V_{ub}|}\right]$$

$$\zeta_J^{B\pi}\Big|_{\gamma=64^\circ} = (0.10 \pm 0.02)\left[\frac{3.9\times 10^{-3}}{|V_{ub}|}\right]$$

$\Longleftarrow$ QCD Fact. used $\zeta^{B\pi} \gg \zeta_J^{B\pi}$

Pert. theory $$\zeta_J^{B\pi} = \frac{\pi\alpha_s C_F}{N_c}\,\frac{f_B f_\pi}{M_B \lambda_B}\,\langle \bar{x}^{-1}\rangle \sim 0.02 \div 0.05$$

Polarization in $B \to VV$ Decays

- In $\bar{B} \to VV$ decays, there are three helicity amplitudes, $\bar{\mathcal{A}}^{(0)}$, $\bar{\mathcal{A}}^{(-)}$ and $\bar{\mathcal{A}}^{(+)}$, in which both vector mesons are longitudinally, negatively and positively polarized.
- In naive factorization supplemented by the large energy form factor relations

$$\bar{\mathcal{A}}^{(0,-,+)}_{V_1 V_2} = -i \frac{G_F}{\sqrt{2}} \lambda_p^{(q)} \, \tilde{a} \, f_{V_2} m_B \left\{ -m_B \zeta_\parallel^{V_1}, \, m_{V_2} \zeta_\perp^{V_1}, \, m_{V_2} \zeta_\perp^{V_1} r_\perp^{V_1} \right\}$$

Here, $\tilde{a}$ is a combination of Wilson coefficients, $\zeta_\parallel^V$ and $\zeta_\perp^V$ are universal $B \to V$ form factors, and $r_\perp^V$ parameterizes the form factor helicity suppression

- The helicity suppression $\bar{\mathcal{A}}^{(\pm)}/\bar{\mathcal{A}}^{(0)} \sim m_V/m_B$ is clearly seen
- In large-energy limit, $r_\perp^V \to 0$ at leading power in $\Lambda/m_b \Longrightarrow \bar{\mathcal{A}}^{(+)}/\bar{\mathcal{A}}^{(-)} \sim \mathcal{O}(1/m_b)$
- Transversity basis $\bar{\mathcal{A}}_L \equiv \bar{\mathcal{A}}^{(0)}$, $\bar{\mathcal{A}}_{\perp,\parallel} \equiv [\bar{\mathcal{A}}^{(-)} \mp \bar{\mathcal{A}}^{(+)}]\sqrt{2}$ occurs more convenient, so

$$1 - f_L = \mathcal{O}(1/m_b^2), \quad f_\perp/f_\parallel = 1 + \mathcal{O}(1/m_b), \quad f_i \equiv \Gamma_i/\Gamma_{\rm tot}$$

- First relation remains formally true when non-factorizable contributions are included in the QCD Factorization approach and is realized in $B \to \rho\rho$ [Kagan, hep-ph/0405134]
- Similar result follows from SCET where at LO only long-distance charming-penguin operator contributes to the $B \to V_1^\perp V_2^\perp$ amplitude [Bauer et al., hep-ph/0401188]
- The observation of $f_L(\phi K^*) \simeq 0.5$ can be accounted for in the Standard Model within QCD-F approach, with large theoretical errors [Rohrer, this School]

$B \to VV$ Decays: Experimental Data

Longitudinal polarization fraction f_L

Mode	BABAR	BELLE	Average [HFAG]
$B^+ \to K^{*0}\rho^+$	$0.79 \pm 0.08 \pm 0.04$	$0.43 \pm 0.11^{+0.05}_{-0.02}$	0.66 ± 0.07
$B^+ \to K^{*+}\rho^0$	$0.96^{+0.04}_{-0.15} \pm 0.04$		$0.96^{+0.06}_{-0.15}$
$B^+ \to \phi K^{*+}$	$0.46 \pm 0.12 \pm 0.03$	$0.52 \pm 0.08 \pm 0.03$	0.50 ± 0.07
$B^+ \to \rho^+\rho^0$	$0.97^{+0.03}_{-0.07} \pm 0.04$	$0.95 \pm 0.11 \pm 0.02$	$0.97^{+0.05}_{-0.07}$
$B^+ \to \omega\rho^+$	$0.88^{+0.12}_{-0.15} \pm 0.03$		$0.88^{+0.12}_{-0.15}$
$B^0 \to \phi K^{*0}$	$0.52 \pm 0.05 \pm 0.02$	$0.45 \pm 0.05 \pm 0.02$	0.48 ± 0.04
$B^0 \to \rho^+\rho^-$	$0.99 \pm 0.03^{+0.04}_{-0.03}$		$0.99^{+0.05}_{-0.04}$

Full angular analysis is also avaluable for $B \to \phi K^*$ modes

Transverse polarization fraction $f_\perp$

Mode	BABAR	BELLE	Average [HFAG]
$B^+ \to \phi K^{*+}$		$0.19 \pm 0.08 \pm 0.02$	0.19 ± 0.08
$B^0 \to \phi K^{*0}$	$0.25 \pm 0.05 \pm 0.02$	$0.31^{+0.06}_{-0.05} \pm 0.02$	0.26 ± 0.04

Polarization in $B \to VV$ Decays

[Li, Mishina, hep-ph/0411146]

- Tree-dominated decays into two light vector mesons $B \to \rho\rho, \ldots$ can be understood by kinematics in the large enery limit; this is also robust under subleading corrections; all are negligible within pQCD; QCD dynamics play a minor role for the polarization
- For penguin-dominated decays, polarization fractions can deviate from the naive counting rules; important annihilation contributions from the $(S-P)(S+P)$ operators; R_L can be decrease up to 0.75 for decays like $B^+ \to \rho^+ K^{*0}, \ldots$; can be accommodated within the SM
- $f_L \simeq 0.5$ for $B \to \phi K^*$. In the SM, suggested solutions – taking into account penguin-annihilation contributions [Kagan, hep-ph/0405134], rescattering effects [Colangelo et al., hep-ph/0406162], etc., are not satisfactory.
- It is too early talk about effects of New Physics as complicated QCD dynamics in $B \to VV$ decays is not properly worked out

Summary

- Experiment requires a deeper understanding of QCD dynamics in hadronic B-meson decays
- Several theoretical approaches (QCD-F, pQCD, SCET, etc) are proposed and experimentally tested
- Majority of experimental data can be successfully explained within these approaches; puzzles still exist and require satisfactory explanation
- SCET – the emerging QCD technology, hold the promise to provide a better theoretical description of B-meson hadronic decays than existing approaches
- Progress in understanding of QCD dynamics in exclusive processes will allow to reduce theoretical errors in determination of the CKM matrix elements and check a mechanism of CP violation