Two-Body Nonleptonic B-Meson Decays – I (Theoretical Approaches)

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Contents

Introduction

The aim of the study of B-meson weak decays:

1. To determine the elements of the CKM matrix and to explore the origin of CP violation at low-energy scale
2. To study the strong interaction dynamics related to the confinement of quarks and gluons inside hadrons
3. To explore possibility of New Physics beyond the Standard Model (SM)

All tasks complement each other

An understanding of the connection between parton and hadron properties is a necessary prerequiste for a precise determination of the CKM matrix elements and CP-violating phase (the Kobayashi-Maskawa phase)

Introduction

Theoretical Approaches for $B \to M_1 M_2$ Decays

1. SU(2)/SU(3) Symmetries, supplemented with phenomenological Ansaetze [Lipkin; Gronau, London; Grossman, Quinn; Charles; Gronau, London, Sinha, Sinha; Fleisher, Mannel; Neubert, Rosner; Buras, Fleisher; Grossman, Legeti, Nir; Buchalla, Safir; Botella, Silva; Lavoura; Fleisher et al.; Buras et al.; Soni et al.; Ali, Lunghi, AP]
2. QCD Factorization (QCD-F) Approach [Beneke, Buchalla, Neubert, Sachrajda (BBNS)]
3. Perturbative QCD (pQCD) Approach [Keum, Li, Sanda]
4. Charming Penguins [Cuichini et al.] using the Renormalization Group (RG) Invariant Topological Approach [Buras, Silvestrini]
5. QCD Light-Cone Sum Rules (QCD-LCSR) [Khodjamirian et al.]
6. Soft-Collinear Effective Theory (SCET) [Bauer et al.; Beneke et al.; Neubert et al.]

The Cabibbo-Kobayashi-Maskawa Matrix

- By convention, the quark mixing is often expressed in terms of a (3×3) unitarity matrix $V_{\rm CKM}$ operating on the charge $Q = -1/3$ quark mass eigenstates d, s and b
- Weak charged current

$$j_\alpha^{\rm W} = (\bar u,\, \bar c,\, \bar t)\, \gamma_\alpha (1 - \gamma_5)\, V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Four parameters completely determine mixing in the quark sector
- Elements of the CKM matrix are expressed as an expansion in λ:

$$V_{\rm CKM} \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda\,(1 + iA^2\lambda^4\eta) & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2\,(1 + i\lambda^2\eta) & 1 \end{pmatrix}$$

- Perturbatively improved version of the Wolfenstein parameterization

$$\bar\rho = \rho\,(1 - \lambda^2/2) \qquad \bar\eta = \eta\,(1 - \lambda^2/2)$$

occurs more convenient in physical applications

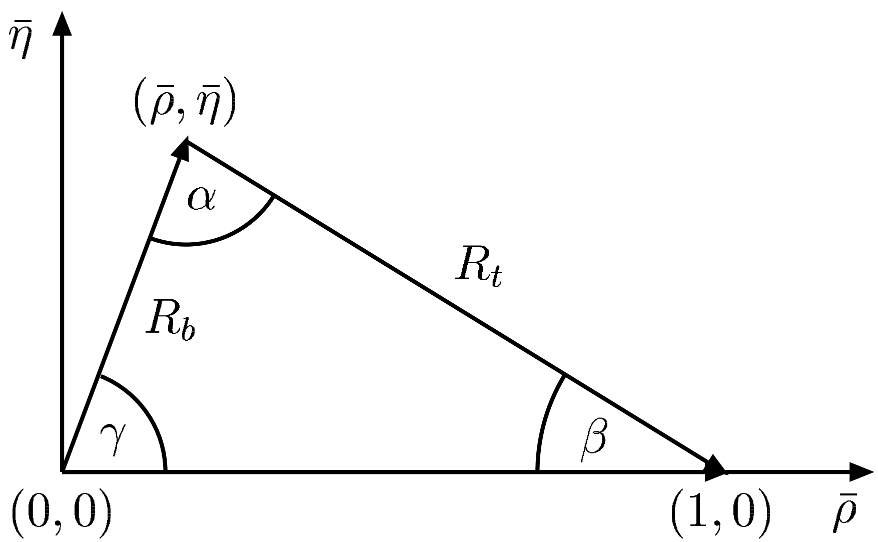
The Cabibbo-Kobayashi-Maskawa Matrix

- All the parameters are of order unity; global CKM fit of experimental data [CKMfitter Group (2005)]

$$\lambda = 0.2265 \pm 0.0020 \quad A = 0.801^{+0.029}_{-0.018}$$

$$\bar{\rho} = 0.204^{+0.036}_{-0.043} \qquad \bar{\eta} = 0.340^{+0.025}_{-0.022}$$

- Rescaled CKM-Unitarity triangle: $V_{cd}V_{cb}^* \simeq -A\lambda^3$ is almost real; basement equal to one; $R_b = |V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$ and $R_t = |V_{td}V_{tb}^*|/|V_{cd}V_{cb}^*|$ are the sides with lengths of order one



global CKM fit of data [CKMfitter Group (2005)]

$$\beta = \phi_1 = (23.1 \pm 1.5)^\circ$$
$$\alpha = \phi_2 = (97.9^{+5.0}_{-6.4})^\circ$$
$$\gamma = \phi_3 = (59.0^{+6.4}_{-4.9})^\circ$$

Decays Modes: Determination of the UT Angles

- Angle β: Time-Dependent CP Asymmetry

$$A_{\rm CP}(t) = -\eta_f \left[\sin(2\beta) + \Delta S_\beta\right] \sin(\Delta M_B t)$$

  - $B_d^0 \to J/\psi\, K^0$ (the "gold-plated" mode)
  - $B_d^0 \to \pi^0\, K_S,\ \eta'\, K_S$
  - $B_d^0 \to f_0\, K_S$
  - $B_d^0 \to \phi\, K_S,\ \omega\, K_S$
  - $B_d^0 \to K^+ K^- K_S,\ K_S K_S K_S$
- Angle α
  - $B_d^0 \to \pi^+ \pi^-$
  - $B_d^0 \to \rho^+ \rho^-$
  - $B_d^0 \to \rho^\pm \pi^\mp$
- Angle γ
  - $B^- \to D^0 K^-, D^{*0} K^-, D^0 K^{*-}$
  - $B_s^0 \to \rho\, K_S,\ \pi^0\, K_S$

Effective Electroweak Theory

- Weak interaction phenomena at energies $E \ll M_W, M_Z$ are most conveniently described in the framework of an effective theory
- This theory is derived from the Standard Model (SM) by integrating out heavy particles – the top quark, W- and Z-bosons
- Lagrangian density includes all the other quark flavors $q = u, d, s, c, b$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD \times QED} + \mathcal{L}_{\rm weak}^{b \to d} + \mathcal{L}_{\rm weak}^{b \to s}$$

- Flavor-changing neutral current (FCNC) term $\mathcal{L}_{\rm weak}^{b \to d}$ describes $b \to d$ transition

$$\mathcal{L}_{\rm weak}^{b \to d} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \sum_j C_j(\mu) \mathcal{O}_j(\mu)$$

- FCNC term $\mathcal{L}_{\rm weak}^{b \to s}$ for $b \to s$ transition can be obtained from $\mathcal{L}_{\rm weak}^{b \to d}$ by replacements:

1. $d \to s$ for the quark fields in all the operators $\mathcal{O}_j(\mu)$
2. $\lambda_d^{(p)} \equiv V_{pb} V_{pd}^* \to \lambda_s^{(p)} \equiv V_{pb} V_{ps}^*$ in the CKM factors

Operator Basis

- For most phenomenological applications, only operators $\mathcal{O}_j(\mu)$ of the dimension $d = 5$ and $d = 6$ are relevant
- The standard basis of four-fermion operators for the $b \to s$ transition
  - Tree Operators

$$\mathcal{O}_1^{(p)} = (\bar{s}_\alpha p_\alpha)_{V-A} (\bar{p}_\beta b_\beta)_{V-A} \qquad \mathcal{O}_2^{(p)} = (\bar{s}_\alpha p_\beta)_{V-A} (\bar{p}_\beta b_\alpha)_{V-A}$$

  - QCD Penguins

$$\mathcal{O}_3 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A} \qquad \mathcal{O}_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}$$

$$\mathcal{O}_5 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \qquad \mathcal{O}_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}$$

  - Electroweak Penguins

$$\mathcal{O}_7 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q \tfrac{3e_q}{2} (\bar{q}_\beta q_\beta)_{V+A} \qquad \mathcal{O}_8 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \tfrac{3e_q}{2} (\bar{q}_\beta q_\alpha)_{V+A}$$

$$\mathcal{O}_9 = (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q \tfrac{3e_q}{2} (\bar{q}_\beta q_\beta)_{V-A} \qquad \mathcal{O}_{10} = (\bar{s}_\alpha b_\beta)_{V-A} \sum_q \tfrac{3e_q}{2} (\bar{q}_\beta q_\alpha)_{V-A}$$

- Electromagnetic and chromomagnetic dipole operators

$$\mathcal{O}_{7\gamma} = \tfrac{e}{8\pi^2} (\bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) b_\alpha) F_{\mu\nu} \quad \mathcal{O}_{8g} = \tfrac{g_s}{8\pi^2} (\bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) T^A_{\alpha\beta} b_\beta) G^A_{\mu\nu}$$

Wilson Coefficients

- Wilson coefficients $C_j(\mu)$ are determined by matching Green's functions of the effective theory and the SM (or its extension) at the electroweak scale $\mu_W = \mathcal{O}(M_W)$
- Application of the Renormalization Group Equation (RGE)

$$\mu \frac{d}{d\mu} C_j(\mu) = \gamma_{kj}(\mu)\, C_k(\mu)$$

allows to evolve $C_j(\mu)$ to the relevant low-energy scale $\mu_b = \mathcal{O}(m_b)$

- Large logarithms $\ln(\mu_W^2/\mu_b^2)$ are resummed from all orders of the perturbation series
- RGE general solution for the Wilson coefficients

$$C_j(\mu_b) = U_{jk}(\mu_b, \mu_W)\, C_k(\mu_W)$$

involves the evolution matrix $U_{jk}(\mu_b, \mu_W)$ which depends on the strong gauge coupling ratio $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$

Wilson Coefficients

- At the matching scale μ_W, Wilson coefficients can be calculated as a perturbative expansion

$$C_j(\mu_W) = \sum_{k=0}^{\infty} \left[\frac{\alpha_s(\mu_W)}{4\pi}\right]^k C_j^{(k)}(\mu_W)$$

- Neglecting QED effects, the Anomalous Dimension Matrix (ADM) $\gamma(\mu)$ has also a perturbative expansion in the strong coupling $\alpha_s(\mu)$

$$\gamma(\mu) = \sum_{k=0}^{\infty} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^{k+1} \gamma^{(k)}$$

- Hierarchy in the Wilson coefficients exists; in the naive dimentional regularization scheme at the next-to-leading logarithmic (NLL) order

$C_1(m_b)$	1.080	$C_3(m_b)$	0.011	$C_7(m_b)$	4.9×10^{-4}
$C_2(m_b)$	-0.177	$C_4(m_b)$	-0.033	$C_8(m_b)$	4.6×10^{-4}
$C_{7\gamma}(m_b)$	-0.317	$C_5(m_b)$	0.010	$C_9(m_b)$	-9.8×10^{-3}
$C_{8g}(m_b)$	0.149	$C_6(m_b)$	-0.040	$C_{10}(m_b)$	1.9×10^{-3}

CKM Factors

- A hierarchy of contributions is also possible due to CKM factors
- $b \to s$ transitions: in the improved Wolfenstein parameterization
  $\lambda_u^{(s)} = V_{ub} V_{us}^* \simeq A\lambda^4(\bar{\rho} - i\bar{\eta})$
  $\lambda_c^{(s)} = V_{cb} V_{cs}^* \simeq A\lambda^2(1 - \lambda^2/2)$
  $\lambda_t^{(s)} = V_{tb} V_{ts}^* \simeq -A\lambda^2(1 - i\lambda^2\bar{\eta})$
- The hierarchy in the CKM factors exists

$$\lambda_u^{(s)} \ll \lambda_c^{(s)},\, \lambda_t^{(s)}$$

- Applying the CKM unitarity relation

$$\lambda_u^{(s)} + \lambda_c^{(s)} + \lambda_t^{(s)} = 0$$

and neglecting doubly Cabibbo-suppressed factor $\lambda_u^{(s)}$, the Lagrangian density is left with the unique overall CKM factor; usual choice is $\lambda_t^{(s)}$

CKM Factors

- $b \to d$ transitions: in the improved Wolfenstein parameterization
  $\lambda_u^{(d)} = V_{ub}V_{ud}^* \simeq A\lambda^3(\bar\rho - i\bar\eta)$
  $\lambda_c^{(d)} = V_{cb}V_{cd}^* \simeq -A\lambda^3$
  $\lambda_t^{(d)} = V_{tb}V_{td}^* \simeq A\lambda^3(1 - \bar\rho + i\bar\eta)$
- No hierarchy: all factors are of the same order in λ
- One factor can be eliminated after the CKM unitarity relation is applied

$$\lambda_u^{(d)} + \lambda_c^{(d)} + \lambda_t^{(d)} = 0$$

- The (most popular) c- or t-convention for amplitudes is used in dependence on either $\lambda_t^{(d)}$ or $\lambda_c^{(d)}$ is eliminated

$$\begin{aligned}
\mathcal{P}^{(EW)} &= \lambda_u^{(d)}\mathcal{P}_u^{(EW)} + \lambda_c^{(d)}\mathcal{P}_c^{(EW)} + \lambda_t^{(d)}\mathcal{P}_t^{(EW)} \\
&= \lambda_u^{(d)}\left[\mathcal{P}_u^{(EW)} - \mathcal{P}_t^{(EW)}\right] + \lambda_c^{(d)}\left[\mathcal{P}_c^{(EW)} - \mathcal{P}_t^{(EW)}\right] \quad \text{c-conv.} \\
&= \lambda_u^{(d)}\left[\mathcal{P}_u^{(EW)} - \mathcal{P}_c^{(EW)}\right] + \lambda_t^{(d)}\left[\mathcal{P}_t^{(EW)} - \mathcal{P}_c^{(EW)}\right] \quad \text{t-conv.}
\end{aligned}$$

Factorization

Color-Transparancy Argument for B-Meson Decays

Since b-quark decays into energetic light quarks ($E > 1$ GeV), the produced quark-antiquark pair does not have enough time to evolve to the real size hadronic entity, but remains a small size bound state with a correspondingly small chromomagnetic moment which suppress the QCD interaction between final state mesons

[Bjorken; Brodsky and Lepage (1980)]

1. Naive Factorization Approach

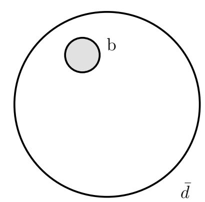
   [Bauer, Stech, Wirbel (1985)]

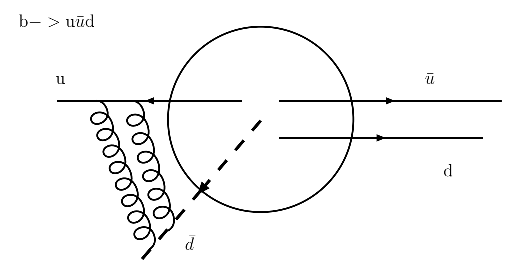
   Factorized part only was considered

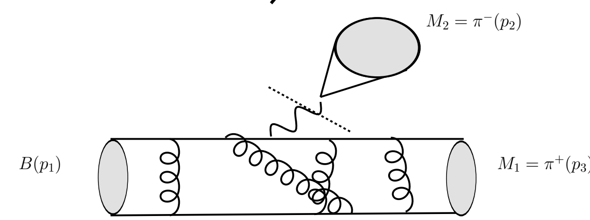
$$\langle \pi^-(p_2)\pi^+(p_3)|(\bar{d}u)_{V-A}(\bar{u}b)_{V-A}|\bar{B}^0(p_1)\rangle =$$

$$\langle \pi^-(p_2)|(\bar{d}u)_{V-A}|0\rangle\langle \pi^+(p_3)|(\bar{u}b)_{V-A}|\bar{B}^0(p_1)\rangle$$

$$f_\pi \qquad \otimes \qquad f^{B\to\pi}(q^2 = m_\pi^2)$$



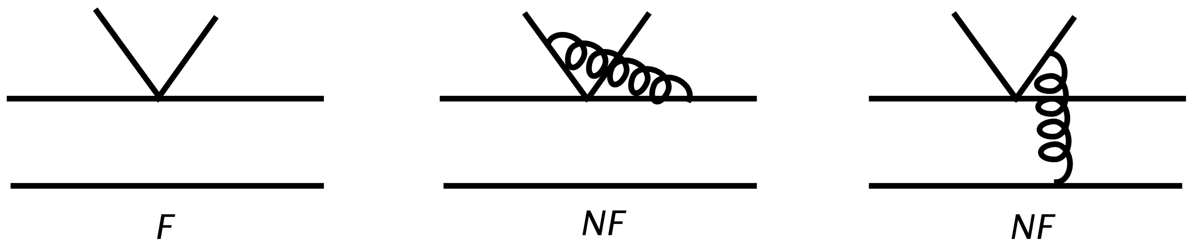




Generalized Factorization Aproach

[Cheng; Soares; Kamal et al.; Ali et al.]

- Nonfactorizable contributions were considered



- Assumption that $NF = \chi \otimes F$ was applied

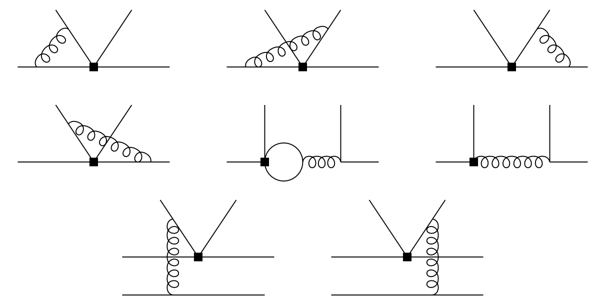
$$a_1^{\text{eff}} = C_1 + C_2 \left[\frac{1}{N_C} + \chi_1 \right] \qquad \text{for color-favored modes}$$

$$a_2^{\text{eff}} = C_2 + C_1 \left[\frac{1}{N_C} + \chi_2 \right] \qquad \text{for color-suppressed modes}$$

- Perturbative QCD corrections in the matrix elements calculated to remove scale dependence of Wilson coefficients
- Hard-spectator corrections were left out
- No proof of the factorization was provided

Basic Idea of QCD Factorization

[Beneke, Buchalla, Neubert, Sachrajada]



- Example: $\bar{B} \to \pi\pi$ decay
  Effective Hamiltonian for $b \to d$ transitions

$$\mathcal{H}_{\rm eff}^{b\to d} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pd}^* \Big[C_1 Q_1^p + C_1 Q_1^p + \sum_{i=3}^{10} C_i Q_i + C_{8g} Q_{8g} \Big]$$

- Energetic π-mesons $E_\pi \sim M_B/2$; soft gluons with momenta $\sim \Lambda_{\rm QCD}$ are decouple in $\Lambda_{\rm QCD}/M_B$

QCD Factorization for $\bar{B} \to M_1 M_2$ decay with recoil M_1 and emitted M_2 implies

- only hard interactions between $(\bar{B} M_1)$ and M_2 survive in $m_b \to \infty$ limit; soft effects are confined to $(\bar{B} M_1)$ system
- In this limit hadronic matrix elements of four-quark operators are simplified

$$\langle M_1 M_2 | j_1^{(i)} \times j_2^{(i)} | \bar{B} \rangle = \langle M_1 | j_1^{(i)} | \bar{B} \rangle \langle M_2 | j_2^{(i)} | 0 \rangle \Big[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\rm QCD}/m_b) \Big]$$

- Nonfactorizable effects are calculable
- Srtong phases $\sim \mathcal{O}(\alpha_s)$

QCD Factorization

Structure of $\mathcal{O}(\alpha_s)$ corrections:

I. Vertex corrections: $\langle M_1|j_1^{(i)}|\bar{B}\rangle \Longrightarrow f^{B\to M_1}(q^2)$; $\langle M_2|j_2^{(i)}|0\rangle \Longrightarrow f_{M_2}\phi_{M_2}(x)$

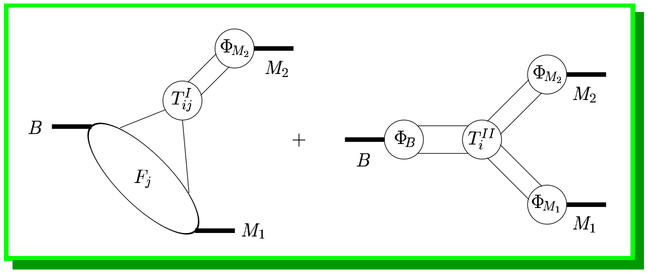
II. Hard-spectator corrections:

$$\langle M_1|j_1^{(i)}|\bar{B}\rangle \Longrightarrow f_B\phi_B(\xi)\, f_{M_1}\phi_{M_1}(u); \quad \langle M_2|j_2^{(i)}|0\rangle \Longrightarrow f_{M_2}\phi_{M_2}(x)$$

Operator Q_{8g} contributes at $\mathcal{O}(\alpha_s)$ to hard-spectator corrections only

Factorization formula for $\bar{B}\to M_1M_2$ hadronic matrix element is proven to $\mathcal{O}(\alpha_s)$ and leading twist; proof extended to all orders in α_s [Bauer et al.]

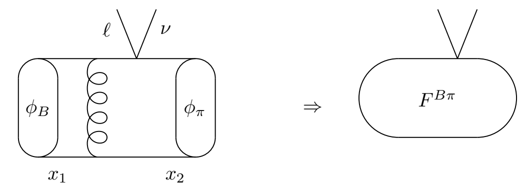
$$\langle M_1M_2|Q_i|\bar{B}\rangle = f^{B\to M_1}(m_{M_2}^2)\int_0^1 dx\, T_i^{\rm I}(x)\, f_{M_2}\phi_{M_2}(x)$$
$$+\int_0^1 dx\int_0^1 du\int d\xi\, T_i^{\rm II}(x,u,\xi)\, f_{M_1}\phi_{M_1}(x)\, f_{M_2}\phi_{M_2}(u)\, f_B\phi_B(\xi)$$



- $T_i^{\rm I}(x)$ and $T_i^{\rm II}(x,u,\xi)$ are perturbatively calculable hard-scattering kernels
- Strong phases generates through Bander, Silverman, Soni (BSS) mechanism and hard-spectator corrections
- Annihilation contributions and Charm penguins are subleading in QCD Factorization
- Infrared logarithms appear at subleading powers; Breakdown of factorization

Perturbative QCD Approach

[Keum, Li, Sanda]

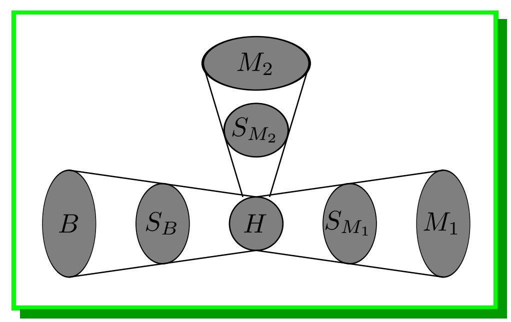


Factorization theorem

- Example: $B \to M$ transition
- Form factor $F^{BM}(q^2)$ is in the kinematic region of fast-recoil meson; soft momentum $\sim \bar{\Lambda} = M_B - m_b \sim 0.5$ GeV of spectator quark in B-meson; energetic spectator quark in the π-meson with momentum $\mathcal{O}(M_B)$; hard-gluon exchange is necessary in the leading order; off-shellness of the gluon $\sim \bar{\lambda} M_B$
- At higher orders, infinitely many gluon exchanges appear; such diagrams can generate infrared diverences: soft and collinear; they can be factorized into LCDAs of B and M-mesons, respectively; remaining finite part is the hard part
- Hard-scattering amplitudes are calculated in perturbation theory taking into account both longitudinal $\{x_i\}$ and transverse $\{\vec{k}_{Ti}\}$ momentum distributions
- Amplitude of non-leptonic B-meson decay

$$\langle M_1 M_2 | C_k(t) \mathcal{O}_k | \bar{B} \rangle = \int [dx] \int [d^2 b] \, C_k(t) \, H_k(x_i, \vec{b}_i, t)$$

$$\times S_t(x_i) \, \mathrm{e}^{-S(x_i, \vec{b}_i, t)} \, \Phi_B(x_1, \vec{b}_1) \, \Phi^*_{M_1}(x_2, \vec{b}_2) \, \Phi^*_{M_2}(x_3, \vec{b}_3)$$



Perturbative QCD Approach

End-point singularities

- Inclusion of $\vec{k}_{Ti}$ brings large double logarithms $\alpha_s \ln^2(k_{Ti}^2/M_B^2)$ through radiative corrections; should be resummed in order to improve perturbative calculations; k_T-resummation (Sudakov suppression) sets a distribution on $\vec{k}_T$ or, equvalently, on varaible $\vec{b}$, $\mathrm{e}^{-S(x_i,\vec{b}_i,t)} = \mathrm{e}^{-S_B(x_1,\vec{b}_1,t)}\mathrm{e}^{-S_{M_1}(x_2,\vec{b}_2,t)}\mathrm{e}^{-S_{M_2}(x_3,\vec{b}_3,t)}$, conjugate to $\vec{k}_T$
- Off-shellness of internal particles $\mathcal{O}(\bar{\Lambda} M_B)$ even in the end-point regions; end-point singularitires are smeared out
- Loop corrections to the weak decay vertex produce double logarithms $\alpha_s \ln^2 x_2$ where x_2 is momentum fraction associated with the spectator quark in the recoil meson
- These logarithms can be factored out from the hard part systematically and grouped into an exclusive quark jet funtcion
- If the end-point region is important, these logarithms need to be resummed (threshold resummation), $S_t(x_i) \sim [x_i(1-x_i)]^{0.3-0.4}$; decrease faster than any power of x_2 as $x_2 \to 0$ and remove singularities
- If pQCD analysis is performed self-consistently, there exist no end-point singularity

Comparison pQCD and QCD-F Approaches with Data

- Dynamical enhancement mechanism in pQCD $\longrightarrow$ Large branching fractions of PP, PV and VV modes [Keum, Li, Sanda]
- New sources of strong phases in pQCD $\longrightarrow$ Large direct CP violation [Keum, Li]

Branching fractions in $B^0 \to \pi^+\pi^-, \pi^0\pi^0$ and $B^0 \to \pi^+ K^-$ modes (in units of 10^{-6})

Modes	BELLE	BABAR	pQCD	QCD-F (BBNS)
$\pi^+\pi^-$	$4.4 \pm 0.6 \pm 0.3$	$4.7 \pm 0.6 \pm 0.2$	$5.9 \div 11.0$	$4.3 \div 14.3$
$\pi^0\pi^0$	$2.5 \pm 0.5 \pm 0.3$	$1.17 \pm 0.32 \pm 0.10$	$0.33 \div 0.65$	$0.05 \div 0.75$
π^+K^-	$18.5 \pm 1.0 \pm 0.7$	$17.9 \pm 0.9 \pm 0.7$	$12.6 \div 19.3$	$8.2 \div 31.2$

Direct CP asymmetry in $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^+ K^-$ modes (%)

Modes	BELLE	BABAR	pQCD	QCD-F (BBNS)
$\pi^+\pi^-$	$58 \pm 15 \pm 7$	$9 \pm 15 \pm 4$	$16.0 \div 30.0$	$-19.8 \div 7.2$
π^+K^-	$-10.1 \pm 2.5 \pm 0.5$	$-13.3 \pm 3.0 \pm 0.9$	$-13.0 \div -22.0$	$-5.4 \div +13.6$

SCET: Introduction

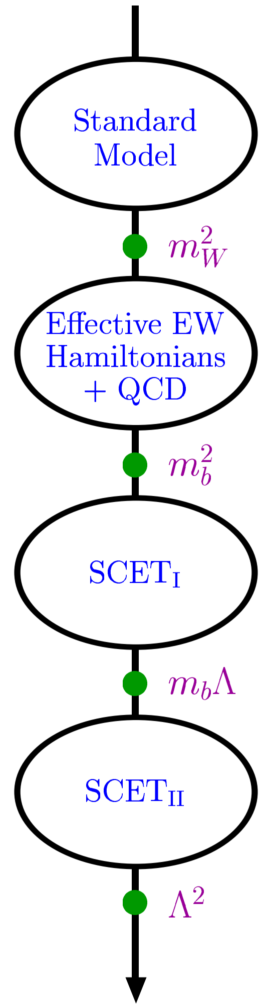
- Soft-Collinear Effective Theory (SCET) – an effective theory for energetic hadrons, $E \gg \Lambda \sim \Lambda_{\rm QCD}$
- An attempt to provide a parameterization of power-suppressed long-distance effects in strong interactions
- For a large class of processes, principal difficulty arises from collinear modes, i.e., highly energetic but almost massless particles
- SCET key idea: Separate perturbative and non-perturbative scales at the level of operators
- SCET involves typically three scales: Q, $Q\lambda$ and $Q\lambda^2$, where Q is a hard scale and $\lambda \ll 1$ is an expansion parameter $\lambda = \sqrt{\Lambda/Q}$ or $\lambda = \Lambda/Q$ depending on the process
- Two formulations of SCET exist:

1. Hybrid momentum-position representation
   [Bauer, Pirjol, Stewart, Fleming, Luke]
2. Conventional position-space representation
   [Beneke, Chapovsky, Diehl, Feldmann; Neubert, Hill]

SCET: Introduction

Defining Effective Field Theories for B-Meson Decays

- $\mu^2 \sim m_W^2$: $A(B \to f) \sim C_i(\mu)\, \langle f|\mathcal{O}_i|\bar{B}\rangle$
  - Matching Standard Model onto effective electroweak Hamiltonians; fluctuations $\sim m_t^2,\ m_Z^2,\ m_W^2$ integrated out $\Longrightarrow$ Wilson coefficients $C_i(\mu)$
  - Anomalous dimensions of $\mathcal{O}_i$ calculated
  - Effective theory evolves down to $\mu \sim m_b$ by RG method
- $\mu^2 \sim m_b^2$: $A(B \to f) \sim \int d\omega_i\, c_j(\omega_i, \mu)\, \langle f|Q_j^{(0)}(\omega_i)|\bar{B}\rangle + \ldots$
  - Matching effective electroweak Hamiltonians onto $\mathrm{SCET_I}$; fluctuations $\sim m_b^2$ integrated out; $\mathrm{SCET_I}$ contains hard-collinear $p_{hc}^2 \sim m_b\Lambda$ and soft $p_s^2 \sim \Lambda^2$ fields $\Longrightarrow$ Wilson coefficients $c_j(\omega_i, \mu)$
  - Anomalous dimensions of $Q_j^{(0)}, \ldots$ calculated
  - Effective theory evolves down to $\mu \sim \sqrt{m_b\Lambda}$ by RG method



SCET: Introduction

Basic Idea of Factorization in B-Meson Decays

- $\mu^2 \sim m_b \Lambda$:

  $A(B \to f) \sim \int d\omega_i \int dk_m \, c_j(\omega_i) \, J_j(\omega_i, k_m) \, \langle f | \tilde{Q}_j^{(0)}(k_m) | \bar{B} \rangle + \ldots$

  – Matching $\mathrm{SCET_I}$ onto $\mathrm{SCET_{II}}$; fluctuations $\sim m_b \Lambda$ integrated out

  – WCs, $J_j(\omega_i, k_m)$, called jet functions, are either non-perturbative or perturbative expansion in $\alpha_s(m_b \Lambda)$

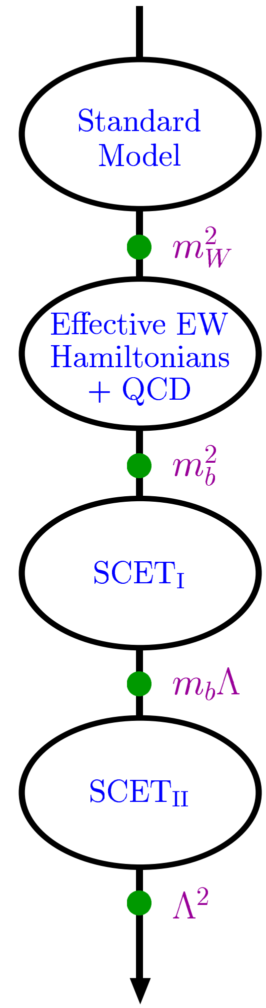
  – Number of $J_j(\omega_i, k_m)$ is restricted by symmetry

  – $\mathrm{SCET_{II}}$ operators and jet functions evolve down to $\mu \sim \Lambda$

- $\mu^2 \sim \Lambda^2$: $\langle f | \tilde{Q}_j^{(0)}(k_m) | \bar{B} \rangle$ factorizes into non-perturbative objects

  – Collinear and soft degrees of freedom decouple $\Longrightarrow$ Factorization

  – Matrix elements are identified with LCDAs, transition FFs, shape functions, etc



SCET Formalism

- SCET formulated in terms of Light-cone variables:

$$a_\mu = (n \cdot a)\,\frac{\bar{n}_\mu}{2} + (\bar{n} \cdot a)\,\frac{n_\mu}{2} + a_\mu^\perp = (a^+, a^-, a^\perp)$$
$$n^2 = \bar{n}^2 = 0, \quad (n \cdot \bar{n}) = 2, \quad (n \cdot a^\perp) = (\bar{n} \cdot a^\perp) = 0$$

- SCET_{I} $\longrightarrow$ Energetic jets $\Lambda^2 \ll Q\Lambda \ll Q^2$

hard-collinear $p_{hc}^\mu \sim Q\,(\lambda^2, 1, \lambda)$ $p_{hc}^2 \sim Q\Lambda$
semi-hard $p_{sh}^\mu \sim Q\,(\lambda, \lambda, \lambda)$ $p_{sh}^2 \sim Q\Lambda$ $\lambda = \sqrt{\Lambda/Q}$
soft $p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$ $p_s^2 \sim \Lambda^2$

- SCET_{II} $\longrightarrow$ Energetic hadrons $\Lambda^2 \ll Q^2$

collinear $p_c^\mu \sim Q(\lambda'^2, 1, \lambda')$ $p_c^2 \sim \Lambda^2$
soft $p_s^\mu \sim Q(\lambda', \lambda', \lambda')$ $p_s^2 \sim \Lambda^2$ $\lambda' = \lambda^2 = \Lambda/Q$

SCET: Formalism

Lagrangian of the Effective Theory for B-Meson Decays

- Lagrangian consists of three parts:

$$\mathcal{L} = \mathcal{L}_{\rm HQET} + \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm SCET}$$

- $\mathcal{L}_{\rm HQET} = \bar{h}_v^{(b)} i(v \cdot D_s) h_v^{(b)}$ is the usual HQET Lagrangian
- $\mathcal{L}_{\rm YM}$ is the λ-expanded pure Yang-Mills Lagrangian

$$\mathcal{L}_{\rm YM} = -\frac{1}{2}\,{\rm Tr}\left(F_c^{\mu\nu} F_{\mu\nu}^c\right) - \frac{1}{2}\,{\rm Tr}\left(F_s^{\mu\nu} F_{\mu\nu}^s\right) + \mathcal{L}_{\rm YM}^{(1)} + O(\lambda^2)$$

- $\mathcal{L}_{\rm SCET}$ is the light-quark Lagrangian [$\xi_n = (\not{n}\not{\bar{n}}/4)\, q_{c,n}$]

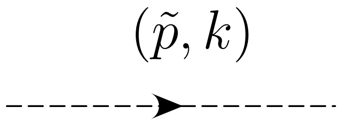
$$\mathcal{L}_{\rm SCET} = \bar{\xi}_n \left[i(n \cdot D) + i\not{D}_c^{\perp} \frac{1}{i(\bar{n} \cdot D_c)}\, i\not{D}_c^{\perp} \right] \frac{\not{\bar{n}}}{2}\, \xi_n + \bar{q}_s\, i\not{D}_s\, q_s + \mathcal{L}_{\xi\xi}^{(1)} + \mathcal{L}_{\xi q}^{(1)} + O(\lambda^2)$$

Power-suppresed interaction terms are (W_c is a collinear Wilson line)

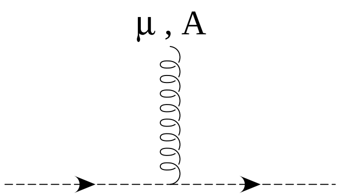
$$\mathcal{L}_{\xi\xi}^{(1)} = \bar{\xi}_n \left[x_\perp^\mu n^\nu W_c g F_{\mu\nu}^s W_c^\dagger \right] \frac{\not{\bar{n}}}{2}\, \xi_n, \qquad \mathcal{L}_{\xi q}^{(1)} = \bar{q}_s W_c^\dagger i\not{D}_c^{\perp} \xi_n - \bar{\xi}_n i \overleftarrow{\not{D}}_c^{\perp} W_c q_s$$

SCET: Formalism

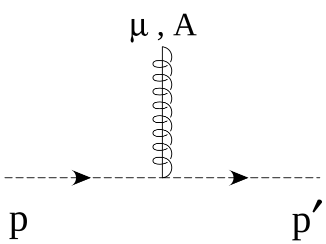
$O(\lambda^0)$ Feynman Rules for collinear quarks and soft/collinear gluons



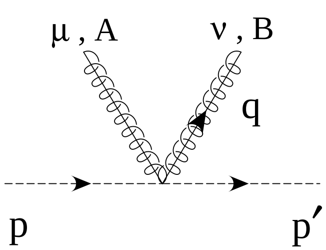
$$= i\,\frac{\not n}{2}\,\frac{\bar n\cdot p}{n\cdot k\,\bar n\cdot p + p_\perp^2 + i\epsilon}$$



$$= i g\, T^A\, n_\mu\, \frac{\bar{\not n}}{2}$$



$$= i g\, T^A \left[n_\mu + \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p} + \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \right] \frac{\bar{\not n}}{2}$$

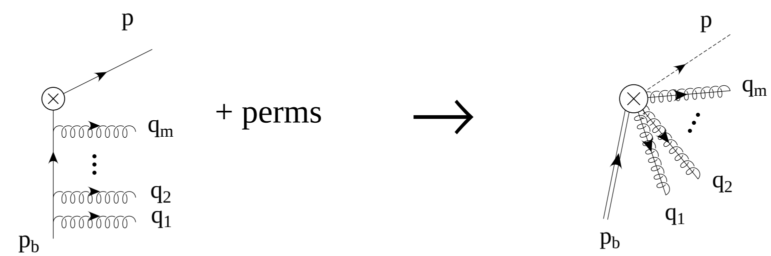


$$= \frac{i g^2\, T^A\, T^B}{\bar n\cdot(p-q)} \left[\gamma_\mu^\perp \gamma_\nu^\perp - \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\nu - \frac{\not p'_\perp \gamma_\nu^\perp}{\bar n\cdot p'}\bar n_\mu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\bar{\not n}}{2}$$

$$+ \frac{i g^2\, T^B\, T^A}{\bar n\cdot(q+p')} \left[\gamma_\nu^\perp \gamma_\mu^\perp - \frac{\gamma_\nu^\perp \not p_\perp}{\bar n\cdot p}\bar n_\mu - \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'}\bar n_\nu + \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n_\mu \bar n_\nu \right] \frac{\bar{\not n}}{2}$$

Matching of Heavy-to-Light Currents

- Naively,
  $(\bar{q}\Gamma b) \to C(\mu)\,(\bar{\xi}^{(q)}_{n,p}\Gamma h^{(b)}_v)$
- An arbitrary number of gluonic fields $(\bar{n}\cdot A_{n,q}) \sim \lambda^0$ can be included without power suppression



- In the position space, the leading-order heavy-to-light current in SCET for $Q\lambda < \mu < Q$ is

$$(\bar{q}\Gamma b) \to C(\mu)\,(\bar{\chi}^{(q)}_n \Gamma h^{(b)}_v)$$

- The jet field of collinear particles is introduced

$$\chi^{(q)}_n(0) \equiv W_c^\dagger \xi^{(q)}_n(0) = P\exp\left[-ig\int_{-\infty}^{0} ds\;(\bar{n}\cdot A_c(s\bar{n}))\right]\xi^{(q)}_n(0)$$

  Here, P denotes path ordering along the $\bar{n}$ direction
- $\chi^{(q)}_n(0)$ is gauge-invariant under the collinear gauge transformations
  $\Longrightarrow$ $(\bar{\chi}^{(q)}_n \Gamma h^{(b)}_v)$ is gauge-invariant under these transformations
- $(\bar{\chi}^{(q)}_n \Gamma h^{(b)}_v)$ is also gauge-invariant under soft gauge transformations

SCET: Formalism

Matching of Heavy-to-Light Currents

- List of heavy-to-light currents (valid to all orders in α_s and LO in λ)

$$
\begin{aligned}
(\bar{q}b) &\to C^{(S)}(\mu)\big[\bar{\chi}_n^{(q)} h_v^{(b)}\big] \\
(\bar{q}\gamma_5 b) &\to C^{(P)}(\mu)\big[\bar{\chi}_n^{(q)}\gamma_5 h_v^{(b)}\big] \\
(\bar{q}\gamma_\nu b) &\to C_1^{(V)}(\mu)\big[\bar{\chi}_n^{(q)}\gamma_\nu^\perp h_v^{(b)}\big] + \Big\{C_2^{(V)}(\mu)\, n_\nu + C_3^{(V)}(\mu)\, v_\nu\Big\}\big[\bar{\chi}_n^{(q)} h_v^{(b)}\big] \\
(\bar{q}\gamma_\nu\gamma_5 b) &\to C_1^{(A)}(\mu)\, i\epsilon_{\nu\rho}^\perp\big[\bar{\chi}_n^{(q)}\gamma^{\perp\rho} h_v^{(b)}\big] + \Big\{C_2^{(A)}(\mu)\, n_\nu + C_3^{(A)}(\mu)\, v_\nu\Big\}\big[\bar{\chi}_n^{(q)}\gamma_5 h_v^{(b)}\big] \\
(\bar{q}i\sigma_{\nu\rho} b) &\to C_1^{(T)}(\mu)\,(v_\nu n_\rho - v_\rho n_\nu)\big[\bar{\chi}_n^{(q)} h_v^{(b)}\big] + C_2^{(T)}(\mu)\, i\epsilon_{\nu\rho}^\perp\big[\bar{\chi}_n^{(q)}\gamma_5 h_v^{(b)}\big] \\
&+ \Big\{C_3^{(T)}(\mu)\,[n_\nu g_{\rho\lambda} - n_\rho g_{\nu\lambda}] + C_4^{(T)}(\mu)\,[v_\nu g_{\rho\lambda} - v_\rho g_{\nu\lambda}]\Big\}\big[\bar{\chi}_n^{(q)}\gamma^{\perp\lambda} h_v^{(b)}\big]
\end{aligned}
$$

Here, $\gamma_\mu^\perp = \gamma_\mu - n^\mu \bar{n}\!\!\!/ /2 - \bar{n}^\mu n\!\!\!/ /2$ and $\epsilon_{\mu\nu}^\perp = \epsilon_{\mu\nu\rho\sigma} v^\rho n^\sigma$

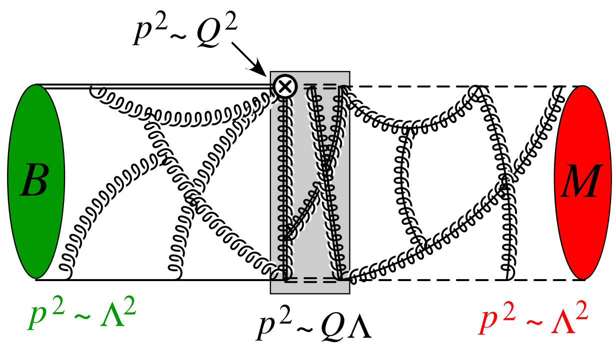
- At tree level, matching gives

$$C^{(S)}(m_b) = C^{(P)}(m_b) = C_{1,2}^{(V)}(m_b) = C_{1,2}^{(A)}(m_b) = C_{1,2,3}^{(T)}(m_b) = 1,$$

$$C_3^{(V)}(m_b) = C_3^{(A)}(m_b) = C_4^{(T)}(m_b) = 0$$

- Matching at one-loop level is also done [Bauer et al., hep-ph/0011336]

$B \to M$ Transition Form Factors



Proof of the factorization in $B \to M$
[Bauer, Pirjol, Stewart, hep-ph/0211069]
[Beneke, Feldmann, hep-ph/0311335]
[Becher, Hill, Lange, Neubert, hep-ph/0211069]

Result at leading-order in Λ/Q, where $Q = \{m_B, E_M\}$, and all orders in α_s
Pseudoscalar FF: $f_+, \; f_0, \; f_T$ Vector FF: $V, \; A_0, \; A_1, \; A_2, \; T_1, \; T_2, \; T_3$

$$F(E) = F^{\rm NF}(E) + F^{\rm F}(E) = C(m_b, E)\, \zeta^{BM}(Q\Lambda, \Lambda^2) + \int_0^1 dz\, T(z, E, m_b)\, \zeta_J^{BM}(z, E)$$

If jet function $J(z, u, k^+, E)$ is known, $\zeta_J^{BM}(z, E)$ can be estimated by

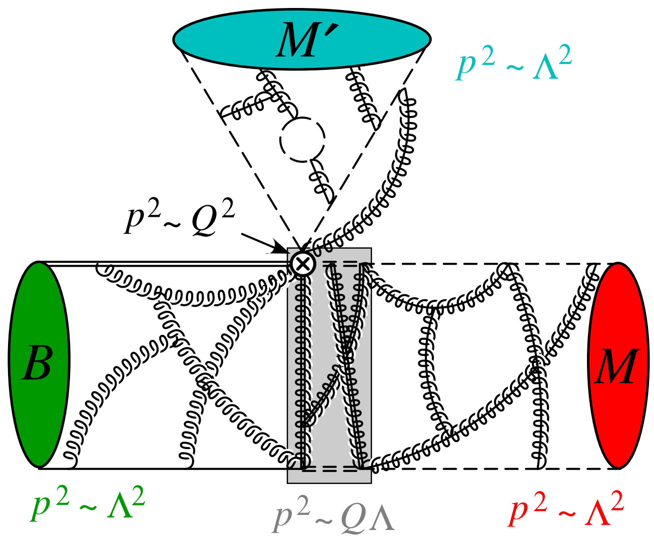
$$\zeta_J^{BM}(z, E) = \frac{f_B f_M m_B}{4E^2} \int_0^1 du \int_0^\infty dk^+\, J(z, u, k^+, E)\, \phi_M(u)\, \phi_B^+(k^+)$$

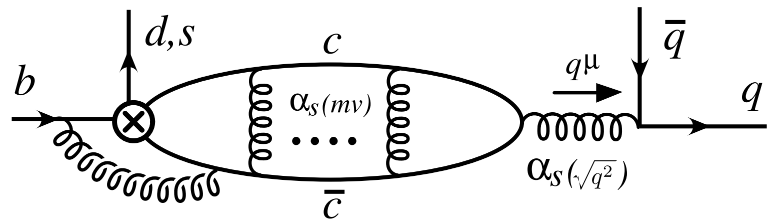
Recent progress:

- one-loop matching: $C(m_b, E)$, $T(z, E, m_b)$, $J(z, u, k^+, E)$
- leading-logarithm resummation

$B \to M_1 M_2$ Factorization in SCET

[Chey, Kim, hep-ph/0301262]
[Bauer, Pirjol, Rothstein, Stewart, hep-ph/0401188]





- form-factor and hard-spectator terms are formally the same as in QCD Factorization Approach
- long-distance charming-penguin contribution appears in LO

$$\Lambda^2 \ll Q\lambda \ll Q^2, \quad Q = \{m_b, E, m_c\}$$

Operators in $B \to \pi\pi$ Decay

Effective weak Hamiltonian for $b \to d$ transition & QCD Lagrangian

$$\mathcal{H}_W^{b\to d} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left(C_1\, \mathcal{O}_1^{(p)} + C_2\, \mathcal{O}_2^{(p)} + \sum_{i=3}^{10} C_i\, \mathcal{O}_i + C_{7\gamma}\, \mathcal{O}_{7\gamma} + C_{8g}\, \mathcal{O}_{8g} \right)$$

SCET$_\mathrm{I}$ Hamiltonian, fluctuations $\sim m_b$ are integrated out

$$\mathcal{H}_W^{b\to d} = \frac{2G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{6} \sum_{j=1}^{3} \int d\omega_j\, c_i^{(d)}(\omega_j)\, Q_{id}^{(0)}(\omega_j) + \sum_{i=1}^{8} \sum_{j=1}^{4} \int d\omega_j\, b_i^{(d)}(\omega_j)\, Q_{id}^{(1)}(\omega_j) + \mathcal{Q}_{\bar{c}c} + \ldots \right\}$$

$$Q_{1d}^{(0)}(\omega_j) = \left[\bar{u}_{n,\omega_1} \bar{\not{n}} P_L b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L u_{\bar{n},\omega_3}\right], \ldots$$

$$Q_{1d}^{(1)}(\omega_j) = \frac{-2}{m_b} \left[\bar{u}_{n,\omega_1} ig \not{\mathcal{B}}_{n,\omega_4}^{\perp} P_L b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L u_{\bar{n},\omega_3}\right], \ldots$$

$$Q_{7d}^{(1)}(\omega_j) = \frac{-2}{m_b} \left[\bar{u}_{n,\omega_1} ig \mathcal{B}_{n,\omega_4}^{\perp\mu} P_L b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} \gamma_\mu^{\perp} P_L u_{\bar{n},\omega_3}\right], \ldots$$

$B \to M_1 M_2$ Factorization Formula in SECT$_{\rm II}$

[Bauer et al., hep-ph/0401188]

$$A(B \to M_1 M_2) = A_{\bar{c}c}^{M_1 M_2} + \frac{G_F}{\sqrt{2}} m_B^2 \int_0^1 du \left\{ f_{M_2}\, \phi_{M_2}(u) \left[\zeta^{BM_1} T_{2\zeta}(u) + \frac{f_B f_{M_1}}{m_B} \int_0^1 dz \int_0^1 dx \int_0^\infty dk^+\, T_{2J}(z,u)\, J(z,x,k^+)\, \phi_{M_1}(x)\, \phi_B^+(k^+) \right] + (1 \leftrightarrow 2) \right\}$$

SCET$_{\rm II}$ results the same jet function $J(z,x,k^+)$ as in $B \to M$ transition

New non-perturbative result in $\alpha_s(\sqrt{Q\Lambda})$, where $\zeta^{BM} \sim \zeta_J^{BM}(z) \sim (\Lambda/Q)^{3/2}$

$$A(B \to M_1 M_2) = A_{\bar{c}c}^{M_1 M_2} + \frac{G_F}{\sqrt{2}} m_B^2 \int_0^1 du \left\{ f_{M_2}\, \phi_{M_2}(u) \times \left[\zeta^{BM_1} T_{2\zeta}(u) + \int_0^1 dz\, \zeta_J^{BM_1}(z)\, T_{2J}(u,z) \right] + (1 \leftrightarrow 2) \right\}$$

For comparison, QCD Factorization Approach gives

$$A(B \to M_1 M_2) \sim \int_0^1 du \left\{ F^{B\to M_1}(0)\, T^{\rm I}_{M_2}(u)\, f_{M_2}\, \phi_{M_2}(u) + F^{B\to M_2}(0)\, T^{\rm I}_{M_1}(u)\, f_{M_1}\, \phi_{M_1}(u) \right\} + \int_0^1 du \int_0^1 dx \int_0^\infty dk^+\, T^{\rm II}(u,x,k^+)\, f_{M_1}\, \phi_{M_1}(u)\, f_{M_2}\, \phi_{M_2}(x)\, f_B\, \phi_B^+(k^+)$$

$B \to \pi\pi$ Decay Amplitudes

[Bauer et al., hep-ph/0401188]

In terms of SCET Wilson coefficients $c_i^{(d)}(u)$ and $b_i^{(d)}(u, z)$

$$A(B^- \to \pi^-\pi^0) = \frac{G_F}{2}\, m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_1^{(d)}(u) + c_2^{(d)}(u) - c_3^{(d)}(u) \right] + \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_1^{(d)}(u,z) + b_2^{(d)}(u,z) - b_3^{(d)}(u,z) \right] \Big\}$$

$$A(\bar{B}^0 \to \pi^+\pi^-) = \frac{G_F}{\sqrt{2}}\, m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_1^{(d)}(u) + c_4^{(d)}(u) \right] + \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_1^{(d)}(u,z) + b_4^{(d)}(u,z) \right] \Big\} + \lambda_c^{(d)}\, A_{\bar{c}c}^{B\pi}$$

$$A(\bar{B}^0 \to \pi^0\pi^0) = \frac{G_F}{\sqrt{2}}\, m_B^2 \int_0^1 du\, f_\pi \phi_\pi(u) \Big\{ \zeta^{B\pi} \left[c_2^{(d)}(u) - c_3^{(d)}(u) - c_4^{(d)}(u) \right] + \int_0^1 dz\, \zeta_J^{B\pi}(z) \left[b_2^{(d)}(u,z) - b_3^{(d)}(u,z) - b_4^{(d)}(u,z) \right] \Big\} - \lambda_c^{(d)}\, A_{\bar{c}c}^{B\pi}$$

In agreement with isospin relation

$$\sqrt{2}\, A(B^- \to \pi^-\pi^0) = A(\bar{B}^0 \to \pi^+\pi^-) + A(\bar{B}^0 \to \pi^0\pi^0)$$

SCET: List of Applications

Processes described by SCET [Stewart, hep-ph/0308185]

Process	Degrees of Freedom (p^2)	Non.-Pert. Functions
$B^0 \to D^+\pi^-, \ldots$	c (Λ^2), s (Λ^2)	$\xi(w)$, $\phi_\pi(u)$
$B^0 \to D^0\pi^0, \ldots$	hc $(Q\lambda)$, c (Λ^2), s (Λ^2)	$S(k_j^+)$, $\phi_\pi(u)$
$B \to X_s^{\rm end}\gamma$	hc $(Q\lambda)$, s (Λ^2)	$f(k^+)$
$B \to X_u^{\rm end}\ell\nu_\ell$	hc $(Q\lambda)$, s (Λ^2)	$f(k^+)$
$B \to \gamma\ell\nu_\ell$	hc $(Q\lambda)$, s (Λ^2)	$\phi_B(k^+)$
$B \to \gamma\gamma$	hc $(Q\lambda)$, s (Λ^2)	$\phi_B(k^+)$
$B \to \pi\ell\nu_\ell, \ldots$	hc $(Q\lambda)$, s (Λ^2), c (Λ^2)	$\phi_B(k^+)$, $\phi_\pi(u)$, $\zeta^{B\pi}(E)$
$B \to \pi\pi, \ldots$	hc $(Q\lambda)$, s (Λ^2), c (Λ^2)	$\phi_B(k^+)$, $\phi_\pi(u)$, $\zeta^{B\pi}(E)$
$B \to K^*\gamma, \ldots$	hc $(Q\lambda)$, s (Λ^2), c (Λ^2)	$\phi_B(k^+)$, $\phi_{K^*}(u)$, $\zeta_\perp^{(K^*)}(E)$
$e^-p \to e^-X$	c (Λ^2)	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$
$e^-\gamma \to e^-\pi^0$	c (Λ^2), s (Λ^2)	$\phi_\pi(u)$
$\gamma^* M \to M'$	c (Λ^2), s (Λ^2)	$\phi_M(u)$, $\phi_{M'}(u')$
$e^+e^- \to J/\psi X^{\rm end}$	hc $(Q\Lambda)$, s (Λ^2)	$S^{(8,n)}(k^+)$
$\Upsilon \to X^{\rm end}\gamma$	hc $(Q\Lambda)$, s (Λ^2)	$S^{(8,n)}(k^+)$

Summary – I

- Experiment requires a deeper understanding of QCD dynamics in hadronic B-meson decays
- Several theoretical approaches (QCD-F, pQCD, SCET, etc) are proposed and experimentally tested
- SCET – the emerging QCD technology, hold the promise to provide a better theoretical description of B-meson hadronic decays than existing approaches, remarkable progress and activity in evidence