Flavour Physics and CP Violation

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(III)
Lecture III

• Rare Decays:
  – Example: $B_{s,d} \rightarrow \mu^+ \mu^-$

• How Could New Physics Enter in the Roadmap of Quark-Flavour Physics?

• What about New Physics in $B_d \rightarrow J/\psi K_S$?

• Challenging the Standard Model through $B_d \rightarrow \phi K_S$

• The $B \rightarrow \pi\pi, \pi K$ Puzzles & Rare $K$ and $B$ Decays:
  
  \begin{center}
  \textbf{Example of a systematic strategy to search for NP}
  \end{center}

  1. “$B \rightarrow \pi\pi$ puzzle”
  2. “$B \rightarrow \pi K$ puzzle”
  3. Connection with rare $K$ and $B$ decays
Rare $B$ Decays → complement CP-B!

- These processes originate from $b \to s$ or $b \to d$ flavour-changing neutral current transitions, i.e. do not receive tree contributions in the SM:
  - $B \to K^*\gamma, B \to \rho\gamma, ...$
  - $B \to K^*\mu^+\mu^-, B \to \rho\mu^+\mu^-, ...$
  - $B_{s,d} \to \mu^+\mu^-$

  ⊕ inclusive decays: $b \to s\gamma, b \to s\ell^+\ell^-, ...$

- Characteristic features in the SM:
  - Exhibit small branching ratios at the $10^{-4}...10^{-10}$ level.
  - Do not – apart from $B \to \rho\gamma$ – show sizeable CP violation in the SM.

Important probes to search for new physics!

[Many reviews: Ali; Buras; Greub; Hurth; Mannel; Misiak; ...]
A More Detailed Example: $B_{s,d} \rightarrow \mu^+\mu^-$

- **Originate from $Z$ penguins and box diagrams in the SM:**

  \[
  \begin{array}{c}
  s, d \xrightarrow{W} t \xrightarrow{Z} \mu \\
  b \quad \quad t \quad \quad \mu \\
  \end{array}
  \]

- **Belong to the cleanest decays in the field of rare $B$ decays:**
  - Only the matrix element of a quark current is required: $f_{Bq}$
  - NLO QCD corrections were calculated.
  - Long-distance contributions are expected to be negligible.


- **Branching ratios in the SM:** $\rightarrow$ LHC

  \[
  \text{BR}(B_s \rightarrow \mu^+\mu^-) = 4.1 \times 10^{-9} \left[ \frac{f_{B_s}}{0.24 \text{ GeV}} \right]^2 \left[ \frac{m_t}{167 \text{ GeV}} \right]^{3.12} \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left[ \frac{\tau_{B_s}}{1.5 \text{ ps}} \right]
  \]

  \[
  \text{BR}(B_d \rightarrow \mu^+\mu^-) : \quad s \rightarrow d \quad \Rightarrow \quad \mathcal{O}(10^{-10})
  \]

• Current experimental upper bounds:

\[
\text{BR}(B_s \to \mu^+\mu^-) < 5.0 \times 10^{-7} \quad \text{[D0 @ 95\% C.L. ('04)]}
\]
\[
\text{BR}(B_d \to \mu^+\mu^-) < \begin{cases} 
8.3 \times 10^{-8} & \text{[BaBar @ 90\% C.L. ('04)]} \\
16 \times 10^{-8} & \text{[Belle @ 90\% C.L. ('03)]}
\end{cases}
\]

• \(f_{B_s}\) and \(f_{B_d}\), which can be fixed through non-perturbative methods or leptonic \(B_{s,d}\) decays, would allow extractions of \(|V_{ts}|\) and \(|V_{td}|\).

• Relations: \[
\frac{\text{BR}(B_d \to \mu^+\mu^-)}{\text{BR}(B_s \to \mu^+\mu^-)} = \left[ \frac{\tau_{B_d}}{\tau_{B_s}} \right] \left[ \frac{M_{B_d}}{M_{B_s}} \right] \left[ \frac{f_{B_d}}{f_{B_s}} \right]^2 \left[ \frac{V_{td}}{V_{ts}} \right]^2
\]

\[
\frac{\Delta M_d}{\Delta M_s} = \left[ \frac{M_{B_d}}{M_{B_s}} \right] \left[ \frac{f_{B_d}}{f_{B_s}} \right]^2 \left[ \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \right] \left[ \frac{V_{td}}{V_{ts}} \right]^2
\]

“bag” parameters

⇒ complementary determinations of the UT side \(R_t\)! Moreover:

\[
\frac{\text{BR}(B_s \to \mu^+\mu^-)}{\text{BR}(B_d \to \mu^+\mu^-)} = \left[ \frac{\tau_{B_s}}{\tau_{B_d}} \right] \left[ \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \right] \left[ \frac{\Delta M_s}{\Delta M_d} \right]
\]

... exhibits smaller theoretical uncertainties since \((f_{B_s}/f_{B_d})^2\) cancels and \(\hat{B}_{B_s}/\hat{B}_{B_d} = 1\) up to tiny \(SU(3)\)-breaking corrections! [Buras (2003)]
How Could New Physics Enter?
Twofold Impact of NP: Effective Hamiltonians ...

- **Possibility I:** Modification of the “Strength” of the SM Operators
  
  - New short-distance functions, which depend on the NP parameters, such as masses of charginos, squarks, \( \tan \beta \equiv v_2/v_1 \) in the MSSM.
  
  - The NP particles enter in new box and penguin diagrams, and are “integrated out”, as the \( W \) and top (see Lecture I):
    \[
    C_k(\mu = M_W) \rightarrow C_k^{\text{SM}} + C_k^{\text{NP}}
    \]
    initial conditions for RG evolution
  
  - The \( C_k^{\text{NP}} \) may also involve new CP-violating phases.

- **Possibility II:** New Operators
  
  - Operators, which are absent or strongly suppressed in the SM, may actually play an important rôle:
    \[
    \{Q_k\} \rightarrow \{Q_k^{\text{SM}}, Q_i^{\text{NP}}\}
    \]
    operator basis
  
  - In general, new sources of flavour and CP violation.
Classification of New Physics

- **Class A:** Models with *Minimal Flavour Violation*

  - The flavour-changing processes are governed by the CKM matrix, in particular no new sources for CP violation, and the only relevant operators are those present in the SM.
  - NP enters therefore only in the Wilson coefficients of the SM operators through new particles in loops.
  - The short-distance structure involves only 7 “master functions”:
    \[ S(v), X(v), Y(v), Z(v), E(v), D'(v), E'(v). \]
  - Interesting tests of this scenario through correlations! Example:
    \[ \text{BR}(B_{d,s} \rightarrow \mu^+ \mu^-) \propto Y_0(x_t)^2 \rightarrow Y(v)^2 \]
    \[ \Delta M_{d,s} \propto S_0(x_t) \rightarrow S(v) \]
    \[ \Rightarrow \text{relations of the } B_{s,d} \rightarrow \mu^+ \mu^- \text{ discussion are still valid!} \]
  - Examples: THDM-II and the constrained MSSM if $\tan \bar{\beta}$ is not too large, models with one extra universal dimension.
• **Class B:**

  – In contrast to Class A, new operators arise. However, there are still no new CP-violating phases beyond the CKM matrix present.
  – Typical examples of new Dirac structures:

    \[(V - A) \otimes (V + A), (S - P) \otimes (S \pm P), \sigma_{\mu\nu}(S - P) \otimes \sigma^{\mu\nu}(S - P),\]

    which correspond to contributions to \(B^0_d, s - \bar{B}^0_d, s\) mixing that become relevant in the MSSM with large \(\tan \beta\).

• **Class C:**

  – Differs from Class A through new CP-violating phases in the Wilson coefficients of the usual SM operators, but we have still negligible contributions from new operators:

    \[C^{NP}_k \rightarrow \text{complex!}\]

  – Example: MSSM with \(\tan \beta\) not too large and with non-diagonal elements in the squark mass matrices.
• **Class D:**

  – Models with new complex phases, new operators and new flavour-changing contributions that are not described by the CKM matrix:

    \[\rightarrow\text{general case, i.e. very involved!}\]

  – Examples: multi-Higgs models with complex phases in the Higgs sector, general SUSY models, models with spontaneous CP violation and left–right-symmetric models.

• **Class E:**

  – The three-generation CKM matrix is not unitary:

    \[\rightarrow\text{unitarity triangle does not close!}\]

  – Example: models with four generations.

[Classification by A.J. Buras, hep-ph/0402191 → more details]
A Brief Roadmap of Quark-Flavour Physics

• CP-B studies through various processes and strategies:

\[ B \to \pi\pi \text{ (isospin), } B \to \rho\pi, B \to \rho\rho \]

\[ R_b \left( b \to u, c\ell\bar{\nu}_\ell \right) \]

\[ R_t \left( B^0_q-\bar{B}^0_q \text{ mixing} \right) \]

\[ B \to \pi K \text{ (penguins)} \]

\[ \left\{ \begin{array}{l} B_u \to K^\pm D \\ B_d \to K^{*0} D \\ B_c \to D_s^\pm D \end{array} \right\} \text{ only trees} \]

\[ B_d \to D^{(*)}\pm\pi^\mp : \gamma + 2\beta \]

\[ B_s \to D_s^\pm K^\mp : \gamma + \phi_s \]

• Moreover “rare” decays: \( B \to K^*\gamma, B_{d,s} \to \mu^+\mu^-, K \to \pi\nu\bar{\nu}, \ldots \)
  
  – Originate from loop processes in the SM.
  
  – Interesting correlations with CP-B studies.

\[
\begin{array}{c|c}
\text{New Physics} & \Rightarrow \text{Discrepancies}
\end{array}
\]
Avenues for New Physics to Manifest Itself...

- $B_q^0 - \overline{B}_q^0$ mixing:

  - Exchange of NP particles in boxes or new tree contributions:
    \[
    \Delta M_q = \Delta M_q^{\text{SM}} + \Delta M_q^{\text{NP}} \quad (\rightarrow R_t)
    \]
    \[
    \phi_q = \phi_q^{\text{SM}} + \phi_q^{\text{NP}} \quad (\rightarrow A_{\text{CP}}^{\text{mix}})
    \]

  - $B_d$ system: data from the $B$ factories are available (see below).
  - $B_s$ system: essentially unexplored $\rightarrow$ LHCb!

- Decay Amplitudes:

  - Typically small effects if SM tree processes play the dominant rôle.
  - Potentially large effects in the penguin sector through new particles in the loop diagrams or new contributions at the tree level.

- Corresponding hints in the current $B$-factory data:

  - ♦ $B_d \rightarrow \phi K_S$: $(\sin 2\beta)_{\phi K_S} \equiv (\sin 2\beta)_{\psi K_S}$
  - ♥ $B \rightarrow \pi K$: puzzling pattern of certain branching ratios!
What About

New Physics in

\[ B_d \rightarrow J/\psi K_S \]?
A possible loop hole, but ...

• **Lecture II:** → impressive agreement between $A_{CP}^{\text{mix}}(B_d \rightarrow J/\psi K_S)$ and the CKM fits for $\sin 2\beta$. Nevertheless, NP could still be hiding there...

• However, the key quantity is actually:

\[
\phi_d = \phi_d^{\text{SM}} + \phi_d^{\text{NP}} = 2\beta + \phi_d^{\text{NP}}
\]

• $(\sin \phi_d)_S = 0.725 \pm 0.037$: \[\Rightarrow \phi_d = (46.5^{+3.2}_{-3.0})^\circ \lor (133.5^{+3.0}_{-3.2})^\circ\]

CKM fits: $40^\circ \lesssim 2\beta \lesssim 50^\circ$

NP

[R.F. & Matias ('02); R.F., Matias & Isidori ('03)]

• Both solutions can be distinguished through the sign of $\cos \phi_d$:

- $\cos \phi_d = +0.7 > 0 \Rightarrow \phi_d \sim 47^\circ \Rightarrow \text{SM}$
- $\cos \phi_d = -0.7 < 0 \Rightarrow \phi_d \sim 133^\circ \Rightarrow \text{NP}$

- **BaBar (2004):** $B_d \rightarrow J/\psi[\rightarrow \ell^+\ell^-] K^* [\rightarrow \pi^0 K_S]$

\[
\cos \phi_d = 2.72^{+0.50}_{-0.79} \pm 0.27 \quad \Rightarrow \quad \text{favours the SM case!}
\]

- Follows also indirectly from $B_d \rightarrow D^{(*)\pm} \pi^\mp$ and $B \rightarrow \pi\pi, \pi K$ decays.

[R.F. (2003); Buras, R.F., Recksiegel & Schwab (2004)]
• NP contributions at the decay amplitude level:

- Have to compete with SM tree-diagram-like topologies, which play the dominant rôle in $B \rightarrow J/\psi K$ modes:

$$\Rightarrow \text{NP effects generically} \lesssim 10\%$$

- Could be detected through appropriate observables, which exploit also direct CP violation and the charged $B^\pm \rightarrow J/\psi K^\pm$ decays:

$$\Rightarrow \text{no indications in the current } B\text{-factory data ...}$$

[R.F. & Mannel (2001)]

• Situation in the $\bar{\rho} - \bar{\eta}$ plane:

$$\Rightarrow \text{space for NP in } B^0_d - \overline{B^0_d} \text{ mixing is getting smaller and smaller ...}$$
Challenging the Standard Model

Through $B_d \rightarrow \phi K_S$

→ Belle data have triggered excitement ...
CP Violation in $B_d \rightarrow \phi K_S$

- **Decay in CP eigenstate:**
  \[
  (+1) \times (+1) \times (−1)^1 = −1.
  \]

- **Structure of the decay amplitude:**
  \[
  [K_S = (\bar{K}^0 + K^0) / \sqrt{2}]
  \]
  \[
  A(B_d^0 \rightarrow \phi K_S) = \lambda_u^{(s)} A_P^u + \lambda_c^{(s)} A_P^c + \lambda_t^{(s)} A_P^t
  \]

- **Unitarity of the CKM matrix:**
  \[
  \lambda_t^{(s)} = −\lambda_c^{(s)} − \lambda_u^{(s)} \Rightarrow
  \]
  \[
  A(B_d^0 \rightarrow \phi K_S) \propto [1 + \lambda^2 be^{i\Theta} e^{i\gamma}]
  \]
  \[
  be^{i\Theta} = \left( \frac{R_b}{1 − \lambda^2} \right) \left[ \frac{A_P^u − A_P^t}{A_P^c − A_P^t} \right] \sim \mathcal{O}(1)
  \]

- **B_d \rightarrow \phi K_S is a pure penguin process!**
• Consequently:  
\[ \xi_{\phi K_S}^{(d)} = +e^{-i\phi_d} \left[ 1 + \frac{2\lambda be^{i\Theta}e^{-i\gamma}}{1 + \lambda^2 be^{i\Theta}e^{+i\gamma}} \right] \]

• Since the essentially “unknown” hadronic parameter \( be^{i\Theta} \) enters in a \textit{doubly Cabibbo-suppressed way}:

\[
\mathcal{A}_{CP}^{\text{dir}}(B_d \to \phi K_S) = 0 + \mathcal{O}(\lambda^2) \\
\mathcal{A}_{CP}^{\text{mix}}(B_d \to \phi K_S) = -\sin \phi_d + \mathcal{O}(\lambda^2)
\]

• On the other hand:  
\[
\mathcal{A}_{CP}^{\text{mix}}(B_d \to J/\psi K_S) = -\sin \phi_d + \mathcal{O}(\lambda^3) \implies
\]

\[
\mathcal{A}_{CP}^{\text{mix}}(B_d \to \phi K_S) = \mathcal{A}_{CP}^{\text{mix}}(B_d \to J/\psi K_S) + \mathcal{O}(\lambda^2) \quad (\ast)
\]

[R.F. ('97); Grossman & Worah ('97); London & Soni ('97)]

• \( B_d \to \phi K_S \) is a sensitive probe for new physics:

– Dominated by QCD penguins  
  [London & Pececi ('89); Deshpande & Trampetic ('90); ...]

– EW penguins have a sizeable impact [R.F. ('94); Deshpande & He ('94)]

– Model-independent NP analyses [R.F. & Mannel ('01)]

\[
\implies (\ast) \text{ could well be violated through NP!}
\]
Experimental Picture of $B_d \rightarrow \phi K_S$

- **Time evolution of the data:**
  
  - **LP ’03:**
    \[
    A_{CP}^{\text{dir}}(B_d \rightarrow \phi K_S) = \begin{cases} 
    -0.38 \pm 0.37 \pm 0.12 & \text{(BaBar)} \\
    +0.15 \pm 0.29 \pm 0.07 & \text{(Belle)} 
    \end{cases}
    \]
    \[
    A_{CP}^{\text{mix}}(B_d \rightarrow \phi K_S) = \begin{cases} 
    -0.45 \pm 0.43 \pm 0.07 & \text{(BaBar)} \\
    +0.96 \pm 0.50^{+0.11}_{-0.09} & \text{(Belle)} 
    \end{cases}
    \]
  
  - **ICHEP ’04:**
    \[
    A_{CP}^{\text{dir}}(B_d \rightarrow \phi K_S) = \begin{cases} 
    +0.00 \pm 0.23 \pm 0.05 & \text{(BaBar)} \\
    -0.08 \pm 0.22 \pm 0.09 & \text{(Belle)} 
    \end{cases}
    \]
    \[
    A_{CP}^{\text{mix}}(B_d \rightarrow \phi K_S) = \begin{cases} 
    -0.50 \pm 0.25^{+0.04}_{-0.07} & \text{(BaBar)} \\
    -0.06 \pm 0.33 \pm 0.09 & \text{(Belle)} 
    \end{cases}
    \]

- **On the other hand:**

  \[
  A_{CP}^{\text{mix}}(B_d \rightarrow J/\psi K_S) = -0.725 \pm 0.037 \Rightarrow
  \]

  - Belle indicates CP-violating NP contributions to $b \rightarrow s\bar{s}s$ processes!
  - But the data moved towards the SM, and no confirmation from BaBar.
  - Hopefully, clarification soon (→ monitor also similar modes).
$B \to \pi\pi, \pi K$ Puzzles

& Connection with

Rare $K$ and $B$ Decays

→ example of a systematic strategy to search for NP:

... leads us to a NP scenario of Class C!

Logical Structure

Step 1.
- B \rightarrow \pi\pi \text{ Decays described within SM (EW ~Penguins small)}
- Isospin Symmetry
- B \rightarrow \pi\pi Data
- Hadronic Parameters in B \rightarrow \pi\pi (d, \theta, x, \Delta)
- Sizable Departures from QCDF, PQCD

Step 2.
- d, \theta, x, \Delta + SU(3)_F
- Hadronic Parameters in B \rightarrow \pi K
- B \rightarrow \pi K Data
- Enhanced EWP with large New Complex Phase
- \gamma (65 \pm 7)^\circ

Step 3.
- Correlations between B \rightarrow \pi K, Rare K and B Decays and other Processes
- Implications for Rare K and B Decays sensitive to EWP
Step 1: \[ B \to \pi\pi \]

\[ B_d^0 \to \pi^+\pi^-, \quad \bar{B}_d^0 \to \pi^+\pi^- \]

\[ B_d^0 \to \pi^0\pi^0, \quad \bar{B}_d^0 \to \pi^0\pi^0 \]

\[ B^+ \to \pi^+\pi^0, \quad B^- \to \pi^-\pi^0 \]
Input Observables

- Two independent ratios of the CP-averaged branching ratios:

\[ R^{\pi\pi}_{+-} \equiv 2 \left[ \frac{\text{BR}(B^\pm \to \pi^\pm \pi^0)}{\text{BR}(B_d \to \pi^+\pi^-)} \right] \frac{\tau_{B^0_d}}{\tau_{B^+}} = 2.20 \pm 0.31 \]

\[ R^{\pi\pi}_{00} \equiv 2 \left[ \frac{\text{BR}(B_d \to \pi^0\pi^0)}{\text{BR}(B_d \to \pi^+\pi^-)} \right] = 0.67 \pm 0.14 \]

- The BRs for \( B_d \to \pi^+\pi^- \) and \( B_d \to \pi^0\pi^0 \) are found to be surprisingly small and large, respectively, whereas that for \( B^\pm \to \pi^\pm \pi^0 \) looks OK.

- CP-violating observables of \( B_d \to \pi^+\pi^- \):

\[ A_{\text{dir}}^{\text{dir}}(B_d \to \pi^+\pi^-) = -0.37 \pm 0.11, \quad A_{\text{mix}}^{\text{mix}}(B_d \to \pi^+\pi^-) = +0.61 \pm 0.14 \]

- Experimental picture is not yet settled (HFAG averages).

- Theoretical interpretation to be discussed below yields constraints for the UT in nice accordance with the SM ...
Hadronic Parameters: Isospin Symmetry

• Observables involve the following hadronic parameters:

  – Ratio of “penguin” to “tree” amplitudes (see Lecture II):

    \[ d e^{i\theta} \equiv \frac{1}{R_b} \left[ \frac{A^c_P - A^t_P}{A^u_T + A^u_P - A^t_P} \right] \equiv \frac{1}{R_b} \left[ \frac{\mathcal{P}_{tc}}{\mathcal{T} - (\mathcal{P}_{tu} - \mathcal{E})} \right] \]

  – Ratio of “colour-suppressed” to “colour-allowed tree” amplitudes:

    \[ x e^{i\Delta} \equiv \left[ \frac{\mathcal{C} + (\mathcal{P}_{tu} - \mathcal{E})}{\mathcal{T} - (\mathcal{P}_{tu} - \mathcal{E})} \right] \]

• Can be cleanly and unambiguously determined from the \( B \to \pi\pi \) data:\footnote{EW penguins have a tiny impact on the \( B \to \pi\pi \) system, but are included in our numerical analysis.}

    \[ d = 0.51^{+0.26}_{-0.20}, \quad \theta = +\left(140^{+14}_{-18}\right)^\circ; \quad x = 1.15^{+0.18}_{-0.16}, \quad \Delta = -(59^{+19}_{-26})^\circ \] (1)

• Theoretical picture: [QCDF: Buchalla & Safir ('04); PQCD: Keum & Sanda ('03)]

    \[ d|_{\text{QCDF}} = 0.29 \pm 0.09, \quad \theta|_{\text{QCDF}} = -(171.4 \pm 14.3)^\circ \]
    \[ d|_{\text{PQCD}} = 0.23^{+0.07}_{-0.05}, \quad +139^\circ < \theta|_{\text{PQCD}} < +148^\circ \]
The hadronic parameters in (1) allow the following predictions:

\[
\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^0\pi^0) \bigg|_{\text{SM}} = -0.28^{+0.37}_{-0.21}
\]

\[
\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \to \pi^0\pi^0) \bigg|_{\text{SM}} = -0.63^{+0.45}_{-0.41}
\]

⇒ exciting perspective of large CP violation!

First \(B\)-factory results were reported @ ICHEP ’04:

\[
\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^0\pi^0) = \begin{cases} 
-(0.12 \pm 0.56 \pm 0.06) & \text{(BaBar)} \\
-(0.43 \pm 0.51^{+0.17}_{-0.16}) & \text{(Belle)}
\end{cases}
\]

⇒ \(\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \to \pi^0\pi^0) = -(0.28 \pm 0.39)\)

⇒ encouraging agreement with our prediction!
Three Lessons from the $B \rightarrow \pi\pi$ Analysis

1. The data indicate large non-factorizable effects.

2. Sizeable CP asymmetries are expected in the $B_d \rightarrow \pi^0\pi^0$ channel.

3. The current data can be nicely accommodated in the SM.

More accurate input data will lead to sharper and sharper pictures ...

[In accordance with analyses by Ali et al. ('04); Bauer et al. ('04); Chiang et al.; ...]
Step 2:

\[ B \rightarrow \pi K \]

\[
\begin{align*}
B^+ &\rightarrow \pi^+ K^0, & B^- &\rightarrow \pi^- \bar{K}^0 \\
B^0_d &\rightarrow \pi^- K^+, & \bar{B}^0_d &\rightarrow \pi^+ K^-
\end{align*}
\]

\textit{colour-suppressed EW penguins} (expected to be tiny)

\[
\begin{align*}
B^+ &\rightarrow \pi^0 K^+, & B^- &\rightarrow \pi^0 K^- \\
B^0_d &\rightarrow \pi^0 K^0, & \bar{B}^0_d &\rightarrow \pi^0 \bar{K}^0
\end{align*}
\]

\textit{colour-allowed EW penguins} (significant)
Main Ingredients of Our $B \rightarrow \pi K$ Analysis

• **Starting point:**
  
  – Hadronic $B \rightarrow \pi\pi$ parameters determined in Step 1.
  – SM CKM fits (insignificantly affected by EW penguins).

• **Working hypothesis:**
  
  i) $SU(3)$ flavour symmetry of strong interactions
  ii) Neglect penguin annihilation and exchange topologies

    Internal consistency checks OK ($\rightarrow$ LHCb)

• **We may then determine the hadronic $B \rightarrow \pi K$ parameters through their $B \rightarrow \pi\pi$ counterparts:**

    $\Rightarrow$ Prediction of the $B \rightarrow \pi K$ observables in the SM
Observables with a Tiny Impact of EW Penguins

- Direct CP violation in $B_d \rightarrow \pi^\mp K^\pm$ (was established @ ICHEP ’04):
  - Average of the corresponding BaBar and Belle data:
    $$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^\mp K^\pm) = 0.113 \pm 0.019$$
  - In our strategy, we obtain the following prediction:
    $$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^\mp K^\pm) = 0.127^{+0.102}_{-0.066}$$
  - Moreover, i) and ii) specified above imply the following relation:
    $$H \propto \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{\text{BR}(B_d \rightarrow \pi^+ \pi^-)}{\text{BR}(B_d \rightarrow \pi^\mp K^\pm)} \right] = - \left[ \frac{A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^\mp K^\pm)}{A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-)} \right] \left[ \frac{0.38 \pm 0.04}{0.31 \pm 0.11} \right]$$
    … gives us further confidence in our working assumptions!
- The $B_d \to \pi^\mp K^\pm$ data allow us also to convert the CP asymmetries of the $B_d \to \pi^+\pi^-$ channel into a range for $\gamma$:

$$\begin{align*}
A_{CP}^{\text{dir}}(B_d \to \pi^+\pi^-) &= G_1(d, \theta; \gamma) \\
A_{CP}^{\text{mix}}(B_d \to \pi^+\pi^-) &= G_2(d, \theta; \gamma, \phi_d) \oplus H = G_3(d, \theta; \gamma) \Rightarrow
\end{align*}$$

\[ \beta \gamma \]

- On the other hand, moderate numerical discrepancy for the ratio $R$ of the CP-averaged $B_d \to \pi^\mp K^\pm$, $B^\pm \to \pi^\pm K$ branching ratios:

- Suggests the sizeable impact of hadronic parameters $(\rho_c, \theta_c)$.
- These quantities can be constrained through the direct CP asymmetry of the decay $B^\pm \to \pi^\pm K$ and the emerging $B^\pm \to K^\pm K$ signal...

\[ \Rightarrow \text{no problems for the SM in this sector!} \]
Observables with a *Sizeable* Impact of EW Penguins

- **The key quantities:** [Buras & R.F. ('98)]

  \[ R_c \equiv 2 \left\{ \frac{\text{BR}(B^+ \to \pi^0 K^+) + \text{BR}(B^- \to \pi^0 K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- \bar{K}^0)} \right\} \text{ Exp } = 1.00 \pm 0.08 \]

  \[ R_n \equiv \frac{1}{2} \left\{ \frac{\text{BR}(B_d^0 \to \pi^- K^+) + \text{BR}(\bar{B}_d^0 \to \pi^+ K^-)}{\text{BR}(B_d^0 \to \pi^0 K^0) + \text{BR}(\bar{B}_d^0 \to \pi^0 \bar{K}^0)} \right\} \text{ Exp } = 0.79 \pm 0.08 \]

- **Features of the EW penguins:**

  - Enter in colour-allowed form through the modes involving \( \pi^0 \)'s.
  - Theoretical description through the following parameters:

    \[ \begin{aligned} 
    q^{\text{SM}} &\equiv 0.69 \ (\rightarrow \text{“strength”}), \\
    \phi^{\text{SM}} &\equiv 0^\circ \ (\rightarrow \text{CP-violating phase}) \\
    \end{aligned} \]

    \( SU(3) \) [Neubert & Rosner ('98)]

  - Provide an interesting avenue for NP to manifest itself...

    [R.F. & Mannel ('97); Grossman, Neubert & Kagan ('99); ...]
• Situation in the $R_n$–$R_c$ plane:

![Diagram showing $R_n$, $R_c$, $\phi$, and experimental region with $q$ and $\phi$ values.]

• Allow for NP in the EW penguin sector to resolve this "$B \to \pi K$ puzzle":

$$R_{c,n}|_{\text{exp}} \Rightarrow q = 1.08^{+0.81}_{-0.73}, \quad \phi = -(88.8^{+13.7}_{-19.0})^\circ$$

$$\Rightarrow \text{prediction of CP violation in } B^\pm \to \pi^0K^\pm \text{ and } B_d \to \pi^0K_S \ldots$$
Compilation of our predictions for the CP asymmetries:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Our Prediction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{CP}}^{\text{dir}}(B_d \to \pi^0\pi^0)$</td>
<td>$-0.28^{+0.37}_{-0.21}$</td>
<td>$-0.28 \pm 0.39$</td>
</tr>
<tr>
<td>$A_{\text{CP}}^{\text{mix}}(B_d \to \pi^0\pi^0)$</td>
<td>$-0.63^{+0.45}_{-0.41}$</td>
<td>$-0.48^{+0.48}_{-0.40}$</td>
</tr>
<tr>
<td>$A_{\text{CP}}^{\text{dir}}(B_d \to \pi^\mp K^\pm)$</td>
<td>$0.127^{+0.102}_{-0.066}$</td>
<td>$0.113 \pm 0.019$</td>
</tr>
<tr>
<td>$A_{\text{CP}}^{\text{dir}}(B^\pm \to \pi^0 K^\pm)$</td>
<td>$0.10^{+0.25}_{-0.19}$</td>
<td>$-0.04 \pm 0.04$</td>
</tr>
<tr>
<td>$A_{\text{CP}}^{\text{dir}}(B_d \to \pi^0 K_S)$</td>
<td>$0.01^{+0.15}_{-0.18}$</td>
<td>$0.09 \pm 0.14$</td>
</tr>
<tr>
<td>$A_{\text{CP}}^{\text{mix}}(B_d \to \pi^0 K_S)$</td>
<td>$-0.98^{+0.04}_{-0.02}$</td>
<td>$-0.34^{+0.29}_{-0.27}$</td>
</tr>
</tbody>
</table>

→ sensitivity on EW penguins!

What about further tests of our NP scenario?
Step 3:

Rare $B$ and $K$ Decays

$Z^0$ penguins

$\Rightarrow$ ... several spectacular NP effects!
Preliminaries

- Enhanced $Z^0$ penguins with a large CP-violating NP phase provide an attractive scenario for NP effects in rare $K$ and $B$ decays:
  - Model-independent analyses
  - Studies within particular supersymmetric scenarios ...
  [Buras & Silvestrini (1999); Buras, Colangelo, Isidori, Romanino & Silvestrini (2000); Buchalla et al. (2001); Atwood & Hiller (2003); Buras, Ewerth, Jäger & Rosiek (2004)]

- We determine the magnitude and phase of the SD (Inami–Lim) function $C$ that characterizes the $Z^0$ penguins through the $B \to \pi K$ data:
  - Performing a renormalization-group analysis yields
    $$C(\bar{q}) = 2.35 \, \bar{q} e^{i\phi} - 0.82, \quad \bar{q} = q \left[ \frac{|V_{ub}/V_{cb}|}{0.086} \right]$$ (1)
  - Evaluating the relevant box-diagram contributions within the SM and using (1), we obtain the following short-distance functions:
    $$X = 2.35 \, \bar{q} e^{i\phi} - 0.09 \quad \text{and} \quad Y = 2.35 \, \bar{q} e^{i\phi} - 0.64,$$ (2)
  which govern rare decays with $\nu \bar{\nu}$ and $\ell^+ \ell^-$ in the final states.
  [Buras, R.F, Recksiegel & Schwab (2003)]
Constraints from Rare Decays

- **Previous $B \rightarrow \pi K$ data:**

  \[ q = 1.75^{+1.27}_{-0.99}, \quad \phi = -(85^{+11}_{-14})^\circ \Rightarrow |X| \approx |Y| \approx |Z| \approx 4.3^{+3.0}_{-2.4} \]

  - $|X|$: compatible with the $K \rightarrow \pi \nu \bar{\nu}$, $B \rightarrow X_{s,d} \nu \bar{\nu}$ data.
  - $|Y|$: violates $|Y| \leq 2.2$ following from the BaBar and Belle data for $B \rightarrow X_s \mu^+ \mu^-$, and the KTeV upper bound on BR($K_L \rightarrow \pi^0 e^+ e^-$).
  - $|Z|$: too large to be consistent with the data on $\varepsilon'/\varepsilon$.

- **Consider only those $(q, \phi)_{B \rightarrow \pi K}$ that satisfy $|Y| = 2.2$:**

  \[ \Rightarrow \tilde{q} = 0.92^{+0.07}_{-0.05}, \quad \phi = -(85^{+11}_{-14})^\circ \]

  - Nicely compatible with the *new* $B \rightarrow \pi K$ data:

    \[ \Rightarrow q = 1.08^{+0.81}_{-0.73}, \quad \phi = -(88.8^{+13.7}_{-19.0})^\circ \]

  - Significant NP effects in several rare decays would emerge...

- Various *predictions* $\Rightarrow$ *Tests of our NP scenario!*
Picture with the Rare-Decay Constraints

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Old Data</th>
<th>Prediction with RDs</th>
<th>New Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>$1.17 \pm 0.12$</td>
<td>$1.00^{+0.12}_{-0.08}$</td>
<td>$1.00 \pm 0.08$</td>
</tr>
<tr>
<td>$R_n$</td>
<td>$0.76 \pm 0.10$</td>
<td>$0.82^{+0.12}_{-0.11}$</td>
<td>$0.79 \pm 0.08$</td>
</tr>
</tbody>
</table>

⇒ data moved accordingly! [see BFRS NPB paper]

• Define CP-violating phases through the following relations:

$$X = |X|e^{i\theta_X}, \quad Y = |Y|e^{i\theta_Y}, \quad Z = |Z|e^{i\theta_Z}$$

$$\beta_X \equiv \beta - \beta_s - \theta_X, \quad \beta_Y \equiv \beta - \beta_s - \theta_Y, \quad \beta_Z \equiv \beta - \beta_s - \theta_Z$$

[$\beta$: usual UT angle, $\beta_s = -\lambda^2 \eta = -1^\circ$]

• Short-distance parameters following from our NP analysis:

$$|C| = 2.24 \pm 0.04, \quad \theta_C = -(105 \pm 12)^\circ$$

$$|X| = 2.17 \pm 0.12, \quad \theta_X = -(86 \pm 12)^\circ, \quad \beta_X = (111 \pm 12)^\circ$$

$$|Y| = 2.2 \text{ (input)}, \quad \theta_Y = -(100 \pm 12)^\circ, \quad \beta_Y = (124 \pm 12)^\circ$$

$$|Z| = 2.27 \pm 0.06, \quad \theta_Z = -(108 \pm 12)^\circ, \quad \beta_Z = (132 \pm 12)^\circ$$

• The SM corresponds to the following values [$\theta_C = \theta_X = \theta_Y = \theta_Z = 0^\circ$]:

$$|C| = 0.79, \quad |X| = 1.53, \quad |Y| = 0.98, \quad |Z| = 0.68$$
Rare $K \to \pi \nu \bar{\nu}$ Decays (→ Very Clean!)

- **The current experimental picture:**

  \[
  \begin{align*}
  \text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) &= (14.7^{+13.0}_{-8.9}) \times 10^{-11} \quad [E949 + E787] \\
  \text{BR}(K_L \to \pi^0 \nu \bar{\nu}) &< 5.9 \times 10^{-7} \quad [KTeV]
  \end{align*}
  \]

- **Branching ratios in the SM:**

  \[
  \begin{align*}
  \text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) \bigg|_{\text{SM}} &= (8.0 \pm 1.1) \times 10^{-11} \\
  \text{BR}(K_L \to \pi^0 \nu \bar{\nu}) \bigg|_{\text{SM}} &= (3.2 \pm 0.6) \times 10^{-11}
  \end{align*}
  \]

- **Branching ratios in our NP scenario:**

  \[
  \begin{align*}
  \text{BR}(K^+ \to \pi^+ \nu \bar{\nu}) &= (7.5 \pm 2.1) \times 10^{-11} \\
  \text{BR}(K_L \to \pi^0 \nu \bar{\nu}) &= (31 \pm 10) \times 10^{-11} \quad [\rightarrow E391(a)?]
  \end{align*}
  \]

- This pattern is dominantly the consequence of $\beta_X \approx 111^\circ$:

  \[
  \frac{\text{BR}(K_L \to \pi^0 \nu \bar{\nu})}{\text{BR}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}} = \left| \frac{X}{X_{\text{SM}}} \right|^2 \left[ \frac{\sin \beta_X}{\sin(\beta - \beta_s)} \right]^2
  \]

  \[
  \frac{\text{BR}(K_L \to \pi^0 \nu \bar{\nu})}{\text{BR}(K^+ \to \pi^+ \nu \bar{\nu})} \approx 4.4 \times (\sin \beta_X)^2 \approx (4.2 \pm 0.2)
  \]
- BR($K_L \rightarrow \pi^0\nu\bar{\nu}$) is close to its absolute upper bound: [Grossman & Nir ('97)]

$$\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu}) \leq 4.4 \times \text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu})$$

- BR($K^+ \rightarrow \pi^+\nu\bar{\nu}$) as a function of BR($K_L \rightarrow \pi^0\nu\bar{\nu}$): [MFV: Buras & R.F. ('01)]

- Moreover:
  - In NP scenarios with MFV, which contain also the SM, the $K \rightarrow \pi\nu\bar{\nu}$ BRs allow a determination of $\sin 2\beta$. [Buchalla & Buras (1994)]
  - However, in our NP scenario, we obtain the following:

$$\frac{(\sin 2\beta)_{\pi\nu\bar{\nu}}^{\text{MFV}}}{-(0.69^{+0.23}_{-0.41})} + (0.725 \pm 0.037) \Rightarrow \text{strong violation of this relation!}$$
Other Spectacular New-Physics Effects ...

- $K_L \to \pi^0 e^+ e^-$:
  
  - SM $\to$ decay is governed by indirect CP violation:
    \[
    \text{BR}(K_L \to \pi^0 e^+ e^-) = (3.2^{+1.2}_{-0.8}) \times 10^{-11}
    \]
  
  - NP $\to$ decay is governed by direct CP violation:
    \[
    \text{BR}(K_L \to \pi^0 e^+ e^-) = (7.8 \pm 1.6) \times 10^{-11}
    \]
  
  [K_L \to \pi^0 \mu^+ \mu^-: Isidori, Smith & Unterdorfer (2004)]

- $B_d \to K^* \mu^+ \mu^-$:

  An integrated forward–backward CP asymmetry [Buchalla et al. ('01)]
  \[
  A_{FB}^{CP} = (0.03 \pm 0.01) \times \tan \theta_Y
  \]

  can be very large in view of $\theta_Y \approx -100^\circ$.

  [See also Choudhury, Gaur & Cornell (2004); ...]

- $B \to X_{s,d} \nu \bar{\nu}$ and $B_{s,d} \to \mu^+ \mu^-$:

  The branching ratios are enhanced by factors of 2 and 5, respectively.
Conclusions and Outlook

• Flavour physics offers interesting avenues to explore the Standard Model and to search for signals of New Physics:
  
  – $B$ system:
    * Data are in remarkable agreement with the KM mechanism!
    * But still several unexplored aspects, and hints for discrepancies...
    
    $\rightarrow$ LHCb, super-$B$ factory (?)

  – $K$ system:
    * Governed the stage of CP violation for more than 35 years!
    * The future lies on rare decays: $K \rightarrow \pi \nu \bar{\nu}$

  – Other important aspects:
    * $D$ system: tiny CP-violating and mixing effects in SM.
    * Search for flavour-violating charged-lepton decays...

    The whole picture is essential ...

• A fruitful interplay with the NP searches/discoveries by ATLAS and CMS at the LHC is expected...

$\Rightarrow$ Exciting Future!