Flavour Physics and CP Violation

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(I)

Lecture I

- Setting the Stage

- CP Violation in the Standard Model:

$$\boxed{\text{Cabibbo–Kobayashi–Maskawa (CKM) Matrix}}$$

- A Closer Look at the B-meson System:

$$\boxed{\text{Low-Energy Effective Hamiltonians}}$$

- Towards Studies of CP Violation in the B-Meson System:
  - Key problems in the exploration of CP violation
  - Classification of the main strategies

Lecture II

- Exploring CP Violation through Amplitude Relations:
  - Example: $B^\pm \to K^\pm D$, $B_c^\pm \to D_s^\pm D$
- Exploring CP Violation through Neutral B Decays:
  - Time Evolution of Neutral B Decays
  - B-Factory Benchmark modes: $B_d \to J/\psi K_{\rm S}$, $B_d \to \pi^+\pi^-$
- The "El Dorado" for Hadron Colliders: $\boxed{B_s \text{ System}}$
  - Basic Features
  - Benchmark Decays:
    - $B_s \to J/\psi\phi$
    - $B_s \to D_s^\pm K^\mp$ (complements $B_d \to D^\pm \pi^\mp$)
    - $B_s \to K^+K^-$ (complements $B_d \to \pi^+\pi^-$)

Lecture III

- Rare Decays:
  - Example: $B_{s,d} \to \mu^+\mu^-$

- How Could New Physics Enter in the Roadmap of Quark-Flavour Physics?

- What about New Physics in $B_d \to J/\psi K_{\rm S}$?

- Challenging the Standard Model through $B_d \to \phi K_{\rm S}$

- The $B \to \pi\pi, \pi K$ Puzzles & Rare K and B Decays:

$\to$ $\boxed{\text{Example of a systematic strategy to search for NP}}$

1. "$B \to \pi\pi$ puzzle"
2. "$B \to \pi K$ puzzle"
3. Connection with rare K and B decays

A Selection of Basic References

- Lecture Notes:
  - R.F.: "Flavour Physics and CP Violation",
    2003 European School on High-Energy Physics [hep-ph/0405091].
  - Y. Nir: "CP Violation: A New Era",
    2001 Scottish Univ. Summer School in Physics [hep-ph/0109090].

- Textbooks:
  - G. Branco, L. Lavoura and J. Silva: "CP Violation",
    International Series of Monographs on Physics 103, Oxford Science Publications (Clarendon Press, Oxford 1999).
  - I.I. Bigi and A. I. Sanda: "CP Violation",
    Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge University Press, Cambridge, 2000).
  - K. Kleinknecht: "CP Violation",
    Springer Tracts in Modern Physics, Vol. 195 (2004).

Setting the Stage

A Brief History of CP Violation

- In 1957, surprising discovery that the weak interactions are *not* invariant under parity transformations (Wu *et al.*):

$$\Rightarrow \boxed{\text{parity violation!}}$$

  - Parity transformation $\mathcal{P}$: space inversion $\vec{x} \to -\vec{x}$

- However, it was believed that the product $\mathcal{CP}$ was preserved:

  - Charge conjugation $\mathcal{C}$: particle $\to$ antiparticle

$$\pi^+ \to e^+\nu_e \xrightarrow{C} \pi^- \to e^-\nu_e^C \xrightarrow{P} \pi^- \to e^-\overline{\nu}_e$$

lefthanded ($\times$) righthanded (OK)

- In 1964, discovery of CP violation in neutral K decays (Christenson *et al.*):

$$\boxed{K_{\mathrm{L}} \to \pi^+\pi^-} \qquad (\text{BR} \sim 2 \times 10^{-3})$$

- These effects are a manifestation of *indirect CP violation:*

$$(\mathcal{CP}) \qquad (-) \qquad (+) \qquad\qquad\qquad (+)$$

direct: ε'

$$K_{\rm L} \;=\; K_2 + \overline{\varepsilon} K_1 \longrightarrow \begin{cases} \pi^+\pi^- \\ \pi^0\pi^0 \end{cases}$$

indirect: ε

$$\boxed{\varepsilon = (2.280 \pm 0.013) \times 10^{-3} \times e^{i\pi/4}}$$

- In 1999, *direct* CP violation could be established [NA48 & KTeV]:

$$\boxed{\mathrm{Re}(\varepsilon'/\varepsilon) = \begin{cases} (14.7 \pm 2.2) \times 10^{-4} & \text{[NA48 (2002)]} \\ (20.7 \pm 2.8) \times 10^{-4} & \text{[KTeV (2002)]} \end{cases}}$$

- In 2001, discovery of CP-violating effects in B decays [BaBar & Belle], i.e. for the first time *outside* of the K system:

$$\boxed{B_d \to J/\psi K_{\rm S}} \to \textit{mixing-induced}\ \text{CP violation!}$$

- In 2004, also observation of *direct* CP violation in $B_d \to \pi^\mp K^\pm$ ...

Why Study CP Violation & Flavour Physics?

- Despite tremendous progress, we have (still!) few insights ...
- New Physics (NP): $\longrightarrow$ typically new sources for flavour & CP violation
  - SUSY, models with extended Higgs sectors, LR-symmetric models...
- ν masses: $\longrightarrow$ origin beyond the Standard Model (SM)!
  - CP violation in the neutrino sector? Neutrino factories...
- Cosmology:
  - CP violation is one of the necessary ingredients for the generation of the matter–antimater asymmetry! [Sacharow 1967]
  - Model calculations: $\Rightarrow$ CP violation too small in SM ...
    * Could be associated with very high energy scales (e.g. "Leptogenesis").
    * *But could also be accessible in the laboratory* ...
- Moreover:
  - The origin of the fermion masses, flavour mixing, CP violation etc. lies completely in the dark $\rightarrow$ *involves new physics, too!*

Challenging the Standard Model ...

- Before searching for NP, we have first to understand the SM picture!

- Key problem for the theoretical interpretation: $\boxed{\text{hadronic uncertainties!}}$

  – Famous example: Re$(\varepsilon'/\varepsilon)$

- The B-meson system is particularly promising in this respect:

  – Offers various strategies: simply speaking, there are many B decays!

  – Search for clean SM relations that may well by spoiled by NP ...

$$\longrightarrow \boxed{\text{our focus!}}$$

- How about the good old K-meson system?

  – Clean tests of the SM are offered by $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$!

  – These "rare" decays are *absent* at the tree level of the SM, i.e. originate there exclusively from loop processes.

CP Violation

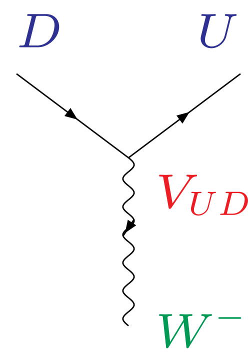
in the

Standard Model

Weak Interactions of Quarks

- Charged-current interactions:

  $(D \in \{d, s, b\}, U \in \{u, c, t\})$



- Possible transitions:

1st gen.	2nd gen.	3rd gen.	
$d \to u$	$s \to u$	$b \to u$	1st gen.
$d \to c$	$s \to c$	$b \to c$	2nd gen.
$d \to t$	$s \to t$	$b \to t$	3rd gen.

- Matrix of couplings:

$$\hat{V}_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo–Kobayashi–Maskawa (CKM) matrix

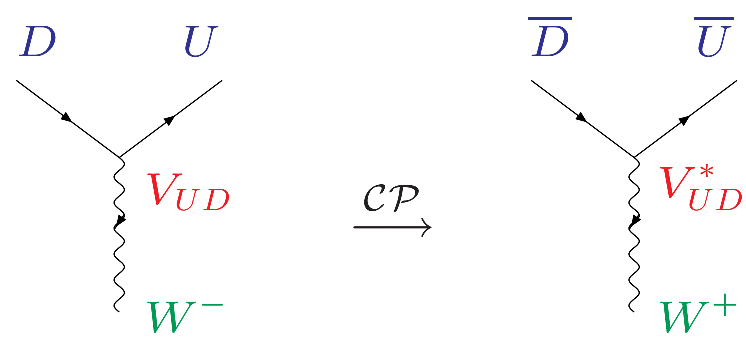
- The CKM matrix connects the electroweak flavour states (d', s', b') with their mass eigenstates (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\mathcal{L}_{\rm int}^{\rm CC} = -\frac{g_2}{\sqrt{2}} \left(\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L}\right) \gamma^\mu \, \hat{V}_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix} W_\mu^\dagger + \text{h.c.}$$

- The CKM matrix is unitary: $\boxed{\hat{V}_{\mathsf{CKM}}^\dagger \cdot \hat{V}_{\mathsf{CKM}} = \hat{1} = \hat{V}_{\mathsf{CKM}} \cdot \hat{V}_{\mathsf{CKM}}^\dagger}$

- CP-conjugate transitions:



$$\boxed{V_{UD} \xrightarrow{\mathcal{CP}} V_{UD}^*}$$

Phase Structure of the CKM Matrix

- Redefinition of the quark-field phases in $\mathcal{L}_{\text{int}}^{\text{CC}}$:

$$\left.\begin{aligned} U &\longrightarrow \exp(i\xi_U)\,U \\ D &\longrightarrow \exp(i\xi_D)\,D \end{aligned}\right\} \Rightarrow \boxed{V_{UD} \longrightarrow \exp(i\xi_U)\,V_{UD}\,\exp(-i\xi_D)}$$

- Parameters of the $N \times N$ quark-mixing matrix:

$$\underbrace{\frac{1}{2}N(N-1)}_{\text{Euler angles}} + \underbrace{\frac{1}{2}(N-1)(N-2)}_{\text{complex phases}} = (N-1)^2$$

- Two generations: $\longrightarrow$ $\boxed{\text{Cabibbo angle } \theta_C \text{ (1963)}}$

$$\hat{V}_C = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \qquad [\sin\theta_C = 0.22 \text{ from } K \to \pi e \overline{\nu}_e]$$

- Three generations: $\longrightarrow$ $\boxed{\text{Kobayashi \& Maskawa (1973)}}$
  - Requires three Euler angles and one complex phase ...
  - Complex phase: origin of CP violation in the SM!

Parametrizations of the CKM Matrix

- "Standard" Parametrization ($\to$ PDG): $[\, c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}]$

$$\hat{V}_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- Kobayashi & Maskawa: $[c_i = \cos\theta_i$ and $s_i = \sin\theta_i]$

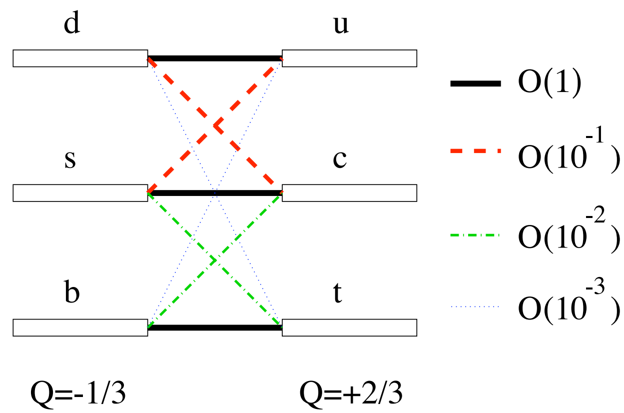
$$\hat{V}_{\rm CKM} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

- Fritzsch & Xing: $[c_{\rm u} = \cos\theta_{\rm u},\ s_{\rm u} = \sin\theta_{\rm u},$ etc.$]$

$$\hat{V}_{\rm CKM} = \begin{pmatrix} s_{\rm u}s_{\rm d}c + c_{\rm u}c_{\rm d}e^{-i\varphi} & s_{\rm u}c_{\rm d}c - c_{\rm u}s_{\rm d}e^{-i\varphi} & s_{\rm u}s \\ c_{\rm u}s_{\rm d}c - s_{\rm u}c_{\rm d}e^{-i\varphi} & c_{\rm u}c_{\rm d}c + s_{\rm u}s_{\rm d}e^{-i\varphi} & c_{\rm u}s \\ -s_{\rm d}s & -c_{\rm d}s & c \end{pmatrix}$$

Wolfenstein Parametrization

- Hierarchy of the quark transitions mediated through charged currents:



- This hierarchy is reflected in the standard parametrization as follows:

$$s_{12} = 0.22 \quad \gg \quad s_{23} = \mathcal{O}(10^{-2}) \quad \gg \quad s_{13} = \mathcal{O}(10^{-3}) \quad \Rightarrow$$

- New parameters: $s_{12} \equiv \lambda = 0.22, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

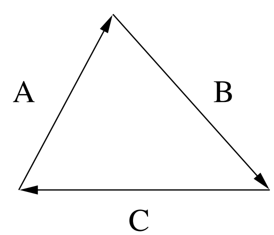
- Go back to the standard parametrization and neglect all terms of $\mathcal{O}(\lambda^4)$:

$$\boxed{\hat{V}_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\,\eta) & -A\lambda^2 & 1 \end{pmatrix}}$$

[Wolfenstein (1983)]

Unitarity Triangle(s) of the CKM Matrix

- Unitarity of the CKM matrix: $\boxed{\hat{V}^{\dagger}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}} = \hat{1} = \hat{V}_{\text{CKM}} \cdot \hat{V}^{\dagger}_{\text{CKM}}}$ $\Rightarrow$
  - 6 normalization relations (columns and rows)
  - 6 orthogonality relations (columns and rows): $\boxed{A + B + C = 0}$
- The orthogonality relations can be represented as 6 triangles:



$\longrightarrow$ $\boxed{\text{Unitarity triangles!}}$

- These triangles have all the same area A_{Δ}, which can be interpreted as a measure of the "strength" of CP violation in the SM:

$$\boxed{2A_{\Delta} \equiv |J_{\text{CP}}| = \lambda^6 A^2 \eta = \mathcal{O}(10^{-5}).}$$

- Columns:

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}V_{ub}^*}_{(\rho+i\eta)A\lambda^3} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{(1-\rho-i\eta)A\lambda^3} = 0$$

- Rows:

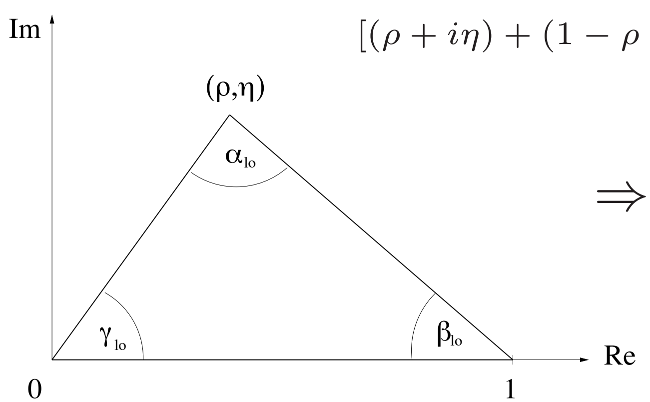
$$\underbrace{V_{ud}^*V_{cd}}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}^*V_{cs}}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}^*V_{cb}}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{cd}^*V_{td}}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}^*V_{ts}}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{cb}^*V_{tb}}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}^*V_{td}}_{(1-\rho-i\eta)A\lambda^3} + \underbrace{V_{us}^*V_{ts}}_{-A\lambda^3} + \underbrace{V_{ub}^*V_{tb}}_{(\rho+i\eta)A\lambda^3} = 0$$

- Only in two relations, all terms are of $\mathcal{O}(\lambda^3)$, and *agee* with one another:

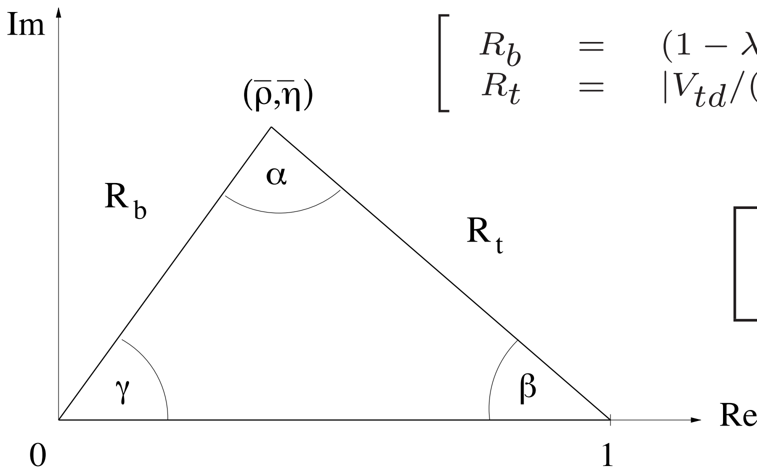
$$[(\rho + i\eta) + (1 - \rho - i\eta) + (-1)]\, A\lambda^3 = 0$$



$\Rightarrow$ $\boxed{\textit{the}\text{ unitarity triangle of the CKM matrix!}}$

- The unitarity triangles at next-to-leading order in λ:

  – $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$: $\Rightarrow$ $\boxed{\text{UT}}$

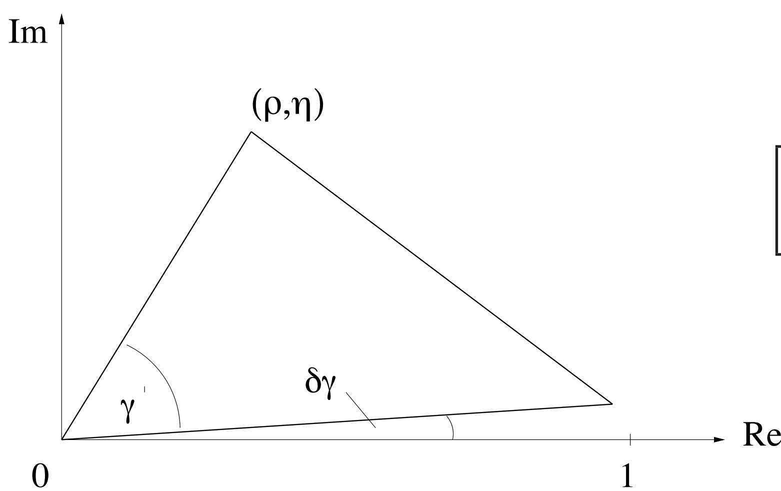


$$\left[\begin{array}{lcl} R_b & = & (1-\lambda^2/2)|V_{ub}/(\lambda V_{cb})| \\ R_t & = & |V_{td}/(\lambda V_{cb})| \end{array}\right]$$

$$\boxed{\bar{\rho} \equiv (1-\lambda^2/2)\,\rho, \quad \bar{\eta} \equiv (1-\lambda^2/2)\,\eta}$$

[Buras *et al.* (1994)]

  – $V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0$:



$$\boxed{\gamma = \gamma' + \delta\gamma, \quad \delta\gamma = \lambda^2\eta = \mathcal{O}(1^\circ)}$$

Determination of the Unitarity Triangle

- Method I: *conventional ("CKM-Fits")* ...

  - Semileptonic $b \to u\ell\overline{\nu}_\ell, c\ell\overline{\nu}_\ell$ decays [$\longrightarrow$ UT side R_b].
  - $B^0_{d,s}$–$\overline{B^0_{d,s}}$ mixing [$\longrightarrow$ UT side R_t].
  - CP violation in the kaon system, ε_K [$\longrightarrow$ hyperbola].

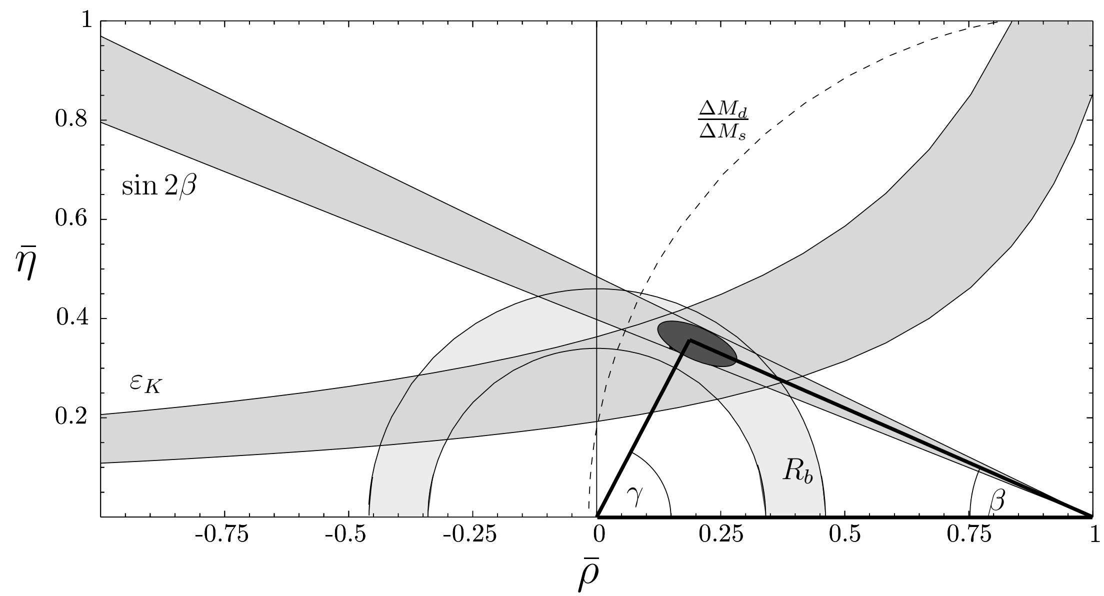
$$\boxed{\text{Theory}} \quad \Rightarrow \quad \boxed{\text{contours in the } \overline{\rho}\text{–}\overline{\eta} \text{ plane}}$$

- Methode II: *future* ...

  - CP-violating effects in B decays [$\longrightarrow \sin 2\beta$, ...]

$$\boxed{\text{Theory}} \quad \Rightarrow \quad \boxed{\text{angles of the unitarity triangle}}$$

- Example of a specific analysis:



[Buras, Schwab & Uhlig, hep-ph/0405132; alternative analyses: `http://ckmfitter.in2p3.fr/`, `http://www.utfit.org`]

- In the future, more contours in the $\overline{\rho}$–$\overline{\eta}$ plane can be added:
  - Alternative determinations of R_t through rare decays.
  - $K^+ \to \pi^+ \nu\bar{\nu} \quad \longrightarrow \quad$ ellipse.
  - $K_{\rm L} \to \pi^0 \nu\bar{\nu} \quad \longrightarrow \quad |\overline{\eta}|$, i.e. horizontal line.

$\oplus$ measurements of the UT angles $\Rightarrow$ $\boxed{\textit{overconstrain}\text{ the UT!}}$

The System of the B Mesons

- Promising experimental perspective:
  - The asymmetric e^+–e^- B factories are currently taking data:
    $\longrightarrow$ already $\mathcal{O}(10^8)$ produced $B\overline{B}$ at BaBar (SLAC) & Belle (KEK);
    first results from CDF-II and D0-II (FNAL).
  - 2nd generation B-decay studies at the Large Hardon Collider (CERN):
    - LHCb; also ATLAS and CMS $\gtrsim$ 2007
  - Discussion of an e^+–e^- super-B factory : $\gtrsim$ 201?
- Interesting playground for theorists:
  - Aspects of strong interactions
  - Aspects of weak interactions
  - Offers probes to search for NP ...

$\longrightarrow$ $\boxed{\text{fruitful interplay between theory and experiment!}}$

Basics of the B-Meson System

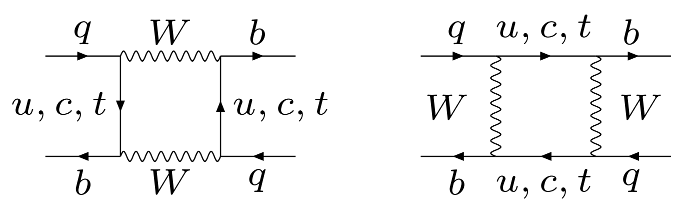
- Charged B mesons: $B^+ \sim u\,\overline{b}$ $\quad B^- \sim \overline{u}\,b$

  $B_c^+ \sim c\,\overline{b}$ $\quad B_c^- \sim \overline{c}\,b$

- Neutral B mesons: $B_d^0 \sim d\,\overline{b}$ $\quad \overline{B_d^0} \sim \overline{d}\,b$

  $B_s^0 \sim s\,\overline{b}$ $\quad \overline{B_s^0} \sim \overline{s}\,b$

  – B_q^0–$\overline{B_q^0}$ mixing:



$$\Rightarrow \quad \boxed{\,|B_q(t)\rangle = a(t)|B_q^0\rangle + b(t)|\overline{B_q^0}\rangle \ :\,}$$

  $*$ Schrödinger equation $\Rightarrow$ mass eigenstates:

$$\boxed{\Delta M_q \equiv M_{\rm H}^{(q)} - M_{\rm L}^{(q)}, \quad \Delta\Gamma_q \equiv \Gamma_{\rm H}^{(q)} - \Gamma_{\rm L}^{(q)}}$$

  $*$ Decay rates: $\Gamma(\overset{(-)}{B_q^0}(t) \to \overset{(-)}{f})$:

$$\cos(\Delta M_q t) \ \& \ \sin(\Delta M_q t) \quad \longrightarrow$$ oscillations!

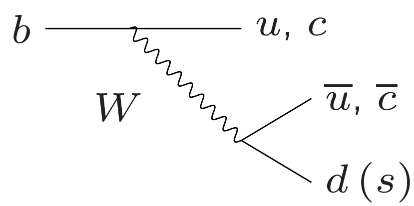
Key Rôle for CP Violation:

Nonleptonic B Decays

$\longrightarrow$ only quarks in the final states!

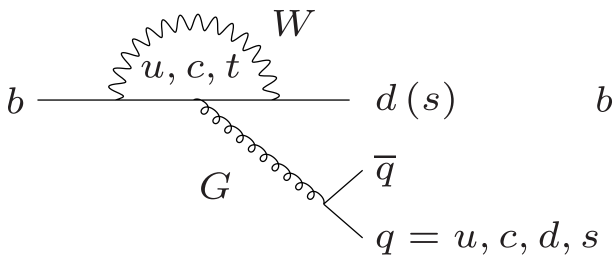
Topologies & Classification

- Tree diagrams:

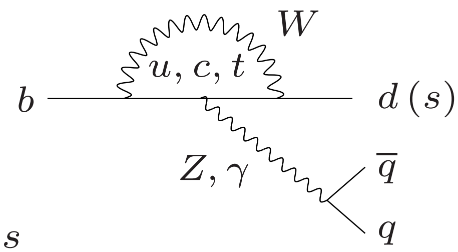


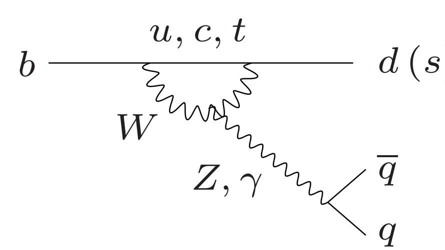
- Penguin diagrams:

QCD penguins:



Electroweak (EW) penguins:





- Classification (depends on the flavour content of the final state):
  - Only tree diagrams.
  - Tree and penguin diagrams.
  - Only penguin diagrams.

Low-Energy Effective Hamiltonians

- Operator product expansion (OPE): $\Rightarrow$

$$\boxed{\langle f|\mathcal{H}_{\text{eff}}|i\rangle = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM} \sum_k C_k(\mu)\, \langle f|Q_k(\mu)|i\rangle}$$

[$G_{\rm F}$: Fermi constant, $V_{\rm CKM}$: CKM factor, μ: renormalization scale]

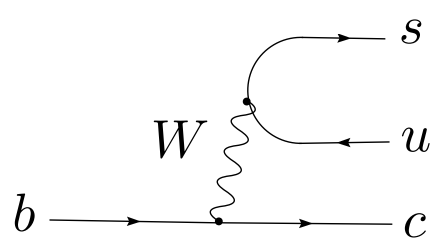
- The operator product expansion allows a separation of the short-distance from the long-distance contributions:
  - *Perturbative* Wilson coefficients $C_k(\mu)$ $\rightarrow$ short-distance physics.
  - *Non-perturbative* hadronic MEs $\langle f|Q_k(\mu)|i\rangle$ $\rightarrow$ long-distance physics.

- The Q_k are local operators, which are generated through the electroweak interactions and QCD, and govern "effectively" the decay in question.

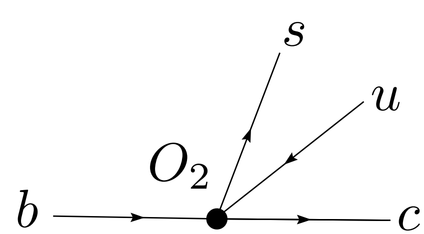
- The Wilson coefficients $C_k(\mu)$ describe the scale-dependent "couplings" of the interaction vertices associated with the Q_k.

- Illustration through an example:

  - Consider a pure "tree" decay: $\boxed{b \longrightarrow c\bar{u}s}$



  - "Integrate out" the W boson:

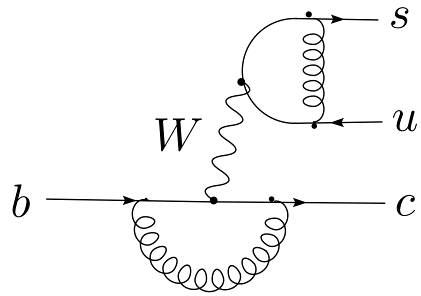


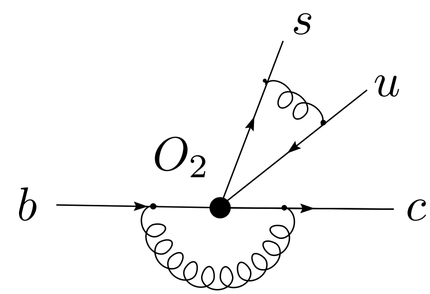
$$\frac{g_{\nu\mu}}{k^2 - M_W^2} \quad \xrightarrow{k^2 \ll M_W^2} \quad -\frac{g_{\nu\mu}}{M_W^2} \equiv -\left(\frac{8G_F}{\sqrt{2}g_2^2}\right) g_{\nu\mu}$$

$$\Rightarrow \mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb} \underbrace{[\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha]\,[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\beta]}_{\text{“current–current” operator } O_2} \equiv \frac{G_F}{\sqrt{2}} V_{us}^* V_{cb} C_2 O_2$$

– Impact of QCD, i.e. exchange of gluons:

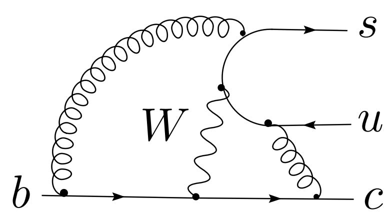
∗ *Factorizable* QCD corrections:

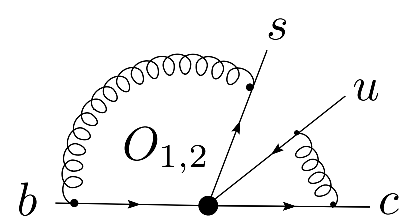




$\longrightarrow$ C_2 acquires a renormalization-scale dependence, i.e. $C_2(\mu) \neq 1$

∗ *Non-factorizable* QCD corrections:





$\longrightarrow$ generation of a second current–current operator:

$$O_1 \equiv [\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\beta]\,[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\alpha]$$

$\longrightarrow$ $\boxed{\textit{operator mixing} \text{ through QCD!}}$

- The results for the $C_k(\mu)$ contain $\log(\mu/M_W)$ terms, which become large for renormalization scales μ in the GeV regime:

$\longrightarrow$ what shall we do?

- Use *renormalization-group improved perturbation theory*:
  - The fact that the transition matrix element $\langle f|\mathcal{H}_{\text{eff}}|i\rangle$ *cannot* depend on the renormalization scale μ implies a renormalization-group equation.
  - Its solution can be written as follows:

$$\boxed{\vec{C}(\mu) = \hat{U}(\mu, M_W) \cdot \vec{C}(M_W)} \tag{1}$$

  - The initial conditions $\vec{C}(M_W)$ describe the *short-distance* physics at the high-energy scales, and are related to the "Inami–Lim functions".
  - The following terms can be systematically summed up through (1):

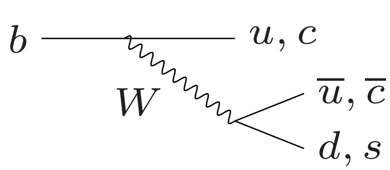
$$\underbrace{\alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^n}_{\text{(LO)}}, \quad \underbrace{\alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^{n-1}}_{\text{(NLO)}}, \quad \ldots$$

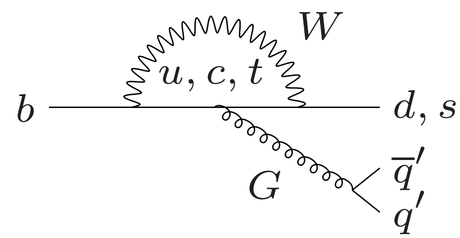
- Low-energy effective Hamiltonians provide a nice tool to deal with weak B- and K-meson decays, as well as with B^0–$\bar{B}^0$ and K^0–$\bar{K}^0$ mixing.

Application to Nonleptonic B Decays

- Particularly interesting: $|\Delta B| = 1,\ \Delta C = \Delta U = 0$

- $\Delta C = \Delta U = 0 \quad \Rightarrow \quad$ $\boxed{\text{tree and penguin processes:}}$





$$\underbrace{V_{uq}^* V_{ub} + V_{cq}^* V_{cb} + V_{tq}^* V_{tb} = 0}_{\text{CKM unitarity } (q \in \{d, s\})} \quad \Rightarrow \quad \boxed{\text{only } \textit{two} \text{ weak amplitudes!}}$$

- Integrate out the W boson and the top quark ($\rightarrow$ penguins):

$$\boxed{\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \left[\sum_{j=u,c} V_{jq}^* V_{jb} \left\{ \underbrace{\sum_{k=1}^{2} C_k(\mu)\, Q_k^{jq}}_{\text{current–current}} + \underbrace{\sum_{k=3}^{10} C_k(\mu)\, Q_k^{q}}_{\text{penguins}} \right\} \right] + \text{h.c.}}$$

- Four-quark operators Q_k^{jq} ($j \in \{u, c\}$, $q \in \{d, s\}$):

  - Current–current operators (tree-like processes):

$$\begin{aligned} Q_1^{jq} &= (\bar{q}_\alpha j_\beta)_{\text{V–A}} (\bar{j}_\beta b_\alpha)_{\text{V–A}} \\ Q_2^{jq} &= (\bar{q}_\alpha j_\alpha)_{\text{V–A}} (\bar{j}_\beta b_\beta)_{\text{V–A}} \end{aligned}$$

  - QCD penguin operators:

$$\begin{aligned} Q_3^{q} &= (\bar{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_4^{q} &= (\bar{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V–A}} \\ Q_5^{q} &= (\bar{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_6^{q} &= (\bar{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \end{aligned}$$

  - EW penguin operators:

$$\begin{aligned} Q_7^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_8^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \\ Q_9^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V–A}} \\ Q_{10}^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{\text{V–A}} \textstyle\sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V–A}} \end{aligned}$$

[Here α, β are $SU(3)_{\text{C}}$ indices, V$\pm$A refers to $\gamma_\mu(1 \pm \gamma_5)$, $q' \in \{u, d, c, s, b\}$ runs over the active quark flavours at $\mu = \mathcal{O}(m_b)$, and the $e_{q'}$ are the electrical charges]

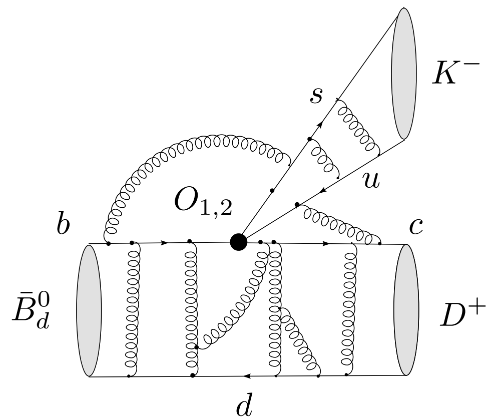
- The Wilson coefficients at $\mu = m_b$ for different renormalization schemes:

	$\Lambda^{(5)}_{\overline{MS}} = 160\text{MeV}$			$\Lambda^{(5)}_{\overline{MS}} = 225\text{MeV}$			$\Lambda^{(5)}_{\overline{MS}} = 290\text{MeV}$		
Scheme	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
C_1	-0.283	-0.171	-0.209	-0.308	-0.185	-0.228	-0.331	-0.198	-0.245
C_2	1.131	1.075	1.095	1.144	1.082	1.105	1.156	1.089	1.114
C_3	0.013	0.013	0.012	0.014	0.014	0.013	0.016	0.016	0.014
C_4	-0.028	-0.033	-0.027	-0.030	-0.035	-0.029	-0.032	-0.038	-0.032
C_5	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.010
C_6	-0.035	-0.037	-0.030	-0.038	-0.041	-0.033	-0.041	-0.045	-0.036
C_7/α	0.043	-0.003	0.006	0.045	-0.002	0.005	0.047	-0.002	0.005
C_8/α	0.043	0.049	0.055	0.048	0.054	0.060	0.053	0.059	0.065
C_9/α	-1.268	-1.283	-1.273	-1.280	-1.292	-1.283	-1.290	-1.300	-1.293
C_{10}/α	0.302	0.243	0.245	0.328	0.263	0.266	0.352	0.281	0.284

[Detailed discussion: A.J. Buras, hep-ph/9806471]

Factorization of Hadronic Matrix Elements

- The problem:



- Transition amplitude:[1]

$$\left[a_2 \equiv C_1 + \frac{C_2}{N_\mathrm{C}}\right]$$

$$\langle D^+K^-|\mathcal{H}_\mathrm{eff}|\overline{B^0_d}\rangle = \frac{G_\mathrm{F}}{\sqrt{2}}V^*_{us}V_{cb}\Big[\underbrace{\left(\frac{C_1}{N_\mathrm{C}}+C_2\right)}_{\equiv a_1}\langle D^+K^-|(\overline{s}_\alpha u_\alpha)_\text{V–A}(\overline{c}_\beta b_\beta)_\text{V–A}|\overline{B^0_d}\rangle$$

$$+2\,C_1\langle D^+K^-|(\overline{s}_\alpha\,T^a_{\alpha\beta}\,u_\beta)_\text{V–A}(\overline{c}_\gamma\,T^a_{\gamma\delta}\,b_\delta)_\text{V–A}|\overline{B^0_d}\rangle\Big]$$

- "Factorization" of the hadronic matrix elements:

$$\langle D^+K^-|(\overline{s}_\alpha u_\alpha)_\text{V–A}(\overline{c}_\beta b_\beta)_\text{V–A}|\overline{B^0_d}\rangle\Big|_\mathrm{fact}$$

$$= \langle K^-|\,[\overline{s}_\alpha\gamma_\mu(1-\gamma_5)u_\alpha]\,|0\rangle\langle D^+|\,[\overline{c}_\beta\gamma^\mu(1-\gamma_5)b_\beta]\,|\overline{B^0_d}\rangle$$

$$\propto f_K[\longrightarrow \text{"decay constant"}] \times F_{BD}[\longrightarrow \text{"form factor"}]$$

$$\langle D^+K^-|(\overline{s}_\alpha\,T^a_{\alpha\beta}\,u_\beta)_\text{V–A}(\overline{c}_\gamma\,T^a_{\gamma\delta}\,b_\delta)_\text{V–A}|\overline{B^0_d}\rangle\Big|_\mathrm{fact} = 0$$

[1] Here we use the well-known $SU(N_\mathrm{C})$ colour-algebra relation $T^a_{\alpha\beta}T^a_{\gamma\delta} = (\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta}/N_\mathrm{C})/2$.

- Long history of factorization:

  Schwinger (1964); Farikov & Stech (1978); Cabibbo & Maiani (1978); Bjorken (1989); Dugan & Grinstein (1991); Politzer & Wise (1991); ...

- Factorization in weak decays in the large-N_C limit:

  Buras, Gérard & Rückl (1986); Buras and Gérard (1988).

- Interesting recent developments: $\rightarrow$ important target $B \rightarrow \pi\pi, \pi K$

  - QCD Factorization (QCDF):

    Beneke, Buchalla, Neubert & Sachrajda (1999–2001); ...

  - Perturbative Hard-Scattering (PQCD) Approach:

    Li & Yu ('95); Cheng, Li & Yang ('99); Keum, Li & Sanda ('00); ...

  - Soft Collinear Effective Theory (SCET):

    Bauer, Pirjol & Stewart (2001); Bauer, Grinstein, Pirjol & Stewart (2003); ...

  - QCD light-cone sum-rule methods:

    Khodjamirian (2001); Khodjamirian, Mannel & Melic (2003); ...

Data indicate large non-factorizable corrections
$\Rightarrow$ remain a theoretical challenge ...

[Buras *et al.*; Ali *et al.*; Bauer *et al.*; Chiang *et al.*; ...]

Towards Studies of

CP Violation in the

B-Meson System

Amplitude Structure

- Because of the unitarity of the CKM matrix, *at most two independent CKM amplitudes* contribute to a given decay, as we have seen above!

- Consequently, we may write the decay amplitudes as follows:

$$\boxed{\begin{aligned} A(\overline{B} \to \overline{f}) &= e^{+i\varphi_1}|A_1|e^{i\delta_1} + e^{+i\varphi_2}|A_2|e^{i\delta_2} \\ A(B \to f) &= e^{-i\varphi_1}|A_1|e^{i\delta_1} + e^{-i\varphi_2}|A_2|e^{i\delta_2} \end{aligned}}$$

  – The $\varphi_{1,2}$ are CP-violating weak phases (CKM matrix)

  – The $|A_{1,2}|e^{i\delta_{1,2}}$ are CP-conserving "strong" amplitudes:

$$\boxed{|A_j|e^{i\delta_j} = \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \overline{f}|Q_k^j(\mu)|\overline{B}\rangle}_{\text{“unknown”}}}$$

$\Rightarrow$ encode the *hadron dynamics* of the decay ...

Direct CP Violation

- The most straightforward CP asymmetry ("direct" CP violation):[2]

$$\mathcal{A}_{\text{CP}} \equiv \frac{\Gamma(B\to f)-\Gamma(\overline{B}\to\overline{f})}{\Gamma(B\to f)+\Gamma(\overline{B}\to\overline{f})} = \frac{|A(B\to f)|^2-|A(\overline{B}\to\overline{f})|^2}{|A(B\to f)|^2+|A(\overline{B}\to\overline{f})|^2}$$

$$= \frac{2|A_1||A_2|\sin(\delta_1-\delta_2)\sin(\varphi_1-\varphi_2)}{|A_1|^2+2|A_1||A_2|\cos(\delta_1-\delta_2)\cos(\varphi_1-\varphi_2)+|A_2|^2}$$

- Provided the two amplitudes satisfy the following requirements:

  i) Non-trivial CP-conserving strong phase difference $\delta_1-\delta_2$.

  ii) Non-trivial CP-violating weak phase difference $\varphi_1-\varphi_2$.

$\Rightarrow$ $\boxed{\text{CP violation originates through interference effects!}}$

- Goal: extraction of $\varphi_1-\varphi_2$ ($\to$ UT angle) from the measured $\mathcal{A}_{\text{CP}}$!

- Problem: uncertainties related to the strong amplitudes $|A_{1,2}|e^{i\delta_{1,2}}$ ...

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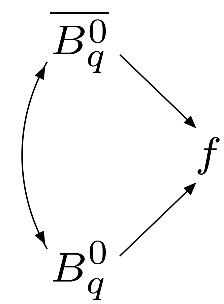
[2] This CP asymmetry is the B-meson counterpart of ε'/ε; established through $B_d\to\pi^\mp K^\pm$ in '04.

Two Main Strategies

- Amplitude relations allow us in fortunate cases to eliminate the hadronic matrix elements ($\to$ typically strategies to determine γ):
  - *Exact* relations: class of pure "tree" decays (e.g. $B \to DK$).
  - *Approximate* relations, which follow from the *flavour symmetries* of strong interactions, i.e. $SU(2)$ isospin or $SU(3)_{\mathrm{F}}$:

$$B \to \pi\pi,\; B \to \pi K,\; B_{(s)} \to KK.$$

- Decays of neutral B_d and B_s mesons:



$$\boxed{\text{Interference effects through } B^0_q\text{–}\overline{B^0_q} \text{ mixing}}$$

  - "Mixing-induced" CP violation!
  - If one CKM amplitude dominates (e.g. $B_d \to \psi K_{\mathrm{S}}$):

$$\Rightarrow \text{ hadronic matrix elements cancel!}$$

  - Otherwise, we have to rely again on amplitude relations ...

The Major Lessons of Lecture I

- Central Target: $\boxed{\text{UT of the CKM matrix}}$

- The B-meson system and ε_K allow us to determine this triangle; in the future also rare B and K decays will enter this game.

- A key rôle is played by non-leptonic B decays:

  $\longrightarrow$ CP violation & direct determination of the UT angles!

- Theoretical description of non-leptonic B decays:

  – Low-energy effective Hamiltonians [$\longrightarrow$ general, very useful tool]
  – Factorization

- Key Problem: $\boxed{\text{hadronic matrix elements}}$

  $\longrightarrow$ two main strategies $\longrightarrow$ Lecture II