Radiative, Semileptonic and Leptonic Rare B-Decays

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Plan of Talk

- Interest in Rare B Decays
- $B \to X_s\gamma$: SM Predictions and Comparison with Data
- Exclusive Decays $B \to K^*\gamma$: NLO Predictions and Current Data
- Exclusive Radiative Decays $B \to (\rho, \omega)\gamma$ in the SM and their Impact on the CKM Phenomenology
- $B \to X_s\ell^+\ell^-$ in NNLO & Current Data
- Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$ & Current Data
- Flavour Structure in Minimal Supersymmetric Standard Model (MSSM)
- Possible Supersymmetric Effects in Rare B-Decays
- $B_s(B_d) \to \mu^+\mu^-$ Decays & SUSY
- A Model-independenet analysis of $B \to X_s\gamma$ & $B \to X_s\ell^+\ell^-$
- Future Prospects in Rare B-Decays & Summary

Interest in Rare B Decays

- Rare B Decays ($b \to s\gamma, b \to d\gamma, b \to s\ell^+\ell^-, ...$) are Flavour-Changing-Neutral-Current (FCNC) processes ($|\Delta B| = 1, |\Delta Q| = 0$)
- In the SM, all electrically neutral bosons (γ, Z^0, H^0, Gluons) have only Flavour-diagonal couplings. Hence, in the SM, FCNC processes are not allowed at the Tree level
- Instead, FCNC processes are governed by the GIM mechanism, which imparts them sensitivity to higher scales (m_t, m_W)
- GIM amplitudes (renormalized by QCD corrections) involve, in particular, CKM matrix elements $V_{ti}; i = d, s, b$; hence rare B-decays play an important role in the determination of these matrix elements
- FCNC processes are sensitive to physics beyond the SM, such as supersymmetry, and the BSM amplitudes can be comparable to the (tW)-part of the GIM amplitudes
- Last, but not least, Rare B-decays enjoy great attention in the ongoing and planned experimental programme in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

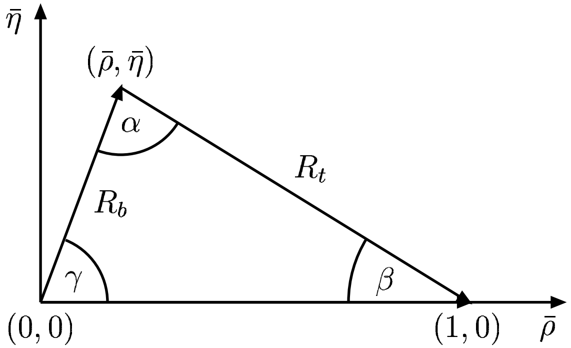
- Customary to use the handy Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2(1+i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters: $A, \ \lambda, \ \rho, \ \eta$
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2), \quad \bar{\eta} = \eta(1-\lambda^2/2)$$

- The CKM-Unitarity triangle $[\phi_1 = \beta; \ \ \phi_2 = \alpha; \ \ \phi_3 = \gamma]$



Courtesy: G. Isidori (FNAL '05 Talk)

Towards a model independent approach to the flavour problem:

th. error $\lesssim 10\%$
(grey circle) = exp. error $\lesssim 10\%$
(white circle) = exp. error ~ 30%

FLAVOUR COUPLING:

ELECTROWEAK STRUCTURE

	$b \to s$ ($\sim\lambda^2$)	$b \to d$ ($\sim\lambda^3$)	$s \to d$ ($\sim\lambda^5$)
$\Delta F=2$ box	ΔM_{B_s}, $A_{CP}(B_s\to\psi\phi)$	ΔM_{B_d}, $A_{CP}(B_d\to\psi K)$	ΔM_K, ε_K
$\Delta F=1$ 4-quark box	$B_d\to\phi K$, $B_d\to K\pi$, ...	$B_d\to\pi\pi$, $B_d\to\rho\pi$, ...	ε'/ε, $K\to 3\pi$, ...
gluon penguin	$B_d\to X_s\gamma$, $B_d\to\phi K$, $B_d\to K\pi$, ...	$B_d\to X_d\gamma$, $B_d\to\pi\pi$, ...	ε'/ε, $K_L\to\pi^0 l^+l^-$, ...
γ penguin	$B_d\to X_s l^+l^-$, $B_d\to X_s\gamma$, $B_d\to\phi K$, $B_d\to K\pi$, ...	$B_d\to X_d l^+l^-$, $B_d\to X_d\gamma$, $B_d\to\pi\pi$, ...	ε'/ε, $K_L\to\pi^0 l^+l^-$, ...
Z^0 penguin	$B_d\to X_s l^+l^-$, $B_s\to\mu\mu$, $B_d\to\phi K$, $B_d\to K\pi$, ...	$B_d\to X_d l^+l^-$, $B_d\to\mu\mu$, $B_d\to\pi\pi$, ...	ε'/ε, $K_L\to\pi^0 l^+l^-$, $K\to\pi\nu\nu$, $K\to\mu\mu$, ...
H^0 penguin	$B_s\to\mu\mu$	$B_d\to\mu\mu$	

Mandatory to explore this corner of the table!

Rare B decays

Two inclusive rare B-decays of current experimental interest

$$\bar{B} \to X_s \gamma \quad \text{and} \quad \bar{B} \to X_s l^+ l^-$$

$X_s =$ any hadronic state with $S = -1$, containing no charmed particles

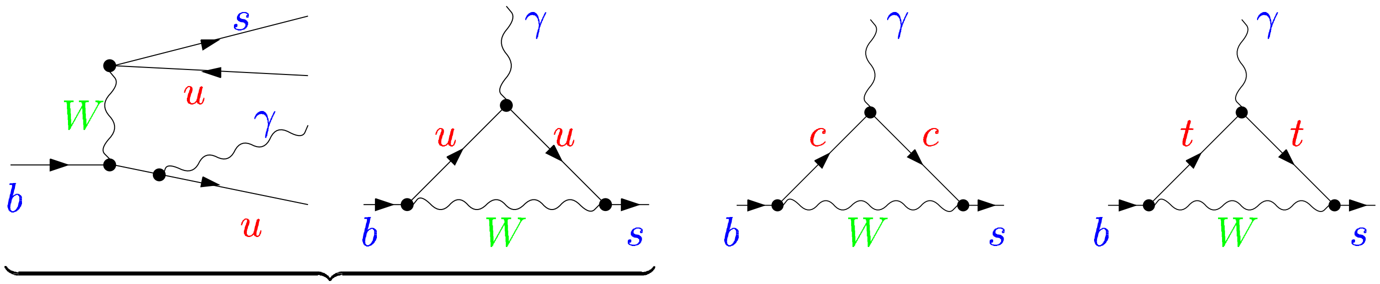
Theoretical Interest:

- Accurate measurements anticipated in near future
- Non-perturbative effects under control
- Sensitivity to new physics

Status of the NNLO perturbative calculations:

- $\bar{B} \to X_s l^+ l^-$: completed
- $\bar{B} \to X_s \gamma$: $\sim \frac{1}{3}$ way through [Misiak, Steinhauser, Greub, Haisch, Gorbahn, Schröder, Czakon,...]

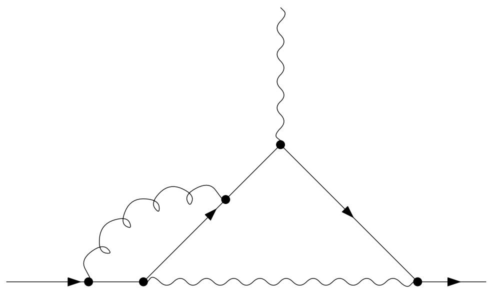
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.



QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($\bar{B} \to X_s\gamma$) more than twice.

Effective field theory method is the most convenient for resummation of such large logarithms.

The effective Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \ \ l = e, \mu)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1,2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\mathbf{\Sigma}_q(\bar{q}\Gamma_i' q), & i = 3,4,5,6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, \mathbf{10} \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Status of the SM calculations for $\bar{B} \to X_s \gamma$ (Courtesy: M. Misiak)

Matching $(\mu_0 \sim M_W, m_t)$:

$$C_i(\mu_0) = C_i^{(0)}(\mu_0) + \frac{\alpha_s(\mu_0)}{4\pi} C_i^{(1)}(\mu_0) + \left(\frac{\alpha_s(\mu_0)}{4\pi}\right)^2 C_i^{(2)}(\mu_0)$$

$i = 1, ..., 6$:	tree	1-loop	2-loop	[Bobeth, Misiak, Urban, NPB 574 (2000) 291]
$i = 7, 8$:	1-loop	2-loop	3-loop	[Steinhauser, Misiak, hep-ph/0401041]

The 3-loop matching has less than 2% effect on BR($\bar{B} \to X_s\gamma$)

Mixing:

$$\hat{\gamma} = \frac{\alpha_s}{4\pi} \begin{pmatrix} 1\mathrm{L} & 2\mathrm{L} \\ 0 & 1\mathrm{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^2 \begin{pmatrix} 2\mathrm{L} & 3\mathrm{L} \\ 0 & 2\mathrm{L} \end{pmatrix} + \left(\frac{\alpha_s}{4\pi}\right)^3 \begin{pmatrix} 3\mathrm{L} & 4\mathrm{L} \\ 0 & 3\mathrm{L} \end{pmatrix}$$

Haisch, Gorbahn, Gambino, Schröder, Czakon

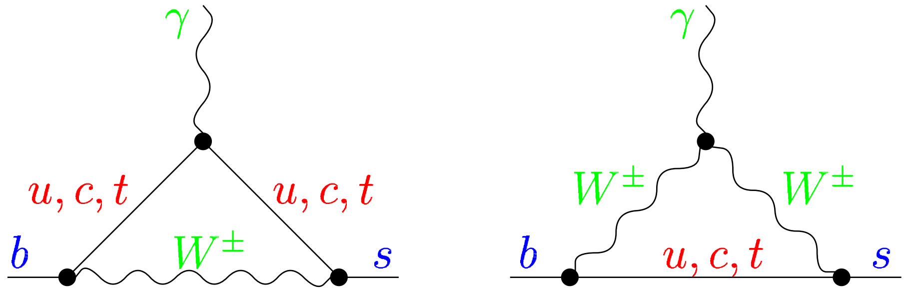
Matrix elements $(\mu_b \sim m_b)$:

$$\langle O_i \rangle(\mu_b) = \langle O_i \rangle^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} \langle O_i \rangle^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 \langle O_i \rangle^{(2)}(\mu_b)$$

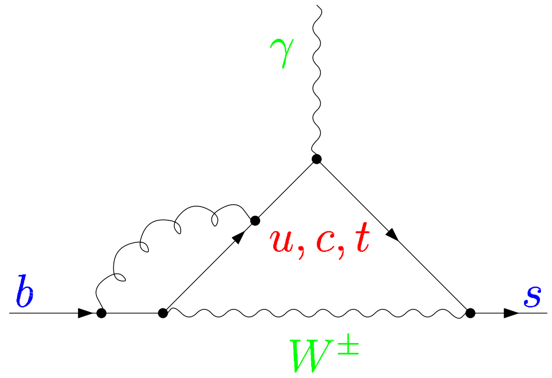
$i = 1, ..., 6$:	1-loop	2-loop	3-loop	[Bieri, Greub, Steinhauser, hep-ph/0302051] $\mathcal{O}(\alpha_s^2 n_f)$, Steinhauser, Misiak
$i = 7, 8$:	tree	1-loop	2-loop	[Greub, Hurth, Asatrian]

Examples of SM diagrams for the matching of $C_7(\mu_0)$:

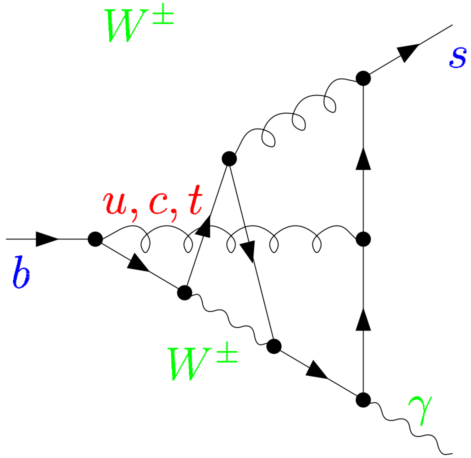
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



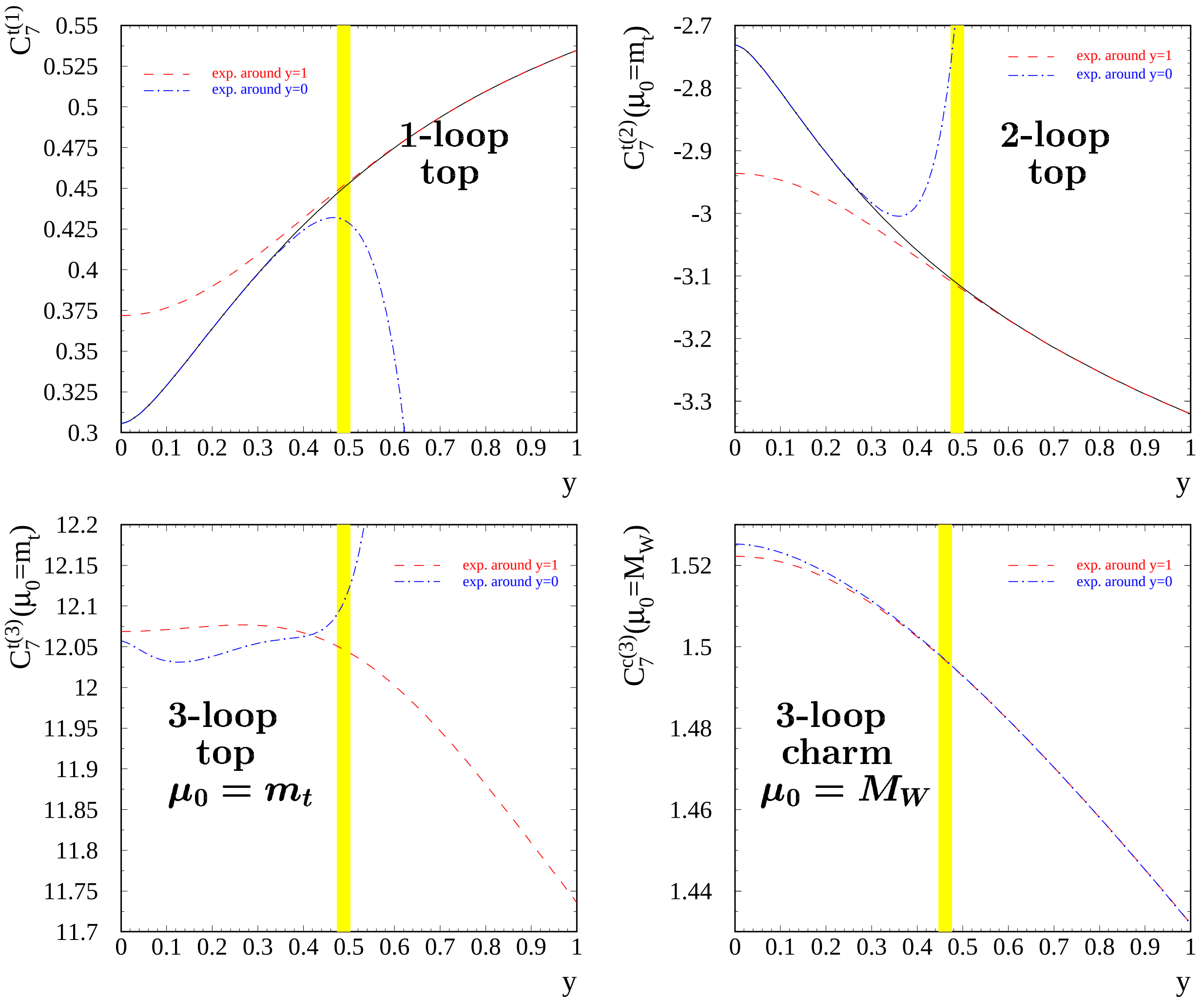
NNLO:
[Steinhauser, Misiak, 2004]



The "flavour-split" matching condition:

$$V_{ts}^* V_{tb} C_7(\mu_0) \equiv V_{ts}^* V_{tb} C_7^t(\mu_0) + (V_{us}^* V_{ub} + V_{cs}^* V_{cb})\, C_7^c(\mu_0)$$

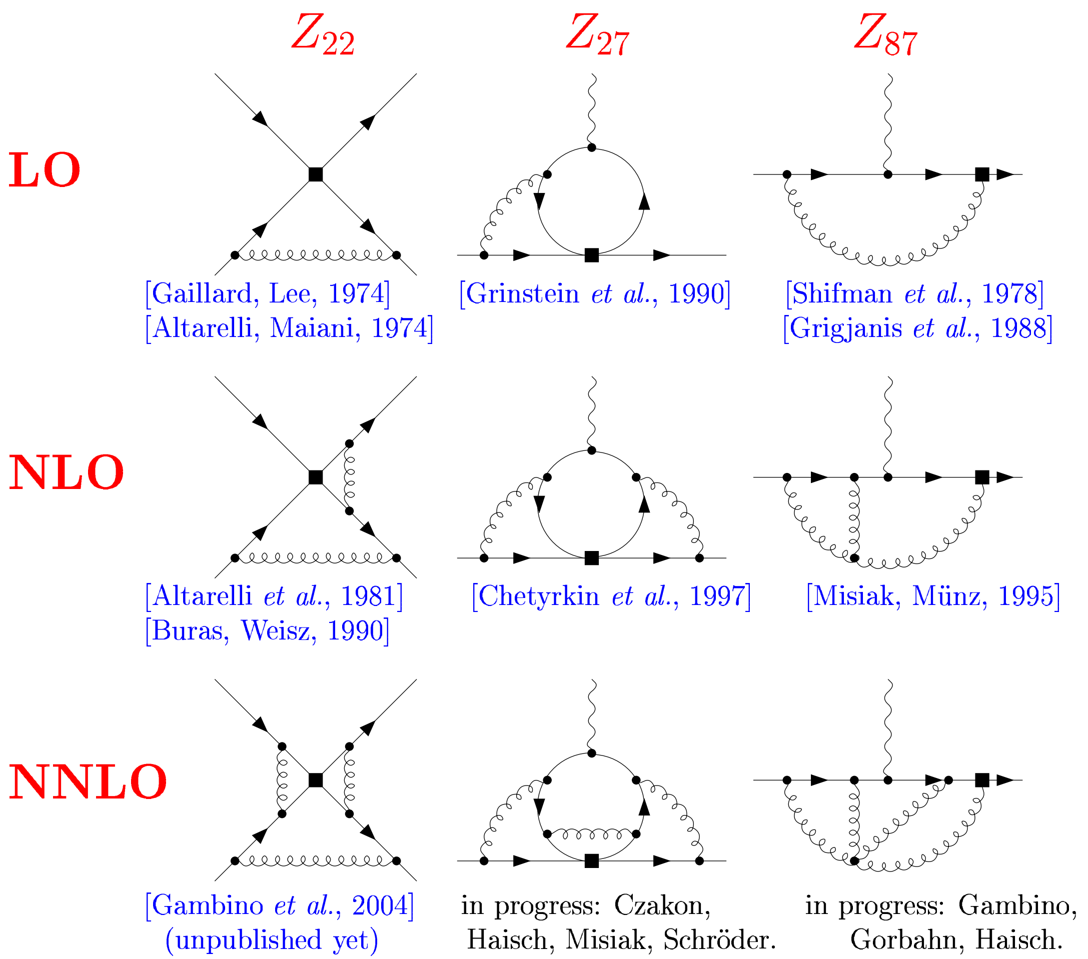
The coefficients $C_7^{Q(n)}(\mu_0)$ as functions of $y = \frac{M_W}{m_t(\mu_0)}$



Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in $b \to s\gamma$ amplitude

RGE for the Wilson coefficients $\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$

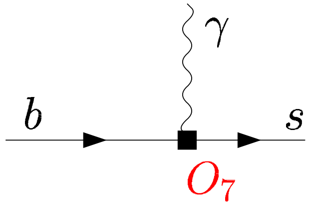
- Renormalization constants $\Longrightarrow$ γ_{ij}



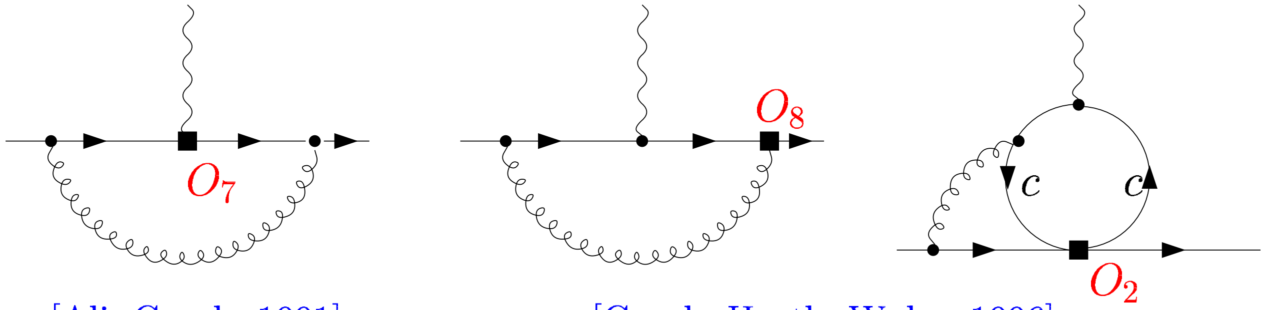
The $b \to s\gamma$ matrix elements

Perturbative on-shell amplitudes

LO

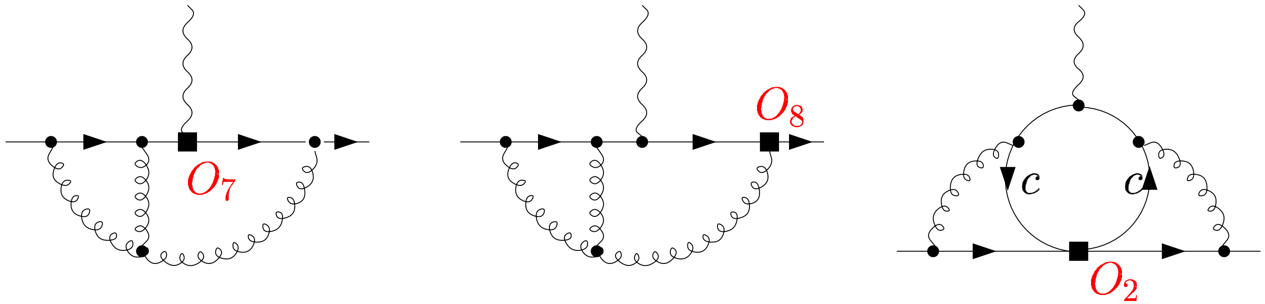


NLO



[Ali, Greub, 1991]   [Greub, Hurth, Wyler, 1996]

NNLO



in progress: Asatrian, Greub, Hurth

[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, Misiak
(extrapolation in m_c)

Non-perturbative effects in $\bar{B} \to X_s\gamma$

We need to sum the matrix elements of the effective Hamiltonian:

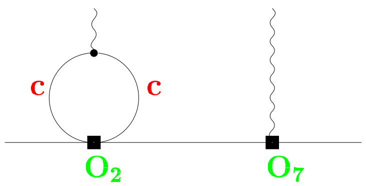
$$\Sigma_{X_s} \left| C_7 \langle X_s\gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s\gamma | O_2 | \bar{B} \rangle + ... \right|^2$$

The "77" term in the above sum can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude

HQET gives us a double expansion:

$$\Sigma_{X_s} \mathrm{BR}(\bar{B} \to X_s\gamma)_{E_\gamma > 1 \ \mathrm{GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + ... \right] + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + ... \right]$$
$$+ \mathcal{O} \left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right] + \text{ [Contributions other than the "77" term]}$$

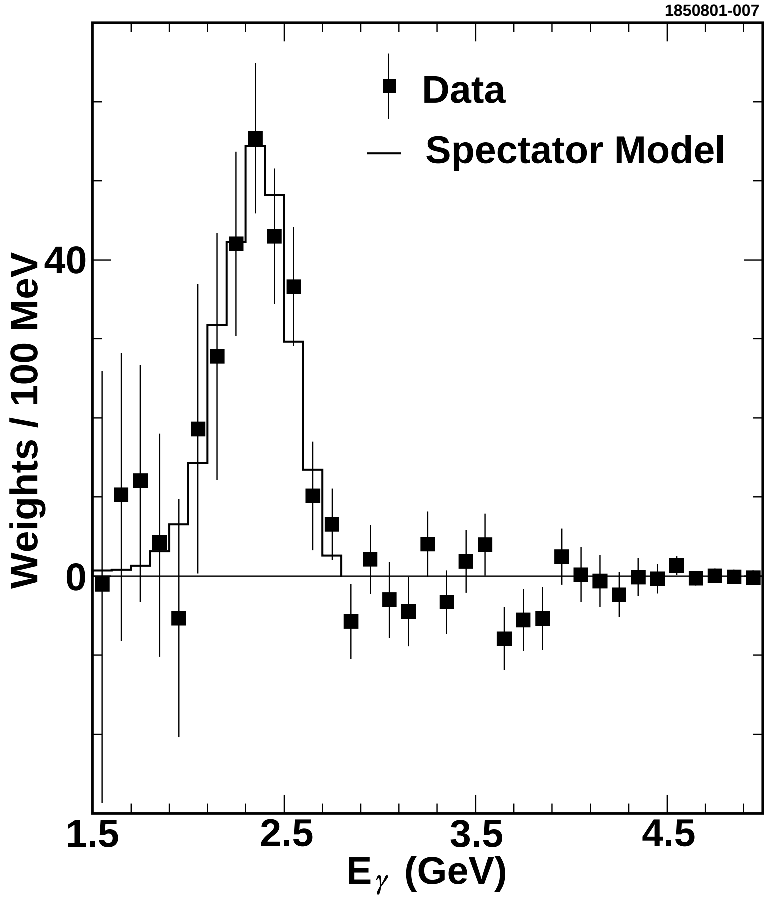
Contributions from Operators containing the charm quark at the leading order in α_s can be expressed as a power series:

$\langle \bar{B} |$  $| \bar{B} \rangle$

$$= \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n ,$$

which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n. The calculable $n = 0$ term makes BR$[\bar{B} \to X_s\gamma]$ increase by around 3%.

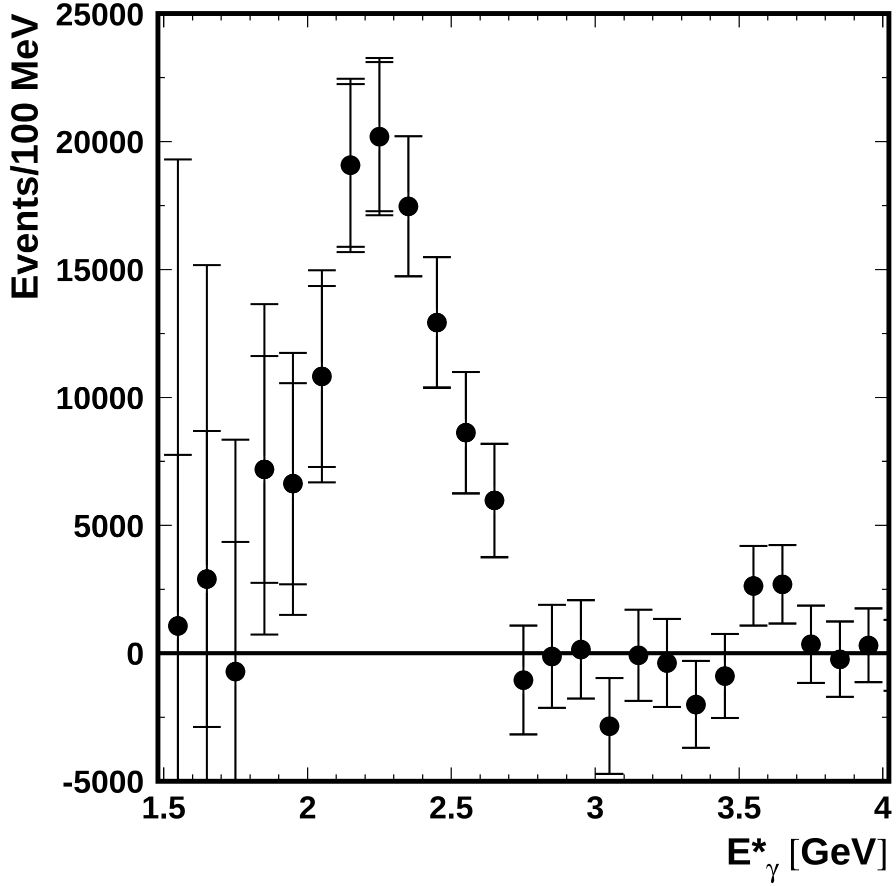
Measurement of $\bar{B} \to X_s \gamma$



CLEO

hep-ex/0108032

PRL 87 (2001) 251807



BELLE

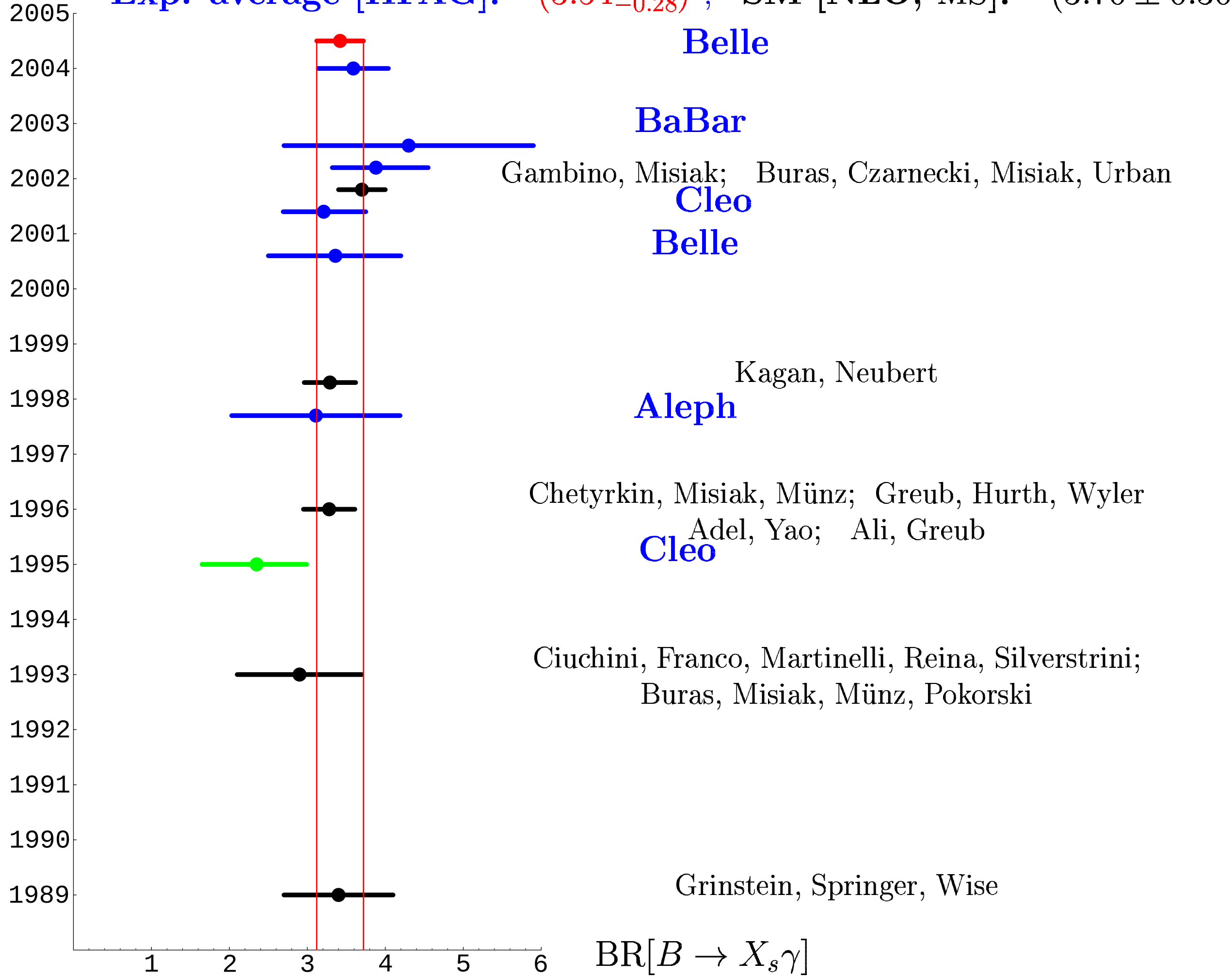
hep-ex/0403004

Evolution in time

$\mathrm{BR}[\bar{B} \to X_s\gamma]$ **(units: 10^{-4})**

Measurements & the SM calculations

Exp. average [HFAG]: $(3.54^{+0.30}_{-0.28})$; **SM [NLO; $\overline{\mathrm{MS}}$]:** (3.70 ± 0.30)



Determination of V_{ts} from BR $(\bar{B} \to X_s \gamma)$

- Unitarity of the CKM Matrix

$$\sum_{u,c,t} \lambda_i = 0, \quad \text{with} \quad \lambda_i = V_{ib} V_{is}^*$$

- $\lambda_u = V_{ub} V_{us}^* \simeq A\lambda^4 (\bar{\rho} - i\bar{\eta}) \simeq O(10^{-2})$
- $\lambda_t = -\lambda_c = -A\lambda^2 + ... = -(41.0 \pm 2.1) \times 10^{-3}$

- Without invoking the CKM unitarity, NLO SM-calculations in the $\overline{\mathbf{MS}}$ scheme and current data imply the following constraint

[Misiak, AA]

$$|1.69\lambda_u + 1.60\lambda_c + 0.60\lambda_t| = (0.94 \pm 0.07)|V_{cb}|$$

$$\Longrightarrow \lambda_t = V_{tb} V_{ts}^* = -(47.0 \pm 8.0) \times 10^{-3}$$

- In future, NNLO calculations will lead to a determination of BR$(\bar{B} \to X_s \gamma)$ to an accuracy of 5%
- With improved data, this will determine V_{ts} to an accuracy of about 10%

$B \to (K^*, \rho)\, \gamma$ decay rates in NLO

- For Large $E_V \sim m_B/2$, symmetries in effective theory $\Longrightarrow$ relations among FFs:

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2)$$

- Symmetries in effective theory broken by perturbative QCD

Factorization Ansatz:

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

Perturbative Corrections:

$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

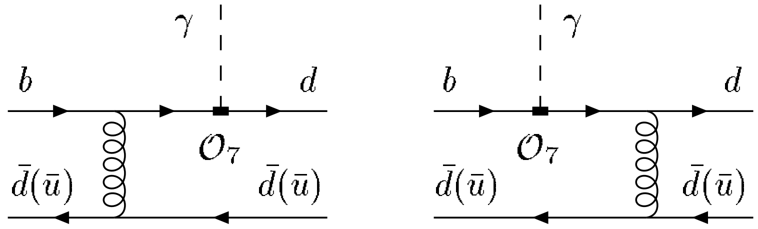
- T_k: Hard Spectator Corrections

$$\Delta\mathcal{M}^{(\mathrm{HSA})} \propto \int_0^1 du \int_0^\infty dl_+\, M^{(B)} M^{(V)}\, T_k$$

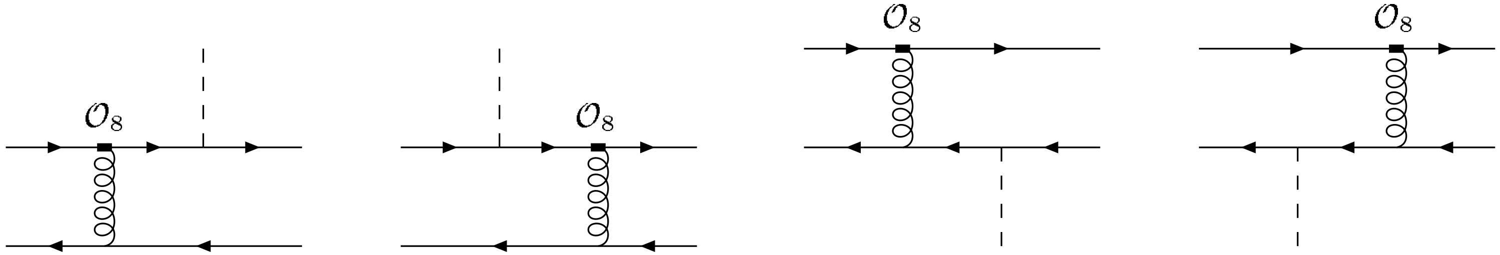
- $M^{(B)}$ and $M^{(V)}$ B-Meson & V-Meson Projection Operators

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

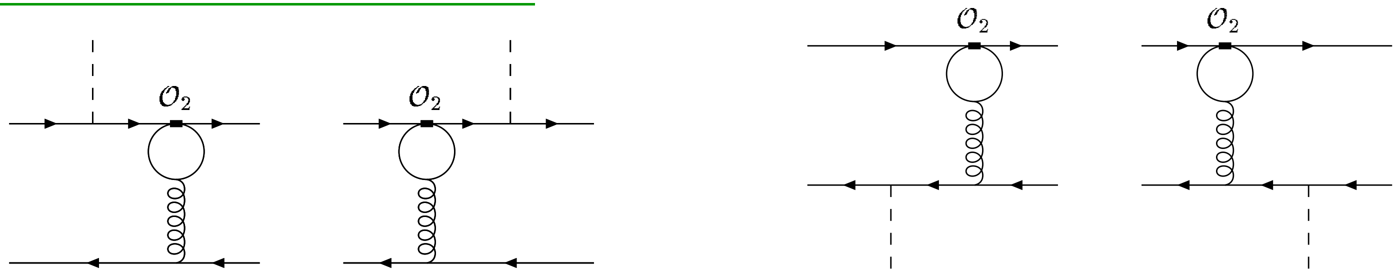
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Comparison with data

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2$$

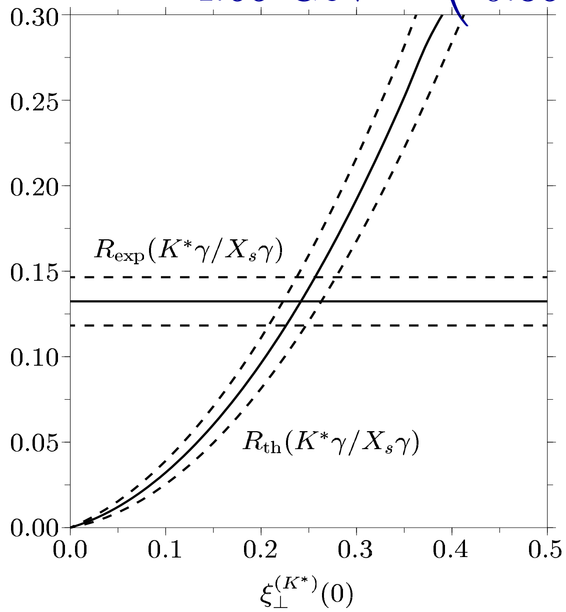
$$K = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \leq K \leq 1.7$$

[Beneke, Feldmann, Seidel; Bosch, Buchalla; Parkhomenko, A.A.]

$$\mathcal{B}_{\rm th}(B^0 \to K^{*0}\gamma) \simeq (6.9 \pm 1.1) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$\mathcal{B}_{\rm th}(B^\pm \to K^{*\pm}\gamma) \simeq (7.4 \pm 1.2) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

- $T_1^{K^*}(0) = (1 + O(\alpha_s))\xi_\perp^{(K^*)}(0)$
  [Beneke, Feldmann]



Current Experimental Average

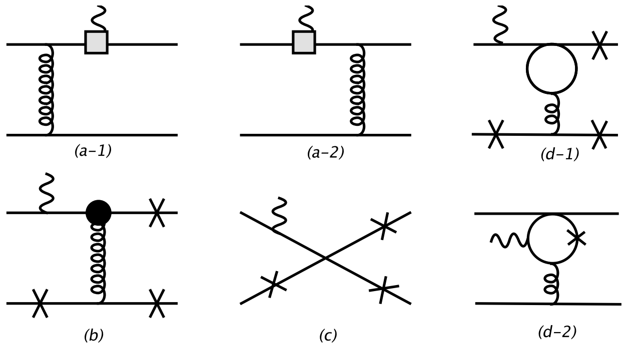
$$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.14 \pm 0.26) \times 10^{-5}$$

$$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (3.98 \pm 0.35) \times 10^{-5}$$

$$\Longrightarrow \quad T_1^{K^*}(0) = 0.27 \pm 0.02$$

$B \to K^*\gamma$ in PQCD

[Keum, Matsumori, Sanda]



$$\begin{aligned} Br(B^0 \to K^{*0}\gamma) &= (4.9 \pm 2.5) \times 10^{-5} \\ Br(B^\pm \to K^{*\pm}\gamma) &= (5.0 \pm 2.5) \times 10^{-5} \end{aligned}$$

$\Longrightarrow$ Form factor: $T_1^{K^*}(0) = 0.23 \pm 0.06$
in agreement with QCDF-based estimates of the same and data

- Isospin Symmetry Breaking :

$$\Delta_{0-} = \frac{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) - Br(B^- \to K^{*-}\gamma)}{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) + Br(B^- \to K^{*-}\gamma)} = (3.0 \pm 0.9)\%$$

[Cf: $\Delta_{0-} = (8 \pm 4)\%$ [Kagan, Neubert (QCDF)]]

- $\Delta_{0-}(K^*\gamma)^{exp} = (3.9 \pm 4.8)\%$

Electromagnetic Radiative Penguins $b \to s\gamma$

- $b \longrightarrow s\gamma$ decay rate

$$\mathcal{B}(B \to X_s\gamma) = (3.52 \pm 0.29) \times 10^{-4} \quad [\mathrm{HFAG'05}]$$

$$SM: (3.70 \pm 0.30) \times 10^{-4} \quad [\mathrm{NLO}; \overline{\mathrm{MS}}]$$

$\Longrightarrow \lambda_t \equiv V_{tb}V_{ts}^* = -(47.0 \pm 8.0) \times 10^{-3}$; in agreement with $\lambda_t \simeq -\lambda_c$

- CP Asymmetry in $b \longrightarrow s\gamma$ transition
  - Direct CPV

$$A_{\mathrm{CP}}(X_s\gamma) \equiv \frac{\Gamma(b \to s\gamma) - \Gamma(\bar{b} \to \bar{s}\gamma)}{\Gamma(b \to s\gamma) + \Gamma(\bar{b} \to \bar{s}\gamma)} = (4.2^{+1.7}_{-1.2}) \times 10^{-3} \quad [\mathrm{SM}]$$

$$A_{\mathrm{CP}}(X_s\gamma) = (5 \pm 36) \times 10^{-3} \quad [\mathrm{HFAG'04}]$$

$$A_{\mathrm{CP}}(K^*\gamma) \leq -0.5\% \quad [\mathrm{SM}]; \quad -0.010 \pm 0.028 \quad [\mathrm{BELLE\ \&\ BABAR}]$$

  - Time-dependent CPV in $B^0 \longrightarrow K^{*0}\gamma$

$$A_{\mathrm{CP}}(t) = S\sin(\Delta m \Delta t) + A\cos(\Delta m \Delta t)$$

$$S \sim 0.04 - 0.10, \quad A \sim 0 \; [\mathrm{SM}]$$

$$S = -0.58^{+0.46}_{-0.38} \pm 0.11; A = 0.03 \pm 0.34 \pm 0.11 [\mathrm{CKM'05}]$$

- ALL CPV measurements in $b \to s\gamma$, $B \to K^*\gamma$ are in agreement with the SM, but still significantly larger than SM CPV effects allowed by data

$B \to \rho\gamma$ decay

Penguin amplitude $\mathcal{M}_{\rm P}(B \to \rho\gamma)$

$$-\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta - i \left[g^{\mu\nu} (q.p) - p^\mu q^\nu \right] \right) T_1^{(\rho)}(0)$$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$

$$e \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_A^{(\rho);\rm p.v.} - i \left[g^{\mu\nu} (q.p) - p^\mu q^\nu \right] F_A^{(\rho);\rm p.c.} \right)$$

- $F_A^{(\rho);\rm p.v.}(0) \simeq F_A^{(\rho);\rm p.c.}(0) = F_A^{(\rho)}(0)$ [e.g., Byer, Melikhov, Stech]

$$\epsilon_{\rm A}(\rho^\pm\gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

- Holds in factorization approximation
- $O(\alpha_s)$ corrections to annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$: Leading-twist contribution vanishes in the chiral limit [Grinstein, Pirjol]; non-factorizing annihilation contribution likely small; testable in $B^\pm \to \ell^\pm \nu_\ell \gamma$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^0 \to \rho^0\gamma)$

• Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\rm A}(\rho^0\gamma) \simeq 0.05$$

$B \to (\rho,\ \omega)\gamma$ decay rates

[Parkhomenko, A.A.; Bosch, Buchalla; Lunghi, Parkhomenko, AA; Beneke, Feldmann, Seidel]

$$R(\rho\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \rho\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \, [1 + \Delta R(\rho/K^*)]$$

$$R(\omega\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \omega\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = 1/2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\omega^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3} \zeta^2 \, [1 + \Delta R(\omega/K^*)]$$

- $S_\rho = 1$ for $B^\pm \to \rho^\pm\gamma$; $= 1/2$ for $B^0 \to \rho^0\gamma$
- $\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.85 \pm 0.10$; $T_1^\omega(0) = T_1^{(\rho)}(0)$ [QCD − SRs, Lattice]
- $\Delta R(\rho^\pm/K^{*\pm}) = 0.12 \pm 0.10$
- $\Delta R(\rho^0/K^{*0}) \simeq \Delta R(\omega/K^{*0}) = 0.1 \pm 0.07$

Theoretical Branching Ratios [Lunghi, Parkhomenko, AA]

- $R(\rho^\pm/K^{*\pm}) = (3.3 \pm 1.0) \times 10^{-2}$
- $R(\rho^0/K^{*0}) \simeq R(\omega/K^{*0}) = (1.6 \pm 0.5) \times 10^{-2}$
- $\mathrm{BR}(\mathrm{B}^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}$
- $\mathrm{BR}(\mathrm{B}^0 \to \rho^0\gamma) \simeq \mathrm{BR}(\mathrm{B}^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}$

Comparison with data

Experimental Average

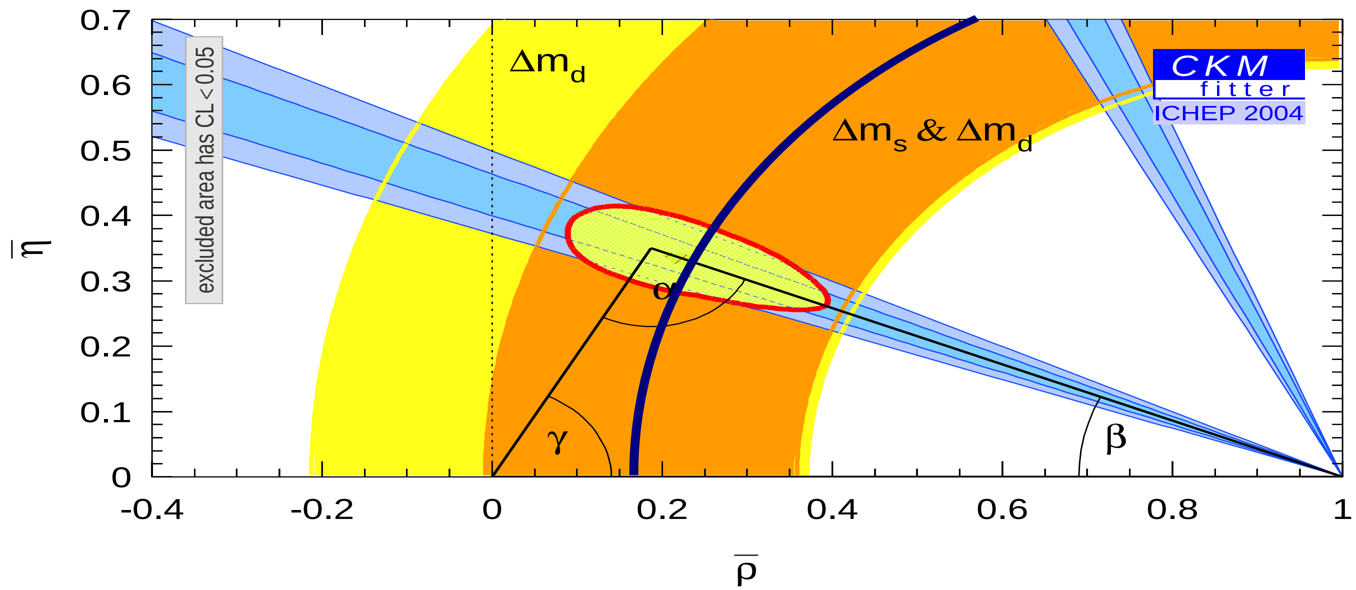
$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$

Upper Limits (90% C.L.)

$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] < 1.4 \times 10^{-6}$; $R[(\rho,\omega)/K^*] < 0.035$; $|V_{td}/V_{ts}| < 0.22$ [BELLE]

$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] < 1.2 \times 10^{-6}$; $R[(\rho,\omega)/K^*] < 0.029$; $|V_{td}/V_{ts}| < 0.19$ [BABAR]

Constraints from $R[(\rho,\omega)/K^*] < 0.029$ on CKM Parameters [Berryhill (BABAR)]



$\bar{B} \to X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

- Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l} \gamma^\mu \gamma_5 l), \qquad i = 9, 10$$

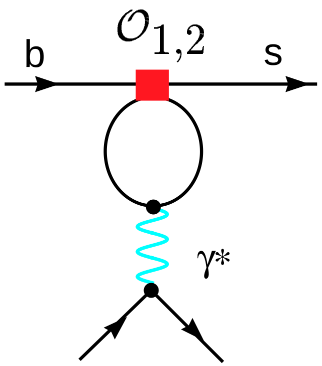
have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

- After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electro-weak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

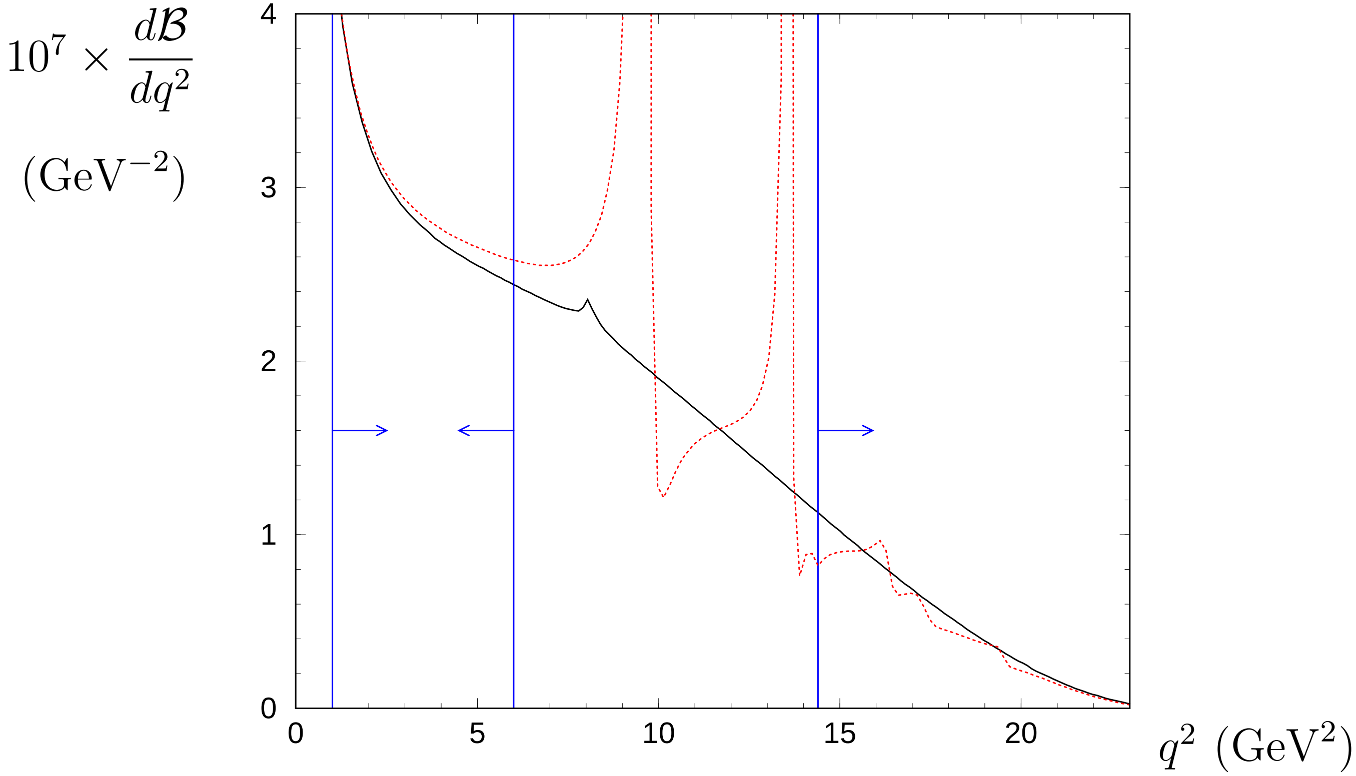
On the other hand: $\quad C_9^{(0)}(m_b) \simeq 2.2$

NNLO Calculations of BR($\bar{B} \to X_s \ell^+ \ell^-$)

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:
    [Asatryan, Asatrian, Greub, Walker;
    Asatrian, Bieri, Greub, Hovhannissyan;
    Ghinculov, Hurth, Isidori, Yao;
    Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays [AA, Greub, Hiller, Lunghi]
  - $\mathrm{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathrm{BR}(\bar{B} \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 \in\ [1,6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathbf{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$,
  in agreement with the earlier NNLO analysis
  [AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003]

Electroweak Penguins $b \to s\ell^+\ell^-$

- $B \to X_s\ell^+\ell^-$ decay rate

$$\mathcal{B}(B \to X_s\ell^+\ell^-) = (4.46^{+0.98}_{-0.96}) \times 10^{-6} \quad [\text{HFAG}'05]$$

$$SM : (4.2 \pm 0.7) \times 10^{-4} \quad [\text{AGHL}'01]; \quad (4.6 \pm 0.8) \times 10^{-4} \quad [\text{GHIY}'04]$$

- Differential distributions in $B \to X_s\ell^+\ell^-$

  - $M(X_s)$-distribution: tests $s \to X_s$ fragmentation model; current FMs provide reasonable fit to data

  - $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', ...$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but the precision is not better than 25%

  - Forward-Backward Asymmetry (FBA) is likewise sensitive to the SM and BSM effects, in particular encoded in the Wilson coefficients C_7, C_9 and C_{10}

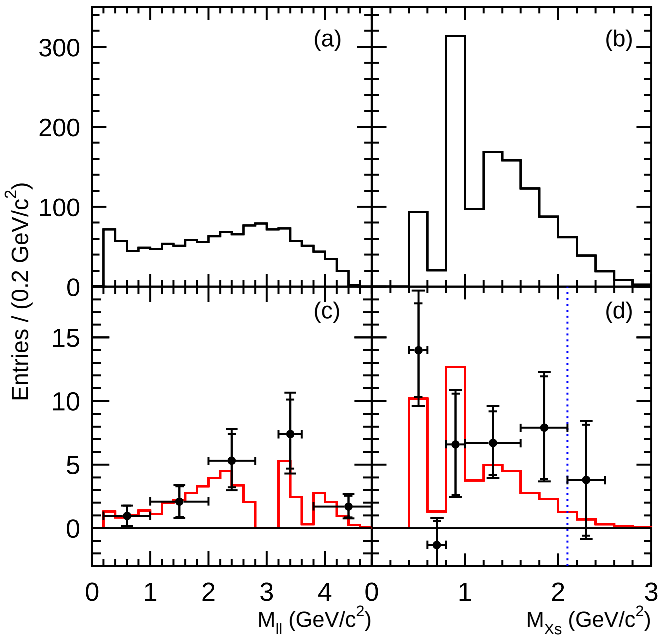
$$A_{\text{FB}}(\hat{s} \sim C_{10}(2C_7 + C_9(\hat{s})\hat{s}); \quad \hat{s} = q^2/M_B^2$$

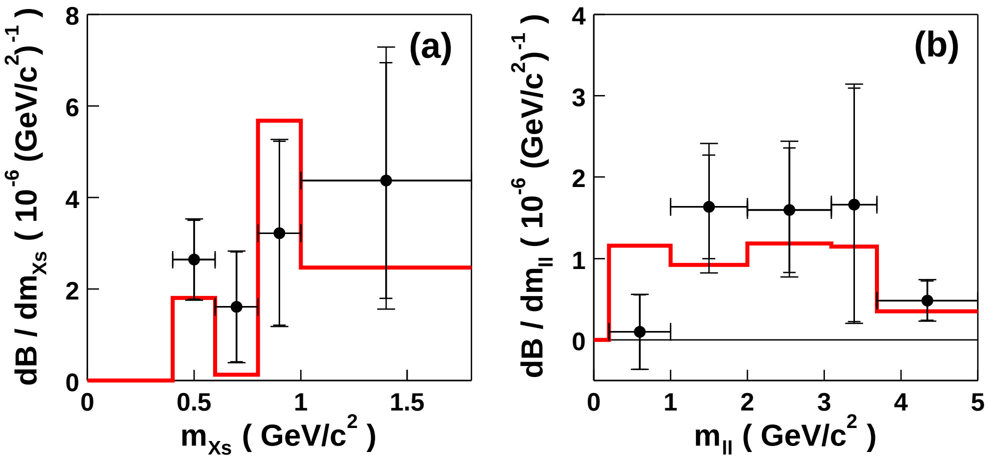
- $A_{\text{FB}}(\hat{s})$ not yet measured; possible only in experiments at B factories

Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE] [BABAR]

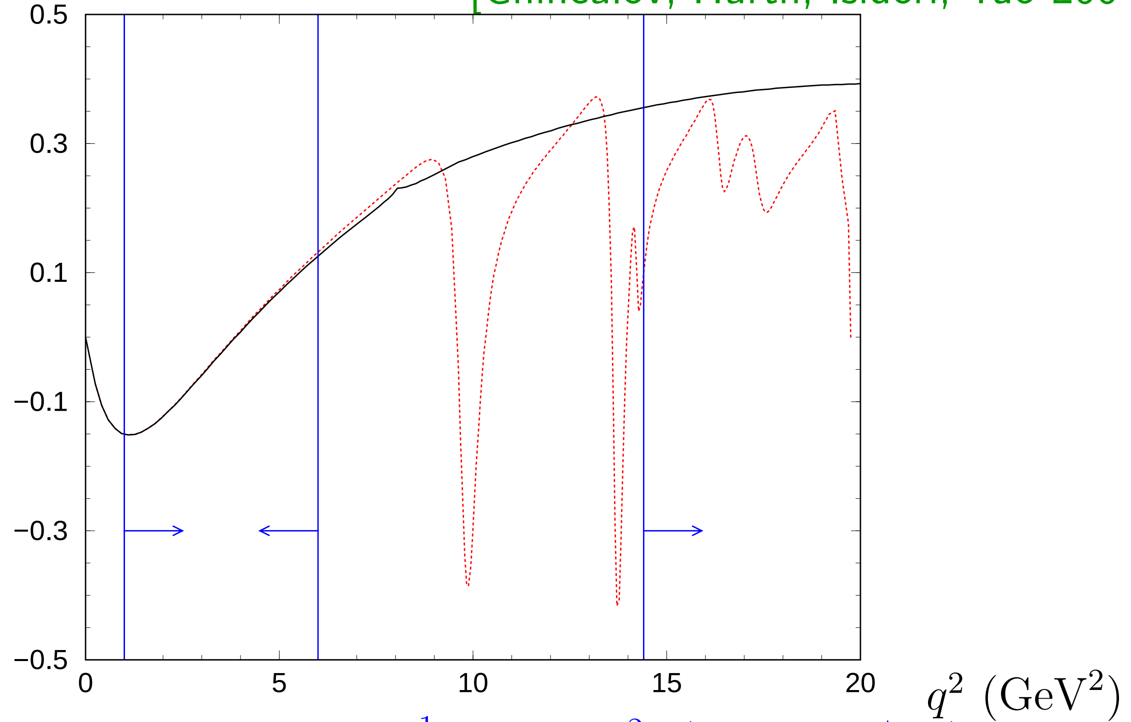




- In agreement with the NNLO SM calculations

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \, d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$q_0^2 = (3.90 \pm 0.25)$ GeV^2 [Ghinculov, Hurth, Isidori, Yao 2004]

$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})$ GeV^2 [Bobeth, Gambino, Gorbahn, Haisch 2003]

Electroweak Penguins $b \to s\ell^+\ell^-$

- $B \to (K, K^*)\ell^+\ell^-$ decay rates

  • Decay rates and distributions depend on the form factors; estimates given below based on Light-cone QCD Sum Rules [Ball, Hiller, Handoko,AA]; Several competing estimates available in the literature [Zhong et al; Melnikov et al.;...]

  $\mathcal{B}(B \to K\ell^+\ell^-) = (0.58 \pm 0.07) \times 10^{-6}$ [HFAG'05]; $(0.35 \pm 0.12) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^*e^+e^-) = (1.44 \pm 0.35) \times 10^{-6}$ [HFAG'05]; $(1.6 \pm 0.5) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^*\mu^+\mu^-) = (1.73^{+0.30}_{-0.27}) \times 10^{-6}$ [HFAG'05]; $(1.2 \pm 0.5) \times 10^{-6}$[SM]

- Differential distributions in $B \to (K, K^*)\ell^+\ell^-$

  • $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', \ldots$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but theoretical precision is not better than 35% due to FF dependence

- The ratio $\mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-)$ sensitive to SUSY effects in the large-$\tan\beta$ region due to Higgs effects

  • $A_{\rm FB}(\hat{s})[B \to K\ell^+\ell^-] \simeq 0$ in the SM and most BSM extensions; in agreement with data which is used as a control sample to measure $A_{\rm FB}(\hat{s})[B \to K^*\ell^+\ell^-]$

  • $A_{\rm FB}(\hat{s})$ in $B \to K^*\ell^+\ell^-$ qualitatively similar to $A_{\rm FB}(\hat{s})$ in $B \to X_s\ell^+\ell^-$, except for FF complication; First measurements from BELLE at hand, appear SM-like; Super-B and LHC-B will measure $A_{\rm FB}(\hat{s})$ precisely

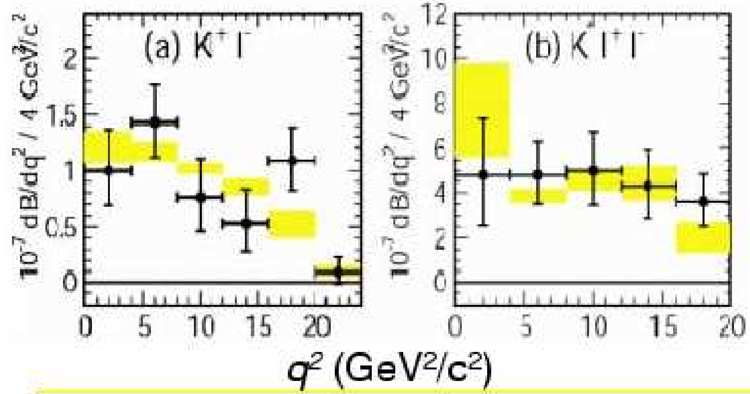
Sakai (FPCP 2004)

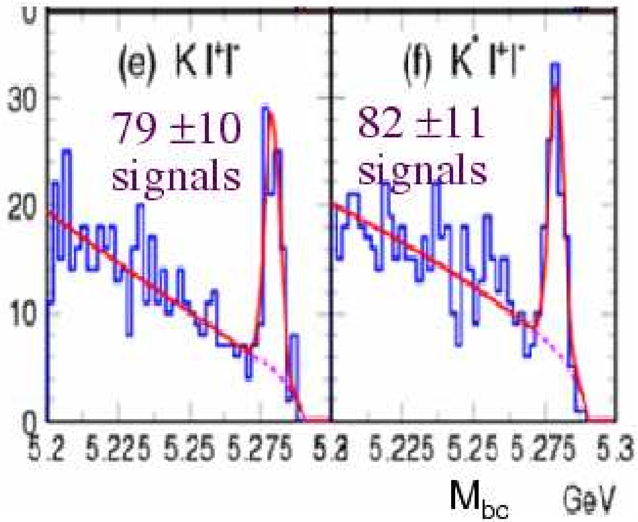
$B \to K^{(*)} l^+ l^-$ [Belle-conf-0415]

LP03: $B \to X_s ll$, $K^{(*)}ll$: Belle/BaBar
Br, A_{CP} ~SM

275M $B\bar{B}$ update >10σ signals

$\mathcal{B}$(Kll)= $(5.50 \pm ^{0.75}_{0.70} \pm 0.27 \pm 0.02)$
$\mathcal{B}$(K*ll)= $(16.5 \pm ^{2.3}_{2.2} \pm 0.9 \pm 0.4\ )$
x 10^{-7}





ICHEP 2004, Beijing 26

Comparison of $\mathcal{B}(B \to (K, K^*)\ell^+\ell^-$ with SM-1

$B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$

Belle branching fractions (253 fb^{-1})

– $K\ell^+\ell^-$: $(5.50\,^{+0.75}_{-0.70} \pm 0.27 \pm 0.02) \times 10^{-7}$

– $K^*\ell^+\ell^-$: $(16.5\,^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7}$

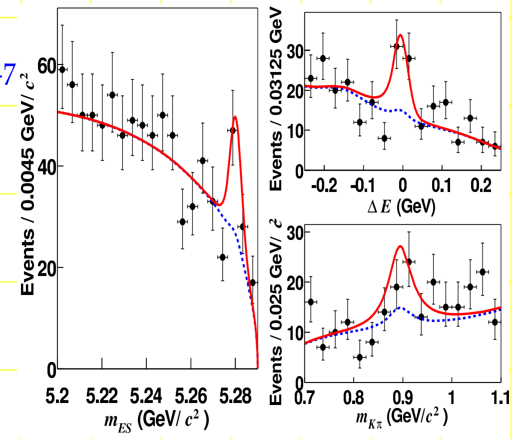
New BaBar results (208 fb^{-1})

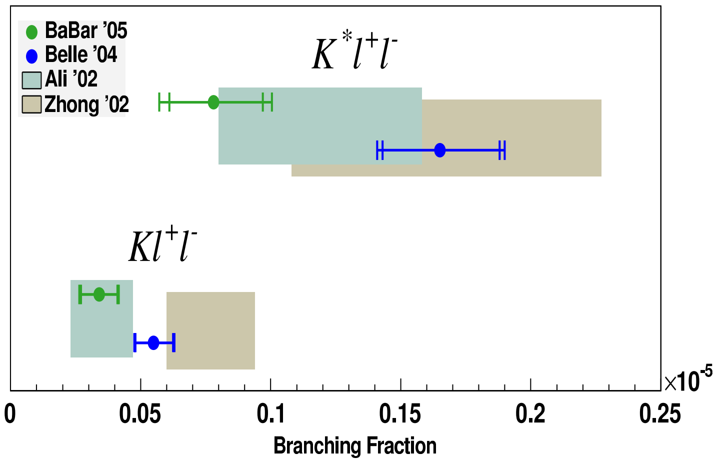
– $K\ell^+\ell^-$: $(3.4 \pm 0.7 \pm 0.3) \times 10^{-7}$

– $K^*\ell^+\ell^-$: $(7.8\,^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7}$

$A_{CP}(B^+ \to K^+\ell^+\ell^-) = -0.08 \pm 0.22 \pm 0.11$

$A_{CP}(B \to K^*\ell^+\ell^-) = +0.03 \pm 0.23 \pm 0.12$

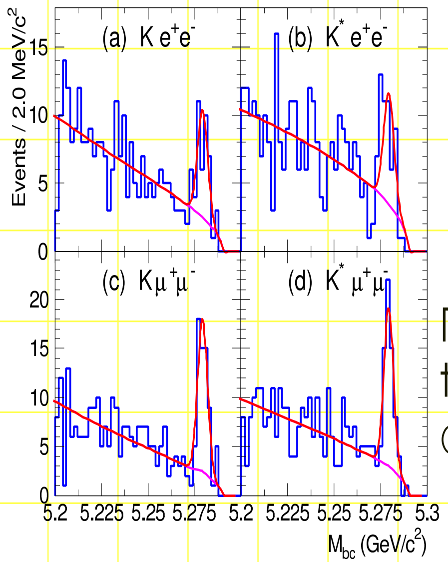


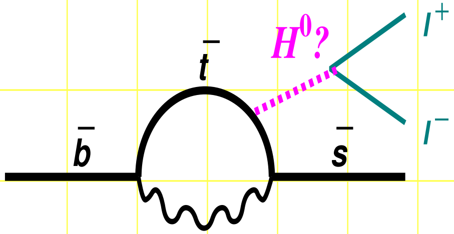


Radiative Pengiuns — Mikihiko Nakao — p.25

Comparison of $\mathcal{B}(B \to (K, K^*)\ell^+\ell^-$ with SM-2

$B \to K^{(*)}e^+e^-$ vs $B \to K^{(*)}\mu^+\mu^-$





Ratio of $K^{(*)}\mu^+\mu^-$ to $K^{(*)}e^+e^-$ is sensitive to neutral SUSY Higgs if $\tan\beta$ is large

($O(1)$ enhancement if $\tan\beta \sim 30$)

Belle ratios:

$\mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-) = 1.38\,^{+0.39}_{-0.41}\,^{+0.06}_{-0.07}$ (1.00 in SM)

$\mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-) = 0.98\,^{+0.30}_{-0.31} \pm 0.08$ ($\sim$ 0.75 in SM)

Babar ratios:

$\mathcal{B}(B \to K\mu^+\mu^-)/\mathcal{B}(B \to Ke^+e^-) = 1.06 \pm 0.48 \pm 0.05$ (1.00 in SM)

$\mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-) = 0.93 \pm 0.46 \pm 0.06$ ($\sim$ 0.75 in SM)

Radiative Pengiuns — Mikihiko Nakao — p.26

Forward-Backward Asymmetry in $B \to K^* \ell^+ \ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff}) V A_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{eff} (V T_2 (1-\hat{m}_V) + A_1 T_1 (1+\hat{m}_V))]$$

- $\boldsymbol{T_1, T_2, V, A_1}$ form factors
  - Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m_{J/\psi}^2$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^* \ell^+ \ell^-$

$$\mathrm{Re}(\mathrm{C_9^{eff}}(\hat{\mathrm{s}}_0)) = -\frac{\hat{\mathrm{m}}_\mathrm{b}}{\hat{\mathrm{s}}_0} \mathrm{C_7^{eff}} (\frac{\mathrm{T_2}(\hat{\mathrm{s}}_0)}{\mathrm{A_1}(\hat{\mathrm{s}}_0)} (1-\hat{\mathrm{m}}_\mathrm{V}) + \frac{\mathrm{T_1}(\hat{\mathrm{s}}_0)}{\mathrm{V}(\hat{\mathrm{s}}_0)} (1+\hat{\mathrm{m}}_\mathrm{V}))$$

- Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
- HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ $(E = \frac{m_B^2 + m_{K^*}^2 - q^2}{2m_B})$ and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1+\hat{m}_V}{1+\hat{m}_V^2 - \hat{s}} (1 - \frac{\hat{s}}{1-\hat{m}_V^2}); \quad \frac{T_1}{V} = \frac{1}{1+\hat{m}_V}$$

- No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]:
  $C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}$

$O(\alpha_s)$ corrections to FB-Asymmetry in $B \to K^* \ell^+ \ell^-$

- $O(\alpha_s)$ corrections to the LEET-symmetry relations lead to substantial perturbative shift in $\hat{s}_0$ [Beneke, Feldmann, Seidel '01]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff} (1 + \frac{\alpha_s C_F}{4\pi}[\ln \frac{m_b^2}{\mu^2} - L] + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)})$$

[AA, A.S. Safir (hep-ph/02054)]

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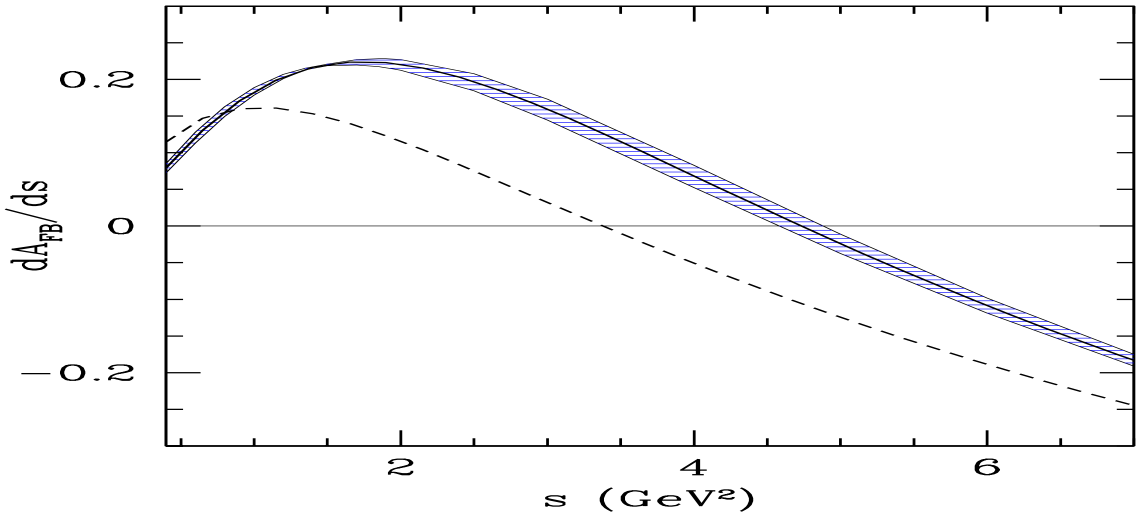


Forward-backward asymmetry $dA_{FB}(B \to K^* l^+ l^-)/ds$ at next-to-leading order (solid center line) and leading order (dashed)

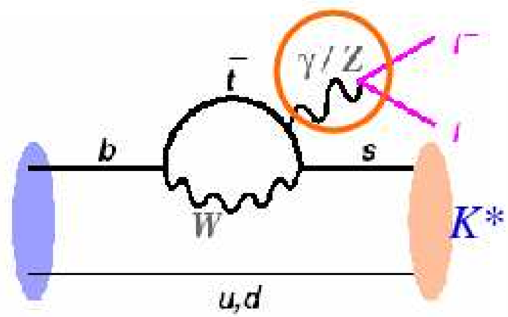
Sakai (FPCP 2004)

$B \to K^* l^+ l^-$:FB Asymmetry

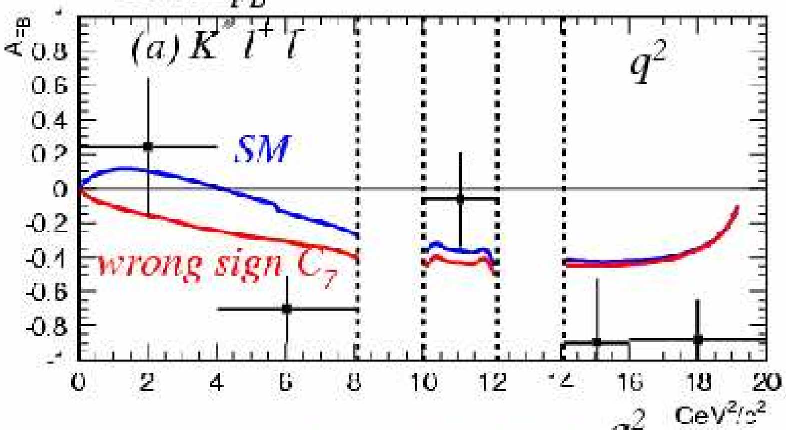
275M $B\bar{B}$

$A_{FB}(K^*ll)$: very sensitive to NP
that may not be seen in $\mathcal{B}(b \to s\gamma)$

$$A_{FB} = \frac{\Gamma(\theta_{Bl^+} < \pi/2) - \Gamma(\theta_{Bl^+} > \pi/2)}{\Gamma(\theta_{Bl^+} < \pi/2) + \Gamma(\theta_{Bl^+} > \pi/2)}$$



raw A_{FB}



First Look !

[Belle-conf-0415]

ICHEP 2004, Beijing 27

LHC-B MC Studies

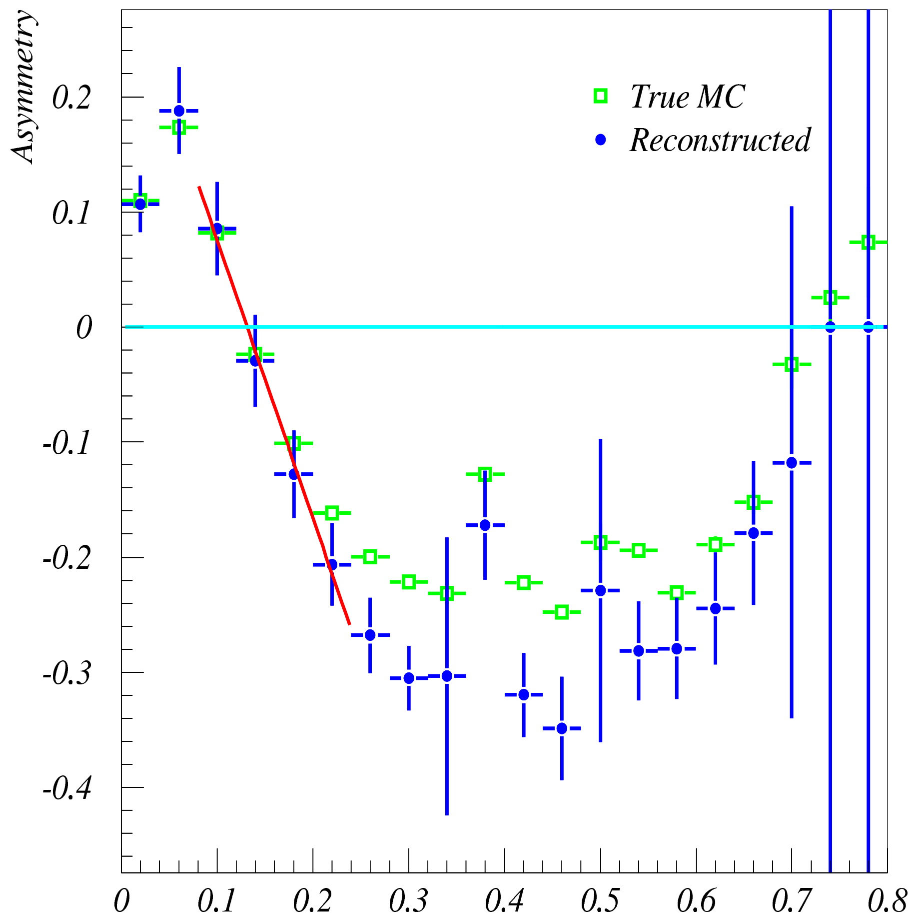


Figure 4: FB Asymmetry versus $\hat{s}$ for $B \xrightarrow{\hat{s}} \mu^+\mu^- K^*$ (from Koppenburg)

The Minimal Supersymmetric Standard Model: MSSM

- Superfields classified according to their $SU(3)_C \otimes SU(2)_I \otimes U(1)_Y$ Quantum Numbers; $i = 1, 2, 3$ a generation index

- Chiral Superfields for Quarks $(\hat{Q}_i, \hat{U}_i^c, \hat{D}_i^c)$

$$\hat{Q}_i(3, 2, 1/6); \quad \hat{U}_i^c(\bar{3}, 1, -2/3); \quad \hat{D}_i^c(\bar{3}, 1, 1/3)$$

$$\hat{Q}_i = (\tilde{Q}_{L_i}, Q_{L_i}); \quad \hat{U}_i^c = (\tilde{U}_{L_i}^c, U_{L_i}^c); \quad \hat{D}_i^c = (\tilde{D}_{L_i}^c, D_{L_i}^c)$$

- Chiral Superfields for Leptons $(\hat{L}_i, \hat{E}_i^c)$

$$\hat{L}_i(1, 2, -1/2); \quad \hat{E}_i^c(1, 1, 1)$$

$$\hat{L}_i = (\tilde{E}_{L_i}, E_{L_i}); \quad \hat{E}_i^c = (\tilde{E}_{L_i}^c, E_{L_i}^c)$$

- Chiral Superfields for Two Higgs Doublets (also denoted as $\hat{H}_1$ & $\hat{H}_2$)

$$\hat{H}_u(1, 2, -1/2); \quad \hat{H}_d(1, 2, 1/2)$$

$$\hat{H}_u = (H_u, \tilde{H}_u); \quad \hat{H}_d = (H_d, \tilde{H}_d)$$

- Vector Superfields $(\hat{G}, \hat{W}, \hat{B})$ (α is an SU(2) index)

$$\hat{G}(8, 1, 1); \quad \hat{W}^\alpha(1, 3, 1); \quad \hat{B}(1, 1, 1)$$

$$\hat{G} = (g, \tilde{g}); \quad \hat{W} = (W^\alpha, \tilde{W}^\alpha); \quad \hat{B} = (B, \tilde{B})$$

Flavour Mixing in the MSSM

- Flavour mixings in the MSSM reside in the Superpotential $W_{\rm MSSM}$ and in the soft supersymmetry-breaking Lagrangian $\mathcal{L}_{\rm soft}$
- $W_{\rm MSSM}$

$$W_{\rm MSSM} = \epsilon_{\alpha\beta}[-\hat{H}_u^\alpha \hat{Q}_i^\beta Y_u^{ij} \hat{U}_j^c + \hat{H}_d^\alpha \hat{Q}_i^\beta Y_d^{ij} \hat{D}_j^c + \hat{H}_d^\alpha \hat{L}_i^\beta Y_e^{ij} \hat{E}_j^c - \mu \hat{H}_d^\alpha \hat{H}_u^\beta]$$

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}; \quad \epsilon_{12} = 1$$

- $\mathcal{L}_{\rm soft}$

$$\begin{aligned}\mathcal{L}_{\rm soft} &= \frac{1}{2}[M_3 \tilde{g}\tilde{g} + M_2 \tilde{W}^\alpha \tilde{W}^\alpha + M_1 \tilde{B}\tilde{B} + h.c.] \\ &+\epsilon_{\alpha\beta}[-b H_d^\alpha H_u^\beta - H_u^\alpha \tilde{Q}_i^\beta \tilde{A}_{u_{ij}} \tilde{U}_j^c + H_d^\alpha \tilde{Q}_i^\beta \tilde{A}_{d_{ij}} \tilde{D}_j^c + H_d^\alpha \tilde{L}_i^\beta \tilde{A}_{e_{ij}} \tilde{E}_j^c + h.c.] \\ &+m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + \tilde{Q}_i^\alpha m_{Q_{ij}}^2 \tilde{Q}_j^{\alpha *} \\ &+\tilde{L}_i^\alpha m_{L_{ij}}^2 \tilde{L}_j^{\alpha *} + \tilde{U}_i^{c*} m_{U_{ij}}^2 \tilde{U}_j^c + \tilde{D}_i^{c*} m_{D_{ij}}^2 \tilde{D}_j^c + \tilde{E}_i^{c*} m_{E_{ij}}^2 \tilde{E}_j^c\end{aligned}$$

- MSSM contains 124 parameters residing in the Superpotential $W_{\rm MSSM}$ (Yukawa couplings) and Soft-SUSY-breaking $\mathcal{L}_{\rm soft}$ (Scalar) terms
- Various realizations of the MSSM differ from each other in the details of $\mathcal{L}_{\rm soft}$

SUGRA and mSUGRA models

- CKM matrix is the only source of Flavour transitions
- In SUGRA models, this is achieved by assuming that the SUSY-breaking parameters have a simple structure at the GUT scale (m_X)

$$(m_Q^2)^i_j = (m_E^2)^i_j = (m_D^2)^i_j = (m_U^2)^i_j = (m_L^2)^i_j = M_0^2 \delta^i_j$$
$$m_{H_d}^2 = m_{H_u}^2 = \Delta_0^2$$
$$M_1 = M_2 = M_3 = M_{1/2}$$

$$A_{d_{ij}} = A_0 (Y_d)_{ij}; \quad Au_{ij} = A_0 (Y_u)_{ij}; \quad Ae_{ij} = A_0 (Y_e)_{ij}$$

- In MSUGRA model, in addition $\Delta_0^2 = M_0^2$
- RG running ($m_X \to m_W$) induces flavour non-diagonal terms, but they are small
- This reduces the number of parameters enormously, leaving the parameters: M_0, $M_{1/2}$, $|A_0|$, $\tan\beta$, ϕ_μ, ϕ_A, where the phases are constrained by the EDMs
- Minimal flavour violation (MFV) models are highly predictive, and hence highly constrained

General Flavour Violating SUSY & The MIA Technique

- In a general SUSY Model, many more sources of Flavour Violation
- A technique to carry out an analysis in a general SUSY framework is the Mass Insertion Approximation (MIA) [Hall, Kostelecky, Raby 1986]
- In the MIA approach, one choses a basis in which the couplings of $\tilde{f}_i \tilde{g} f_j$ are flavour-diagonal $(\propto \ \delta_{ij})$; FC take place on the sfermion propagators by mass insertions: Δ^u_{ij}, Δ^d_{ij} etc.

$$(m_0^2)_i \delta_{ij} + \Delta_{ij}$$

- Need not know the full diagonalization of the sfermion $(\tilde{f})$ mass matrices; sufficient to compute the ratios ($\langle m_0^2 \rangle$ is an average sfermion mass squared):

$$\delta_{ij} = \frac{\Delta_{ij}}{\langle m_0^2 \rangle}$$

- All FC effects can be parametrized in terms of a limited number of complex MIA parameters: $(\delta^u_{ij})_{AB}$ & $(\delta^u_{ij})_{AB}$, $(A, B = L, R)$
- Typically, one expects $(\delta^f_{ij})_{AB} \leq 1$
- Analysis for FV processes can then be carried out in terms of the SUSY-MFV contributions and the MIA parameters [Masiero et al.,...]

Yasuhiro Okada (Super-B Workshop, Hawaii, '05)

Different assumptions on the SUSY breaking sector

SUSY breaking

- Minimal Flavor Violation (ex. mSUGRA)
  $V_{squark} \sim V_{CKM}$
- SUSY GUT with see-saw neutrinos
  $(m^2_{\tilde{q}})_{ij} \ni c(y^\dagger_\nu y_\nu)_{ij}$
- Flavor symmetry — U(2) etc. for Y_q and $m^2_{\tilde{q}}$
- Effective SUSY — $m_{\tilde{q}_3} \ll m_{\tilde{q}_1}, m_{\tilde{q}_2}$
- etc.

How to distinguish these models from B factory observables?

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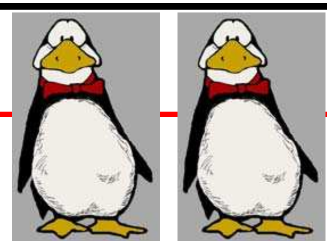
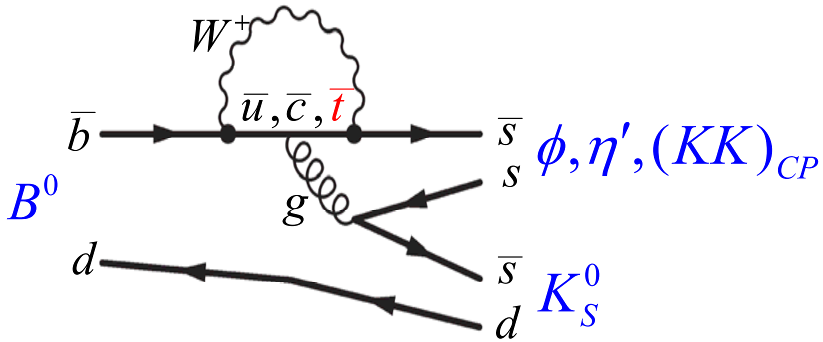
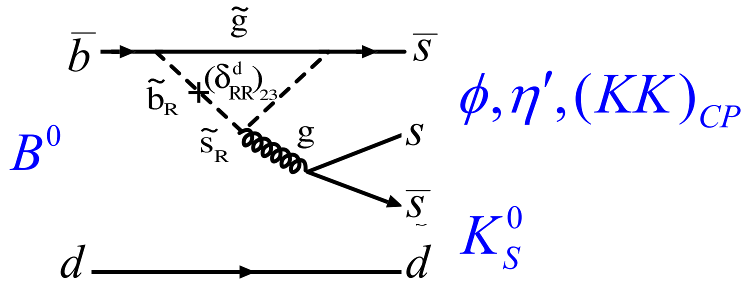
$\sin 2\beta$ and...

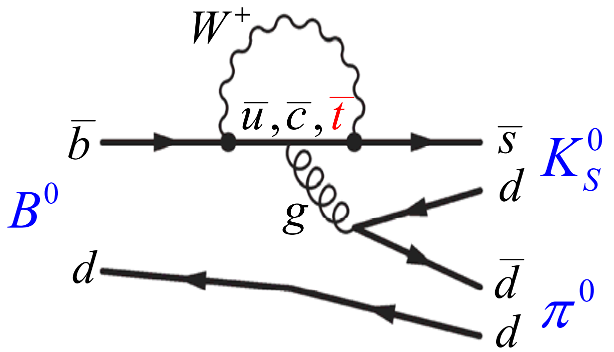
and....



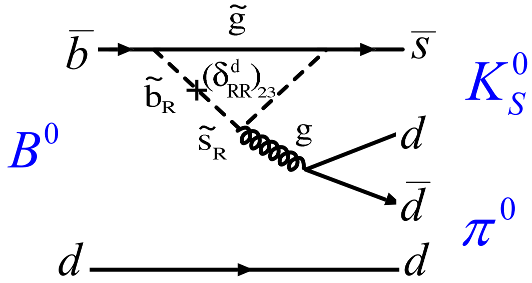
In SM interference between B mixing, K mixing and Penguin $b \to s\bar{s}s$ or $b \to s\bar{d}d$ gives the same $e^{-2i\beta}$ as in tree process $b \to c\bar{c}s$. However loops can also be sensitive to New Physics!







New phases from SUSY?



ICHEP04-北京
August 20, 2004

Marcello A. Giorgi

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$S_{b\to q\bar{q}s}$ and $C_{b\to q\bar{q}s}$ [HFAG 2005; hep-ex/0505100]

Table 30: $S_{b\to q\bar{q}s}$ and $C_{b\to q\bar{q}s}$.

Experiment		$-\eta S_{b\to q\bar{q}s}$	$C_{b\to q\bar{q}s}$
		ϕK^0	
BABAR	[188]	$0.50 \pm 0.25\,^{+0.07}_{-0.04}$	$0.00 \pm 0.23 \pm 0.05$
Belle	[189]	$0.06 \pm 0.33 \pm 0.09$	$-0.08 \pm 0.22 \pm 0.09$
Average		0.34 ± 0.20	-0.04 ± 0.17
Confidence level		0.30	0.81
		$\eta' K^0_S$	
BABAR	[190]	$0.30 \pm 0.14 \pm 0.02$	$-0.21 \pm 0.10 \pm 0.02$
Belle	[189]	$0.65 \pm 0.18 \pm 0.04$	$0.19 \pm 0.11 \pm 0.05$
Average		0.43 ± 0.11	-0.04 ± 0.08
Confidence level		$0.13\ (1.5\sigma)$	$0.011\ (2.5\sigma)$
		$f_0 K^0_S$	
BABAR	[191]	$0.95\,^{+0.23}_{-0.32} \pm 0.10$	$-0.24 \pm 0.31 \pm 0.15$
Belle	[189]	$-0.47 \pm 0.41 \pm 0.08$	$0.39 \pm 0.27 \pm 0.08$
Average		0.39 ± 0.26	0.14 ± 0.22
Confidence level		$0.008\ (2.7\sigma)$	$0.16\ (1.4\sigma)$
		$\pi^0 K^0_S$	
BABAR	[192]	$0.35\,^{+0.30}_{-0.33} \pm 0.04$	$0.06 \pm 0.18 \pm 0.03$
Belle	[189]	$0.30 \pm 0.59 \pm 0.11$	$0.12 \pm 0.20 \pm 0.07$
Average		$0.34\,^{+0.27}_{-0.29}$	0.09 ± 0.14
Confidence level		0.94	0.83
		ωK^0_S	
BABAR	[193]	$0.50^{+0.34}_{-0.38} \pm 0.02$	$-0.56\,^{+0.29}_{-0.27} \pm 0.03$
Belle	[189]	$0.75 \pm 0.64\,^{+0.13}_{-0.16}$	$-0.26 \pm 0.48 \pm 0.15$
Average		$0.55\,^{+0.30}_{-0.32}$	-0.48 ± 0.25
Confidence level		0.74	0.61
		$K^+K^-K^0_S$	
BABAR	[188]	$0.55 \pm 0.22 \pm 0.04 \pm 0.11$	$0.10 \pm 0.14 \pm 0.06$
Belle	[189]	$0.49 \pm 0.18 \pm 0.04\,^{+0.17}_{-0.00}$	$0.08 \pm 0.12 \pm 0.07$
Average		0.53 ± 0.17	0.09 ± 0.10
Confidence level		0.72	0.92
		$K^0_S K^0_S K^0_S$	
BABAR	[194]	$0.71\,^{+0.32}_{-0.38} \pm 0.04$	$-0.34\,^{+0.28}_{-0.25} \pm 0.05$
Belle	[195]	$-1.26 \pm 0.68 \pm 0.20$	$-0.54 \pm 0.34 \pm 0.09$
Average		0.26 ± 0.34	-0.41 ± 0.21
Confidence level		$0.014\ (2.5\sigma)$	
Average of all $b \to q\bar{q}s$		0.43 ± 0.07	-0.021 ± 0.049
Confidence level		$0.17\ (1.4\sigma)$	$0.15\ (1.4\sigma)$
Average including $b \to c\bar{c}s$		0.665 ± 0.0033	0.018 ± 0.025
Confidence level		$0.006(2.7\sigma)$	$0.17\ (1.4\sigma)$

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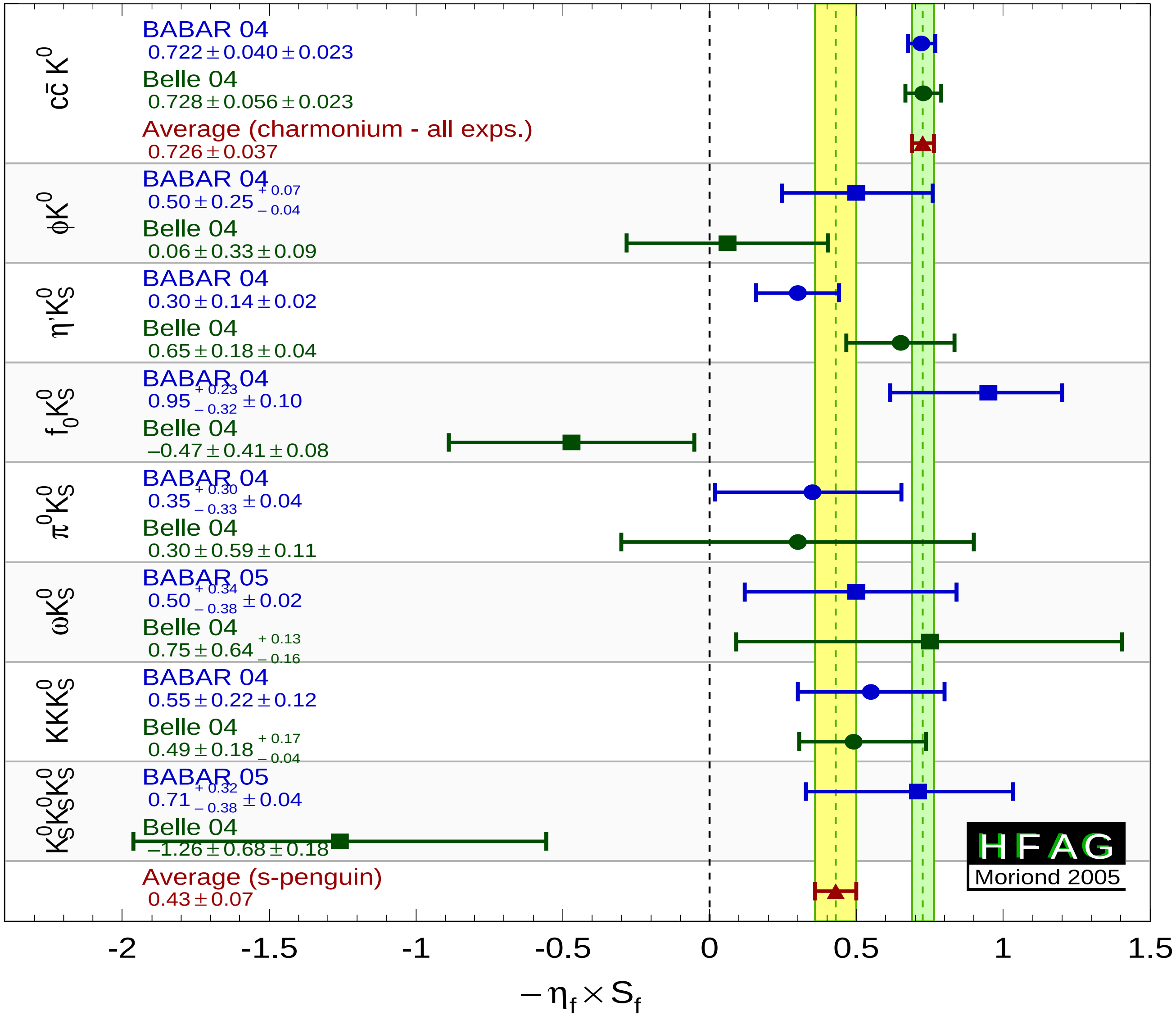
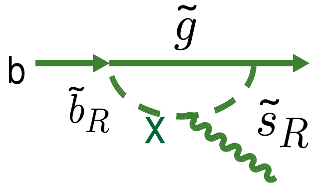
Comparison of $\sin 2\beta(c\bar{c})$ and $\sin 2\beta(s\text{-penguins})$

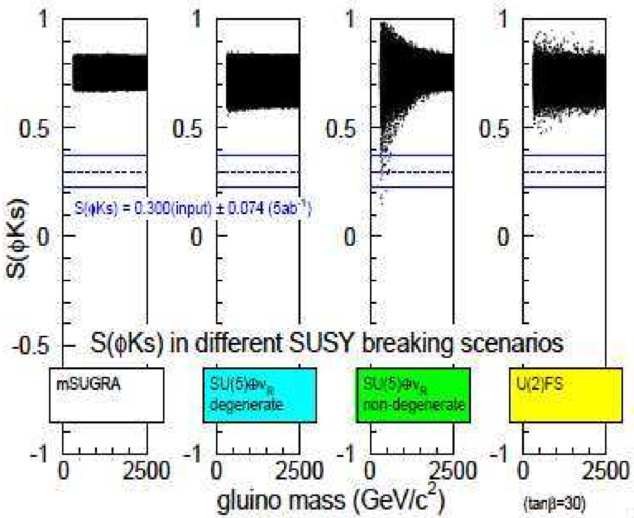


Y. Okada et al. (Super-B Workshop, Hawaii, ’05)

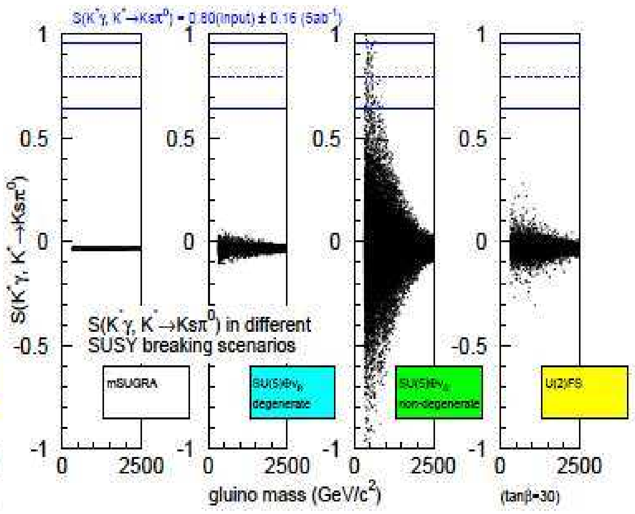
Mixing-induced CP asymmetries in B →φKs and B→K*γ



A_CP(B –>φ Ks)



A_CP(B –> K*γ)



Large CPV in φKs and K*γ for the SUSY GUT with non-degenerate RHN.

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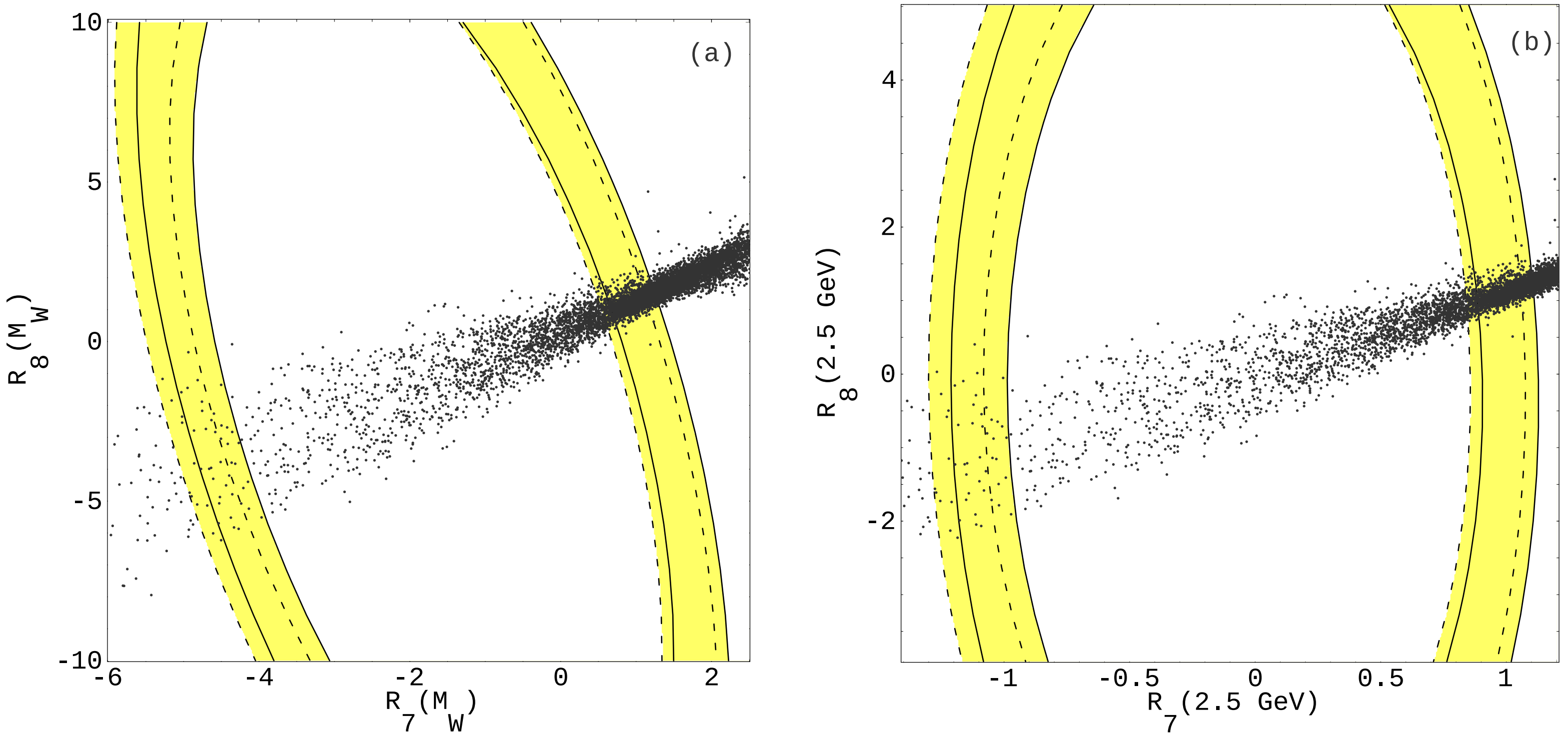
A Model-independent Analysis of $B \to X_s \gamma$ & $B \to X_s \ell^+ \ell^-$

- Assume $\mathcal{H}_{eff}^{\mathrm{SM}}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM [BSM] physics only in $C_7(\mu_W)$,$C_8(\mu_W)$, $C_9(\mu_W)$, and $C_{10}(\mu_W)$
- BSM Coefficients: $R_7 - 1$, $R_8 - 1$, C_9^{NP}, & C_{10}^{NP}
- Define: $R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{\mathrm{tot}}(\mu_W)}{C_{7,8}^{\mathrm{SM}}(\mu_W)}$
  with $C_{7,8}^{\mathrm{tot}}(\mu_W) = C_{7,8}^{\mathrm{SM}}(\mu_W) + C_{7,8}^{\mathrm{NP}}(\mu_W)$
- Set the scale $\mu_W = M_W$, and use RGE to evolve
  $$R_{7,8}(\mu_W) \to R_{7,8}(\mu_b) = \frac{A_{7,8}^{\mathrm{tot}}(\mu_b)}{A_{7,8}^{\mathrm{SM}}(\mu_b)}$$
- Impose constraints from $R_7(\mu_b)$ and $R_8(\mu_b)$ from $B \to X_s \gamma$ Data
- Use Data on $B \to (X_s, K^*, K)\ell^+\ell^-$ BRs to constrain C_9^{NP} and C_{10}^{NP}
- Two-fold ambiguity due to the sign of C_7^{eff} can be resolved by data on $\boldsymbol{B \to (X_s, K, K^*)\ell^+\ell^-}$

Simulation of $B \to X_s\gamma$ in SUSY-MFV Models

- 90% C.L. bounds in the $[R_7(\mu), R_8(\mu)]$ plane from the $\mathcal{B}(B \to X_s\gamma)$
  $\mu = m_W$ (left-hand plot); $\mu = 2.5$ GeV (right-hand plot)



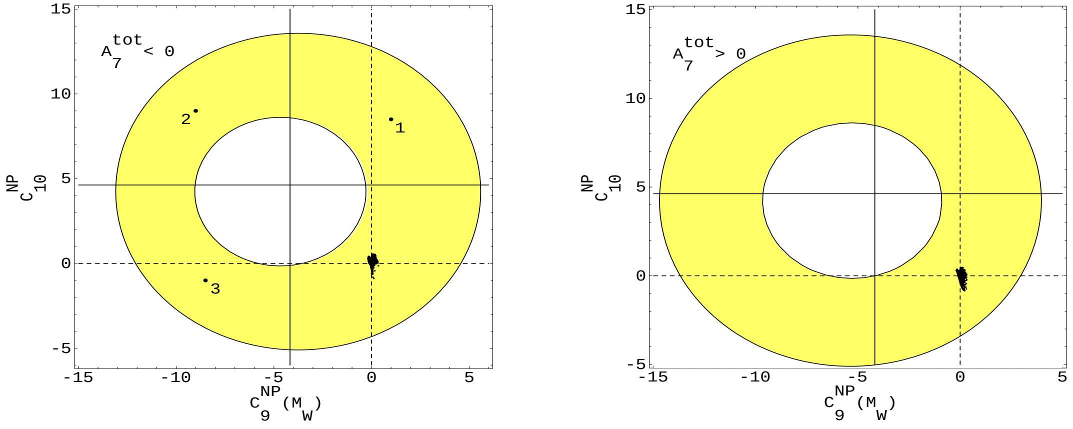
$$A_7^{\rm tot} - \text{negative} : -0.37 \leq A_7^{\rm tot,<0}(2.5\ \text{GeV}) \leq -0.17$$

$$A_7^{\rm tot} - \text{positive} : 0.21 \leq A_7^{\rm tot,>0}(2.5\ \text{GeV}) \leq 0.43$$

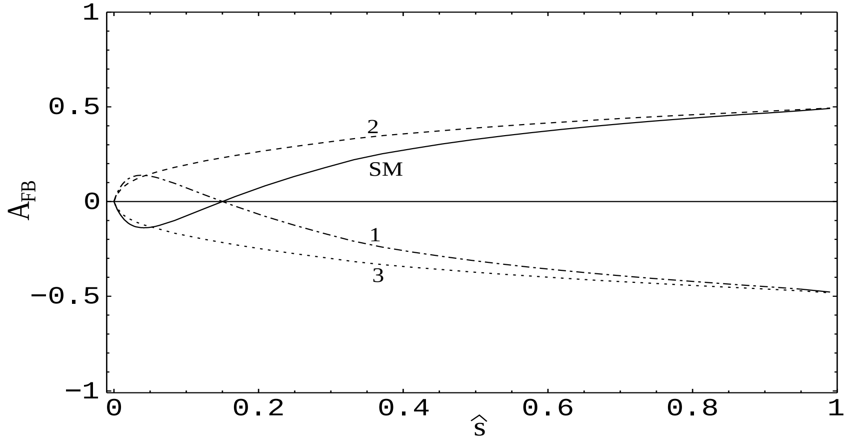
Combined analysis of $B \to X_s\gamma$ & $B \to (X_s, K, K^*)\ell^+\ell^-$

[A.A., Lunghi, Greub, Hiller; DESY 01-217; hep-ph/0112300]

- Constraints from radiative and semileptonic rare decays (Points: SUSY-MFV Model)



- FB asymmetry for $\bar{B} \to X_s\ell^+\ell^-$, corresponding to the points indicated above

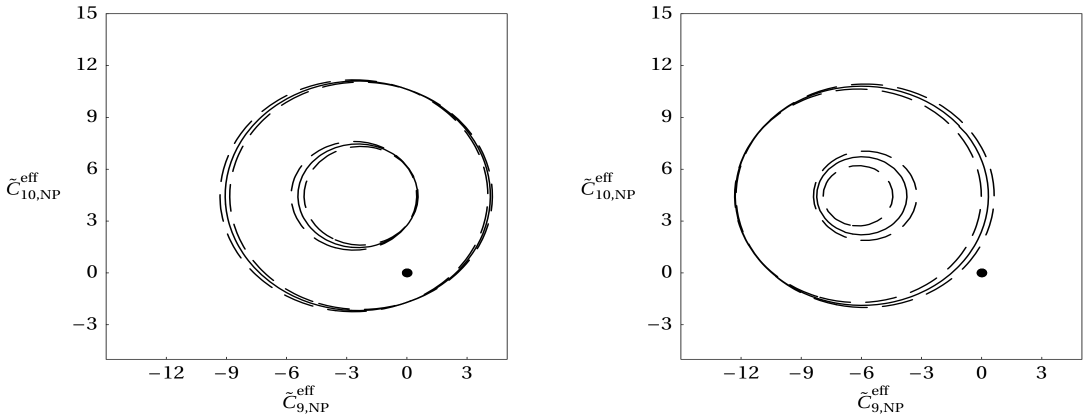


First hints on the sign of the $B \to X_s\gamma$ amplitude

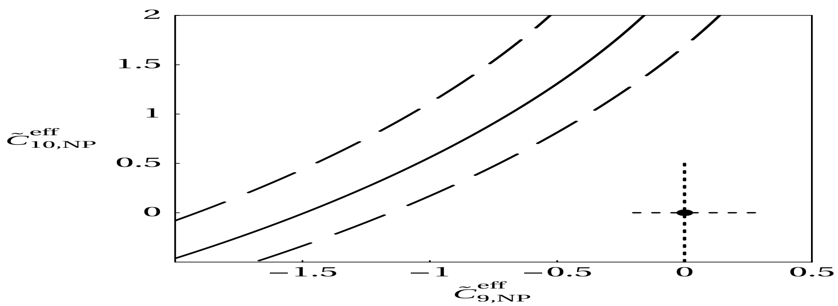
[Gambino, Haisch, Misiak; hep-ph/0410155]

90% C.L. constraints from $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

C_7 SM-like (left frame) C_7 opposite sign (right frame)



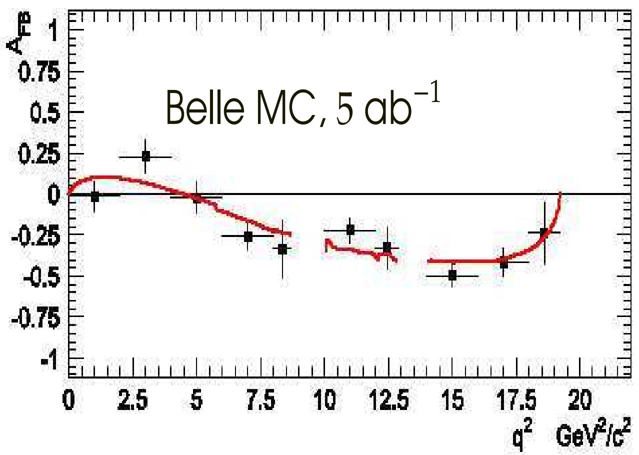
Surroundings of the origin in the right frame above; dashed lines: MFV-MSSM

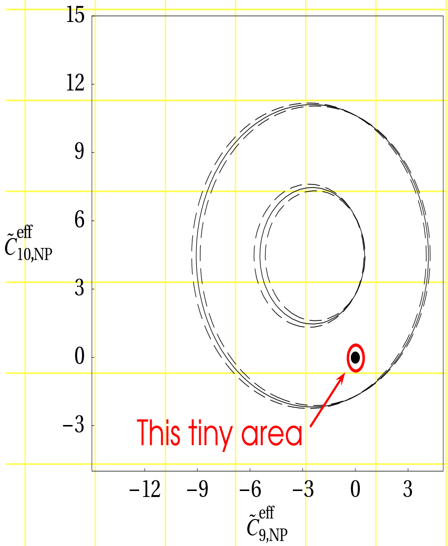


Prospects of precise determination of C_9, C_{10} at Super-B Factory

Extracting C_9 and C_{10} from $B \to K^* \ell^+ \ell^-$

- Precise determination of C_9 and C_{10} is possible
- $\Delta C_9/C_9 \sim 11\%$, $\Delta C_{10}/C_{10} \sim 13\%$ at 5 ab^{-1}, C_7 fixed from $b \to s\gamma$
  - Current branching fraction / background extrapolated
  - Fit to 2-dim q^2 vs angular distribution, not simple A_{FB}
  - Systematic error is neglected





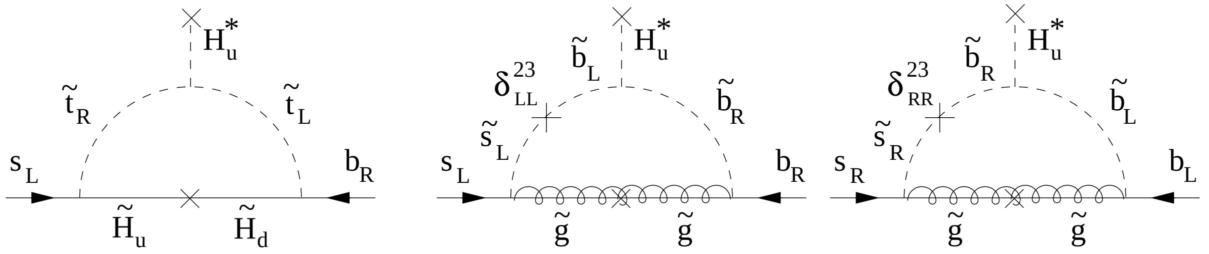
Radiative Pengiuns — Mikihiko Nakao — p.28

$B_s \to \mu^+\mu^-$ in Supersymmetric Models

- The decay $B_s \to \mu^+\mu^-$ probes essentially the Higgs sector of Supersymmetry, a type-II two-Higgs doublet model; One Higgs field (H_u) couples to the up-type quarks, the other (H_d) couples to the down-type quarks

$$\mathcal{L} = \overline{Q} Y_U U_R H_u + \overline{Q}_L Y_D D_R H_d$$

- Supersymmetry does not have discrete symmetries to protect the alignment of the Higgs boson interaction eigenbasis with the fermion mass eigenbasis; Higgs-induced FCNC interactions are generated through loops



(a) (b)

- As H_u gets a VEV (v_u), it contributes an off-diagonal piece to the down-type fermion mass matrix, mixing s_L and b_L by an angle θ

$$\sin\theta = y_b \epsilon v_u / m_b; \quad \text{as } m_b = y_b v_d, \ \sin\theta = \epsilon \tan\beta$$

- $\mathcal{A}(b\bar{s} \to \mu^+\mu^-) \simeq \sin\theta \mathcal{A}(b\bar{b} \to \mu^+\mu^-) \ \propto \tan\beta / \cos^2\beta \Longrightarrow \tan^3\beta$ for large-$\tan\beta$

Constraints from $BR(B_s \to \mu^+\mu^-)$

$B_s \to \mu\mu$: Physics Reach

D0 $B_s \to \mu^+\mu^-$ result: 240pb^{-1}

$$BF(B_s \to \mu^+\mu^-) < 3.8 \times 10^{-7}\ 90\%\ \text{CL}$$

CDF $B_{(s,d)} \to \mu^+\mu^-$ results: 171pb^{-1}

$$BF(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-7}\ 90\%\ \text{CL}$$

$$BF(B_d \to \mu^+\mu^-) < 1.5 \times 10^{-7}\ 90\%\ \text{CL}$$

Combined: Bayesian approach with a flat prior. Systematic error on fs correlated. Combination by M. Herndon

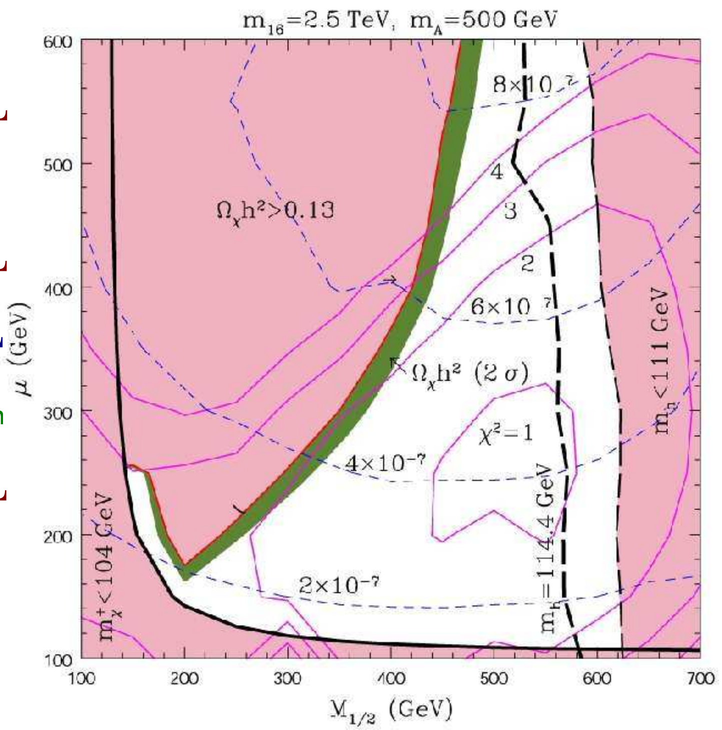
$$BF(B_s \to \mu^+\mu^-) < 2.7 \times 10^{-7}\ 90\%\ \text{CL}$$

SM predictions

$BF(B_{s(d)} \to \mu^+\mu^-)$ 3.5x10^{-9} (1.0x10^{-10})

- No sensitivity for SM decay rate

- BSM predictions Limiting many models
- Example SUSY S0(10)
  - Allows for massive neutrino
  - Accounts for relic density of cold dark matter



$BF\ B_s \to \mu^+\mu^-$: Dashed blue

Excludes scenarios where M_A is light and tanβ ~ 50: M_A > 450GeV/c^2

R. Dermisek hep-ph/0304101,2003

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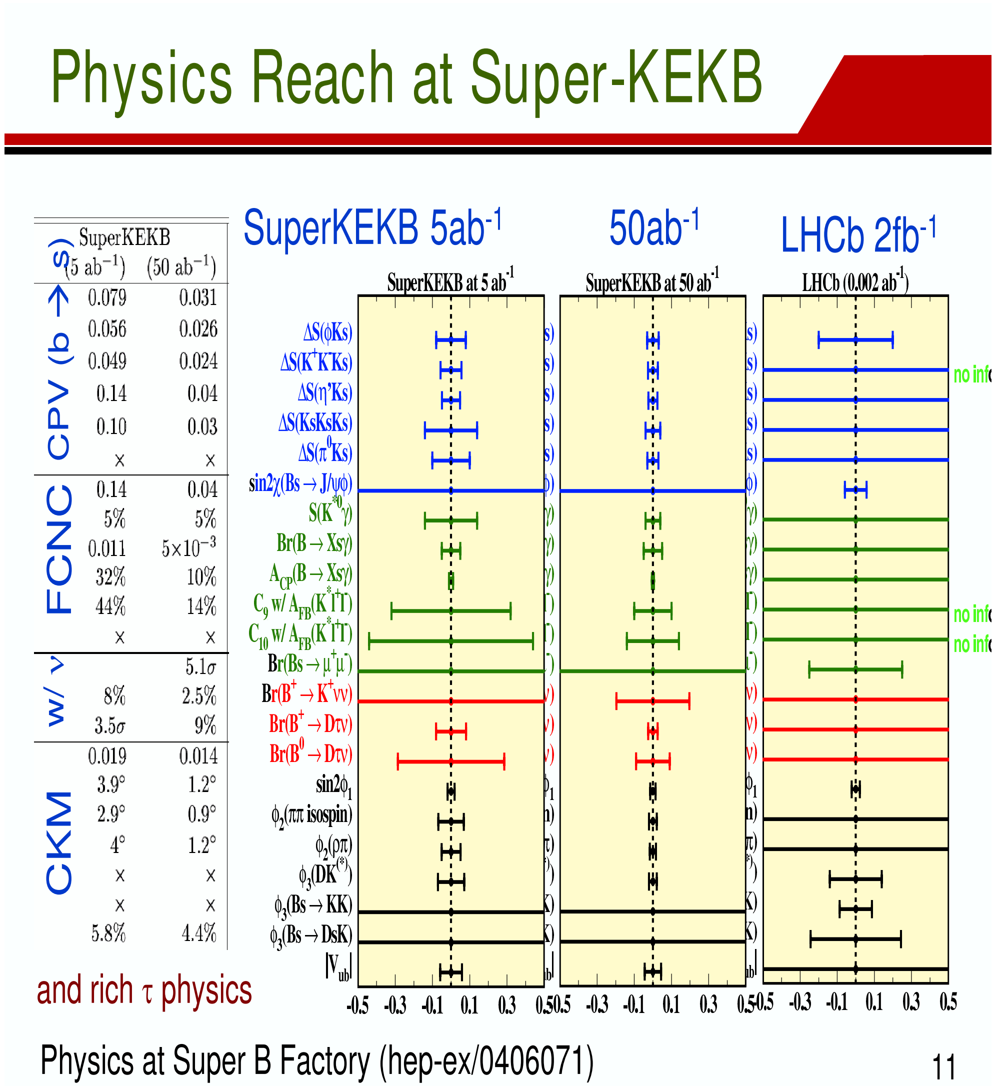
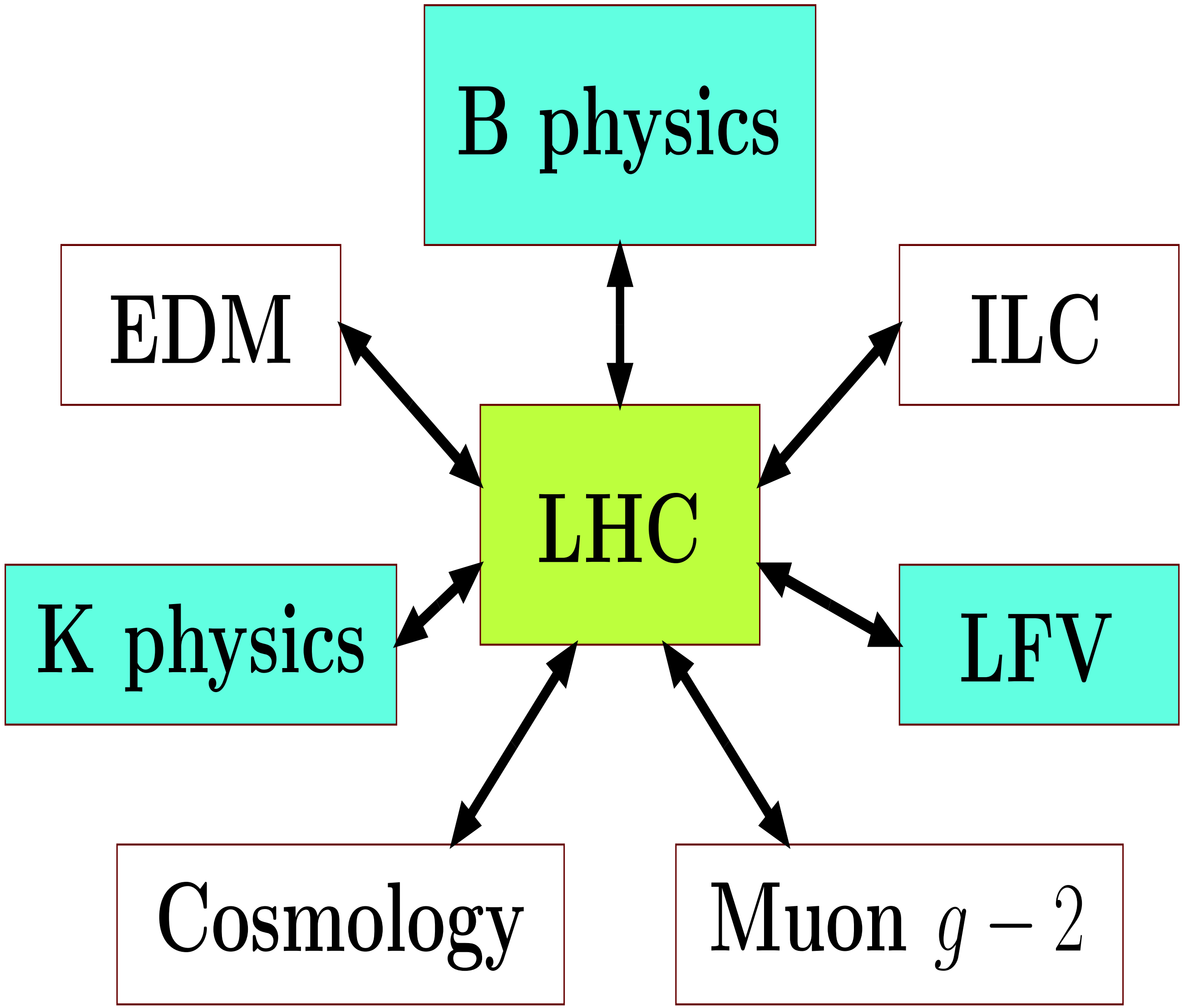
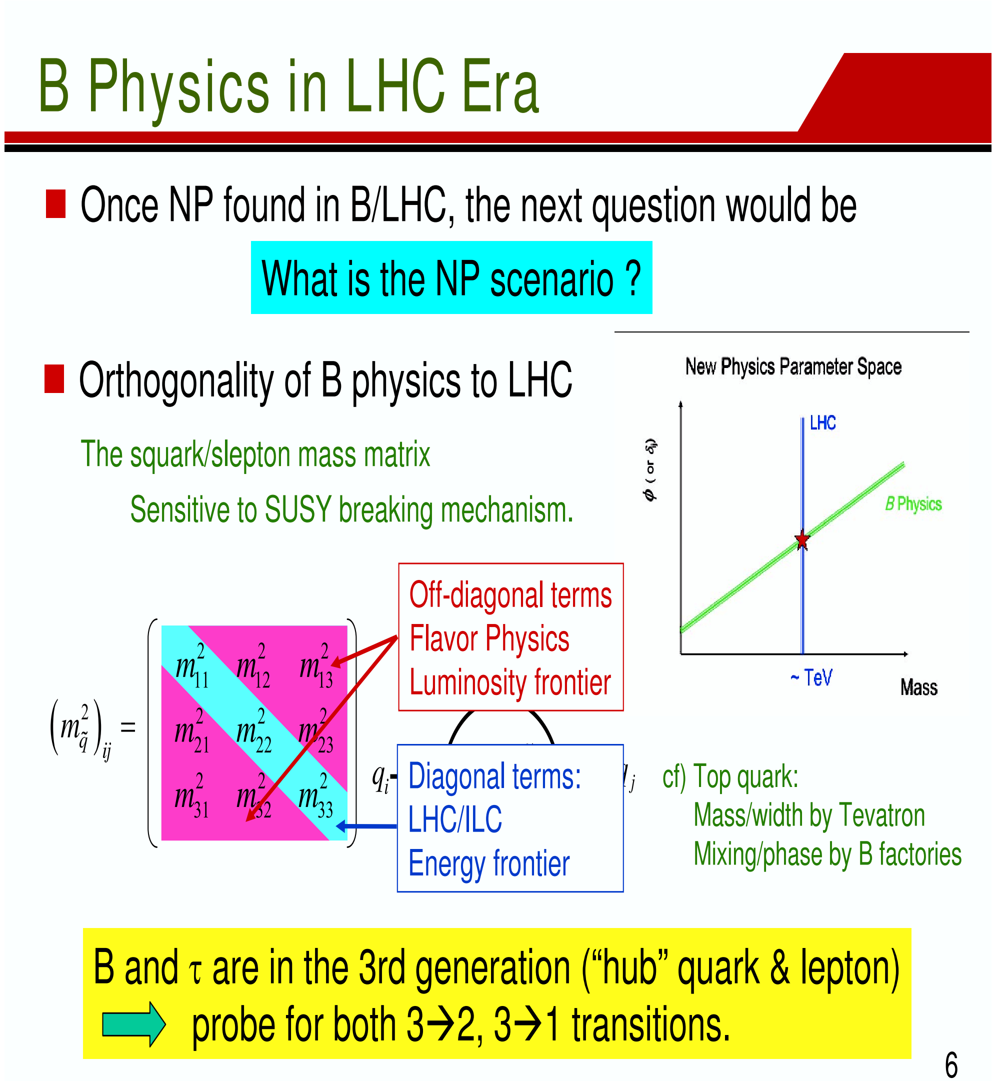
Courtesy Iijika San (Super-B Workshop, Hawaii, '05)



Synergy of Various Approaches in Search of BSM Physics

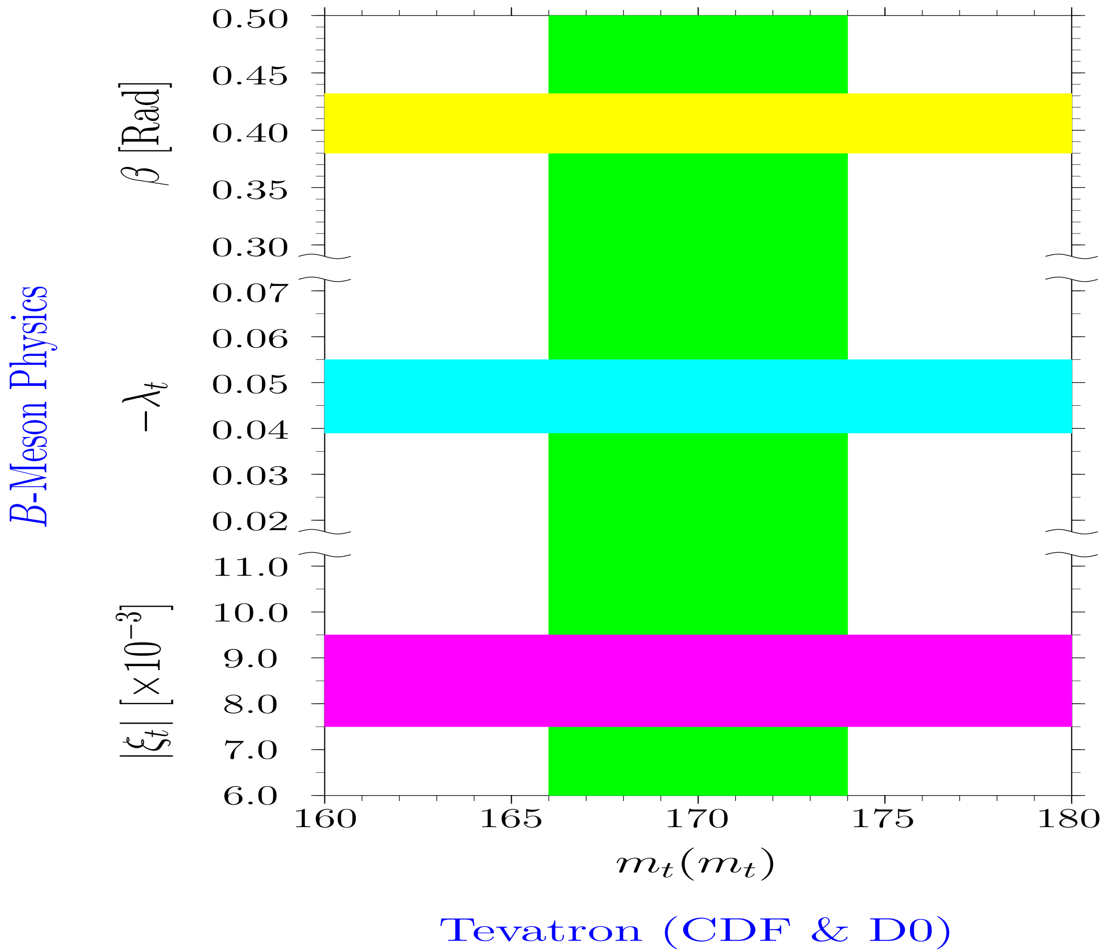


LHC/B-Physics Synergy?



Complentarity of Tevatron & B-Physics Experiments

- m_t measured by Tevatron experiments
- Off-diagonal V_{tj} measured in B-decays



Summary

- All current measurements involving FCNC processes (decay rates and distributions) are in agreement with the SM expectations
- Rare B-decays and B^0 - $\overline{B^0}$ mixings have made a great impact on the determination of the CKM matrix elements in the third row of $V_{\rm CKM}$
- A number of benchmark measurements remain to be done. These include, among others, $\mathcal{B}(B_s \to \mu^+\mu^-)$ and ΔM_{B_s}, which will be carried out at Fermilab and LHC
- Discovery of SUSY at LHC but continued absence of observable effects in FCNC and CPV beyond SM would point to a flavour-blind SUSY (such as mSUGRA,MFV)
- However, data on CPV in $\boldsymbol{b \longrightarrow s\bar{s}s}$ penguins puzzling; currently deviation from the SM is a tantalizing $\boldsymbol{3.5\ \sigma}$ effect; need to clarify this effect experimentally - a motivation to build a Super-B factory
- Let us hope that the synergy of high energy frontier and low energy precision physics, which worked out so well in piecing together the SM, will continue to hold sway in the LHC/ILC-era, providing valuable information about the flavour aspects of the BSM physics

Backup Slides

Rare K-Decays sensitive to BSM Physics; G. Isidori (FNAL '05)

Present status and impact on the UT plane:

$K^+ \to \pi^+ \nu\nu$ $\quad BR(K^+)^{[SM]} = C_+ |V_{cb}|^4 [(\bar{\rho} - \rho_c)^2 + (\sigma\bar{\eta})^2] = (8.0 \pm 1.0) \times 10^{-11}$

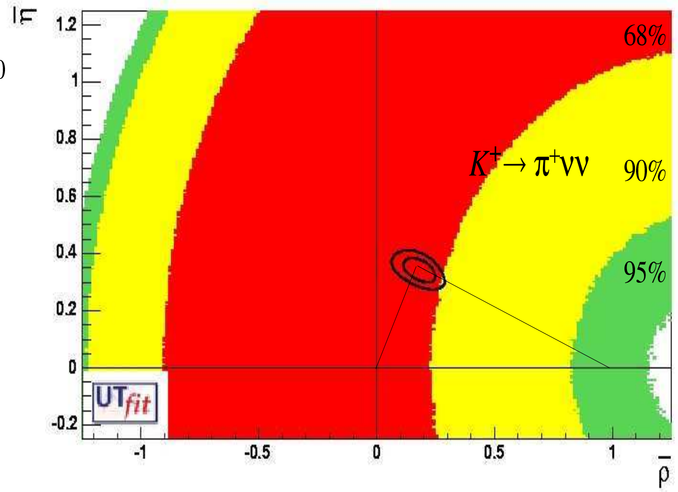
$K_L \to \pi^0 \nu\nu$ $\quad BR(K_L)^{[SM]} = C_0 \left[\frac{\mathrm{Im}(V_{ts}^* V_{td})}{10^{-4}} \right]^2 = (3.0 \pm 0.6) \times 10^{-11}$

$BR(K^+)^{exp} = (1.47^{+1.9}_{-0.9}) \times 10^{-10}$

E787+E949 [BNL] '04

$BR(K_L)^{exp} < 5.9 \times 10^{-7}$

KTeV '99



G. Isidori (FNAL '05)

Possible (optimistic...) future scenario:

$K^+ \to \pi^+ \nu\nu$ $\quad \mathrm{BR}(K^+)^{[SM]} = C_+ \, |V_{cb}|^4 \, [(\bar{\rho} - \rho_c)^2 + (\sigma\bar{\eta})^2] = (8.0 \pm 1.0) \times 10^{-11}$

$K_L \to \pi^0 \nu\nu$ $\quad \mathrm{BR}(K_L)^{[SM]} = C_0 \left[\frac{\mathrm{Im}(V_{ts}{}^* V_{td})}{10^{-4}} \right]^2 = (3.0 \pm 0.6) \times 10^{-11}$

$\mathrm{BR}(K^+)^{\mathrm{exp}} = (1.47^{+1.9}_{-0.9}) \times 10^{-10}$ → $\pm 10\ \%$