An Overview and Current Status of the CKM Matrix

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Plan of Talk

- Introduction to Quark Flavour Mixing & the CKM Matrix
- Present Status of the First Two Rows of V_{CKM}
  with emphasis on $|V_{cb}|$ and $|V_{ub}|$
- Status of the Third Row of V_{CKM}
- Current Knowledge of the Phases α, β and γ
- Summary and Future Prospects

Flavour Mixing in the Standard Model

- Flavour mixings in the SM reside in the Yukawa sector of the theory

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge}(A_i, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi_i, A_i, \psi_i)$$

- 3 Quark families: $Q_{L_i} = (u_L,\ d_L); (c_L,\ s_L); (t_L;\ b_L); \bar{u}_R,\ \bar{d}_R; ...$
- Flavour symmetry broken by Yukawa interactions

$$Q_i Y_d^{ij} d_j \phi \ \longrightarrow\ Q_i M_d^{ij} d_j$$

$$Q_i Y_u^{ij} u_j \phi^c \ \longrightarrow\ Q_i M_u^{ij} u_j$$

$$M_d = \mathrm{diag}(m_d,\ m_s,\ m_b);\ \ M_u^\dagger = \mathrm{diag}(m_u,\ m_c,\ m_t) \times V_{\rm CKM}$$

- $V_{\rm CKM}$ a (3×3) unitary matrix is the only source of Flavour Violation, as all gauge interactions (involving $\gamma,\ Z^0,\ g$) are Flavour diagonal
- All observed phenomena involving flavour changes in the hadrons are consistently described by the CKM framework; i.e., in terms of 10 fundamental parameters: 6 quark masses, 3 mixing angles and 1 phase
- Understanding the observed patterns of quark masses and mixings (as well as the lepton masses and neutrino mixings) requires an organizing principle which is certainly outside of the SM

The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\mathrm{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

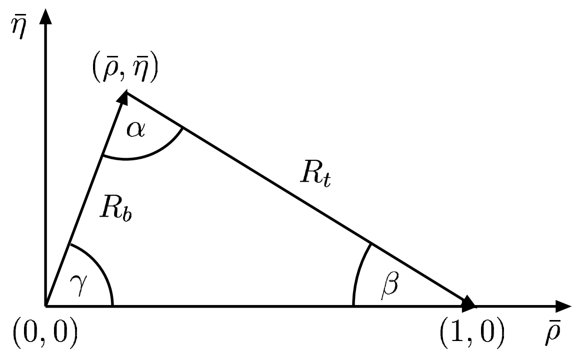
- Customary to use the handy Wolfenstein parametrization

$$V_{\mathrm{CKM}} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2(1+i\lambda^2\eta) & 1 \end{pmatrix}$$

- Four parameters: A, λ, ρ, η
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2), \quad \bar{\eta} = \eta(1-\lambda^2/2)$$

- The CKM-Unitarity triangle $[\phi_1 = \beta; \ \phi_2 = \alpha; \ \phi_3 = \gamma]$



Phases and sides of the UT

$$\alpha \equiv \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right)$$

- β and γ have simple interpretation

$$V_{td} = |V_{td}| e^{-i\beta}, \qquad V_{ub} = |V_{ub}| e^{-i\gamma}$$

- α defined by the relation: $\alpha = \pi - \beta - \gamma$

- The Unitarity Triangle (UT) is defined by:

$$R_b \mathrm{e}^{i\gamma} + R_t \mathrm{e}^{-i\beta} = 1$$

$$R_b \equiv \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|$$

$$R_t \equiv \frac{|V_{tb}^* V_{td}|}{|V_{cb}^* V_{cd}|} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left|\frac{V_{td}}{V_{cb}}\right|$$

Present Status of the CKM Matrix Elements

$|V_{ud}|$

- From $O^+ \to O^+$ Nuclear Superallowed Fermi Transitions:

$$|V_{ud}| = 0.9740 \pm 0.0005 \quad (\text{Townsend\&Hardy})$$

- From Neutron β-decays:

  - Great progress in precise measurements of τ_n and neutron polarization (e.g., at Grenoble), but g_A/g_V an issue at present; Restricting to experiments with neutron polarization of more than 90% :

$$|V_{ud}| = 0.9725 \pm 0.0013; \quad g_A/g_V = -1.2720 \pm 0.0018$$

- From Pion β-decay: $\pi^+ \to \pi^0 e^+ \nu_e$
  - New Result from PIBETA Collaboration (hep-ex/0307258)

$$BR(\pi^+ \to \pi^0 e^+ \nu_e) = (1.044 \pm 0.007(\text{stat}) \pm 0.009(\text{syst})$$

$$\Rightarrow \quad |V_{ud}| = 0.9771 \pm 0.0056$$

- Present World Average [PDG 2004]: $|V_{ud}| = 0.9738 \pm 0.0005$

Current Status of $|V_{ud}|$

- From $O^+ \to O^+$ β-decays

$$\langle p_f; O^+ | \bar{u}\gamma_\mu d | p_i; O^+ \rangle = \sqrt{2}(p_i + p_f)_\mu$$

• ME depends on the vector part of the Weak current; isospin symmetry $(d \to u)$ protects from large corrections
• Adding Nucleus-dependent radiative (δ_R) and isospin (δ_C) corrections to obtain process-independent Ft values for these transitions

$$ft(1 + \delta_R)(1 - \delta_C) \equiv Ft = \frac{K}{2G_F^2 |V_{ud}|^2 (1 + \Delta_R)}$$

• Δ_R: Nucleus-independent Rad. Corr. [Marciano-Sirlin '86; Townsend '92]

$$|V_{ud}|^2 = \frac{K}{2G_F^2 (1 + \Delta_R)\overline{Ft}}$$

Current Values:
$K = (8120.271 \pm 0.012)$ GeV^{-4} s
$\overline{Ft} = (3072.3 \pm 2.0)$ s
$\Delta_R = (2.40 \pm 0.08)\%$
$\Longrightarrow \quad |V_{ud}| = 0.9740 \pm 0.0005$

Neutron β-decay: $n \to p e^- \nu_e$

$$\langle p|\bar{u}\gamma_\mu(1-\gamma_5)d|n\rangle = \bar{u}_p\gamma_\mu(g_V + g_A\gamma_5)u_p$$

- Advantage: No nuclear-structure-dependent corrections
- Disadvantage: ME depends on both Vector and Axial-vector currents
- Requires two measurements, such as the Neutron-lifetime (τ_n) and a correlation measurement to determine g_A/g_V
  - β-emission probability $W(E_e,\theta)$ relative to the neutron spin direction

$$W(E_e,\theta) = F(E_e)(1 + A\beta\cos\theta); \quad \beta = \frac{v}{c}$$

$$A = \frac{-2\lambda(\lambda+1)}{1+3\lambda^2}; \quad \lambda = \frac{g_A}{g_V}; \quad g_V = G_F V_{ud}$$

$$|V_{ud}|^2 = \frac{1}{C\tau_n(1+3\lambda^2)f^{\mathrm{R}}(1+\Delta_R)}$$

with $C = G_F^2 m_e^5/(2\pi)^3$; $f^{\mathrm{R}} = 1.71482(15)$ (rad. corrected phase space)

- $\langle\tau_n\rangle_{\mathrm{WA}} = 885.5 \pm 0.9$ s, $\langle\lambda\rangle = -1.2720 \pm 0.0018$

$\Longrightarrow \quad |V_{ud}| = 0.9725 \pm 0.0013$

Pion β-decay $\pi^+ \to \pi^0 e^+ \nu_e$

- Advantages: No nuclear-structure dependent corrections
- Handicap: Very small branching ratio, $O(10^{-8})$

$$|V_{ud}|^2 = \frac{C\mathcal{B}(\pi^+ \to \pi^0 e^+ \nu_e)}{G_F^2(1+\Delta_R^V) f (1+\Delta_R)\tau_\pi}$$

- $f = \frac{1}{30}(\frac{\Delta}{m_e})^5$; $\Delta = m_{\pi^+} - m_{\pi^0} = (4.5936 \pm 0.0005)$ MeV
- $\tau_{\pi^+} = (2.6033 \pm 0.0005) \times 10^{-8}$ s
- $\Delta_R^V = ((1+\delta^\pi_{SU(2)})(1+\delta^\pi_{e^2p^2}))^2$
- Chiral Perturbation theory Calculations [Cirigliano et al.; hep-ph/0209226]

$$\delta^\pi_{SU(2)} \sim 10^{-5}; \quad \delta^\pi_{e^2p^2} = (0.46 \pm 0.05)\%$$

- $\mathcal{B}(\pi^+ \to \pi^0 e^+ \nu_e)$ (PIBETA Coll.; D. Pocanic et al.; hep-ex/0307258)

$$\mathcal{B}(\pi^+ \to \pi^0 e^+ \nu_e) = (1.044 \pm 0.007(\text{stat}) \pm 0.009(\text{syst})) \times 10^{-8}$$

$$\Rightarrow \quad |V_{ud}| = 0.9771 \pm 0.0056$$

- In agreement with the SFT- and neutron β-decay results; Final Precision on $\mathcal{B}(\pi^+ \to \pi^0 e^+ \nu_e)$ expected to be a factor 3 better

Theoretical Issues in $K_{\ell 3}$ Decays and $|V_{us}|$

- $K_{\ell 3}$ Decays

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us}^* C_K \left[f_+^K(t)(p_K + p_\pi)_\mu + f_-^K(t)(p_K - p_\pi)_\mu \right] L^\mu$$

$L^\mu = \bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_\ell)$; $t = (p_K - p_\pi)^2$; $C_K = 1[\frac{1}{\sqrt{2}}]$ (for K^0 $[K^+]$)

- Partial Width

$$\Gamma = C_K^2 \frac{G_F^2 |V_{us}|^2 M_K^5}{128\pi^3} \cdot |f_+^K(0)|^2 \cdot I_K(f_+, f_-)$$

- Accurate determination of $|V_{us}|$ requires:
  - Evaluation of $f_+^K(0) - 1$ (enters QCD)
  - Momentum dependence of $f_\pm(t) \longrightarrow I_K(f_+, f_-)$
  - Photonic radiative corrections [Ginsberg; Bytev et al.; Cirigliano et al.]
- Integrating out W and Z fields $\Longrightarrow$ Effective Low Eneregy Theory (LET)

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}}(1 + \frac{\alpha}{\pi} \ln \frac{M_Z}{\mu}) \times H_\mu L^\mu$$

- $\mu \sim \mathcal{O}(M_\rho)$; incomplete matching [Marciano, Sirlin '80]

Theoretical Estimates of $f_+(0) - 1$ in χPT

- Matrix elements calculated in LET using Chiral perturbation theory;
  Calculational tool: Chiral symmetry and expansion in the order parameter p/Λ_χ
- Low Energy Constants (LEC's) encode physics order by order in χPT; have to be determined from experiments

Theoretical Developments

- No linear corrections in $(m_s - m_u)$ [Ademollo-Gatto-Sirlin Theorem '64]
- In LO χPT [i.e., in $\mathcal{O}(p^2)$]: $\delta \equiv f_+^K(0) - 1 = 0$
- In $\mathcal{O}(p^4)$: finite non-polynomial corrections induced by meson loops; numerically small: $\delta^{(4)} = -2.2\%$ [Gasser-Leutwyler '85]
- In $\mathcal{O}(p^6)$: Appearance of $(m_s - m_u)^2/\Lambda_\chi^4$ terms; model-dependent
  $\boldsymbol{\delta^{(6)} = (-1.6 \pm 0.8)\%}$ [Leutwyler-Roos, '84]
- Recent Estimates (including $\boldsymbol{\mathcal{O}(e^2p^2)}$ terms) [Cirigliano et al. '01]
  $\boldsymbol{\delta = -(4.0 \pm 0.8)\% \quad \Longrightarrow \quad f_+^K(0) = 0.961 \pm 0.008}$
- Isospin-conserving part of $\boldsymbol{\mathcal{O}(p^6)}$ corrections [Bijnens-Talavera; hep-ph/0303103]
  $\boldsymbol{f_+^K(0) = 0.976 \pm 0.010}$

$|V_{us}|$ from $K_{\ell 3}$ Decays

- $|V_{us}| = 0.2201 \pm 0.0024$ [Cirigliano; hep-ph/0305154; older $K_{\ell 3}$ data] [almost coincides with the PDG 2004 value: $|V_{us}| = 0.2200 \pm 0.0026$]
- New Result E865(Brookhaven) [2.3σ higher than the 2002 PDG Value]

$$\mathcal{B}(K^+_{e3[\gamma]}) = (5.13 \pm 0.02(\mathrm{stat}) \pm 0.09(\mathrm{syst}) \pm 0.04(\mathrm{norm}))\%$$

$$\Rightarrow \quad |V_{us}| = 0.2272 \pm 0.0023(\mathrm{rate}) \pm 0.0018(f^+) \pm 0.0007(\lambda_+)$$

- KLOE: $K_{\ell 3}$ BRs for the K_S and K_L mesons [hep-ex/0307016; hep-ex/0505089]

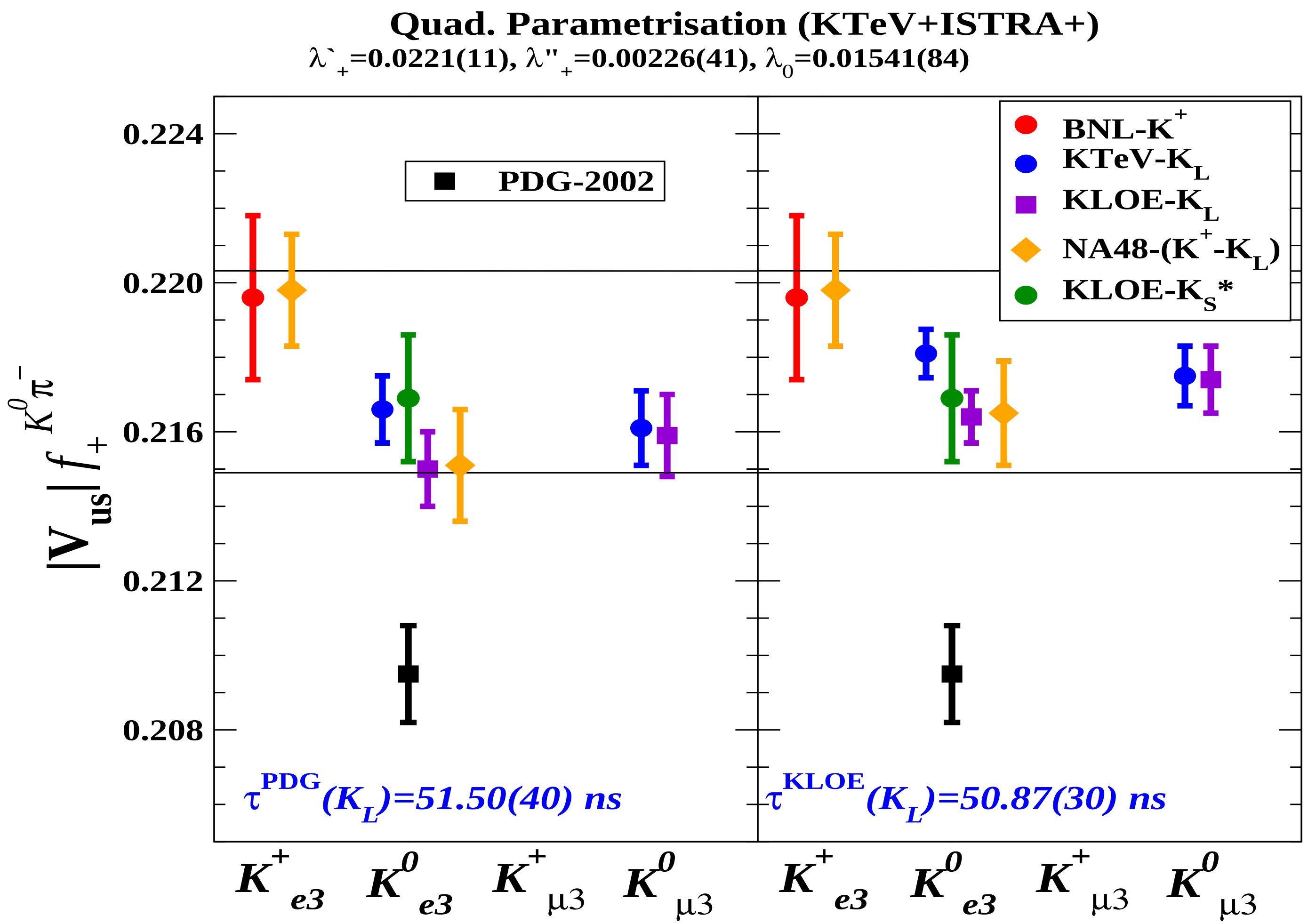
  • $|V_{us}|$ from $\mathcal{B}(K_S \to \pi^\pm e^\mp \nu) = (7.09 \pm 0.07 \pm 0.08) \times 10^{-4}$ and $f_+^{K^0\pi}(0) = 0.961 \pm 0.008$ [Leutwyler-Roos '84]

$$\Rightarrow \quad |V_{us}| = 0.2194 \pm 0.0030$$

- Recent measurements of $\mathcal{B}(K_L \to \pi^\pm e^\mp \nu)$
  • $\mathcal{B}(K_L \to \pi^\pm e^\mp \nu) = (40.67 \pm 0.11)\%$ [KTeV]
  • $\mathcal{B}(K_L \to \pi^\pm e^\mp \nu) = (40.10 \pm 0.45)\%$ [NA48]
  • $\mathcal{B}(K_L \to \pi^\pm e^\mp \nu) = (40.07 \pm 0.15)\%$ [KLOE]
- These and the improved measurement of the K_L-lifetime $\tau^{\mathrm{KLOE}}(K_L) = (50.87 \pm 0.17 \pm 0.25)$ ns
  $\Longrightarrow$ better agreement with the unitarity and the K_S-data

$|V_{us}| \times f_+^{K^0\pi}$ from K-semileptonics

[G. Lanfranchi (KLOE Collaboration); hep-ex/0505089]



$|V_{us}|$ from Non-$K_{\ell 3}$ Decays

- $|V_{us}|$ from Hyperon decays; Recent analysis by Cabibbo et al. (hep-ph/0307214), assuming $SU(3)$ symmetry for the form factors

$$\Rightarrow \quad |V_{us}| = 0.2250 \pm 0.0027$$

- Determination of $|V_{us}|$ from τ-decays [Gamiz et al.; hep-ph/0212230]

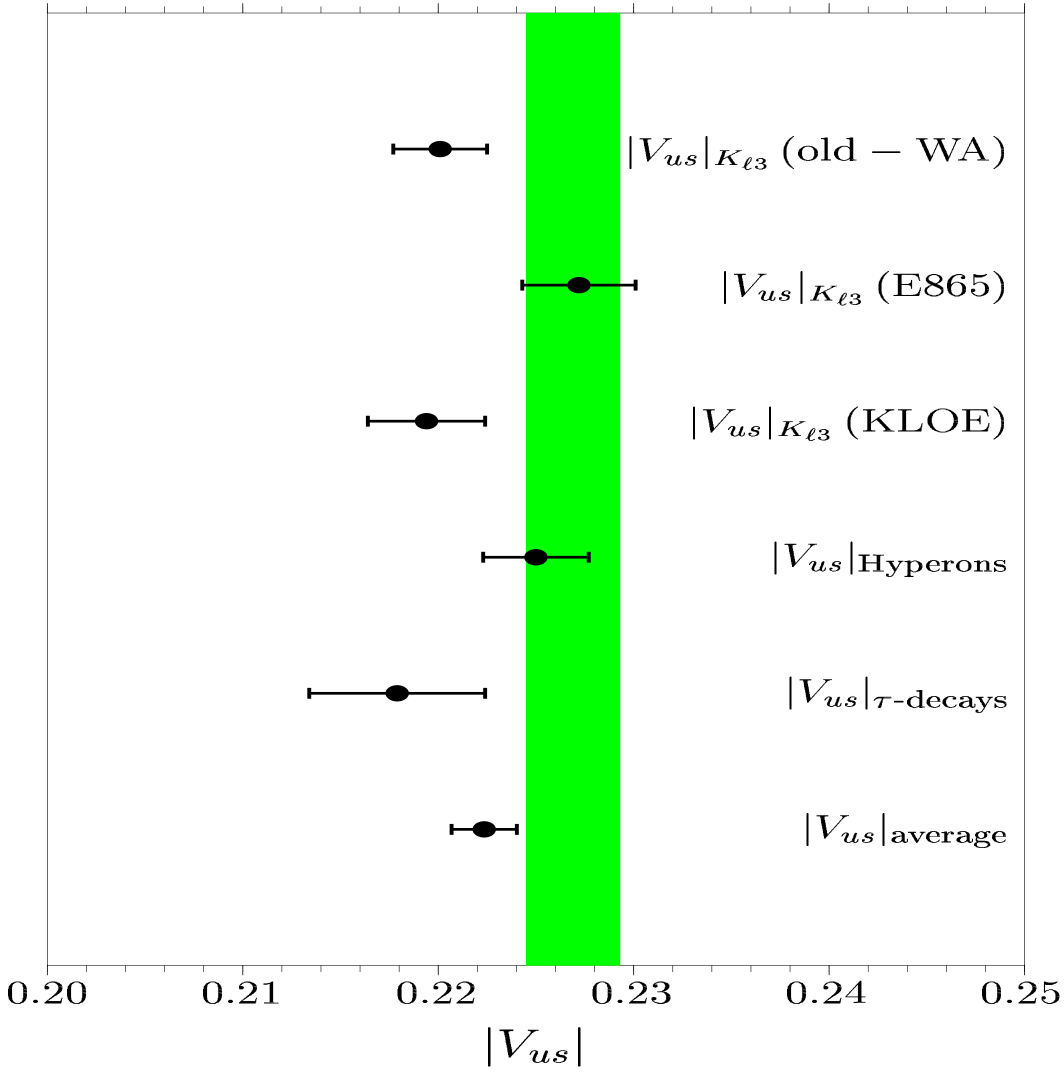
$$\Rightarrow \quad |V_{us}| = 0.2179 \pm 0.0045$$

- Determination of $|V_{us}|$ from the unitarity constraints and the measured value of $|V_{ud}|$

$$\Rightarrow \quad |V_{us}| = 0.2269 \pm 0.0024$$

- My Conclusion: Recent $\boldsymbol{K_{\ell 3}}$ Data and analyses, as well as the Hyperon decay analysis, in agreement with the UT constraint within $\boldsymbol{\pm 1\sigma}$, and hence no problem with the unitarity of $\boldsymbol{V_{\rm CKM}}$ involving the first row

A compilation of Direct & Indirect Determinations of $|V_{us}|$



Current Estimates of $|V_{cd}|$ and $|V_{cs}|$

- $|V_{cd}|$ still determined from the old dimuon data in Neutrino-Nucleon scattering:

$$\nu_\mu + d \to \mu^- c; \quad c \to s\mu^+\nu_\mu \quad \Longrightarrow \quad \nu_\mu \to \mu^+\mu^- X$$

$$\bar{\nu}_\mu + \bar{d} \to \mu^+ \bar{c}; \quad \bar{c} \to \bar{s}\mu^-\bar{\nu}_\mu \quad \Longrightarrow \quad \bar{\nu}_\mu \to \mu^+\mu^- X$$

- Using the relation

$$\frac{\sigma(\nu_\mu \to \mu^+\mu^- X) - \sigma(\bar{\nu}_\mu \to \mu^+\mu^- X)}{\sigma(\nu_\mu \to \mu^- X) - \sigma(\bar{\nu}_\mu \to \mu^+ X)} = \frac{3}{2}\mathcal{B}(c \to \mu^+ X)|V_{cd}|^2$$

- With L.H.S.$= (0.49 \pm 0.05) \times 10^{-2}$ and $\mathcal{B}(c \to \mu^+ X) = 0.099 \pm 0.012$ [PDG 2004] $\Longrightarrow$ $|V_{cd}| = 0.224 \pm 0.016$

- $|V_{cs}|$

  - From $W^+ \to c\bar{s}(g)$ and $W^- \to \bar{c}s(g)$ at LEP
  $\Longrightarrow$ $|V_{cs}| = 0.97 \pm 0.09 \pm 0.07$
  - From the ratio $\Gamma(W^\pm \to \text{hadrons})/\Gamma(W^\pm \to \ell^\pm\nu)$
  $\Longrightarrow$ $|V_{cs}| = 0.996 \pm 0.013$
  provides a quantitative test of the unitarity of V_{CKM} involving the first 2 rows

$|V_{cb}|$ from Inclusive decays $B \to X_c \ell \nu_\ell$

- Theoretical Method
  Heavy Quark Mass Expansion and Operator Product Expansion (OPE)
  [Chay, Georgi, Grinstein; Voloshin, Shifman; Bigi et al.; Manohar, Wise; Blok et al.]

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element
- Decay rate for $B \to X_c \ell \bar{\nu}_\ell$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + \cdots$$

- Γ_i are power series in $\alpha_s(m_b)$ $\to$ Perturbaton theory
- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to Luke's theorem
- Γ_2 is expressed in terms of two non-perturbative parameters

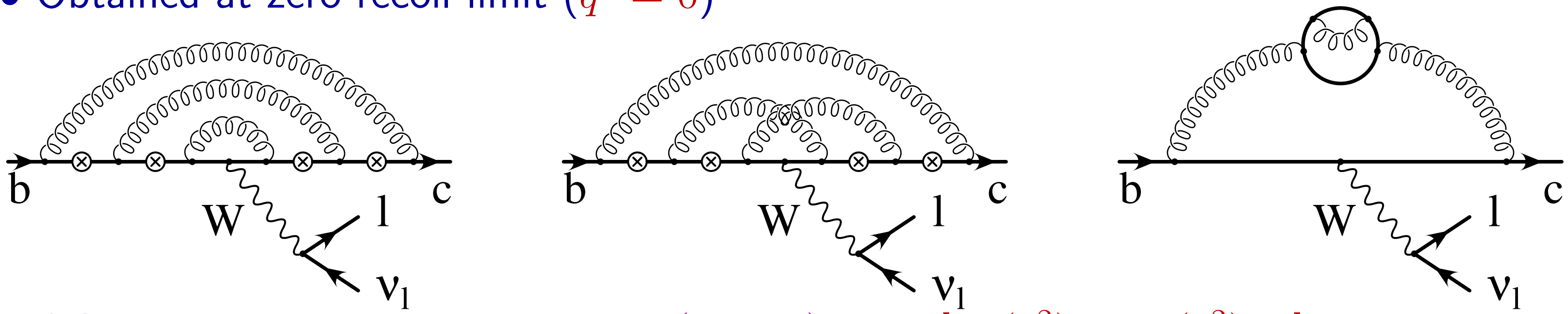
$$2M_B\lambda_1 = \langle B(v)|\bar{Q}_v(iD)^2 Q_v|B(v)\rangle, \qquad 6M_B\lambda_2 = \langle B(v)|\bar{Q}_v\sigma_{\mu\nu}[iD^\mu, iD^\nu]Q_v|B(v)\rangle$$

  λ_1: Kinetic energy, λ_2: Chromomagnetic moment (also called as μ_π^2 and μ_G^2)

- Γ_3 is currently under investigation; involves several new Non-perturbative parameters

Three-loop QCD corrections to $b \to c\ell\nu_\ell$ decays

- Obtained at zero recoil limit ($q^2 = 0$)



- QCD corrections to $W_{\bar{c}b}$ vertex: $\gamma_\mu(1-\gamma_5) \to \gamma_\mu[\eta_V(q^2) - \eta_A(q^2)\,\gamma_5]$

$$\eta_V(q^2=0) = 1$$

$$\eta_A(q^2=0) \equiv \eta_A = 1 + \frac{\alpha_s}{\pi} C_F \eta_A^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \eta_A^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 C_F \eta_A^{(3)} + \mathcal{O}(\alpha_s^4)$$

- $\eta_A^{(1)}$ and $\eta_A^{(2)}$ known since long ago [Shifman, Voloshin; Paschalis, Gounaris; Czarnecki]
- $\eta_A^{(3)}$ calculated recently [Archambault, Czarnecki]

$$\eta_A \simeq 1 - 0.667\frac{\alpha_s}{\pi} - 1.85\left(\frac{\alpha_s}{\pi}\right)^2 - 11.1\left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

$$\simeq 1 - 0.0510 - 0.0108 - 0.00495 \simeq 0.933 \qquad (\text{for} \quad \alpha_s(\sqrt{m_b m_c}) = 0.24)$$

- Effect of $\eta_A^{(3)} \simeq -\frac{1}{2}\%$

$$C_F \eta_A^{(3)}(\text{BLM}) = \left(4 - \frac{33}{2}\right)^2 C_F T_R^2 \left(\frac{25}{324} - \frac{\pi^2}{27}\right) = -15.0$$

$$C_F \eta_A^{(3)}(\text{non-BLM}) = -11.1 + 15.0 = 3.9$$

- $\eta_A^{(3)}$ dominated by BLM correction

Moment analysis of $B \to X_c \ell \nu_\ell$ with lepton energy cut

Lepton-energy and hadron mass moments

[Gambino, Uraltsev; Benson et al.]

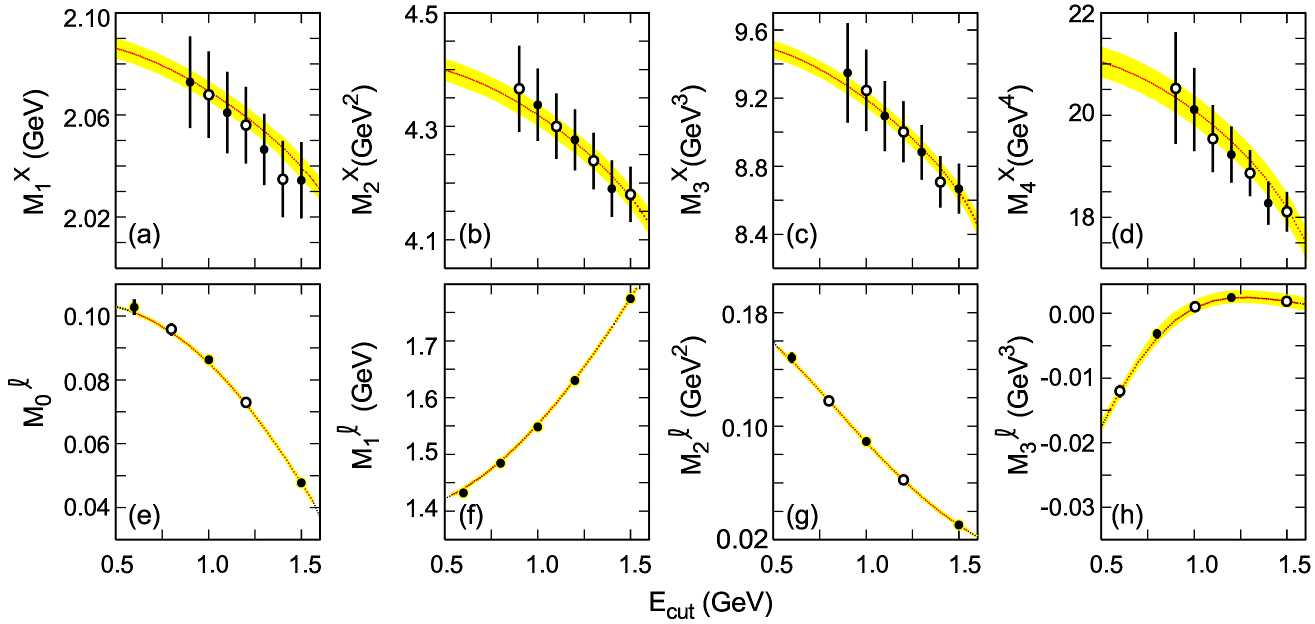
$$M_\ell^{(n)}(E_{\rm cut}) = \frac{\int_{E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{E_{\rm cut}} \frac{d\Gamma}{dE_\ell}\, dE_\ell}, \quad \langle M_X^\nu \rangle = \left(\langle M_X^2 \rangle\right)^{\frac{\nu}{2}} \left[1 + \sum_{k=2}^{\infty} C_{\frac{\nu}{2}}^k \frac{\langle (M_X^2 - \langle M_X^2 \rangle)^k \rangle}{\langle M_X^2 \rangle^k}\right]$$

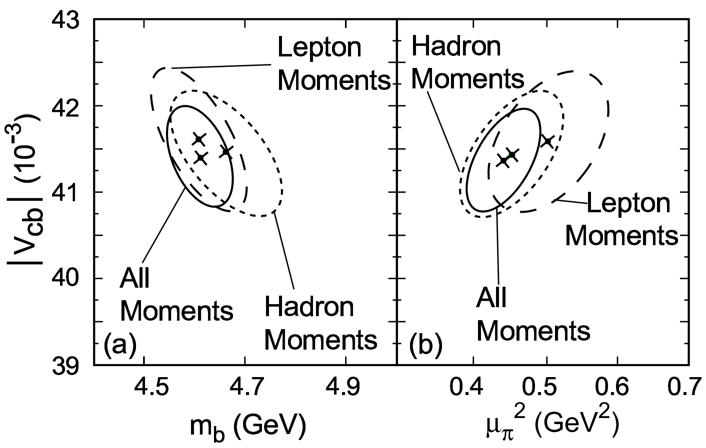
- Combined with the decay $B \to X_s \gamma$

$$\langle m_X^{2n} \rangle_{E_{\rm cut}} = \frac{\int_{E_{\rm cut}} (m_X^2)^n \frac{d\Gamma}{dm_X^2}\, dm_X^2}{\int_{E_{\rm cut}} \frac{d\Gamma}{dm_X^2}\, dm_X^2}, \qquad \langle E_\gamma^n \rangle_{E_{\rm cut}} = \frac{\int_{E_{\rm cut}} E_\gamma^n \frac{d\Gamma}{dE_\gamma}\, dE_\gamma}{\int_{E_{\rm cut}} \frac{d\Gamma}{dE_\gamma}\, dE_\gamma}$$

- Kinematic-mass scheme, $\mu \simeq 1$ GeV
- No Expansion in $1/m_c$
- Theory depends on $m_c(\mu), m_b(\mu)$, $\underbrace{\mu_\pi^2(\mu), \mu_G^2,}_{\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)}$ $\underbrace{\rho_{\rm LS}^3(\mu), \rho_{\rm D}^3(\mu)}_{\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)}$

BABAR hadronic-mass and lepton-energy moments analysis





$$|V_{cb}| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.6_{th}) \times 10^{-3}$$

$$\mathcal{B}_{ce\nu} = (10.61 \pm 0.16_{exp} \pm 0.06_{HQE})$$

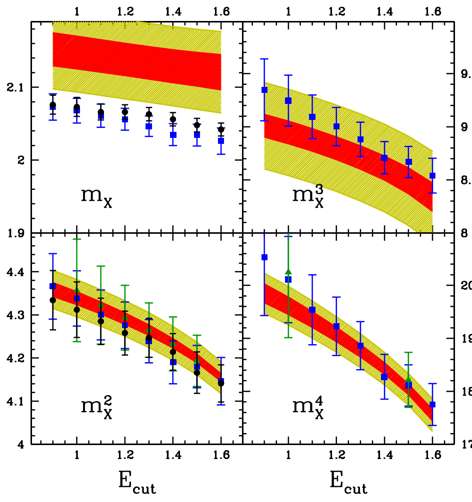
$$m_b\ (1\mathrm{GeV}) = (4.61 \pm 0.05_{exp} \pm 0.04_{HQE} \pm 0.02_{th})\ \mathrm{GeV}$$

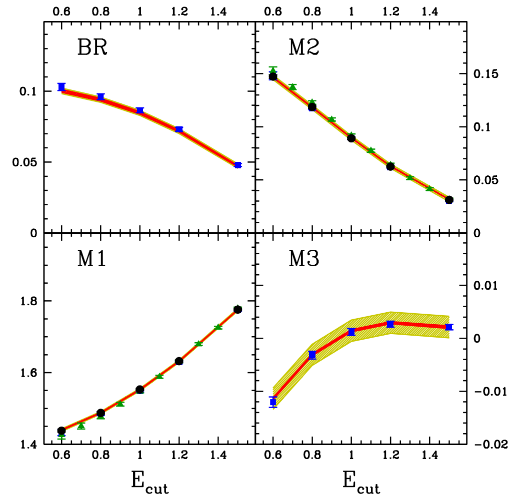
$$m_c\ (1\mathrm{GeV}) = (1.18 \pm 0.07_{exp} \pm 0.06_{HQE} \pm 0.02_{th})\ \mathrm{GeV}$$

Analysis of the moments by Bauer et al.

[Bauer, Ligeti, Luke, Manohar, Trott, hep/ph/0408002]

- Global fit of data from BABAR, BELLE, CDF, CLEO, DELPHI
- Theory precision: up to $\mathcal{O}(\alpha_s^2\beta_0)$, $\alpha_s\Lambda_{\rm QCD}/m_b$, $\Lambda_{\rm QCD}^3/m_b^3$
- Parameters: $m_b(\mu)$, $\underbrace{\lambda_1}_{\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)}$, $\underbrace{\rho_1, \tau_1 - 3\tau_4, \tau_2 + \tau_4, \tau_3 + 3\tau_4}_{\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)}$
- Scheme dependence: 1S, PS, $\overline{\rm MS}$, kinematic, pole
- Find: pole and $\overline{\rm MS}$ schemes significantly worse than others
- Analyse: $m_X^n \equiv \langle m_X^n \rangle$, $\langle E_\ell^n \rangle$, ($n = 1,...4$) $(B \to X_c \ell \nu_\ell)$; $\langle E_\gamma^n \rangle$ $(B \to X_s \gamma)$





$$|V_{cb}| = (41.4 \pm 0.6 \pm 0.1_{\tau_B}) \times 10^{-3},$$

$$m_b^{1S} = (4.68 \pm 0.03)\ \text{GeV},$$

$|V_{cb}|$ from $B \to (D, D^*)\, \ell \nu_\ell$ decays

$B \to D^* \ell \nu_\ell$ decays

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{4\pi^3} \left(\omega^2 - 1\right)^{1/2} m_{D^*}^3 \left(m_B - m_{D^*}\right)^2 \mathcal{G}(\omega)\, |V_{cb}|^2\, |\mathcal{F}(\omega)|^2$$

- $\mathcal{G}(\omega)$ phase space factor: $\mathcal{G}(1) = 1$, $\mathcal{F}(\omega) =$ Isgur–Wise function: $\mathcal{F}(1) = 1$;
- Leading $\Lambda_{\rm QCD}/m_b$ corrections absent Luke's theorem
- Theoretical issues: precise determination of the second order correction to $\mathcal{F}(\omega = 1)$, slope ρ^2 and curvature c

$$\mathcal{F}(\omega) = \mathcal{F}(1)\ \left[1 + \rho^2 (\omega - 1) + c\, (\omega - 1)^2 + ...\right] .$$

- Sum rules: $\rho^2 > \frac{3}{4}$ [Bjorken]; $c > 15/32$ [Uraltsev; Orsay group]

HFAG (Summer 2004 Update)

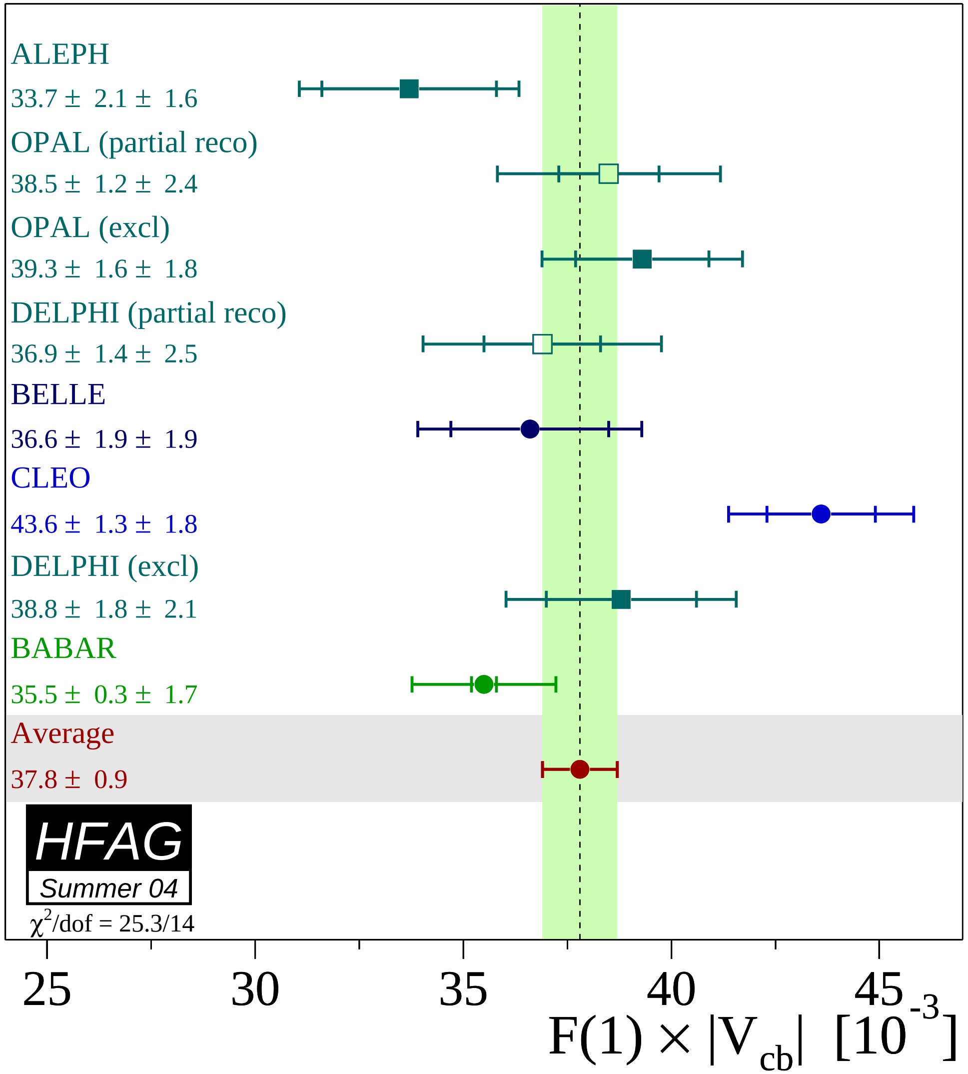
$$\mathcal{F}(1)|V_{cb}| = (37.8 \pm 0.8) \times 10^{-3}\,, \qquad \rho^2 = 1.54 \pm 0.14 \quad (\chi^2 = 25.3/14)$$

Current values of $\mathcal{F}(1)$

$$\begin{aligned} \mathcal{F}(1) &= 0.91 \pm 0.04 \quad \text{[BABAR book]} \\ & \ 0.919^{+0.030}_{-0.035} \quad \text{[Lattice QCD (Hashimoto et al.)]} \end{aligned}$$

With $\mathcal{F}(1) = 0.91 \pm 0.04$: $|V_{cb}|_{B \to D^* \ell \nu_\ell} = (41.6 \pm 0.9_{\rm exp} \pm 1.8_{\rm theo}) \times 10^{-3}$

$\mathcal{F}(1)|V_{cb}|$ (Summer 2004)



$|V_{ub}|$

From End-point spectra in $B \to X_u \ell \nu_\ell$ and $B \to X_s \gamma$

- To remove the background from $B \to X_c \ell \nu_\ell$, need to impose a large E_ℓ-cut

  Kinematics: $p_b^\mu = m_b v^\mu + k^\mu$; v^μ: 4-velocity of the b-quark, $k^\mu \sim O(\Lambda_{\rm QCD})$;
  $m_X^2 = (m_b v + k - q)^2 = (m_b v - q)^2 + 2E_X k_+ + \ldots$, $k_+ = k_0 + k_3$
- Decay rate in the cut-region depends on the shape function $f(k_+)$
- Use of OPE to calculate inclusive spectra:

  Example: Photon Spectrum in $B \to X_s \gamma$ $\quad (x = \frac{2E_\gamma}{m_b})$

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 \left(\delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \delta'(1-x) + \frac{\lambda_1}{6m_b^2} \delta''(1-x) + \cdots \right)$$

- Leading terms can be resummed into a Shape function: [Neubert; Bigi et al.]

$$\frac{d\Gamma}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts} V_{tb^*}|^2 |C_7|^2 f(1-x)$$

  - $2M_B f(\omega) = \langle B | \bar{Q}_v \delta(\omega + n \cdot (iD)) Q_v | B \rangle$; n a light-like vector, $n.v = 1$, $n^2 = 0$
  - E_ℓ - and M_{X_u} -spectra in $B \to X_u \ell \nu_\ell$ governed also by $f(x)$
  - $f(x)$ can be measured in $B \to X_s \gamma$

Model independent determination of $|V_{ub}/(V_{ts}^* V_{tb})|$

$\longrightarrow$ Define Observables (E_c – energy cut)

$$\Gamma_u(E_c) = \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma_u}{dE_\ell}, \qquad \Gamma_s(E_c) = \frac{2}{m_b}\int_{E_c}^{m_B/2} dE_\gamma (E_\gamma - E_c)\frac{d\Gamma_s}{dE_\gamma}$$

Including subleading Shape functions [Bauer, Luke, Mannel]

- $b \longrightarrow s\gamma$:

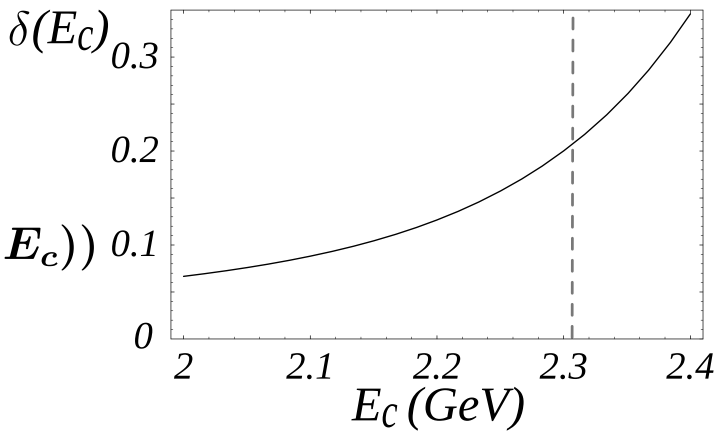
$$\frac{d\Gamma}{dE_\gamma} = \frac{\Gamma_0^s}{m_b}\left[(4E_\gamma - m_b)F(m_b - 2E_\gamma) + \frac{1}{m_b}\left[h_1(m_b - 2E_\gamma) + H_2(m_b - 2E_\gamma)\right]\right]$$

- $b \longrightarrow u\ell\bar{\nu}_\ell$

$$\frac{d\Gamma}{dE_\ell} = \frac{2\,\Gamma_0}{m_b}\int d\omega\,\theta(m_b - 2E_\ell - \omega)\left[F(\omega)\left(1 - \frac{\omega}{m_b}\right) - \frac{1}{m_b}h_1(\omega) + \frac{3}{m_b}H_2(\omega)\right]$$

- Ratio receives $1/m_b$ corrections

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right| = \left(\frac{3\alpha}{\pi}|C_7^{\rm eff}|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)}\right)^{\frac{1}{2}}(1 + \delta(E_c))$$



- $\delta(E_c)$ causes a shift of $\mathcal{O}(15\%)$ in $|V_{ub}|$

$|V_{ub}|$ from inclusive decays

Theoretical uncertainties [Bauer, Luke, Mannel; Leibovich, Ligeti, Wise; Neubert]

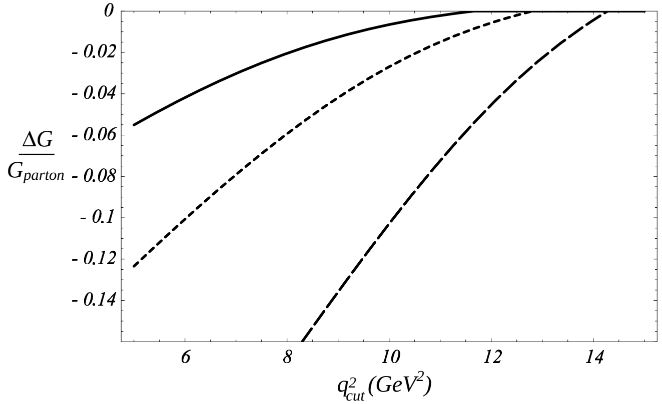
- Weak Annihilation (WA) contribution independent of $q^2_{\rm cut}$ and $m_{\rm cut}$; depends on the magnitude of Factorization violation

$$\frac{d\Gamma_{WA}}{dq^2} \sim (B_2 - B_1)\,\delta\,(q^2 - m_b^2)$$

$$\Gamma(q^2 < q^2_{\rm cut}, m_X < m_{\rm cut}) \equiv \frac{G_F^2 |V_{ub}|^2 \,(4.7\,{\rm GeV})^5}{192\pi^3}\; G(q^2_{\rm cut}, m_{\rm cut})$$

- Effect of $O(\Lambda^3_{\rm QCD}/m_b^3)$ grows as q^2 is increased [Bauer, Luke, Mannel]

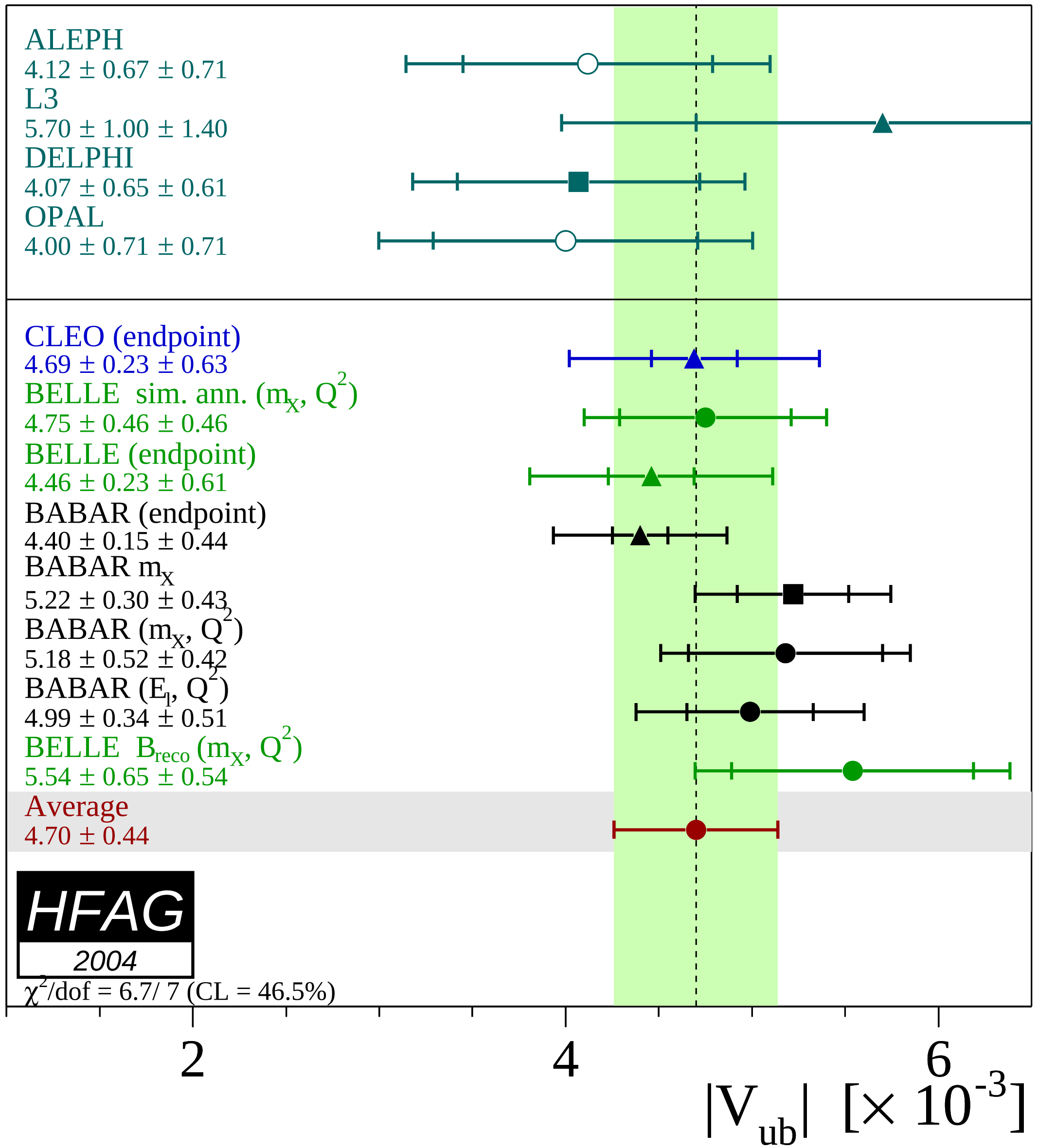
$\frac{\Delta G}{G}(q^2)_{\rm cut}$ for $m_{\rm cut} = 1.86$ GeV (top) to $m_{\rm cut} = 1.50$ GeV (bottom)



Experimental cuts

- $q^2 > (m_B - m_D)^2$: insensitive to $f(x)$; sensitive to m_b; WA corrections
- $m_X < m_D$: lots of rates; depends on $f(x)$
- $E_\ell > \frac{m_B^2 - m_D^2}{2m_B}$: simplest to measure; depends on shape functions

$|V_{ub}|$ from inclusive $B \to X_u \ell \nu_\ell$ decays



$|V_{ub}|$ from exclusive decays $B \to \pi \ell \nu_\ell$

$$\langle \pi(p_\pi)|\bar{b}\gamma_\mu q|B(p_B)\rangle = \left((p_B + p_\pi)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right) F_+(q^2) \; + \; \frac{m_B^2 - m_\pi^2}{q^2} F_0(q^2)\, q_\mu \,,$$

Techniques used to determine $F_+(q^2)$, $F_0(q^2)$

- Light-cone QCD sum rules [Colangelo, Khodjamirian]
- Lattice-QCD (Quenched) [APE, UKQCD, FNAL, JLQCD]
- Lattice-QCD (Unquenched) [HPQCD, FNAL]
- Lattice-QCD and phenomenological models [Becirevic, Kaidalov]

BELLE Analysis [Iijima, ICHEP'04]

[Becirevic]

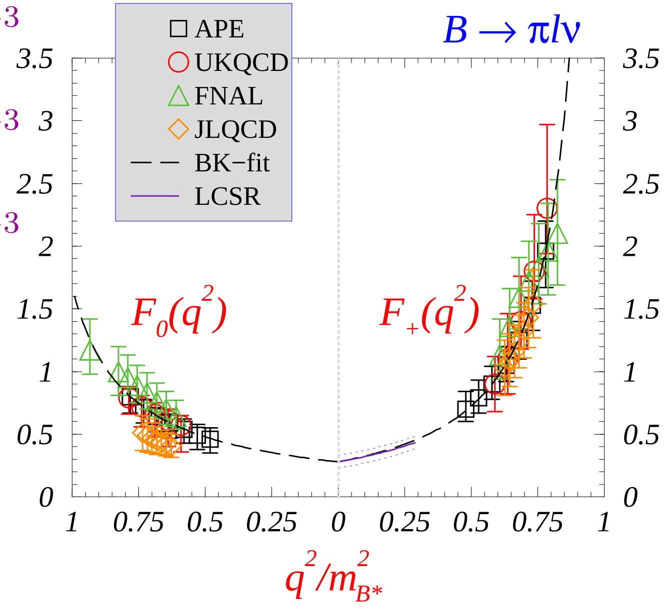
$$|V_{ub}|_{\text{Quenched}} = (3.90 \pm 0.71 \pm 0.23^{+0.62}_{-0.48}) \times 10^{-3}$$

$$|V_{ub}|_{\text{FNAL'04}} = (3.87 \pm 0.70 \pm 0.22^{+0.85}_{-0.51}) \times 10^{-3}$$

$$|V_{ub}|_{\text{HPQCD}} = (4.73 \pm 0.85 \pm 0.27^{+0.74}_{-0.50}) \times 10^{-3}$$

- Errors still large!
- $|V_{ub}|$ from BELLE's inclusive analysis

$$|V_{ub}|_{\text{BELLE}} = (5.54 \pm 0.42 \pm 0.50 \pm 0.12 \pm 0.19 \pm 0.42 \pm 0.27) \times 10^{-3}$$



Status of the Third Row $V_{\rm CKM}$

$|V_{tb}|$

- From direct production and decays of the top quark (hep-ex/0505091)

$$R \equiv \frac{\mathcal{B}(t \to W + b)}{\mathcal{B}(t \to W + \sum_q q)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$

$$R = 1.12^{+0.21}_{-0.19}\ (\text{stat})^{+0.17}_{-0.13} (\text{syst.})$$

- Assuming CKM unitarity & CDF Data $\Longrightarrow |V_{tb}| > 0.78$ (95% C.L.)

$|V_{td}|$

- From B_d^0 - $\overline{B_d^0}$ Mixing; $\Delta M_d = (0.505 \pm 0.005)$ ps^{-1} [HFAG 2005]
- SM (Box contribution with NLO QCD corrections) ($x_t = m_t^2/m_W^2$)

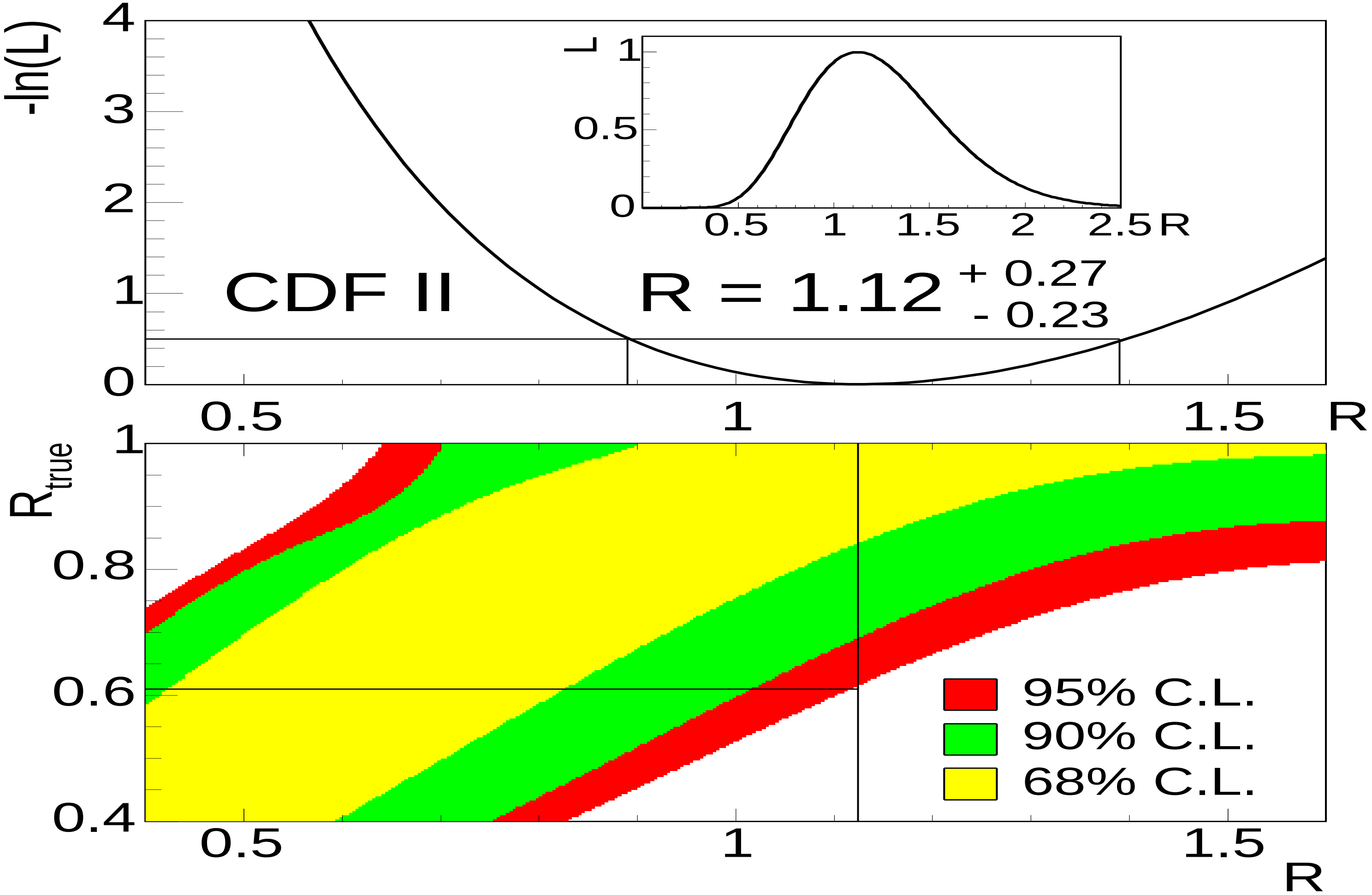
$$\Delta M_d = \frac{G_F^2}{6\pi^2} \hat{\eta}_B |V_{td} V_{tb}^*|^2 M_{B_d} (f_{B_d}^2 \hat{B}_{B_d}) M_W^2 S_0(x_t)$$

$$S_0(x) = x \cdot \left[\frac{1}{4} + \frac{9}{4} \frac{1}{(1-x)} - \frac{3}{2} \frac{1}{(1-x)^2} - \frac{3}{2} \frac{x^2 \ln x}{(1-x)^3} \right]$$

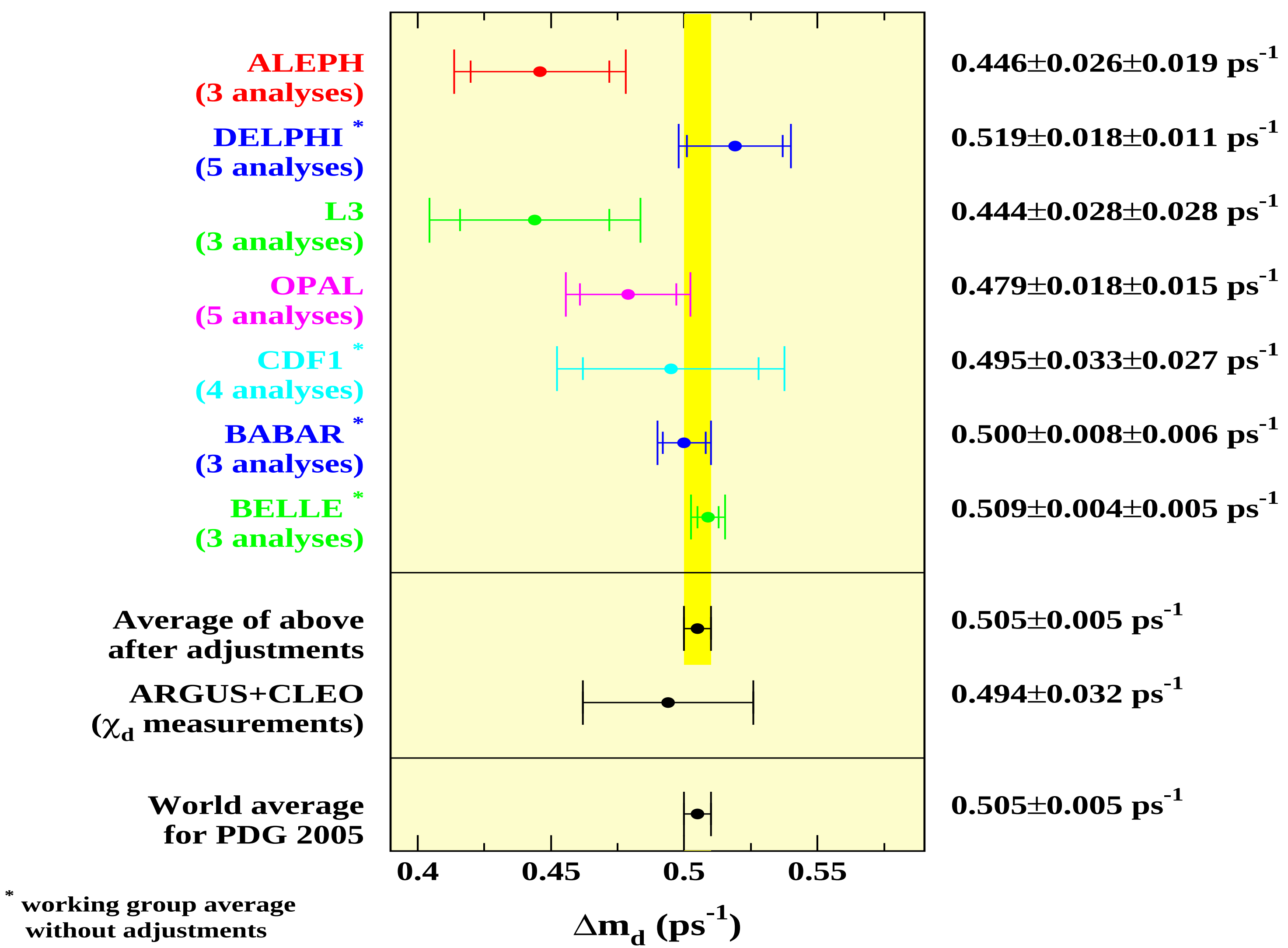
$$\langle \bar{B}_q^0 | (\bar{b} \gamma_\mu (1-\gamma_5) q)^2 | B_q^0 \rangle \equiv \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$$

$-ln(L)$ vs. R from t-quark decays

[D. Acosta et al. (CDF Collaboration); hep-ex/0505091]



Δm_d (HFAG 2005)



V_{td} and V_{ts} with Lattice-QCD

$|V_{td}|$

- Lattice-QCD [Updated H. Wittig, DPG, Berlin, '05]:
  $\sqrt{\hat{B}_{B_d}} F_{B_d} = 216 \pm 30^{+0}_{-21} \;\; \text{(chiral)}$ MeV

$$|V_{td}V_{tb}^*| = 8.5 \times 10^{-3} [\frac{210 \text{ MeV}}{\sqrt{\hat{B}_{B_d}} F_{B_d}}] \sqrt{\frac{2.40}{S_0(x_t)}}$$

- Lattice-QCD & SM $\Longrightarrow$ $|V_{td}V_{tb}^*| = (8.5 \pm 1.0) \times 10^{-3}$

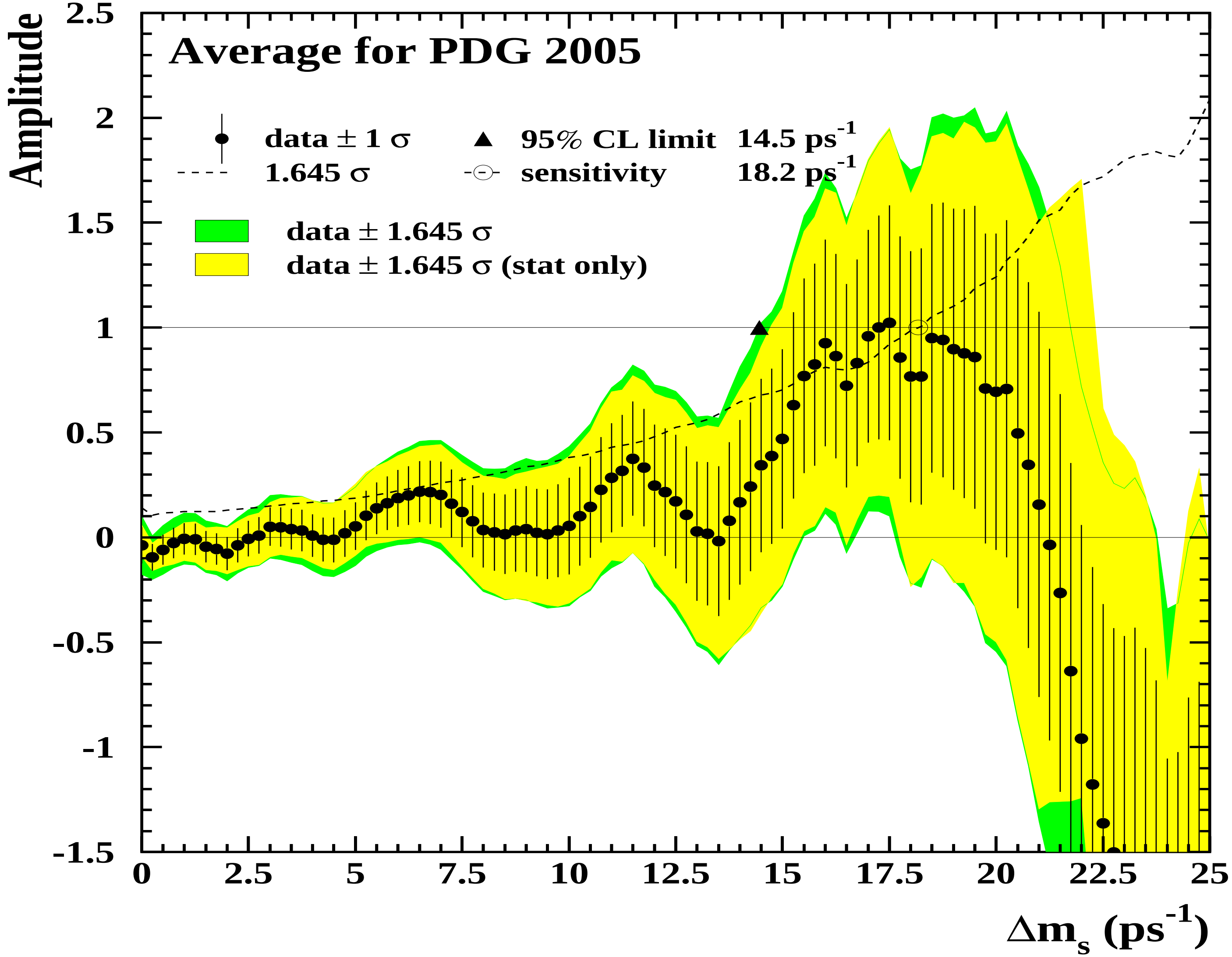
$|V_{ts}|$

- B_s^0 - $\overline{B_s^0}$ Mixing: $\Delta M_s > 14.5$ ps^{-1} (at 95% CL) [HFAG 2005]
- SM: $\Delta M_s = \frac{G_F^2}{6\pi^2} \hat{\eta}_B |V_{ts}V_{tb}^*|^2 M_{B_s} (f_{B_s}^2 \hat{B}_{B_s}) M_W^2 S_0(x_t)$
- Lattice-QCD: $\sqrt{\hat{B}_{B_s}} F_{B_s} = 249 \pm 34$ MeV
- The ratio $\Delta M_s / \Delta M_d$ has a smaller non-perturbative uncertainty

$$\frac{\Delta M_s}{\Delta M_d} = \xi \frac{M_{B_s}}{M_{B_d}} \frac{|V_{ts}|^2}{|V_{td}|^2}; \quad \xi = \sqrt{\frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}}}$$

- Lattice-QCD: $\xi = 1.15 \pm 0.05^{+0.12}_{-0.0}$ (Chiral extr.) $\Longrightarrow$ $|V_{ts}V_{tb}^*| > 0.033$

Δm_s (HFAG 2005)



Source: H. Wittig, DPG'05, Berlin

- Starting point: f_{B_s} for $N_f = 0$
  - 5% error, except quenching *[ALPHA, hep-lat/0309072]*
  - central values vary between 180 and $210\,\mathrm{MeV}$, depending on scale

$\Rightarrow \quad f_{B_s}^{N_f=0} = 195 \pm 10 \pm 15\,(\text{scale})\,\mathrm{MeV}$

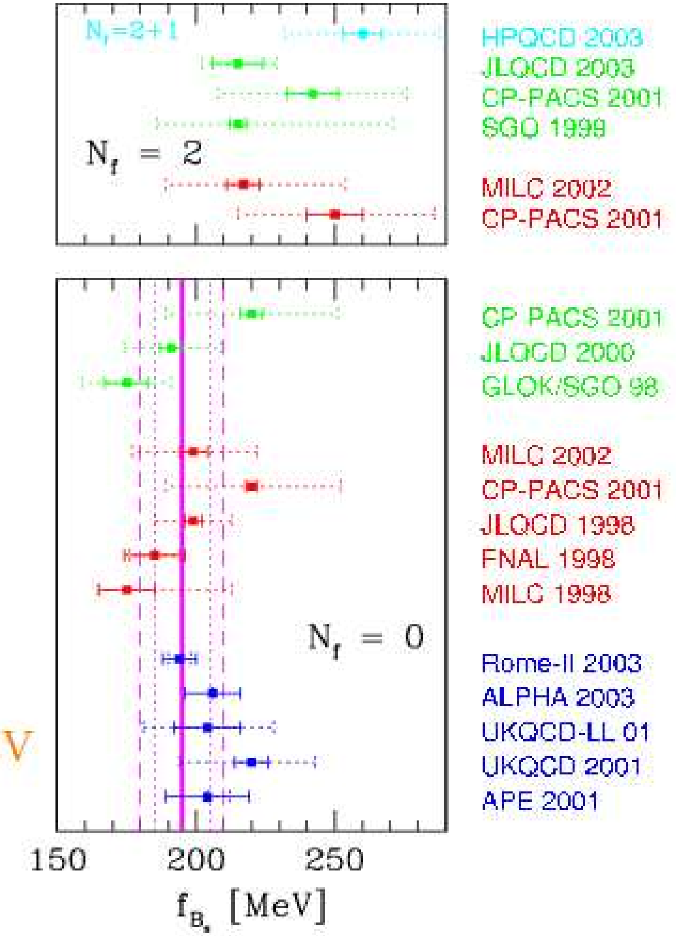
- Multiply by $f_{B_s}^{N_f=2}/f_{B_s}^{N_f=0} = 1.10 \pm 0.10$:

$\Rightarrow \quad f_{B_s}^{N_f=2} = 215 \pm 11 \pm 26\,(\text{quen})\,\mathrm{MeV}$

- Divide by $f_{B_s}/f_{B_d} = 1.15(3)^{-0.12}_{-0.00}$

$\Rightarrow \quad f_{B_d}^{N_f=2} = 187 \pm 11 \pm 23\,(\text{quen})^{\,0}_{-18}\,(\text{chir})\,\mathrm{MeV}$

- No effort to estimate result for $N_f = 3$



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Source: H. Wittig, DPG'05, Berlin

- New data (improved staggered quarks, $N_f = 2+1$): weak evidence for chiral logs

$$\frac{f_{B_s}}{f_{B_d}} = 1.22^{+0.06}_{-0.05} \qquad \text{(fit with chiral log)}$$

[Hashimoto @ ICHEP04, hep-ph/0411120]

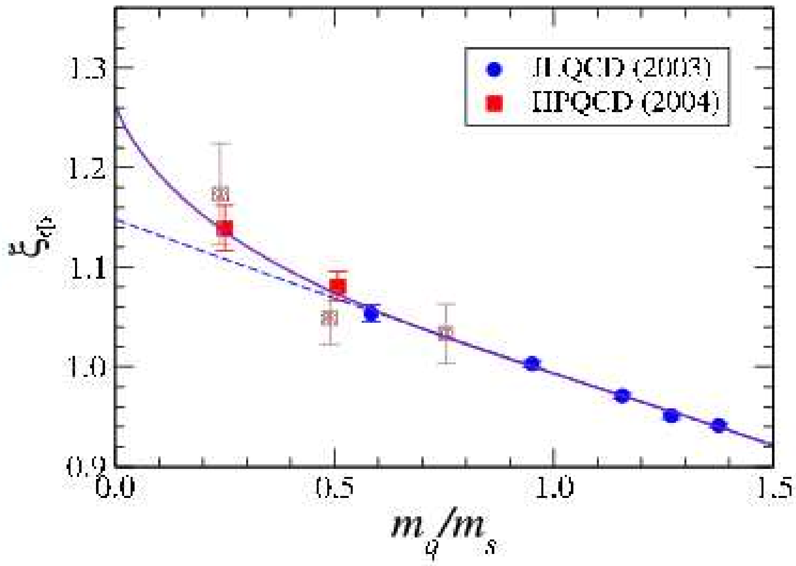
- Curvature due to combining two datasets with different systematics?

- UKQCD find (Wilson quarks, $N_f = 2$)

$$\frac{f_{B_s}}{f_{B_d}} = 1.38(13)(8)$$

[McNeile & Michael, JHEP 0501 (2005) 011]

but curvature not constrained by data



- Issue not settled . . .

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Source: H. Wittig, DPG'05, Berlin

- B_{B_d}, B_{B_s} and ξ
  - fewer results available; only one for $N_f = 2$
  - systematics not as well understood; no continuum extrapolation
  - weak dependence on lattice artefacts and heavy quark treatment

- "Global representation":

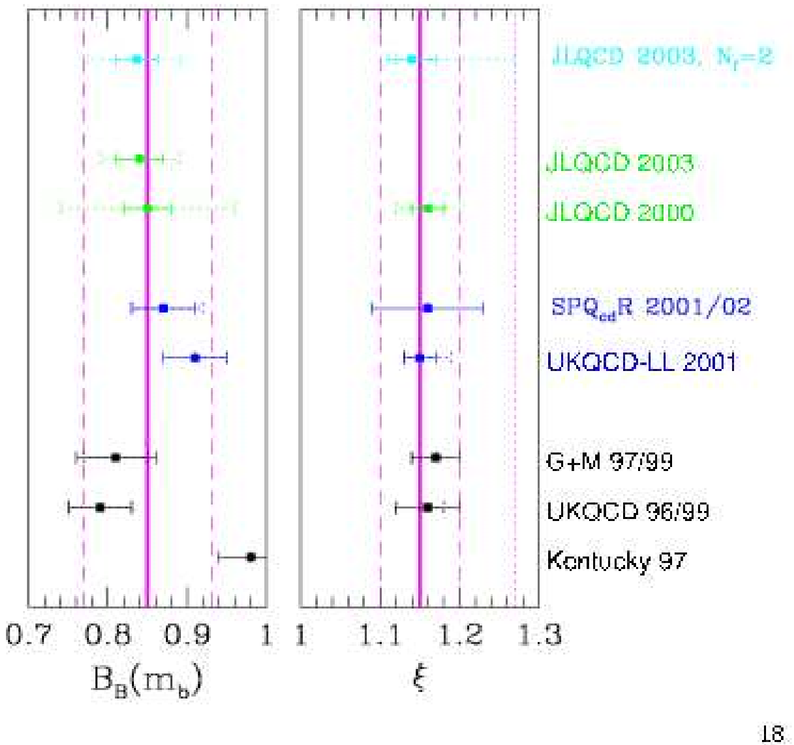
$$B_{B_d}^{\overline{MS}}(m_b) = 0.85 \pm 0.08$$

$$\Rightarrow \quad \widehat{B}_{B_d} = 1.34 \pm 0.12$$

$$B_{B_s}/B_{B_d} = 1.00 \pm 0.03$$

$$\xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}} = 1.15 \pm 0.05^{+0.12}_{-0.00}$$

→ Purple band in plot



Determination of V_{ts} from BR ($\bar{B} \to X_s\gamma$)

- Unitarity of the CKM Matrix

$$\sum_{u,c,t} \lambda_i = 0, \quad \text{with} \quad \lambda_i = V_{ib}V_{is}^*$$

- $\lambda_u = V_{ub}V_{us}^* \simeq A\lambda^4(\bar{\rho} - i\bar{\eta}) \simeq O(10^{-2})$
- $\lambda_t = -\lambda_c = -A\lambda^2 + ... = -(41.0 \pm 2.1) \times 10^{-3}$
- Without invoking the CKM unitarity, NLO SM-calculations in the $\overline{\mathrm{MS}}$ scheme and current data imply the following constraint

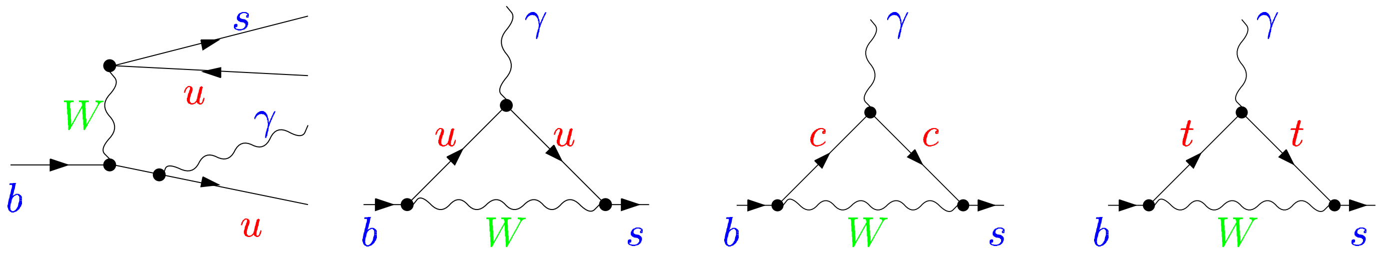
[Misiak, AA]

$$|1.69\lambda_u + 1.60\lambda_c + 0.60\lambda_t| = (0.94 \pm 0.07)|V_{cb}|$$

$$\Longrightarrow \lambda_t = V_{tb}V_{ts}^* = -(47.0 \pm 8.0) \times 10^{-3}$$

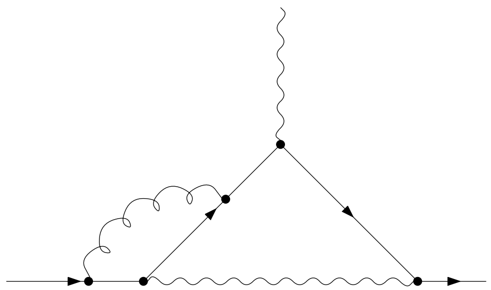
- In future, NNLO calculations will lead to a determination of BR($\bar{B} \to X_s\gamma$) to an accuracy of 5%
- With improved data, this will determine V_{ts} to an accuracy of about 10%

Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\% \qquad \sim +200\% \qquad \sim -100\%$$

In the amplitude, after including LO QCD effects.



QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR($\bar{B} \to X_s\gamma$) more than twice.

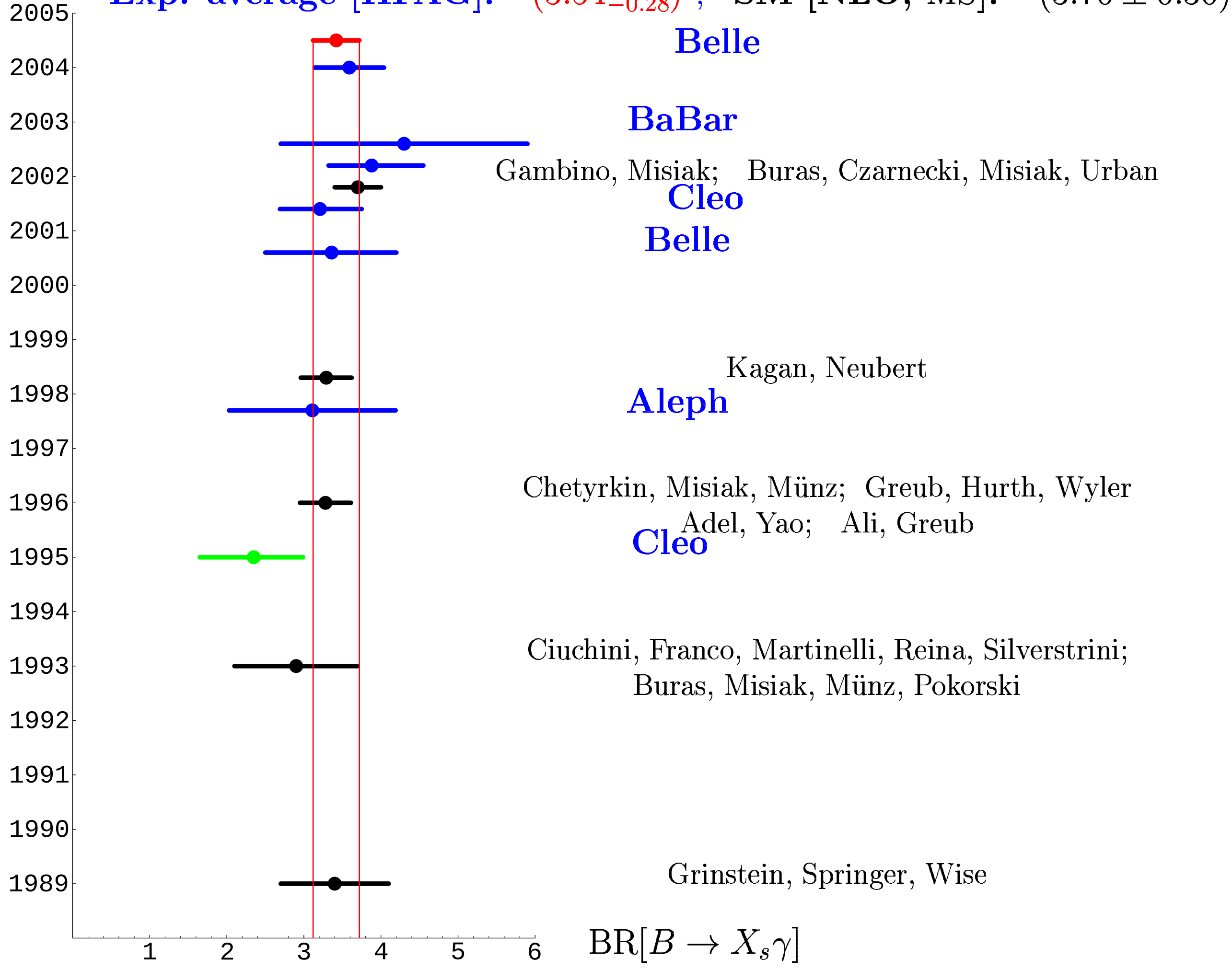
Effective field theory method is the most convenient for resummation of such large logarithms.

Evolution in time

$\mathrm{BR}[\bar{B} \to X_s\gamma]$ **(units: 10^{-4})**

Measurements & the SM calculations

Exp. average [HFAG]: $(3.54^{+0.30}_{-0.28})$; **SM [NLO; $\overline{\mathrm{MS}}$]:** (3.70 ± 0.30)



Constraints from the sides and angles of the Unitarity Triangle

- $|\epsilon_K| = (2.280 \pm 0.013) \times 10^{-3}$ [PDG 2004]

$$\bar{\eta}[(1-\bar{\rho})\eta_2^{\rm QCD} S_0(x_t) + P_c] A^2 \hat{B}_K = 0.187$$

$P_c = 0.29 \pm 0.07$ [Herrlich, Jamin, Nierste]; $S_0(x_t) \simeq 2.4$; $\eta_2^{\rm QCD} = 0.57 \pm 0.01$ [Buras et al.]

- $\Delta M_d = (0.505 \pm 0.005)$ ps^{-1} [HFAG 2005]

$$|V_{td}V_{tb}^*| = 8.5 \times 10^{-3} [\frac{210 \text{ MeV}}{\sqrt{B_{B_d} F_{B_d}}}] \sqrt{\frac{2.40}{S_0(x_t)}}$$

  - Lattice-QCD & SM $\Longrightarrow$ $|V_{td}V_{tb}^*| = (8.5 \pm 1.0) \times 10^{-3}$

- $\Delta M_s > 14.5$ ps^{-1} (at 95% CL) [HFAG 2005]
  - Lattice-QCD & SM $\Longrightarrow$ $|V_{ts}V_{tb}^*| > 0.033$;

- These measurements ($+|V_{ub}|$ & $|V_{cb}|$) $\Longrightarrow$ $\sin 2\beta(c\bar{c}s) = 0.7$ - 0.8 (Unitarity fits in the SM)
  - in remarkable agreement with $\sin 2\beta(c\bar{c}s) = 0.726 \pm 0.037$

- Compatibility between the SM & Experiment implies that CP Violation in the $|\Delta S| = 2$ & $|\Delta B| = 2$ transitions is dominated by the phase of the CKM matrix, but current errors do admit an additional subdominant contribution in $M_{12}(K)$ and $M_{12}(B_d, B_s)$

Weak Hadronic Matrix Elements on the Lattice

II. $K^0 - \overline{K}^0$ mixing and $\widehat{B}_{\rm K}$

- $\widehat{B}_{\rm K}$ parameterises non-perturbative contribution to indirect CP violation:

$$B_{\rm K}(\mu) = \frac{\left\langle \overline{K}^0 \left| O^{\Delta S=2}(\mu) \right| K^0 \right\rangle}{\frac{8}{3} f_{\rm K}^2 m_{\rm K}^2}$$

$$O^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] = O^{\Delta S=2}_{\rm VV+AA} - O^{\Delta S=2}_{\rm VA+AV}$$

- Renormalisation and mixing:

$$O^{\rm R}_{\rm VV+AA} = Z\left\{ O^{\rm bare}_{\rm VV+AA} + \sum_{i=1}^{4} \Delta_i O_i^{\rm bare} \right\}$$

Wilson fermions: explicit chiral symmetry breaking: $\Delta_i \neq 0$

Staggered fermions: Remnant chiral symmetry: $\Delta_i = 0$

Domain Wall/Overlap: chiral symmetry preserved; expensive to simulate

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Source: H. Wittig, DPG'05, Berlin

- Quenched result for $B_K^{\overline{\rm MS}}(2\,{\rm GeV})$ translates into

$$\widehat{B}_K = 0.81 \pm 0.06$$

- Dynamical quark effects: simulations with $N_f = 2, 2+1$ indicate decrease

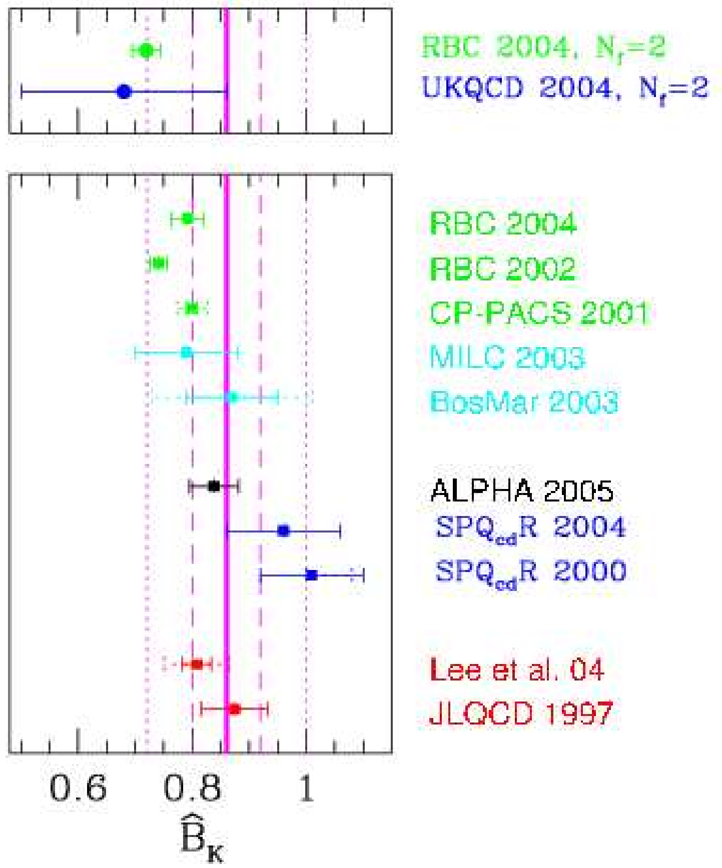
[UKQCD, hep lat/0406013; RBC, hep lat/0410044]

→ Requires confirmation at $m^{\rm sea} < m_s/2$ and in continuum limit

- Purple band: used in CKM fit

$$\widehat{B}_K = 0.86 \pm 0.06\,(\text{gauss}) \pm 0.14\,(\text{flat})$$

[UTfit Collab. (Bona et al.), hep ph/0501199]



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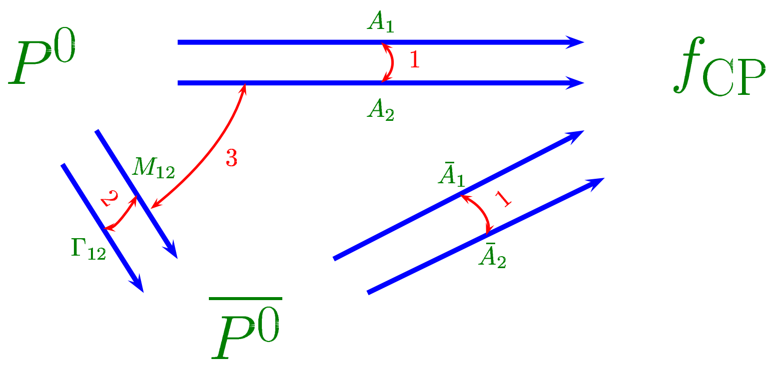
CP violation in neutral meson decay into a CP eigenstate



1. In decay: $\bar{A}/A \neq 1$ $\left(\frac{\bar{A}}{A} = \frac{\bar{A}_1+\bar{A}_2}{A_1+A_2}\right)$

   (For example, A_1 is a Tree amplitude & A_2 is Penguin)
2. In mixing: $|q/p| \neq 1$ $\left(\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma}\right)$
3. In interference: $\mathcal{I}m\lambda \neq 1$ $\left(\lambda = \frac{q}{p}\frac{\bar{A}}{A}\right)$

- The case theorists love!
  - Decay dominated by a single CPV phase: $|\frac{\bar{A}}{A}| = 1$;
  - CPV in mixing negligible $|\frac{q}{p}| = 1$;
  - The remaining eefect is: $S_f \sim \sin[\arg(M_{12}) - 2\arg(A)] = 1$

Interplay of Mixing & Decays of B^0 and $\overline{B^0}$ to CP Eigenstate

- Involving tree-dominated B-decays ($b \to c\bar{c}s$), such as $B^0/\overline{B^0} \to J/\psi K_s;\ J/\psi K_L$

$$\mathcal{A}_f(t) = \frac{\Gamma(\overline{B^0}(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\overline{B^0}(t) \to f) + \Gamma(B^0(t) \to f)}$$

$$= C_f \cos(\Delta M_B t) + S_f \sin(\Delta M_B t)$$

$$C_f = \frac{(|\lambda_f|^2)-1}{(|\lambda_f|^2+1)}; \qquad S_f = \frac{2\ \mathrm{Im}\lambda_f}{(|\lambda_f|^2+1)}$$

- Definitions:

$$\lambda_f \equiv (q/p)\ \rho(f); \quad \rho(f) = \frac{\bar{A}(f)}{A(f)}$$

$$A(f) = \langle f|H|B^0\rangle; \quad \bar{A}(f) = \langle f|H|\overline{B^0}\rangle$$

$$q/p = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = \mathrm{e}^{-2i\phi_{\mathrm{mixing}}} = \mathrm{e}^{-2i\beta}$$

- If only a Single Amplitude dominant, then one can write:

$$\rho(f) = \eta_f \mathrm{e}^{-2i\phi_{\mathrm{decay}}}$$

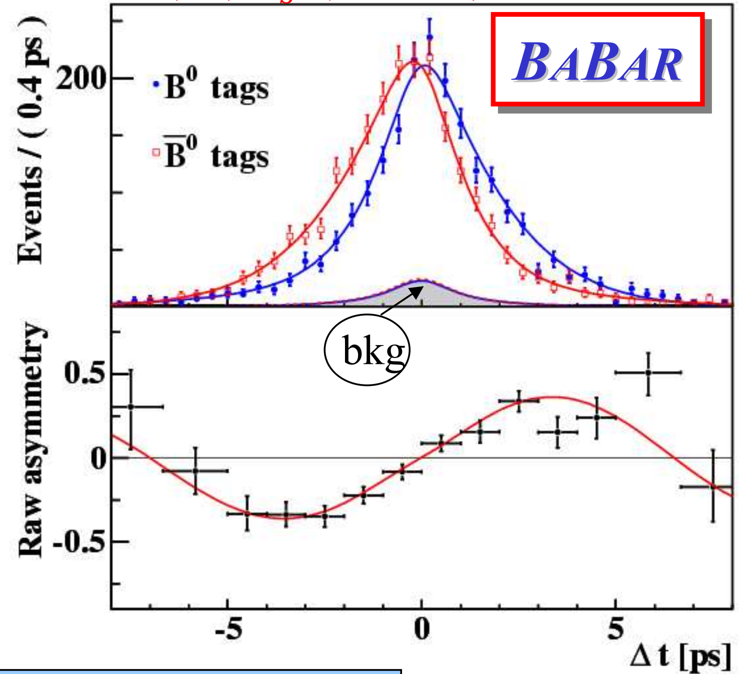
where $\eta_f = \pm 1$ is the intrinsic CP-Parity of the state $f \Rightarrow |\rho(f)| = 1$

$$\mathcal{A}_f(t) = S_f \sin(\Delta M_B t);\ S_f = -\eta_f \sin 2(\phi_{\mathrm{mixing}} + \phi_{\mathrm{decay}});\ C_f = 0$$

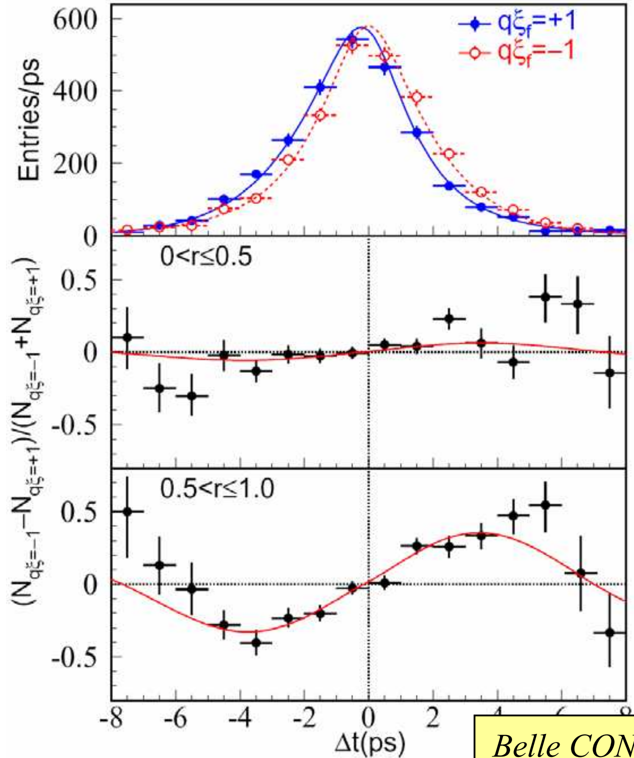
CPV in B-Decays-1

$\sin 2\beta$ results from charmonium modes

$(c\bar{c})K_S^0$ (CP odd) modes



BABAR



Belle 2003

Update for ICHEP04

BABAR PUB-04/038

Belle CONF-0436

BABAR		Belle
$\sin 2\beta = +0.722 \pm 0.040 \pm 0.023$	$(c\bar{c})K_S^0 +$	$\sin 2\beta = +0.728 \pm 0.056 \pm 0.023$
$\lvert\lambda\rvert = \lvert\bar{A}/A\rvert = 0.950 \pm 0.031 \pm 0.013$	$(c\bar{c})K_L^0$	$\lvert\lambda\rvert = \lvert\bar{A}/A\rvert = 1.007 \pm 0.041 \pm 0.033$
$205\, fb^{-1}$ on peak or $227M$ $B\bar{B}$ pairs		$140\, fb^{-1}$ on peak or $152M$ $B\bar{B}$ pairs
7730 CP events (tagged signal)		4347 CP events (tagged signal)

Limit on direct CPV

ICHEP04-北京
August 20, 2004

Marcello A. Giorgi

M.Bruinsma, T. Higuchi, CP-3

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Unitarity Triangles from CKMfitter and UTfit (2005)

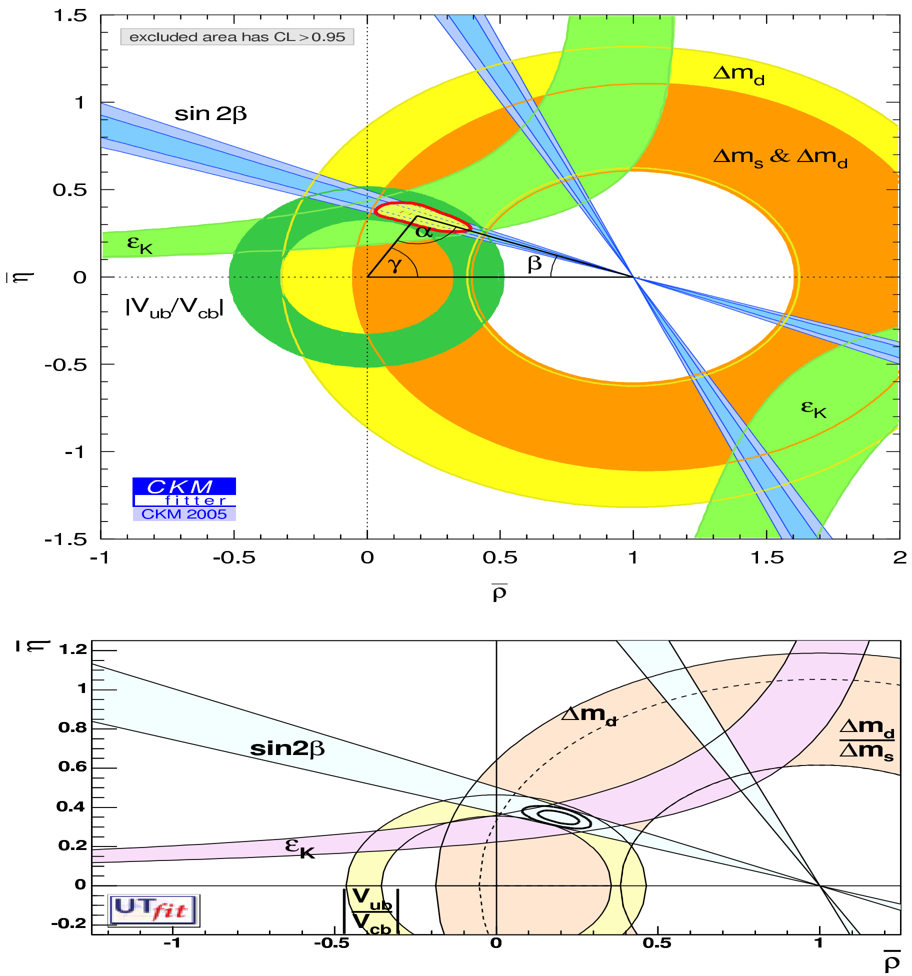

Figure 16: Standard Model constraints on the $(\overline{\rho}, \overline{\eta})$ plane, from (top) [183] and (bottom) [184].

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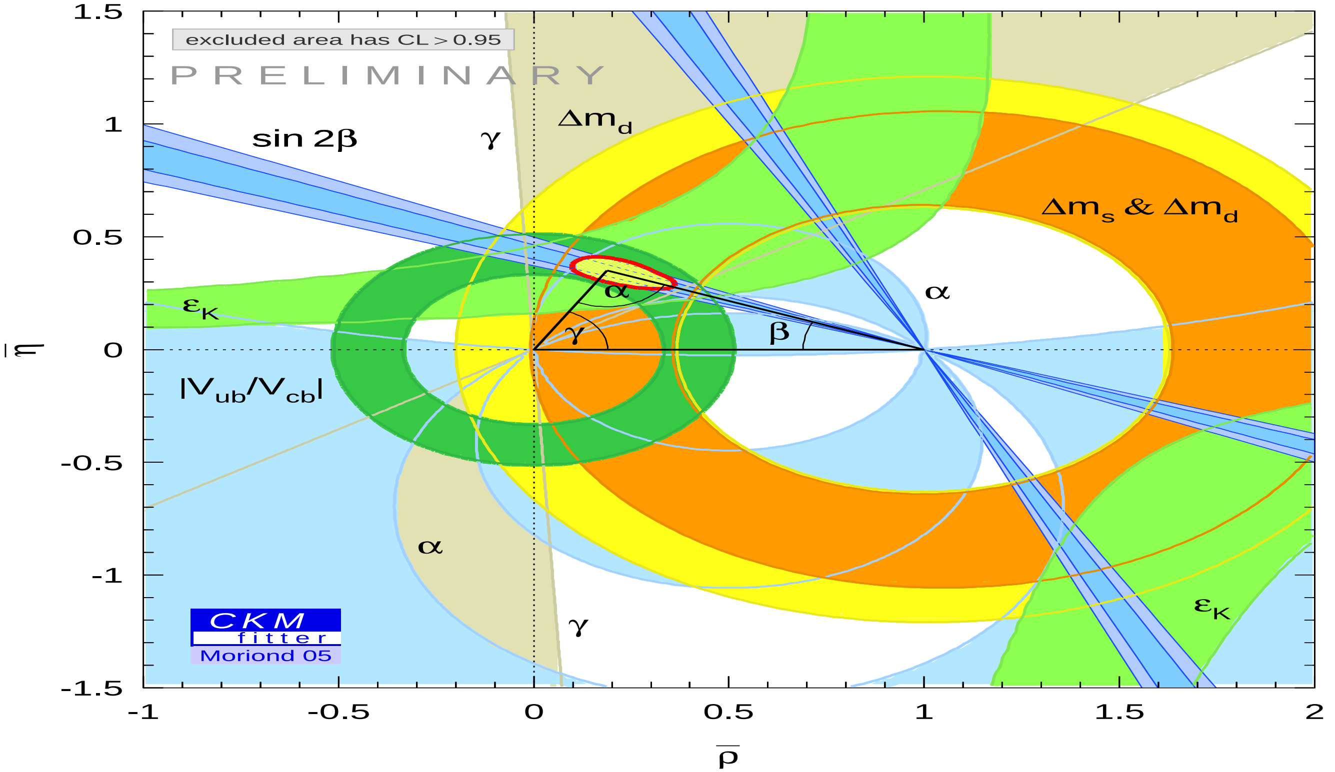
SM confronts measurements of $\sin 2\beta, \quad \alpha, \quad \gamma$



- $\sin 2\beta = 0.725 \pm 0.037 (\beta = [23.2 \pm 1.5]^\circ)$
- $\alpha = [101^{+16}_{-9}]^\circ$
- $\gamma = [63^{+15}_{-13}]^\circ$
- Direct and indirect measurements of angles agree very well
- Unconstrained sum of angles $= 187^\circ$, consistent with unitarity sum within errors

Summary

- Thanks to dedicated experiments and progress in theoretical techniques (χPT, Lattice-QCD, QCD Sum Rules, Heavy quark expansion) $V_{\rm CKM}$ now well measured
- Precision on V_{ij} ranges from $\delta|V_{ud}|/|V_{ud}| = 5 \times 10^{-4}$ (best measurement) to $\delta|V_{tb}|/|V_{tb}| = 0.2$ (current CDF measurement), which will be vastly improved at the LHC and ILC
- $|V_{cb}|$ determined precisely: $\frac{\delta|V_{cb}|}{|V_{cb}|} \sim 2\%$; close on the heels of $\frac{\delta|V_{us}|}{|V_{us}|}$!
- Current precision on $|V_{ub}|$ about 14%; many theoretical proposals to improve our knowledge of $|V_{ub}|$; require lot more data; forthcoming from B factories
- Radiative rare B-decays in agreement with the SM rates; determine $|V_{ts}|$ (and in principle also $|V_{td}|$); Current precision on $|V_{td}|$ from B^0 - $\overline{B^0}$ mixing is about 10%
- A non-trivial test of the CKM paradigm for CP violation in the K- and B-meson sectors has been carried out at the current B-factories by overconstraining the CKM unitarity triangle
- B-factories have measured all three inner angles of the UT triangle: $\alpha = (100 \pm 11)^\circ$; $\beta = (23.3 \pm 1.5)^\circ$; $\gamma = (63 \pm 14)^\circ$
- Largest current discrepancy from SM is in CPV $b \to s\bar{s}s$ penguins; $3.5\ \sigma$ effect
- We look forward to new data from the ongoing and planned experiments at CERN, Fermilab, Frascati, BNL, KEK, and ILC

Backup Slides

More on CP Violation in B-decays sensitive to BSM Effects

- In addition to the $c\bar{c}s$ final state, CP asymmetries have been measured in a number of B decays, involving direct CP violation & interplay of mixings and decay amplitudes
- Direct CP asymmetries provide tests of QCD dynamics in B-decays

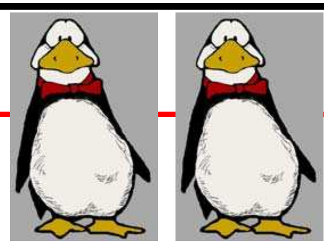
$$A_{\rm CP}(K^{\pm}\pi^{\mp}) = -0.101 \pm 0.020 \quad [\text{BABAR \& BELLE}]$$

- More interesting for the BSM searches are measurements involving penguin amplitudes and B^0 -$\overline{B^0}$ mixing in CP eigenstates
  - $B^0 \to \phi K_s^0$; $B^0 \to \eta' K_S^0$; $B \to f_o K_s^0$;...
- Current experiments (BELLE & BABAR) seem to measure a different effective angle $\sin 2\beta_{\rm eff}$ in the Penguin-dominated amplitudes $b \to s\bar{s}s$

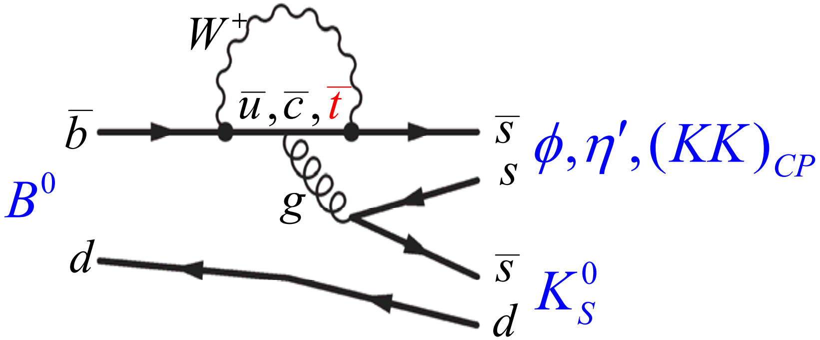
$$\sin 2\beta_{\rm eff}(s\bar{s}s; s\bar{d}d) = 0.43 \pm 0.07 \quad (\sim 3.7\sigma \text{ BSM Effect, theor. uncertainties??})$$

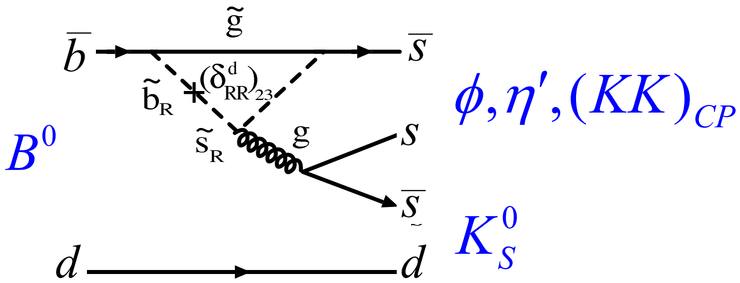
- If confirmed by more data, this would imply the existence of BSM physics in $b \to s$ transitions

Feynman Diagrams for $\sin 2\beta$ from Penguins

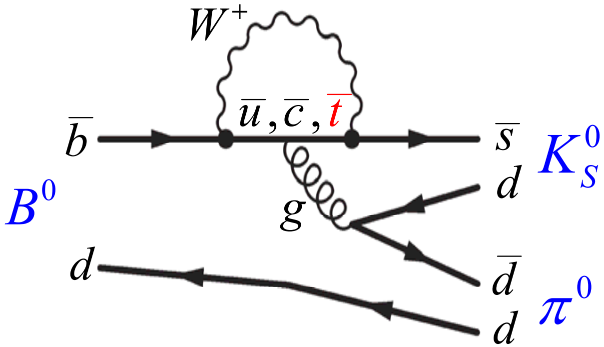
$\sin 2\beta$ and...  and.... 

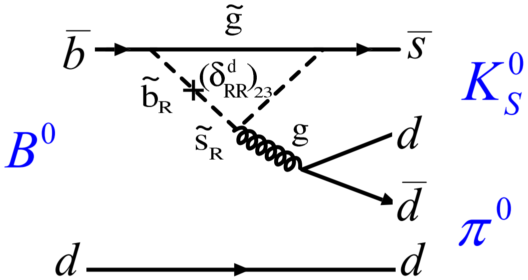
In SM interference between B mixing, K mixing and Penguin $b \to s\bar{s}s$ or $b \to s\bar{d}d$ gives the same $e^{-2i\beta}$ as in tree process $b \to c\bar{c}s$. However loops can also be sensitive to New Physics!





New phases from SUSY?





$S_{b\to q\bar{q}s}$ and $C_{b\to q\bar{q}s}$ [HFAG 2005; hep-ex/0505100]

Table 30: $S_{b\to q\bar{q}s}$ and $C_{b\to q\bar{q}s}$.

Experiment		$-\eta S_{b\to q\bar{q}s}$	$C_{b\to q\bar{q}s}$
		ϕK^0	
BABAR	[188]	$0.50 \pm 0.25\,^{+0.07}_{-0.04}$	$0.00 \pm 0.23 \pm 0.05$
Belle	[189]	$0.06 \pm 0.33 \pm 0.09$	$-0.08 \pm 0.22 \pm 0.09$
Average		0.34 ± 0.20	-0.04 ± 0.17
Confidence level		0.30	0.81
		$\eta' K^0_S$	
BABAR	[190]	$0.30 \pm 0.14 \pm 0.02$	$-0.21 \pm 0.10 \pm 0.02$
Belle	[189]	$0.65 \pm 0.18 \pm 0.04$	$0.19 \pm 0.11 \pm 0.05$
Average		0.43 ± 0.11	-0.04 ± 0.08
Confidence level		$0.13\ (1.5\sigma)$	$0.011\ (2.5\sigma)$
		$f_0 K^0_S$	
BABAR	[191]	$0.95\,^{+0.23}_{-0.32} \pm 0.10$	$-0.24 \pm 0.31 \pm 0.15$
Belle	[189]	$-0.47 \pm 0.41 \pm 0.08$	$0.39 \pm 0.27 \pm 0.08$
Average		0.39 ± 0.26	0.14 ± 0.22
Confidence level		$0.008\ (2.7\sigma)$	$0.16\ (1.4\sigma)$
		$\pi^0 K^0_S$	
BABAR	[192]	$0.35\,^{+0.30}_{-0.33} \pm 0.04$	$0.06 \pm 0.18 \pm 0.03$
Belle	[189]	$0.30 \pm 0.59 \pm 0.11$	$0.12 \pm 0.20 \pm 0.07$
Average		$0.34\,^{+0.27}_{-0.29}$	0.09 ± 0.14
Confidence level		0.94	0.83
		ωK^0_S	
BABAR	[193]	$0.50^{+0.34}_{-0.38} \pm 0.02$	$-0.56\,^{+0.29}_{-0.27} \pm 0.03$
Belle	[189]	$0.75 \pm 0.64\,^{+0.13}_{-0.16}$	$-0.26 \pm 0.48 \pm 0.15$
Average		$0.55\,^{+0.30}_{-0.32}$	-0.48 ± 0.25
Confidence level		0.74	0.61
		$K^+K^-K^0_S$	
BABAR	[188]	$0.55 \pm 0.22 \pm 0.04 \pm 0.11$	$0.10 \pm 0.14 \pm 0.06$
Belle	[189]	$0.49 \pm 0.18 \pm 0.04\,^{+0.17}_{-0.00}$	$0.08 \pm 0.12 \pm 0.07$
Average		0.53 ± 0.17	0.09 ± 0.10
Confidence level		0.72	0.92
		$K^0_S K^0_S K^0_S$	
BABAR	[194]	$0.71\,^{+0.32}_{-0.38} \pm 0.04$	$-0.34\,^{+0.28}_{-0.25} \pm 0.05$
Belle	[195]	$-1.26 \pm 0.68 \pm 0.20$	$-0.54 \pm 0.34 \pm 0.09$
Average		0.26 ± 0.34	-0.41 ± 0.21
Confidence level		$0.014\ (2.5\sigma)$	
Average of all $b \to q\bar{q}s$		0.43 ± 0.07	-0.021 ± 0.049
Confidence level		$0.17\ (1.4\sigma)$	$0.15\ (1.4\sigma)$
Average including $b \to c\bar{c}s$		0.665 ± 0.0033	0.018 ± 0.025
Confidence level		$0.006 (2.7\sigma)$	$0.17\ (1.4\sigma)$

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Comparison of $\sin 2\beta(c\bar{c})$ and $\sin 2\beta(s\text{-penguins})$