EQUILIBRATION AND THERMALIZATION IN RELATIVISTIC HEAVY-ION COLLISIONS

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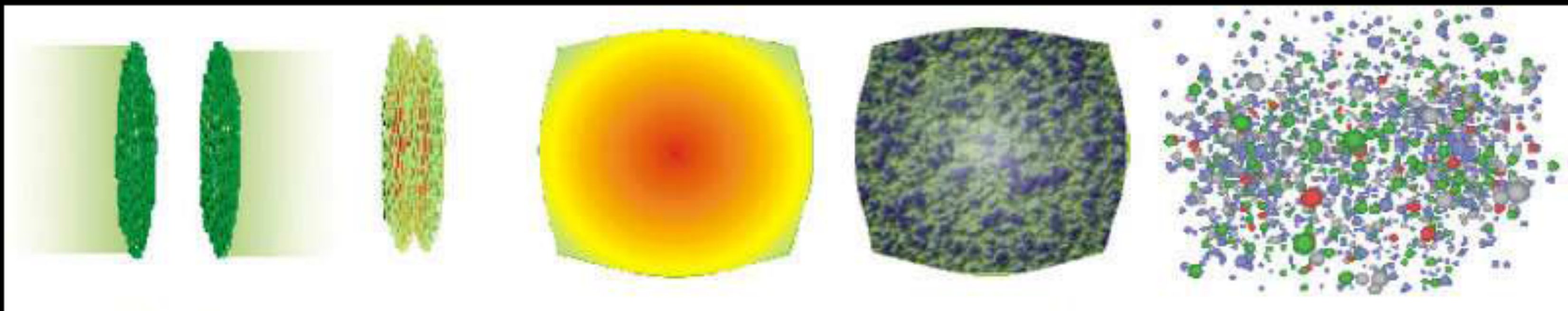
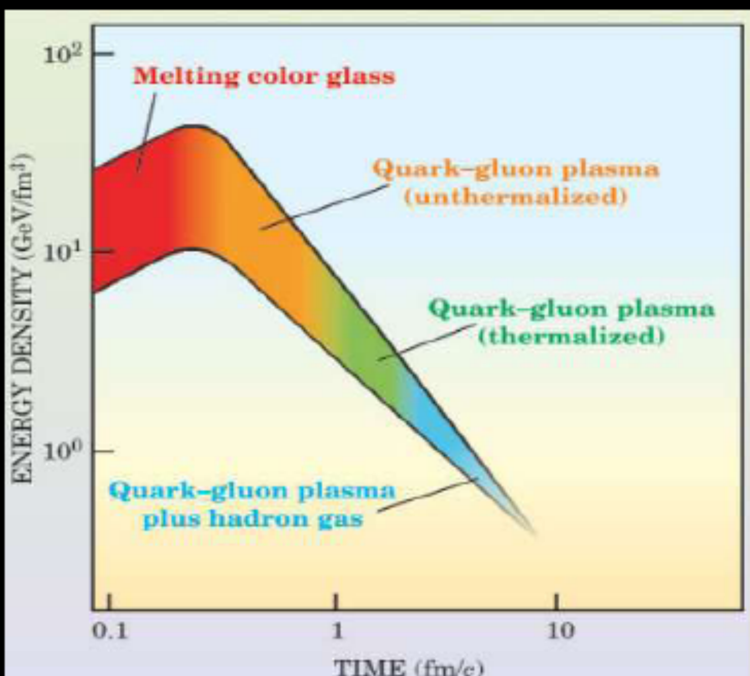
Stages of an ultrarelativistic heavy-ion collision



Initial state in colliding nuclei | Gluon-dominated preequilibrium | Quark-gluon matter | Hadron gas



- Initial stage: Hard partonic collisions described by pQCD. Partons materializing from gluon fields.
- Formation of a quark-gluon plasma, approaching equilibrium through multiple interactions. Initial temperature of several hundred MeV. Violent expansion and cooling.
- Hadronization when the plasma reaches $T_c \sim 170$ MeV, through parton fragmentation or quark coalescence.
- Chemical and finally kinetic freezeout. Only variables surviving to this stage are observable.

Phase diagram

SIS	2 AGeV	(FOPI, TAPS, HADES)
AGS	10 AGeV	(E896, E917, E910)
SPS	30-160 AGeV	(NA49, NA50, NA60, WA98…)
RHIC	7-200 GeV	(BRAHMS, PHENIX, PHOBOS, STAR)
LHC	2.76-5.1 TeV	(ALICE, CMS, ATLAS)
FAIR	30 AGeV	(CBM)
NICA	10 GeV	(MPD, SPD)

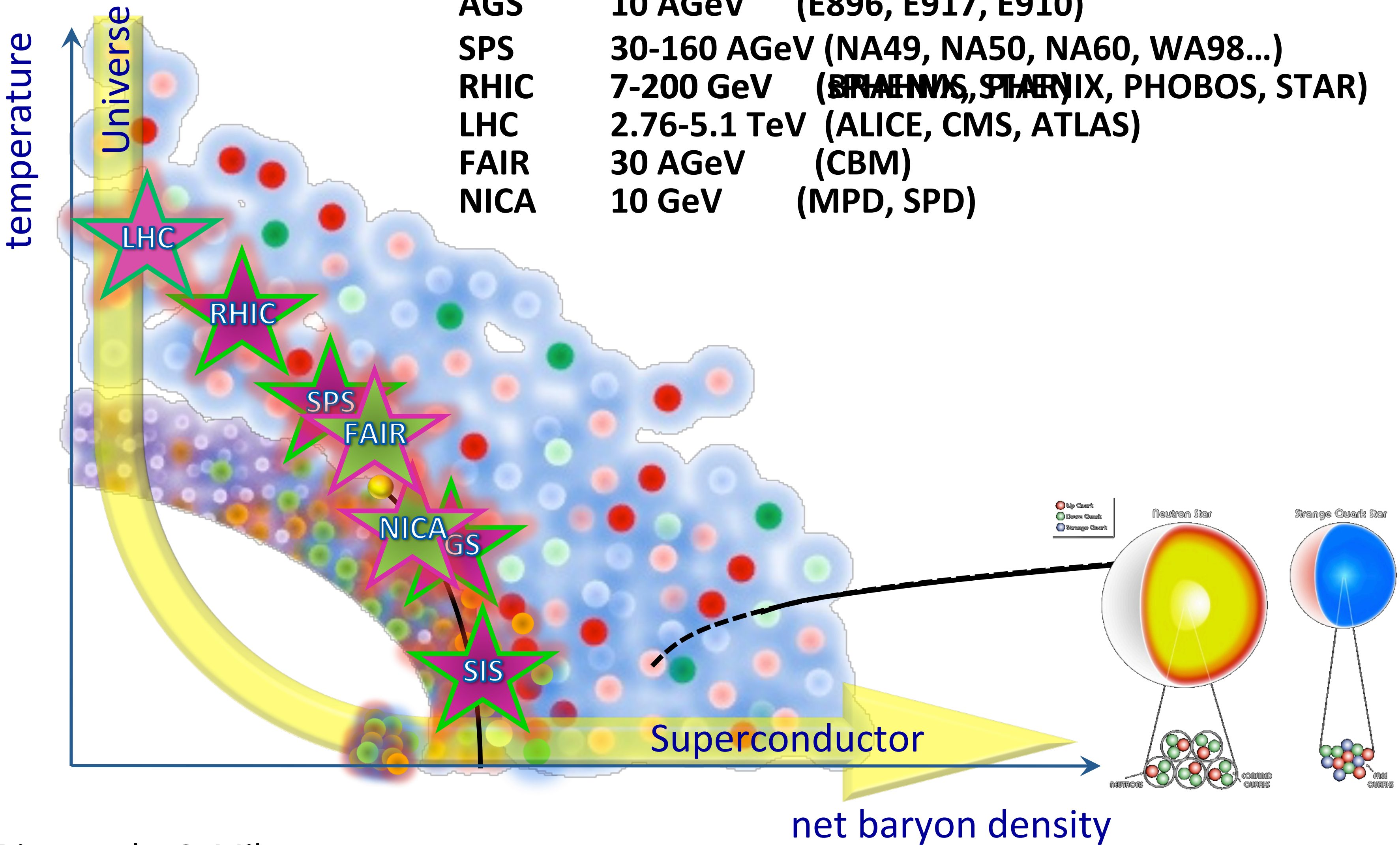
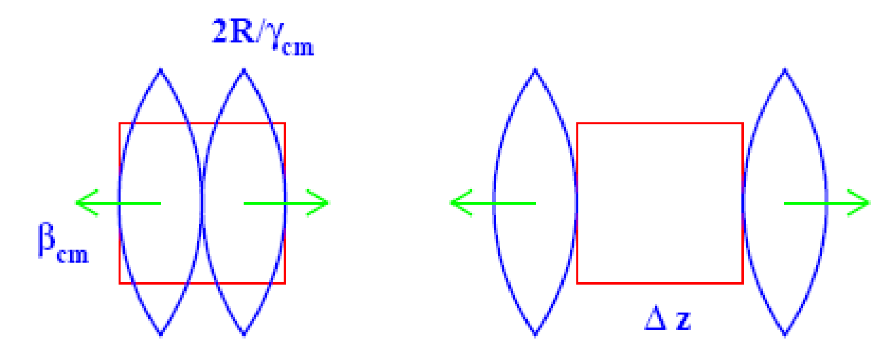


Diagram by S. Milov

Central cell:
Relaxation to equilibrium

Equilibration in the Central Cell



Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

L.Bravina et al., PLB 434 (1998) 379; JPG 25 (1999) 351

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Statistical model of ideal hadron gas

input values

output values

$$\varepsilon^{\mathrm{mic}} = \frac{1}{V}\sum_i E_i^{\mathrm{SM}}(T, \mu_{\mathrm{B}}, \mu_{\mathrm{S}}),$$

$$\rho_{\mathrm{B}}^{\mathrm{mic}} = \frac{1}{V}\sum_i B_i \cdot N_i^{\mathrm{SM}}(T, \mu_{\mathrm{B}}, \mu_{\mathrm{S}}),$$

$$\rho_{\mathrm{S}}^{\mathrm{mic}} = \frac{1}{V}\sum_i S_i \cdot N_i^{\mathrm{SM}}(T, \mu_{\mathrm{B}}, \mu_{\mathrm{S}}).$$

Multiplicity

Energy

Pressure

Entropy density

$$N_i^{\mathrm{SM}} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p, m_i)dp,$$

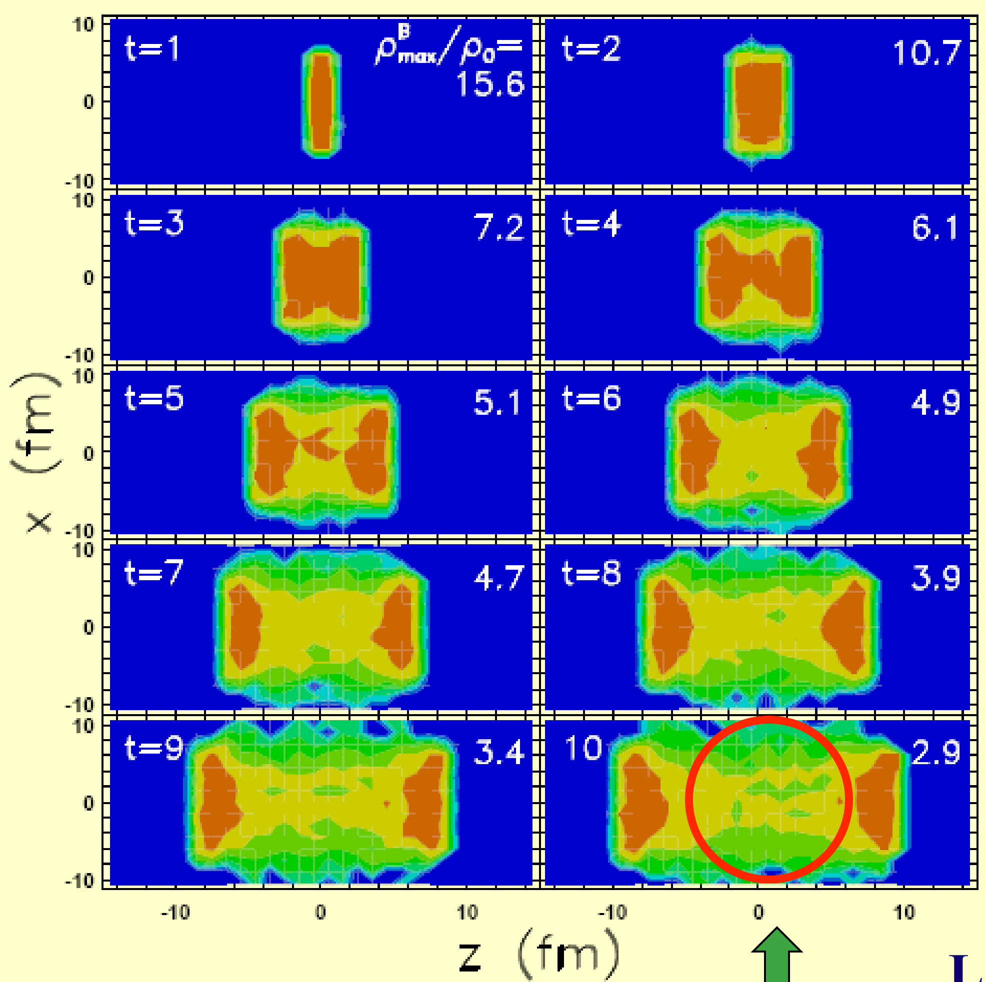
$$E_i^{\mathrm{SM}} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \sqrt{p^2 + m_i^2}\, f(p, m_i)dp$$

$$P^{\mathrm{SM}} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}}\, f(p, m_i)dp$$

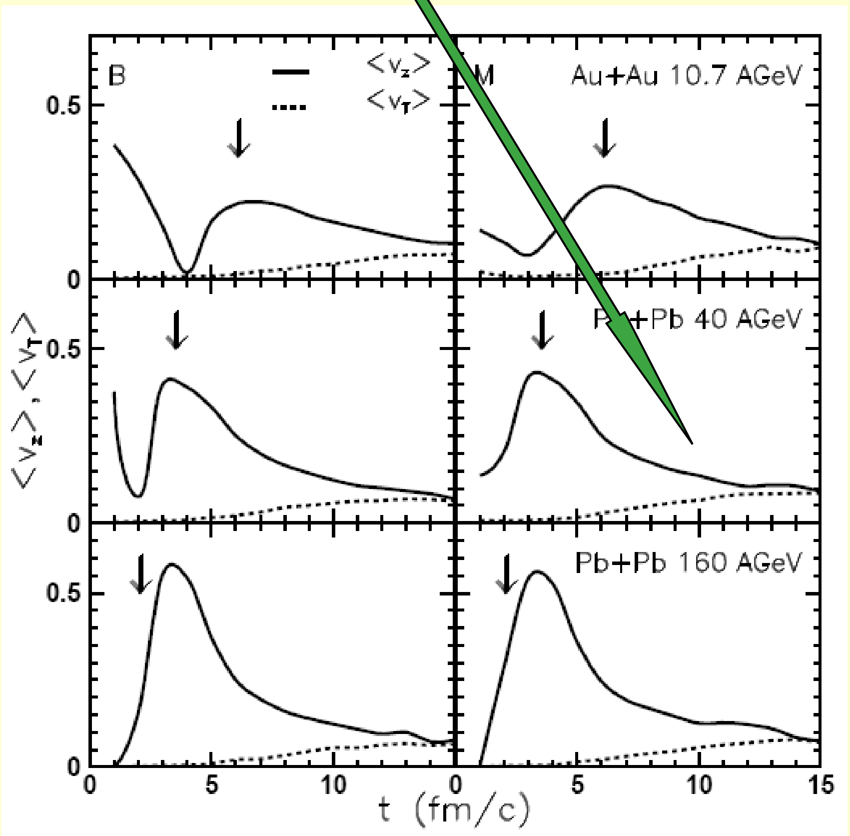
$$s^{\mathrm{SM}} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p, m_i)\,[\ln f(p, m_i) - 1]\, p^2 dp$$

Pre-equilibrium Stage

Homogeneity of baryon matter



Absence of flow



L.Bravina et al., PRC 60 (1999) 024904

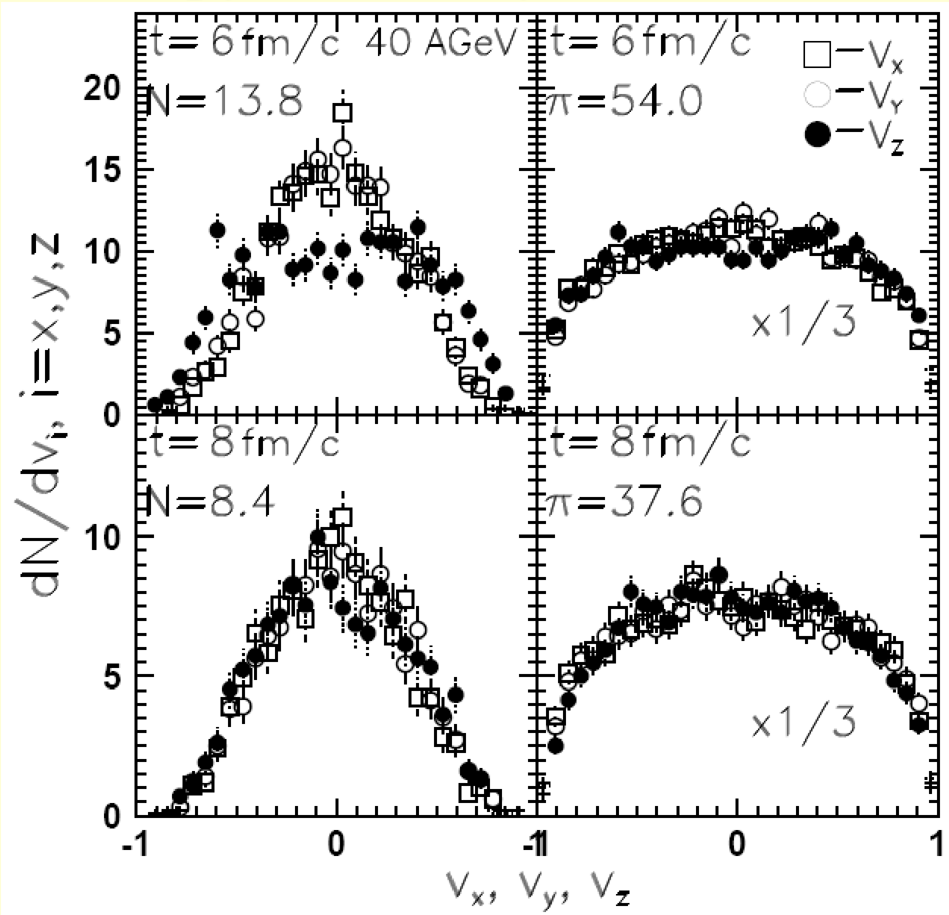
The local equilibrium in the central zone is quite possible

Models employed:

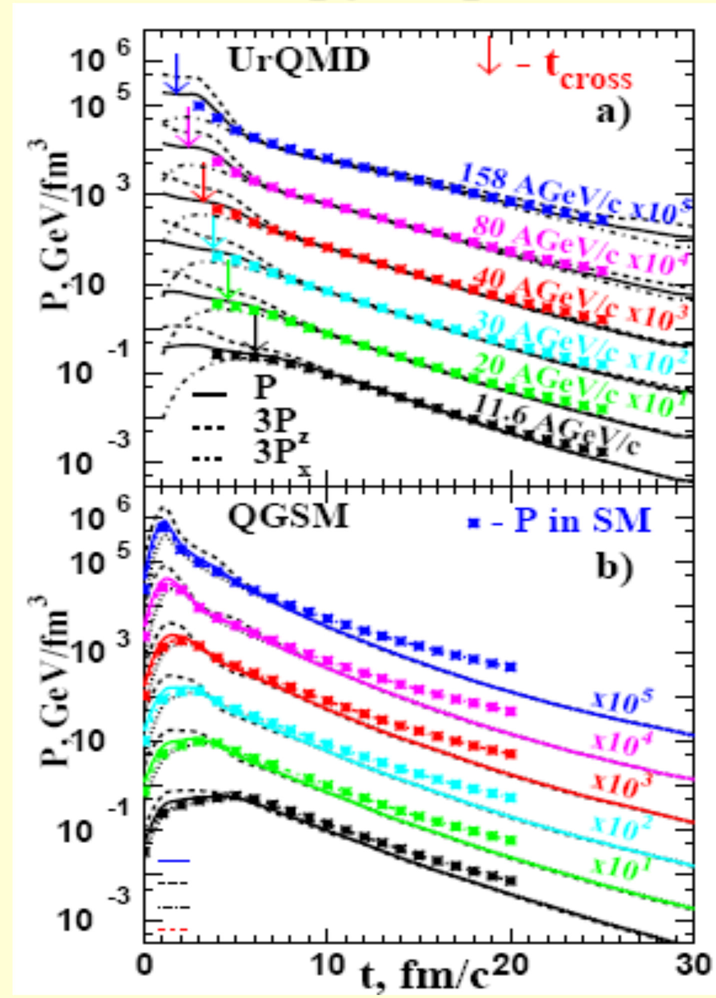
UrQMD,QGSM, also PHSD,GiBUU, 3-fluid hydro, SU(3)

Kinetic Equilibrium

Isotropy of velocity distributions



Isotropy of pressure

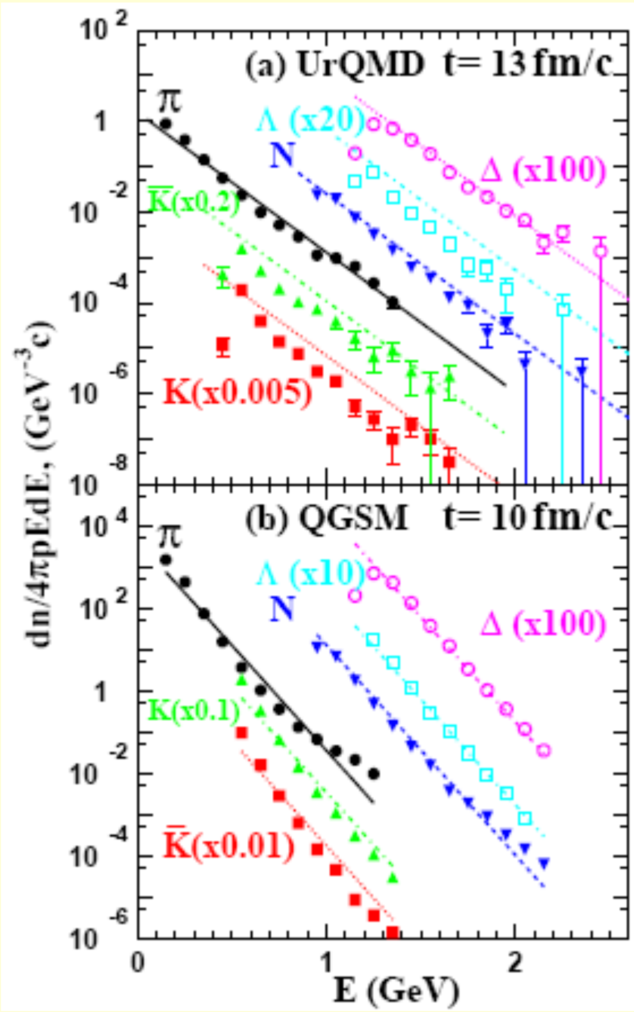


L.Bravina et al., PRC 78 (2008) 014907

Velocity distributions and pressure become isotropic for all energies

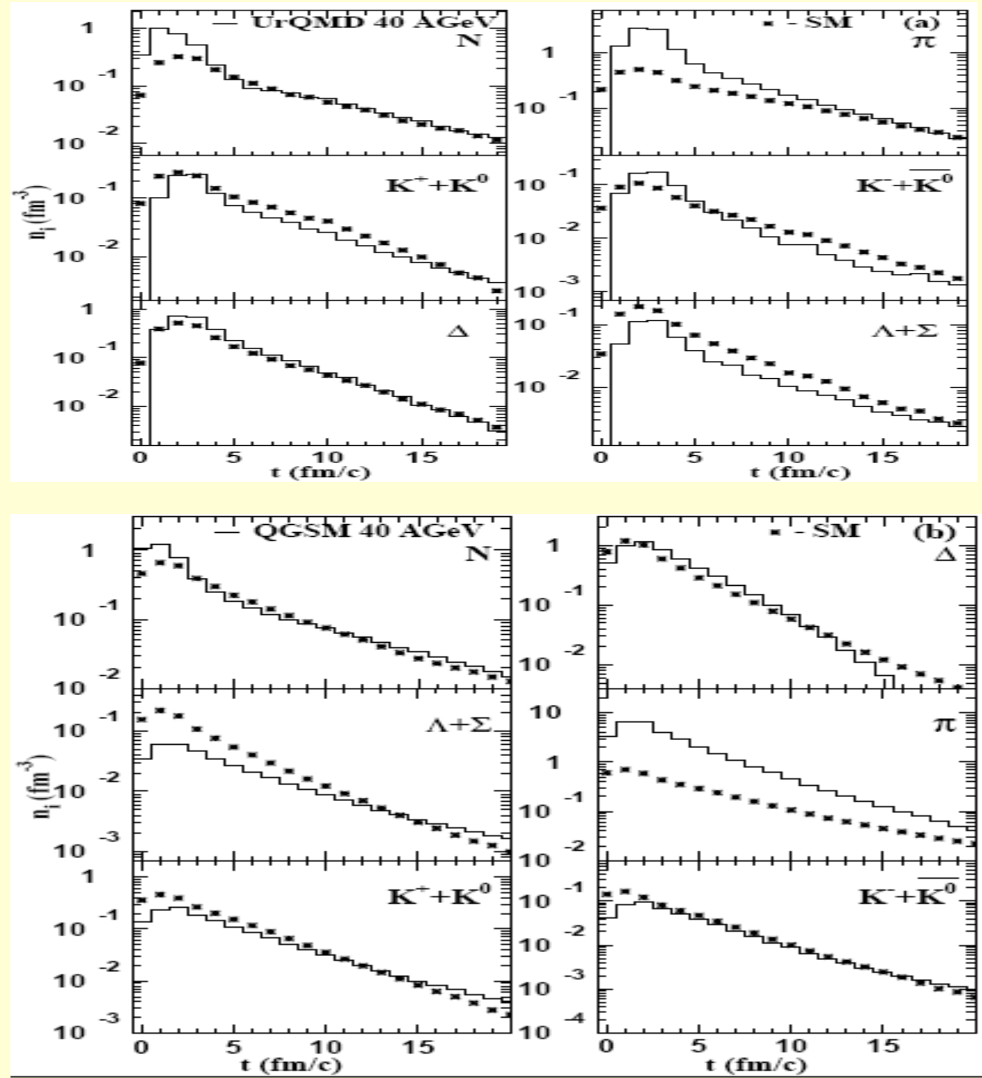
Thermal and Chemical Equilibrium

Boltzmann fit to the energy spectra



Particle yields

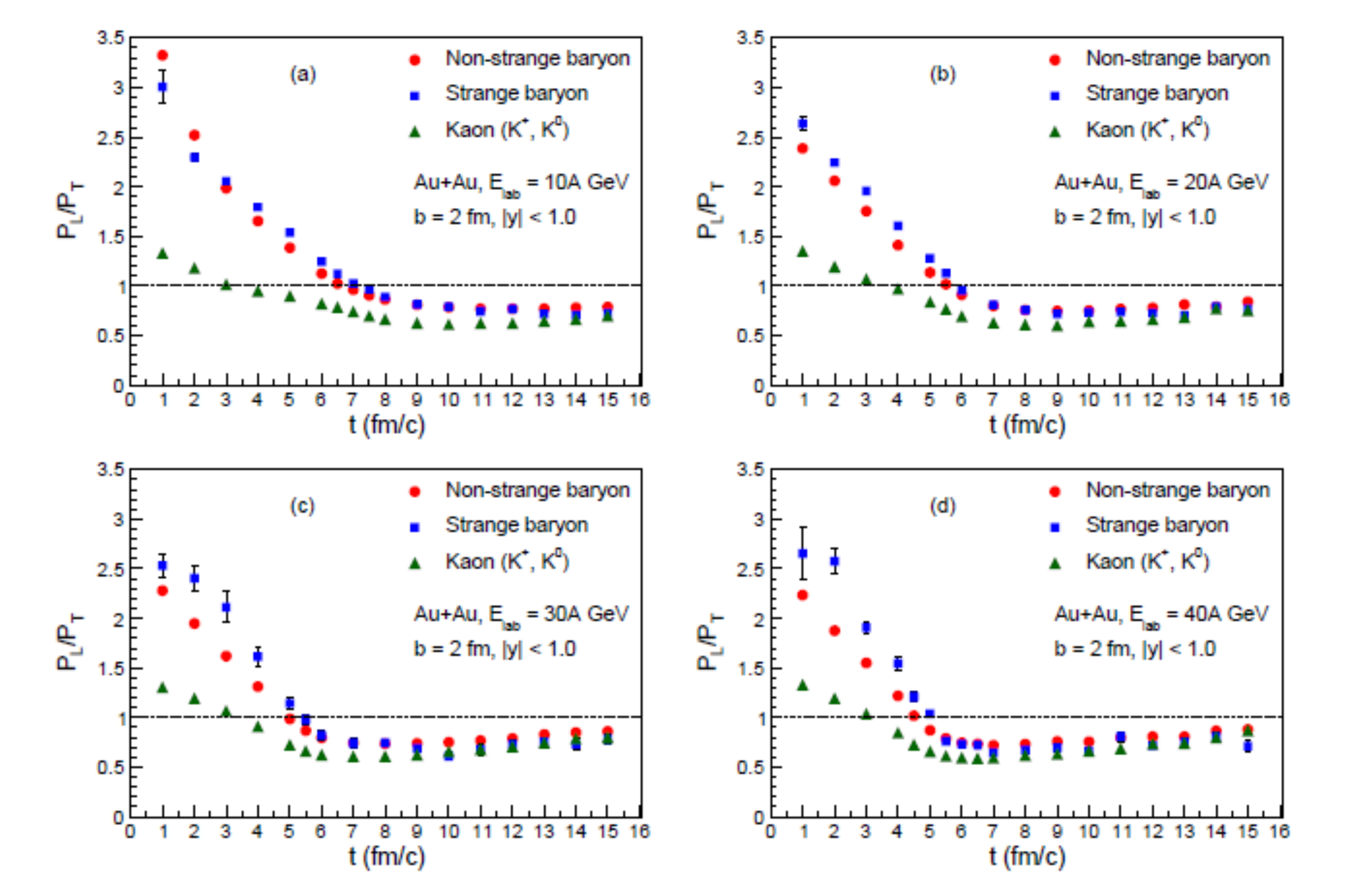
PRC 78 (2008) 014907



Thermal and chemical equilibrium seems to be reached

Other approaches

Isotropy of pressure for hadron species

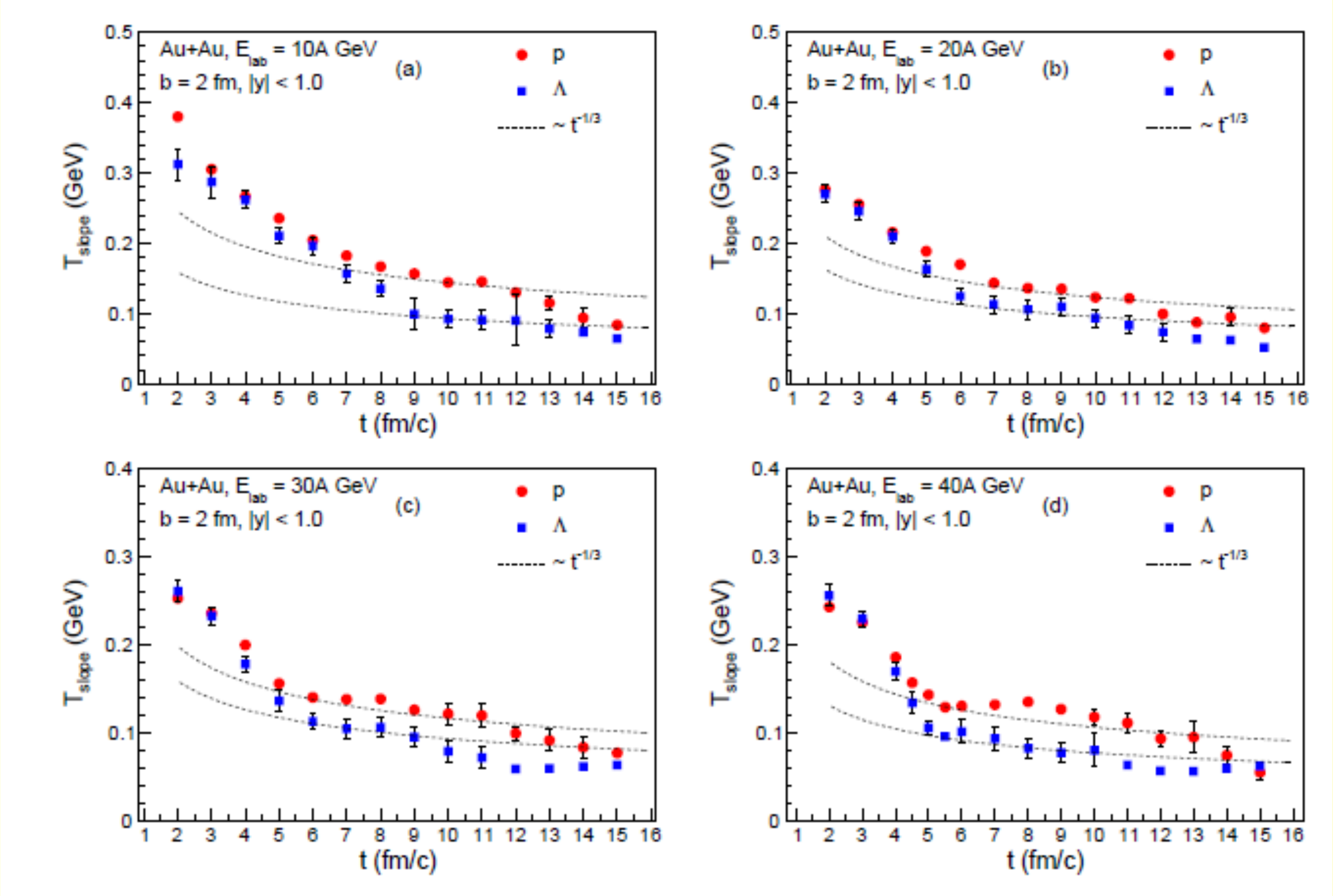


S. De et al., arXiv:1510.01456v2

UrQMD 3.3; Au+Au @ 10, 20, 30, 40 AGeV; central cell: 2x2x2 fm^3

Other approaches

Fit of inverse slope parameter T_{slope} to Bjorken model

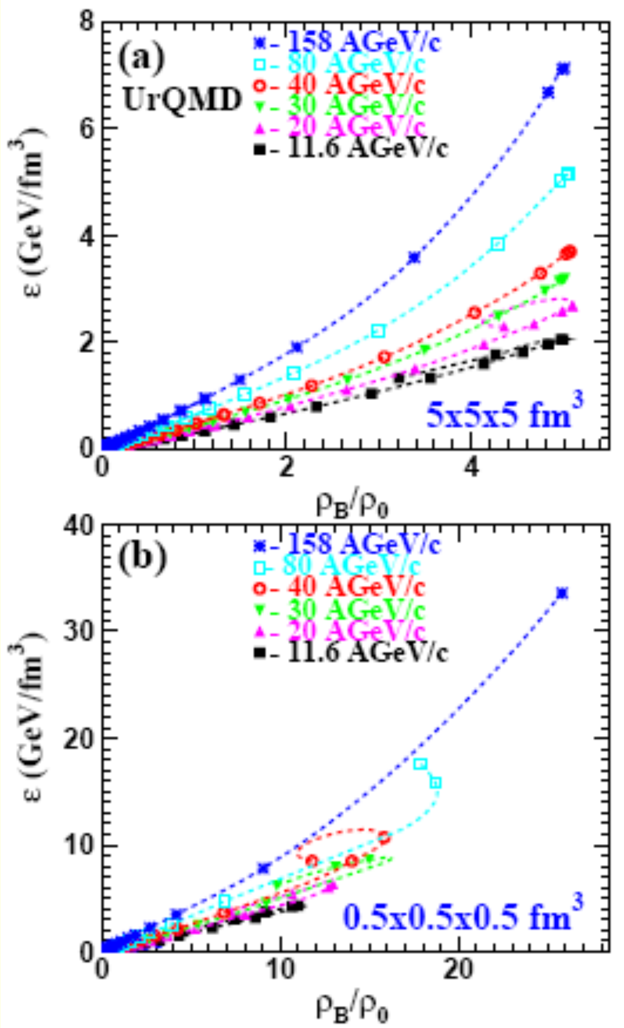


S. De et al., arXiv:1510.01456v2

Fit to $t^{-1/3}$ (1D, dotted line) and t^{-1} (3D, dashed line)

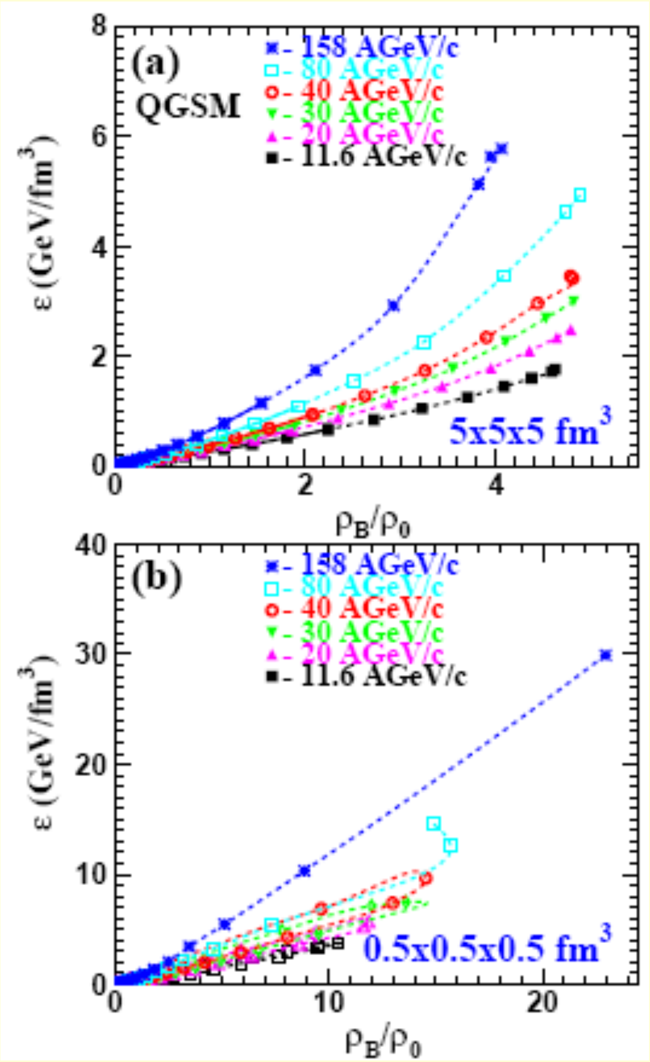
How dense can be the medium?

PRC 78 (2008) 014907



"Big" cell (V = 5x5x5 fm^3)

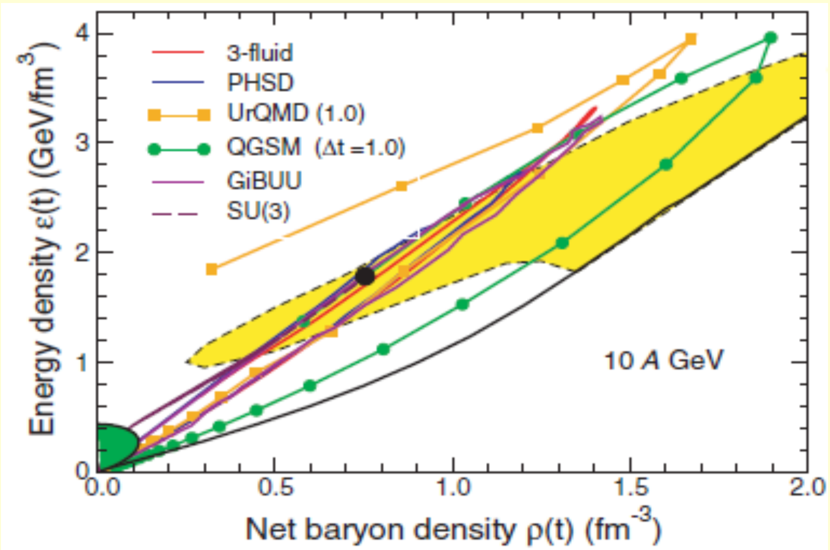
"Small" cell (V => 0)



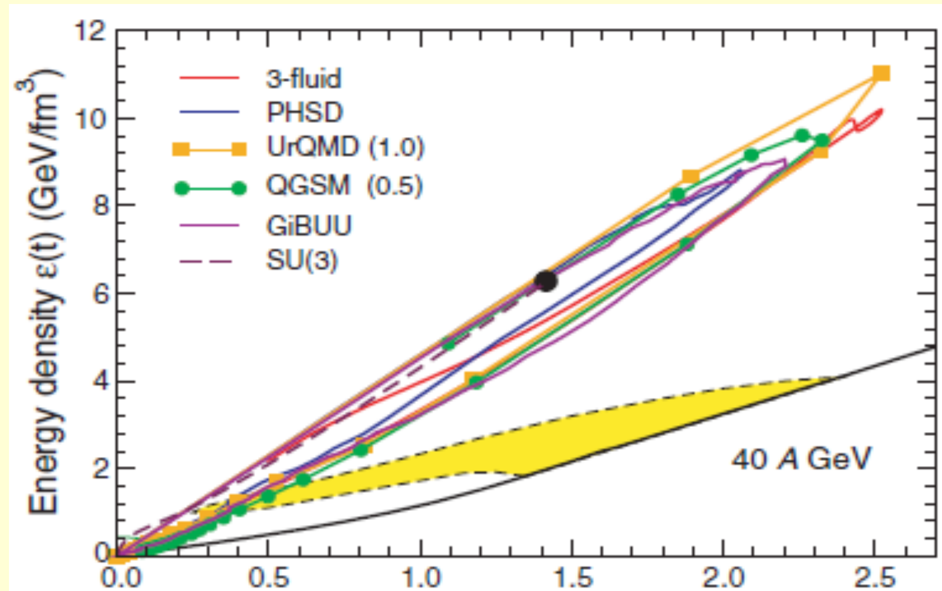
Dramatic differences at the non-equilibrium stage; after beginning of kinetic equilibrium the energy densities and the baryon densities are the same for "small" and "big" cell

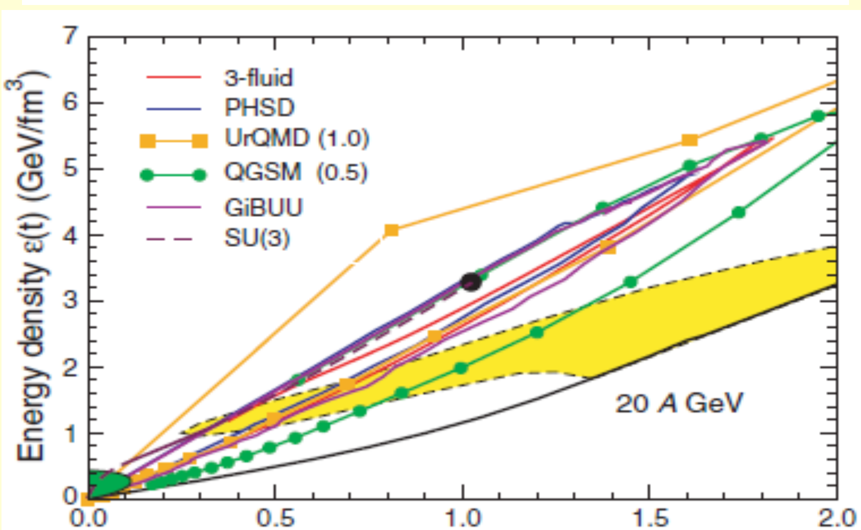
Comparison between models

The phase trajectories at the center of a head-on Au+Au collisions



I. Arsene et al., PRC 75 (2007) 034902





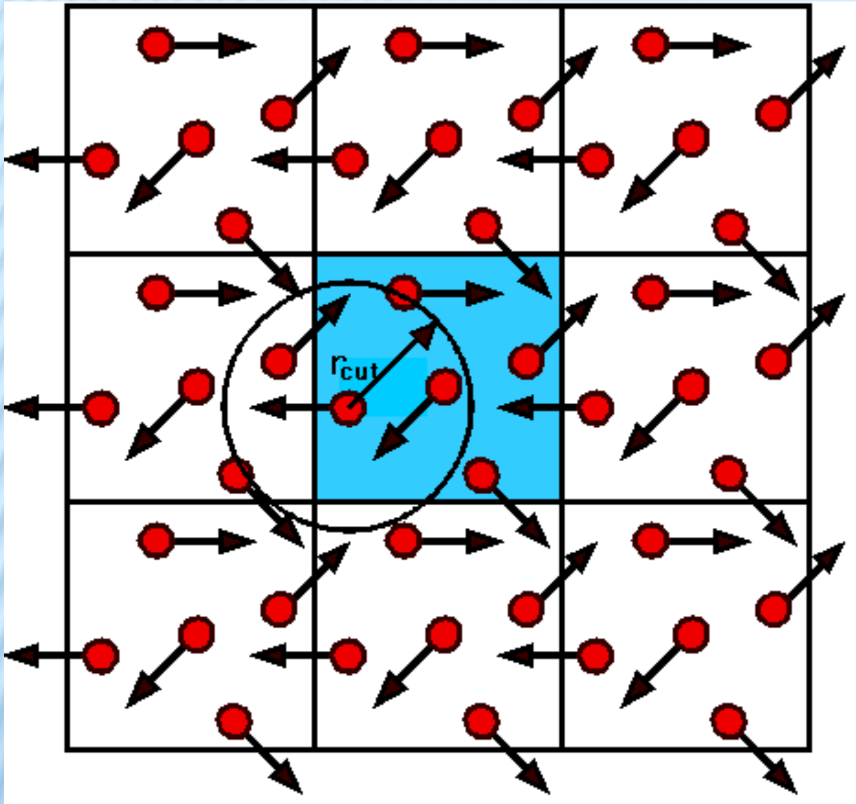
Green area : freeze-out region;
Yellow area : the phase coexistence region from schematic EOS that has a critical point at final density

Different models exhibit a large degree of mutual agreement

Infinite hadron gas:

a box with periodic boundary conditions

BOX WITH PERIODIC BOUNDARY CONDITIONS



M.Belkacem et al., PRC 58, 1727 (1998)

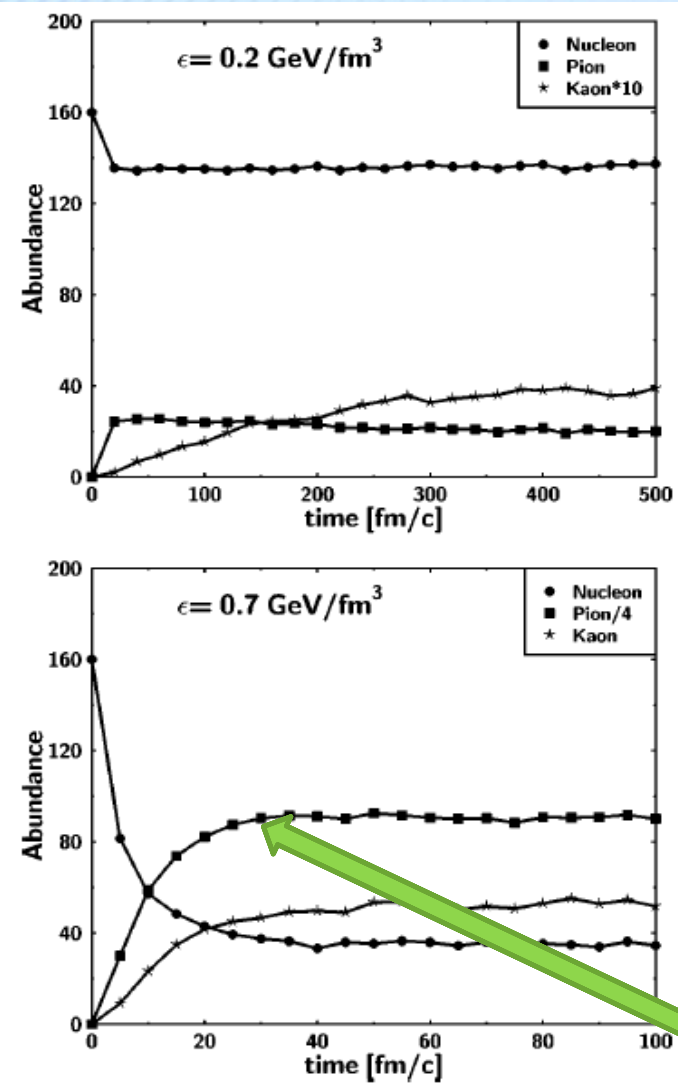
Model employed: UrQMD
55 different baryon species (N, Δ, hyperons and their resonances with $m \leq 2.25$ GeV/c^2),
32 different meson species (including resonances with $m \leq 2$ GeV/c^2) and their respective antistates.
For higher mass excitations a string mechanism is invoked.

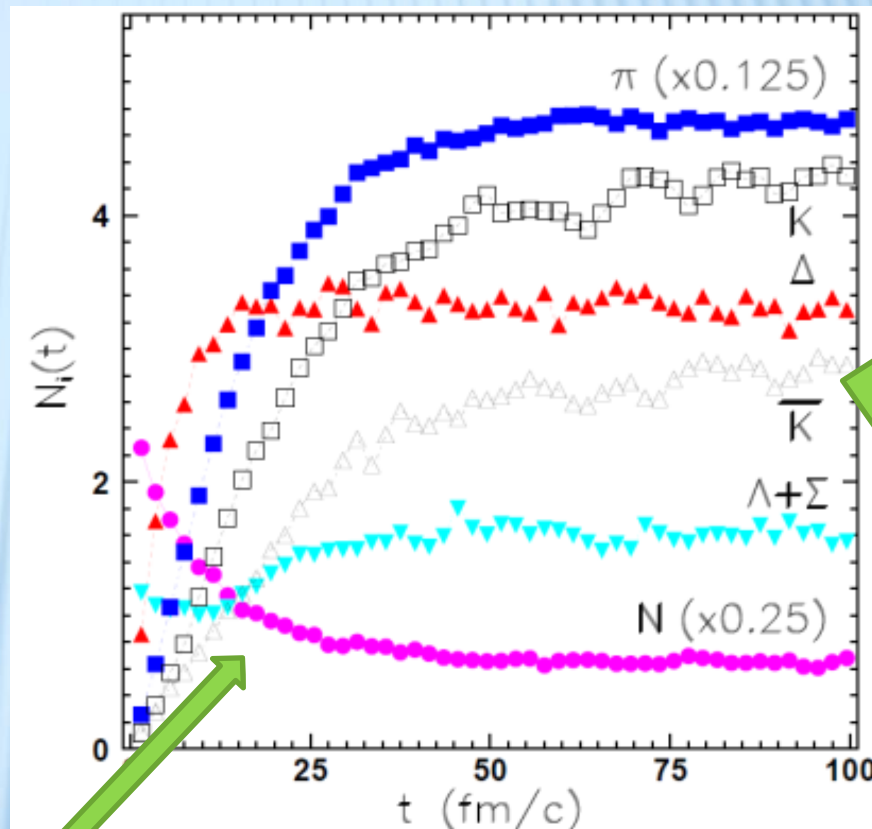
Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

BOX: PARTICLE ABUNDANCES

M.Belkacem et al., PRC 58, 1727 (1998)





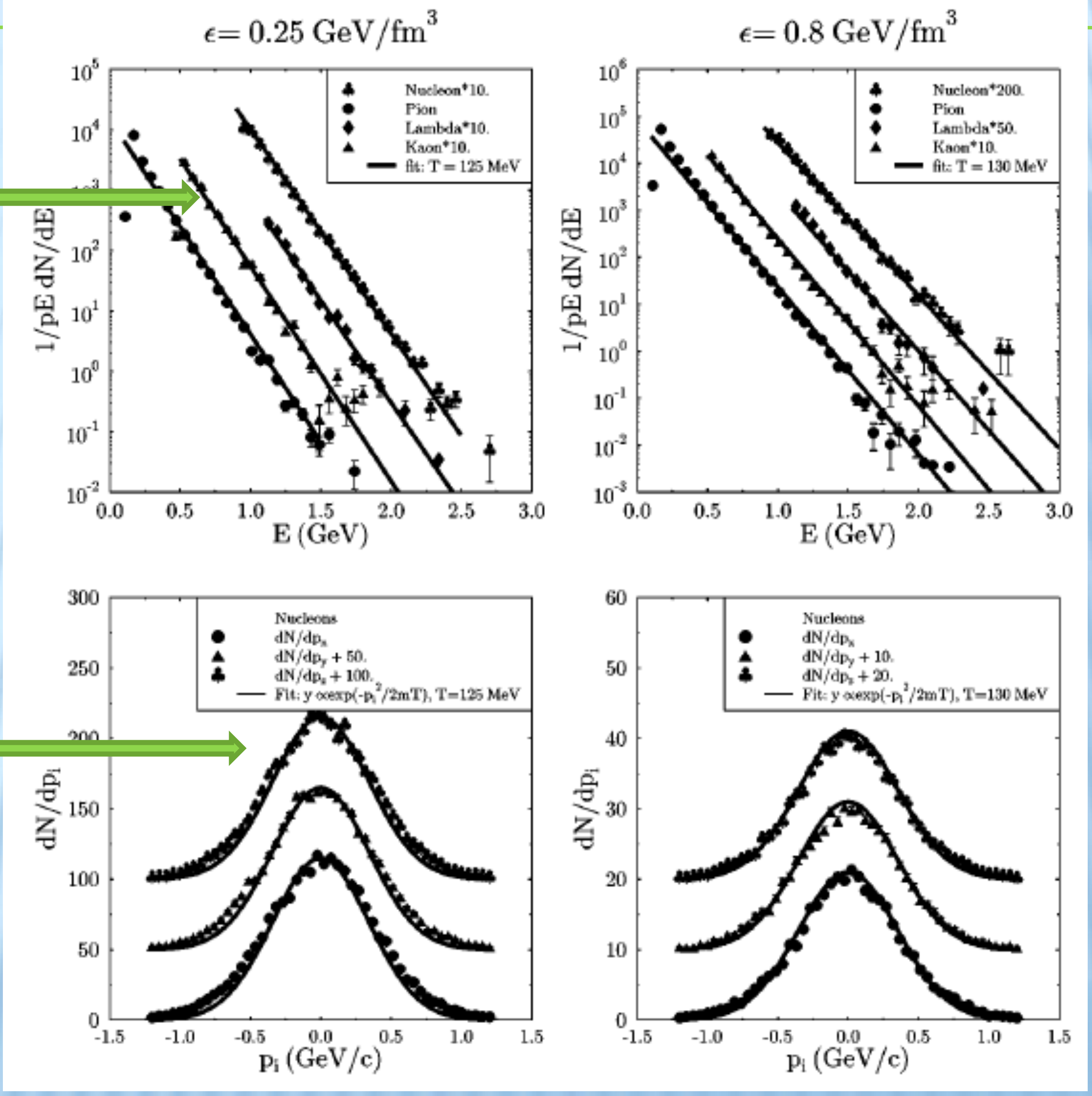
L.Bravina et al., PRC 62, 064906 (2000)

Saturation of yields after a certain time. Strange hadrons are saturated longer than others .

BOX: ENERGY SPECTRA AND MOMENTUM DISTRIBUTIONS

Fit to Boltzmann distributions ~exp(-E/T)

Fit to Gaussian distributions ~exp(-p^2/2mT)



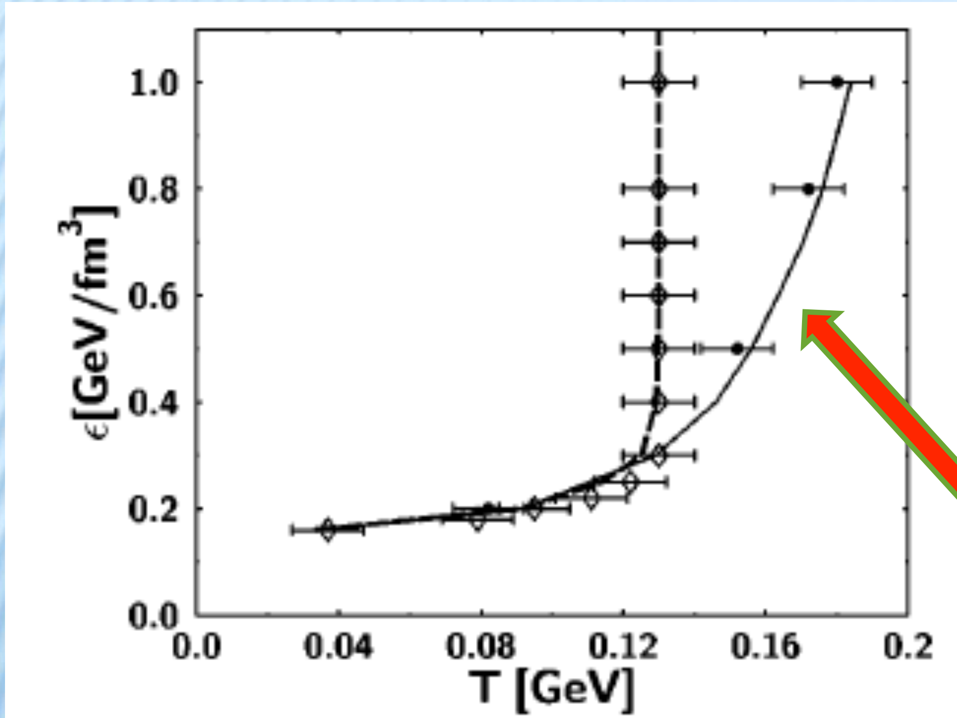
M.Belkacem et al., PRC 58, 1727 (1998)

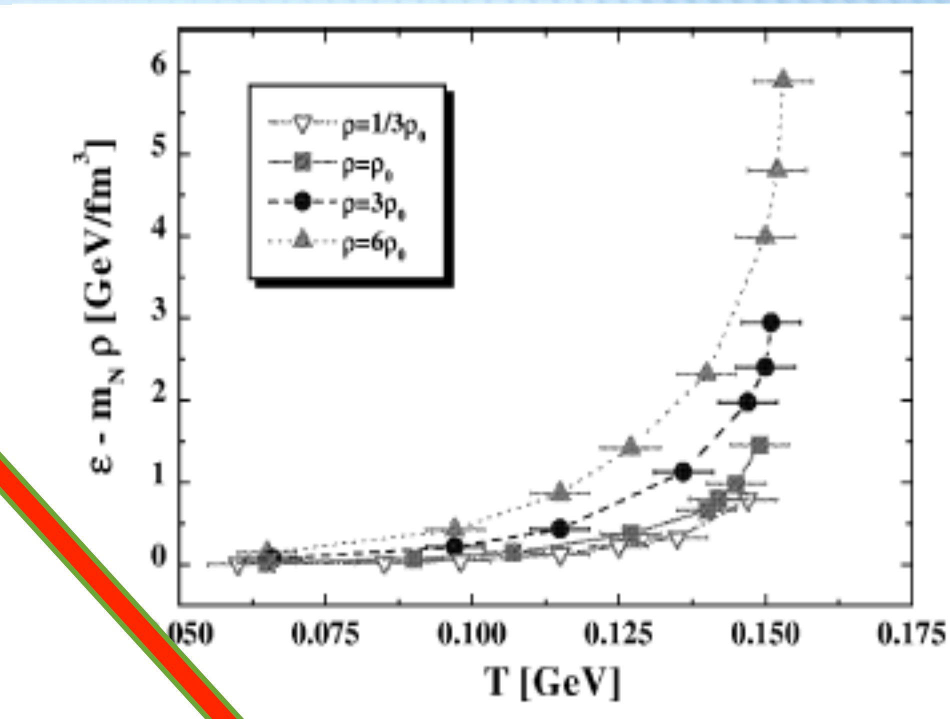
Nearly the same temperature and complete isotropy of dN/dp_T

BOX: HAGEDORN-LIKE LIMITING TEMPERATURE

M.Belkacem et al., PRC 58, 1727 (1998)

HSD





E.Bratkovskaya et al., NPA 675, 661 (2000)

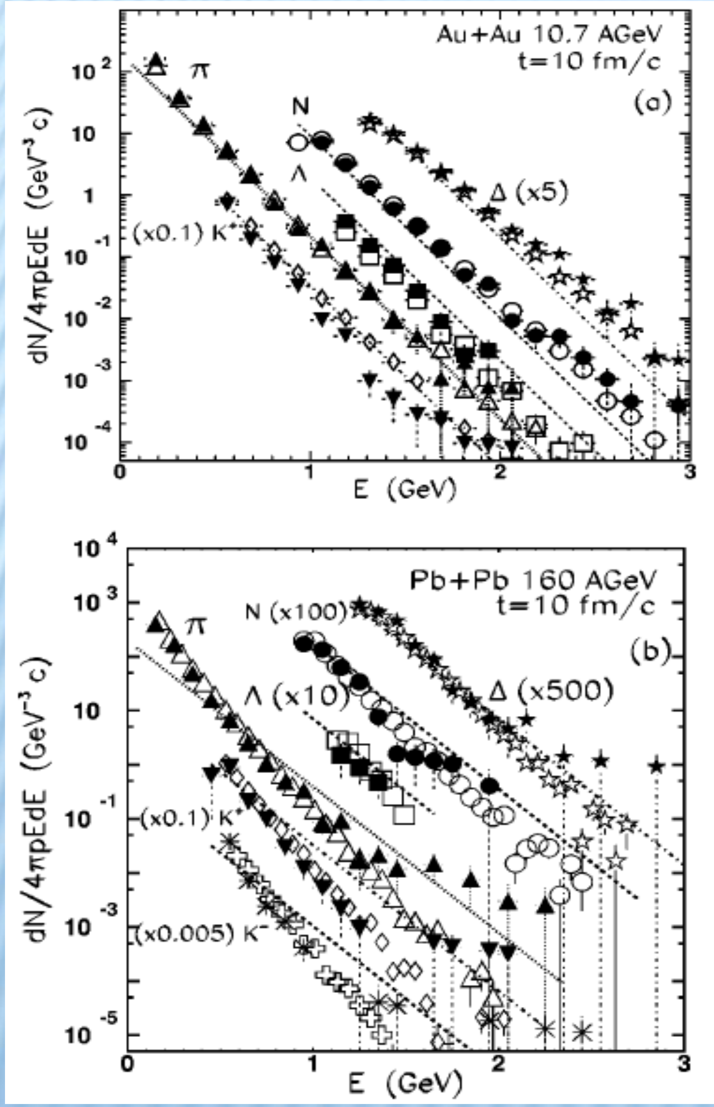
UrQMD

A rapid rise of T at low ε and saturation at high energy densities. Saturation temperature depends on number of resonances in the model. W/o strings and many-N decays – no limiting T is observed.

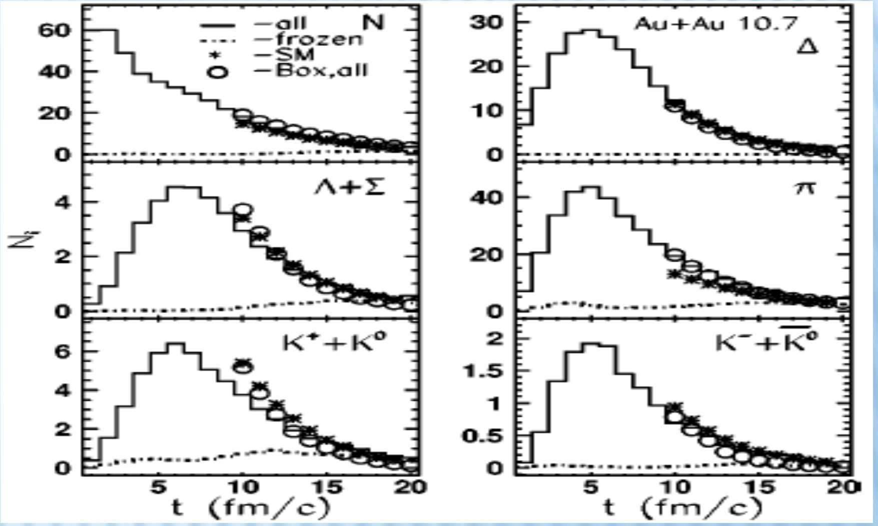
Comparison of cell and box results

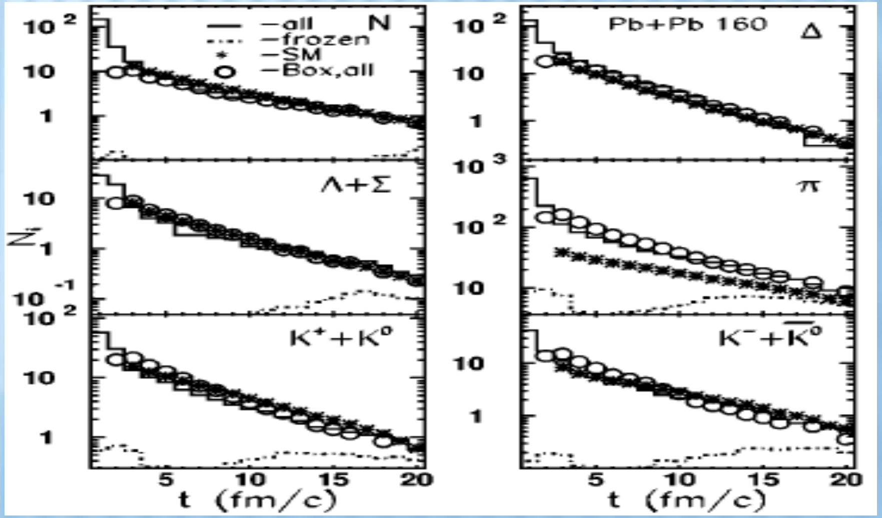
THERMAL AND CHEMICAL EQUILIBRIUM

Boltzmann fit to the energy spectra **Hadron yields**



L.Bravina et al., PRC 62, 064906 (2000)

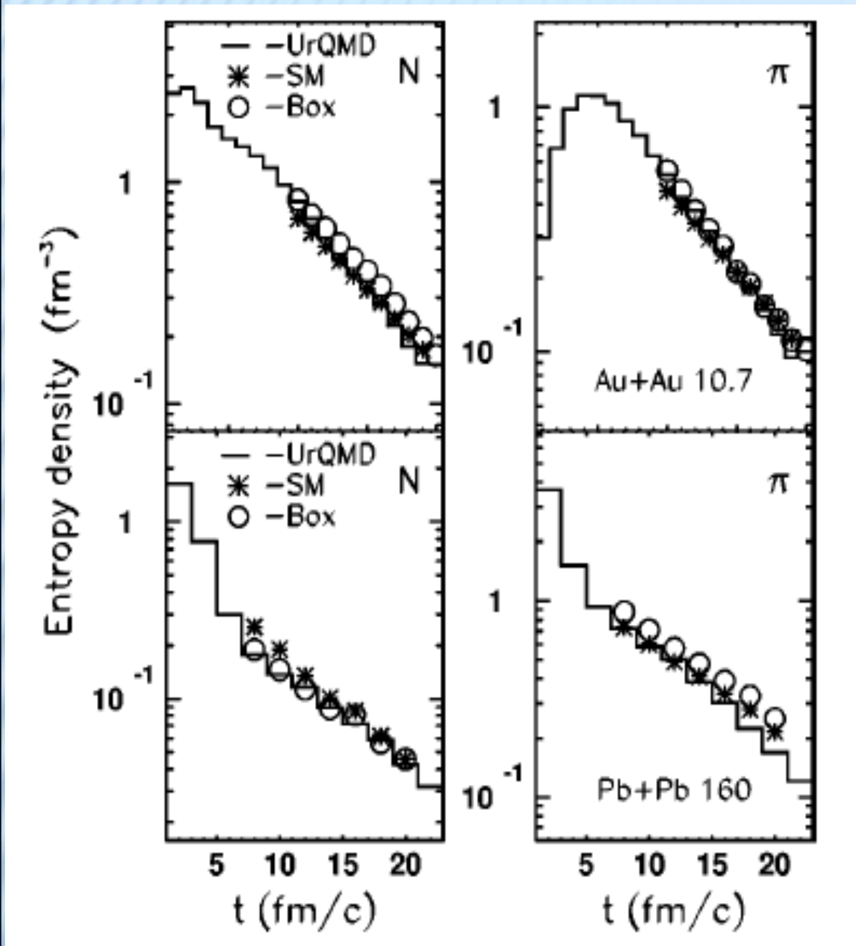




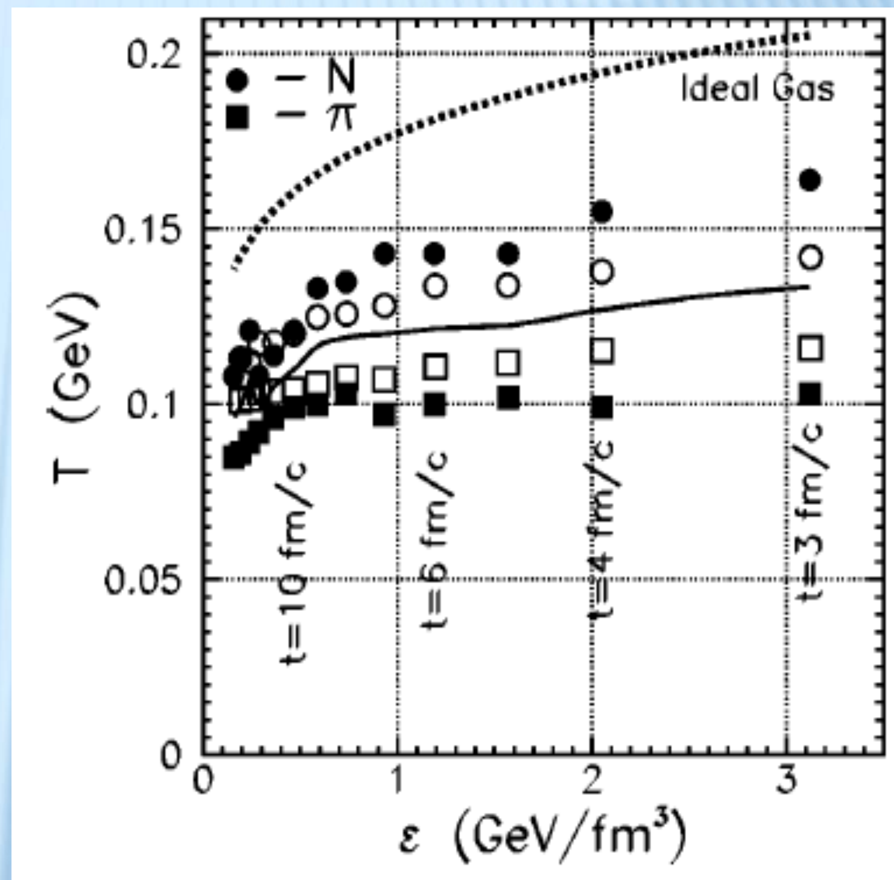
Box calculations are on the top of the cell results

THERMAL AND CHEMICAL EQUILIBRIUM

Partial entropy densities **T vs ε**



L.Bravina et al., PRC 62, 064906 (2000)



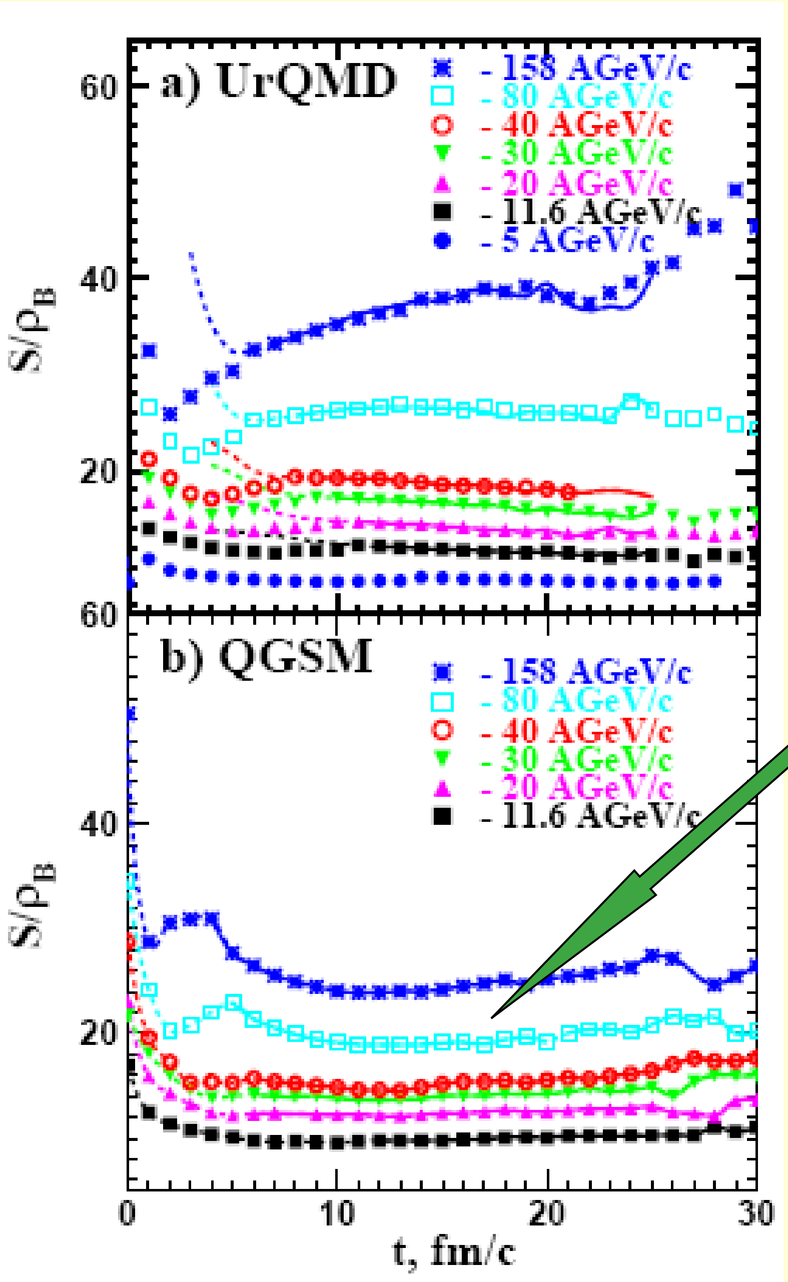
Note the difference between T_N and T_π

Box calculations are on the top of the cell results

Equation of State

T vs. energy,
etc

Isentropic expansion

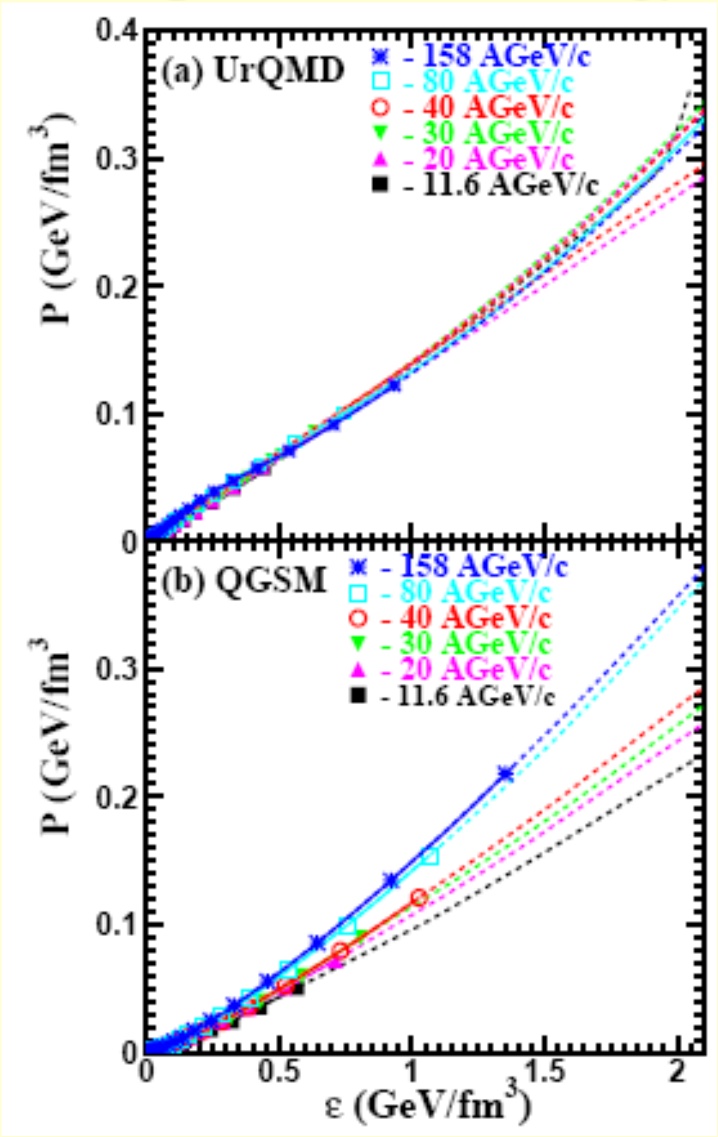


Expansion proceeds isentropically (with constant entropy per baryon). This result supports application of hydrodynamics

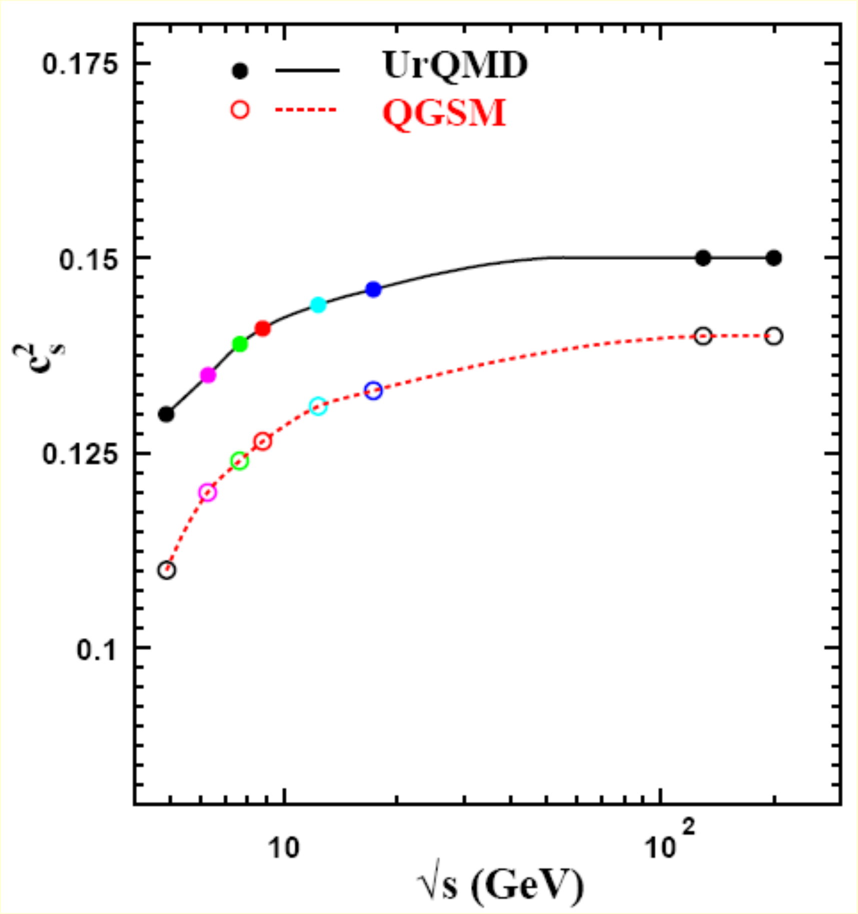
$$s/\rho_B = \text{const} = 12(\text{AGS}),\ 20(40),\ 38(\text{SPS})$$

Equation of State in the cell

pressure vs. energy



sound velocity

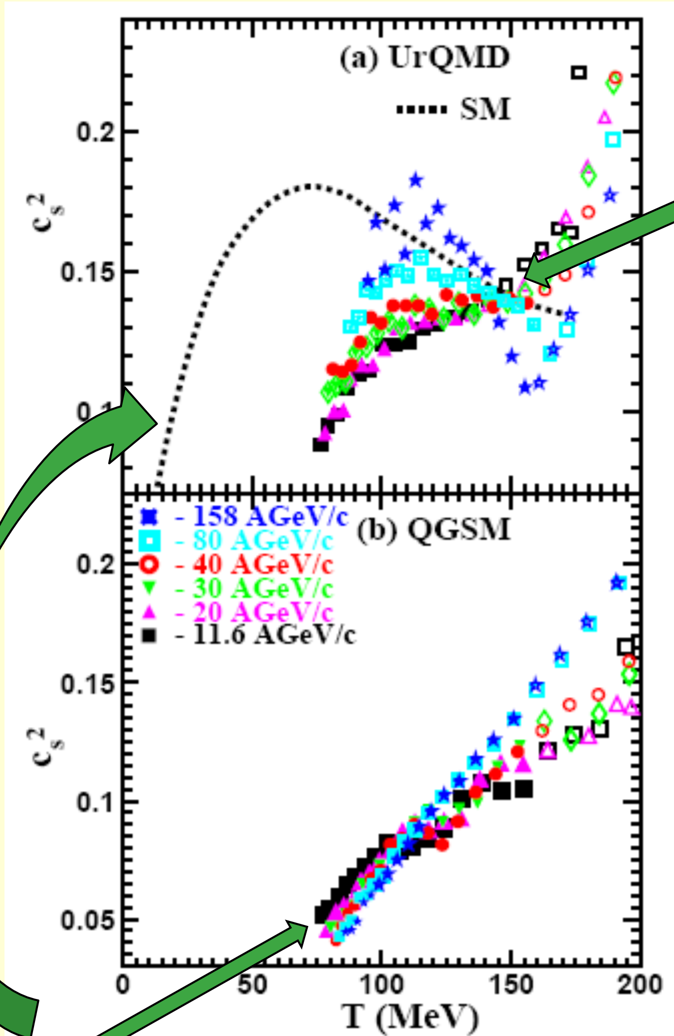


$$P/\varepsilon = 0.13(\mathrm{AGS}),\ 0.14(40),\ 0.146(\mathrm{SPS}),\ 0.15(\mathrm{RHIC})$$

Equation of State:
comparison with Hagedorn model

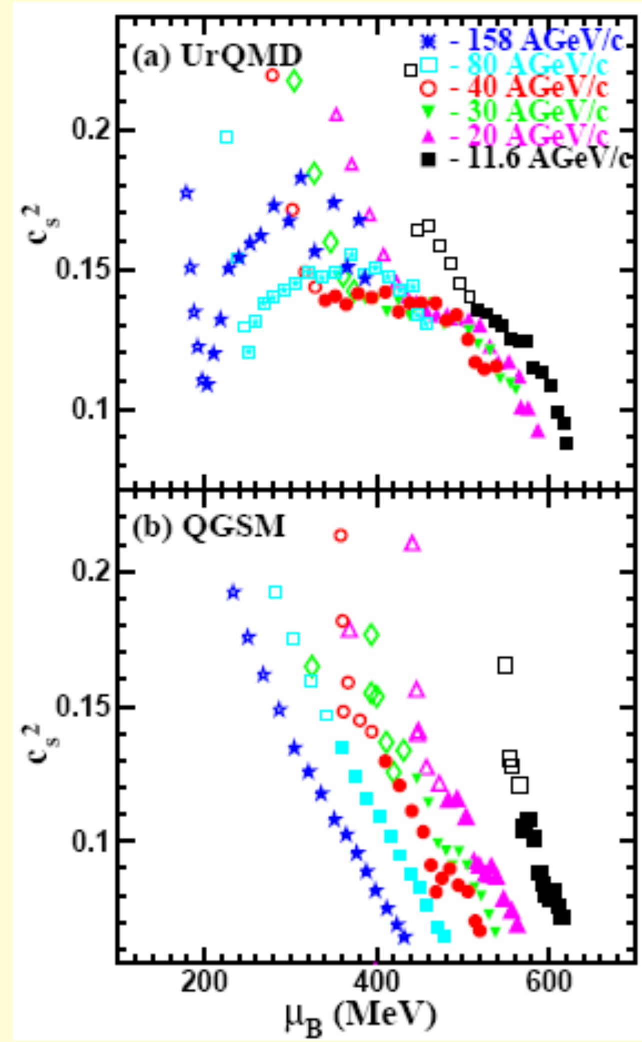
sound velocity vs. T and chemical potential

M. Chojnacki et al., PRC 71 (2005) 044902



Heavy resonances

Big difference between models with and w/o heavy resonances



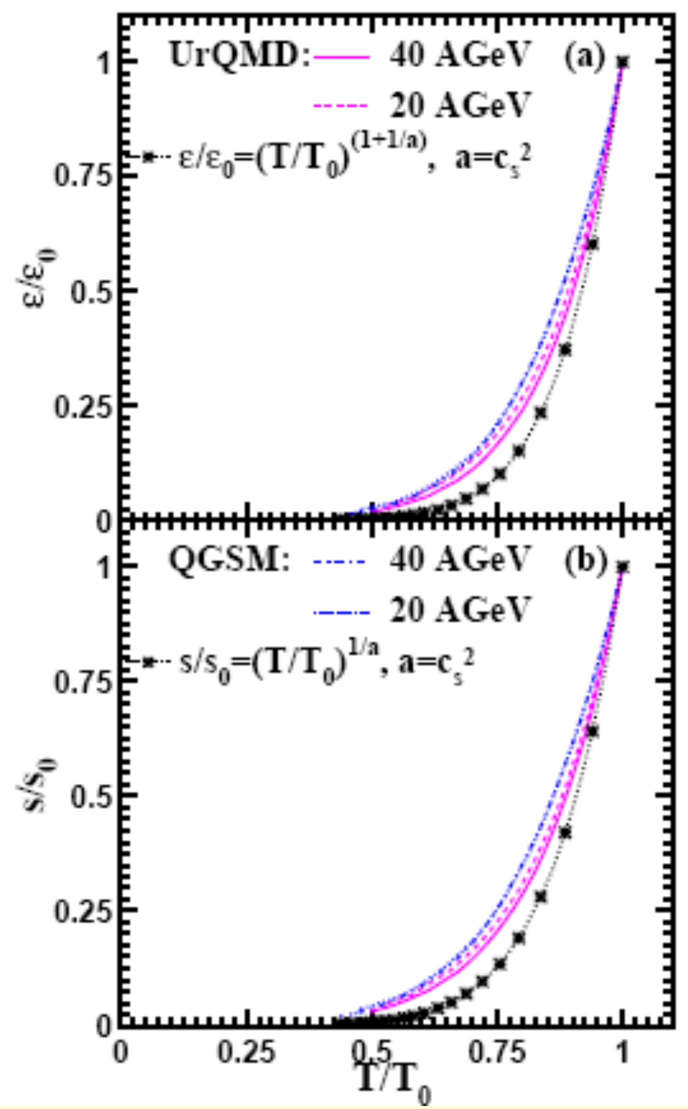
L.B. et al., PRC 78 (2008) 014907

Still sonic velocity drops faster than in Hagedorn model . Non-zero chemical potential ?

The difference increases with bombarding energy

Equation of State:

energy and entropy densities vs. T



$$dP = a\,d\varepsilon$$
$$d\varepsilon = T\,ds$$
$$dP = s\,dT$$

Zero chem. potential

$$\frac{\varepsilon}{\varepsilon_0} = \left(\frac{T}{T_0}\right)^{\frac{1+a}{a}}$$
$$\frac{s}{s_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{a}} \qquad (1)$$

$$dP = a\,d\varepsilon$$
$$d\varepsilon = T\,ds + \mu\,d\rho$$
$$dP = s\,dT + \rho\,d\mu$$
$$ds = b\,d\rho$$

Non-zero chem. potential

$$\frac{\varepsilon}{\varepsilon_0} = \left(\frac{b\,T+\mu}{b\,T_0+\mu_0}\right)^{\frac{a+1}{a}}$$
$$\frac{s}{s_0} = \left(\frac{b\,T+\mu}{b\,T_0+\mu_0}\right)^{\frac{1}{a}} \qquad (2)$$

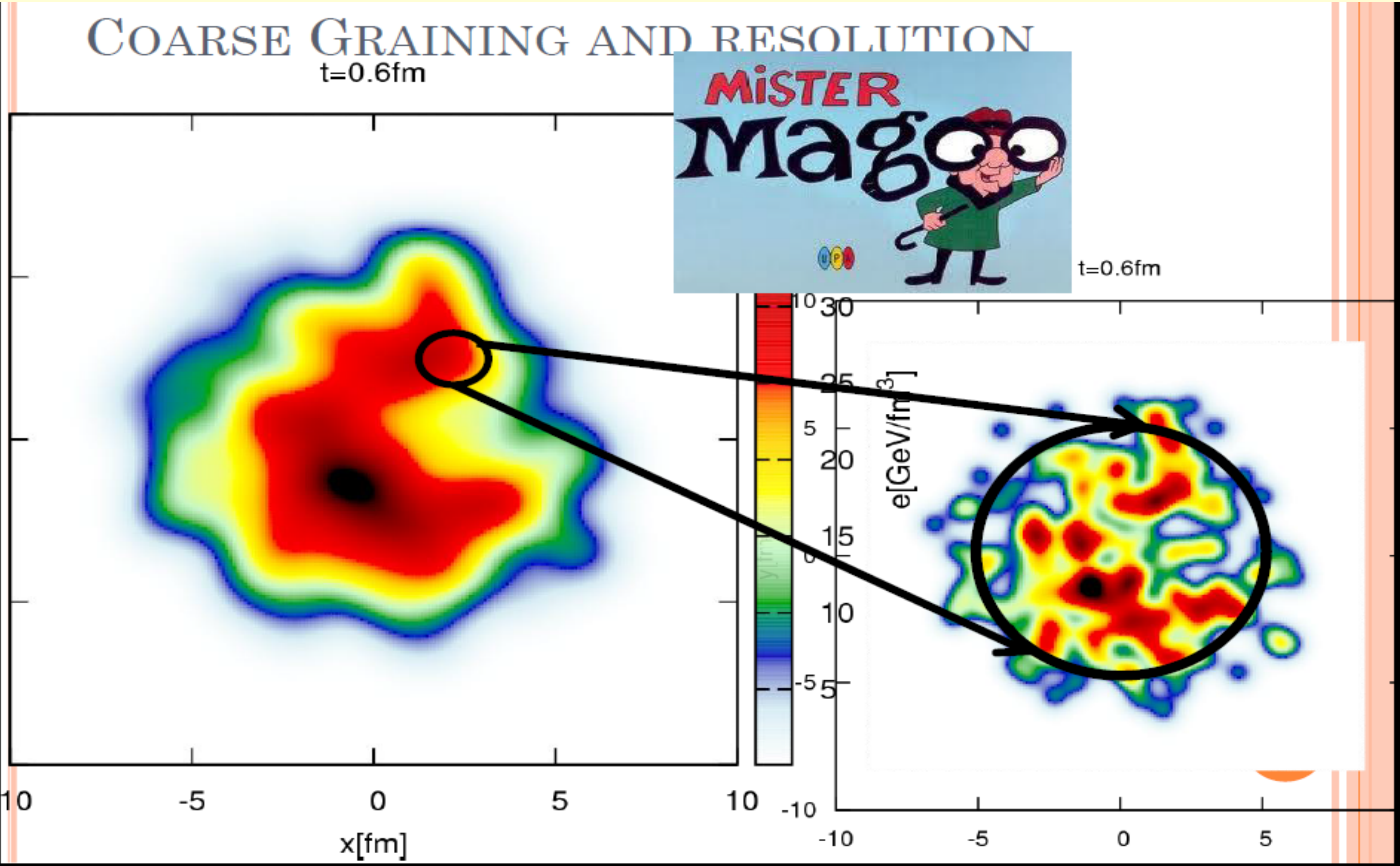
No difference between the models

If $\mu = c\,T$ **then (2) is transformed to (1)**

Modification of the analysis (small cells or coarse-graining)

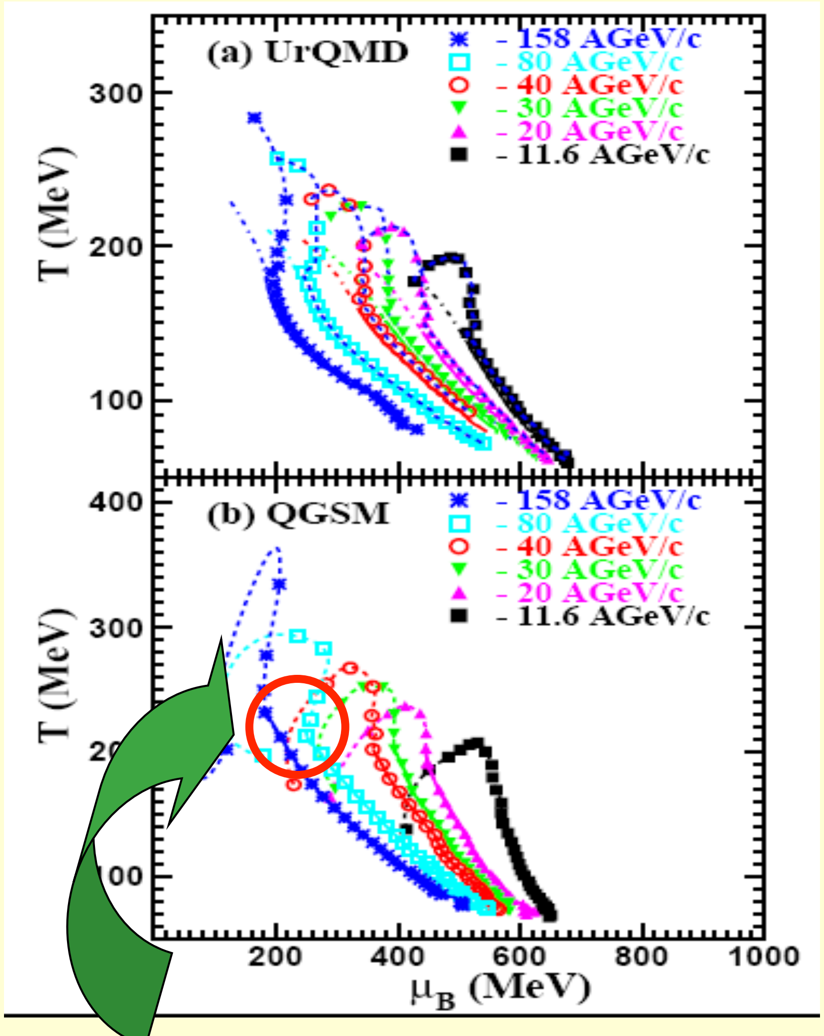
Example of coarse-graining

T. Kodama, conf. NeD/TURIC'2012

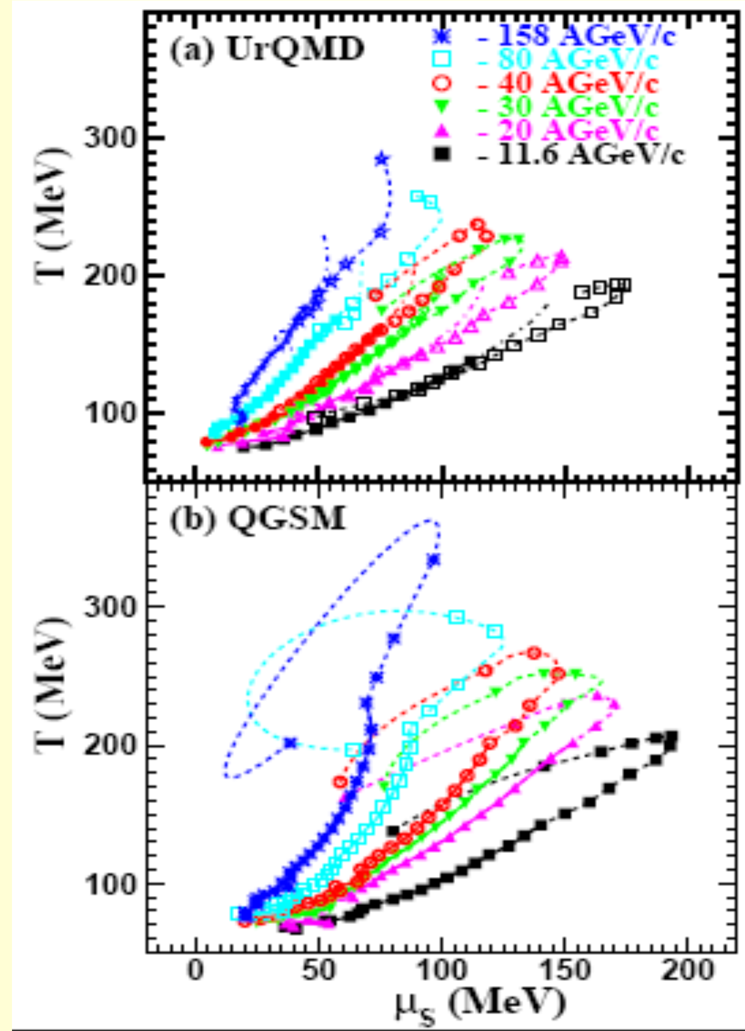


EOS in the cell: observation of knee

temperature vs. chemical potentials



L.B. et al., PRC 78 (2008) 014907

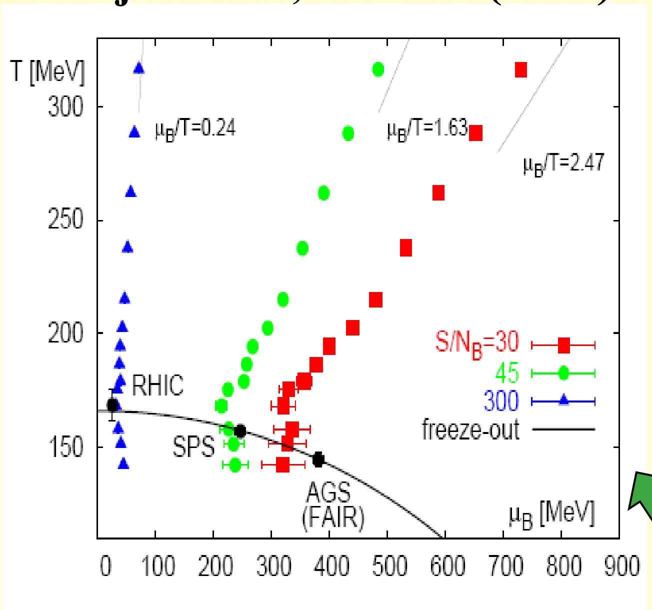


The "knee" in MC simulations appears at chemical FO

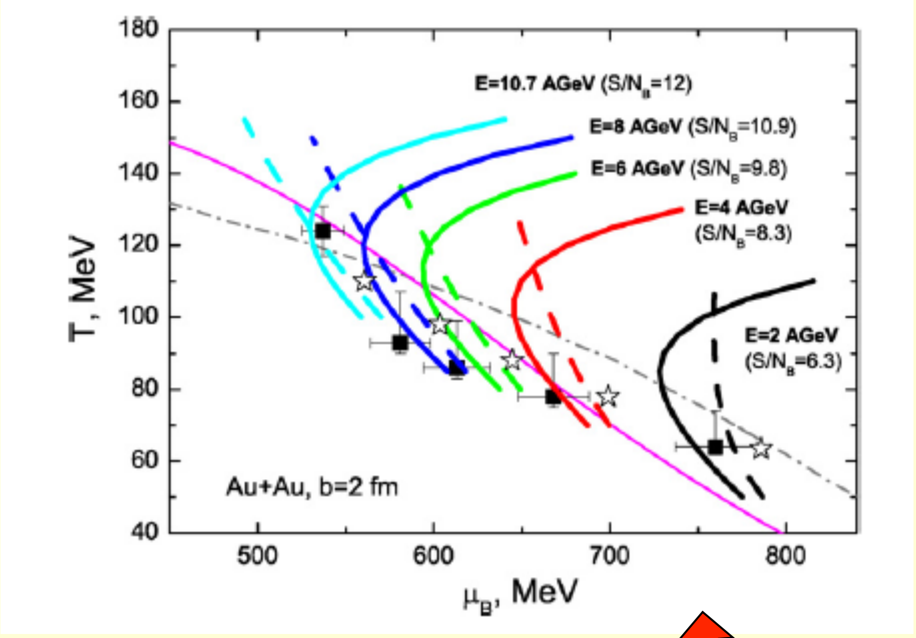
Observation of the 'knee' in other models

temperature vs. chemical potentials

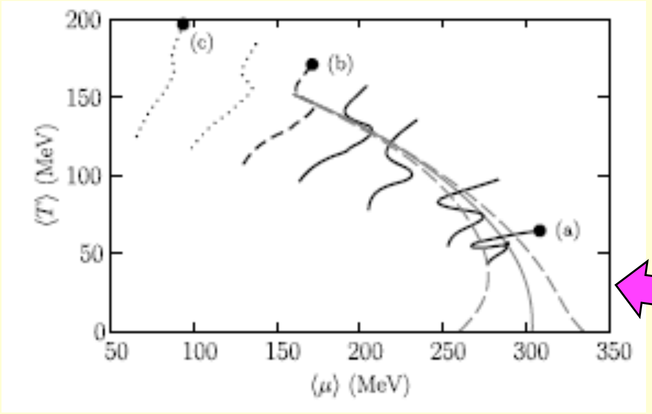
S. Ejiri et al., PRD 73 (2006) 054506



A. Khvororsukhin et al., NPA 791 (2007) 180



C. Herold et al., NPA 925 (2014) 14



The «knee» is observed in

(i)2-flavor lattice QCD;

(ii)σ-ω-ρ model with scaled hadron masses;

(iii) chiral fluid dynamics model with Polyakov loop

Conclusions

- *MC models favor formation of equilibrated matter for a period of 10-15 fm/c*
- *During this period the expansion of matter in the central cell proceeds isentropically with constant S/B (hydro!)*
- *The EOS has a simple form: $P/\varepsilon = const$, where the speed of sound squared varies from 0.12 (AGS) to 0.14 (40 AGeV), and to 0.15 (SPS & RHIC) => onset of saturation*
- *Heavy resonances: are seeing in $c_s^2(T)$ or $c_s^2(\mu_b)$, but not in energy(entropy) vs. T - distributions*
- *T vs. mu: the knee structure which appears at the onset of equilibrium is related to chemical freeze-out*

Back-up Slides

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Local thermodynamical equilibration in central Au + Au collisions at AGS

L.V. Bravina et al., Phys. Lett. B434 (1998) 379-387; DOI: 10.1016/S0370-2693(98)00624-8

Equation of state, spectra and composition of hot and dense infinite hadronic matter in a microscopic transport model

M. Belkacem et al.,Phys. Rev. C58 (1998) 1727-1733; DOI: 10.1103/PhysRevC.58.1727

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L.V. Bravina et al., Nucl. Phys. A661 (1999) 600-603; DOI: 10.1016/S0375-9474(99)85097-0

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L.V. Bravina et al., Phys. Rev. C60 (1999) 024904; DOI: 10.1103/PhysRevC.60.024904

Equation of state of resonance rich matter in the central cell in heavy ion collisions at S^(1/2)=200-A/GeV

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Equilibration of matter near the QCD critical point

L.V. Bravina et al., J. Phys. G32 (2006) S213-S221; DOI: 10.1088/0954-3899/32/12/S27

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L. Bravina et al., Int. J. Mod. Phys. E16 (2007) 777-786; DOI: 10.1142/S0218301307006277

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L.V. Bravina et al.. Phys. Rev. C78 (2008) 014907 ; DOI: 10.1103/PhysRevC.78.014907

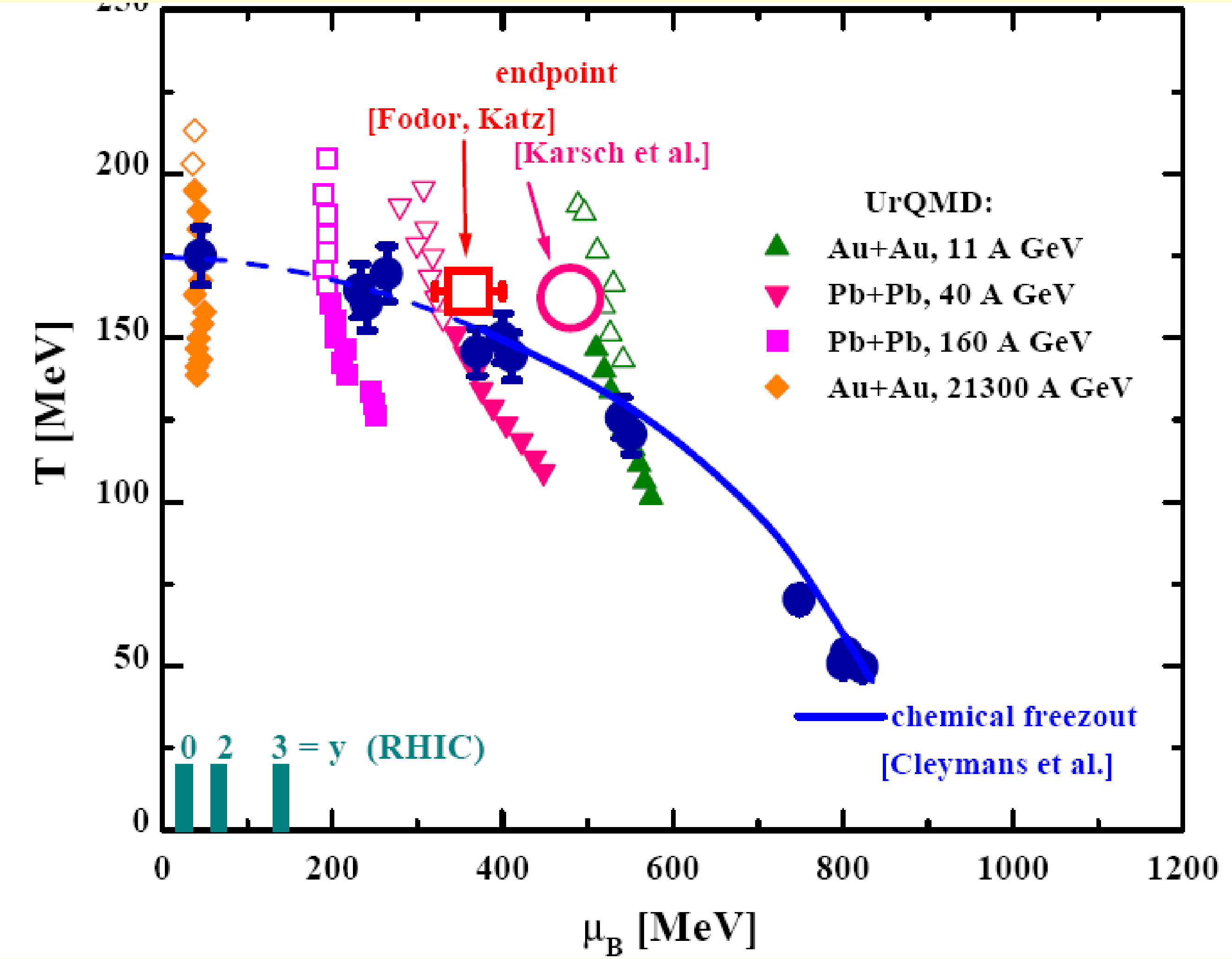
Equation of state at FAIR energies and the role of resonances

E E Zabrodin et al.. J. Phys. G36 (2009) 064065; DOI: 10.1088/0954-3899/36/6/064065

Effective equation of state of hot and dense matter in nuclear collisions around FAIR energy

L. Bravina , E. Zabrodin , EPJ Web Conf. 95 (2015) 01003; DOI: 10.1051/epjconf/20159501003

Phase Diagram



Tricritical point is located around 10-40 GeV (LQCD)

We have to explore this energy range to study the possible phase transition

QGP can be formed already at low energies

H. Stoecker, J. Phys. Conf. Ser. 50 (2006) 300
L. Bravina et al., PRC 60 (1999) 024904; 63 (2001) 064902

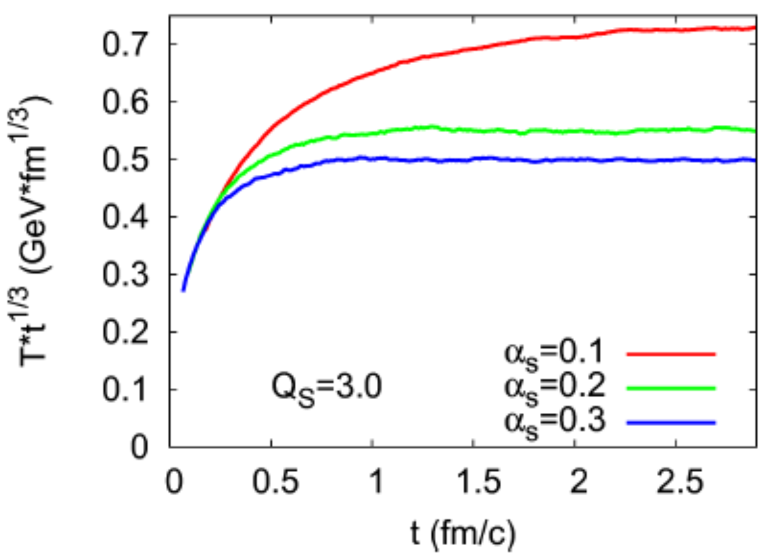
Other possible mechanisms of fast equilibration

gg <=> ggg

A. El, C. Greiner, Z. Xu, NPA 785 (2007) 132

> Solving the Boltzmann equation for on-shell gluons (and quarks) using Monte Carlo techniques
>
> **2<->3 processes included** (detailed balance)
>
> stochastic interpretation of collision rates

Boltzmann **A**pproach for **M**ulti-**P**arton **S**catterings



or fast thermalization via the Hagedorn States

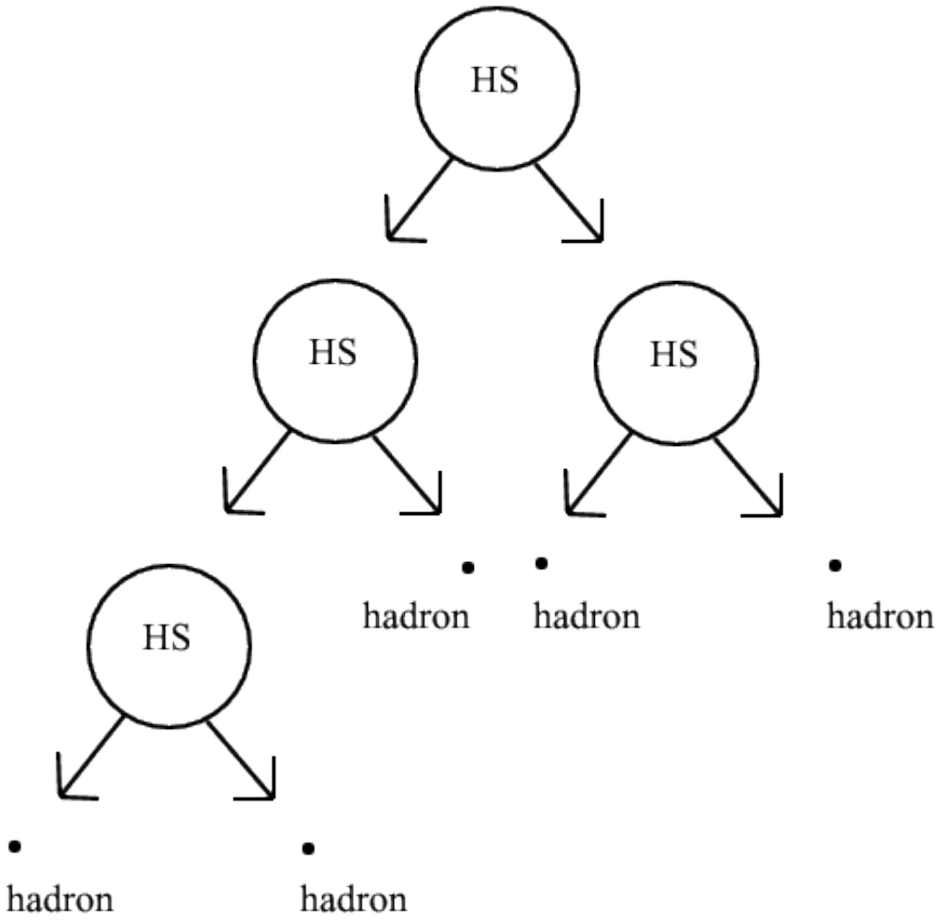
J.Noronha-Hostler et al., PRL 100 (2008) 252301

$$g(m) = \int_{M_0}^{M} \frac{A}{[m^2 + (m_0)^2]^{(5/4)}} e^{(m/T_H)} dm$$

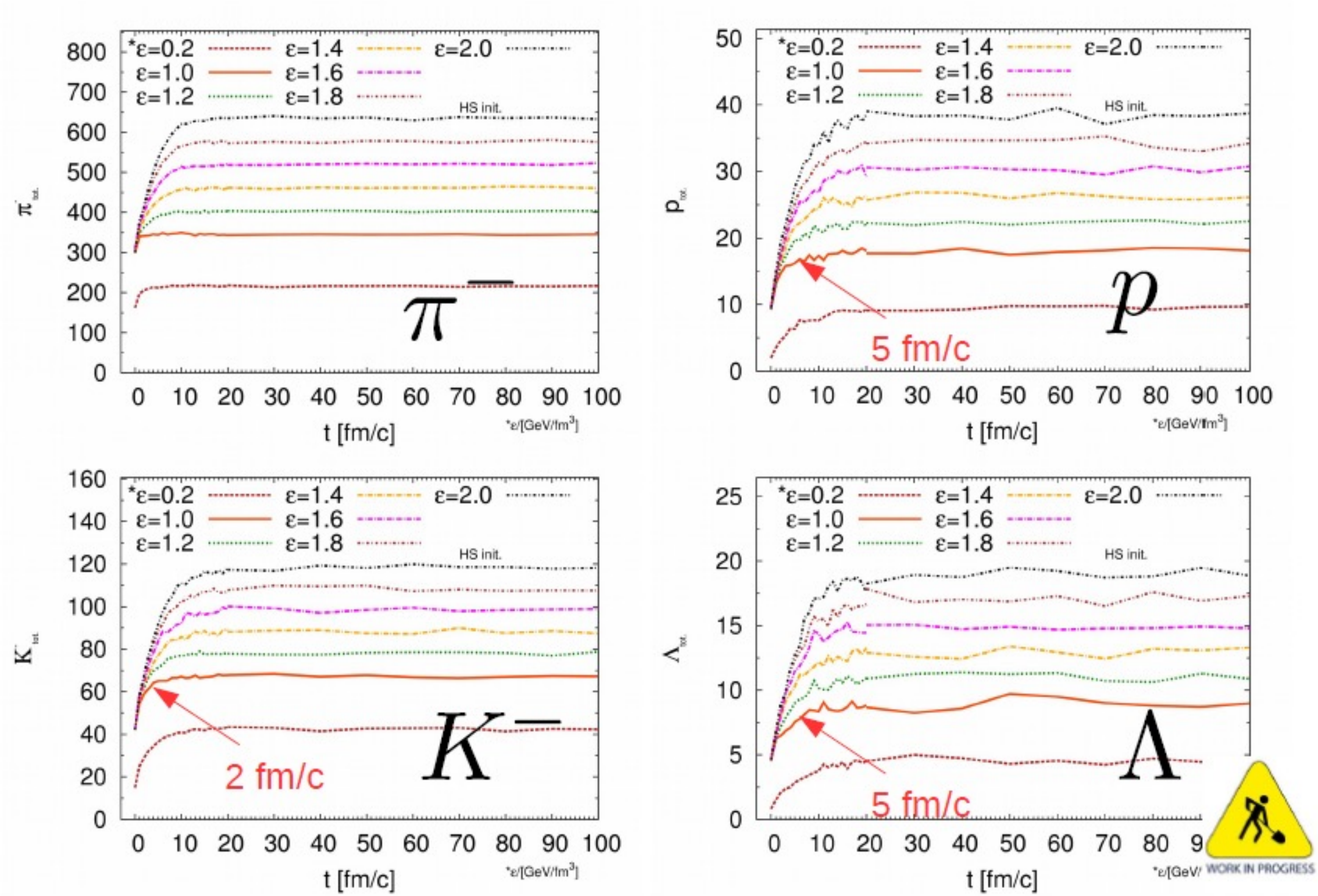
$$n\pi \leftrightarrow HS \leftrightarrow n'\pi + B\bar{B}, \quad n\pi \leftrightarrow HS \leftrightarrow n'\pi + K\bar{K},$$

These mechanisms, however, are more appropriate for A+A collisions at RHIC and LHC

One possible Hagedorn state decay chain



UrQMD-Box, HS init



K. Gallmeister, talk at SQM'15 (JINR, Dubna)