

# Clusters and higher moments of proton number fluctuations

Boris Tomášik

Univerzita Mateja Bela, Banská Bystrica, Slovakia  
and FNSPE, České vysoké učení technické, Praha, Czech Republic

*boris.tomasik@umb.sk*

with

Zuzana Paulínyová-Fecková, Jan Steinheimer, Marcus Bleicher

Hadronic matter under extreme conditions  
JINR Dubna

31.10.2016

- ① (Net) Proton number fluctuations and the influence of deuterons
- ② Production mechanism of deuterons and their scaled moments

## Part 1: proton number fluctuations

- particle number fluctuations are sensitive to the phase transition
- higher-order cumulants are sensitive to the vicinity of the critical point
- however, cumulants can be influenced:
  - conservation laws
  - final state hadronic interactions
  - acceptance effects
  - efficiency effects
  - ...

# The influence of deuteron formation

- deuterons are formed **after** the fireball breakup
- some of the originally produced protons are eaten up by deuterons
- deuteron numbers scales with  $n_p^2$  – non-linear coupling to proton number
- therefore, if proton number in an event is high, many protons disappear in deuterons
- this must be seen in higher moments of multiplicity distributions

# The measures

$n_p$  is the number of protons  
 $\langle \dots \rangle =$  averaging over events

variance

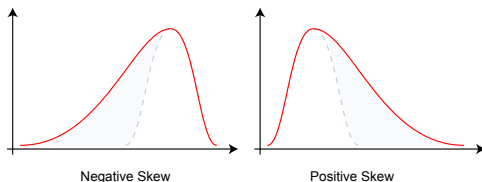
$$\sigma^2 = \langle (n_p - \langle n_p \rangle)^2 \rangle$$

skewness

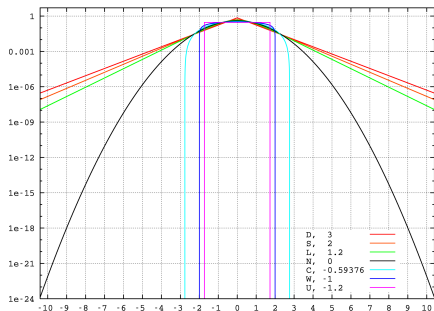
$$S = \frac{\langle (n_p - \langle n_p \rangle)^3 \rangle}{\sigma^3}$$

kurtosis

$$\kappa = \frac{\langle (n_p - \langle n_p \rangle)^4 \rangle}{\sigma^4} - 3$$



kurtosis measures the tails:



# Underlying distributions of protons and deuterons

- **initial** proton number distribution Poissonian

$$P_i(n_i) = \lambda_p^{n_i} \frac{e^{-\lambda_p}}{n_i!}$$

NB:  $n_i$  is **not** measurable – it still includes protons which go into deuterons

- **average** number of deuterons is proportional to  $n_i^2$

$$\lambda_d = Bn_i^2$$

- number of deuterons fluctuates according to Poisson distribution

$$P_d(n_d | n_i) = \lambda_d^{n_d} \frac{e^{-\lambda_d}}{n_d!} = (Bn_i^2)^{n_d} \frac{e^{-Bn_i^2}}{n_d!}$$

# The observed distributions of protons and deuterons

- the **observed** number of deuterons is distributed as

$$P_d(n_d) = \sum_{n_i \geq n_d} P_d(n_d | n_i) P_i(n_i)$$

- the measured proton number is obtained after subtracting protons in deuterons  $n_p = n_i - n_d$
- observed** proton number distribution

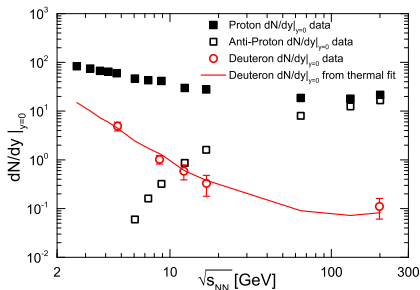
$$P(n_p) = \sum_{n_i \geq n_p} P_i(n_i) P_d(n_i - n_p | n_i)$$

# Parameters fixed by observables

- parameters are:
  - mean initial proton number  $\lambda_p$
  - coalescence factor  $B$
- can be fixed with the help of observed multiplicities:
  - $\langle n_p \rangle = \sum_{n_p} n_p P(n_p)$
  - $\langle n_d \rangle = \sum_{n_d} n_d P(n_d)$

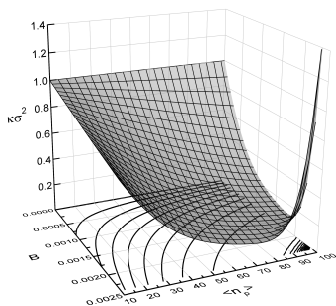
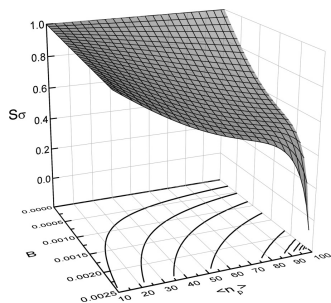
parametrisation of the mean  
deuteron/proton ratio

$$\frac{d}{p} = 0.8 \left[ \frac{\sqrt{s_{NN}}}{1 \text{ GeV}} \right]^{-1.55} + 0.0036$$

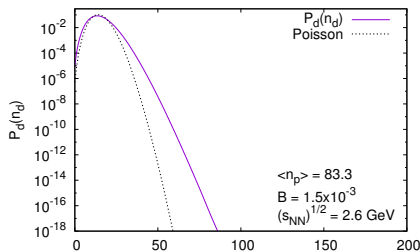
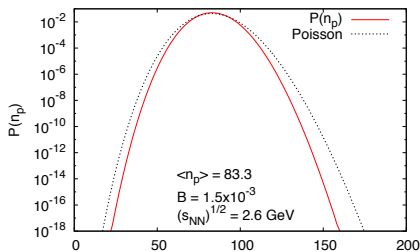




# Skewness and kurtosis after subtraction of deuterons

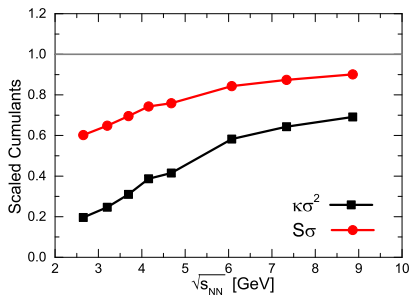


Example: distributions of p and d for  $\langle n_p \rangle = 83.3$  and  $B = 1.5 \times 10^{-3}$

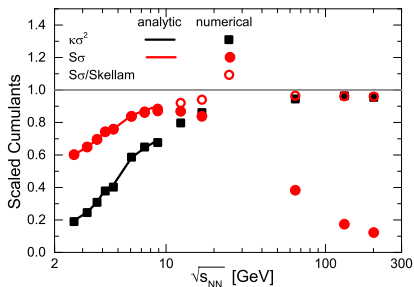


# Results for scaled cumulants as functions of $\sqrt{s_{NN}}$

low energy: no antiprotons



higher energy:  
also antiprotons fluctuate



Formation of deuterons has an important impact on proton number fluctuations.

## Part 2: thermal production vs coalescence of deuterons

- due to their fragility, deuterons can hardly exist in the dense hadronic system
- mean production numbers are well described by [coalescence](#)
- good description of data is also obtained with the help of [Statistical model](#)

Such models could be distinguished by fluctuations

- in Statistical model fluctuations of all species are Poissonian
- coalescence leads to non-Poissonian fluctuations of clusters (deuterons)

## Deuteron number distribution

Model A: fully correlated proton and neutron numbers (as previously)

$$\begin{aligned}\lambda_d &= Bn_i^2 \\ P_d(n_d|n_i) &= \frac{\lambda_d^{n_d} e^{-\lambda_d}}{n_d!} = (Bn_i^2)^{n_d} \frac{e^{-Bn_i^2}}{n_d!} \\ P_d(n_d) &= \sum_{n_i \geq n_d} P_d(n_d|n_i) P_i(n_i)\end{aligned}$$

Model B: independent proton number  $n_i$  and neutron number  $n_j$

$$\begin{aligned}\lambda_d &= Bn_i n_j \\ P_d(n_d|n_i, n_j) &= \frac{\lambda_d^{n_d} e^{-\lambda_d}}{n_d!} = (Bn_i n_j)^{n_d} \frac{e^{-Bn_i n_j}}{n_d!} \\ P_d(n_d) &= \sum_{n_i, n_j \geq n_d} P_d(n_d|n_i, n_j) P_i(n_i) P(n_j)\end{aligned}$$

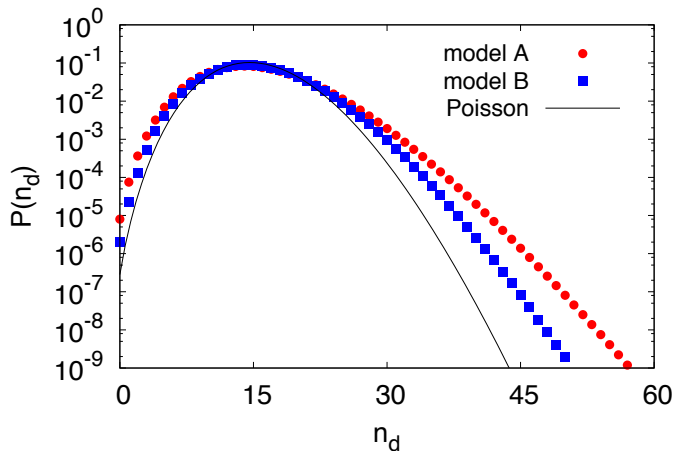
## An example of deuteron number distribution

calculated for  $\sqrt{s_{NN}} = 2.6$  GeV

correlated p and n:  $\sigma^2/\langle n_d \rangle = 1.609$ ,  $S\sigma = 2.218$ ,  $\kappa\sigma^2 = 6.915$

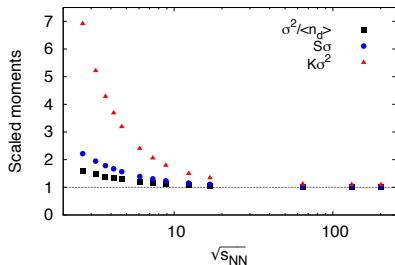
independent p and n:  $\sigma^2/\langle n_d \rangle = 1.308$ ,  $S\sigma = 1.616$ ,  $\kappa\sigma^2 = 3.422$

Poissonian values are 1.

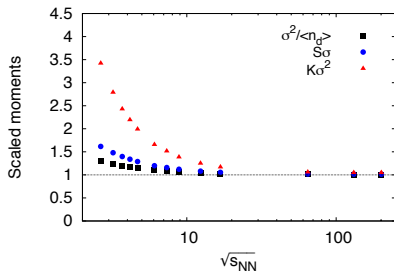


# Predictions for the deuteron scaled moments

correlated p and n number



independent p and n number



Poissonian values (Statistical model) would be 1!

Particularly higher moments can clearly distinguish production by coalescence!

# Summary

Deuteron formation has large influence on proton number fluctuations, especially at NICA energies.

Z. Fecková, J. Steinheimer, B. Tomášik, M. Bleicher, Phys. Rev. C **92**, 064908 (2015)

Higher moments of deuteron number distribution can help to distinguish between statistical production and coalescence.

Z. Fecková, J. Steinheimer, B. Tomášik, M. Bleicher, Phys. Rev. C **93**, 054906 (2016)