Chiral Thermodynamics in a box

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in collaboration with **Ana Juricic**

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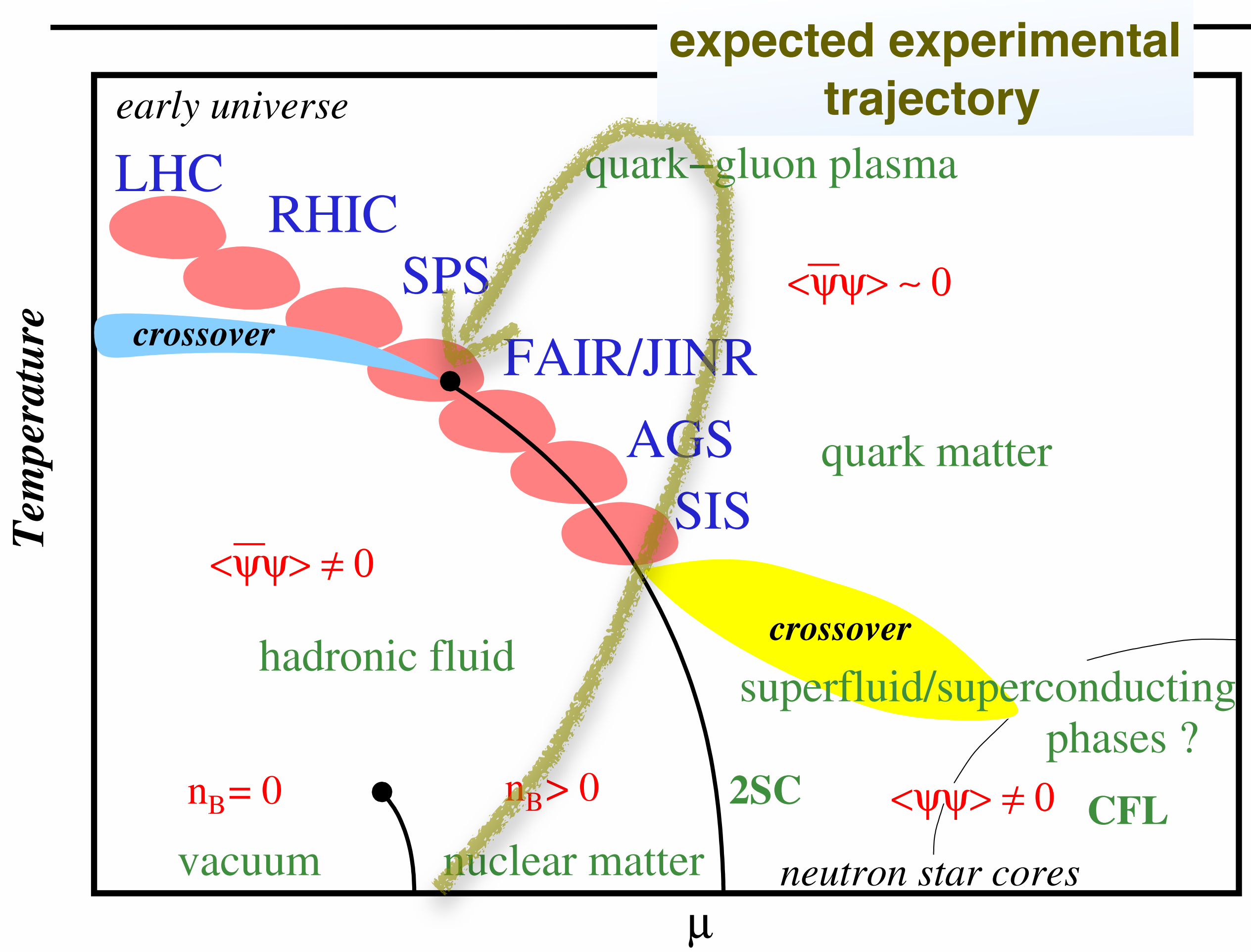
Germany

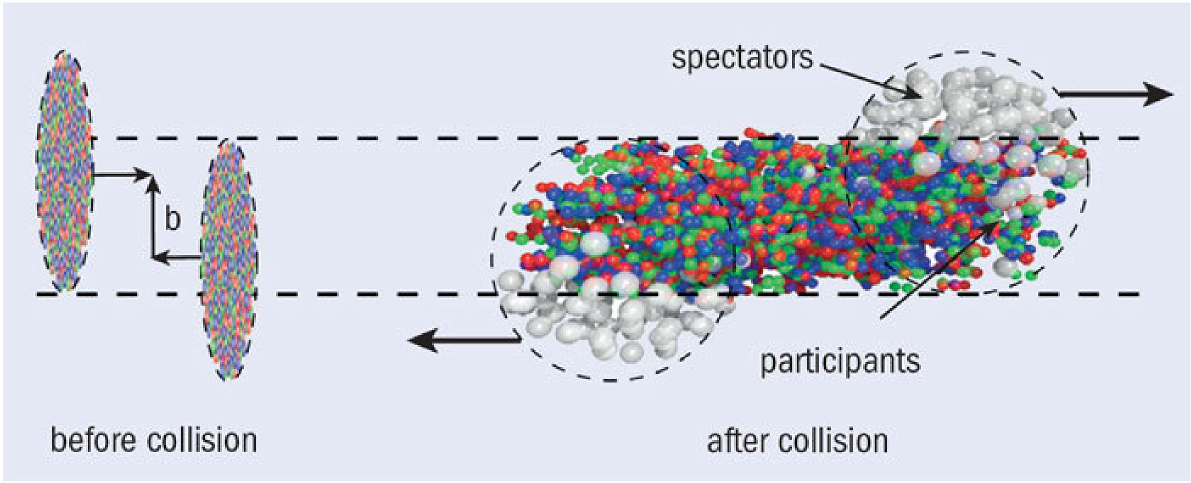
Nov 1st, 2016

Meeting of the working group on theory of hadronic matter under extreme conditions
Dubna, October 31 - November 3, 2016

Conjectured QC₃D phase diagram



Experiment:



Theory:

➜ Lattice: but simulations restricted to small μ

➜ Functional QFT methods: FRG, DSE, nPI

➜ Models: effective theories parameter dependency

Experiment: (non-equilibrium)

➜ in a finite box (HBT radii: freeze-out vol. ~ 2000-3000 fm^3)

(UrQMD ($\sqrt{s}$): system vol. ~ 50 - 250 fm^3)

Theoretical aim:

deeper understanding & more realistic HIC description

➜ existence of critical end point(s)?

Non-trivial physical issues!

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Agenda

complementary to

- **Motivation: physics in a finite volume**

- **Generalized susceptibilities**

  ➜ towards chiral phase transition

- **Role of Fluctuations:**
  from mean-field approximations to RG

- **Comparison:**
  Finite/infinite volume effects

many open theoretical issues

to clarify ➜ long term project

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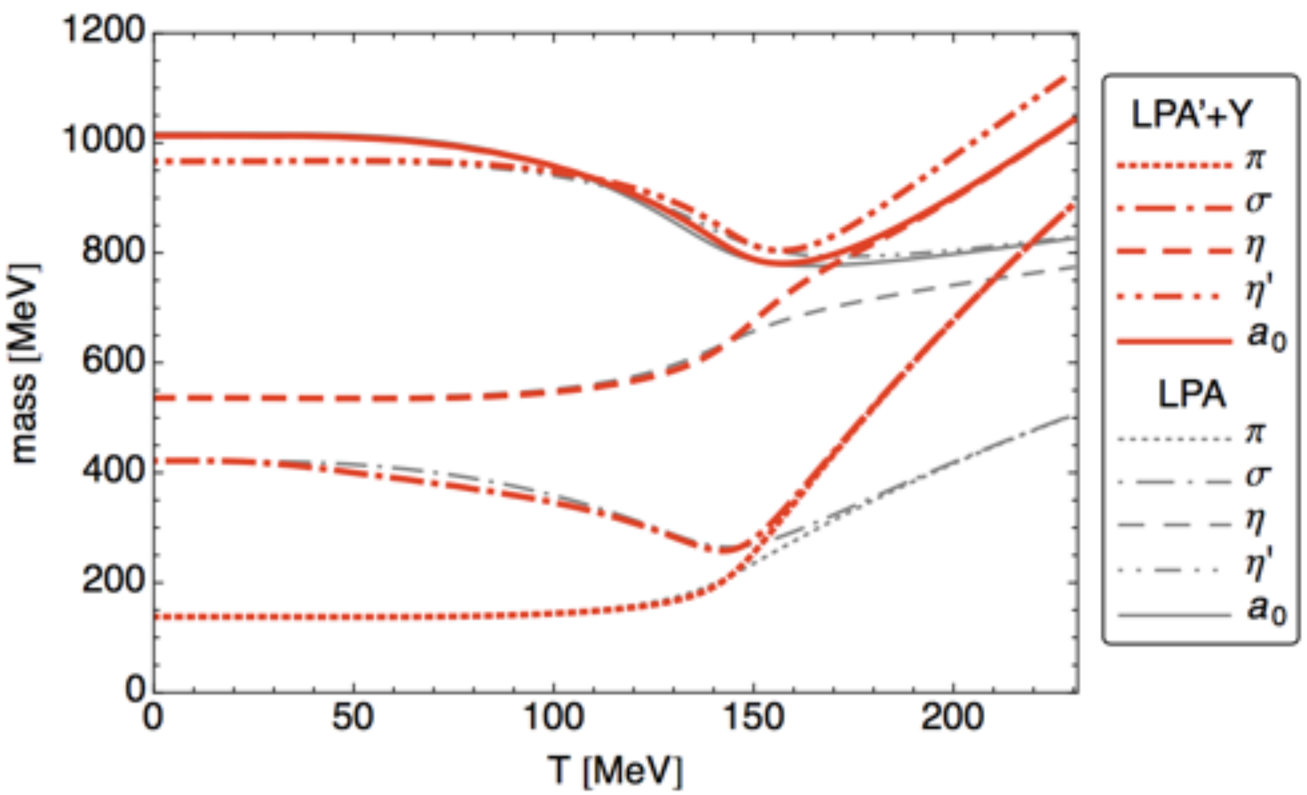
Fluctuations are important

F. Rennecke, BJS 1610.08748
FRG Nf=2+1 beyond “usual” LPA truncation



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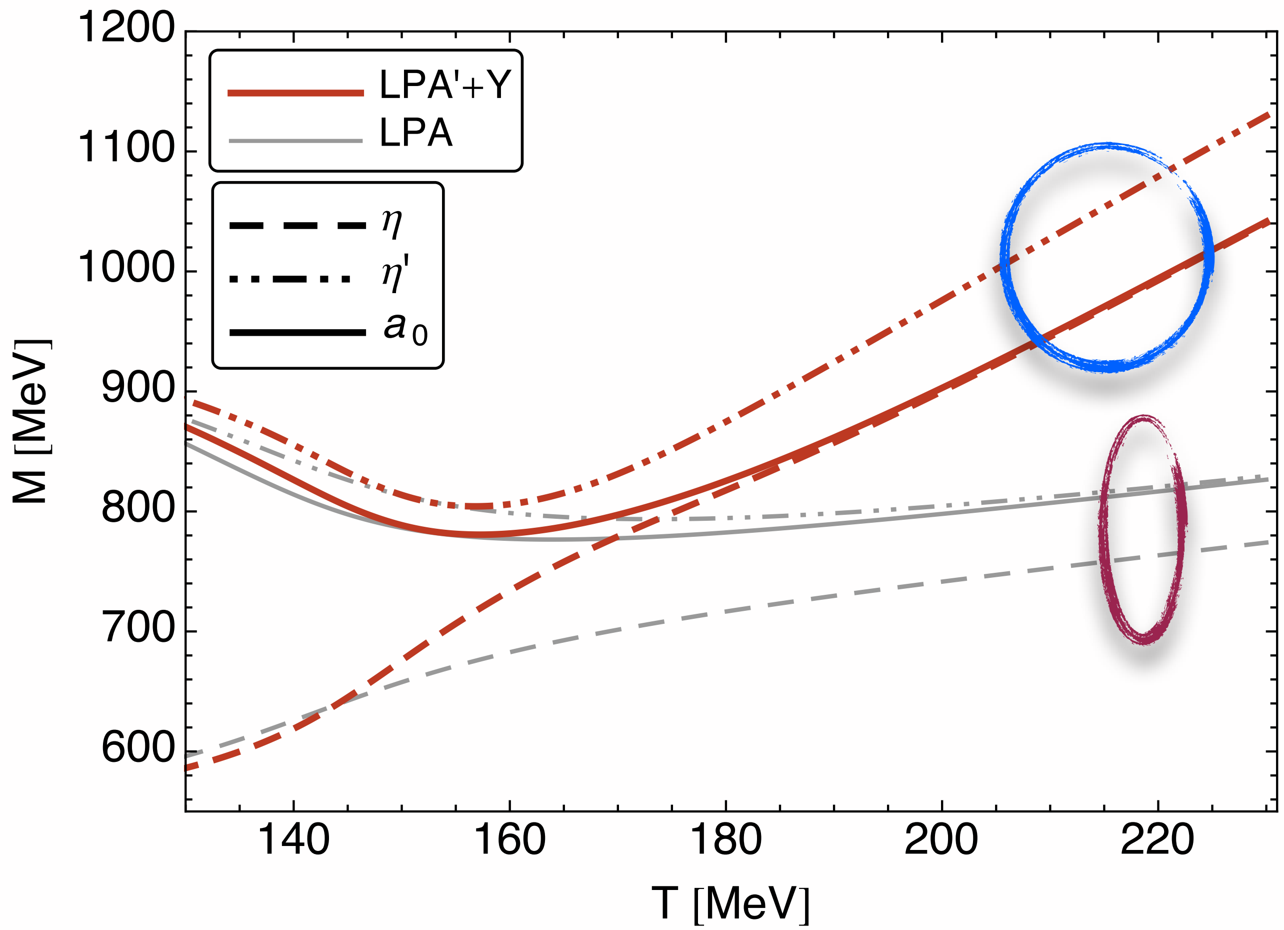

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Physics in a finite Volume

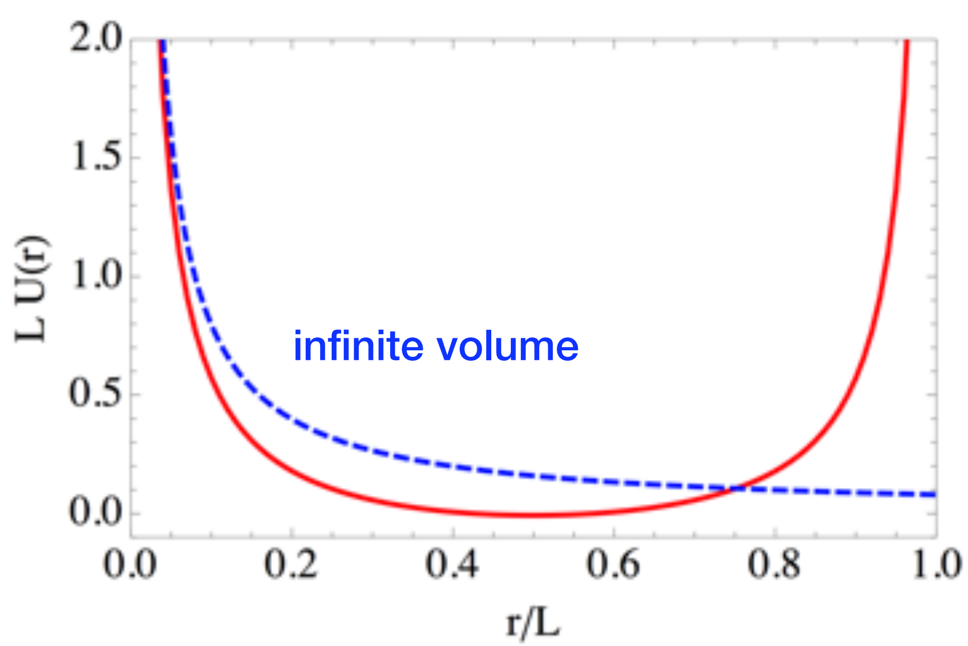
[Davoudi, Savage 2014]

Lattice simulations:

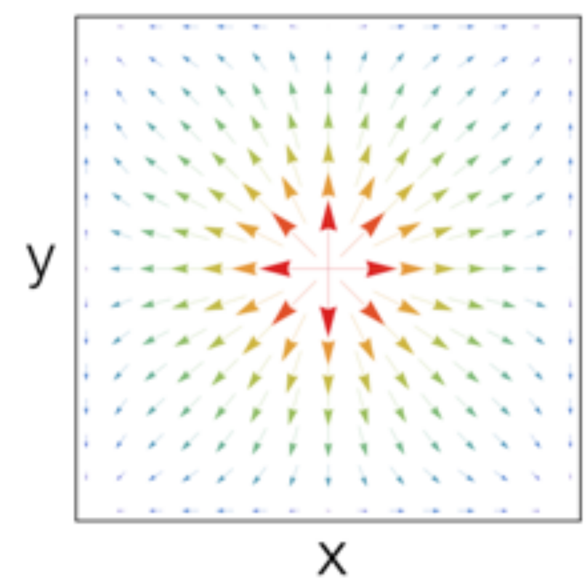
QCD (short-ranged) with QED (long-ranged ➜ truncated) corrections

➜ violation Gauss's & Ampere's law

if EM gauge field subject to periodic boundary condition



finite volume Coulomb potential between two charges



point charge at the center

circumvent this problem:

introduce uniform background charge density

similar to many-body physics

➜ equivalent to

removing zero modes of the gauge field

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Physics in a finite Volume

Quantum Field Theory in a finite volume:

➜ no spontaneous symmetry breaking

if only finite number of degrees of freedom

QCD: [Gasser, Leutwyler 1988]

chiral condensate: non-perturbative phenomenon

e.g.

chiral symmetry

$$N_f = 2 : SU(2) \times SU(2) \cong O(4)$$

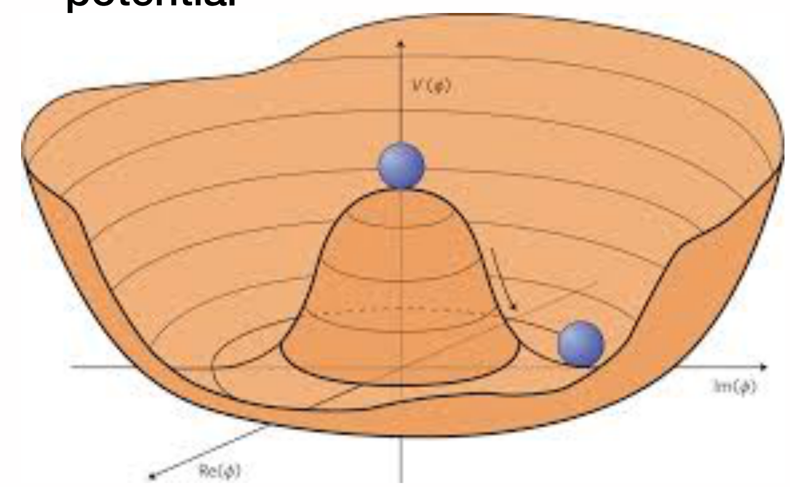
$O(4) \to O(3)$ infinite volume

massless Goldstone Bosons

finite volume:

fluctuations of Goldstone bosons always restore symmetry

potential



minimum: zero-momentum mode of the field

$$Z_2 : \varphi \to -\varphi$$

probability of tunneling: $P_{\text{tunnel}} \sim e^{-L}$

exponentially suppressed with volume

$O(N)$ - case: rotation ➜ averaging to zero (no breaking)

infinite volume ➜ no tunneling ➜ symmetry broken

Physics in a finite Volume

result so far:

long-range correlations are necessary to obtain spontaneous SB (for a continuous symmetry)

chiral limit: massless Goldstone boson fluctuations in a finite box avoid symmetry breaking

but

symmetry breaking in mean-field approximations are possible:

Goldstone-fluctuations are absent

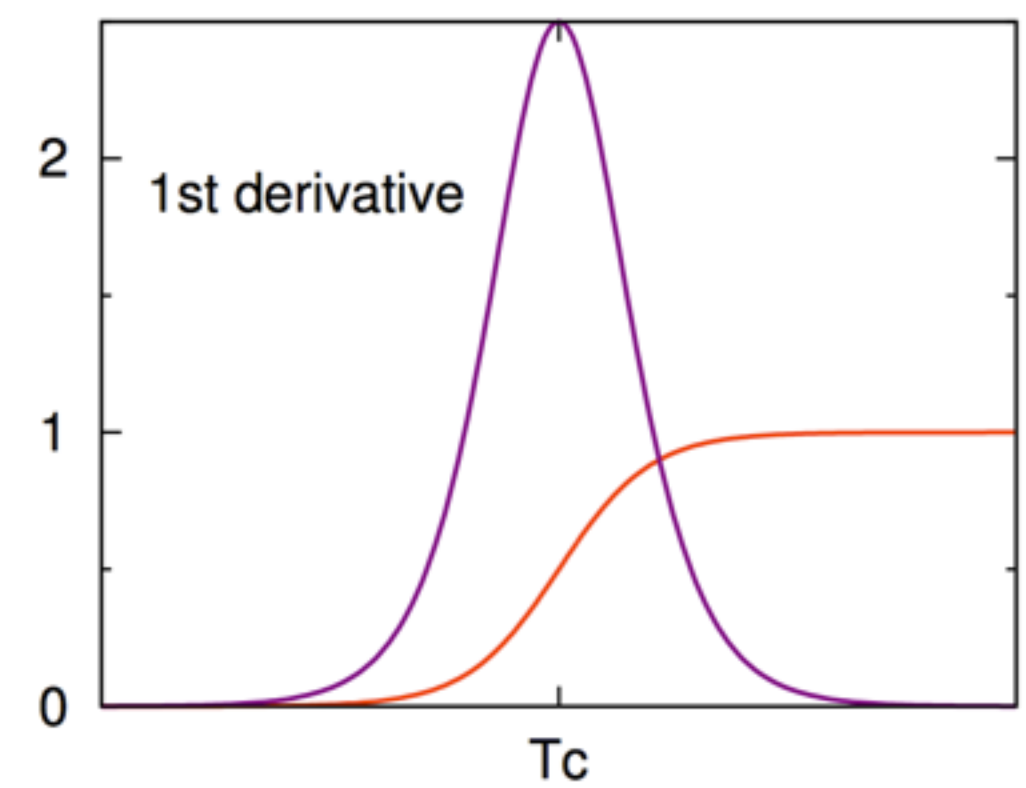
<u>Thermodynamics on a torrus:</u>

correlation length always finite ➜ no real 2^{nd} order phase transition

criterion for phase transition: (generalized) susceptibilities

➜ derivatives of order parameter reveal more details

derivatives of thermodynamic quantities ↔ fluctuations



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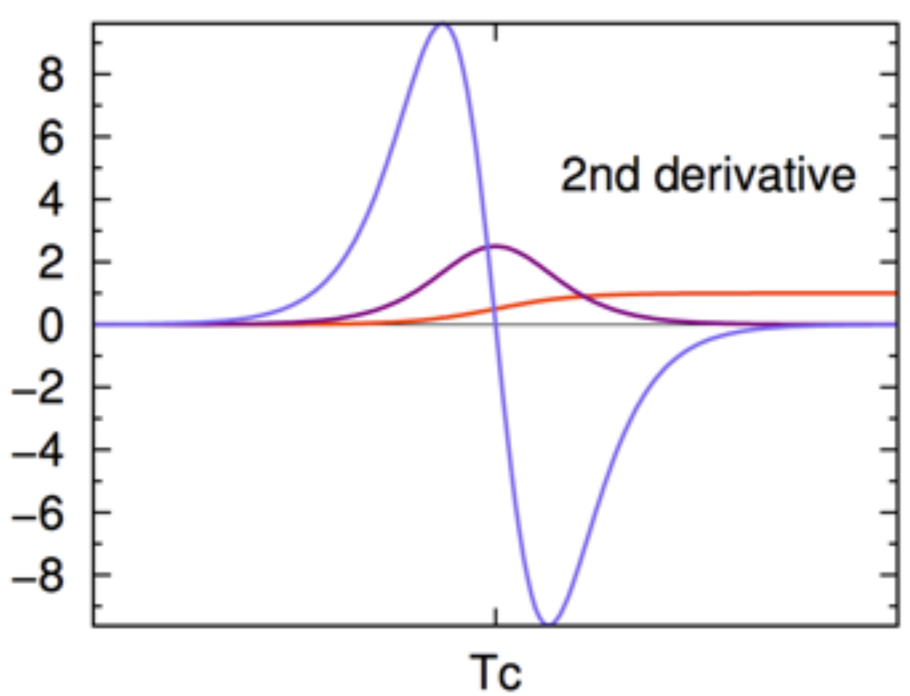

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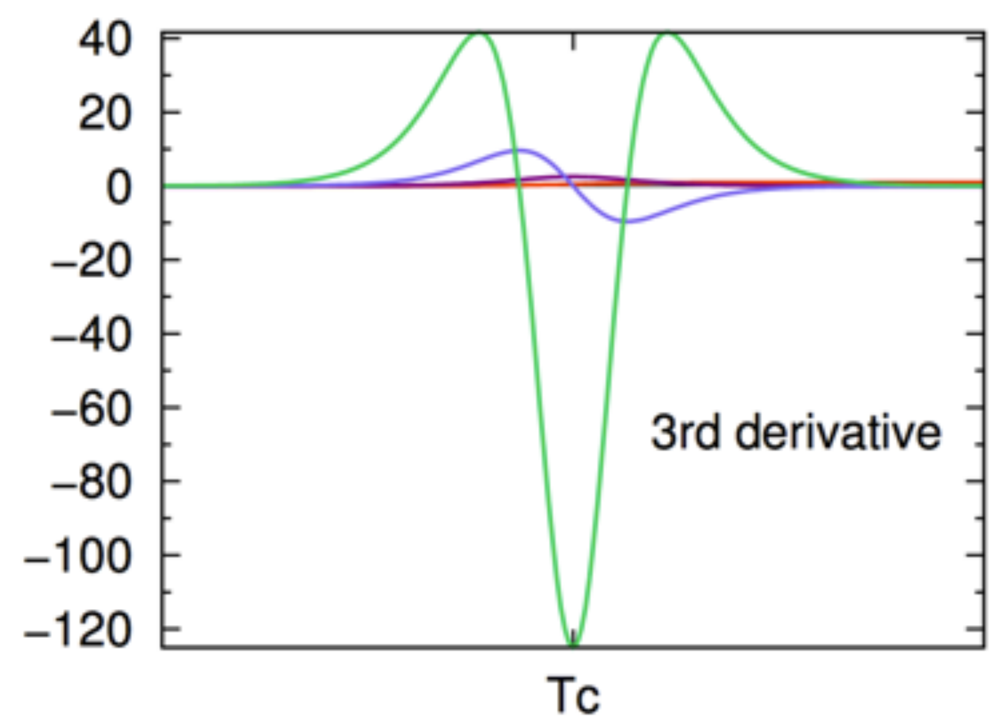

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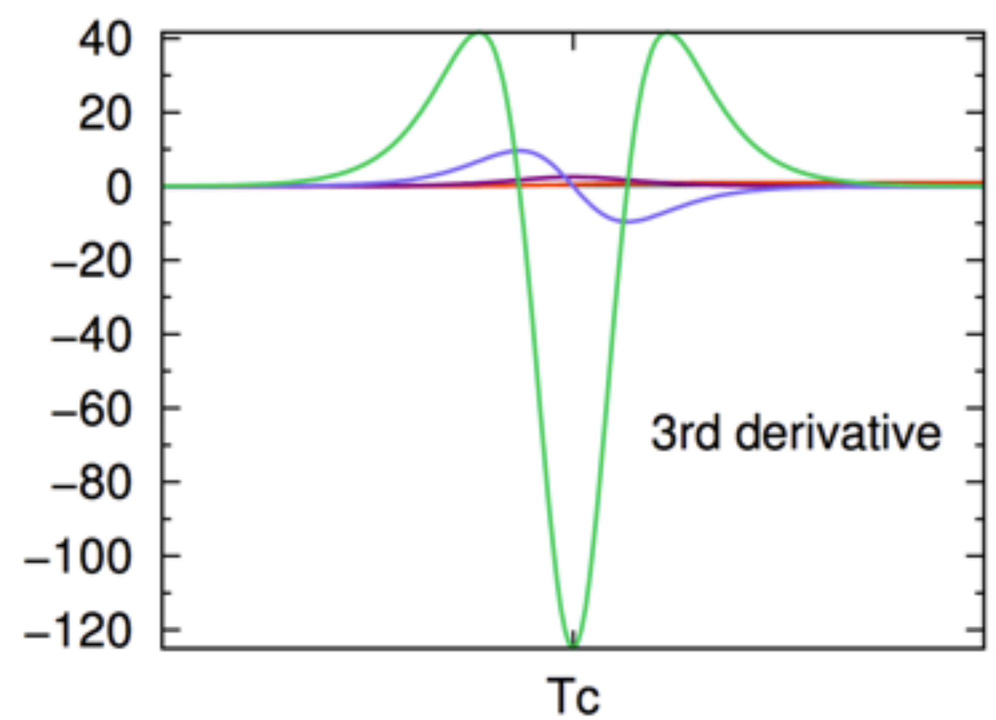


Fluctuation observables

- generalized susceptibilities:

$$\chi_n = \left. \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n} \right|_T$$

- Fluctuations of conserved charges

$$\delta Q_X = Q_X - \langle Q_X \rangle \qquad X = Q, B, S, \ldots$$

mean value: $\chi_1 \sim \langle Q \rangle$

$\chi_2 \sim \langle (\delta Q)^2 \rangle$

$\chi_3 \sim \langle (\delta Q)^3 \rangle$

- Measured in event-by-event multiplicity distributions

variance: $\sigma^2 \sim \frac{\chi_2}{\chi_1}$

skewness: $S\sigma = \frac{\chi_3}{\chi_2}$

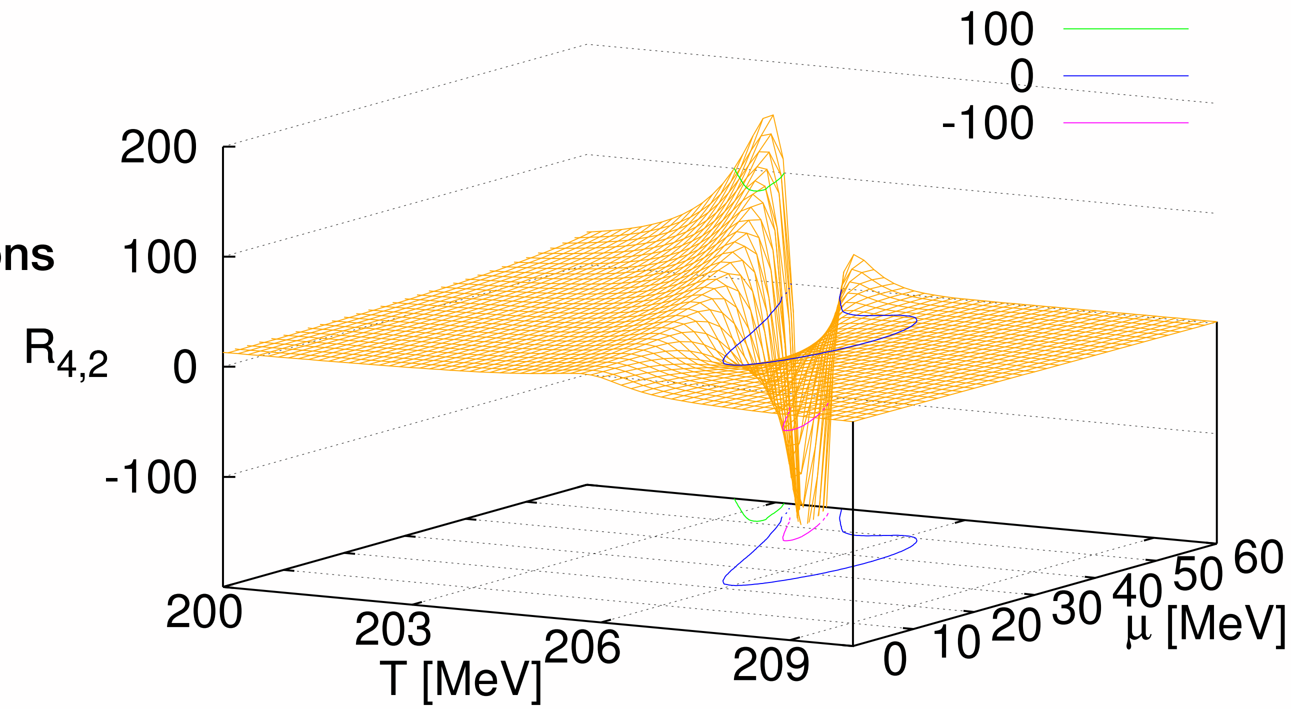
kurtosis: $\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$

[STAR Coll. 2014; PHENIX Coll 1506.07834]

strong temperature & density

dependence of ratios

[BJS, M. Wagner 2012]



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Fluctuation observables

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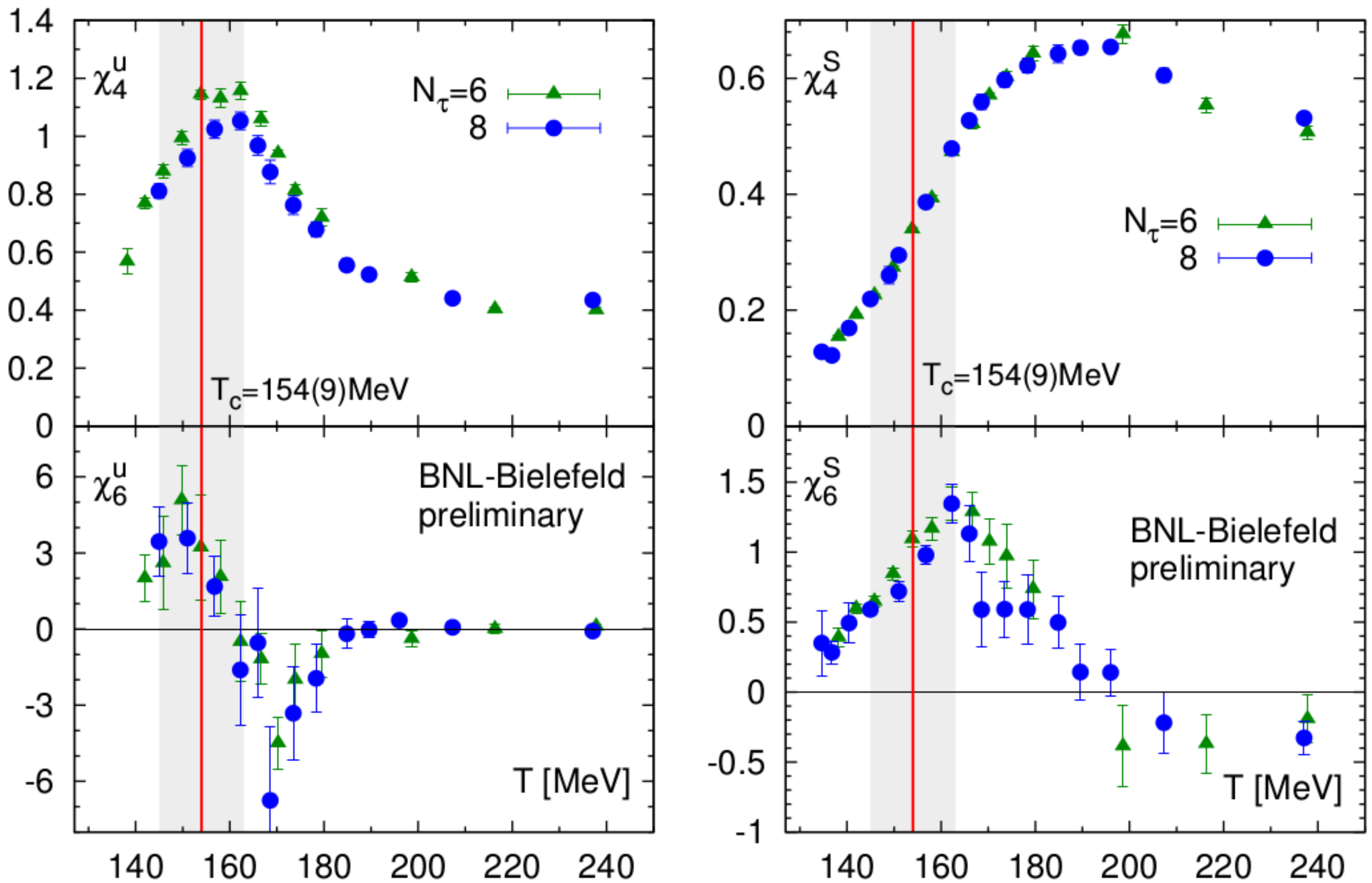
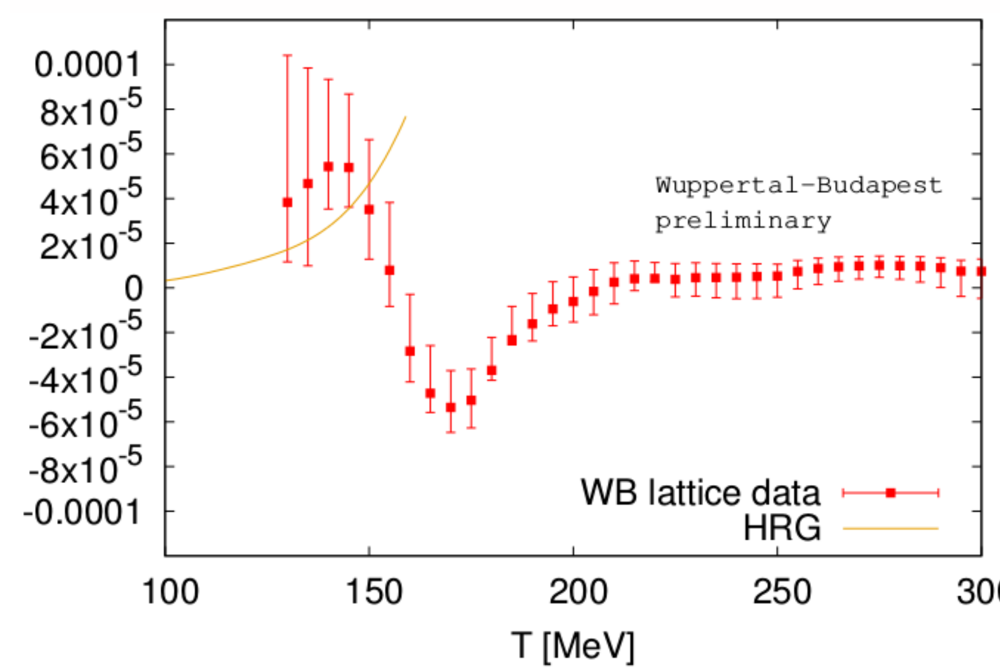
change of sign ➜ deviations from HRG ➜ criticality

[Schmidt et al. 2015]



[Bellwied et al. 2016]



Grand potential

Low energy QCD model: $\phi = (\sigma, \vec{\pi})$

$$\mathcal{L} = \bar{\psi}\left(\not{\partial} + g(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)\right)\psi + \frac{1}{2}(\partial_\mu \phi)^2 + V_{\text{Meson}}(\phi)$$

Partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi\, e^{-\int d^4x\, \mathcal{L}(\bar{\psi},\psi,\phi)}$$

replace with (const.) condensate σ

Integration of quarks, neglect bosonic fluctuations: ➜ mean-field approximations MFA

Integration of quarks and bosonic fluctuations: ➜ renormalization group treatment FRG

Grand potential (Polyakov-)quark-meson model (quark loop)

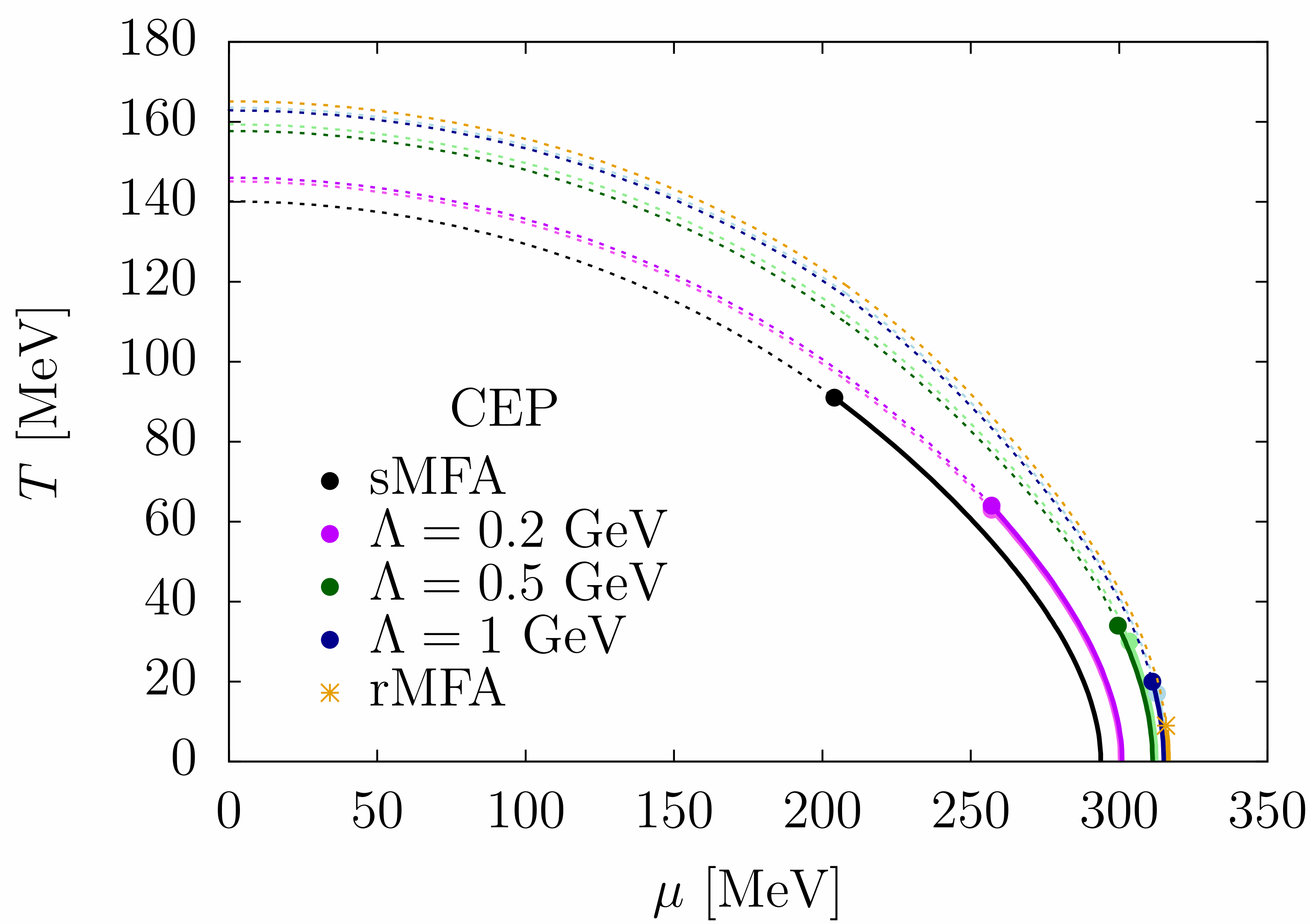
$$\Omega(T,\mu;\sigma) = \Omega_{\text{vac}} + \Omega_{\text{T}} + V_{\text{MF}}(\sigma) \qquad (+\mathcal{U}_{\text{Poly}}(\Phi)\ )$$

$$\Omega_{\text{vac}} = -4\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2 + m_q^2}$$

Vacuum term: regularize e.g. with sharp O(3)-momentum cutoff

Infinite volume

[A Juricic, BJS to be published]



sMFA: no vacuum fluctuations

⬇

rMFA: renormalized MFA

e.g.

Pauli-Villars regularization

sharp O(3)-momentum cutoff

proper-time regularization

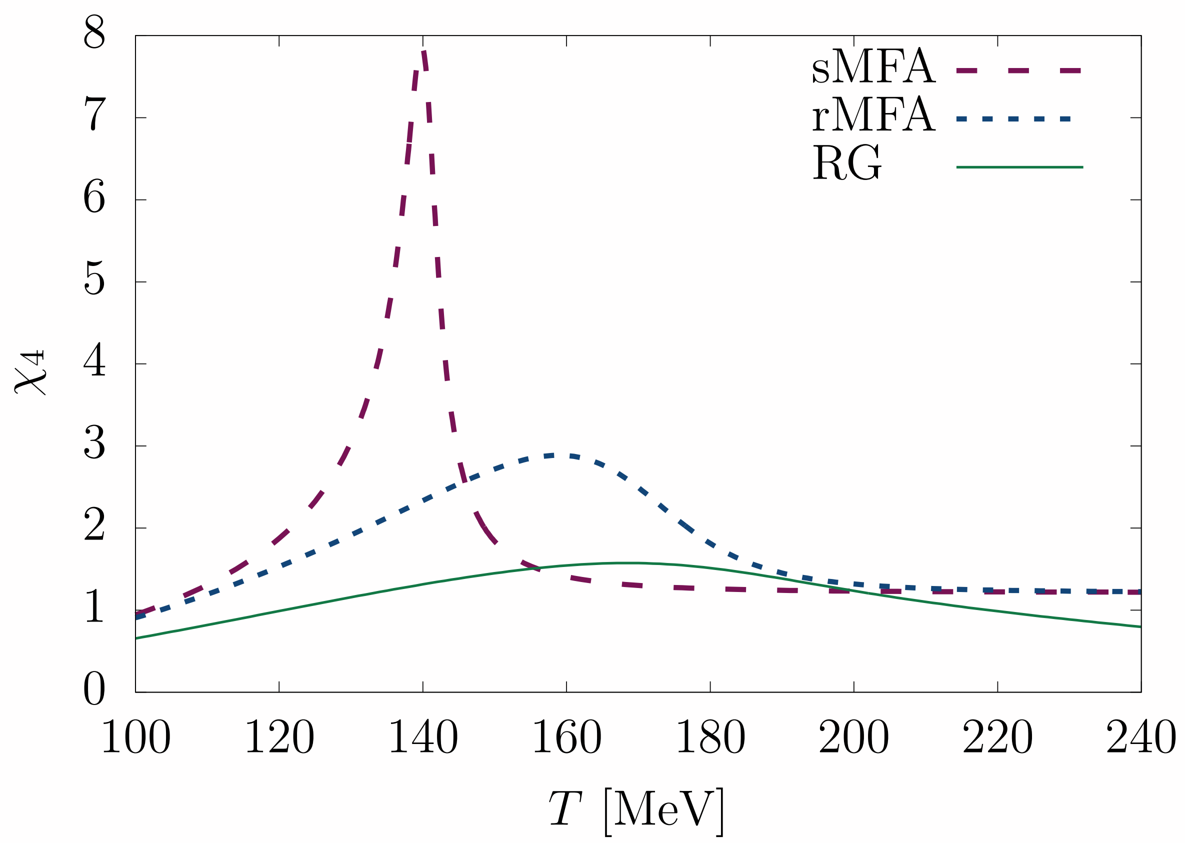
Infinite volume

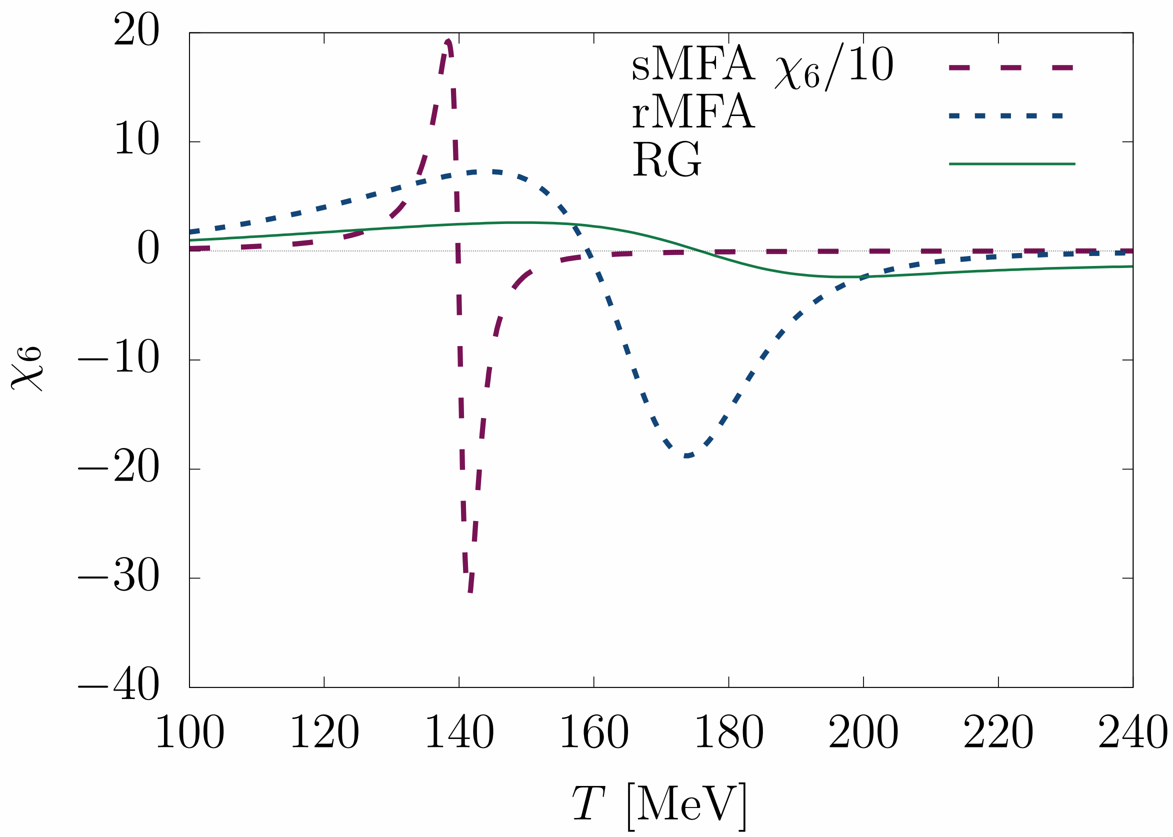
generalized susceptibilities

standard MFA:
no quark vacuum fluctuations

renormalized MFA:
including quark vacuum fluctuations

RG:
quark + meson fluctuations





[A Juricic, BJS to be published]

Higher cumulants

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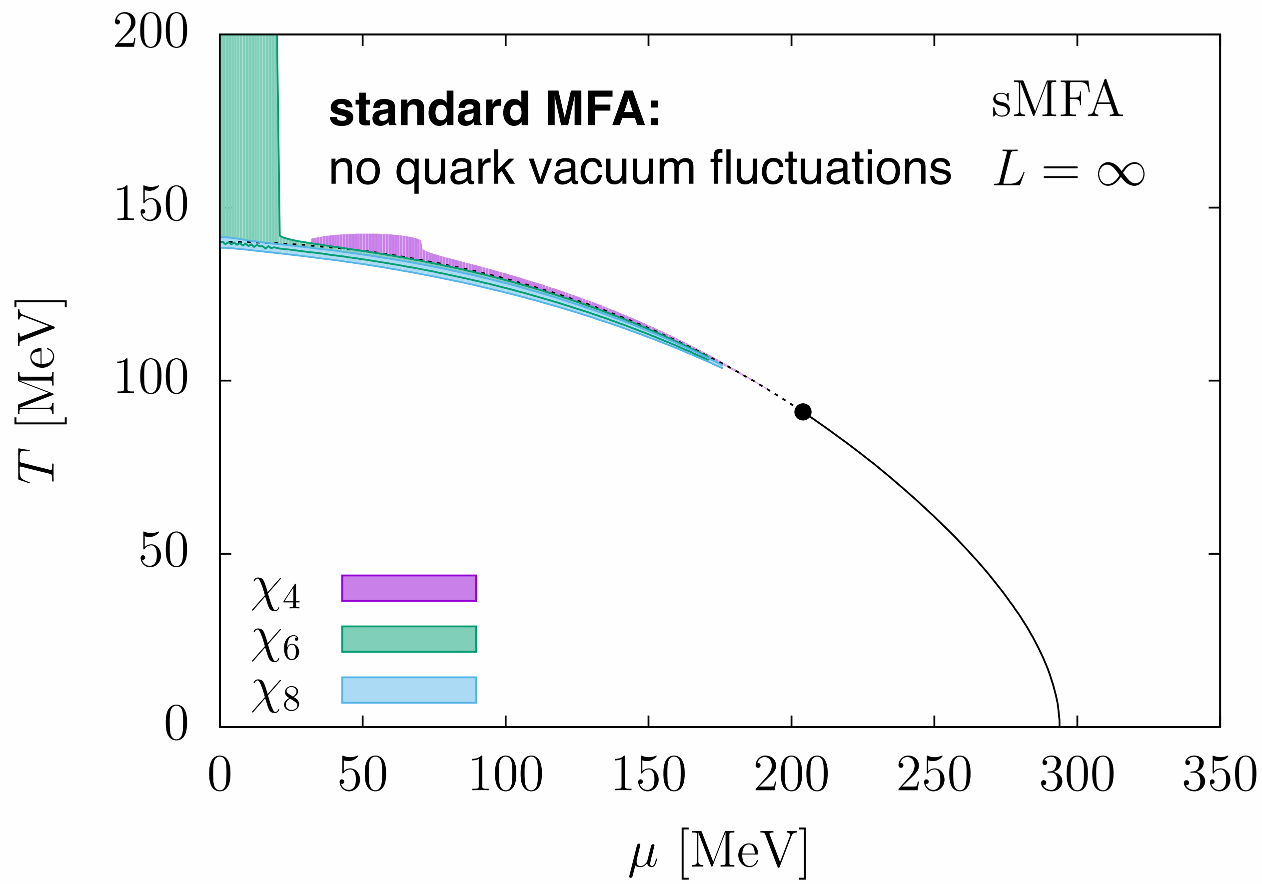
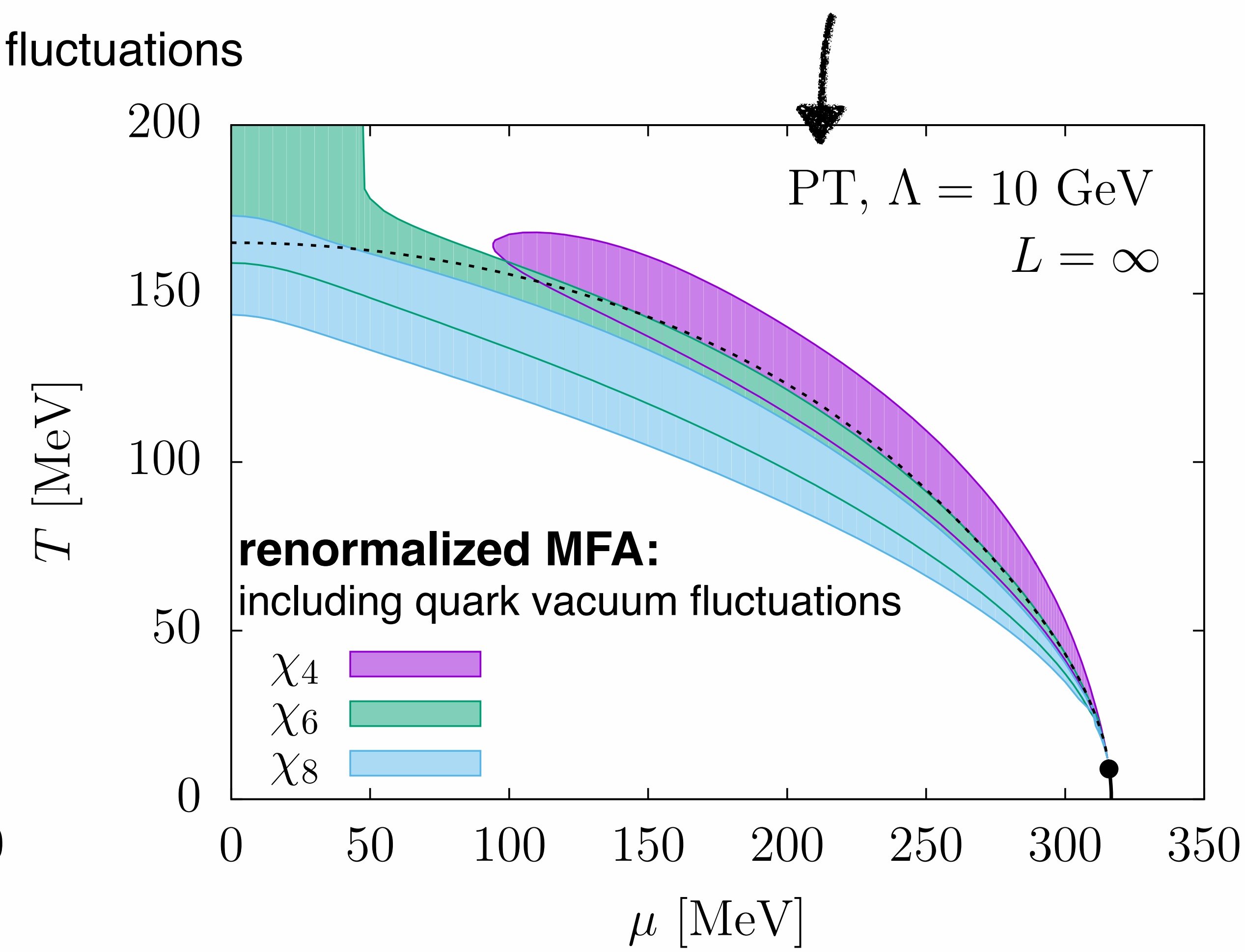
infinite volume: influence of fluctuations





findings:

(quark) fluctuations pushes CEP to smaller T and bigger μ

Fluctuations wash out phase transition ➜ broader negativer regions

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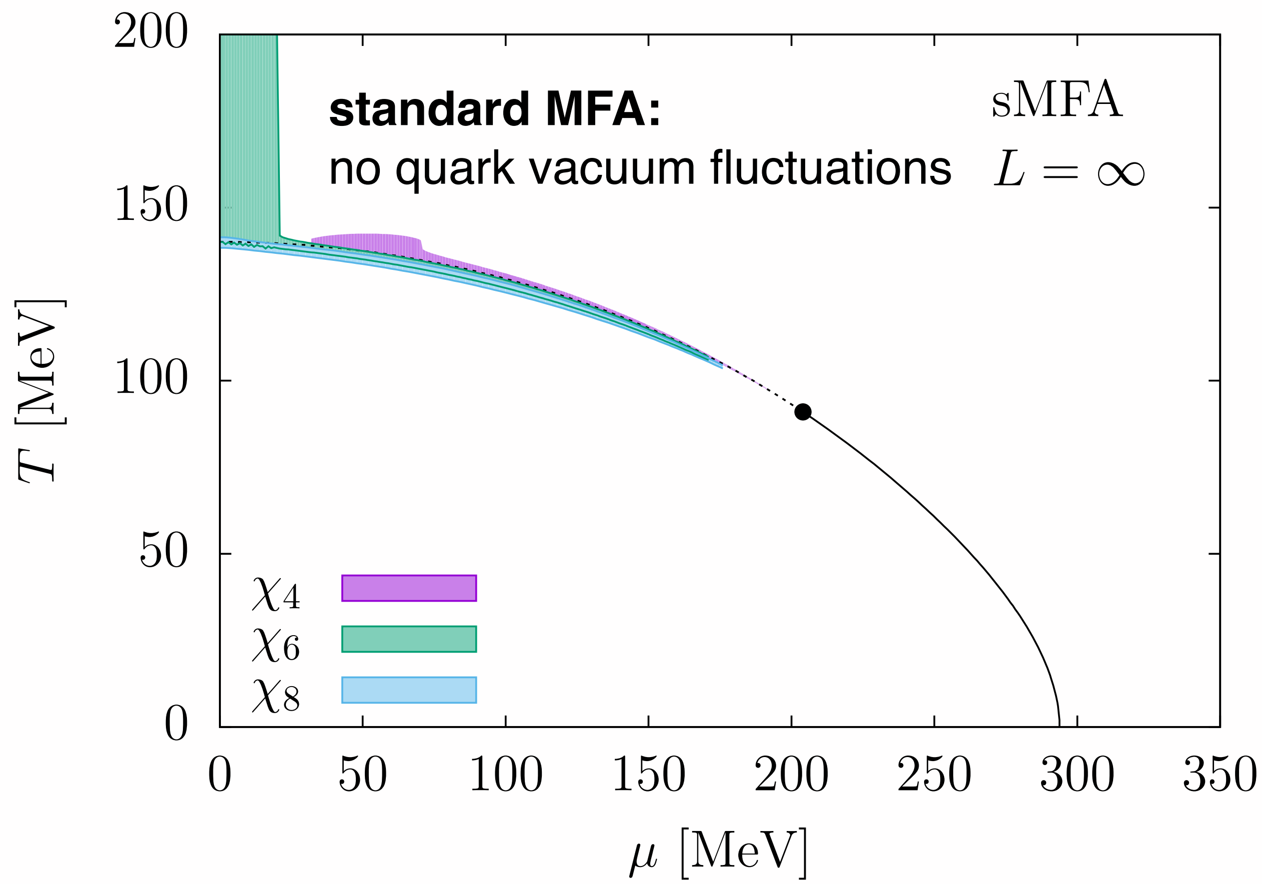

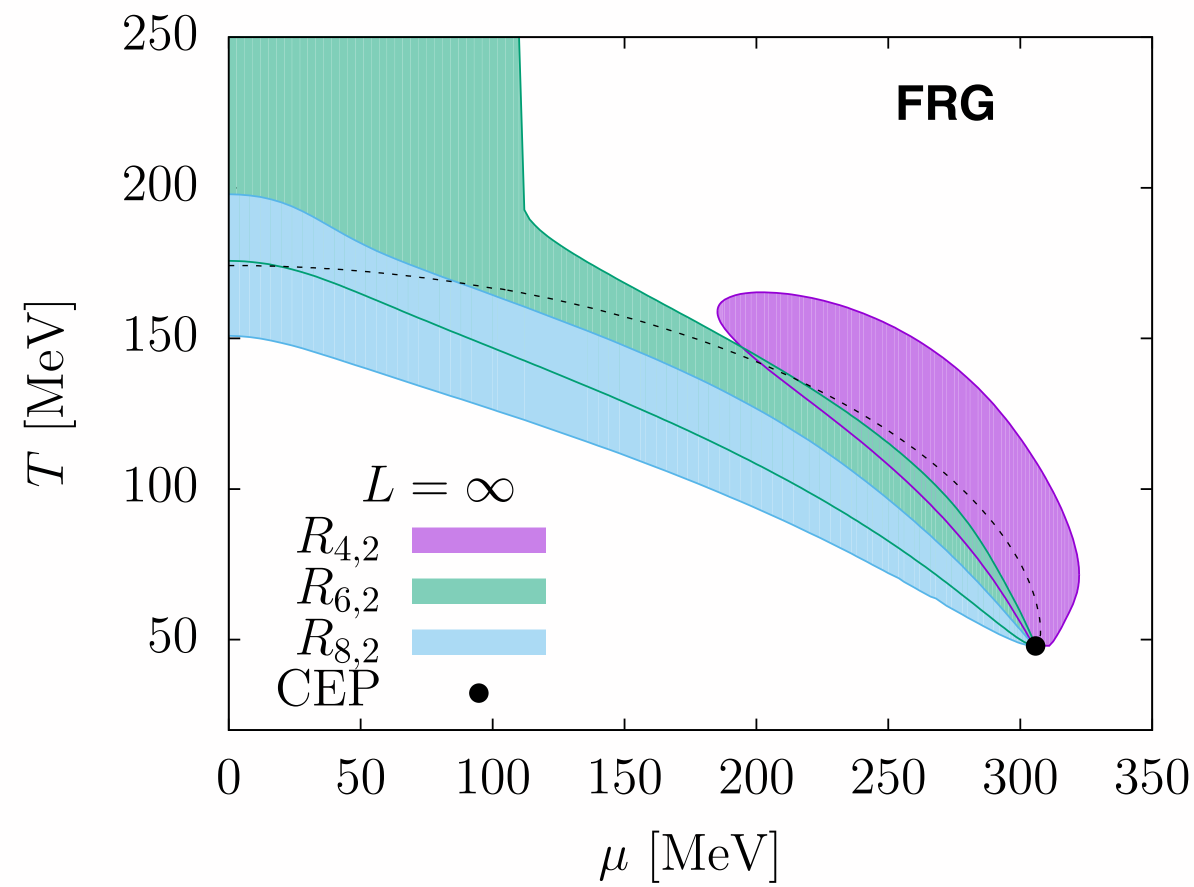

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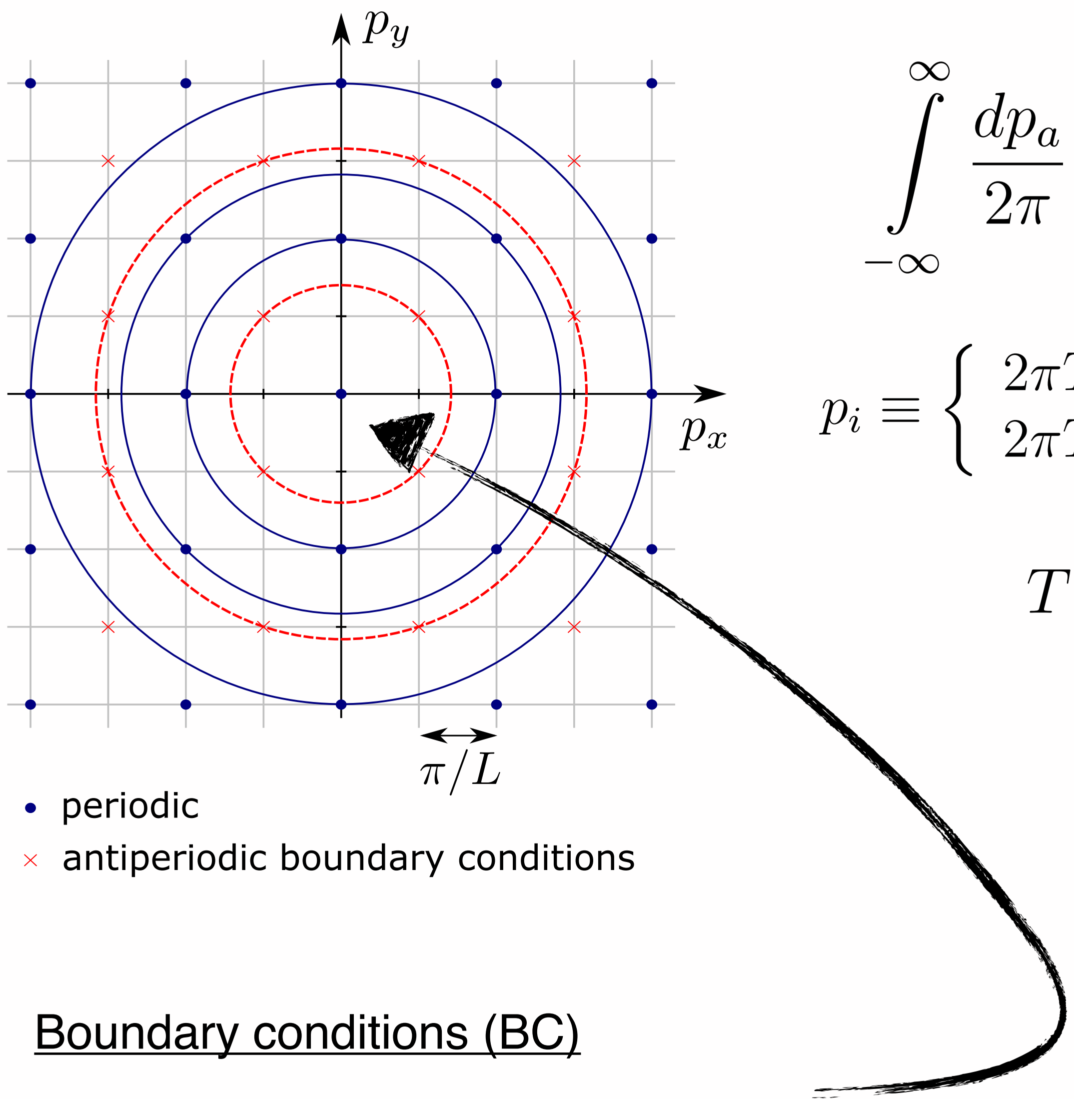
Finite volume



- periodic
- × antiperiodic boundary conditions

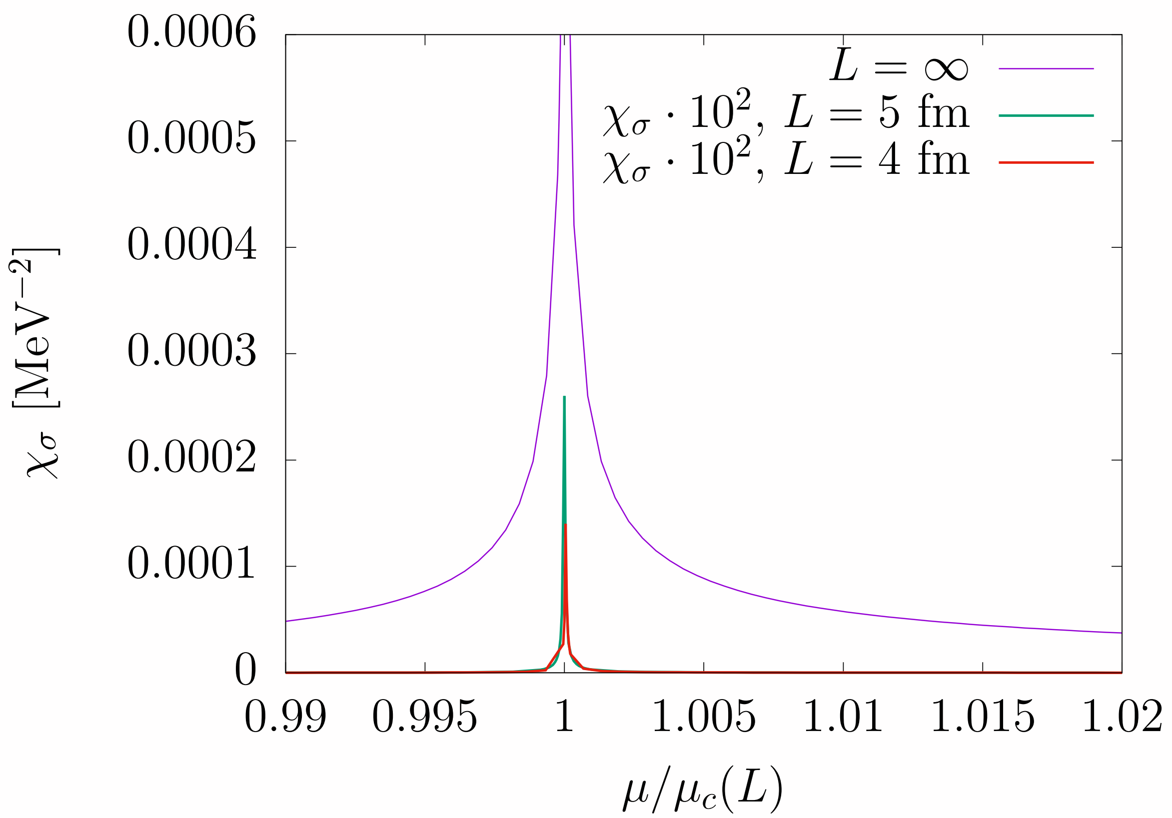
$$\int_{-\infty}^{\infty} \frac{dp_a}{2\pi} \ldots \to \frac{1}{L} \sum_{n_a}$$

$$p_i \equiv \begin{cases} 2\pi T n_i \\ 2\pi T (n_i + \frac{1}{2}) + i\mu \end{cases}$$

$$T \leftrightarrow 1/L$$

Longitudinal susceptibility:

$$\chi_\sigma = \frac{1}{m_\sigma^2} \sim \frac{\partial \langle \bar{q} q \rangle}{\partial m_q}$$



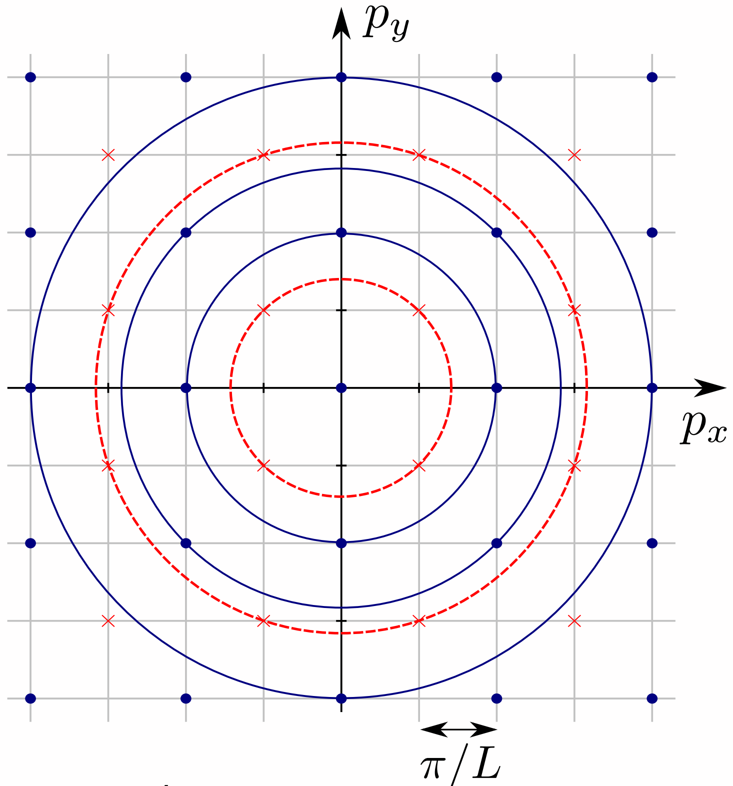
<u>Boundary conditions (BC)</u>

PBC: periodic including zero mode

PBC*: star means without zero mode

ABC: antiperiodic

Finite volume



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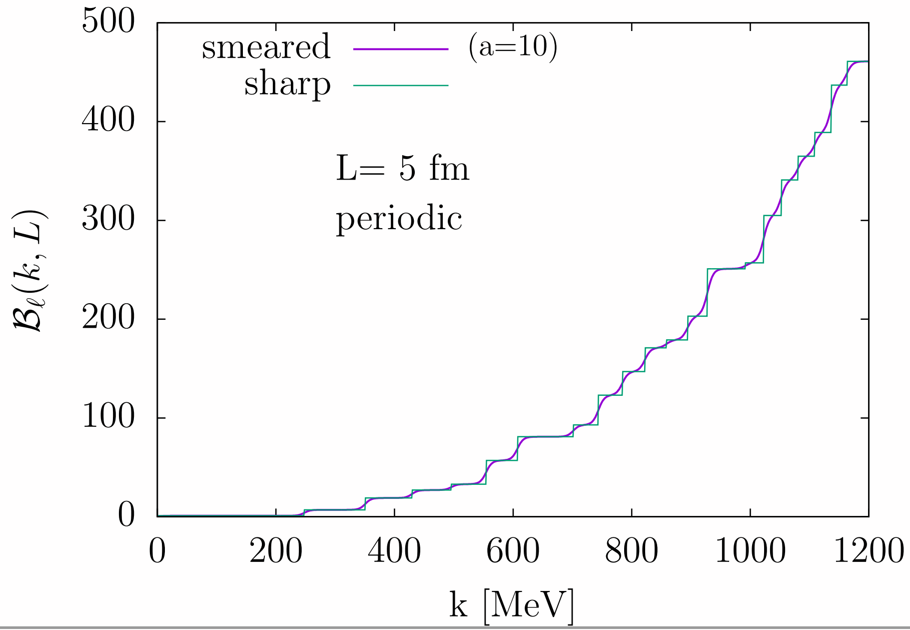


Flow for sharp Litim regulator (not suitable for finite volume)

$$\partial_k U_k(T,L) \sim \mathcal{B}_\ell \cdot \partial_k U_k(T,\infty)$$

$$\mathcal{B}_\ell(k,L) = \frac{6\pi^2}{(kL)^3} \sum_{\vec{n}} \Theta(k^2 - \vec{p}_\ell^{\,2})$$

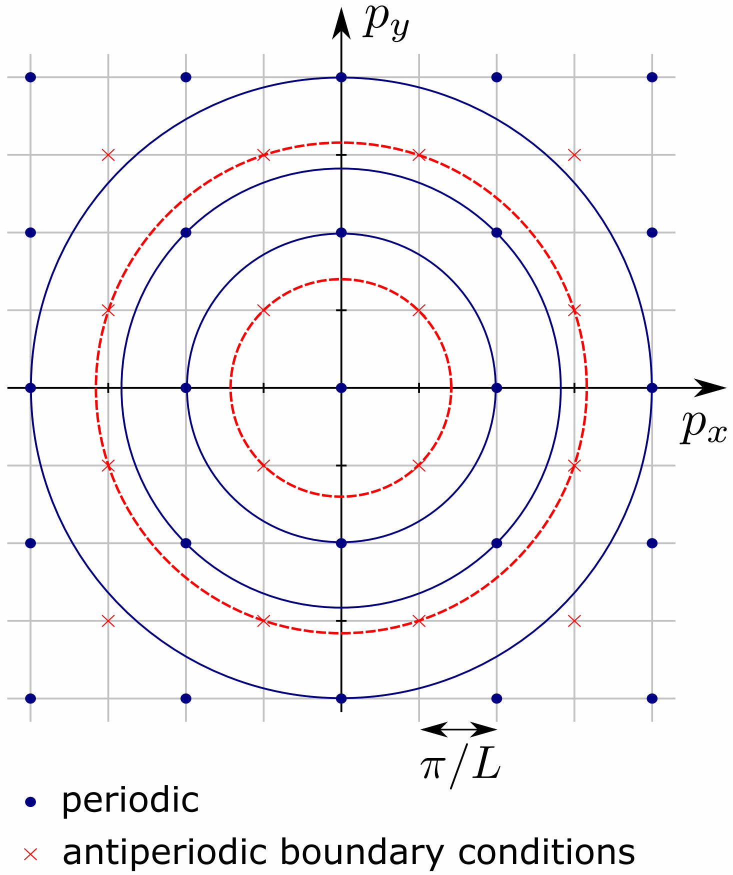
➜ use smeared regulator

[Juricic, BJS in preparation]

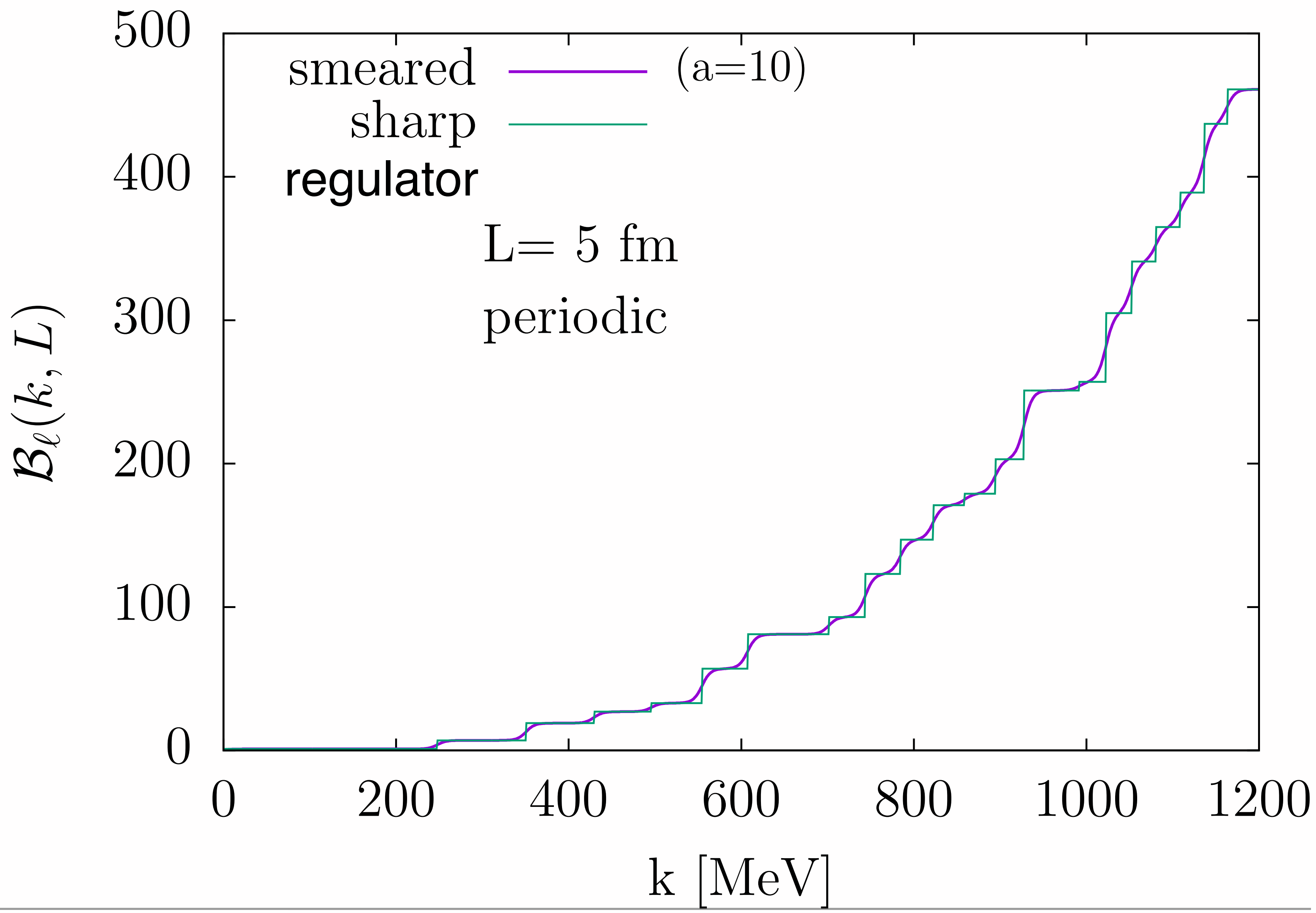
[Fister, Pawlowski 2015]

[Tripolt, Braun, Klein, BJS 2012, 2014]

Finite volume



mode counting function:



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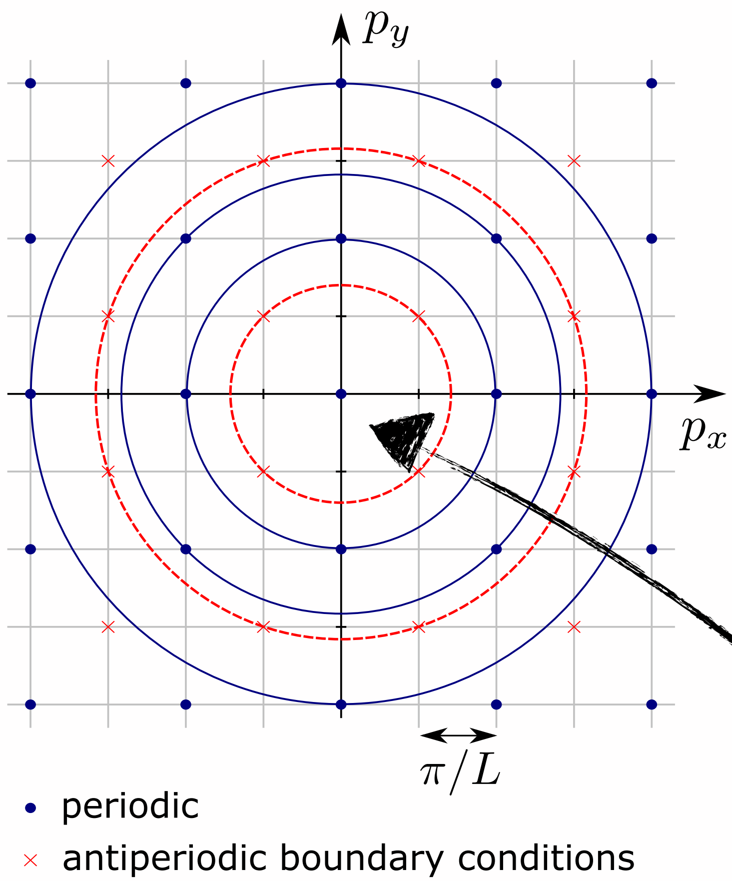

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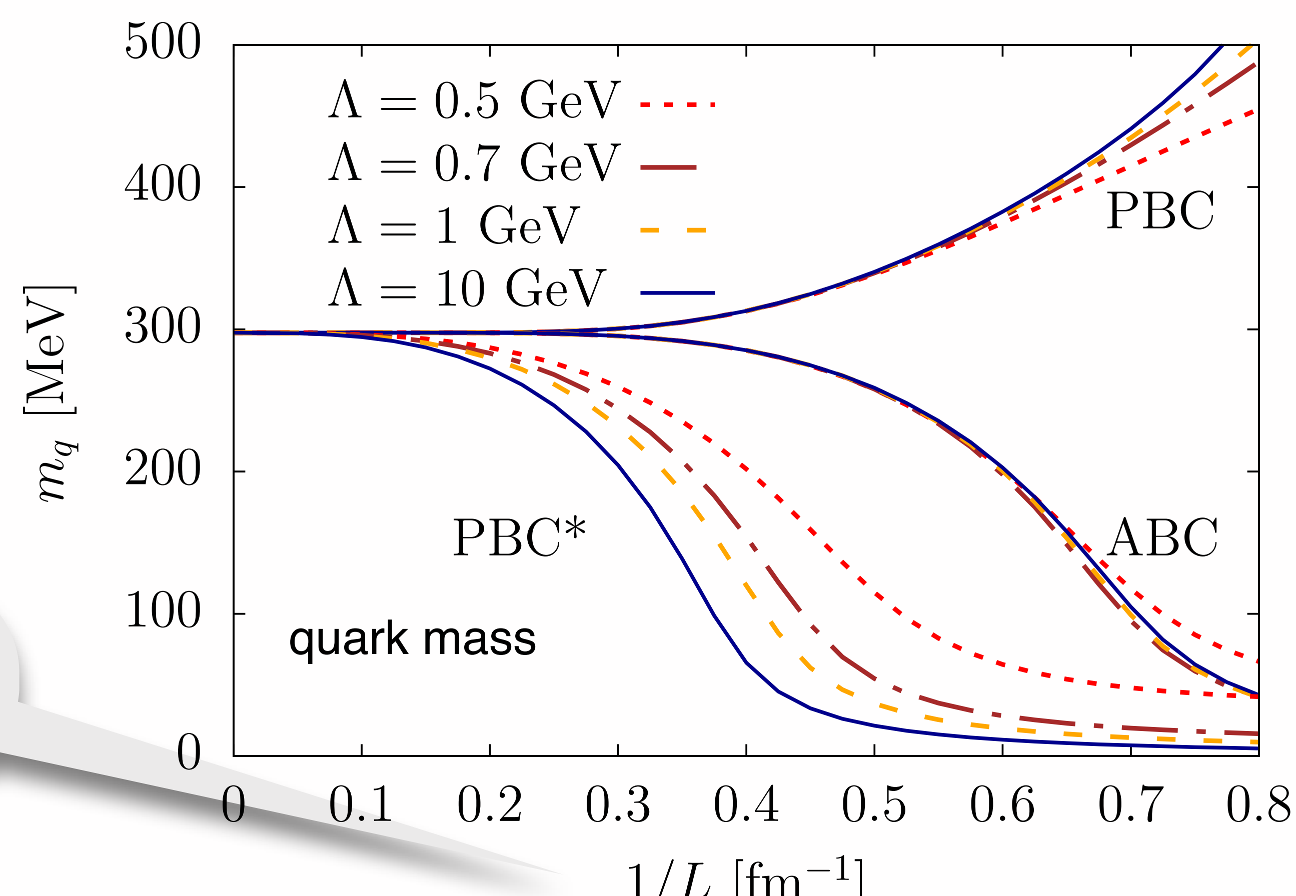
[A Juricic, BJS in preparation]



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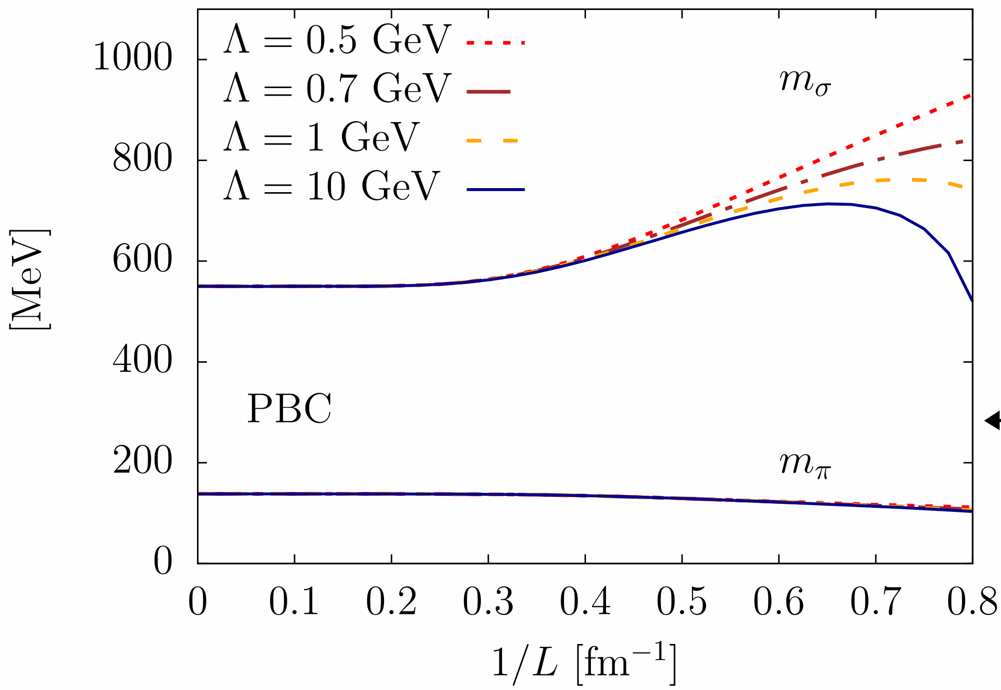
Vacuum meson masses

[A Juricic, BJS in preparation]

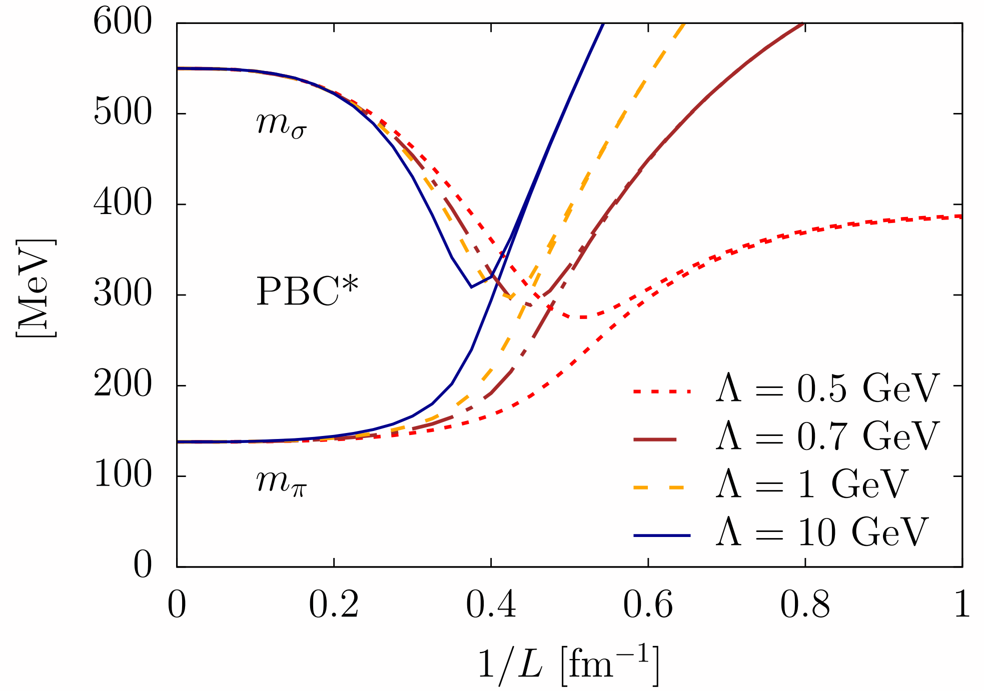


curvature masses (not pole masses)

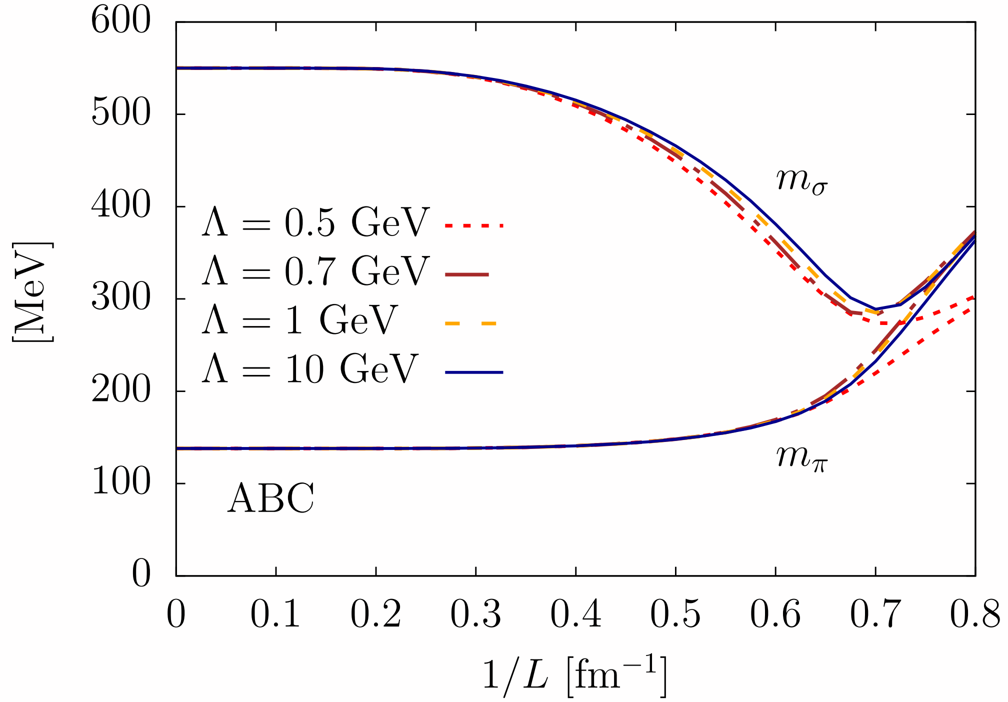
← periodic including zero-mode (PBC)

periodic without zero-mode (PBC*)

↙



antiperiodic (ABC)



Thermodynamics on a torus

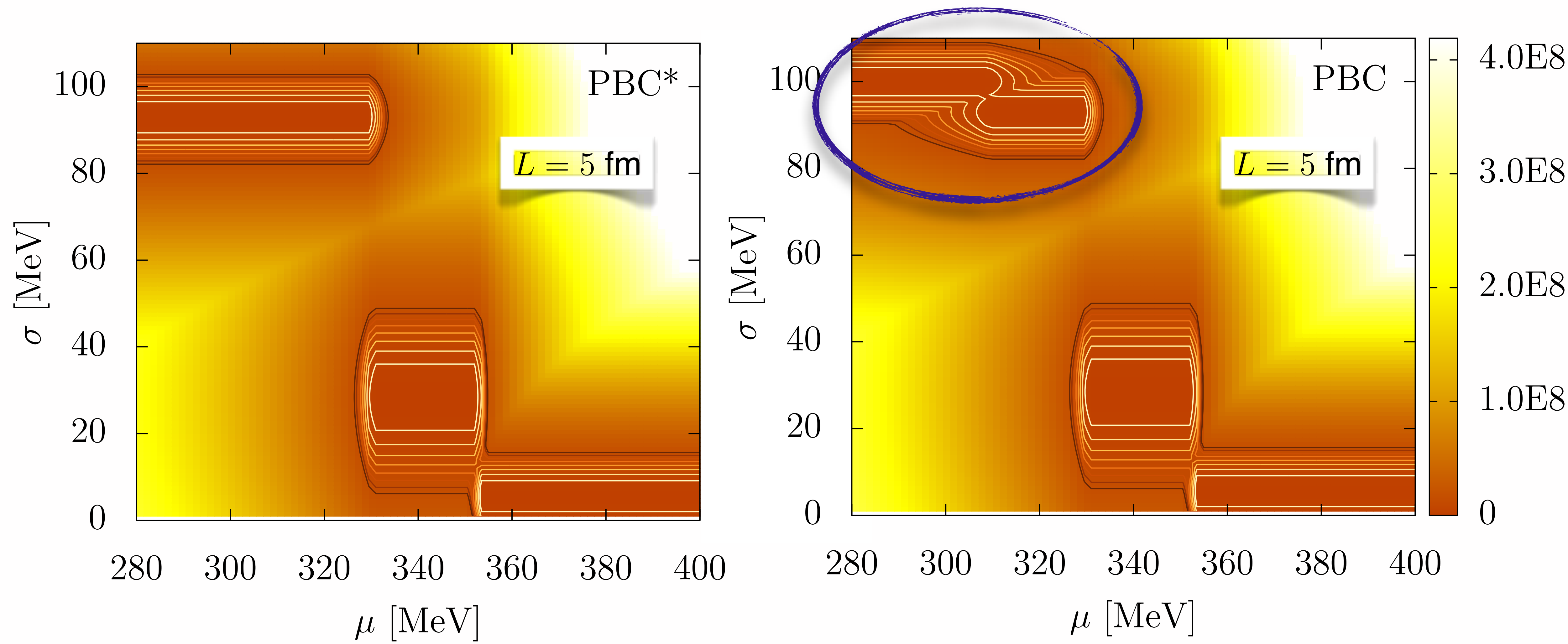
grand potential T=0 & μ>0:

[A Juricic, BJS in preparation]

$$U^{\text{therm}} = 2N_cN_f\frac{1}{L^3}\sum_{\vec{n}\in\mathbb{Z}^3}(\mu - E_{q,\ell})\Theta(\mu - E_{q,\ell})$$

for each mode: discontinuous jumps in potential

contour plot of potential



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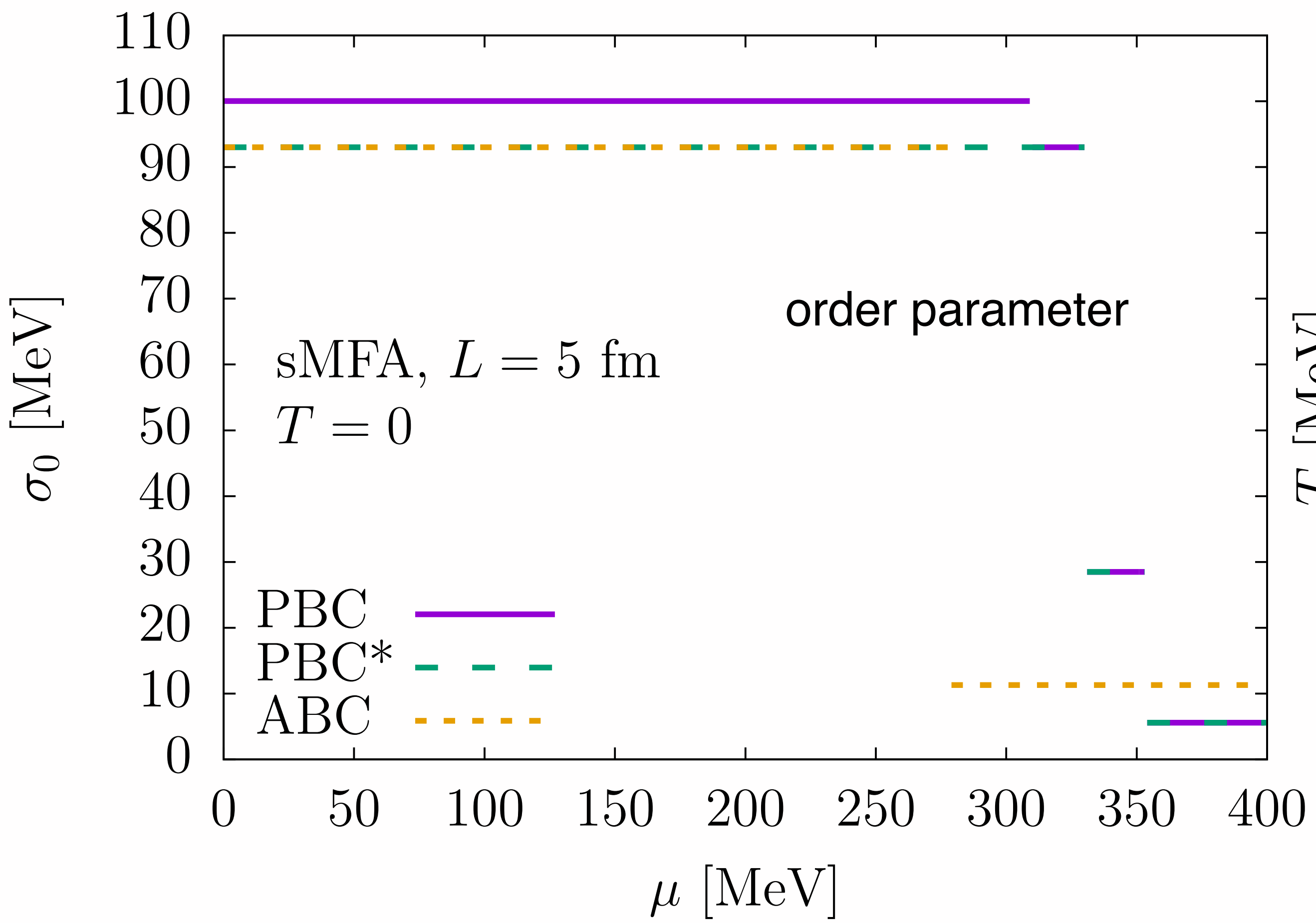
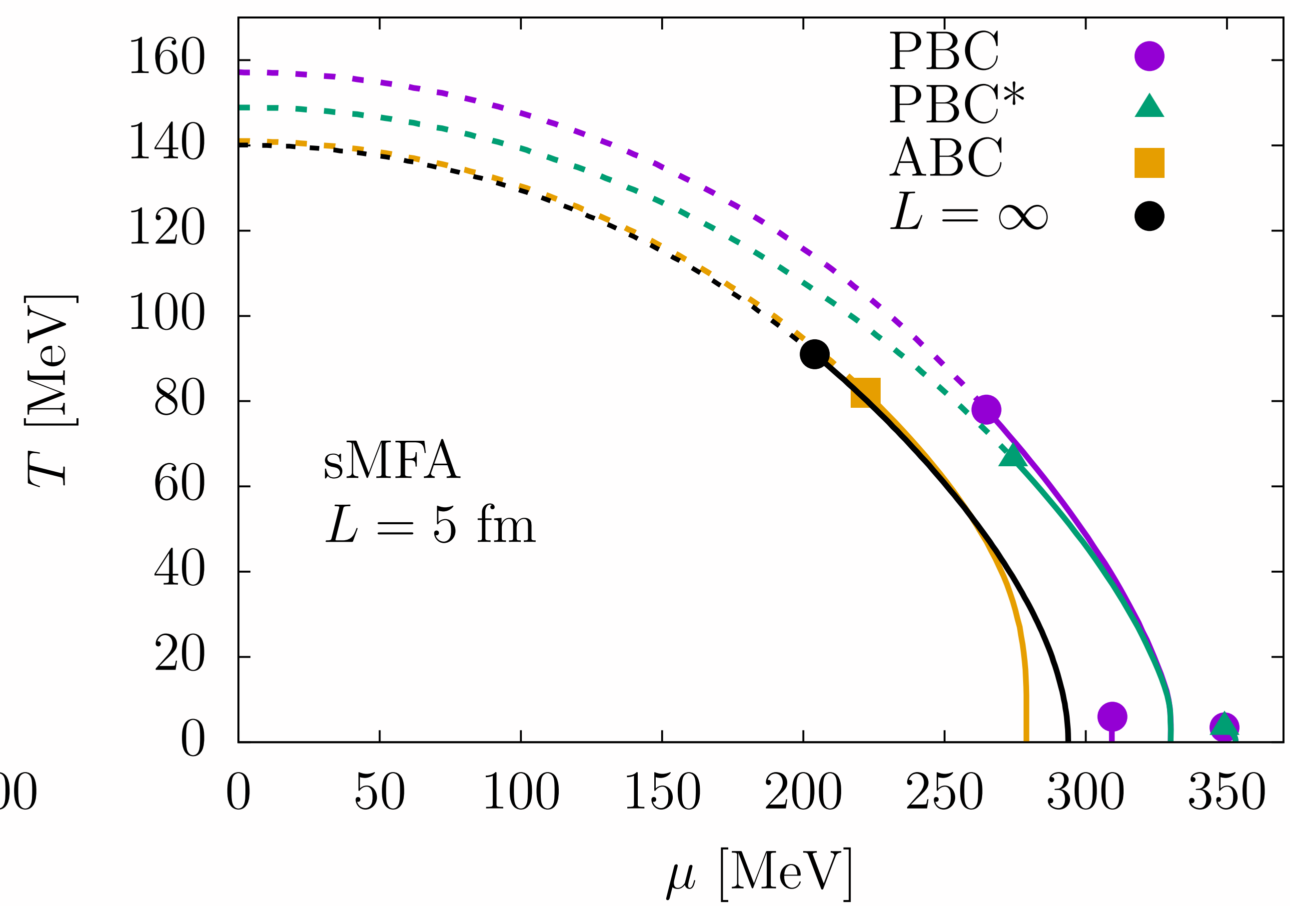
discontinuous jumps washed out at T>0



phase diagram without vacuum fluctuations



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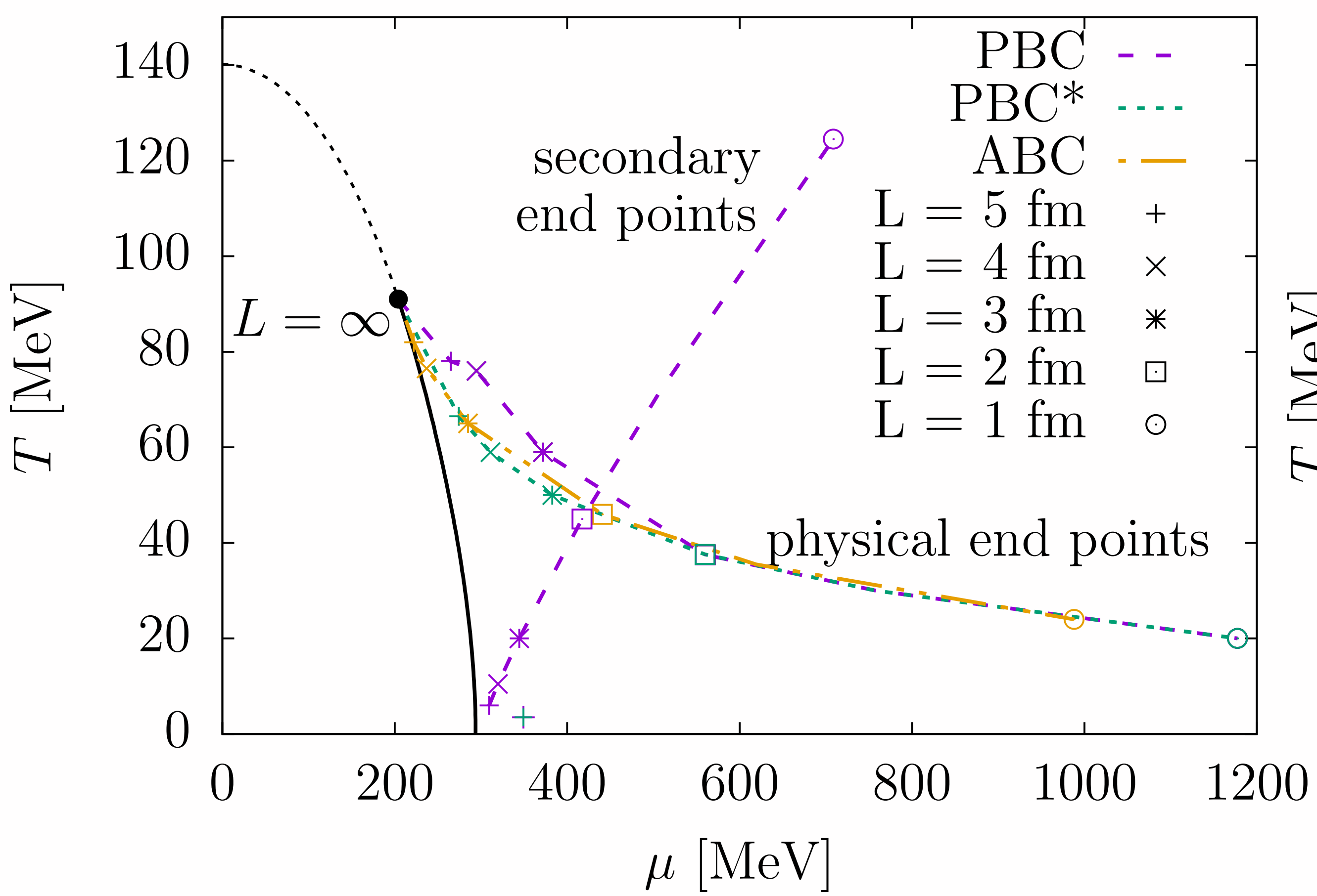
movement of the CEP's

standard MFA (no vacuum fluctuations)



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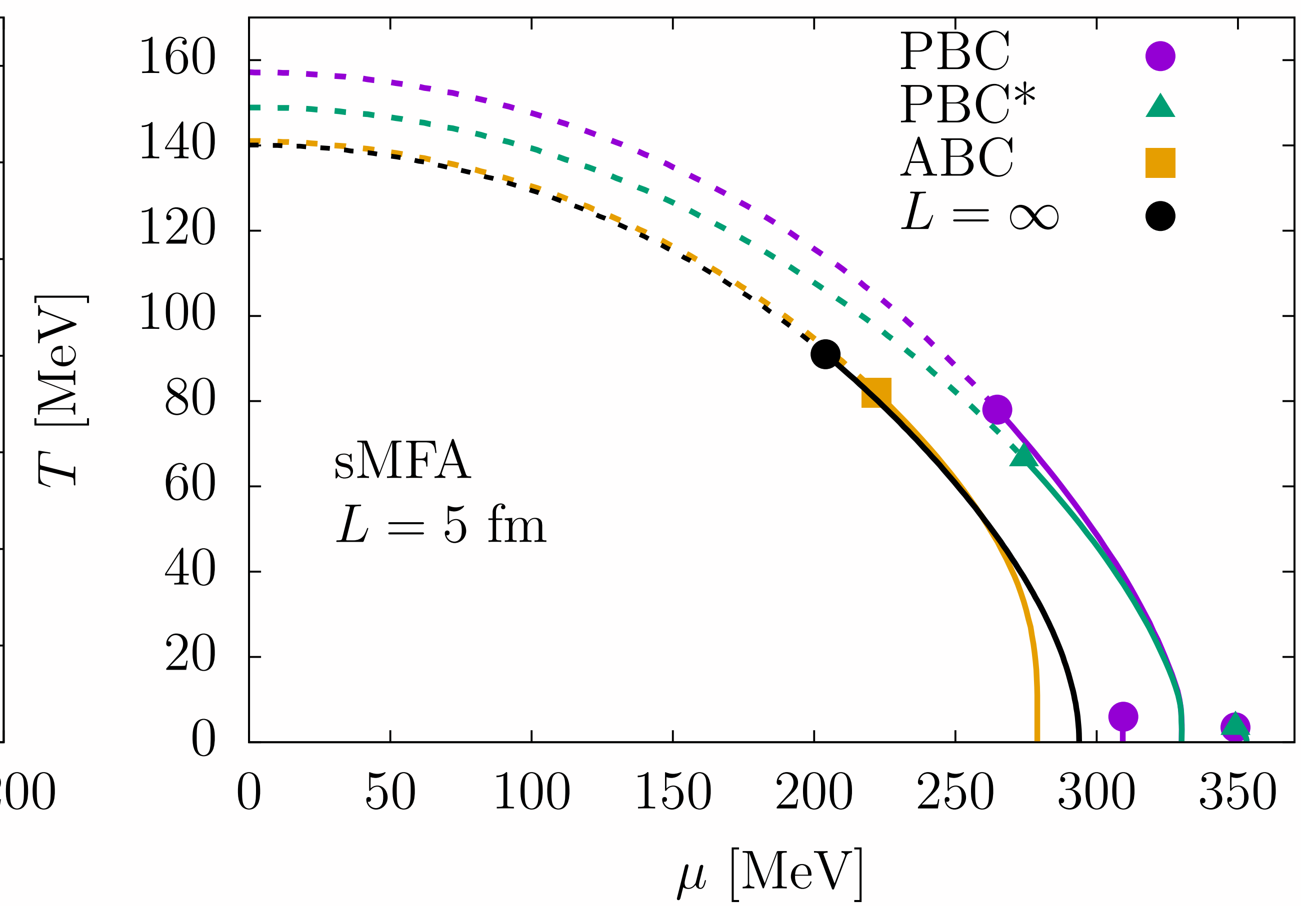

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Vacuum fluctuations

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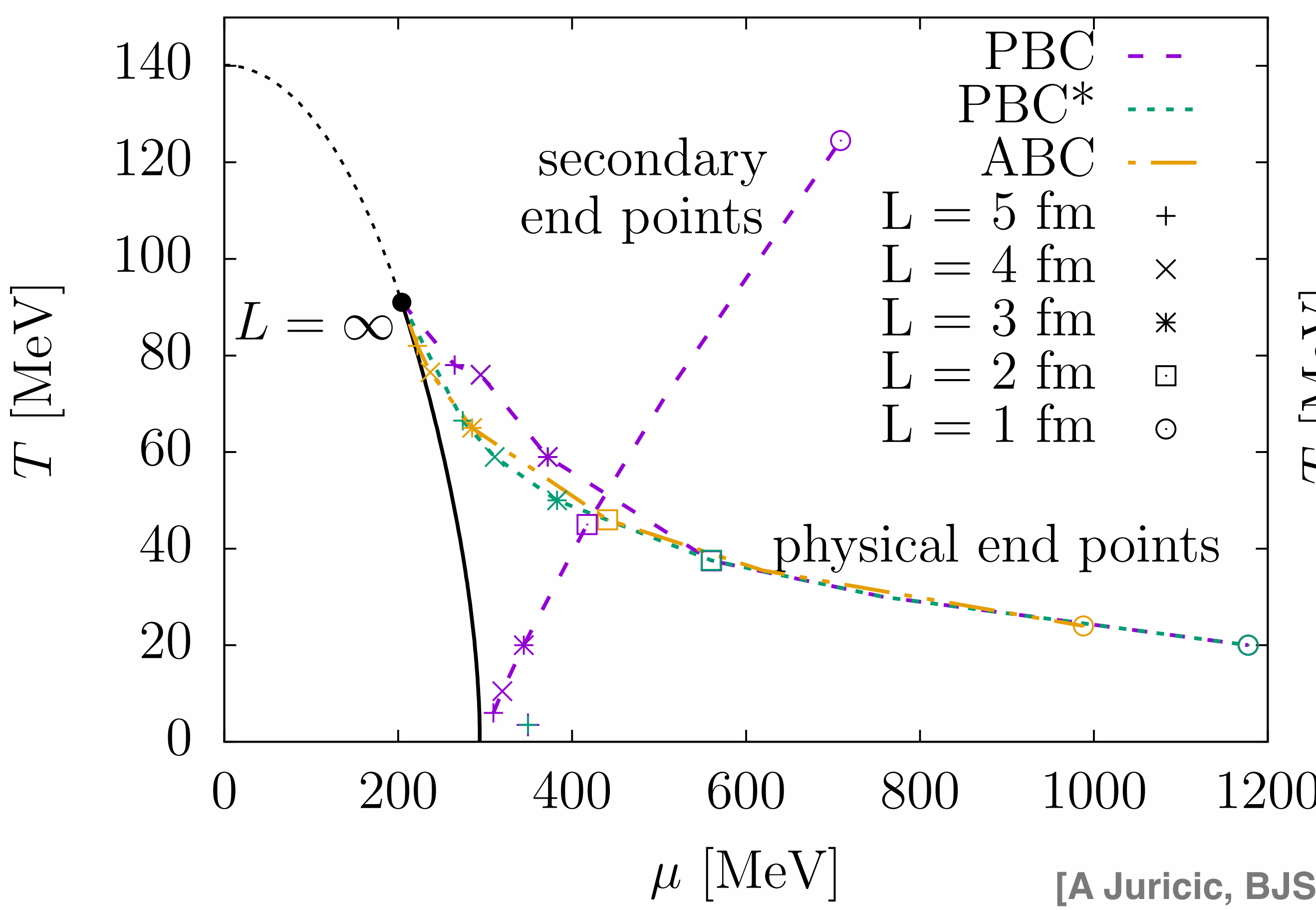
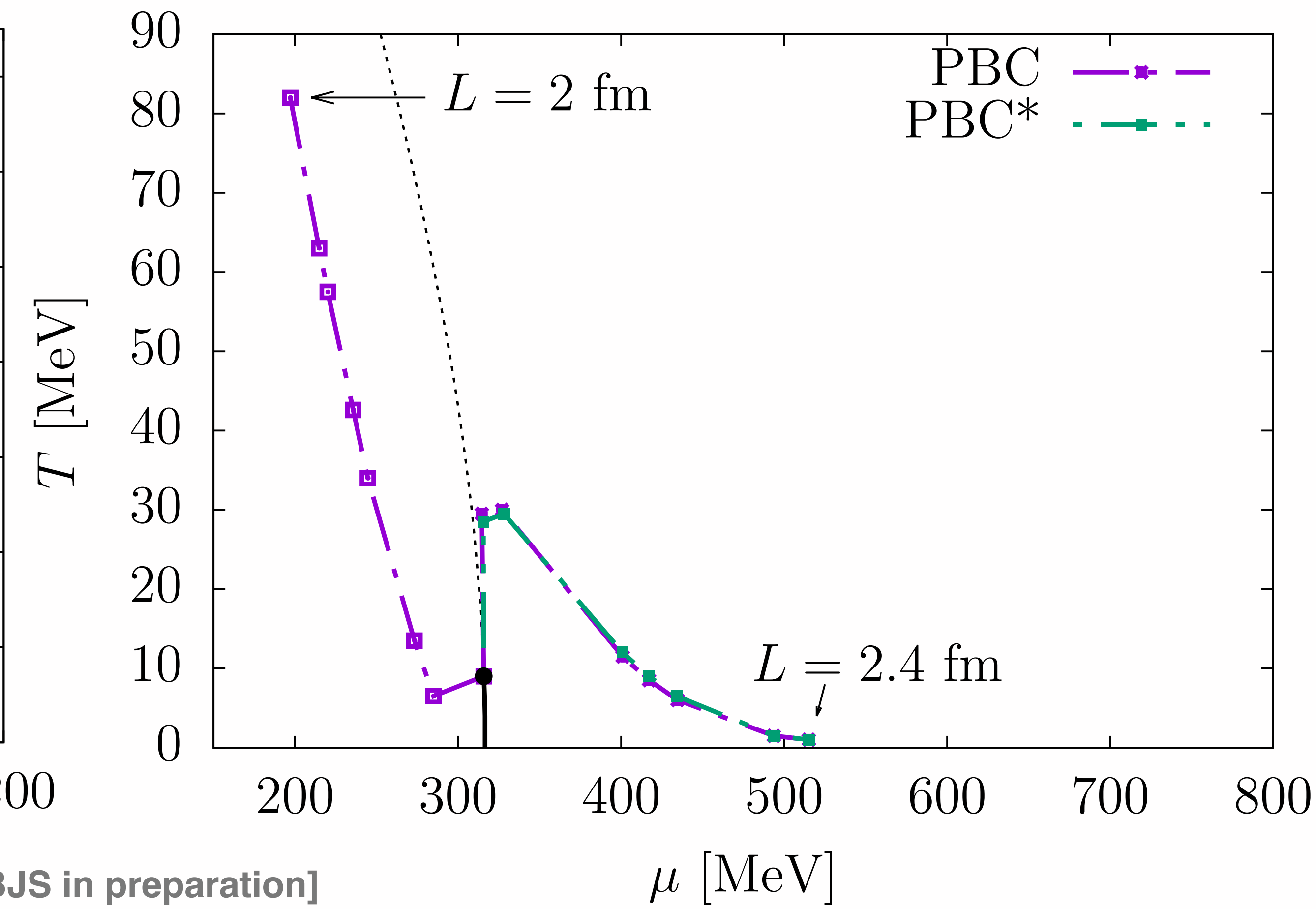


renormalizes MFA (with vacuum fluctuations)



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Vacuum fluctuations

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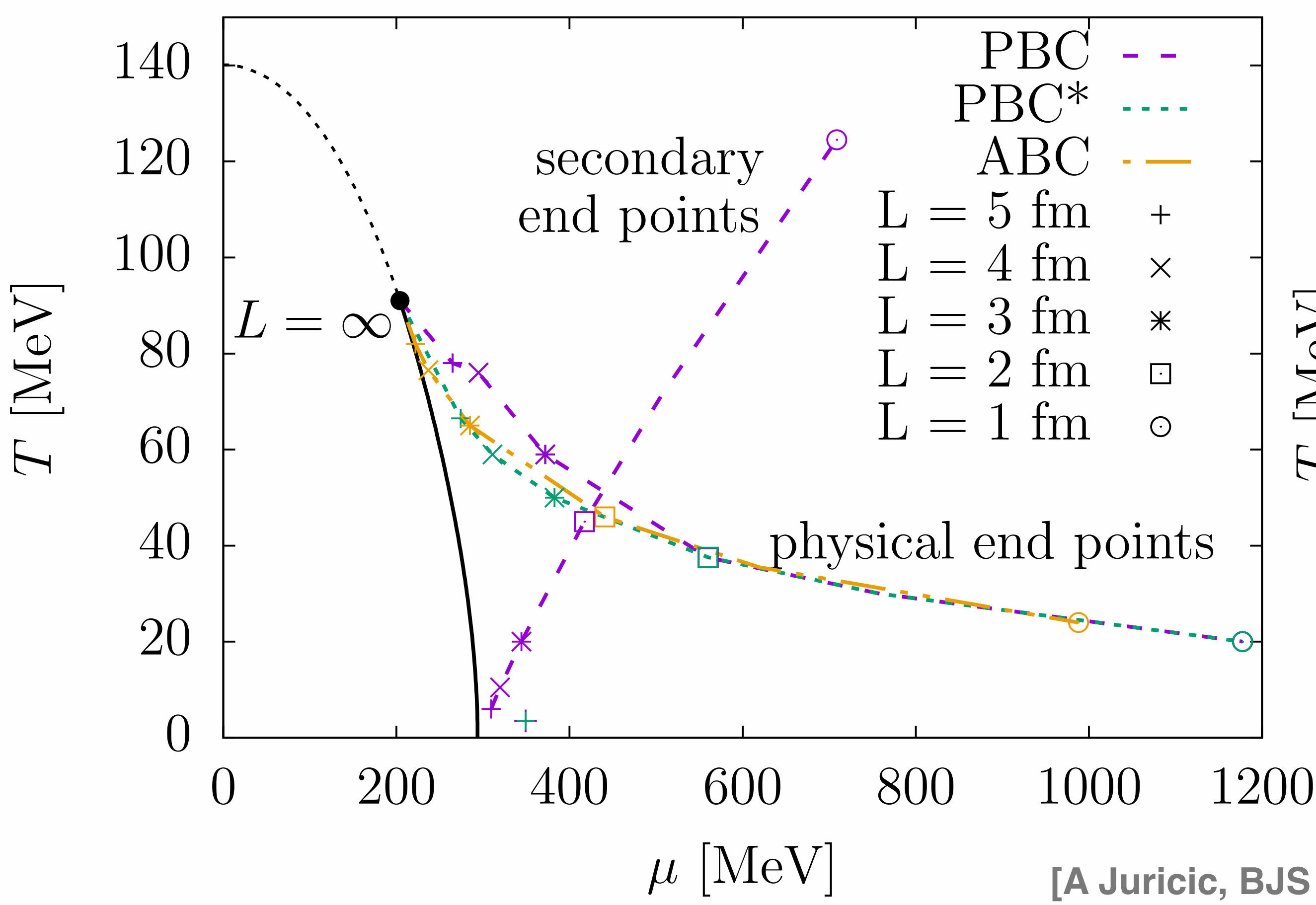

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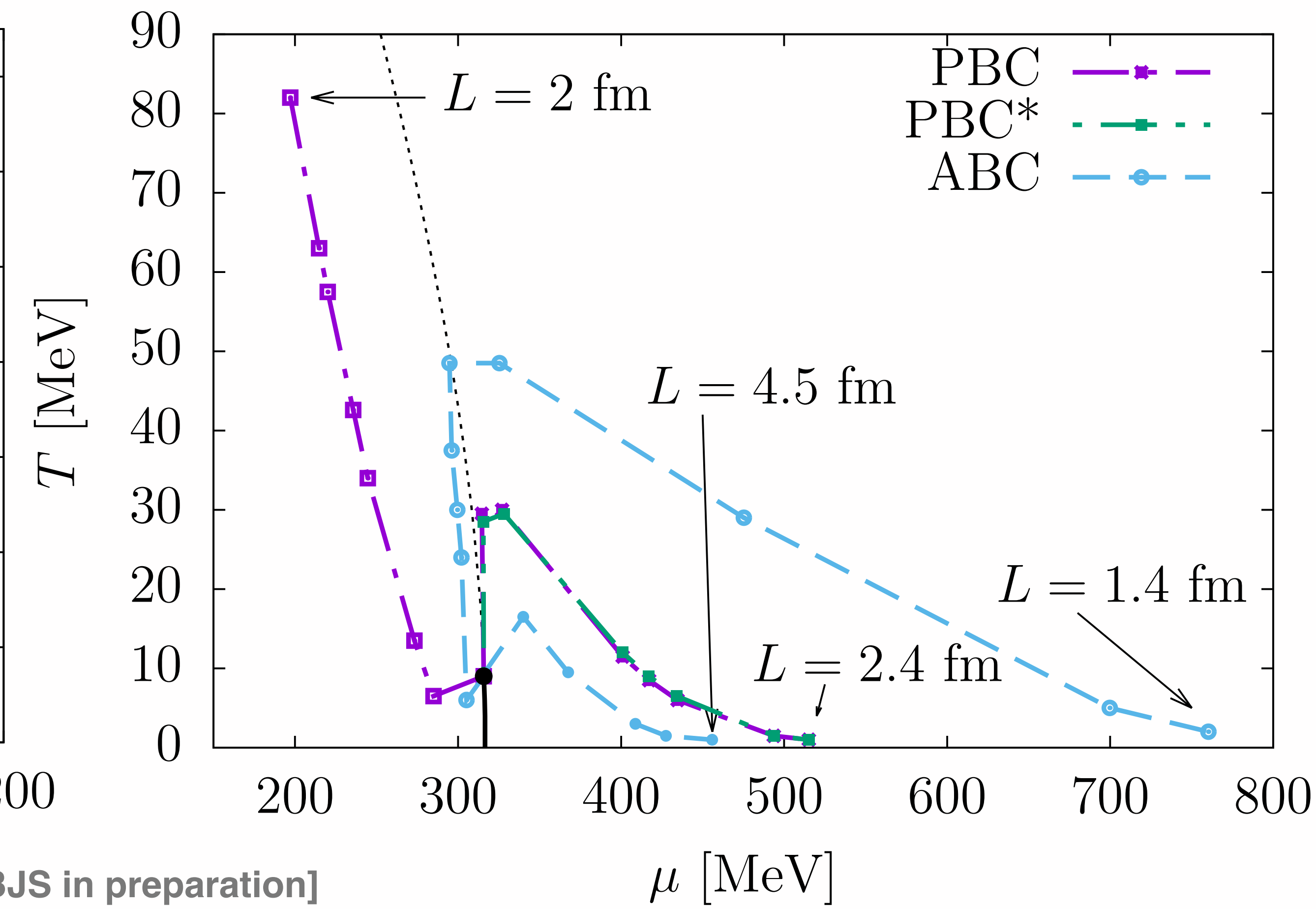

[A Juricic, BJS in preparation]

Summary & Conclusions

- effects of quantum and thermal fluctuations in a box

  comparison: sMFA, rMFA, RG

  ➜ fluctuations wash out the phase transition

- existence of critical points in phase diagram
  in finite volume

- crossover curvature changes in a box

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