Dynamical restoration of Z(N) symmetry in SU(N)+Higgs theories

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Outline

Motivation

- Confinement-deconfinement transition in $SU(N)$ gauge theories can be studided by studying the Z_N symmetry.
- In presence of fundamental matter fields coupled to gauge fields, Z_N symmetry of the theory is believed to be explicitly broken.
- This explicit breaking affects the nature of the CD transition and the thermodynamic behavior of the phases themselves.
- It weakens the CD transition and in the deconfined phase all but only one of the N phases become metastable.
- In the case of fermions with small mass, 1-loop perturbative calculations suggests that the explicitly breaking is so large that there are no metastable states.
- But non-perturbative studies of the effects of the matter fields are required near the CD transition.

Z(N) symmetry in SU(N) gauge theories

Euclidean $SU(N)$ action for the gauge fields

$$S = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} Tr\left(F^{\mu\nu} F_{\mu\nu}\right) \right\} \tag{1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \qquad A_\mu = A_\mu^a T^a. \tag{2}$$

In Euclidean theory,

$$A_\mu^a(\vec{x}, 0) = A_\mu^a(\vec{x}, \beta) \tag{3}$$

The transformation of the gauge fields under $SU(N)$ is

$$A_\mu \longrightarrow U A_\mu U^{-1} + \frac{1}{g}\left(\partial_\mu U\right) U^{-1}. \tag{4}$$

The invariance of the pure gauge action and the periodicity of the gauge fields can be satisfied by,

$$U(\vec{x}, \tau = 0) = zU(\vec{x}, \tau = \beta). \tag{5}$$

The Polyakov loop (L) which is the path ordered product of links in the temporal direction,

$$L(\vec{\mathbf{x}}) = \frac{1}{N} Tr \left\{ P e^{\left(-ig \int_0^\beta A_0 d\tau \right)} \right\} \tag{6}$$

transforms as $L \longrightarrow zL$ under a gauge transformation

In pure $SU(N)$ gauge theories, this is the ideal candidate for an order parameter for confinement - deconfinement transition since it is zero in the confined phase and acquires non-zero value in the deconfined phase.

Thus in deconfined phase the Z_N symmetry is spontaneously broken and we have N degenerate vacua.

The action in presence of Higgs field Φ in fundamental representation is,

$$S = \int_V d^3x \int_0^\beta d\tau \left\{ \frac{1}{2} Tr \left(F^{\mu\nu} F_{\mu\nu} \right) + \frac{1}{2} |D_\mu \Phi|^2 + \frac{m^2}{2} \Phi^\dagger \Phi + \frac{\bar{\lambda}}{4!} (\Phi^\dagger \Phi)^2 \right\}. \tag{7}$$

where $\Phi(\vec{x}, 0) = \Phi(\vec{x}, \beta)$.
Under a gauge transformation $U(\vec{\mathbf{x}}, \tau)$ the Φ field transforms as,

$$\Phi' = U\Phi. \tag{8}$$

Φ' is periodic only when the gauge transformations are periodic. Thus Z_N symmetry is explicitly broken by the presence of the Higgs field.

The simulation

The discretized lattice action is,

$$S = \beta \sum_p Tr(1 - U_p - U_p^\dagger) - \kappa \sum_\mu Re\left[(\Phi^\dagger_{n+\mu} U_{n,\mu} \Phi_n)\right] + \frac{1}{2}\left(\Phi^\dagger_n \Phi_n\right) + \lambda \left(\frac{1}{2}\left(\Phi^\dagger_n \Phi_n\right) - 1\right)^2 \quad (9)$$

Initialise the configurations Φ_n and $U_{\mu,n}$

Generate a Monte Carlo history by updating the old configuration according to the Boltzmann probability factor e^{-S} and the principle of detailed balance.

We use pseudo heat-bath algorithm for the Φ field and the standard heat-bath algorithm for the link variables U_μ's.

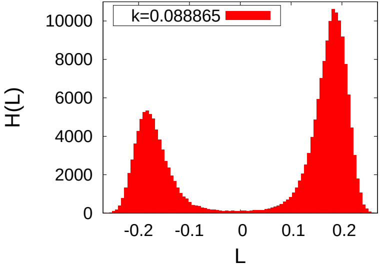
The Z_N rotation can be effected by multiplying the temporal links on a fixed temporal slice of the $4-D$ lattice by an element of the Z_N group.

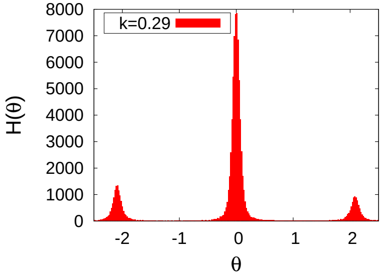
For a fixed (λ, β):
For $\kappa > \kappa_c$ the system is in the Higgs phase
For $\kappa < \kappa_c$ the condensate vanishes leading to the Higgs symmetric phase.

For a given (λ,κ):
$\beta < \beta_c$ corresponds to the confinement phase and $\beta > \beta_c$ to deconfined phase.

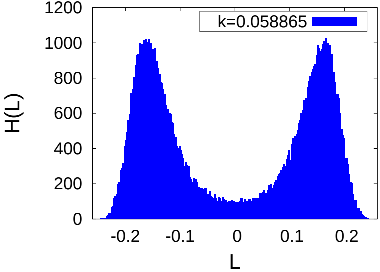
We study the distribution of Polyakov loop in the deconfined phase in the Higgs broken and symmetric phase

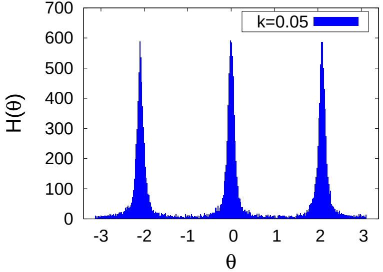
Distribution of Polyakov loop

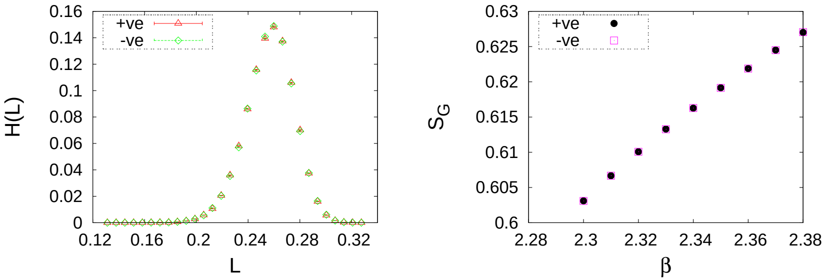




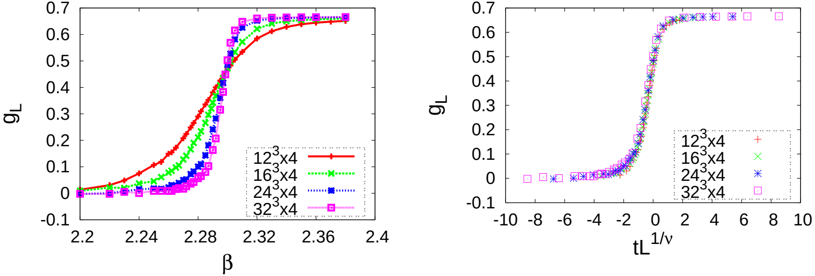
Higgs broken phase for $SU(2)$ and $SU(3)$ cases.







(a)The Polyakov loop distributions for $\beta = 2.38$,$\kappa = 0.056$,$\lambda = 0.005$ and lattice is $16^3 \times 4$ and (b) the gauge action for the two sectors



(a)Binder cumulant and (b) and its scaling for *SU*(2)

Summary and conclusions

We have studied the Z_N symmetry in $SU(N)$+Higgs theories for $N = 2, 3$ using Monte Carlo simulations.

The strength of the explicit symmetry breaking of Z_N seems to vanish in the Higgs symmetric case.

The patterns of explicit symmetry breaking observed in $N = 2$ and $N = 3$ are very similar.

Conventionally it is expected that the explicit symmetry breaking will vanish only when κ is zero.

In our simulations it is found that the explicit symmetry breaking vanishes for small but non-zero values of κ in the Higgs symmetric phase. This will have very interesting consequences like the

appearance of Z_N domain walls in the system.

What about QCD?

There have been results which suggest that Z_3 metastable states appear at high temperatures in QCD.

We could expect the explicit breaking to vanish, i.e, the Z_3 vacua to be degenerate when the chiral symmetry is restored.

In this case $\bar{\psi}\gamma_0\psi$ which couples to the A_0 field may play the role similar to the Higgs field.

In QCD at high densities, we expect di-fermion condensation in the color channel 3^*.

In this case qq will form spin singlet Cooper pairs and will form a BCS color superconductor known as the CFL phase.

There has been discussions on the possibilty of phases without the Bose Einstein condensation, but with qq bound states (pre-CFL).

Thus we have a gauge theory coupled to bosonic degrees of freedom in 3^* representation. Similar restoration of Z_N may should happen in this case.