The Asymmetry of the $\langle A^2 \rangle$ Condensate and Propagators in the $SU(2)$ Gluodynamics at $T > T_c$

V.G. Bornyakov[1,3,4], V.K. Mitrjushkin[2], and R.N. Rogalyov[1]

[1] IHEP, Protvino; [2] JINR, Dubna [3] FEFU, Vladivostok [4] ITEP, Moscow

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- Motivation
  - Why $\langle A^2 \rangle$ is of interest?
  - Does semi-QGP make sense?
- Domains of chromoelectric and chromomagnetic dominance
- Propagators and the asymmetry
  are to set boundary between them
- Problem of small lattices at high T
  - Gribov copies and flip sectors
  - Finite-volume effects
- Temperature dependence of the asymmetry and propagators
- Unexpected goodness of perturbatively motivated fits

Dimension 2 operators in gauge theories.

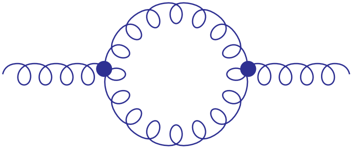
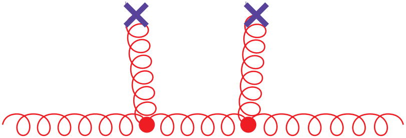
We begin with the Semenov-tyan-Shanskii–Franke functional:

$$\mathcal{F}(A) = \frac{1}{V}\int_V A^a_\mu A^a_\mu dx$$

$$min_{g(x)}\mathcal{F}(A^g) \qquad \Longrightarrow \qquad \partial_\mu A^a_\mu = 0$$

The operator $A^a_\mu A^a_\mu$ in the Landau gauge is

- local;
- Lorentz and BRST invariant;
- needed in the Operator Product Expansion in PT
- $\langle A^2 \rangle$ is gauge-invariant [A.A.Slavnov, 2004]

$$D^{ab}_{\mu\nu}(p) \simeq \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \left(D_{pert}(p^2) + \frac{3g^2 \langle A^2 \rangle}{4(N_c^2 - 1)p^2} + \ldots \right)$$

$$\alpha_s^{MOM}(q^2) \simeq \left(\alpha_s^{MOM}(q^2) \right)_{pert} \left[1 + \frac{9g^2 \langle A^2 \rangle}{4(N_c^2 - 1)q^2} + \ldots \right]$$

$\langle A^2 \rangle$ was computed numerically
from fits to lattice data for the gluon and ghost propagators.

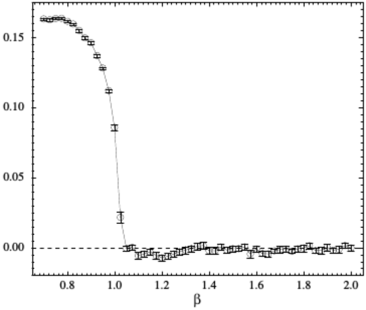
For example, in the Taylor renormalization scheme
(defined by zero incoming ghost momentum) at $\mu = 10$ GeV,
the following values for the $N_f = 2+1+1$ QCD were found
[Blossier, Boucaud et al., 2013]:

$$\langle A^2 \rangle = 2.8(8)\ \text{GeV}^2 \qquad (\text{OPE up to } \frac{1}{p^4})$$

$$\langle A^2 \rangle = 3.8(6)\ \text{GeV}^2 \qquad (\text{OPE up to } \frac{1}{p^6})$$

in order to obtain the QCD coupling constant
$\alpha_{\overline{MS}}(M_Z) = 0.1198(4)(8)(6)$!

Interest in $A_\mu^a A_\mu^a$ aroused in 2001



Rapid change of

$$\langle A^2 \rangle_{noncompact} - \langle A^2 \rangle_{compact}$$

is correlated with the confinement-deconfinement transition in the compact $U(1)$ theory.

F.V.Gubarev, L.Stodolsky, V.I.Zakharov, Phys.Rev.Lett.(2001)

At nonzero temperatures there are two condensates,

$$\langle A_E^2 \rangle = g^2 \langle A_4^a(x) A_4^a(x) \rangle, \qquad \langle A_M^2 \rangle = g^2 \langle \sum_{i=1}^{3} A_i^a(x) A_i^a(x) \rangle.$$

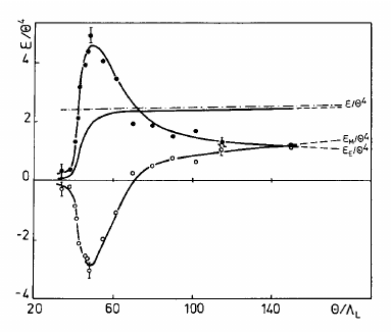
The A^2 asymmetry is defined by the formula

$$\langle \Delta_{A^2} \rangle \equiv \langle A_E^2 \rangle - \frac{1}{3} \langle A_M^2 \rangle \qquad \mathcal{A} = \frac{\langle \Delta_{A^2} \rangle}{T^2}.$$

The asymmetry in terms of the propagators:

$$\mathcal{A} = \frac{4N_t}{\beta a^2 N_s^3} \left[3(D_L(0) - D_T(0)) + \sum_{p \neq 0} \left(\frac{3|\vec{p}|^2 - p_4^2}{p^2} D_L(p) - 2 D_T(p) \right) \right]$$

Postconfinement domain



[Mitrjushkin, Zadorozhny, Zinoviev 1988]

- Polaykov loop behavior
- Failure of PT and old effective theories to evaluate pressure at $T_c < T < 2 \div 3T_c$.
- Monopole density (condensate-liquid-gas)

We are interested not only
in the interplay of $\langle E^2 \rangle$ and $\langle B^2 \rangle$,
but also in the radii of action
of the chromoelectric and chromomagnetic forces

Yet another candidate to distinguish between
the postconfinement and deconfinement domains
is an interplay between $\left\langle A_E^2 \right\rangle$ and $\left\langle A_E^2 \right\rangle$

$$D_{\mu\nu}(p) = D_L(p) P^L_{\mu\nu} + D_T(p) P^T_{\mu\nu} + \alpha \frac{p_\mu p_\nu}{p^4}$$

We consider propagators only for <u>*soft modes*</u> $p_4 = 0$, where

$$P^T_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} - \frac{p_i p_j}{|\vec{p}|^2} \end{pmatrix} \qquad P^L_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$D_L(p) = \frac{1}{p^2 + F(p)}, \qquad D_T(p) = \frac{1}{p^2 + G(p)}$$

$D_L(0) \simeq \frac{1}{m_e^2} \simeq r_e^2$ — chromoelectric forces

$D_T(0) \simeq \frac{1}{r_e^2} \simeq r_m^2$ — chromomagnetic forces

Screening mass in QED

We consider two charges in QED plasma,

$$\vec{E}_1^{cl} = -i\frac{\vec{p}}{|\vec{p}|^2} Q_1 e^{-i\vec{p}\vec{x}_1} \qquad \vec{E}_2^{cl} = -i\frac{\vec{p}}{|\vec{p}|^2} Q_2 e^{-i\vec{p}\vec{x}_2}$$

Each of them can be considered as a small perturbation in the linear response theory:

$$h = \int d\vec{x}\ \vec{E}^{cl}(\vec{x})\vec{E}(\vec{x})\ ,$$

the resulting field has the form

$$E_i^{tot}(\vec{p}) = E_i^{cl} + \langle\langle \delta E_i \rangle\rangle = \frac{p_i p_j E_j^{cl}(\vec{p})}{|\vec{p}|^2 + F(0,\vec{p})}\ .$$

$$
\begin{aligned}
V &\simeq \frac{1}{2}\int d\vec{x}\ \left(\langle\!\langle \vec{E}_1^{tot}\rangle\!\rangle \vec{E}_2^{cl} + \langle\!\langle \vec{E}_2^{tot}\rangle\!\rangle \vec{E}_1^{cl}\right) \\
&= Q_1 Q_2 \int \frac{d\vec{k}}{(2\pi)^3}\ \frac{e^{i\vec{k}(\vec{x}_1-\vec{x}_2)}}{|\vec{k}|^2 + F(0,\vec{k})} \\
&\simeq \frac{Q_1 Q_2}{4\pi}\ \frac{e^{-\,m_e|\vec{x}_1-\vec{x}_2|}}{|\vec{x}_1 - \vec{x}_2|}
\end{aligned}
$$

$$
m_e = \frac{eT}{\sqrt{3}}
$$

Screening in $SU(N_c)$ theories to one loop

Feynman gauge:

$$
\begin{aligned}
F(0,\vec{p}\to 0) &= \frac{1}{3}g^2T^2N_c - \frac{1}{4}g^2TN_c|\vec{p}| \\
G(0,\vec{p}\to 0) &= -\frac{3}{16}g^2TN_c|\vec{p}|
\end{aligned}
$$

Temporal axial gauge ($A_0 = 0$ with the PV pole prescription):

$$
\begin{aligned}
F(0,\vec{p}\to 0) &= \frac{1}{3}g^2T^2N_c - \frac{1}{4}g^2TN_c|\vec{p}| - \frac{11g^2}{48\pi^2}N_c|\vec{p}|^2\ln\left(\frac{|\vec{p}|^2}{T^2}\right) \\
G(0,\vec{p}\to 0) &= -\frac{5}{16}g^2TN_c|\vec{p}|
\end{aligned}
$$

Lattice settings

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\;\; U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}$$

$$U_{x,\mu} \;=\; u_0 \;+\; i \sum_{a=1}^{3} u_a \sigma_a, \qquad (1)$$

$$A^a_\mu = -\,\frac{2Z\; u^a_\mu}{ga}, \qquad (2)$$

$$\Lambda : \quad U_{x,\mu} \to \Lambda^\dagger_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

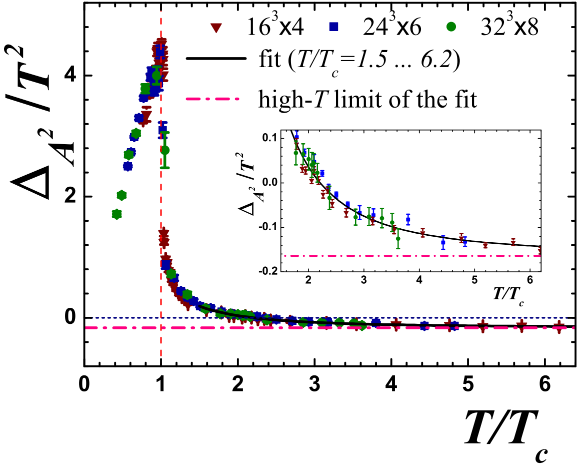
We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \frac{1}{2} \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \qquad (3)$$

Stationarity condition:

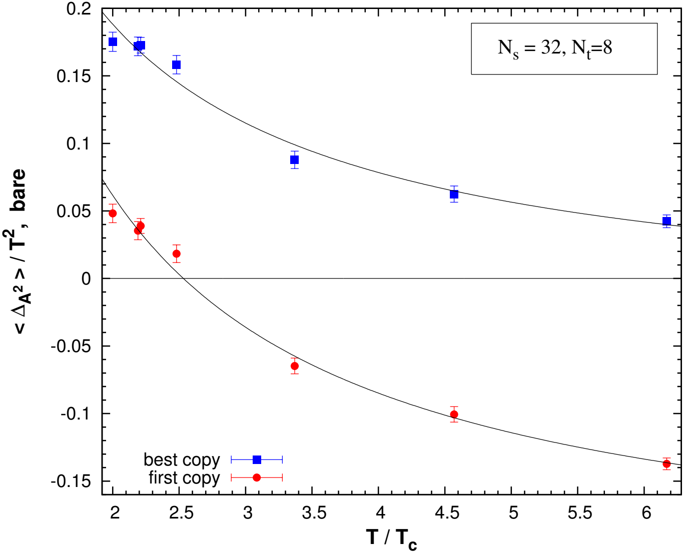
$$\partial_\nu A^a_\nu = 0.$$

We use the simulated annealing algorithm with subsequent over-relaxation



M.N. Chernodub and E.-M. Ilgenfritz, Phys.Rev.D (2008)

main result



Lattice size decreases from 1.3 fm to 0.4 fm

our result

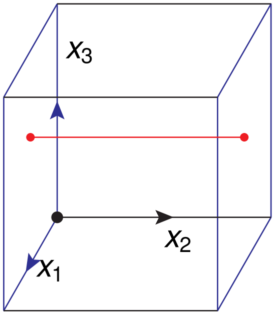
$$A_\mu \to A_\mu^\Lambda = (\Lambda Z)^\dagger A_\mu (\Lambda Z) + \frac{i}{g} (\Lambda Z)^\dagger \partial_\mu (\Lambda Z).$$

$$A_\mu \to A_\mu^\Lambda = \Lambda^\dagger A_\mu \Lambda + \frac{i}{g} \Lambda^\dagger \partial_\mu \Lambda.$$

For $SU(3)$, as an example:

$$Z \in \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^{\frac{2i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{2i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{2i\pi}{3}} \end{pmatrix}, \begin{pmatrix} e^{\frac{4i\pi}{3}} & 0 & 0 \\ 0 & e^{\frac{4i\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{4i\pi}{3}} \end{pmatrix} \right\}$$

Gauge transformation is the same on both sides!



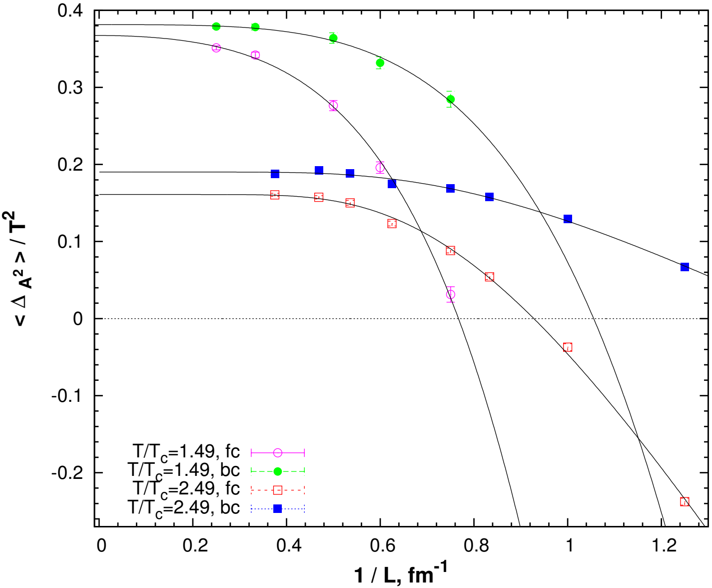
We extend the gauge group by nonperiodic gauge transformations:

$$\Lambda(x_1, b, x_3) = Z\Lambda(x_1, 0, x_3) \text{ etc.}$$

$$P\exp\left(ig\int_0^b A_2(x_1, z, x_3)dz\right) =$$

$$= L(x_1, x_3) \quad \longrightarrow \quad L(x_1, x_3)Z$$

Thus the Hilbert space is broken into 8 superselection sectors



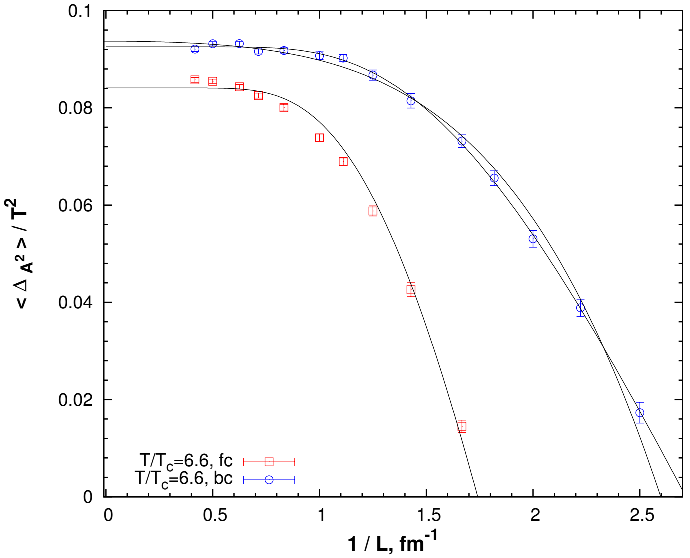
Finite-volume effects

$$\overline{\mathcal{A}}(L) = \overline{\mathcal{A}}_{\infty}^{pol} - \frac{c_2}{L^2} - \frac{c_4}{L^4} \ ,$$

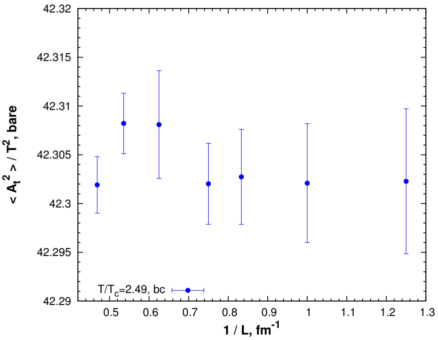
$$\overline{\mathcal{A}}(L) \simeq \overline{\mathcal{A}}_{\infty}^{exp} - c \exp\left(- \ L/L_0 \right)$$

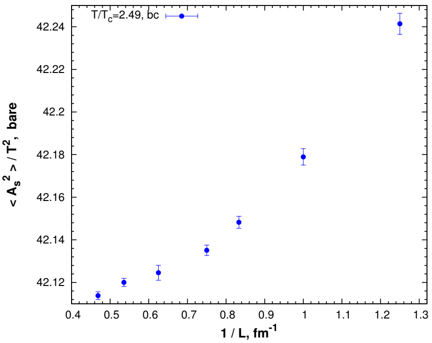
$\frac{T}{T_c}$	Gauge fixing algorithm	$\overline{\mathcal{A}}_{\infty}^{exp}$	c	L_0 (fm)	$\frac{\chi^2}{N_{dof}}$
1.49	*bc*	0.380(2)	1.7(1.0)	0.41(5)	0.34
1.49	*fc*	0.352(1)	4.7(1.0)	0.47(8)	0.06
2.49	*bc*	0.190(2)	1.7(5)	0.31(3)	1.71
2.49	*fc*	0.161(2)	5.6(5)	0.31(1)	2.60
6.60	*bc*	0.09254(21)	1.06(11)	0.151(5)	0.89

Table: Parameters are given for the exponential fit. The quadratic fit function works worse: at $T/T_c = 6.6$ quality of the exponential fit $Q = 0.55$, polynomial - $Q = 0.00072$.



For the best copy both exponential and polynomial fit functions are shown ($N_t = 4$).

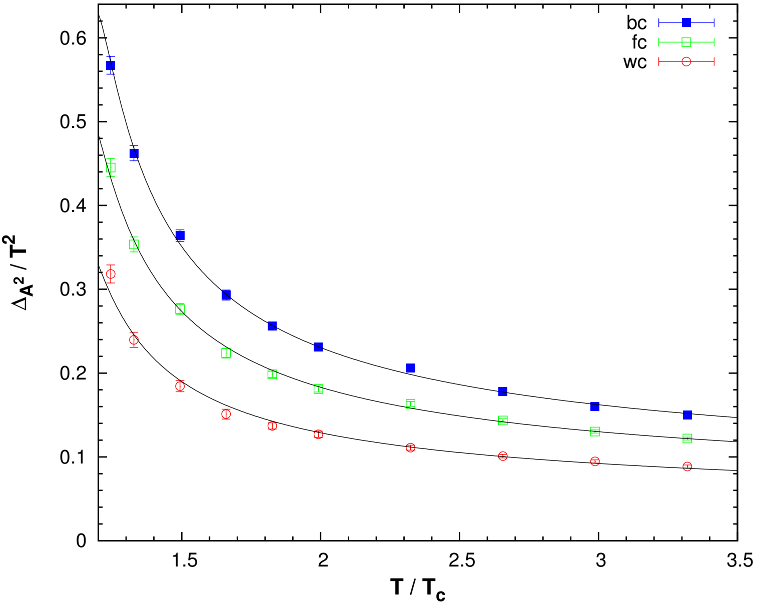




“Chromoelectric” condensate

“Chromomagnetic” condensate

$$\langle \Delta_{A^2} \rangle \equiv \langle A_t^2 \rangle - \langle A_s^2 \rangle \; .$$



bc
fc
wc
0.6
0.5
0.4
0.3
0.2
0.1
0
1.5
2
2.5
3
3.5
Δ_A^2 / T^2
T / T_c

Fitting high-temperature behavior

$$\bar{\mathcal{A}} \simeq b_0 + b_2 \left(\frac{T_c}{T}\right)^2$$

Gauge fixing algorithm	b_0	b_2	$\frac{\chi^2}{N_{dof}}$
bc	0.1036(27)	0.517(16)	1.40
fc	0.0893(22)	0.372(13)	0.92
wc	0.0682(5)	0.231(3)	0.05

Table: $1.65 < T/T_c < 3.32$, fixed lattice size $L = 2$fm.

$b_0 > 0$ in all cases in agreement with perturbation theory

One-loop estimate at high temperatures [Vercauteren *et al.*, 2010]

$$\langle \Delta_{A^2} \rangle \simeq c\, g^2 T^2 \left(1 - \frac{g}{3\pi} \sqrt{\frac{2}{3}} \right) \tag{4}$$

- Perturbation theory (2010): c>0
- Lattice simulations (2008): c<0

$$\bar{\mathcal{A}} \simeq \frac{zg^2(T)}{4}\left(1 - \frac{g(T)}{3\pi}\sqrt{\frac{2}{3}}\right) \; ,$$

where the running coupling is taken in the two-loop approximation,

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2}\;\left(\frac{11}{6}\ln\left(\frac{T^2}{\Lambda^2}\right) + \frac{17}{11}\ln\ln\left(\frac{T^2}{\Lambda^2}\right)\right)\; ,$$

z and Λ are the fit parameters, $1.24 < \frac{T}{T_c} < 3.32$.

$$z = 0.1284(14), \quad \Lambda/T_c = 0.845(7), \quad \frac{\chi^2}{N_{dof}} = 1.50$$

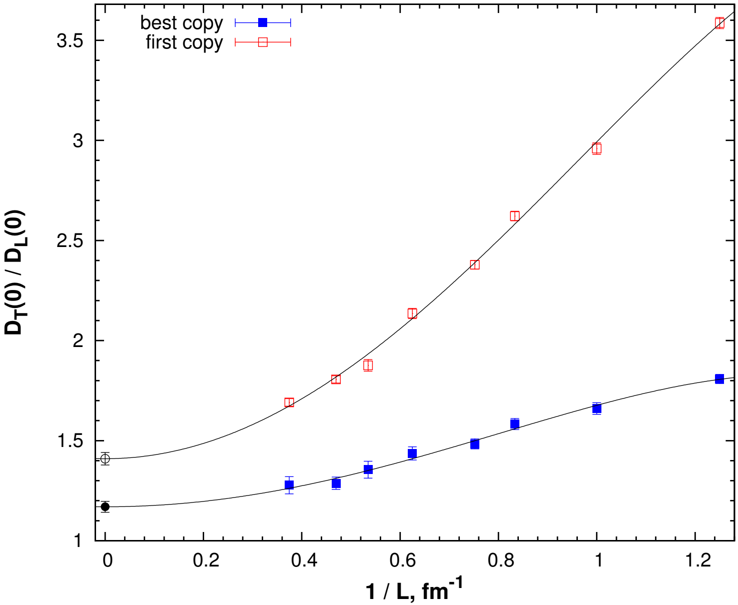
Another definition of screening masses [Heller, Karsch, Rank 97]:

$$\begin{aligned} \tilde{D}_L(p_\perp = 0, x_3) &\sim \exp(-m_e|x_3|), \\ \tilde{D}_T(p_\perp = 0, x_3) &\sim \exp(-m_m|x_3|), \ |x_3| \to \infty \end{aligned}$$

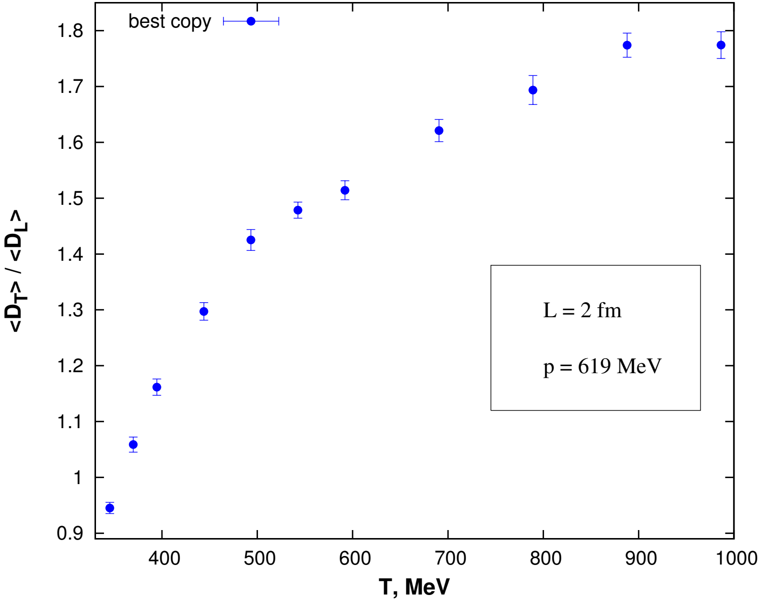
Approximations $m_e = \sqrt{\frac{2}{3}} g(T)T + ...$ and $m_m \sim g^2(T)T$ suggest the fit function ($T > 2T_c$)

$$\frac{m_e^2(T)}{m_m^2(T)} = \frac{C}{g^2(T)} = 1 \quad \text{at} \quad \frac{T}{T_c} = 0.9(1)$$

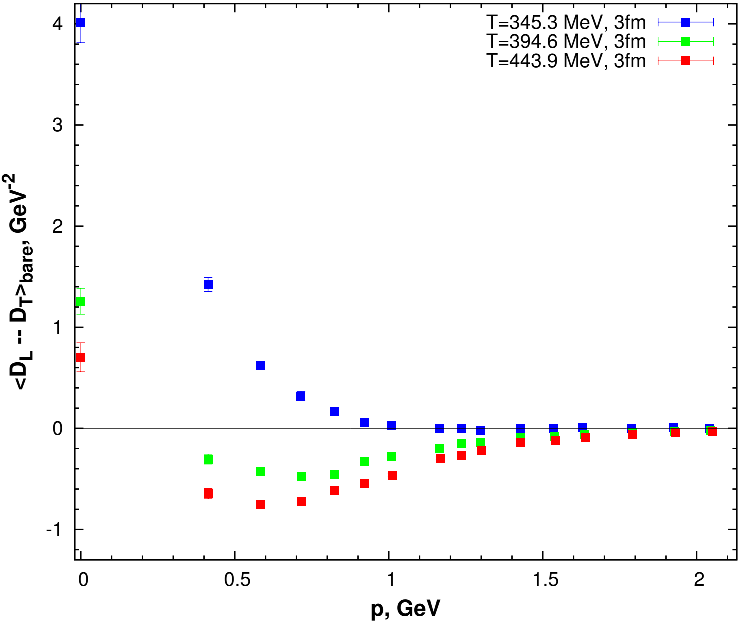
we consider the ratio $r(T) = \frac{D_T(0)}{D_L(0)}$ instead of $\frac{m_e^2}{m_m^2}$



best copy
first copy
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1
1.2
1 / L, fm^-1
D_T(0) / D_L(0)



Ratio of the “magnetic” to the “electric” propagator at $p = p_{min}$



T=345.3 MeV, 3fm
T=394.6 MeV, 3fm
T=443.9 MeV, 3fm
4
3
2
1
0
-1
0
0.5
1
1.5
2
p, GeV
$<D_L - D_T>_{bare}$, GeV^{-2}

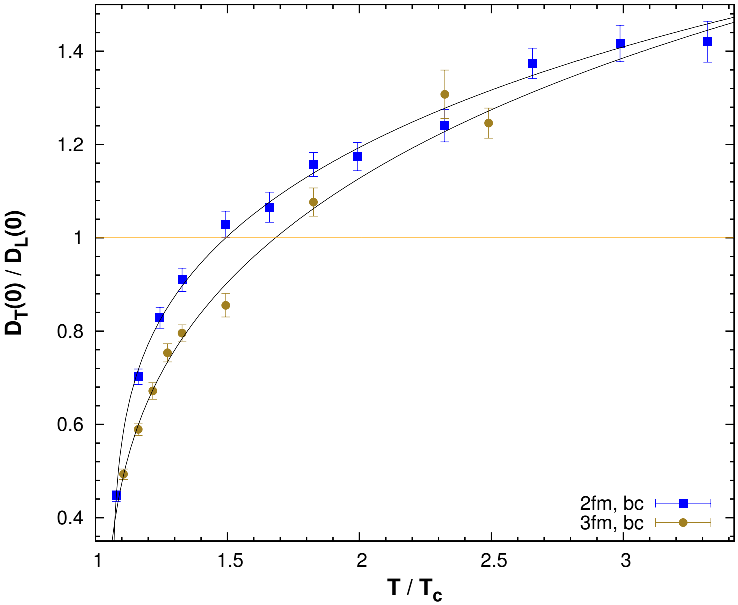
$$r(T) \simeq r_0 + \frac{r_1}{g^2(T)}$$

where

$$\frac{1}{g^2(T)} = \frac{1}{4\pi^2} \left(\frac{11}{6} \ln\left(\frac{T^2}{\Lambda^2}\right) + \frac{17}{11} \ln\ln\left(\frac{T^2}{\Lambda^2}\right) \right) ,$$

Lattice size	r_0	r_1	Λ/T_c	T_p/T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	0.94(1)	3.78(12)	1.060(3)	1.494(30)	0.64
3 fm	0.79(3)	4.59(37)	1.02(2)	1.68(12)	1.42

Table: Fit parameters for the best-copy values of $r(T)$.

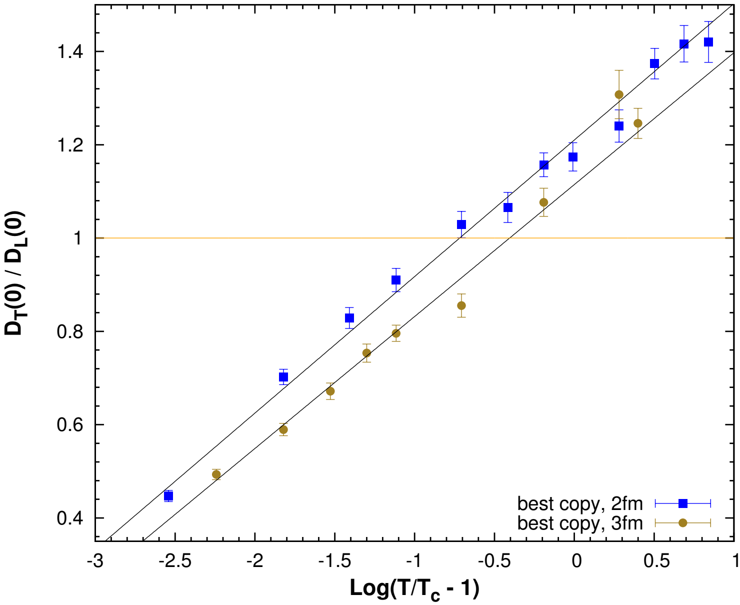


$D_T(0) / D_L(0)$
T / T_c
2fm, bc
3fm, bc
1.4
1.2
1
0.8
0.6
0.4
1
1.5
2
2.5
3

$$r(T) \simeq R_0 + R_1 \ln\left(\frac{T}{T_c} - 1\right) = R_1 \ln\left(\frac{T - T_c}{Q}\right) \ .$$

Lattice size	R_0	R_1	T_p/T_c	$\frac{\chi^2}{N_{dof}}$
2 fm	1.21(1)	0.293(6)	1.488(13)	1.35
3 fm	1.115(15)	0.283(9)	1.667(27)	1.92

Table: Fit parameters for the best-copy values of $r(T)$.



$D_T(0) / D_L(0)$
1.4
1.2
1
0.8
0.6
0.4
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
$\mathrm{Log}(T/T_c - 1)$
best copy, 2fm
best copy, 3fm

Conclusions

- The flip-sector effect is substantial at $L \simeq 2$ fm and crucial at $L < 1$ fm. In the latter case, it dramatically changes the behavior of the asymmetry.
- Finite-volume effects for $\overline{\mathcal{A}}$ and r are significant at lattice sizes < 2 fm .
- The data can be fitted to the function motivated by perturbation theory down to temperatures as low as $1.25 T_c$
- Contrary to the conclusions by Chernodub and Ilgenritz (2008), $\overline{\mathcal{A}} > 0$ at all temperatures under consideration
- Boundary of the postconfinement domain T_p is indicated by the condition $D_T(0)/D_L(0) = 1$ rather than by criteria based on $\overline{\mathcal{A}}$. At $L = 3fm$ $T_p = 1.68(12) T_c$.