Universität Regensburg

Real-time simulations of anomaly induced transport in external magnetic field

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Dubna

Chiral plasmas

Chiral plasma: medium consist of massless fermions

Quark-gluon plasma

Hadronic matter

Leptons, neutrinos at early stages of Universe

Weyl semimetals

Liquid He3

Chiral quantum anomaly: classical action is invariant under chiral rotations, but the measure of the path integral is not:

$$\mathcal{L} = \bar{\psi}\not{D}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-i\int dx_\mu \mathcal{L}[\bar{\psi},\psi,A_\mu]}$$

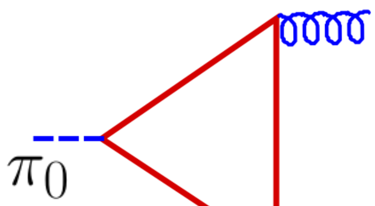
$$\xrightarrow[e^{i\theta\gamma_5}]{} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-i\int dx_\mu \mathcal{L}[\bar{\psi},\psi,A_\mu]-iS_\theta}$$

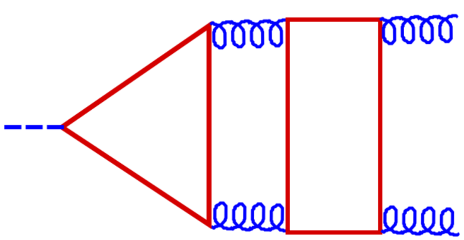
Non-conservation of axial current:

$$\partial_\mu j^\mu_A = \frac{1}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$$

$$\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2}\int d^3x \vec{E}\cdot\vec{B}$$

$$Q_A = N_R - N_L \qquad J_A = J_R - J_L$$

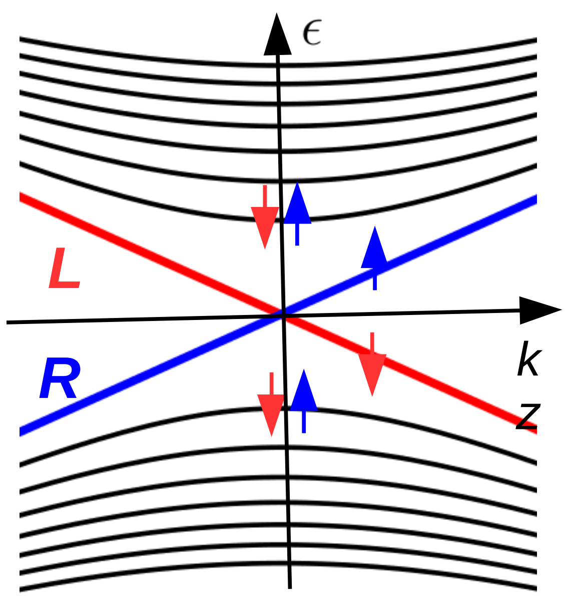




In a medium there might appear non-trivial correction

Chiral anomaly as Schwinger effect in 1D

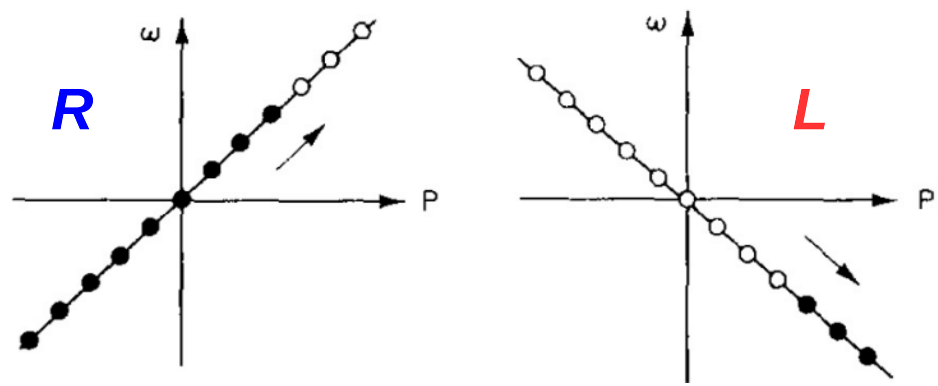
In the magnetic field motion of fermions is effectively 1D:



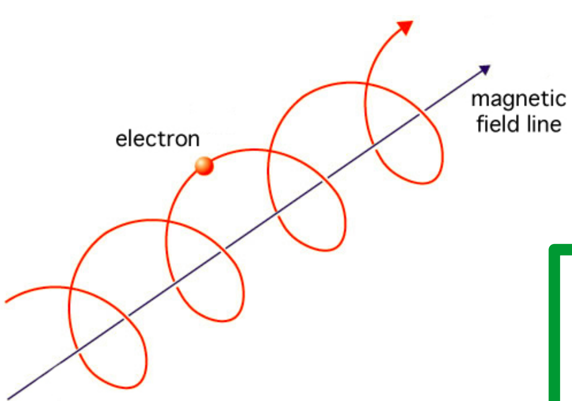
Landau levels in the magnetic field

$$\epsilon_{n,\sigma} = \pm\sqrt{k_z^2 + 2B(n - 1 + \sigma)}$$

In external electric field E || B there is a pair production on the topological lowest Landau level (n = 0):



$$\omega = \pm p \quad \dot{p} = eE/2\pi \;\rightarrow\; dn_{R,L}/dt = \pm eE/2\pi$$

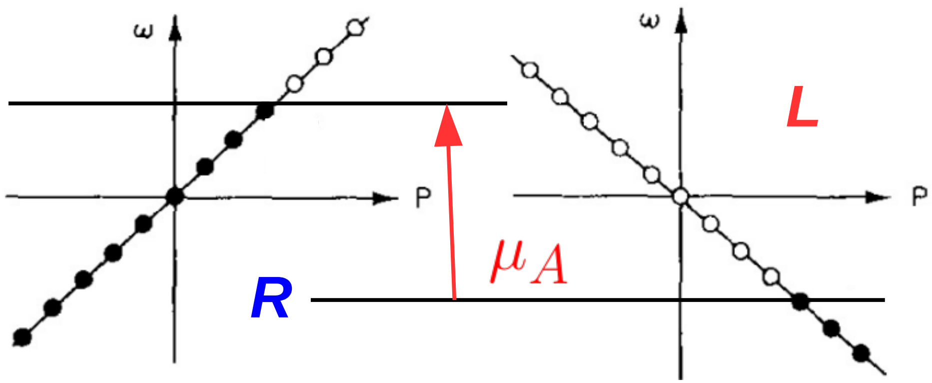


Degeneracy per unit area of each Landau level is $B/2\pi$

$$\frac{dQ_A}{dt} = \frac{d(n_R - n_L)}{dt}\frac{B}{2\pi} = \frac{eEB}{2\pi^2}$$

$$J_z(t) = \frac{eEBt}{2\pi^2}$$

Negative magnetoresistivity as manifestation of chiral anomaly



Suppose that there is a chirality-flipping process in the system with typical scattering time τ :

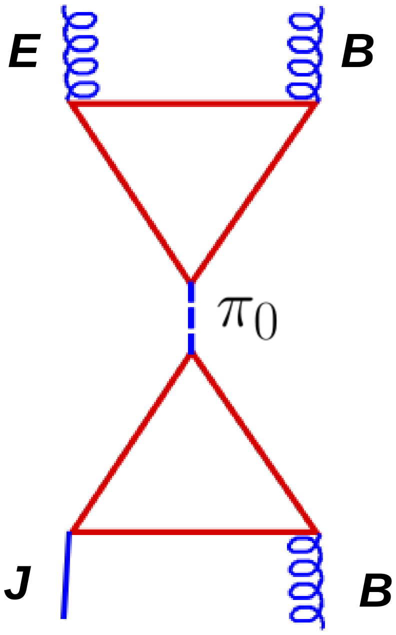
$$\dot{N}_{pairs} = \dot{N}_{scat}$$

Then steady state is described by chiral chemical potential μ_A.

$$\frac{dQ_A}{dt} = \frac{EB}{2\pi^2} - \frac{Q_A}{\tau} \longrightarrow \mu_A = \frac{3}{4\pi^2}\frac{EB}{T^2+\mu^2/\pi^2}\tau$$

$$J_z = \frac{\mu_A}{2\pi^2}B \longrightarrow J_z = \frac{3}{8\pi^4}\frac{EB^2}{T^2+\mu^2/\pi^2}\tau$$

Chiral Magnetic Effect (CME)



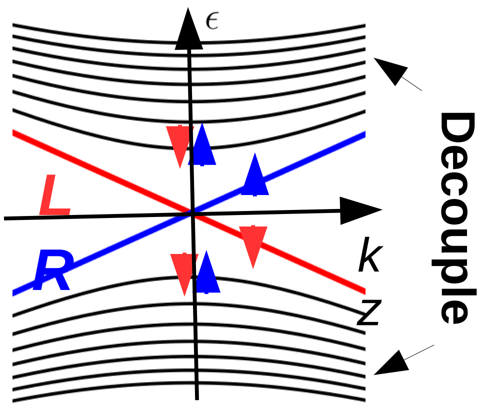
Nielsen, Ninomiya,
Phys. Lett. B 130 (1983) 389

K. Fukushima et al,
Phys. Rev. D,78, 074033(2008)

Effect of Interactions? QED in strong magnetic field, plasmons

Large magnetic field B → ***Dimensional reduction***

3 + 1 → ***1 + 1*** $N_f = B/2\pi$ ***flavors***



$$J_z(z,t) = Q_5(z,t) \qquad Q(z,t) = J_{5z}(z,t)$$

Maxwell equations and anomaly:

$$\partial_t Q_5(z,t) + \partial_z J_{5z}(z,t) = \kappa B E_z(z,t)$$

$$\partial_z E_z(z,t) = Q(z,t) \qquad \partial_t E_z(z,t) = -J_z(z,t)$$

Wave equation:

$$\partial_t^2 E_z(z,t) - \partial_z^2 E_z(z,t) + \kappa B E_z(z,t) = 0$$

Plasmon dispersion relation:

$$\omega^2 = k_z^2 + \omega_A^2 \qquad \omega_A = \sqrt{\kappa B}$$

Plasmon is Chiral Magnetic Wave!

Introduces a time scale of wave formation τ_{CMW}

Competition of time scales τ **and** τ_{CMW} **?**

Effect of Interactions? QED in strong magnetic field, plasmons

Homogeneous electric field: $E(t) = E_0 \cos(t\sqrt{\kappa B})$

$$Q_5(t) = \sqrt{\kappa B} E_0 \sin(t\sqrt{\kappa B}) \qquad Q_5(t) \xrightarrow[t \to 0]{} \kappa B E_0 t$$

Time scale of wave formation: $\tau_{CMW} \sim 1/\sqrt{\kappa B} = 1/\omega_A$

Applicability of static chiral chemical potential: $\tau \ll \tau_{CMW}$

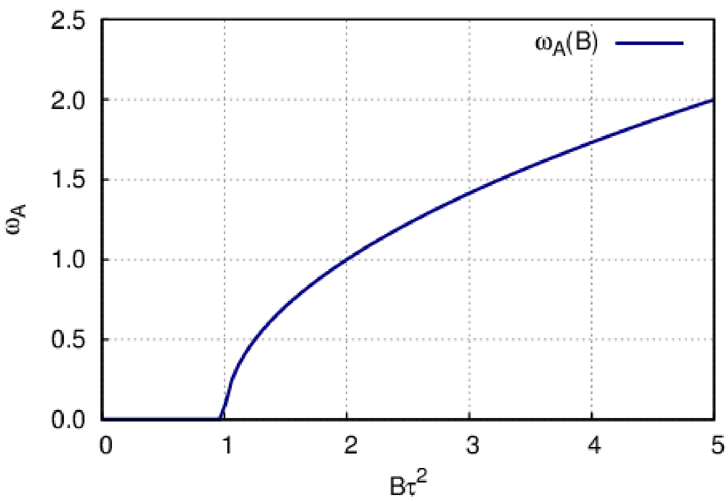
Example: effect of Ohmic resistivity

$$\partial_t E_z(z,t) = -\sigma E_z(z,t) - j_z(z,t)$$

higher Landau levels...

$$\partial_t^2 E_z(z,t) - \partial_z^2 E_z(z,t) + \kappa B E_z(z,t) + \sigma \partial_t E_z(z,t) = 0$$

$$\omega_A = \sqrt{\kappa B - \sigma^2/4}$$

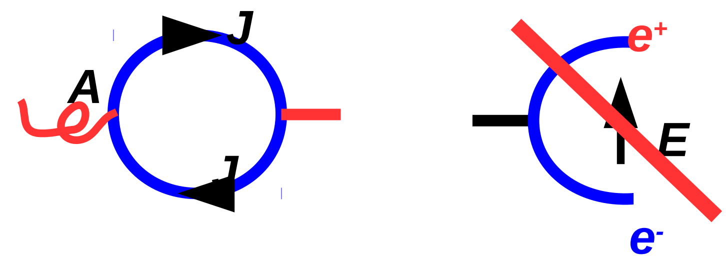


CMW dissapear if $2/\sigma \equiv \tau < \tau_{CMW}$

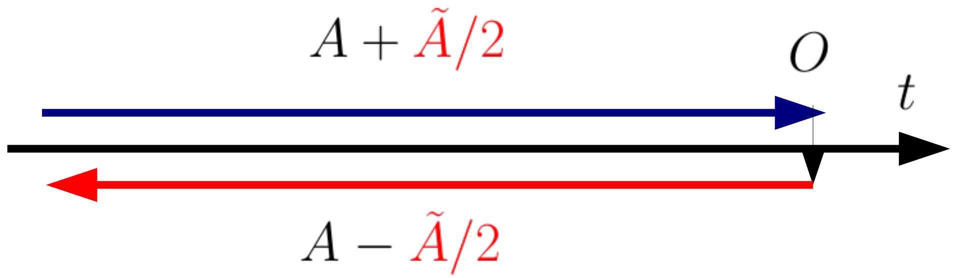
Classical-statistical real-time simulations

In Euclidean lattice gauge theory:

DIfficult analytical continuation to real-time



Out-of equlibrium real-time classical-statistical approximation:



$$\langle O(t) \rangle = \mathrm{Tr}\left[\rho_0 U_+(0,t) O U_-(t,0)\right]$$

$\tilde{A} \ll A$

$$U_\pm(0,t) = \mathcal{T} \exp\left(-i \int H_\pm(t') dt'\right)$$

$$H = \psi^\dagger h \psi + H_g \qquad O = \psi^\dagger o \psi$$

Managable!

Maxwell equations

$$\partial_t \vec{E}(t) = -\langle j(t) \rangle - \nabla \times \vec{B}(t)$$

Fermionic current

$$\langle j(t) \rangle = \mathrm{tr}\left[\rho_0 u(0,t) j u^\dagger(0,t)\right]$$

$$\partial_t u(0,t) = -ih[\vec{A}(t)] u(0,t)$$

$$j = \partial h / \partial A$$

Occupation numbers of bosonic fields have to be sufficiently high

Susskind, '93
G. Aarts, '99

J. Berges, F. Hebenstreit, N. Mueller
P. Buividovich, M. Ulybyshev

Lattice fermions and chiral symmetry

Wilson-Dirac hamiltonian: $h^{wd} = \gamma_0 D_m^{wd}$

$$D_m^{wd} = -i\gamma_i \nabla_i + m + r\Delta$$

+ Good for condensed matter

- Chiral symmetry is broken

$$\nabla_{i,xx'} = \frac{1}{2}\left(\delta_{x+e_i,x'} e^{iA_{x,i}} - \delta_{x-e_i,x'} e^{-iA_{x-e_i,i}}\right)$$

$$\Delta_{xx'} = \delta_{x,x'} - \frac{1}{2}\sum_i \left(\delta_{x+e_i,x'} e^{iA_{x,i}} + \delta_{x-e_i,x'} e^{-iA_{x-e_i,i}}\right)$$

Overlap hamiltonian: $h^{ov} = \gamma_0 D^{ov}$ **Creutz, Neuberger hep-lat/0110009**

$$D^{ov} = 1 + \gamma_5 \mathrm{sign}\left[\gamma_5 D_{m-1}^{wd}\right]$$

+ Exact lattice chiral symmetry

- Very expensive

$$q_5 = \gamma_5 \left(1 - \frac{D_{ov}}{2}\right) \qquad [q_5, h^{ov}] = 0 \qquad \{\gamma_5, D^{ov}\} = aD^{ov}\gamma_5 D^{ov}$$

First time experience with real time Overlap!

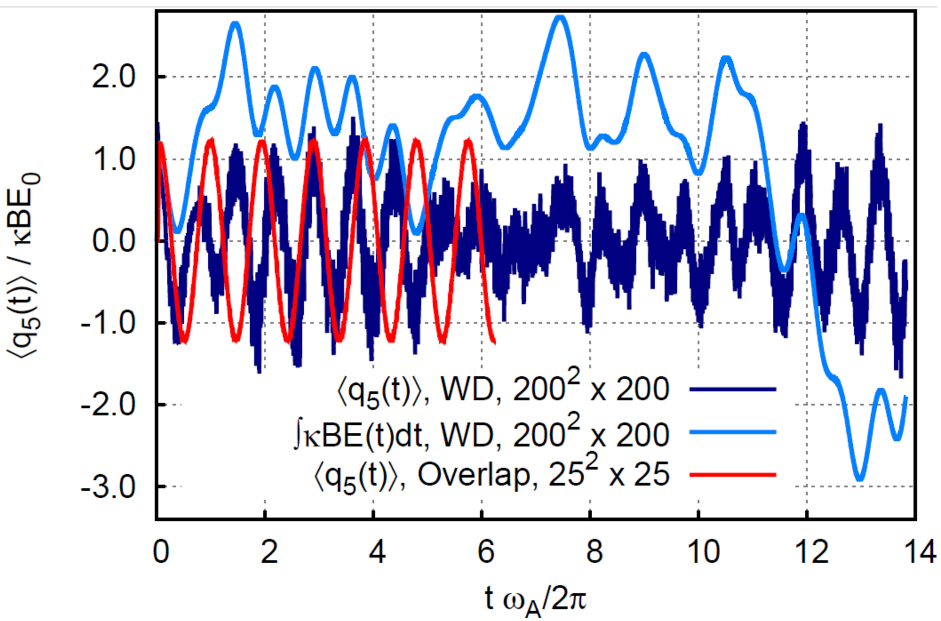
Plasmons with lattice fermions

CMW: $Q_5(t) = Q_5^0 \sin(t\sqrt{\kappa B})$ $\omega_A = \sqrt{\kappa B}$ $\tau_{CMW} \sim 1/\sqrt{\kappa B} = 1/\omega_A$

External perturbation: $E_z^{ext}(t) = E_0 \exp\left(-\frac{(t-t_0)^2}{2\tau^2}\right)$

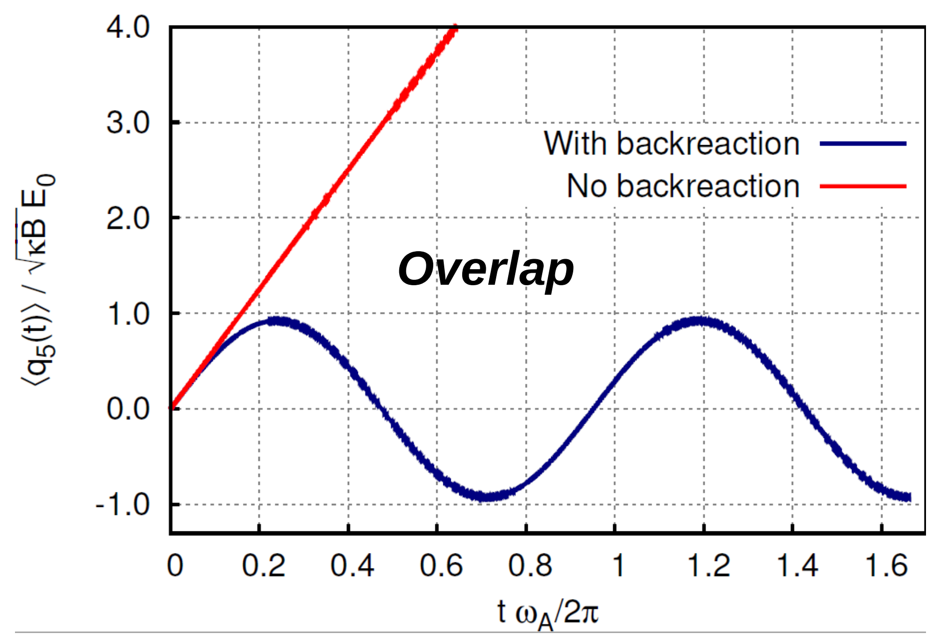
$\tau \ll \tau_{CMW}$: $Q_5^0 = \kappa B E_0 \int E_z^{ext}(t)dt$

Overlap and Wilson-Dirac



$\tau \gg \tau_{CMW}$ **(constant** $E_z^{ext}(t) = E_0$**):**

$Q_5(t) = \sqrt{\kappa B} E_0 \sin(t\sqrt{\kappa B})$



Effect of Wilson term in finite volume is very large for WD fermions

Overlap: practicaly exact anomaly!

$\langle Q_5(t) \rangle = \int \kappa B E(t')dt'$

Plasmons with lattice fermions

Possible sources of dissipation in QED:

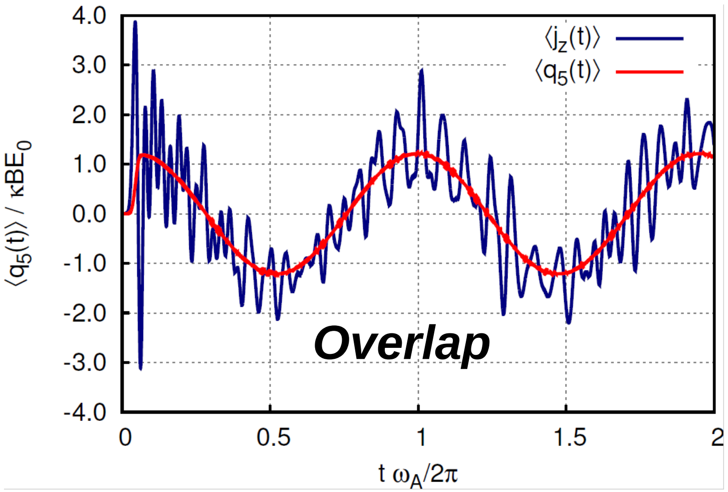
I. Higher Landau levels contrubute to current.

Ohmic resistivity, diffusion...

$$\partial_t E_z(z,t) = -\sigma E_z(z,t) - j_z(z,t)$$

Even more important at very begining:

$$E_z^{ext} \sim \theta(t-t_0) \qquad J_z(t) \sim \delta(t-t_0)$$



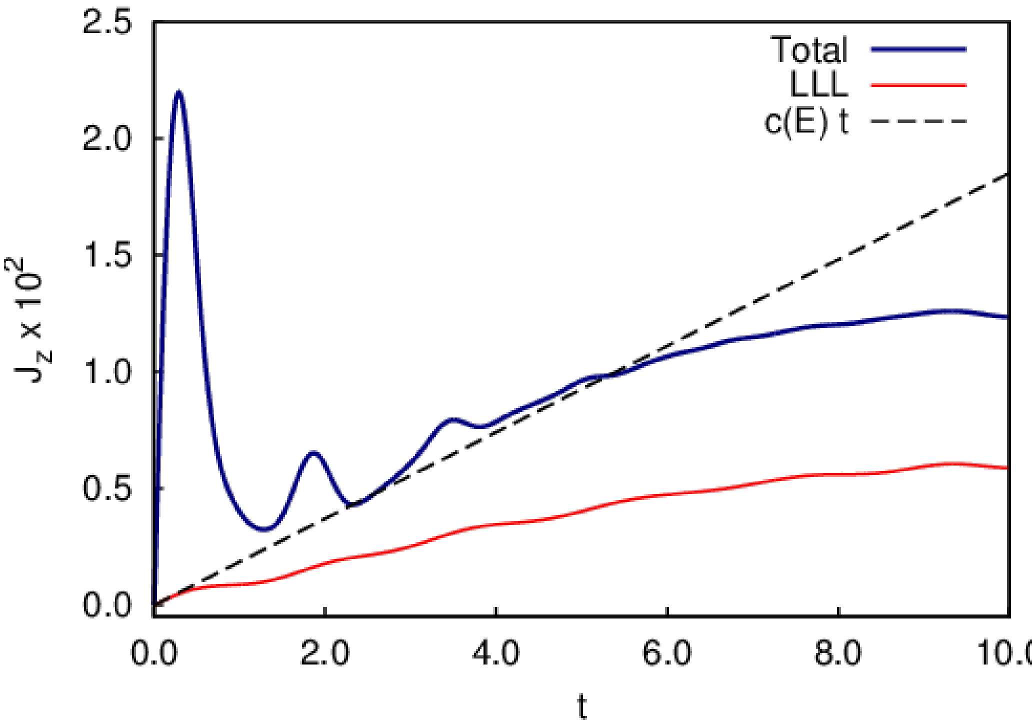
Dimensional reduction holds quiet well, however influence of HLL is visible.

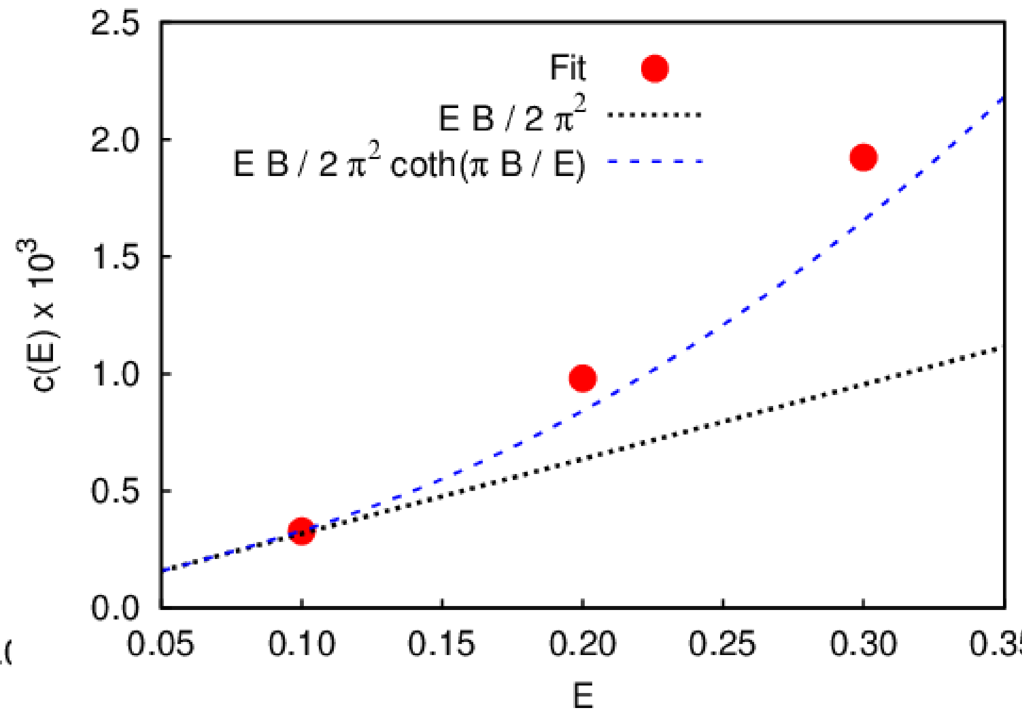
II. Landau damping? ***Simulation at finite density are needed***

Currently we don't see any dissipation within our simulation times...

Vector current and anomaly in constant electric field

Evolution at very strong electric fields E ~ B

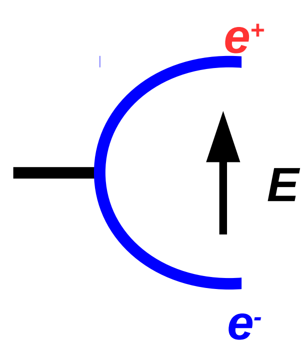


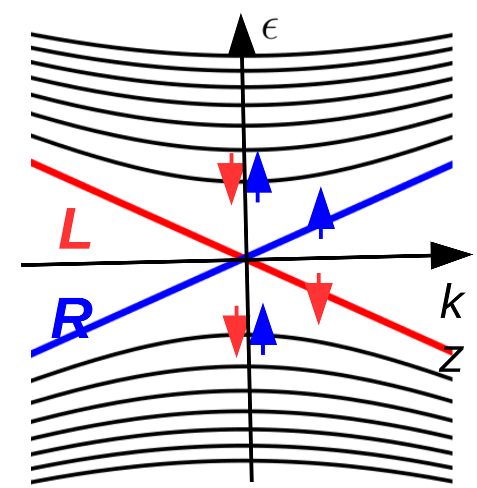


We see NP contributions from Schwinger pair production at higher, massive Landau levels:

$$J_z(t) = \frac{BE}{2\pi^2} \boxed{\coth\left(\frac{\pi B}{E}\right)} t$$

Zubkov, arXiv:1605.02379





Conclusions

1) **Corrections** due to dynamical bosonic fields are very **large** when **CMW** is formed

2) Effects of **higher Landau levels** are expected to be **important** in quickly changing environments

3) In lattice real-time simulations the **Overlap** operator is **essential** choice where exact **chiral symmetry** is important