Investigation of the static and dynamic properties of the quantum phase transition: the RG approach.

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Green functions at finite temperature.
Quantum non-relativistic models will be described.
Let us start with the quantum – field model with the action

$$S = \psi_i^+(\partial_t - \frac{\Delta}{2m} - \mu)\psi_i - \frac{\lambda}{2}(\psi_i^+\psi_i)(\psi_j^+\psi_j) \qquad (2.1)$$

Here t is an imaginary "time"belongs to temperature dependent range $[0, \beta = 1/kT]$. ψ, ψ^+ fields are usual or Grasman variables related to particles wave–functions. All necessary integrations in t and **x** are assumed. The periodic boundary conditions for Bose – particles and anti-periodic for Fermi ones are implied.

2.1 gives rise to the standard free-field temperature propagator

$$G^{(0)}(t,x;t',x') = T\sum_n \int \frac{d\,\mathbf{p}}{(2\pi)^d}\,\frac{e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')+i\omega_n(t-t')}}{i\omega_n+\epsilon}$$
$$= \int \frac{d\,\mathbf{p}}{(2\pi)^d}\, e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-\epsilon(t-t')}\left[\theta(t-t')\pm\frac{1}{e^{\epsilon\beta}\mp 1}\right], \quad (2.2)$$

where $\epsilon = \dfrac{p^2}{2m} - \mu$.
Let us start from the Bose system. At critical region $\epsilon \to 0$ then $G^{(0)} \to 1/(p^2/(2m)-\mu)$. Fields are t independent. 2.1 became a usual ϕ^4 theory that is usual for the descriptions of continuous phase transitions.

This transitions is described by Renormalization Group (RG) approach and $4-\epsilon$ expansion.

RG approach:

1. The construction of the models.
2. The calculations of diagrams.
3. The investigation of the higher order asymptotes.
4. The ressumations.

Competition: the calculations of the renormalization constants from the difficult diagrams up to some level.

1-loop calculations — Wilson,... Vladimirov, Vasiljev, Kozakov,

Chetyrkin – 5-loops calculations.

New progress: Adzhemyan, Kompaniets – 6-loops.

The possibility of the obtaining of critical indexes up to ϵ^6 (ϵ^7 for *eta*). The problem: ϵ expansion is divergent.

Lipatov's asymptotic:

$$G^{(N)} \sim N! a^N n^b(G) C(G),$$

where $G^{(N)}$ is N-th order of the ϵ expansion of G.
Borel-resummation:

$$\sum (G^{(N)} \epsilon^N) \to \int dt e^{-t} \sum (\frac{G^{(N)}}{N!} (\epsilon t)^N).$$

Conform transformations is needed for calculation of $\int dt$. The Lipatov's asymptotic is needed for conform transformation.

The Lipatov's asymptotic of the perturbation expansion in the quantum Bose system as for as the Goldstone singularities here were described in
"Bose-Einstein condensation beyond perturbation theory: Goldstone singularities and instanton solution" *Juha Honkonen, Marina V. Komarova and Mikhail Yu. Nalimov European Physical Journal B, 2014,87: 75*

Fermions with high spines.
Ultra-cold atoms: s=3/2, 5/2,..., 9/2. Graphene example: sublattice, symmetry of the zonal structure..
The action is the same $S=\psi_i^+(\partial_t-\frac{\Delta}{2m}-\mu)\psi_i-\frac{\lambda}{2}(\psi_i^+\psi_i)(\psi_j^+\psi_j)$
To describe the phase transition in a supercondactive state let us introduce additional fields χ_{ij}, χ_{ij}^+ *and rewrite action (2.1) in a form:*

$$S=\psi_i^+(\partial_t-\frac{\Delta}{2m}-\mu)\psi_i+\chi_{ji}^+\chi_{ij}+\sqrt{\frac{\lambda}{2}}\chi_{ij}(\psi_i^+\psi_j^+)+\sqrt{\frac{\lambda}{2}}\chi_{ij}^+(\psi_i\psi_j). \tag{5.1}$$

χ^+,χ *fields are antisymmetric matrices with range* r *determined by the number of components of spinor* ψ*.*
Let us integrate theory with the action (5.1) over fields ψ, ψ^+*.*

As a result we obtain

$$S = \chi^+_{ji}\chi_{ij} + \frac{1}{2}$$  $$+ \frac{1}{4}$$  (5.2)

An effective action is obtained by taking all graph in a form of expansion in external momenta (Ginzburg-Landau Hamiltonian).

$$S = \partial_\alpha\chi^+_{ji}\partial_\alpha\chi_{ij} + \tau\chi^+_{ji}\chi_{ij} + \frac{g_1}{2}(tr(\chi^+\chi))^2 + \frac{g_2}{2}tr(\chi^+\chi\chi^+\chi), \quad (5.3)$$

The parameters of the action are calculated.

The stability conditions for the action are

$$g_2 > 0, \quad g_2 + rg_1 > 0. \tag{5.4}$$

Beta functions of the theory are obtained in five-loops approximations(about 120000 diagrams); one-loop approximation:

$$\beta_{g1} = -\epsilon g_1 + \frac{3}{4}(r^2 - r + 6)g_1^2 + 3(r-1)g_1 g_2 + \frac{9}{4}g_2^2; \tag{5.5}$$

$$\beta_{g2} = -\epsilon g_2 + 9 g_1 g_2 + a\frac{3}{4}(2r-5)g_2^2.$$

This system has only one fixed point $g_{1*} = 0$, $g_2 = 9\epsilon/(3(r^2+8))$. It is IR stable in $r = 2$ case correspondent to usual fermions with 1/2 spin.
For $r > 2$ case the system (5.5) has no IR stable fixed points. Usually in such situation I-st order phase transition is suspected.
The resummation process requires knowledge about the asymptotic behavior $B_i^{(N)}(\bar{g}_1, \bar{g}_2)$ at $N \to \infty$.

$$a = a(n, 2ng_1 + g_2). \tag{5.6}$$

$$\beta_i^{(N)}(g_1, g_2) = const_i N! N^{b_n} \left(-a\right)^N \left(1 + O\left(N^{-1}\right)\right), \tag{5.7}$$

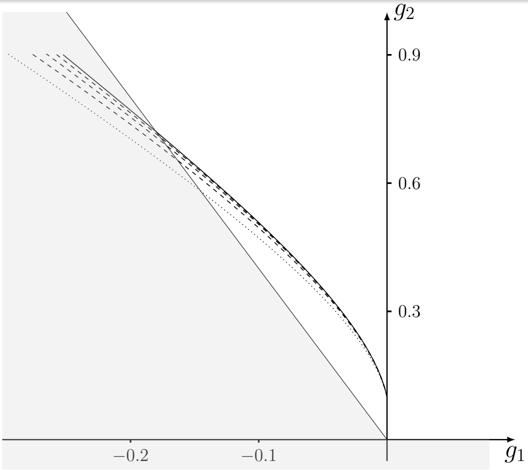


Рис.: The solutions of the RG equations($D=3, r=4$) at different loops.

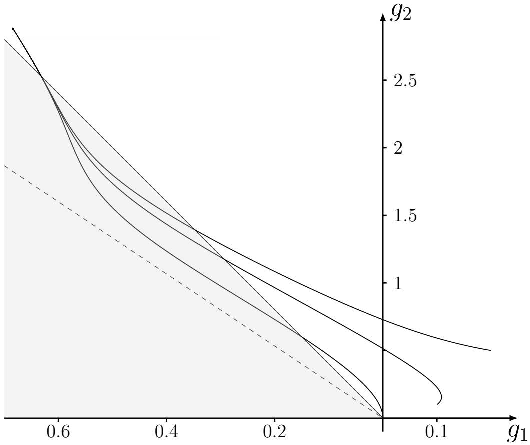


Рис.: Trajectories of the running coupling constants at $D = 3$ and $r = 4$.

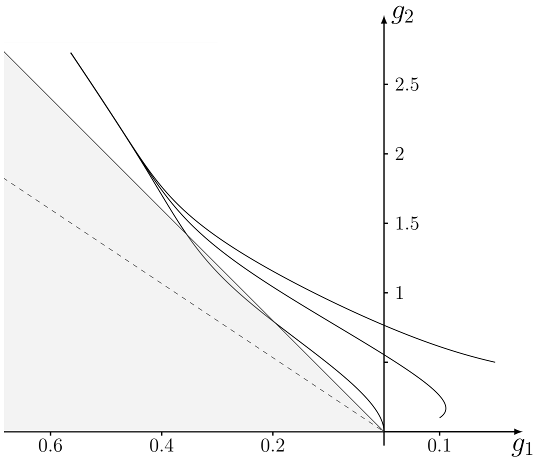


Рис.: Trajectories of the running coupling constants at $D=2$ and $r=4$.

The loss of the action stability is usually considered as a mark of the first-order phase transition. Because the interaction terms ($\sim \chi^4$) of the action are not positively defined now, we have to take into account the next term ($\sim \chi^6$) of the "bubble" expansion of the action (5.2).

Let us consider an effective action with an additional $F_3 \equiv \mathrm{tr}(\chi^\dagger\chi)^3$ *term. In renormalization procedure in* $4-\varepsilon$ *scheme* F_3 *will be considered as a composite operator of canonical dimension* $\Delta_3 = 6 - 3\varepsilon$*. Also, there are composite operators* $F_2 \equiv \mathrm{tr}(\chi^\dagger\chi)^2\,\mathrm{tr}(\chi^\dagger\chi)$ *and* $F_1 \equiv (\mathrm{tr}\,\chi^\dagger\chi)^3$ *with the same canonical dimension as* F_3*, therefore they may be mixed in the process of renormalization.*

The matrix Z *was calculated in three-loop approximation.*

$\beta \equiv \langle \chi \rangle$ is an order parameter of phase transition in the model considered. A non-zero value of β leads to phase transition to the superfluid phase. The value for magnitude β can be calculated by minimization of free energy $-\Gamma$. In the framework of the Landau mean field theory this functional can be written in the form

$$-\Gamma = \tau \, \mathrm{tr}\, \beta^\dagger \beta + \frac{g_{01}}{4}\left(\mathrm{tr}\, \beta\beta^\dagger\right)^2 + \frac{g_{02}}{4}\mathrm{tr}\, \beta\beta^\dagger\beta\beta^\dagger + \tag{5.8}$$

$$+\frac{\lambda_{01}}{36}\left(\mathrm{tr}\, \beta\beta^\dagger\right)^3 + \frac{\lambda_{02}}{36}\mathrm{tr}\left(\beta\beta^\dagger\right)^2 \mathrm{tr}\, \beta\beta^\dagger + \frac{\lambda_{03}}{36}\mathrm{tr}\left(\beta\beta^\dagger\right)^3 .$$

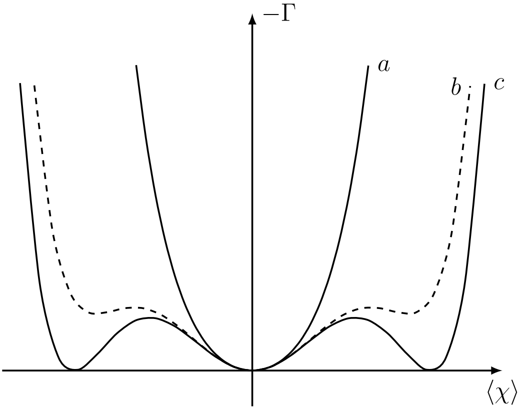


Рис.: Thermodynamics potential as a function of order parameter.

Then we can state that in the model considered the first-order phase transition takes place at a temperature higher than the predictions for continuous phase transitions.

$$\frac{\Delta T}{T_0} = \zeta_0 \frac{6912\pi^6}{7\zeta(3)} \left(\frac{T_0}{T_F}\right)^4 . \tag{5.9}$$

In the standart terminology the model F of critical dynamics is described by the order parameter of conjugated fields $\psi(\mathbf{x}, t)$, $\psi^{+}(\mathbf{x}, t)$ that are averages of the Bose-particle field operators, and a field $m(\mathbf{x}, t)$ connected with temperature fluctuations in the system. The dynamics of all these fields is given by the Langevin equations

$$\partial_t \psi = f_\psi + \lambda_0(1 + ib_0)[\partial^2 \psi - g_{01}(\psi^{+}\psi)\psi/3 + g_{02} m\psi] + i\lambda_0 g_{03}\psi[g_{02}\psi^{+}\psi - m], \quad (6.1)$$

$$\partial_t m = f_m - \lambda_0 u_0 \partial^2 [g_{02}\psi^{+}\psi - m] + i\lambda_0 g_{03}[\psi^{+}\partial^2\psi - \psi\partial^2\psi^{+}], \quad (6.2)$$

and equation for ψ^{+} field is given by complex conjugation of (6.1). $g_{20} = 0$ in E model.

The random forces f_ψ, f_m are assumed to be Gaussian random variables with zero means and correlators D_ψ, D_m with the white-noise correlations in time.

$$D_\psi(p,t,t') = \lambda_0\delta(t-t'), \quad D_m(p,t,t') = \lambda_0 u_0 p^2\delta(t-t'). \quad (6.3)$$

The constants g_{01}, g_{02} and g_{03} define the intensity of (self)interactions of the order parameter and m field; the parameters λ_0 and u_0 relate to the diffusion coefficient, b_0 is an intermode coupling.

MSR field-theoretic action reads

$$
\begin{aligned}
S = {} & 2\lambda_0\psi^{+'}\psi' - \lambda_0 u_0 m'\partial^2 m' + \psi^{+'}\{-\partial_t\psi + \\
& + \lambda_0(1+ib_0)[\partial^2\psi - g_{01}(\psi^+\psi)\psi/3 + g_{02} m\psi] \\
& + i\lambda_0\psi[g_{02}g_{03}\psi^+\psi - g_{03}m + g_{03}h_0]\} + \psi'\{-\partial_t\psi^+ \\
& + \lambda_0(1-ib_0)[\partial^2\psi^+ - g_{01}(\psi^+\psi)\psi^+/3 + g_{02} m\psi^+] \\
& - i\lambda_0\psi^+[g_{02}g_{03}\psi^+\psi - g_{03}m + g_{03}h_0]\} \\
& + m'\{-\partial_t m - \lambda_0 u_0\partial^2[-m + g_{02}\psi^+\psi \\
& + h_0] + i\lambda g_{03}[\psi^+\partial^2\psi - \psi\partial^2\psi^+]\},
\end{aligned}
$$

At the present time, there is no general consensus which dynamic model (E or F) is genuine from the point of view of experimentally measurable quantities. There exist two RG fixed points in the dynamic model E, which are candidates to the possible IR stable regimes. Two-loop calculations do not lead to the decision about the true fixed point because of the lack of accuracy in the ω calculation. The value of the ω index depends on the chosen dynamical model. For the same fixed point the ω value obtained in the framework of the model F can differ from the analogous ω in the model E. The presence of non-perturbation charges leads to difficulties of the applications of multi-loop algorithms. But we have calculated all first order graphs and investigate the influence of hydrodynamic fluctuations. We developing the methods of multi-loop numerical calculations now.

Moreover The higher order asymptotes of perturbation series in dynamical models with Gibbsian static limit are determined in our articles.

The problem discussed is a model dependent. Then it is worth while discussing the alternative direct approach.
The Green functions

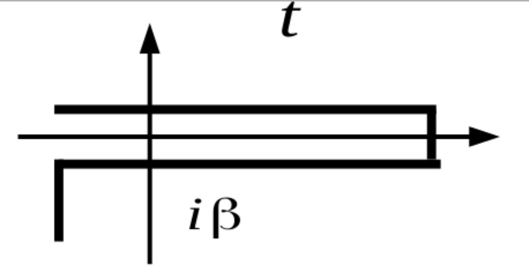
$$G(\mathbf{x_1}, t_1, ..., \mathbf{x_n}, t_n) \equiv < T(\hat{\psi}(\mathbf{x_1}, t_1)...\hat{\psi}(\mathbf{x_n}, t_n)) >$$

where

$$< ... > \equiv Sp(...e^{-\beta \hat{H}}).$$

can be represented in the form

$$< \phi(\mathbf{x})|e^{-i\hat{H}(t-t')}|\phi(\mathbf{x}')|) >= \int_{\psi(\mathbf{x}',t_1)=\phi(\mathbf{x}')}^{\psi(\mathbf{x},t)=\phi(\mathbf{x})} D\psi D\psi^+ e^{iS}$$



Propagators for Bose model are

$$G_{RR}=e^{-i\epsilon(t-t')}(\Theta_{ij}(t-t')+n(\epsilon)).\quad G_{RA}=e^{-i\epsilon(t-t')}n.$$

$$G_{RT}=e^{-i\epsilon(t-t')}n.\quad G_{AA}=e^{-i\epsilon(t-t')}(\Theta(t'-t)+n).$$

$$G_{AR}=e^{-i\epsilon(t-t')}(n+1).\quad G_{AT}=e^{-i\epsilon(t-t')}n.$$

$$G_{TR} = e^{-i\epsilon(t-t')}(n+1). \quad G_{TA} = e^{-i\epsilon(t-t')}(n+1).$$

$$G_{TT} = e^{-i\epsilon(t-t')}(\Theta(t-t')+n). \quad n(\epsilon) = 1/(e^{\beta\epsilon}-1).$$

There are unusual singularites in the perturbation expansion:

$$= ign \int dt' e^{-i\epsilon(t_1-t')}(\Theta(t_1-t')+n(\epsilon))$$

$$e^{-i\epsilon(t_2-t')}(\Theta(t_2-t')+n(\epsilon)) \sim t_{out}-t_{in}$$

But we have found a dissipation due to the renormalization in second order of the perturbation expansion.

Critical behavior:

After the change of variables

$$\xi = \frac{1}{\sqrt{2}}(\psi_R+\psi_A), \quad \eta = \frac{1}{\sqrt{2}}(\psi_R-\psi_A)$$

in the IR region the propagators are

$$< \xi\xi^{+} >= 2e^{-i\epsilon(t-t')-\delta|t-t'|}\frac{1}{\beta k^2}$$

$$< \xi\eta^{+} >= e^{-i\epsilon(t-t')-\delta|t-t'|}\Theta_(t-t')$$

$$< \eta\xi^{+} >= -e^{-i\epsilon(t-t')-\delta|t-t'|}\Theta_(t'-t)$$

$$< \psi_T\xi^{+} >= \frac{1}{\sqrt{2}}e^{-i\epsilon(t-t')}\frac{2}{\beta k^2}$$

$$< \xi\psi_T^{+} >= \frac{1}{\sqrt{2}}e^{-i\epsilon(t-t')}\frac{2}{\beta k^2}$$

$$< \eta\eta^{+} >=< \psi_T\eta^{+} >=< \eta\psi^{+} >= 0, \quad < \psi_T\psi_T^{+} >= \frac{1}{\beta k^2}.$$

And the action is

$$S=\int d\mathbf{x}\Bigg(\int dt[\eta\frac{\alpha}{\beta}\eta^{+}+\eta(\partial_t-(i+\alpha)\Delta)\xi^{+}ig_1\eta\xi\xi^{+}\xi^{+}]-\frac{1}{4}\int dt[\eta](\beta$$

$$+\frac{\alpha^2}{\beta})\Delta\int dt[\eta^{+}]-\sqrt{2}\int dt[\eta](i+\alpha)\psi_T^{+}-\frac{1}{2}\beta\psi_T\Delta\psi_T^{+}+g_2\beta\psi_T\psi_T^{+}\psi_T\psi_T^{+}\Bigg)$$

with the additional interactions:

$$+g_3\int d\mathbf{x}\Big(\psi_T\psi_T\int dt[\eta\xi]\Big)+g_4\int d\mathbf{x}\Big(\int dt[\eta\xi]\int dt[\eta\xi]\Big)$$

$$+g_5\int d\mathbf{x}\Big(\psi_T\int dt[\eta\xi\xi]\Big)+g_6\int d\mathbf{x}\Big(\psi_T\psi_T\psi_T\int dt[\eta]\Big)$$

$$+g_7\int d\mathbf{x}\Big(\psi_T\int dt[\eta]\int dt[\eta\xi]\Big)+g_8\int d\mathbf{x}\Big(\psi_T\psi_T\int dt[\eta]\int dt[\eta]\Big)$$

$$+g_9\int d\mathbf{x}\Big(\psi_T\int dt[\eta]\int dt[\eta]\int dt[\eta]\Big)+g_{10}\int d\mathbf{x}\Big(\int dt[\eta\xi]\int dt[\eta]$$

$$\int dt[\eta]\Big),+g_{11}\int d\mathbf{x}\Big(\int dt[\eta]\int dt[\eta]\int dt[\eta]\int dt[\eta]\Big).$$

We investigating this model now.