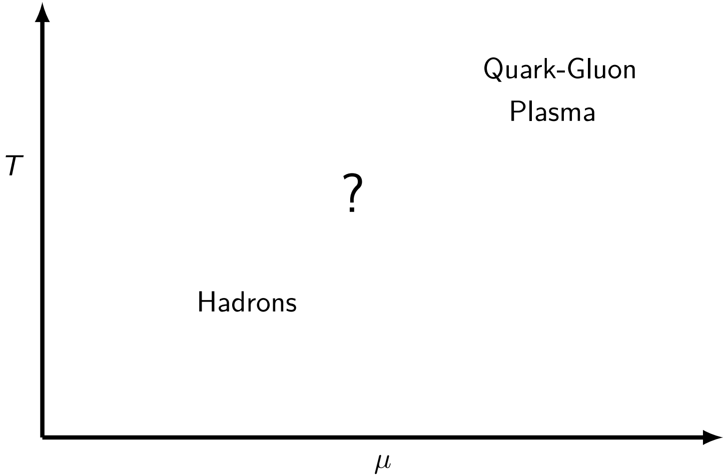
Towards the QCD phase diagram using Complex Langevin

Benjamin Jäger

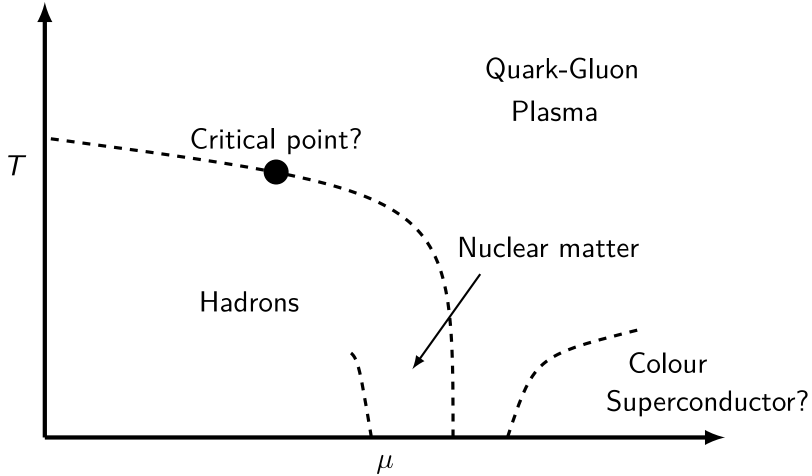
ETH zürich

In collaboration with G. Aarts, F. Attanasio, D. Sexty

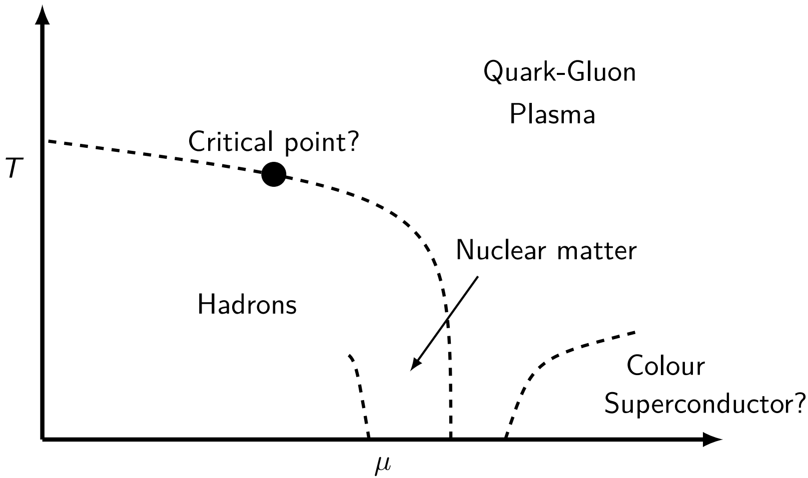
Phase diagram for QCD



Phase diagram for QCD



Phase diagram for QCD



- Sign problem $\leftrightarrow$ det $D(\mu \neq 0) \in \mathbb{C}$

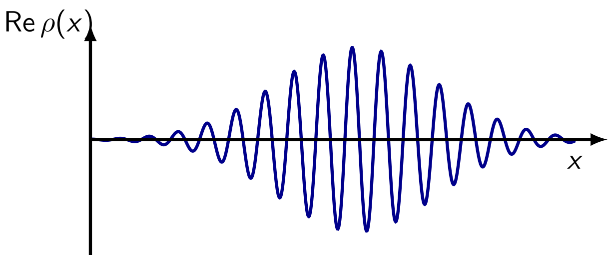
The sign problem

- Finite chemical potential → Sign problem

$$(\det D(\mu))^* = \det D(-\mu^*) \;\rightarrow\; \det D(\mu \neq 0) \in \mathbb{C}.$$

- Importance Sampling based Monte Carlo methods fail

$$\langle A \rangle = \frac{1}{Z} \int \mathrm{D}U\, A(U)\; |\det D|\; \mathrm{e}^{i\Theta}\; \mathrm{e}^{-S_G(U)}$$



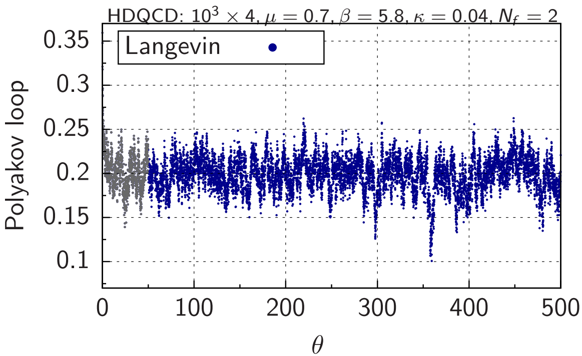
Complex Langevin simulations

- Complexify degrees of freedom $SU(3) \rightarrow SL(3, \mathbb{C})$

$$U_{x,\mu} = \exp\left[\mathrm{i}\, a\, \lambda^c \left(A^c_{x,\mu} + \mathrm{i}\, B^c_{x,\mu}\right)\right]$$

- Evolve links according (1st order) Langevin equation

$$U_{x,\mu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\, \lambda^a \left(-\varepsilon\, D^a_{x,\mu}\, S + \sqrt{\varepsilon}\, \eta^a_{x,\mu}\right)\right] U_{x,\mu}(\theta)$$



Complex Langevin simulations

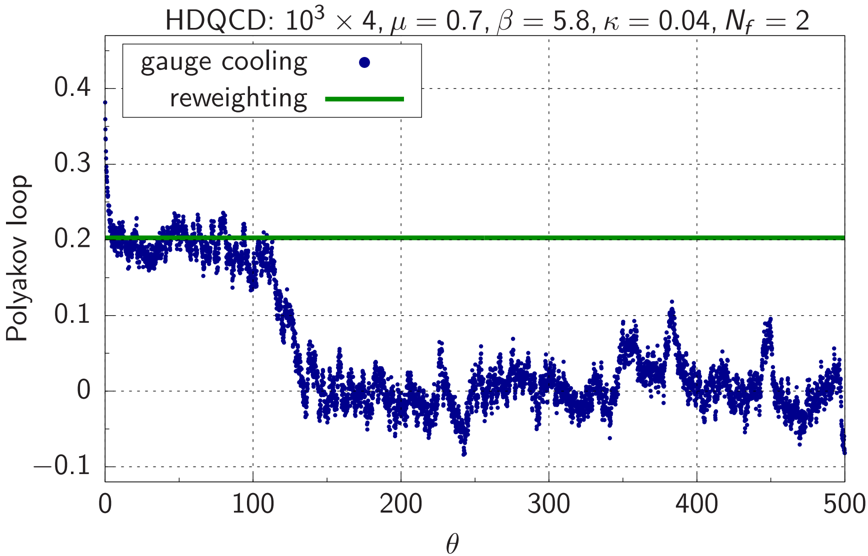
- However, $SL(3, \mathbb{C})$ is not a compact group...
- Convergence $\Leftrightarrow$
  - Action S is holomorphic
  - "Imaginary" direction of $SL(3, \mathbb{C})$ falls off quickly enough
- Measure distance to SU(3) manifold

$$\text{unitnorm} = \text{Tr}\left(U_{x,\mu}\, U^{\dagger}_{x,\mu} - 1\right)^2$$

- Gauge cooling is essential, but sometimes not sufficient...

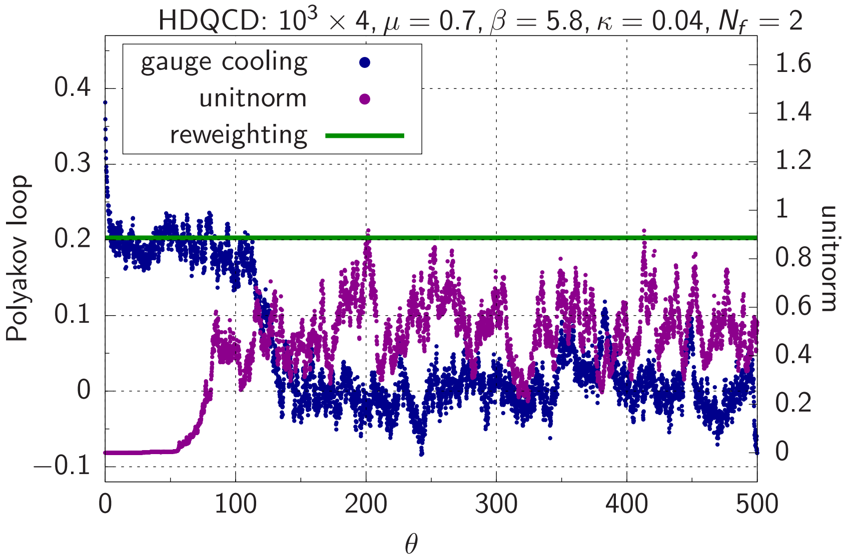
$$U_{x,\mu} \to \Omega_x\, U_{x,\mu}\, \Omega^{-1}_{x+\mu}$$

Gauge cooling



- Tunneling to wrong results.

Gauge cooling



- Tunneling to wrong results, when unitnorm grows too large.

Dynamic stablization

- Adding a trivial force to the Langevin dynamics

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\lambda^a \left(\varepsilon\, K^a_{x,\nu} + \mathrm{i}\,\varepsilon\,\alpha_{DS}\, M^a_x + \sqrt{\varepsilon}\,\eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

where

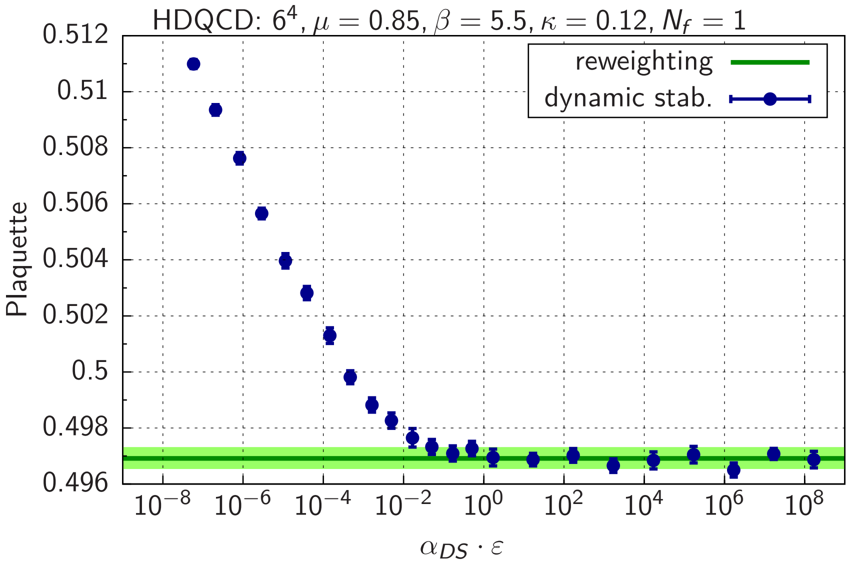
$$M^a_x = \mathrm{i} b^a_x \left(\sum_c b^c_x\, b^c_x\right)^3 \text{ and } b^a_x = \mathrm{Tr}\left[\lambda^a \sum_\nu U_{x,\nu} U^\dagger_{x,\nu}\right].$$

- Expanding the force in terms of gauge fields A and B

$$M^a_x \sim a^7 \left(\overline{B}^c_y\, \overline{B}^c_y\right)^3 \overline{B}^a_x + \mathcal{O}(a^8)\,.$$

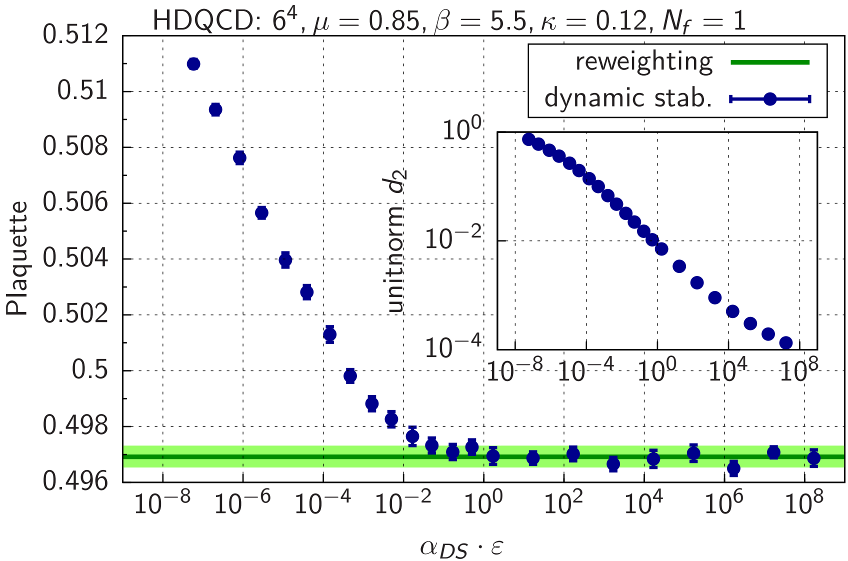
- Dynamic stabilization is numerically cheap and can be combined with gauge cooling (Here: 1 step)

Dynamic stablization



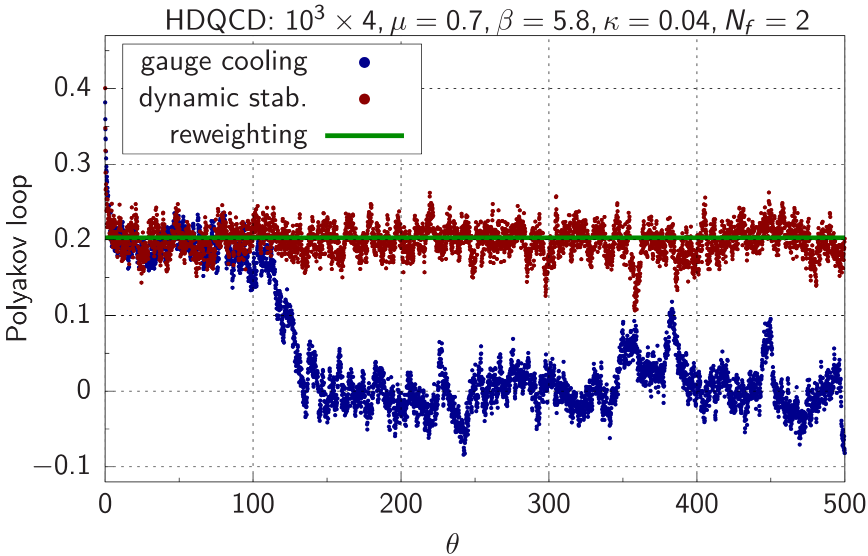
- For sufficient large α_{DS} we find agreement with reweighting.

Dynamic stablization



- For sufficient large α_{DS} we find agreement with reweighting.

Dynamic stablization

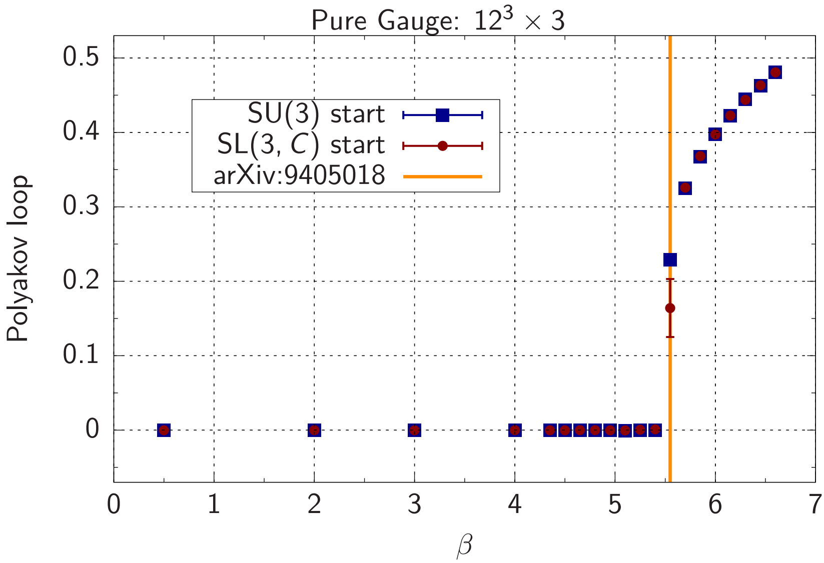


- Improved stability using dynamic stablization

Results

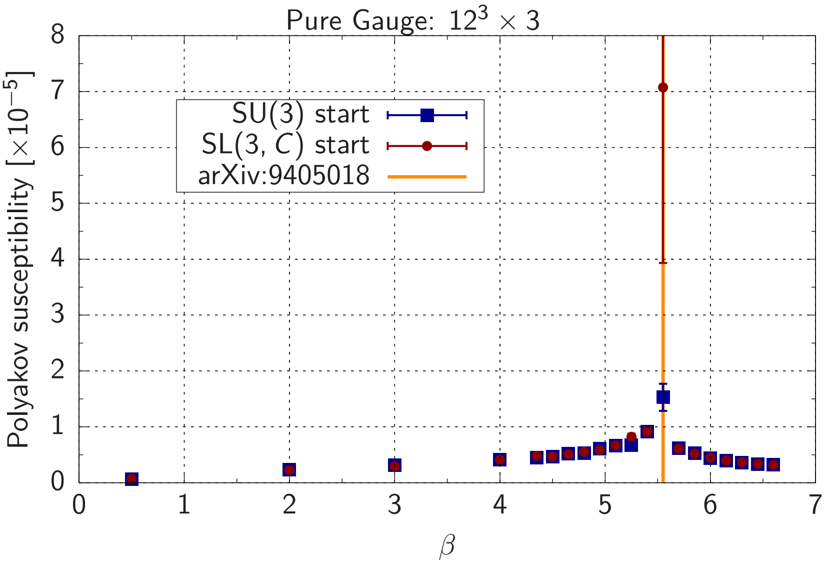
- **Pure Yang-Mills** $\Leftrightarrow$ No sign problem
  - Checking Langevin simulations
  - Compare to standard Hybrid Monte Carlo methods
- **Heavy dense QCD** (HDQCD)
  - Heavy and dense approximation of QCD
  - Has a sign problem and phase transitions
  - Numerical cheap :)
- **Full (Staggered) QCD**
  - Including light dynamical fermions
  - Very preliminary results

Pure Yang-Mills



- The correct transition is obtained, even for $SL(3,\mathbb{C})$ start.

Pure Yang-Mills



- The correct transition is obtained, even for $SL(3,\mathbb{C})$ start.

Heavy Dense QCD

- Here: QCD in the limit of heavy quarks (HDQCD).
- Fermion determinant simplifies with the Polyakov loops $P_{\vec{x}}$ and $P_{\vec{x}}^{-1}$ as

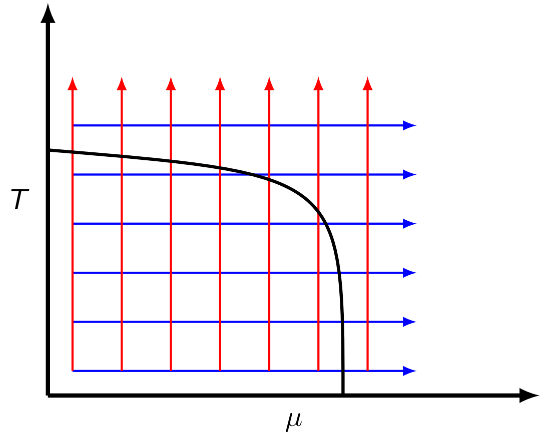
$$\det D(\mu) = \prod_{\vec{x}} \det\left(1 + C\,P_{\vec{x}}\right)^2 \det\left(1 + C'\,P_{\vec{x}}^{-1}\right)^2,$$

where the Polyakov loop $P_{\vec{x}}$ is defined as

$$P_{\vec{x}} = \frac{1}{V}\sum_t U_0(\vec{x}, t)$$

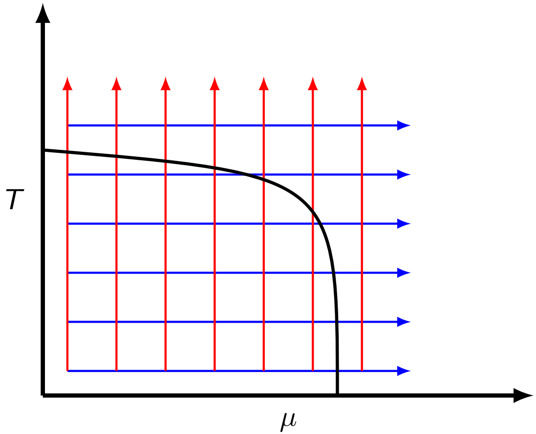
- For the gluonic part, we use the full Wilson gauge action.
- Map out the phase diagram for HDQCD.
- Expected transition: $\mu_c^0 = -\ln(2\kappa)$

Heavy Dense QCD



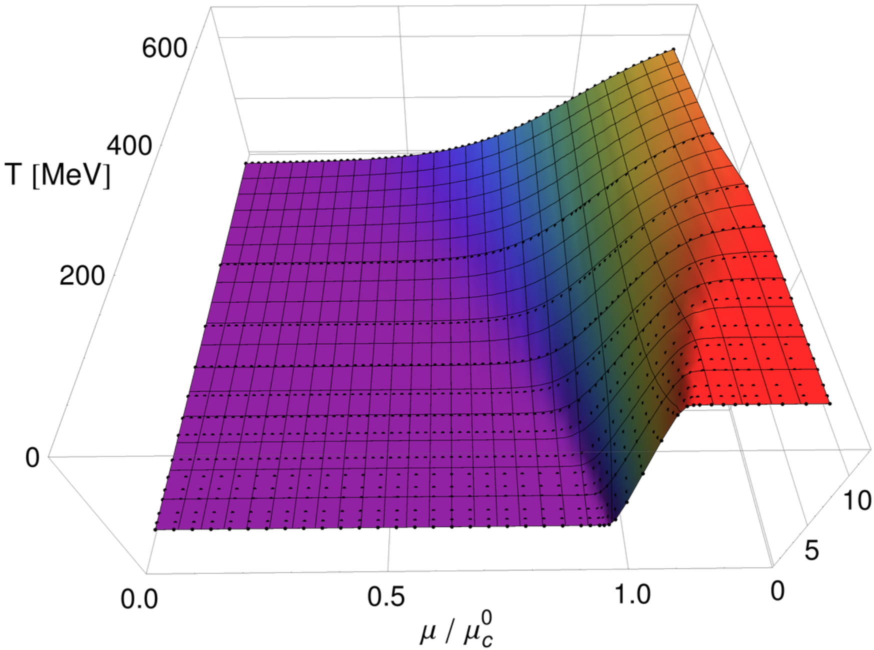
- Strategy:
  - Determine μ-transition in Fermion density
  - Determine T-transition in Polyakov loop

Heavy Dense QCD



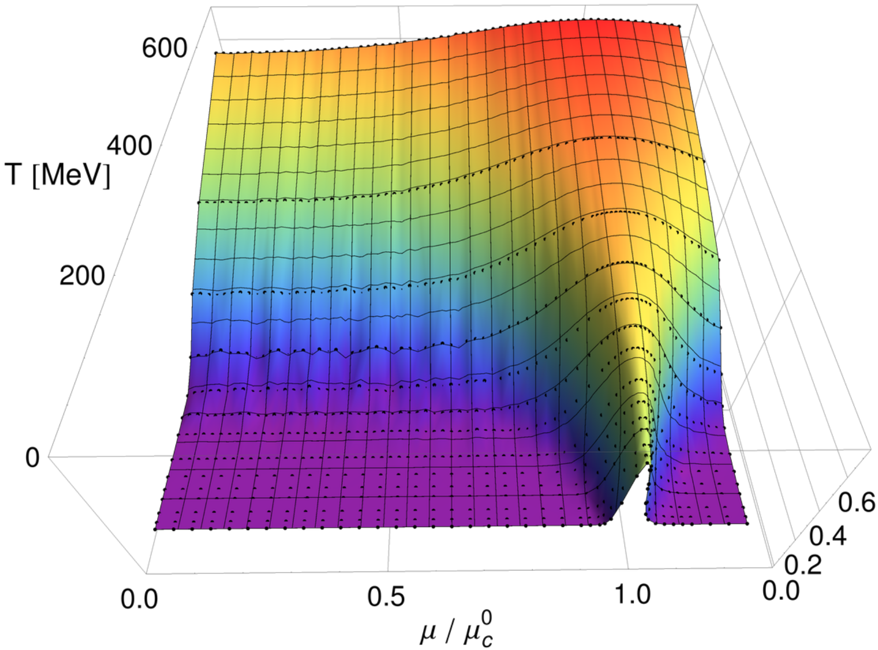
$\beta = 5.8$	$V = 6^3, 8^3, 10^3$	$a \sim 0.15$ fm
$\kappa = 0.04$	$N_f = 2$	$\mu_c^0 = 2.53$
N_τ	28 - 2	
T [MeV]	48 - 671	

Heavy Dense QCD



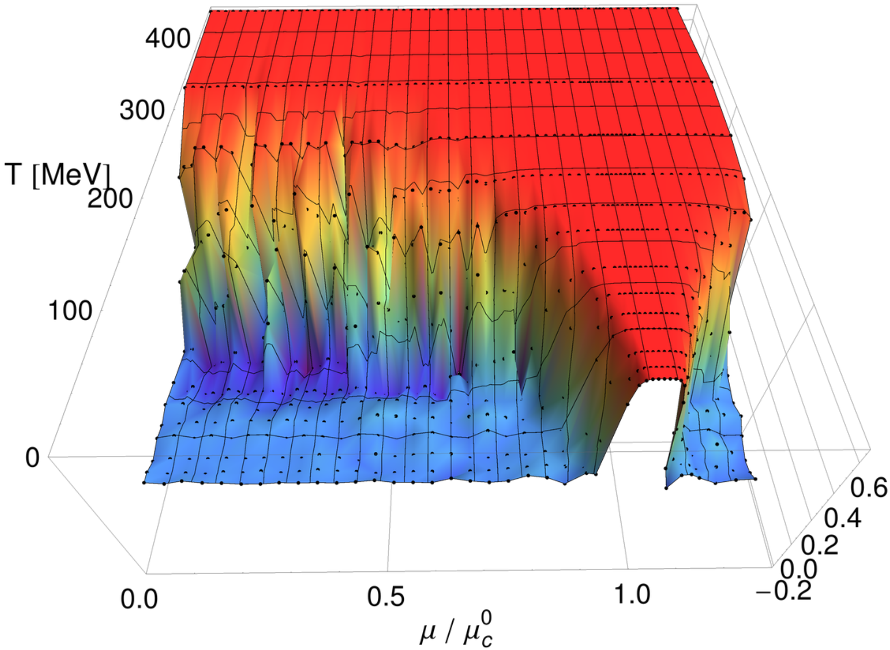
- Fermion density: $n = \frac{1}{N_t}\frac{\partial \ln Z}{\partial \mu}$

Heavy Dense QCD



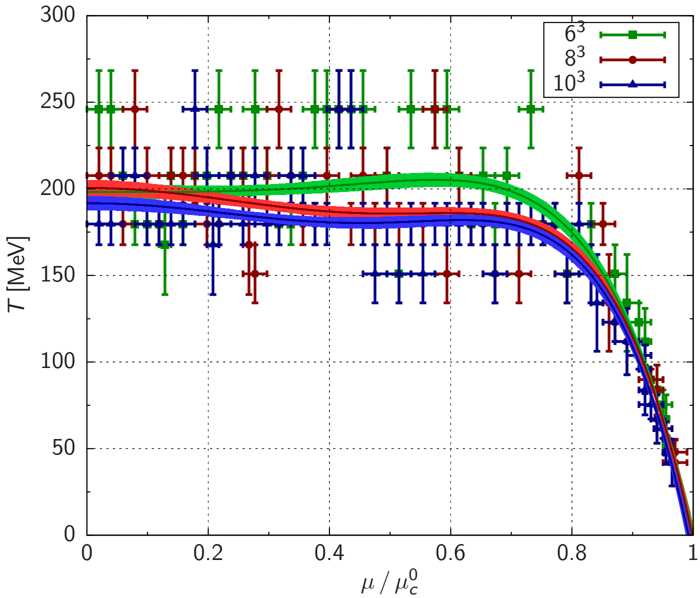
- Polyakov loop

Heavy Dense QCD



- Binder cumulant of the Polyakov loop $B = 1 - \frac{\langle P^4 \rangle}{3 \langle P^2 \rangle^2}$

Heavy Dense QCD



- Fit the phase boundary using $T_c(\mu) = \sum_k b_k \, (1 - \mu/\mu_c)^k$

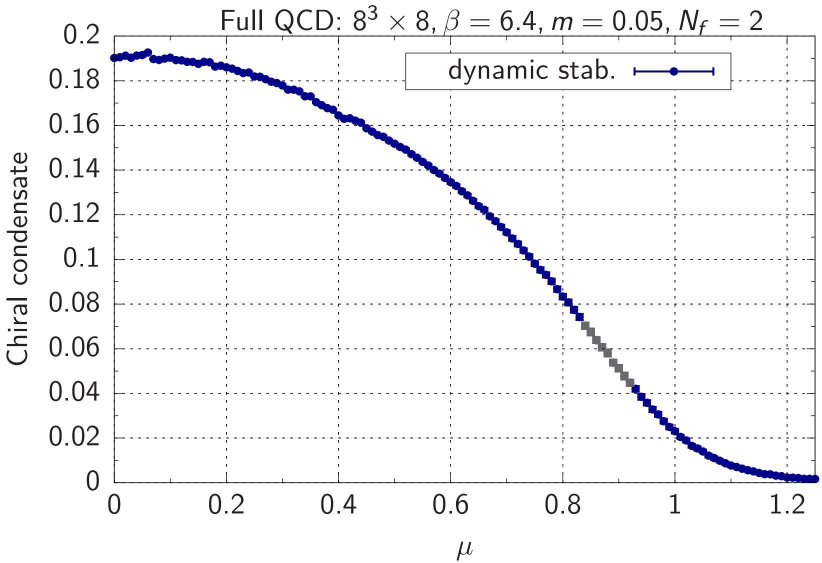
Full (Staggered) QCD

- Here: QCD using unimproved Staggered quarks
- Large gauge coupling $\beta = 6.4$
- Small volume $V = 8^3$
- (Small) pion masses $\sim \mathcal{O}(700)\,\text{MeV}$
- Rough inversion of Dirac operator $M \sim 10^{-3}$

$$D^a_{x,\mu}\, S = D^a_{x,\mu}\, S_{YM} - \frac{N_f}{4} \text{Tr}\left[M^{-1}\, D^a_{x,\mu} M\right]$$

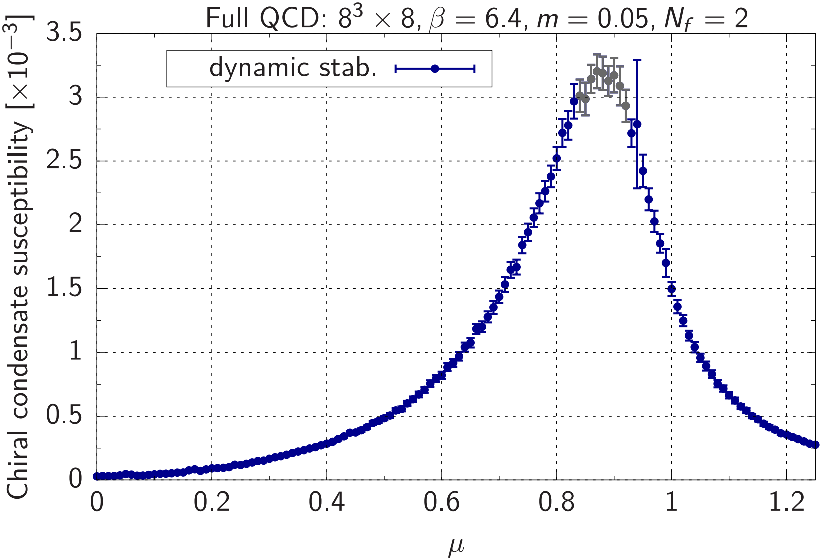
- Every Langevin update needs inversion of the Dirac operator...

Full (Staggered) QCD



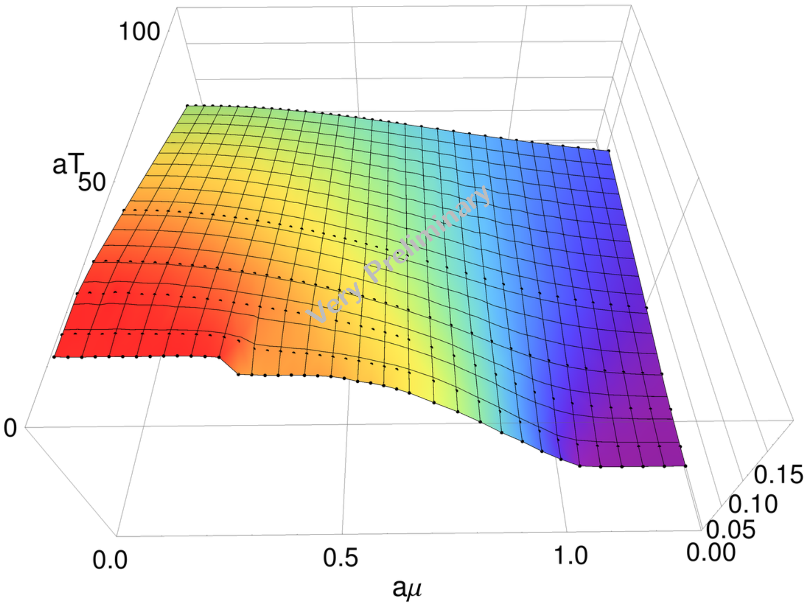
- Unimproved Staggered quarks

Full (Staggered) QCD



- Unimproved Staggered quarks

Full (Staggered) QCD

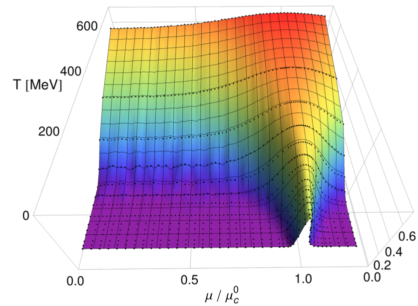


- Unimproved Staggered quarks

Future work

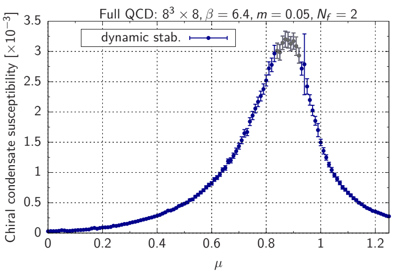
Conclusion

- Complex Langevin simulation can be used to study the QCD phase diagram
- Dynamical stabilisation improves convergence
- Work on the convergence, especially around μ_c.



Future work

- Start proper Full QCD simulations to identify phase structure of QCD.
- A lot of work to be done!



Thank you for your attention!