Charged Pion Condensates in an External Electromagnetism Environment

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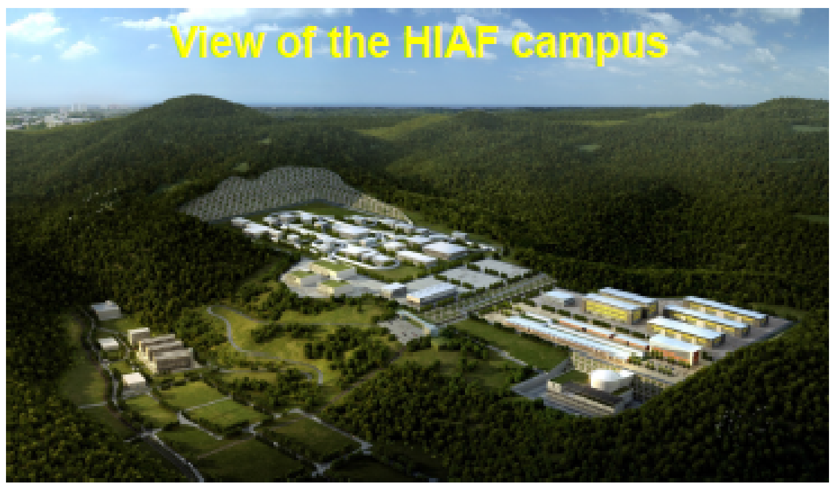
based on JC and Mei Huang, arXiv:1609.04966; JC, Kun Xu and Mei Huang, in preparation

Meeting of the working group on theory of hadronic matter under extreme conditions@JINR, Dubna

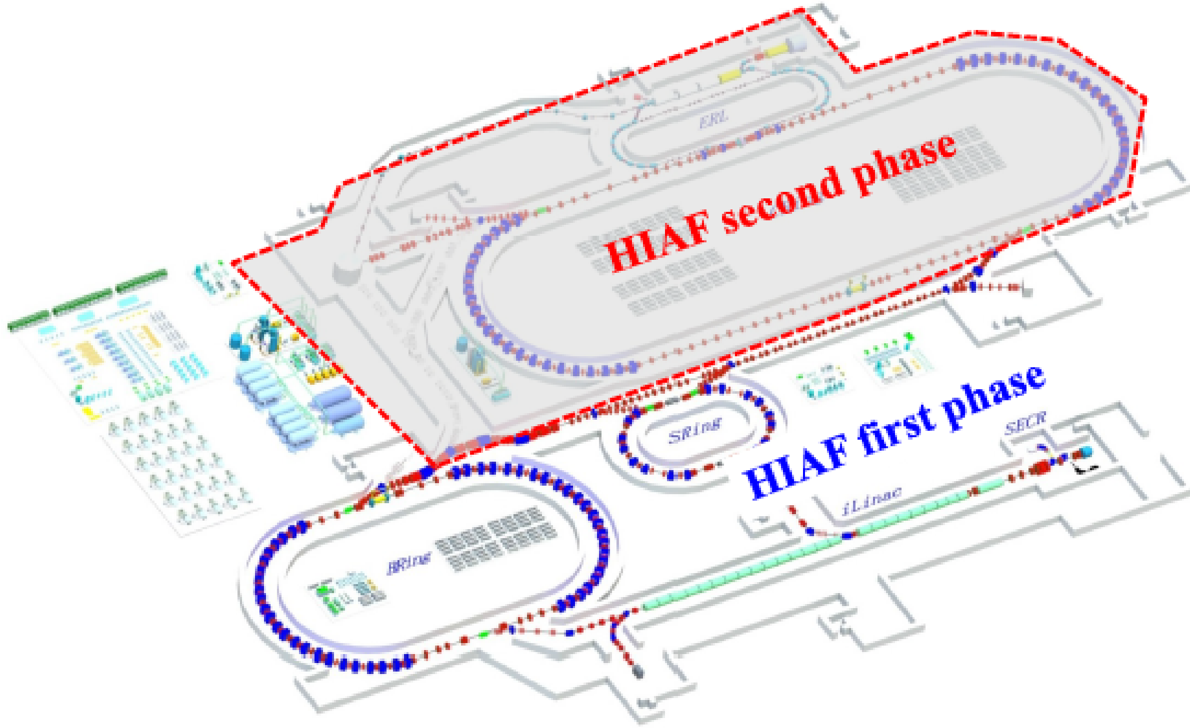
Site of HIAF project-new campus



General description & status

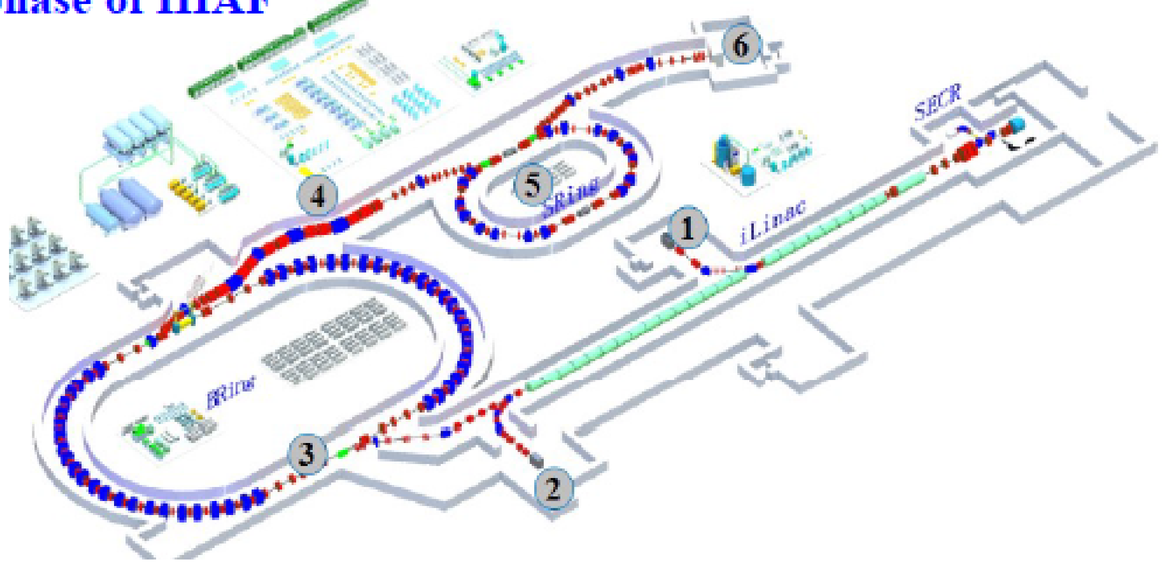
Two phases plan of HIAF

Considering the science goals, technology development, project cost and other factors, the HIAF project will be divided into two phases.



HIAF General description & status

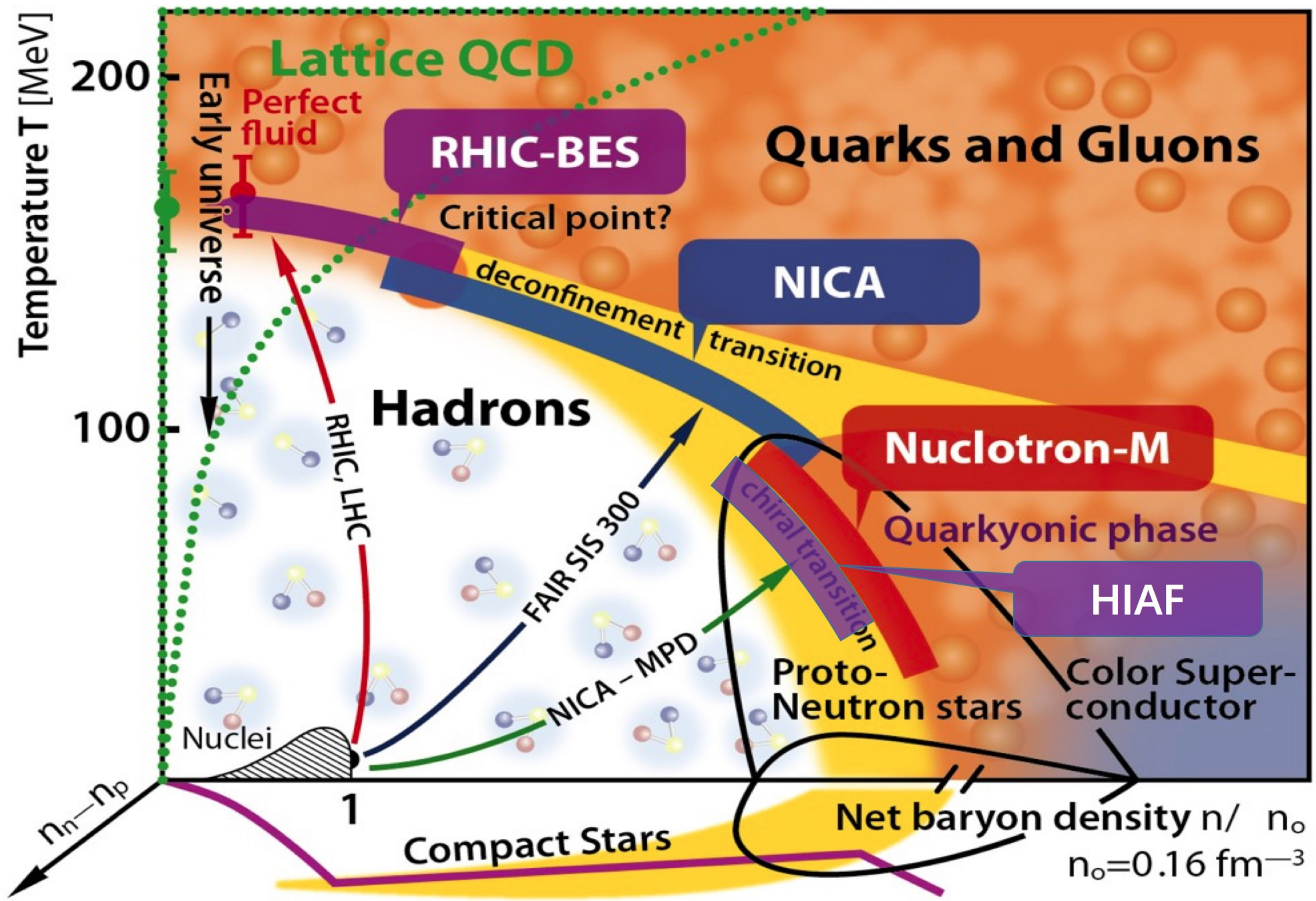
First phase of HIAF



	Ions	Energy	Intensity
SECR	U^{34+}	14 keV/u	0.05 pmA
iLinac	U^{34+}	17 MeV/u	0.028 pmA
BRing	U^{34+}	0.8 GeV/u	~1.0×10^{11} ppp
CRing	U^{34+}	1.1 GeV/u	~5.0×10^{11} ppp
	U^{92+}	4.1 GeV/u	~2.0×10^{11} ppp

1. Nuclear structure spectrometer
2. Low energy irradiation target
3. Electron-ion recombination spectroscopy
4. RIBs beam line
5. High precision spectrometer ring
6. External target station

QCD Phase Diagram

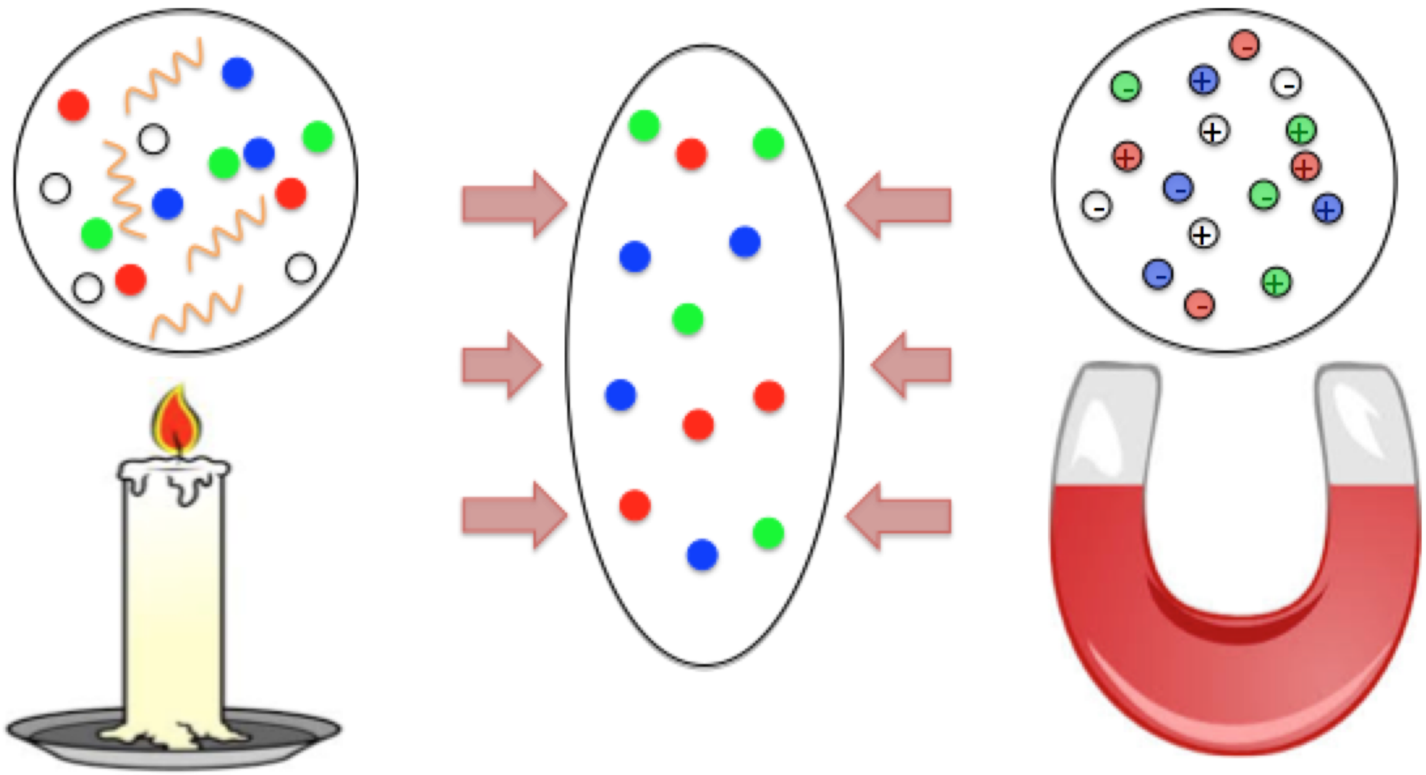


Outline

- Motivations
  - ✓ Different phases of QCD occur in the universe
  - ✓ QCD simplifies in extreme environments
  - ✓ The behaviors of different matter can be similar at the regime of transition
- Proper time method in two flavors space
- Results

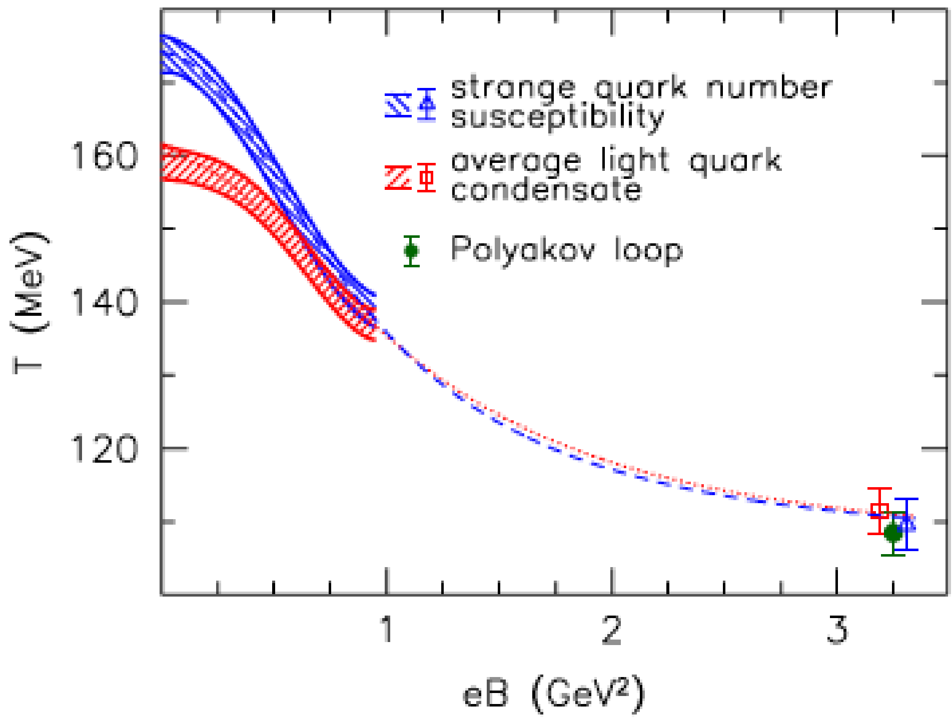
Why Electromagnetic Fields

Heavy ion collisions create the strongest magnetic fields in the Laboratory.



Different excited freedoms at different environments

QCD Phase Diagram in the $T - B$ Plane



T_χ and T_c investigated as a function of magnetic field. c.f. (G. Endrodi, JHEP, 07, 173 (2015).)

Finite Temperature Consequences

Heavy ion collisions and most phase transitions happen at finite temperatures. For example, vacuum gluon (photon) polarization tensor $\Pi^{\mu\nu}$ will become more complicated. c.f. (JC and M. Huang, arXiv:1609.04966)

Six second order tensors:

- The velocity of the fluid u + particle momentum $p \longrightarrow$ three second order tensor $u \otimes u$, $u \otimes q$ and $q \otimes q$.
- Metric $g^{\mu\nu}$.
- The electromagnetic tensor $F_{\mu\nu}$ and dual tensor $\tilde{F}_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu}F^{\alpha\beta}$.

Two constrains:

- Free choice of $u^2 = 1$.
- Wald identity $\Pi^{\mu\nu}q_\nu = 0$.

$\Longrightarrow$ $\Pi^{\mu\nu}$ contains four independent structures

Tensor Structures

Set up four mutual orthogonal four momentums:

$$x_0 = q^\mu; \quad x_1 = \tilde{F}^{\mu\rho} q_\rho; \quad x_2 = F^{\mu\rho} q_\rho;$$
$$x_3 = u^\mu - x_0^\mu \frac{u \cdot x_0}{x_0^2} - x_1^\mu \frac{u \cdot x_1}{x_1^2} - x_2^\mu \frac{u \cdot x_2}{x_2^2}.$$

Hence, the associated transversed symmetric tensors are

$$P_1^{\mu\nu} = \frac{x_1^\mu x_1^\nu}{x_1^2}; \quad P_2^{\mu\nu} = \frac{x_2^\mu x_2^\nu}{x_2^2}; \quad P_3^{\mu\nu} = \frac{x_3^\mu x_3^\nu}{x_3^2},$$

which satisfy following relationship

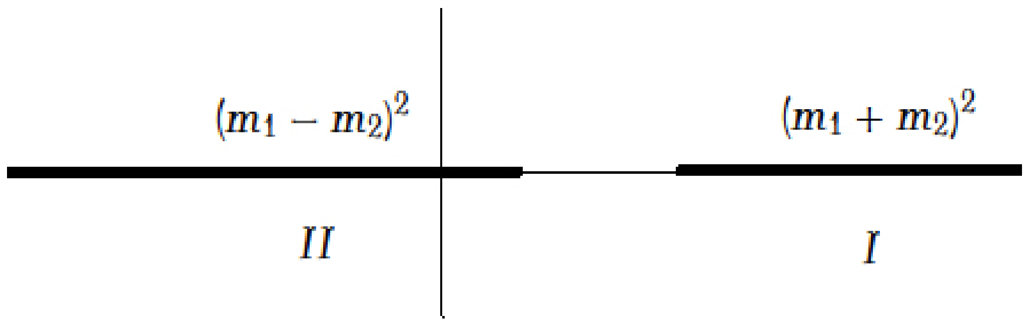
$$P_i^{\mu\nu} = P_i^{\nu\mu}; \quad P_i^{\mu\nu} q_\nu = 0; \quad P_i^2 = P_i; \quad P_i P_j = 0; \quad \sum_{i=1}^{3} P_i^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}.$$

Since $\Pi^{\mu\nu}(q) = \Pi^{\nu\mu}(-q)$, the antisymmetric tensors are allowed:

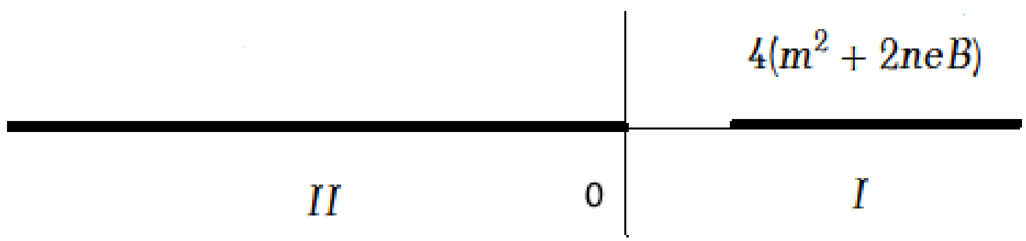
$$P_4^{\mu\nu} = -P_4^{\nu\mu} = \frac{u^\mu x_2^\nu - x_2^\mu u^\nu}{u \cdot q} + F^{\mu\nu}, \quad \text{for} \quad P_4 P_i = 0.$$

Branch Cuts in the Complex Q^2-Plane

- without B:



- turning on B:



Landau Damping from the Finite Landau Levels in Strong B

The first branch cut, $q_0^2 > q_3^2 + 4M^2$,

$$\mathrm{Disc}\,\pi_1(q_0) \simeq \frac{(2eB)|q_{\shortparallel}|^3}{2^{11}\cdot \pi T^3}$$

The second branch cut, $q_0^2 < q_3^2 + 2M_n^2 - 2\sqrt{M_n^4 + q_3^2 M_n^2} < q_3^2$,

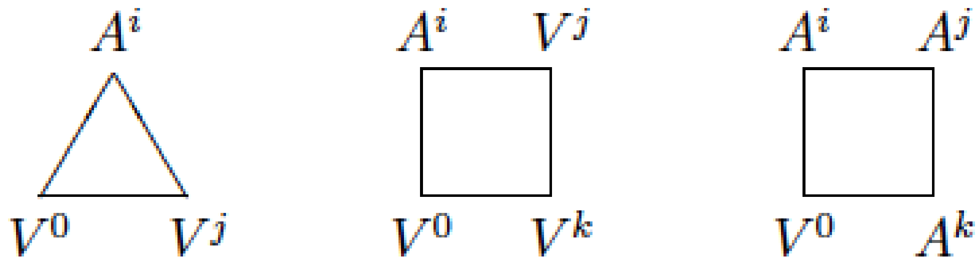
$$\mathrm{Disc}\,\pi_1(q_0) \simeq \left(\sum_{n=0}^{j} + \sum_{n=1}^{j}\right) \frac{-1}{4\pi^{\frac{3}{2}}} \frac{(2eB)\left(q_{\shortparallel}^4 - 4q_0^2 M_n^2\right)^{\frac{1}{2}}}{T^{\frac{1}{2}}|q_0|^{\frac{3}{2}}} \mathrm{Li}_{-1}\left(-\,\mathrm{e}^{-\frac{q_3^2}{2|q_0|T}}\right)$$

where $j = \lfloor q_{\shortparallel}^4/(8eBq_0^2) - \hat{M}^2 \rfloor$ and $M_n^2 = M^2 + 2neB$. The classification of the energy scale is universal in the hard-loop action, where loop momenta $\boldsymbol{k} \sim M_n$; the external momenta $q_3 \sim \lambda^{-\frac{1}{2}} T^{\frac{1}{2}} M_n^{\frac{1}{2}}$, $q_0 \sim \lambda^{-\frac{3}{2}} T$. $\mathrm{Disc}\,\pi_1$ is at the order of $\lambda^{\frac{7}{4}} \mathrm{Li}_{-1}(-\,\mathrm{e}^{-\lambda})$, which is not monotonically decreasing as λ increasing. c.f. (JC and M. Huang, arXiv:1609.04966)

$$\gamma \rightleftharpoons q + q, \qquad \gamma + q \rightleftharpoons q$$

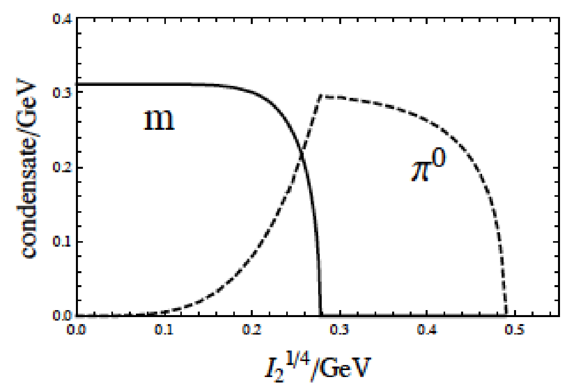
CP Odd Effects in QCD

$$\pi^0 \to \gamma\gamma,\ \pi^- \to e^-\bar{\nu}_e\gamma,\ \gamma\pi^0 \to \pi^+\pi^-$$



Anomalous loop diagrams in two flavour QCD

When $\mathrm{Tr}[\gamma_5 \ldots] \neq 0$, the pion condensates are allowed due to the odd parity domain being created. c.f. (G. Cao and X. G. Huang, Phys. Lett. B, 757, 1 (2016).)



Neutral Pion Condensation and Chiral Density Wave

Put in a periodic ansatz

$$\sigma = M \cos bx; \quad \pi_3 = M \sin bx; \quad \pi_{1,2} = \Delta.$$

c.f. (Y. Hidaka, K. Kamikado, T. Kanazawa and T. Noumi, Phys. Rev. D 92, 034003 (2015); H. Abuki, arXiv:1609.04605; P. Adhikari and J. O. Andersen, arXiv:1610.01647; S. Carignano, L. Lepori, A. Mammarella, M. Mannarelli and G. Pagliaroli, arXiv:1610.06097)

inhomogeneous is energetically favored

Two Flavors NJL Model

For $M = m_0 + \sigma + i\gamma_5\pi_a\tau_a$ and $D_\mu = p_\mu - q_f A_\mu$, one has

$$S_{eff} = \frac{\sigma^2 + \pi_a^2}{4G} + \ln\det\left(i\not{D} - M\right) = \frac{\sigma^2 + \pi_a^2}{4G} + \mathcal{L}_{eff}$$

and the fermions propagator obeys a second order equation via

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{1}{2}\ln\det\left[\left(i\not{D} - M\right)\left(-i\not{D} - M\right)\right] \\ &= \frac{1}{2}\ln\det\left(\not{D}^2 + 2\gamma^5\gamma^\mu\pi_a\left\{D_\mu, \tau_a\right\} + M^2\right)\end{aligned}$$

Without loss of generality, let $\pi_a = (\Delta, 0, 0)$ for charged pions. Therefore, $\left\{q_f A_\mu \delta_{ab}^f, \tau_1\right\} = \left(Q\tau_0 + q\tau_2\right) A_\mu$ where $Q = q_u + q_d$ and $q = q_u - q_d$.

Effective actions from Schwinger proper time

The key of Schwinger proper-time formalism is applying the mathematical identity

$$\frac{i}{A+i\epsilon} = \int_0^\infty ds\, e^{-(A+i\epsilon)s}$$

In the proper time representation:

$$G(x,x') = \int_0^\infty ds\, e^{-iM^2 s}\, e^{-\epsilon s}\mathrm{Tr}\left[\left\langle x'\right| e^{-i\hat{H}s}\left|x\right\rangle\right]$$

It is the amplitude for a particle to propagate from x to x' in the proper time s and then integrate all the trajectory. $\mathcal{L}_{eff}$ is extracted by integrating over M^2.

Ansatz and Solution

We are seeking a solution of the Green function: c.f. (M. R. Brown and M. J. Duff, Phys. Rev. D, 11, 2124 (1975).)

$$\left[\partial_x^2 + \rho_\nu \partial_x^\nu + \alpha(x') + \theta_\nu(x')(x-x')^\nu + \frac{1}{4}\gamma_{\mu\nu}^2(x')(x-x')^\mu(x-x')^\nu\right] \mathcal{G}(x,x';s) = \delta(x,x')$$

where $\rho_\nu = 2\Delta\gamma_5\gamma_\nu\tau_0$ and $\theta_\nu = 2\Delta\gamma_5\gamma_\mu F_\nu^\mu\,(Q\tau_0 + q\tau_2)$ in our study.

Apply the Fourier transformation and make the ansatz that $G(p)$ being in the form

$$\mathcal{G}(p;s) = \int_0^\infty ds\, e^{-\alpha(s)} \exp\left[p^\mu p^\nu A(s)_{\mu\nu} + p^\mu B(s)_\mu + C(s)\right].$$

Under the change of variable $q \to p + \frac{1}{2}A^{-1}\cdot B$, the integration with respect to q becomes an elementary Gaussian. Hence,

$$G(x,x) = \frac{i}{(4\pi)^2}\int_0^\infty \frac{ds}{s^2}\exp\left[-\alpha s + C - \frac{1}{4}B\cdot A^{-1}\cdot B - \frac{1}{2}\mathrm{tr}\ln\left(As^{-1}\right)\right]$$

Equations of Motion

It remains us to determine the unknown A, B and C, By setting

$$\begin{aligned}
\frac{\partial A}{\partial s} &= 1 + A\gamma^2 A; \\
\frac{\partial B}{\partial s} &= 2\, i\theta \cdot A + B \cdot \gamma^2 \cdot A + \rho; \\
\frac{\partial C}{\partial s} &= \frac{1}{2}\mathrm{tr}(\gamma^2 A) + i\theta \cdot B + \frac{1}{4} B \cdot \gamma^2 \cdot B,
\end{aligned}$$

These equations admit the solutions in term of

$$\begin{aligned}
A &= \gamma^{-1} \tan \gamma s \\
B &= -\,i\gamma^{-2}\,(1 - \sec \gamma s)\,(2\theta - i\rho\gamma \cot \gamma s) \\
C &= -\frac{1}{2}\mathrm{tr} \ln \cos \gamma s - \theta \cdot \gamma^{-3}\,(\tan \gamma s - \gamma s) \cdot \theta
\end{aligned}$$

Gap Equations

$$\mathcal{L}_{eff} = \frac{1}{2(4\pi)^2}\int_0^\infty \frac{ds}{s^3}\, e^{-M^2 s - \frac{1}{2}L(s) - q_f\delta^f_{ab}\sigma F s}$$

$$L(s) = \mathrm{tr}\ln\left[(\gamma s)^{-1}\sin\gamma s\right] + \beta\cdot\gamma^{-3}\left(\tan\frac{\gamma s}{2} - \frac{\gamma s}{2}\right)\cdot\beta.$$

Here $\beta = 2\theta - \rho\gamma\cot\gamma s$ and $\gamma = q_f F_{\mu\nu}\delta^f_{ab}$.

Obviously, $\mathcal{L}_{eff}$ reduced to what Schwinger obtained for $\Delta = 0$. c.f. (J. Schwinger, Phys. Rev. 82, 664 (1951).)

Gap equations are derived easily:

$$\frac{\delta S_{eff}}{\delta\sigma} = 0 = \frac{\sigma}{2G} + \frac{\delta\mathcal{L}_{eff}}{\delta\sigma};$$

$$\frac{\delta S_{eff}}{\delta\Delta} = 0 = \frac{\Delta}{2G} + \frac{\delta\mathcal{L}_{eff}}{\delta\Delta}.$$

Flipped Term in the Flavor Space

Trace is taking over spinor and flavor.

Remind $\theta_\nu = 2\Delta\gamma_5\gamma_\mu F^\mu_\nu\left(Q\tau_0 + q\tau_2\right)$.

Under the help of the projection operator:

$$P_\pm = \frac{1}{2}\left(\tau_0 \pm \tau_3\right)$$

we have

$$\tau_2\gamma\tau_2 = \tau_2\left(2q_u P_+ F + 2q_d P_- F\right)\tau_2 = 2q_d P_+ F + 2q_u P_- F$$

Thus, one part of θ in the second term of $L(s)$ is flipped in the u, d flavor space which indicating the formation of charged pion condensates.

But

Numerical simulation suffers a large oscillation in terms of above formula. If you have little time,

Equivalent Replacement

How to get a quick physics answer in phenomena?

☞ As one of candidate to explain the inverse magnetic catalysis, QCD phase transition has been intensively studied at finite chiral chemical potential μ_5. c.f. (JC, P. Chu and M. Huang, Phys. Rev. D, 88, 054009 (2013).)

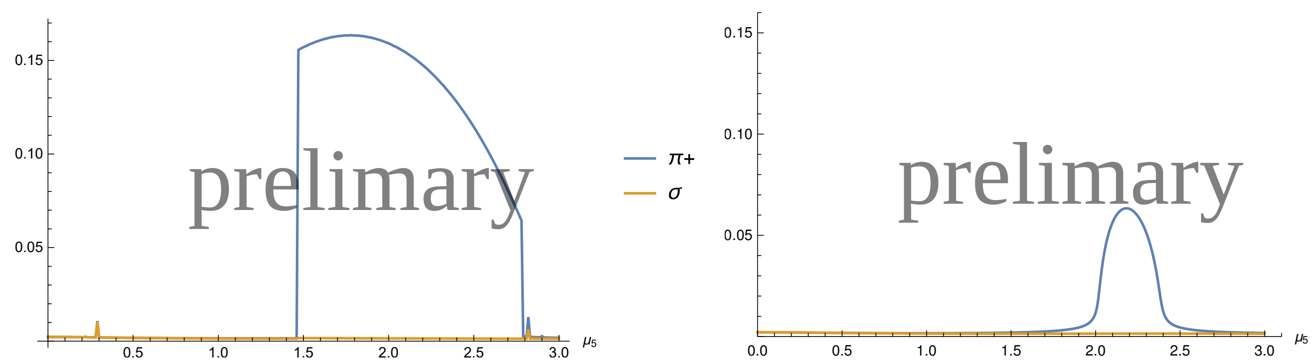
Indeed, nonzero n_5 inducing in electromagnetic field is expressed by: c.f. (K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008))

$$\frac{d^4 Q_5}{dt\,dV} = \frac{q_f^2}{2\pi^2} E \cdot B$$

Asymmetric $\mu_5 = \mathrm{diag}\left(q_u^2, q_d^2\right)\hat{\mu}_5$ is applied in two flavor NJL model. c.f. (JC, K. Xu and M. Huang, in preparation.)

At Finite Baryon Density

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu + \frac{q_u^2+q_d^2}{2}\mu_5\gamma^0\gamma^5 + \frac{q_u^2-q_d^2}{2}\mu_5\gamma^0\gamma^5\tau_3 + \mu_B\gamma^0 - M - i\gamma^5\tau_a\Delta_a\right)\psi - \frac{(M-m_0)^2+\Delta_a^2}{4G}$$



The behaviors of sigma and charged pion condensates as a function of μ_5 at $T = 0, \mu_B = 0.4\text{GeV}$ (left) and $T = 10\text{MeV}, \mu_B = 0.4\text{GeV}$ (right).

c.f. (JC, K. Xu and M. Huang, in preparation.)

Summary

- Thermalize different degrees.
- Construct a $\mathcal{L}_{HBL}$ so that the free propagator gives thermal quasi particles, screening (m_D) and Landau damping.
- Explore new possible phases after turning on $E \cdot B$.

Thank You for Your Attention!