Lattice QCD Studies of Nucleon Structure from χQCD collaboration

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HMEC, 2016

November 3, 2016

Outline

- Lattice approach in brief
- Current status in Nucleon Spin Structure
- Recent work
  - Quark spin
  - Glue spin
  - Strange quark magnetic moment
- Future plans

- Starts with QCD action in continuum

$$S = \int d^4x\overline{\psi}(\gamma^\mu D^\mu + m)\psi + \frac{1}{2}\mathrm{Tr}\int d^4x F^{\mu\nu}F_{\mu\nu}$$

- Euclidean rotation: $t \to -it$.
- $A_\mu \Rightarrow U_\mu(x) \equiv e^{igA_\mu(x+a\hat{\mu}/2)}$.
- Fermionic fields: $\psi(x) \to \psi(na)$.
- Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ with the weight $[\mathrm{Det}\, M]\, e^{-s_g}$.
- Quenched Approximation: $\mathrm{Det}\, M = 1$.

- Gauge configurations, U.
- Quark propagator, M^{-1}, on every configuration.

Construct and analyze:

- Two-point correlation functions

$$\begin{aligned}\mathrm{Tr}[\Gamma\, G_{NN}(t_2,\vec{p})] &= \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} \langle 0|T\left[\chi(x_2)\,\bar{\chi}(x_0)\right]|0\rangle \\ &= \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} \sum_{\{U\}} \mathrm{Tr}\{f\left[M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0);U\right]\}\end{aligned}$$

- Three-point correlation functions

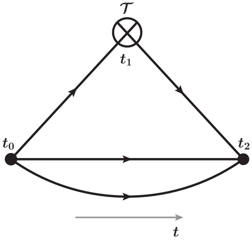
$$\begin{aligned}G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0) &= \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}.\vec{x}_2} e^{i\vec{q}.\vec{x}_1} e^{i\vec{p}'.\vec{x}_0} \langle 0|\mathrm{T}(\chi(x_2)\mathcal{T}(x_1)\bar{\chi}(x_0))|0\rangle \\ &= \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} e^{+i\vec{q}.(\vec{x}_1-\vec{x}_0)} \sum_{\{U\}} \mathrm{Tr}\{f\;\left[M^{-1}\,M^{-1}\,M^{-1}\,M^{-1};U\right]\}\end{aligned}$$

$$\mathrm{Tr}\left[\Gamma G_{N\mathcal{T}N}(\vec{p}, t_2; \vec{q}, t_1; \vec{p}, t_0)\right] \xrightarrow{t_1 \gg t_0,\ t_2 \gg t_1} A\, e^{-E_p(t_2 - t_1)} e^{-E_{p'}(t_1 - t_0)} \mathcal{M}(q^2)$$

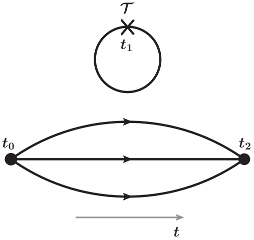
- $\mathcal{M}(q^2) = \langle p, s | \mathcal{T} | p', s' \rangle$
- q^2 is the momentum transfer squared
- Ratio:

$$\frac{\text{3-pt}}{\text{2-pt}} \xrightarrow{t_1 \gg t_0,\ t_2 \gg t_1} \mathcal{M}(q^2) f(E_p, E_{p'}, m)$$

Connected and Disconnected Insertions



(a)



(b)

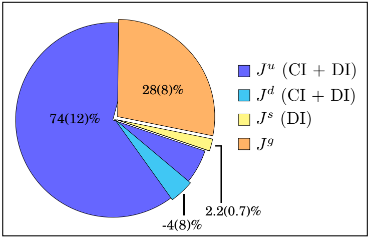
- DI three-point functions are computationally challenging: Involves propagators from *all-to-all* points.
- *strange* quark contributions come only from DI. *up* and *down* quarks have contributions both from CI and DI.
- DI and disconnected diagrams are not the same.

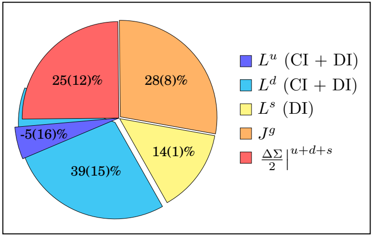
$$G_{N\mathcal{T}N}\big|_{\mathrm{DI}} \equiv \langle \chi \mathcal{T} \bar{\chi} \rangle - \langle \mathcal{T} \rangle \langle \chi \bar{\chi} \rangle$$

Nucleon Spin

- Spin sum rule: $\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma_q + L_q + J_g$ [X. Ji, 1997]
- Quark Spin: $\langle p, s|\overline{\psi}_q \gamma_\mu \gamma_5 \psi_q |p, s\rangle \sim \Delta\Sigma_q$
- Operators for J_q and J_g: $\mathcal{T}^q_{\{4i\}} = \overline{\psi}_q \gamma_{\{4}(-i \overleftrightarrow{D})_{i\}} \psi_q$, $\mathcal{T}^g_{\{4i\}} = -i\, F_{4\alpha}\, F_{i\alpha}$
- Matrix Elements: $\langle p, s|\mathcal{T}^{q,g}_{\{4i\}}|p', s'\rangle \sim [a_1 T_1(q^2) + a_2 T_2(q^2) + a_3 T_3(q^2)]$
- $J_{q,g} = \frac{1}{2}[T_1(0) + T_2(0)]$
- Quark orbital angular momenta: $L_q = J_q - \frac{1}{2}\Sigma_q$

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma_q + L_q + J_g \,, \; L_q = J_q - \frac{1}{2}\Sigma_q$$



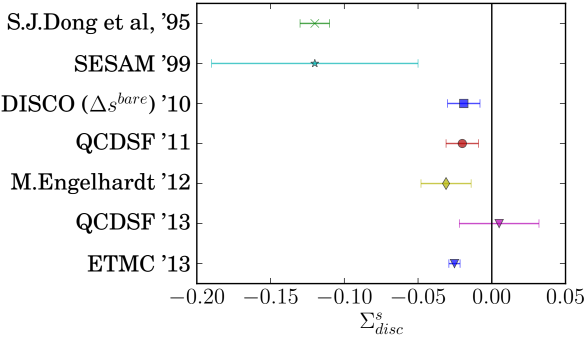


[M. Deka *et al.*, 2015]

Few drawbacks:

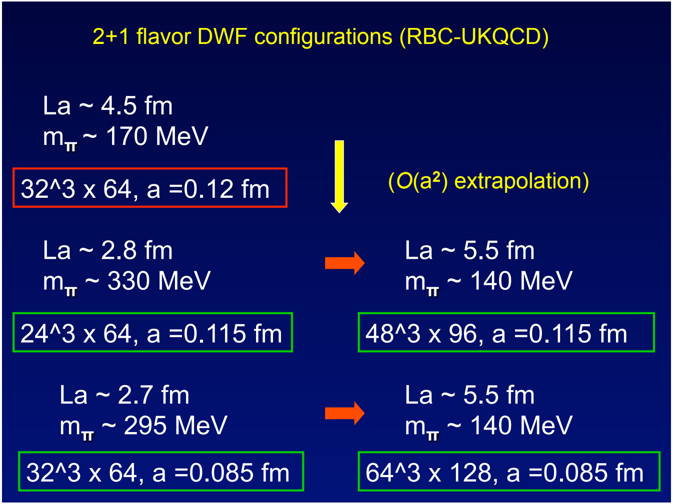
- Quenched approximation
- Higher pion masses, lowest being 478 (4) MeV
- Smaller lattice, $16^3 \times 24 \Rightarrow$ Finite volume effect

[Syritsyn, Lattice 2013]



[D. de Florian, *et al.*, 2009]

Spin	Global Analysis
Δu	0.814(28)(34)
Δd	−0.456(35)(25)
Δs	−0.112(28)(31)
$\Delta\Sigma_q$	0.245(42)(62)



- Overlap valence quarks.

Strange Quark Spin

- Using Anomalous Ward Identity:

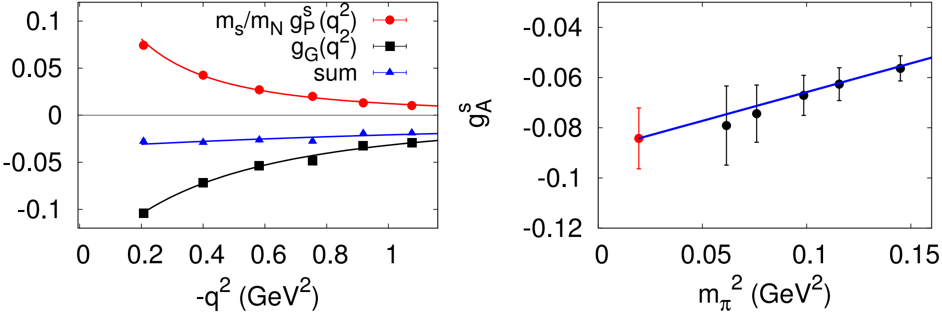
$$\partial_\mu \kappa_A A^0_\mu = \sum_{f=1}^{N_f} 2m_f P^f - 2iN_f q$$

$A^0_\mu = \sum_{f=u,d,s} \overline{\psi}_f i\gamma_\mu\gamma_5(1-\frac{1}{2}D_{ov})\psi_f$

$P^f = \overline{\psi}_f i\gamma_5(1-\frac{1}{2}D_{ov})\psi_f$, $q(x) = \mathrm{Tr}\gamma_5(\frac{1}{2}D_{ov}(x,x)-1)$

- $g^s_A = \lim_{|\vec{q}|\to 0} \frac{i|\vec{s}|}{\vec{q}\cdot\vec{s}} \frac{\langle p', s|2m_s P^s - 2iq\,|p,s\rangle}{\langle p', s|p,s\rangle} = \frac{m_s}{m_N} g^s_P(0) + g_G(0)$

$g^s_A\, s_\mu = \langle p,s\,|A^s_\mu|\,p,s\rangle / \langle p,s|p,s\rangle$



[Ming Gong *et al.*, 2015]

- $24^3 \times 64$ lattice
- $\Delta s = -0.084(12)\,,\ \kappa_A \sim 2.6$
- $\Delta s = -0.112(28)(31)$ [D. de Florian, *et al.*, 2009]

Gluon Helicity

- $J_g \stackrel{?}{=} S_g + L_g$

  [Chen *et al.*, 2008; X. Ji, 2008; Wakamatsu, 2010; E. Leader, 2011; Y. Hatta, 2011; $\cdots$]

- $A_\mu = A_\mu^{phys} + A_\mu^{pure}$

- Satisfy the Gauge transformations:

$$A_\mu^{\prime phys} = g(x) A_\mu^{phys} g^{-1}(x) \,, \;\; A_\mu^{\prime pure} = g(x) A_\mu^{pure} g^{-1}(x) + \frac{i}{g_0} g(x) \partial_\mu g^{-1}(x)$$

- Set the conditions:

$$F_{\mu\nu}^{pure} = \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} + i g_0 [A_\mu^{pure}, A_\nu^{pure}] = 0$$

$$D_i A_i^{phys} = \partial_i A_i^{phys} - i g_0 [A_i, A_i^{phys}] = 0.$$

- In the IMF, the forward matrix element of the longitudinal $\vec{S}_g$ corresponds to the glue helicity, ΔG. [X. Ji *et al.*, 2013]

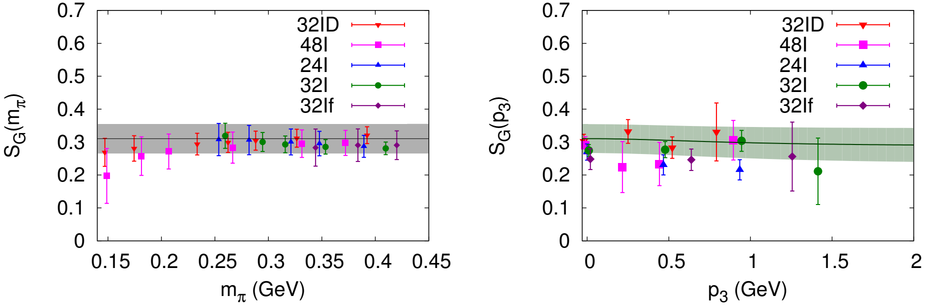
- ΔG is being experimentally measured at STAR, PHENIX and COMPASS.

- Under Coulomb gauge condition, $U_\mu(x) = g_C(x) U^C_\mu(x) g_C^{-1}(x + a\hat{\mu})$

$$A^{phys}_{c,\mu} = \left[\frac{U^c_\mu(x) - U^{c\dagger}_\mu(x) + U^c_\mu(x - a\hat{\mu}) - U^{c\dagger}_\mu(x - a\hat{\mu})}{4iag} \right]_{\text{traceless}} \quad (1)$$

[Y. Hatta *et al.*, 2013; C. Lorcé *et al.*, 2013; Y. Zhao *et al.*, 2015]

- Compute $\vec{S}_g = \int d^3x \, \mathrm{Tr}(\vec{E}^C \times \vec{A}^C)$ for various momenta.



[Yi-Bo Yang, *et al.*, 2016]

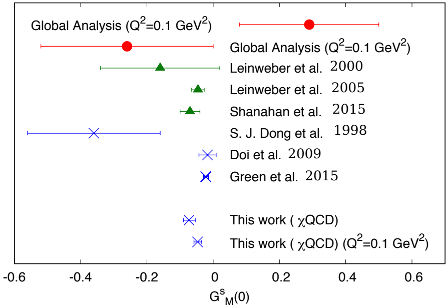
- Mild dependence on quark masses, lattice spacings and volumes.
- Mild dependence on proton momenta.
- $\Delta G(\mu^2 = 10 \text{ Gev}^2) \simeq S_g(\mu^2 = 10 \text{ Gev}^2) = 0.287(55)$.
- Global Analysis: $\Delta G = 0.10^{+0.06}_{-0.07}$ [NNPDF Collaboration, 2014]

Nucleon Strangeness Magnetic Moment

Ensemble	$L^3 \times T$	a (fm)	$m_s^{(s)}$(MeV)	N_{config}
24I	$24^3 \times 64$	0.1105(3)	120	203
32I	$32^3 \times 64$	0.0828(3)	110	309
48I	$48^3 \times 96$	0.1141(2)	94.9	81

- 17 valence quark masses in total in the range $m_\pi \in (135, 400)$ MeV.
- $G_M^s(0) = -0.073(19)\mu_N$, $G_M^s(Q^2 = 0.1\,\text{GeV}^2) = -0.047(11)$

[R. S. Suffian *et al.*, 2016]



Future Plans

Investigate

- All the components of Nucleon Spin, both Connected and Disconnected insertions.
- Moments of PDF's such as $\langle x \rangle$, $\langle x^2 \rangle$, etc.
- $\pi N \sigma$ and σ_{sN} terms.
- Transverse quark Spin.
- PDF's from the Hadronic Tensor, etc.

Thank you !!