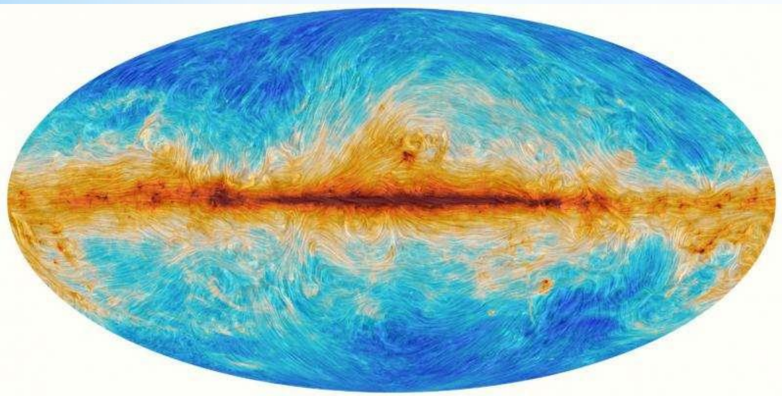
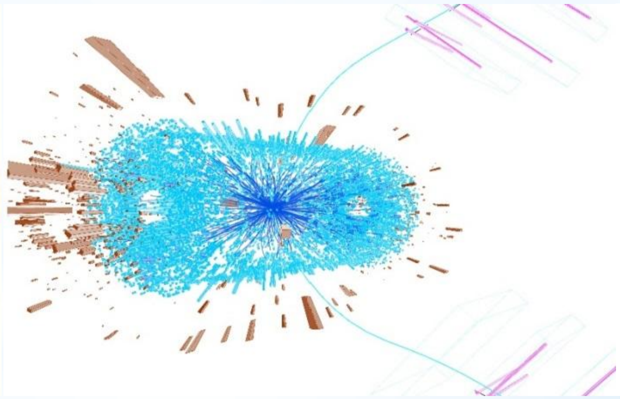
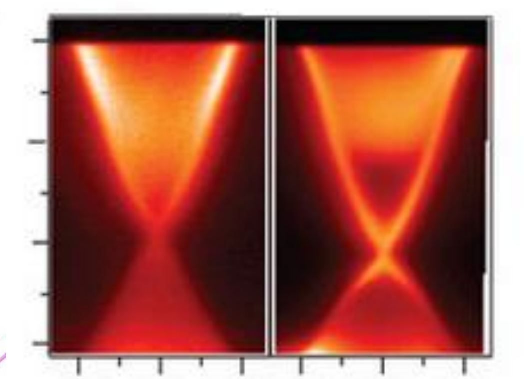
# Real-time simulations of non-equilibrium chiral plasma


Pavel Buividovich
(Regensburg)



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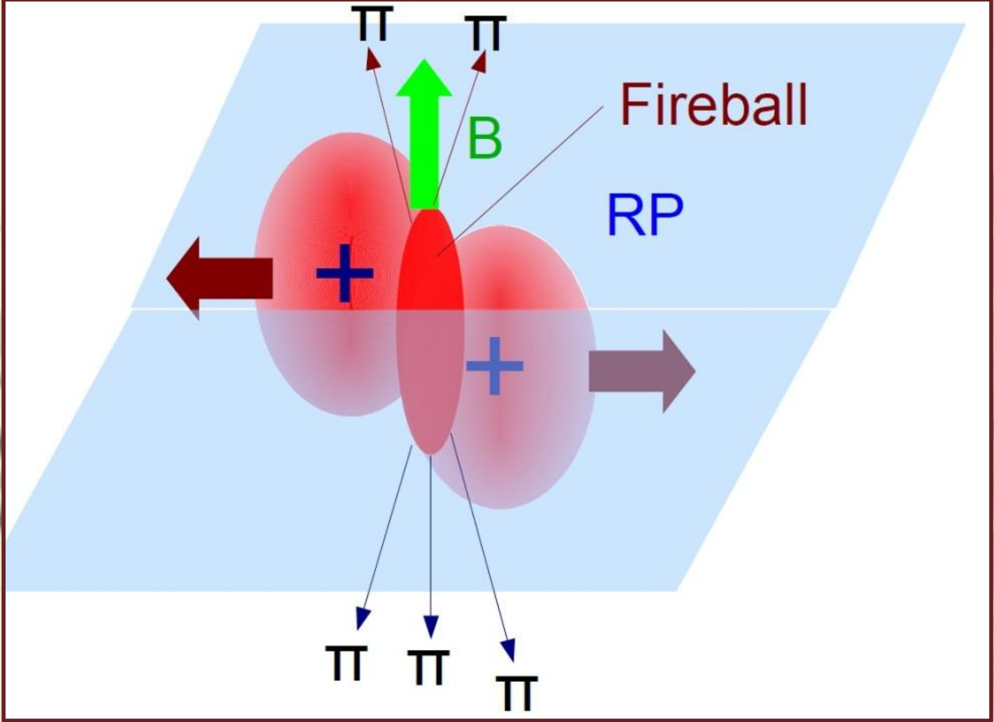
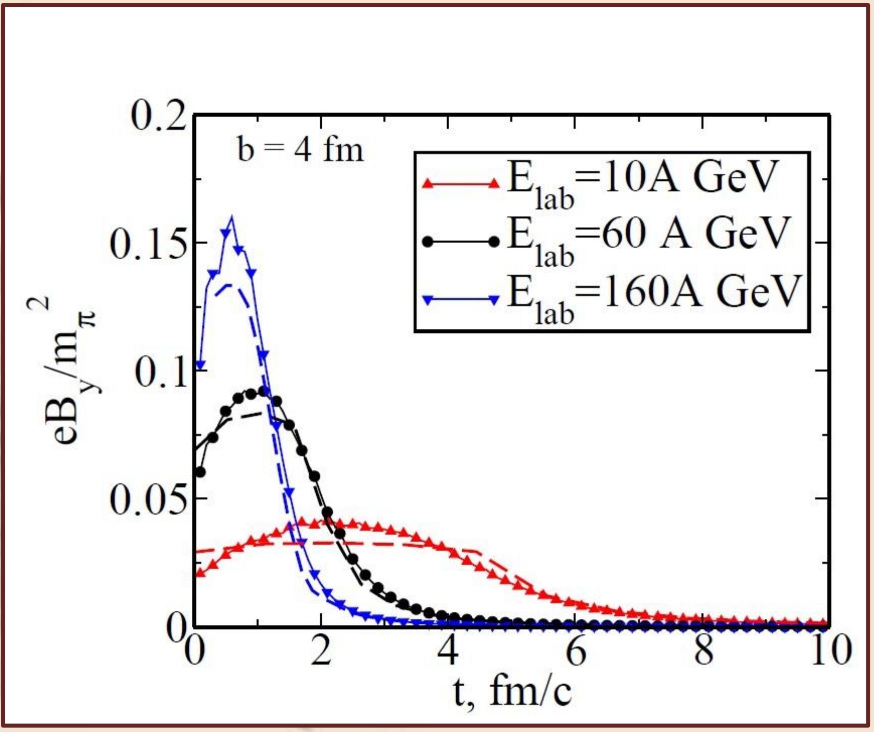
# Generation of magnetic fields in heavy-ion collisions




Relative motion
of two large charges (Z ~ 100)

↓

Large magnetic field and vorticity
in the collision region

URQMD simulations Au+Au
No backreaction
From [Skokov, Toneev, ArXiv:0907.1396]
**Weak energy dependence!!!**

# Anomalous transport: CME, CSE, CVE

**Chiral Magnetic Effect**
[Kharzeev, Warringa, Fukushima]

$$j^i_V = \sigma^{\mathcal{B}}_{VV}\, B^i = \frac{N_c e\, \mu_A}{2\pi^2}\, B^i$$

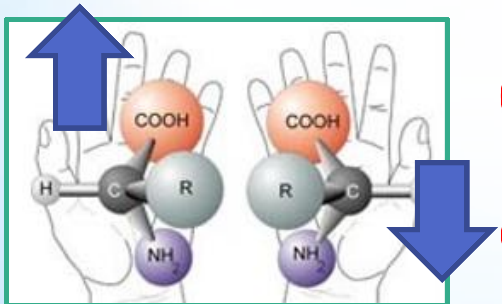
**Chiral Separation Effect**
[Son, Zhitnitsky]

$$j^i_A = \sigma^{\mathcal{B}}_{AV}\, B^i = \frac{N_c e\, \mu_V}{2\pi^2}\, B^i$$

**Chiral Vortical Effect**
[Erdmenger *et al.*,
Teryaev, Banerjee *et al.*]

$$j_V = \sigma^{\mathcal{V}}_V w = \frac{N_c e}{2\pi^2}\, \mu_A\, \mu_V\, w$$

$$j_A = \sigma^{\mathcal{V}}_A w = N_c e \left( \frac{\mu_V^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12} \right) w$$




Flow vorticity

**Origin in quantum anomaly!!!**

# Anomalous transport: signatures???

- Negative magnetoresistivity (CME+anomaly) [this talk, S. Valgushev]
- Charge separation (CME)
- Chiral waves (CME + CSE, CVE…) [S. Valgushev]
- Chiral plasma instability (CME + anomaly) [this talk]

All these effects (except CSE) are non-equilibrium, real-time processes

… Both in cold and hot QCD matter!!!

# Chirality pumping and magnetoresistivity

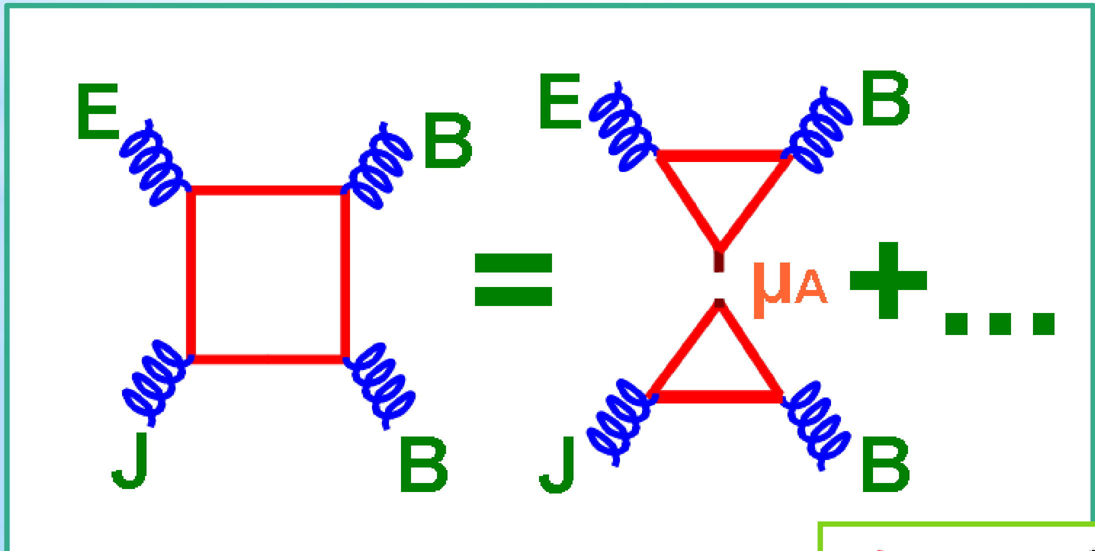
$$\partial_\mu j^A_\mu = \frac{e^2}{8\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} \Rightarrow \frac{d}{dt} Q_A = \frac{e^2}{2\pi^2} \int d^3x\, \vec{E}\cdot\vec{B}$$

**Relaxation time approximation:**

$$\frac{d}{dt} Q_A = \frac{e^2}{2\pi^2} \int d^3\vec{x}\, \vec{E}\cdot\vec{B} - \frac{Q_A}{\tau_A}$$

$$Q_A(t \gg \tau_A) = \frac{e^2 \tau_A}{2\pi^2} \int d^3\vec{x}\, \vec{E}\cdot\vec{B}$$

$$\frac{Q_A}{V} = \frac{\mu_A T^2}{3 v_F^3}, \quad \mu_A \ll T, \mu = 0$$




$$\mu_A = \frac{3 v_F^3 \tau}{2\pi^2} \frac{\vec{E}\cdot\vec{B}}{T^2}$$

$$\vec{j} = \frac{\mu_A}{2\pi^2} \vec{B}$$

$$\vec{j} = \frac{3 v_F^3 \tau}{4\pi^4 T^2} \vec{B} \left(\vec{B}\cdot\vec{E}\right)$$

[S. Valgushev's talk for real-time]

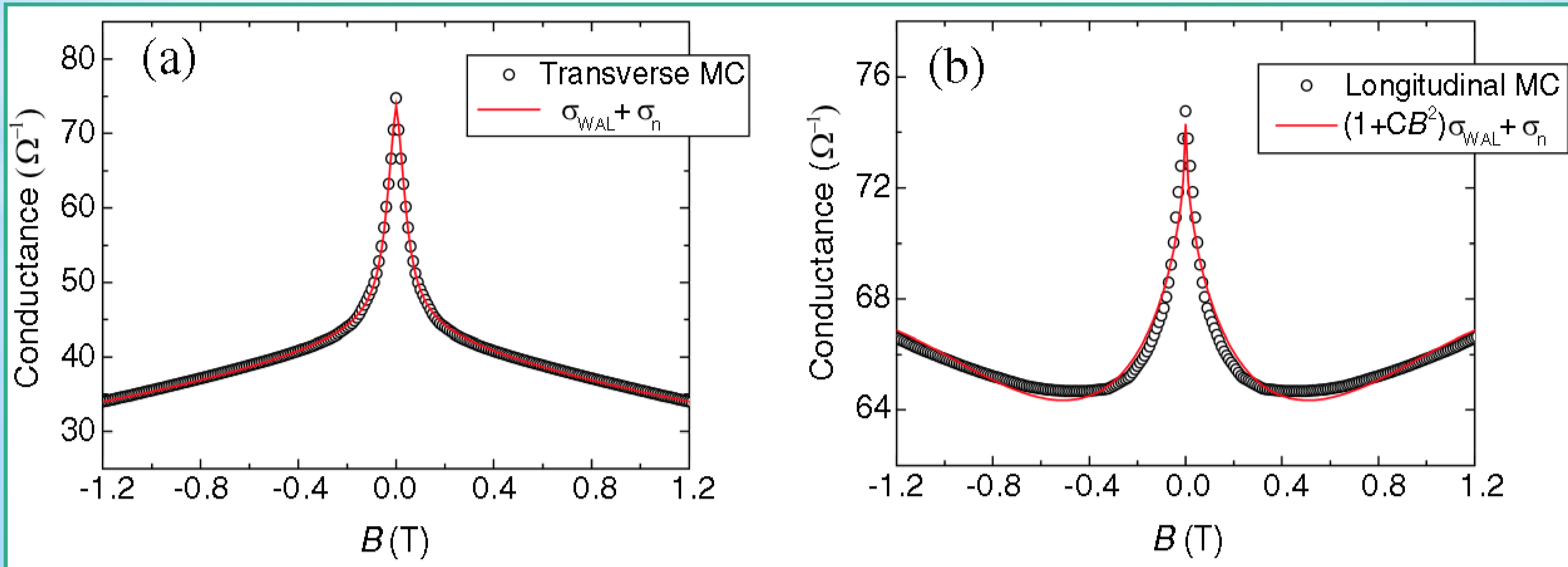
# Negative magnetoresistivity

## Experimental signature of axial anomaly, $Bi_{1-x}Sb_x$ , T ~ 4 K

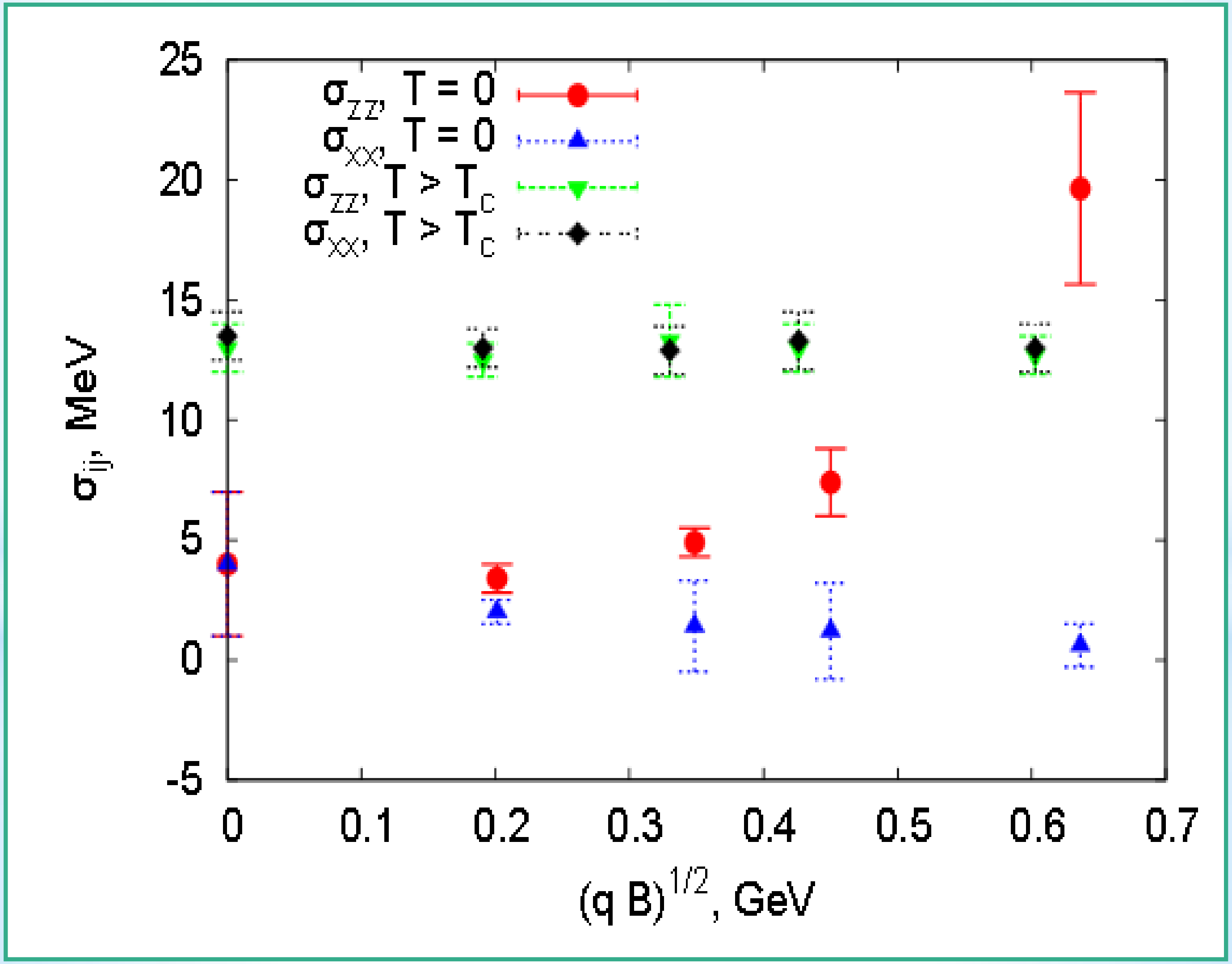
PRL 111, 246603 (2013) PHYSICAL REVIEW LETTERS week ending 13 DECEMBER 2013

**Dirac versus Weyl Fermions in Topological Insulators: Adler-Bell-Jackiw Anomaly in Transport Phenomena**

Heon-Jung Kim,[1,*] Ki-Seok Kim,[2,3,†] J.-F. Wang,[4] M. Sasaki,[5] N. Satoh,[6] A. Ohnishi,[5] M. Kitaura,[5] M. Yang,[4] and L. Li[4]



# Magnetoresistivity in lattice QCD

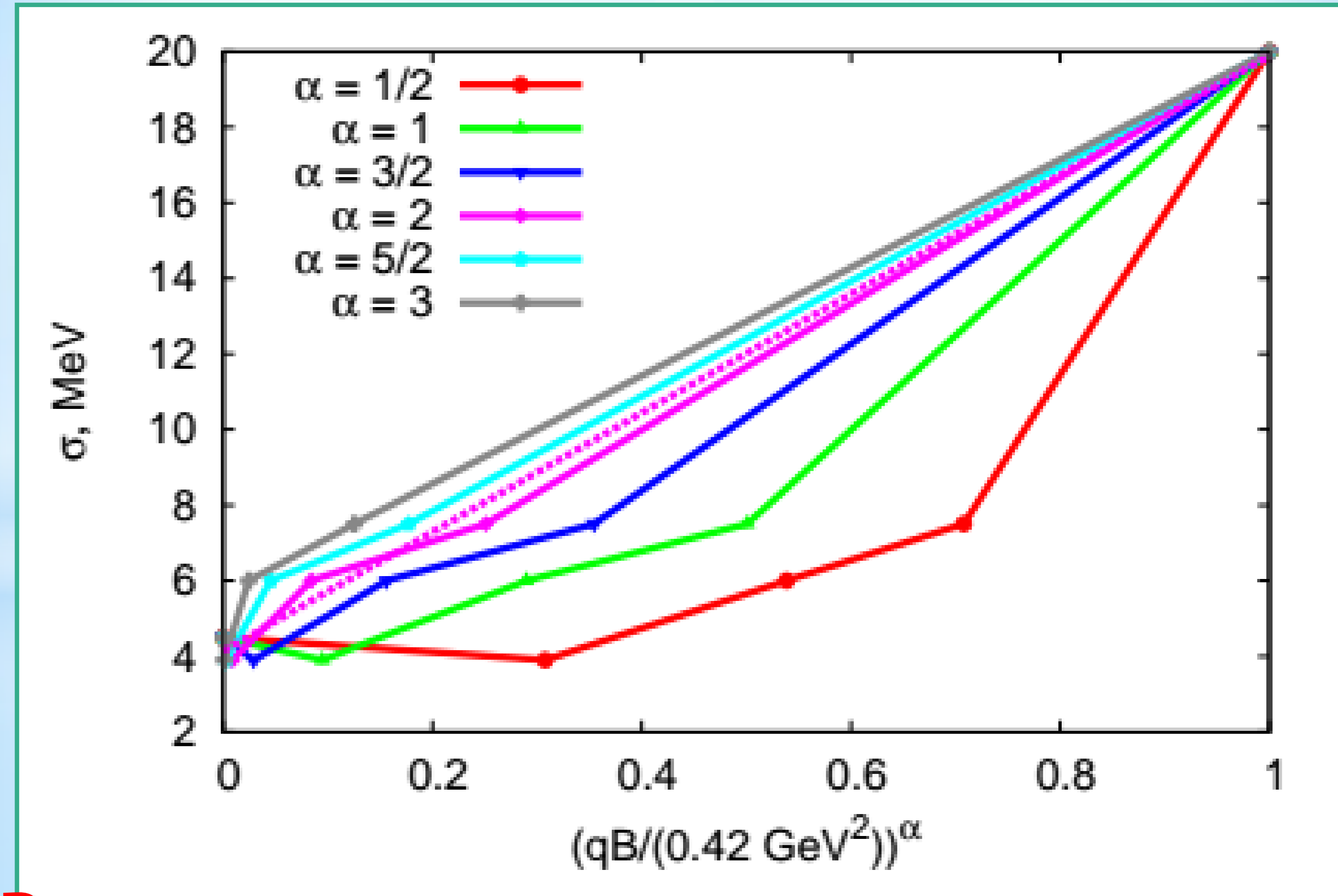



[P.B.,Chernodub,Luschevskaya,Kalaydzhyan,Kharzeev, Polikarpov]

# Magnetoresistivity in QCD

- At high temperatures, anomaly contribution supressed as $1/T^2$
- Small compared to normal conductivity (grows with T) if magnetic fields are realistic
- At small temperatures, anomaly is the only contribution to conductivity
- Effect mostly important for hadronic phase in off-central HIC

# Magnetoresistivity in QCD




**Best fit**

$$\sigma = 0.0157\,\mathrm{GeV}\left(qB/(0.42\,\mathrm{GeV}^2)\right)^2 + 0.004\,\mathrm{GeV}.$$

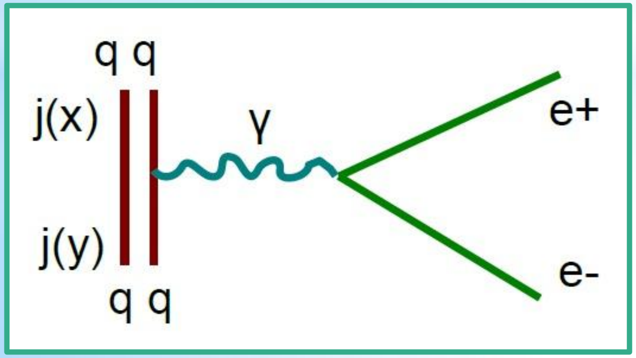
# Magnetoresistivity in QCD

## Estimate of chiral relaxation time

$$\sigma_{\parallel} = \frac{3\tau}{4\pi^4 T^2}$$

$$\tau \sim 0.1\,\mathrm{fm}/c$$

## Experimental signatures in dilepton emission rate



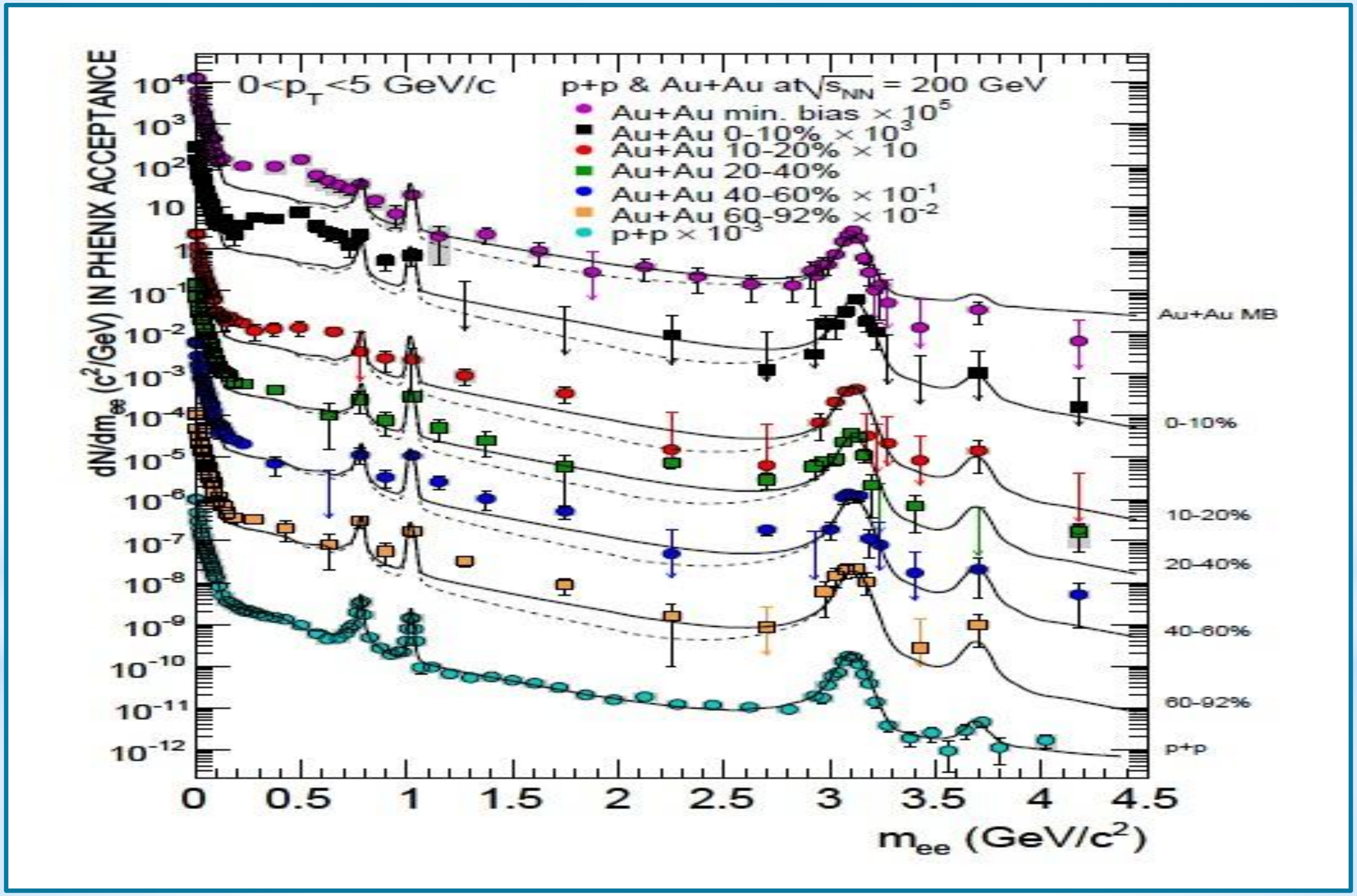

$$\frac{R}{V} = -4e^4 \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} L^{\mu\nu}(p_1, p_2) \frac{\rho_{\mu\nu}(q)}{q^4},$$

$$L^{\mu\nu} = \left(\left(p_1 \cdot p_2 + m^2\right)\eta^{\mu\nu} - p_1^\mu p_2^\nu - p_2^\mu p_1^\nu\right)$$

$$\sigma_{ij} \sim B_i B_j$$

$$\frac{R}{V} \sim \int \frac{d^3 p}{p^2} \left(\vec{B}^2 - \left(\vec{B} \cdot \vec{n}\right)^2\right) \sim B^2 \sin^2(\theta)$$

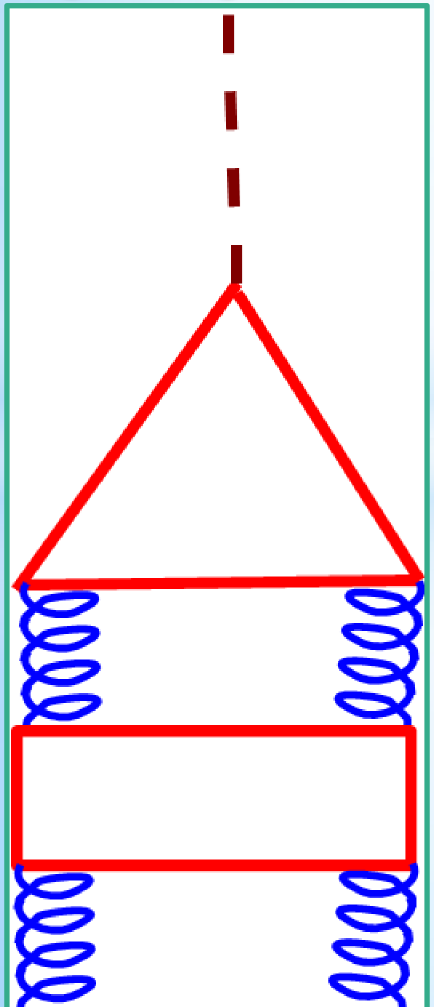
# Magnetoresistivity and dilepton yield?

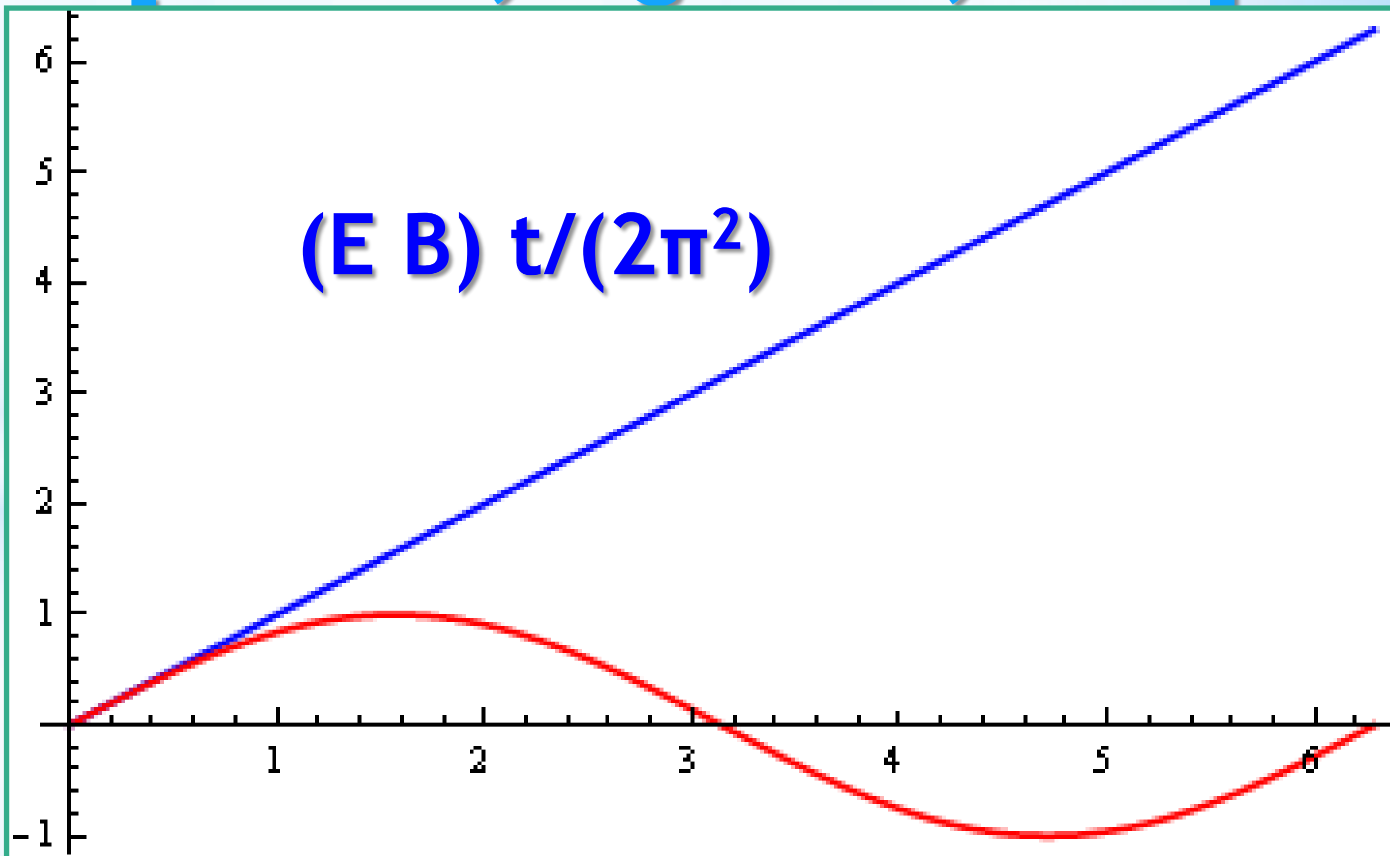



**Experimental data [PHENIX, ArXiv:0912.0244]:**
**More dileptons for central collisions…**
**Advantageous to look at cold matter?**

# Magnetoresistivity and dynamic EM fields

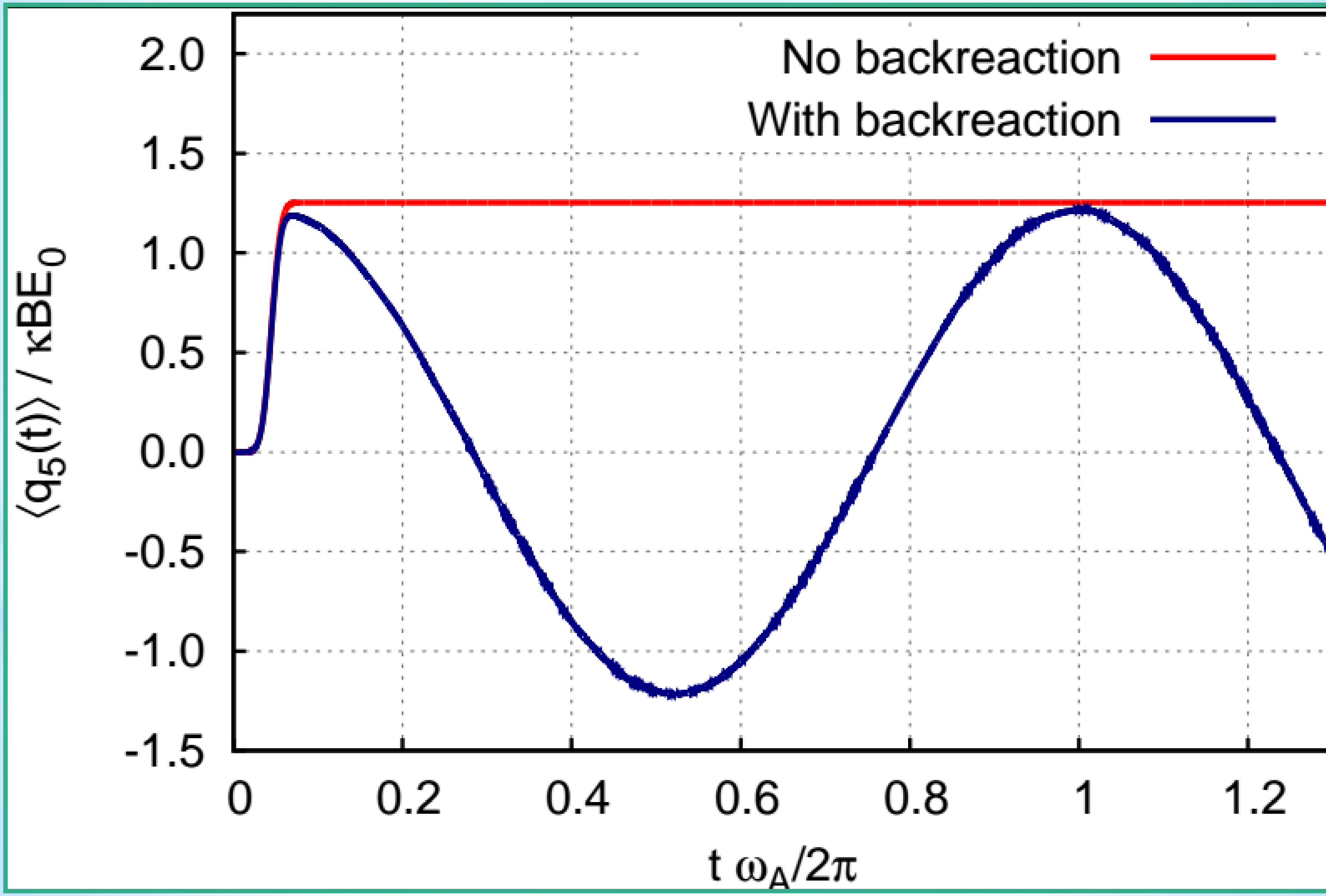
## Corrections to anomaly due to dynamic gauge fields [Anselm, Iogansen, 1989]






$$q_A(t) = E\sqrt{\frac{B}{2\pi^2}}\sin\left(\sqrt{\frac{B}{2\pi^2}}\,t\right)$$

# Magnetoresistivity and dynamic EM fields




**Real-time study with overlap fermions**

**[More in S. Valgushev's talk]**

# Magnetoresistivity and dynamic EM fields

$\sqrt{\frac{B}{2\pi^2}}$ Is the mass of the Chiral Magnetic Wave, due to EM interactions

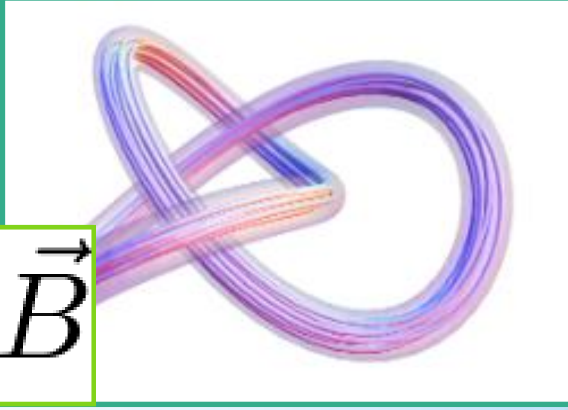
- For processes longer than $\tau \sim \sqrt{\frac{2\pi^2}{B}}$ effects of dynamical electromagnetism are of order 100%
- BUT for shorter processes, higher Landau levels are important
- For HIC, $\tau$ ~ *10* fm/c, large scale

# Chiral plasma instability

## $\mu_A$, $Q_A$- not "canonical" charge/chemical potential

$$\frac{d}{dt}Q_A = \frac{e^2}{2\pi^2}\int d^3\vec{x}\vec{E}\cdot\vec{B} = \frac{d}{dt}\left(\frac{e^2}{4\pi^2}\int d^3\vec{x}\vec{A}\cdot\vec{B}\right)$$

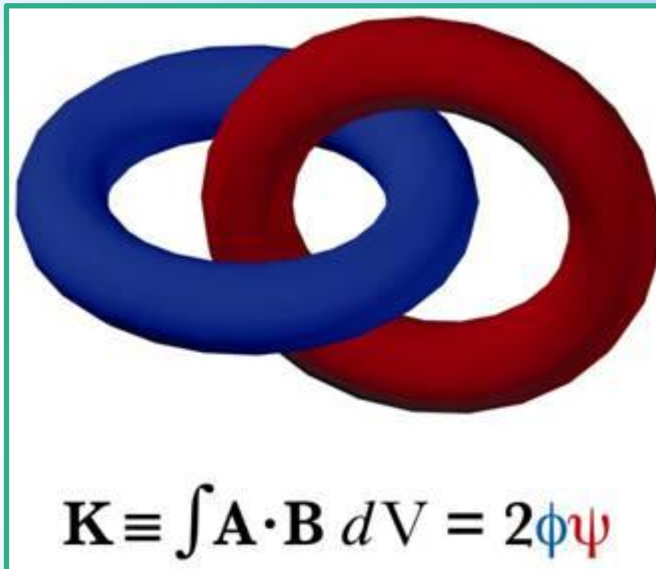
## "Conserved" charge:



$$\tilde{Q}_A = Q_A - \frac{e^2}{4\pi^2}\int d^3\vec{x}\vec{A}\cdot\vec{B}$$

**Chern-Simons term (Magnetic helicity)**

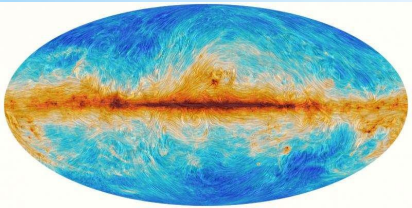
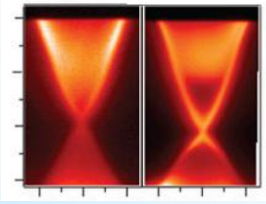
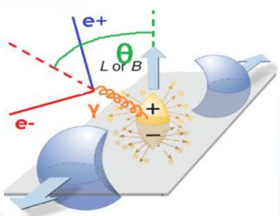
**Integral gauge invariant (without boundaries)**



# Chiral instability and Inverse cascade

**Energy of large-wavelength modes grows**
**… at the expense of short-wavelength modes!**

- **Generation of cosmological magnetic fields [Boyarsky, Froehlich, Ruchayskiy, 1109.3350]**
- **Circularly polarized, anisotropic soft photons in heavy-ion collisions [Hirono, Kharzeev,Yin 1509.07790][Torres-Rincon,Manuel,1501.07608**
- **Spontaneous magnetization of topological insulators [Ooguri,Oshikawa,1112.1414]**
- **THz circular EM waves from Dirac/Weyl semimetals [Hirono, Kharzeev, Yin 1509.07790]**

# Anomalous Maxwell equations

**Maxwell equations + ohmic conductivity + CME**

$$\partial_t \vec{B} = -\mathrm{rot}\,\vec{E}, \quad \partial_t \vec{E} = \mathrm{rot}\,\vec{B} - \sigma \vec{E} - \chi \vec{B}$$

**Ohmic conductivity** (→ $\sigma$)

**Chiral magnetic conductivity** (→ $\chi$)

**Assumption:** $\sigma\left(w, \vec{k}\right) = \mathrm{const}$ $\quad \chi\left(w, \vec{k}\right) = \mathrm{const}$

## Plane wave solution

$$iw\vec{B} = -i\vec{k} \times \vec{E} \quad iw\vec{E} = i\vec{k} \times \vec{B} - \sigma \vec{E} - \chi \vec{B}$$

# Chiral plasma instability

## Dispersion relation

$$w = i\sigma/2 \pm \sqrt{k^2 - \chi k - \sigma^2/4}$$

**At $k < \chi = \mu_A/(2\pi^2)$: Im(w) < 0**

***Unstable solutions!!!***

**Cf. [Hirono, Kharzeev, Yin 1509.07790]**

## Real-valued solution:

$$E_1 = f e^{\kappa t} \cos(k x_3),\ E_2 = -f e^{\kappa t} \sin(k x_3),$$
$$B_1 = -f \tfrac{k}{\kappa} e^{\kappa t} \cos(k x_3),\ B_2 = f \tfrac{k}{\kappa} e^{\kappa t} \sin(k x_3),$$

$$\kappa \equiv -iw = -\frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} - k^2 + \chi k}$$

# Helical structure

$$E_1 = f e^{\kappa t} \cos(kx_3),\ E_2 = -f e^{\kappa t} \sin(kx_3),$$
$$B_1 = -f \frac{k}{\kappa} e^{\kappa t} \cos(kx_3),\ B_2 = f \frac{k}{\kappa} e^{\kappa t} \sin(kx_3),$$

**Helical structure of unstable solutions**

**Helicity only in space - no running waves**

**E || B - ``topological" density**

**Note: E || B not possible for oscillating ``running wave" solutions, where *E•B=0***

***What can stop the instability?***

# What can stop the instability?

$$\partial_t Q_A = \frac{g^2}{2\pi^2} \int d^3x \vec{E} \cdot \vec{B}$$

**For our unstable solution with $\mu_A > 0$:**

$$\vec{E} \cdot \vec{B} = -f^2 \frac{k}{\kappa} e^{2\kappa t} \Rightarrow \partial_t Q_A < 0$$

**Instability depletes $Q_A$**

**$\mu_A$ and chi decrease, instability stops**

---

**Energy conservation:**

$$\partial_t \int d^3\vec{x} \left( \vec{E}^2 + \vec{B}^2 \right) = \int d^3\vec{x} \left( -\sigma \vec{E}^2 - \chi \vec{E} \cdot \vec{B} \right)$$

**Keeping constant $\mu_A$ requires work!!!**

# Real-time dynamics of chiral plasma

## Approaches used so far:

- Anomalous Maxwell equations
- Hydrodynamics (long-wavelength)
- Holography (unknown real-world system)
- Chiral kinetic theory (linear response, relaxation time, long-wavelength…)

## What else can be important:

- Nontrivial dispersion of conductivities
- Developing (axial) charge inhomogeneities
- Nonlinear responses

**Let's try to do numerics!!!**

# Real-time simulations: classical statistical field theory approach

[Son'93, Aarts&Smit'99, J. Berges&Co]

- Full real-time quantum dynamics of fermions
- Classical dynamics of electromagnetic fields
- Backreaction from fermions onto EM fields

$$\partial_t \vec{A} = -\vec{E}$$
$$\partial_t \vec{E} = \vec{\nabla} \times \vec{B} - \langle \vec{j} \rangle - \vec{j}_{ext}$$

$$\partial_t \hat{\psi} = i \left[ \hat{H} \left[ \vec{A} \right], \hat{\psi} \right]$$

$$\langle \vec{j} \rangle = \langle \psi^\dagger j \psi \rangle$$

Vol X Vol matrices,

**Bottleneck for numerics!**

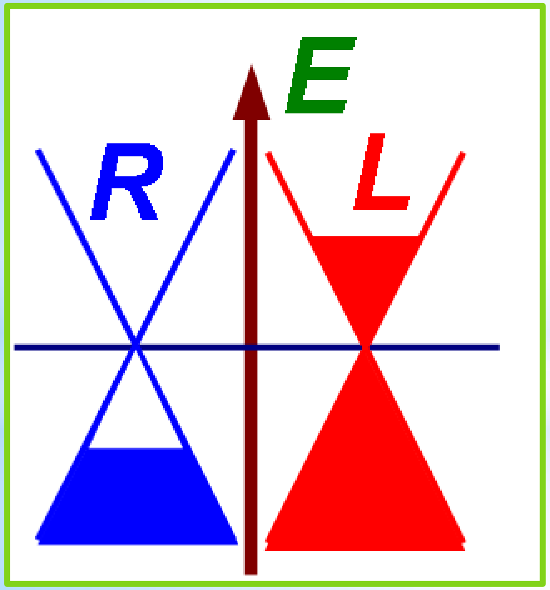
$$\partial_t U = ih \left[ \vec{A} \right] U$$

$$\langle \vec{j} \rangle = \mathrm{Tr} \left( \rho_0 U \vec{j} U^\dagger \right)$$

# Decay of axial charge and inverse cascade

[PB, Ulybyshev 1509.02076]

Excited initial state with chiral imbalance




Hamiltonian is CP-symmetric, State is not!!!

[No momentum separation]

$\mu_A < 1$ on the lattice

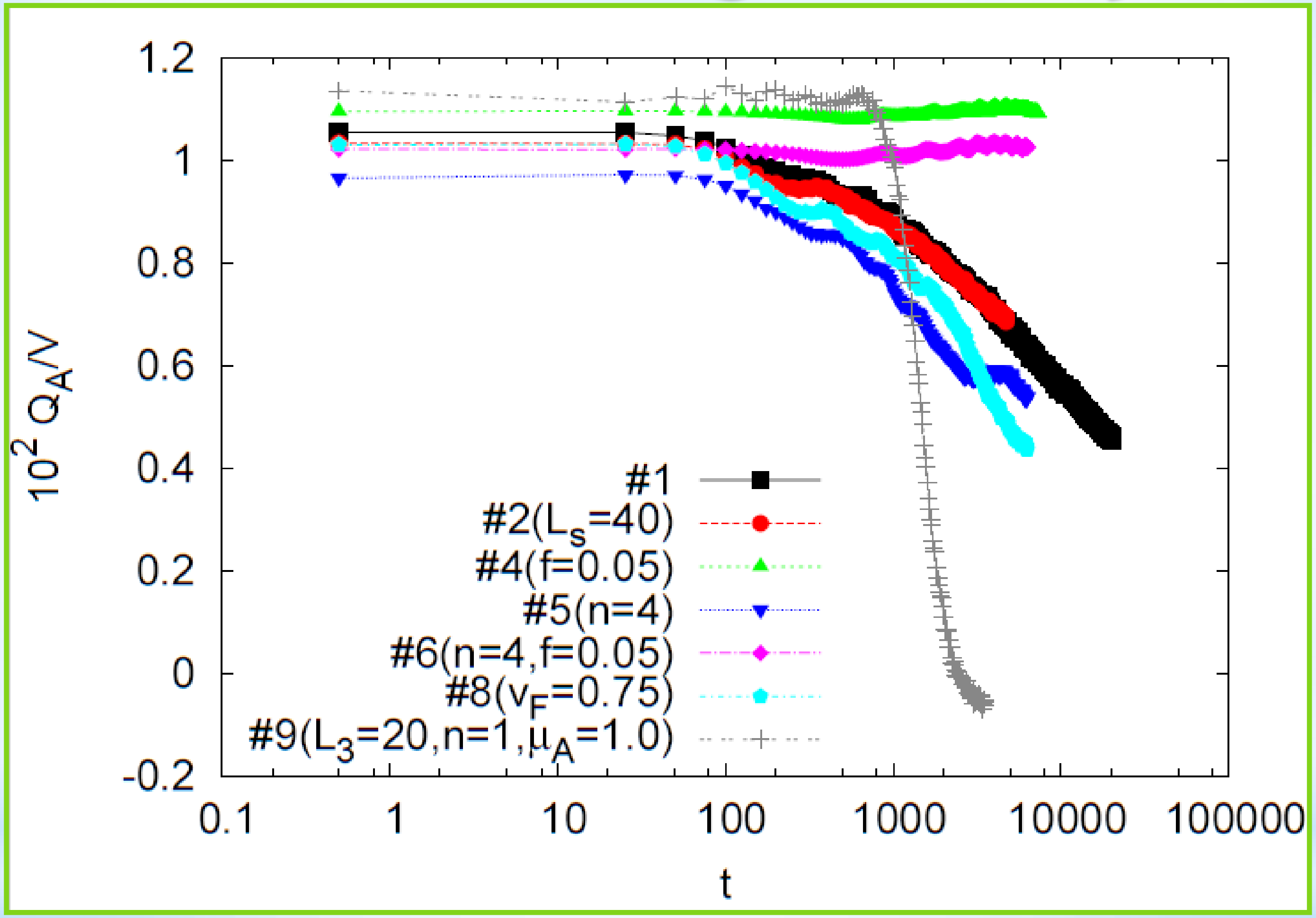
To reach $k < \mu_A/(2\pi^2)$:

- *200x20x20* lattices
- Translational invariance in 2 out of 3 dimensions

To detect instability and inverse cascade:

- Initially n modes of EM fields with equal energies and random linear polarizations

# Axial charge decay



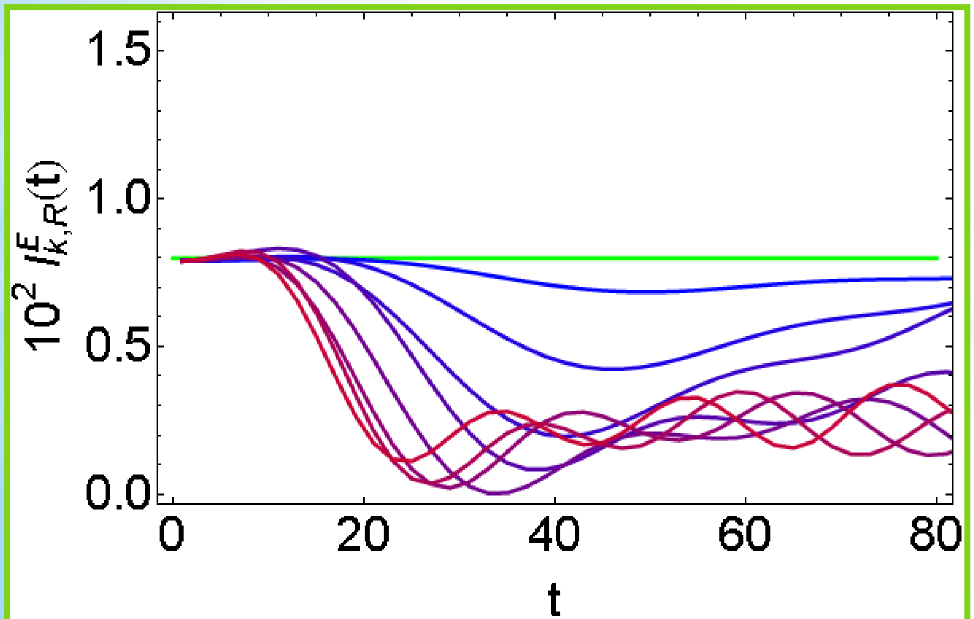

# 200 x 20 x 20 lattice, $\mu_A$= 0.75
# Note the amplitude dependence

# Power spectrum and inverse cascade

## Fourier transform the fields

$$E_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} E_{x,i}(t),$$

$$B_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} B_{x,i}(t),$$




## Basis of helical components

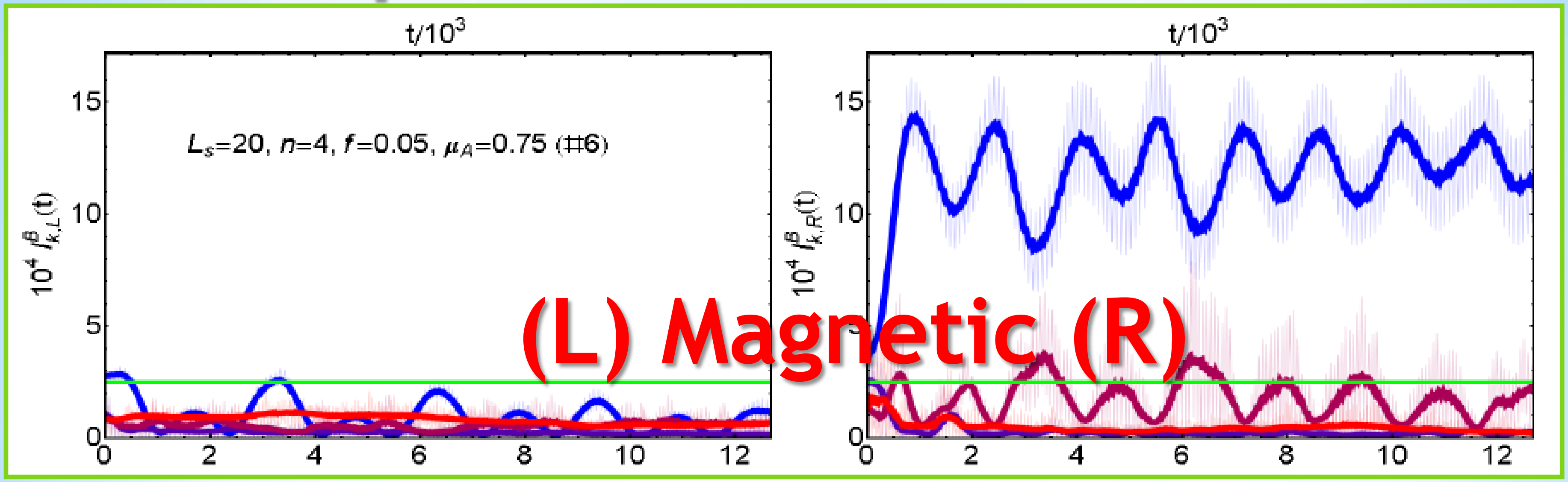
$$B_{k,R}(t) = \frac{1}{2}\left(B_{k,1}(t) + B_{-k,1}(t)\right) + \\ + \frac{1}{2i}\left(B_{k,2}(t) - B_{-k,2}(t)\right),$$

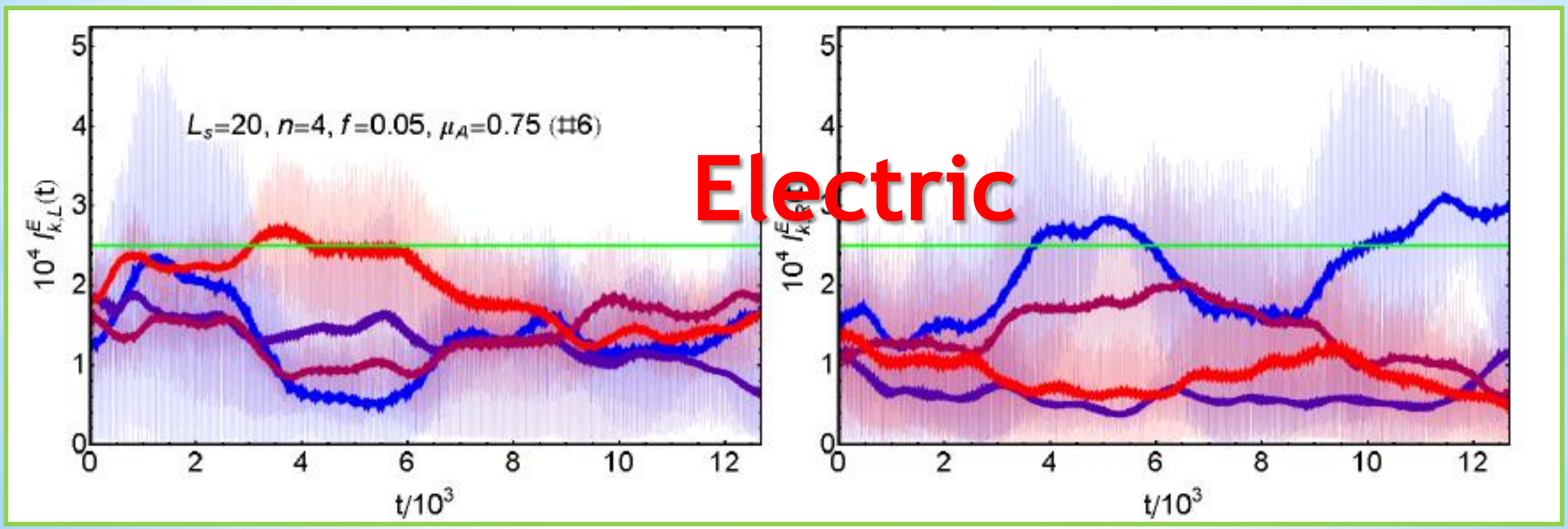
$$B_{k,L}(t) = \frac{1}{2i}\left(B_{k,1}(t) - B_{-k,1}(t)\right) + \\ + \frac{1}{2}\left(B_{k,2}(t) + B_{-k,2}(t)\right).$$

## Smearing the short-scale fluctuations

$$\bar{I}^{E,B}_{k,R/L}(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} dt' \, I^{E,B}_{k,R/L}(t').$$

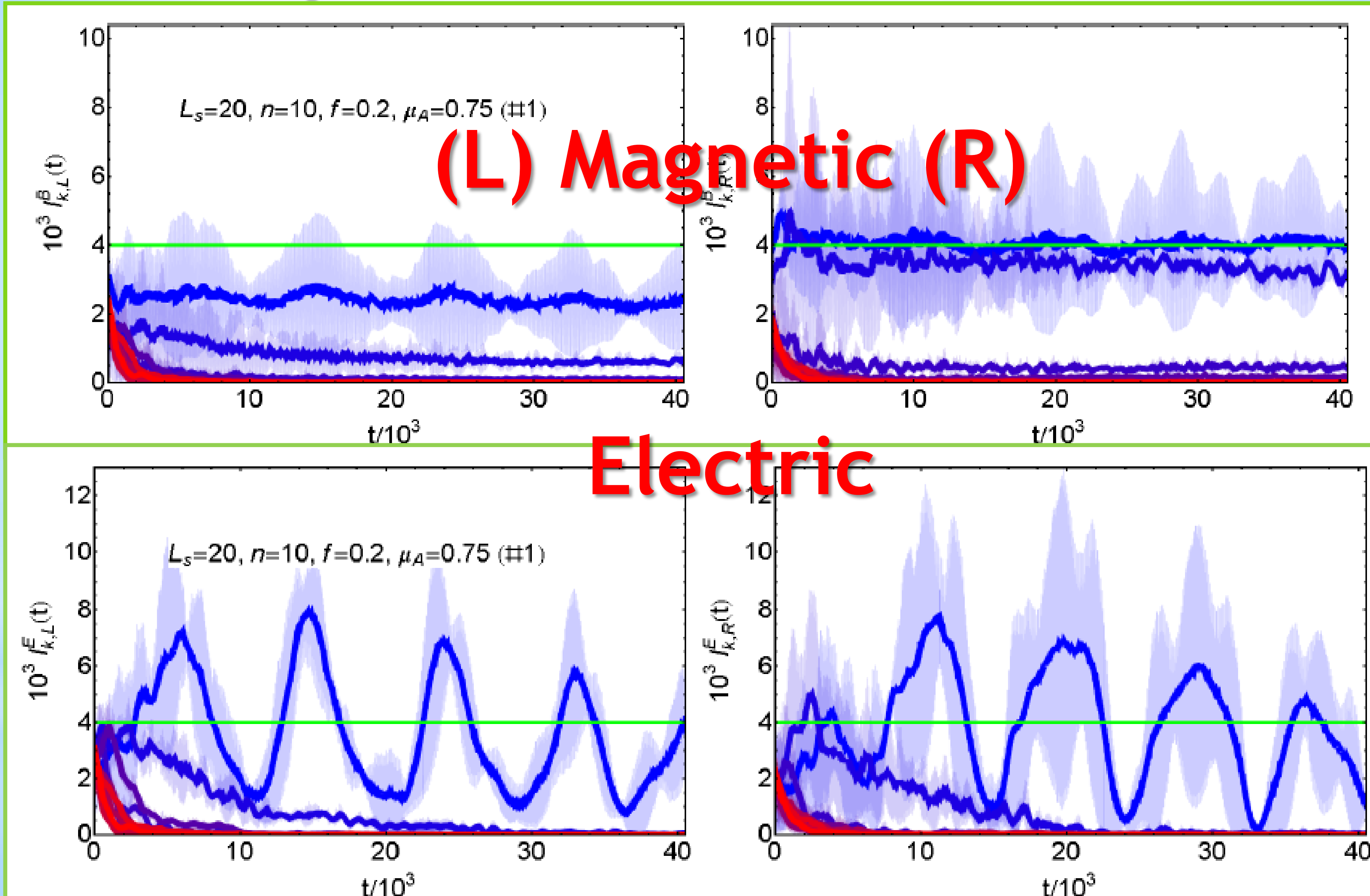
# Power spectrum and inverse cascade




## Small amplitude, $Q_A$ ~ const

# Power spectrum and inverse cascade

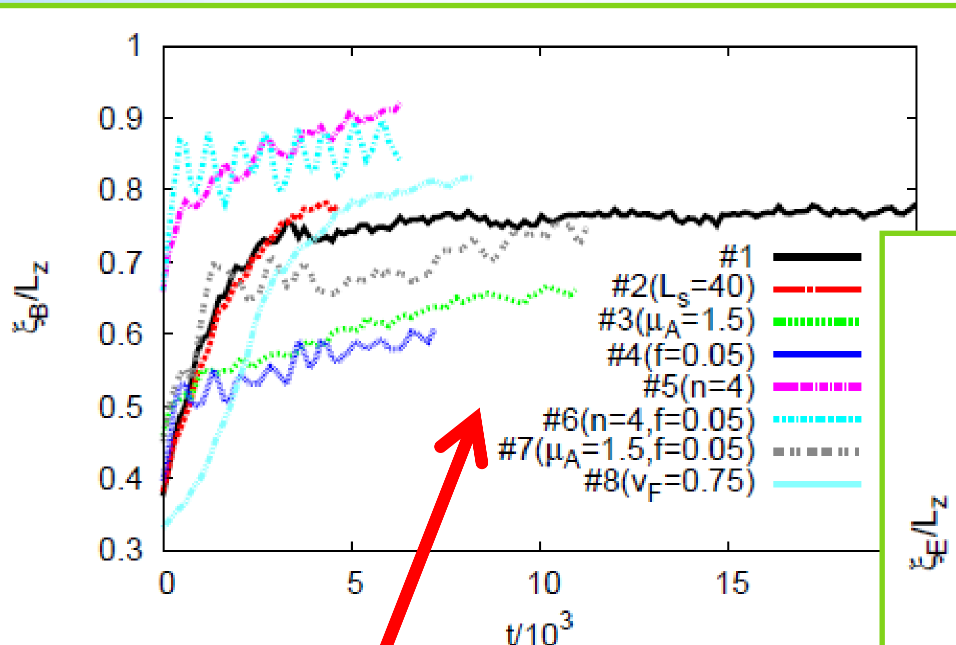



**Large amplitude, $Q_A$ decays**

# Inverse cascade: correllation lengths

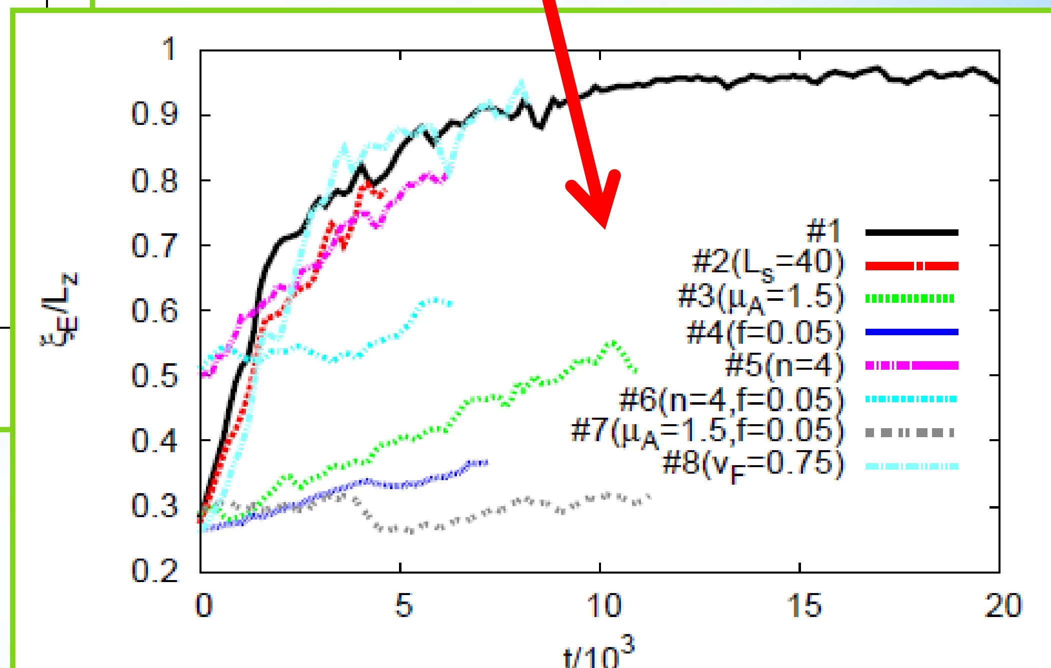
$$\xi_{E,B}(t) = \frac{\sum\limits_k \frac{2\pi}{k} I_k^{E,B}(t)}{\sum\limits_k I_k^{E,B}(t)},$$

**Average wavelength of electric or magnetic fields**

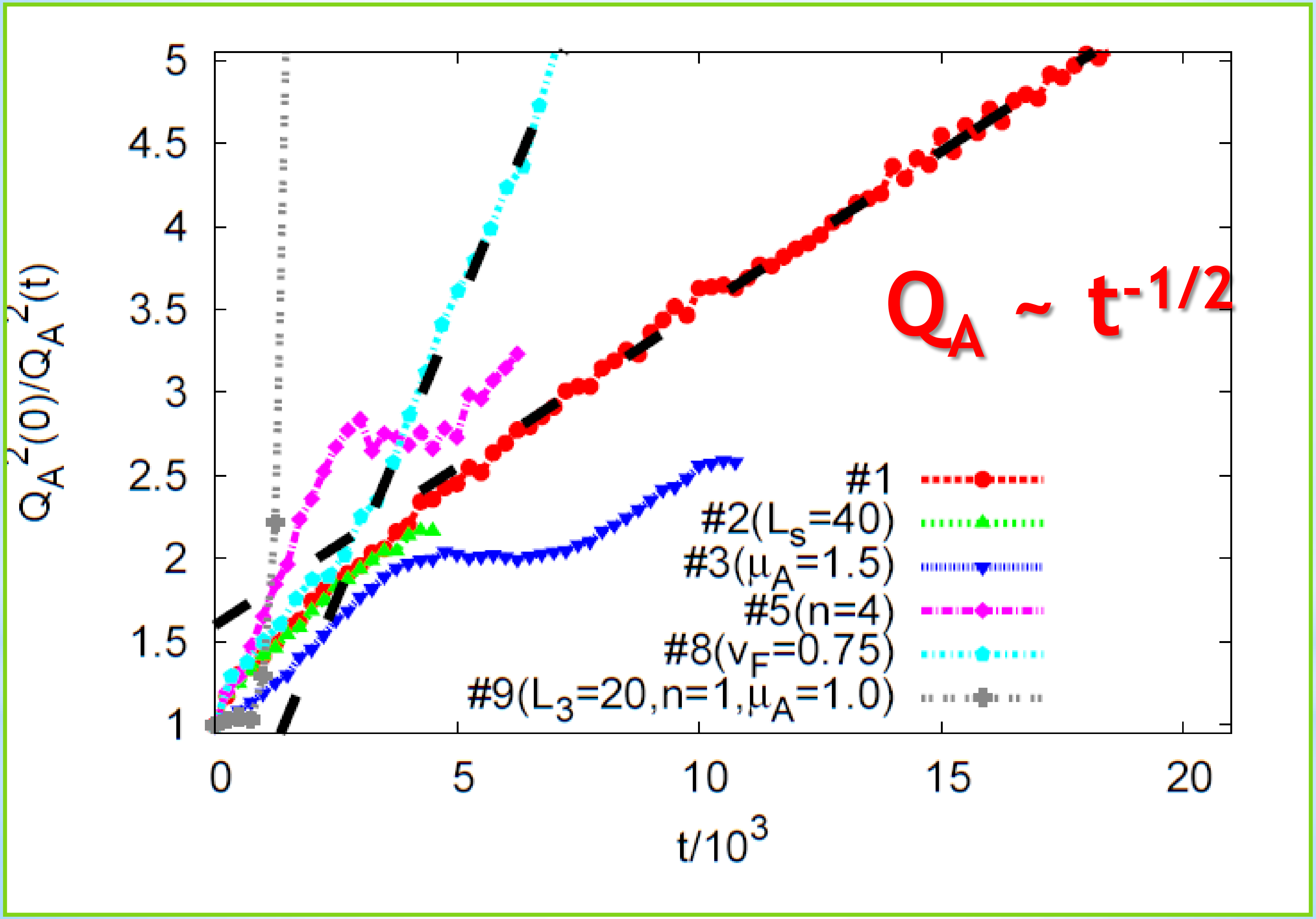



**Magnetic**

**Electric**



# Universal late-time scaling

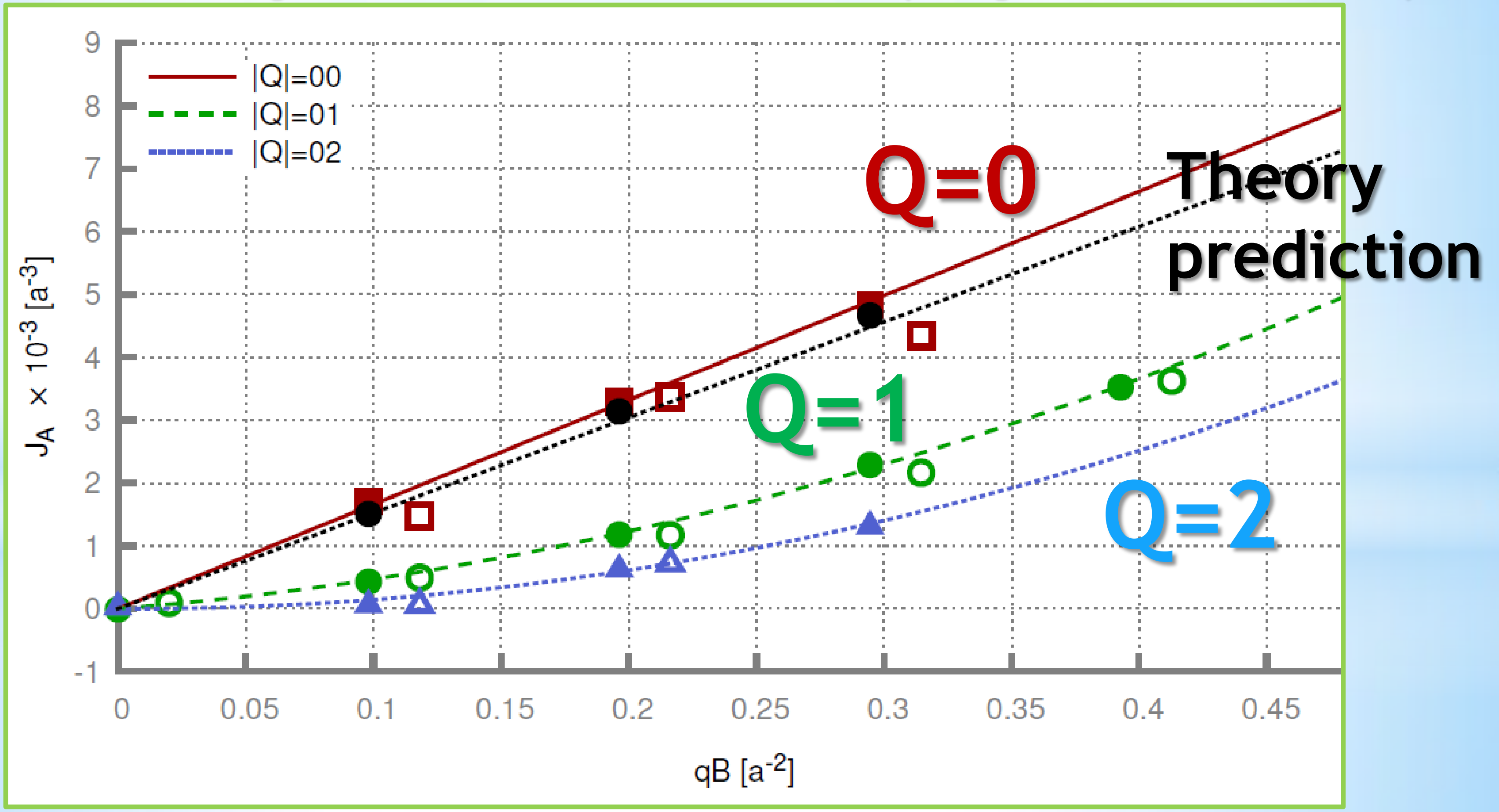


# Effects of anomaly in cold QCD matter

- Anomalous transport coefficients strongly constrained in hydro approximation [Son,Surowka'2009]
- If chiral symmetry spontaneously broken, hydro breaks down (superfluidity/Goldstones)
- Non-perturbative corrections to CME and CSE are possible
- Their magnitude is not known so far! (At least from first principles)

# Effects of anomaly in cold QCD matter

# Chiral Separation Effect (equilibrium!)




## Non-perturbative corrections from nonzero topology sectors! [PB, M. Puhr]

# Discussion and outlook

- Effects of axial anomaly strongly affect conductivity for cold QCD
- Specific anisotropy of conductivity and dilepton yield
- Interesting effects of charge separation in cold QCD
- Non-perturbative corrections to CSE

# Discussion and outlook

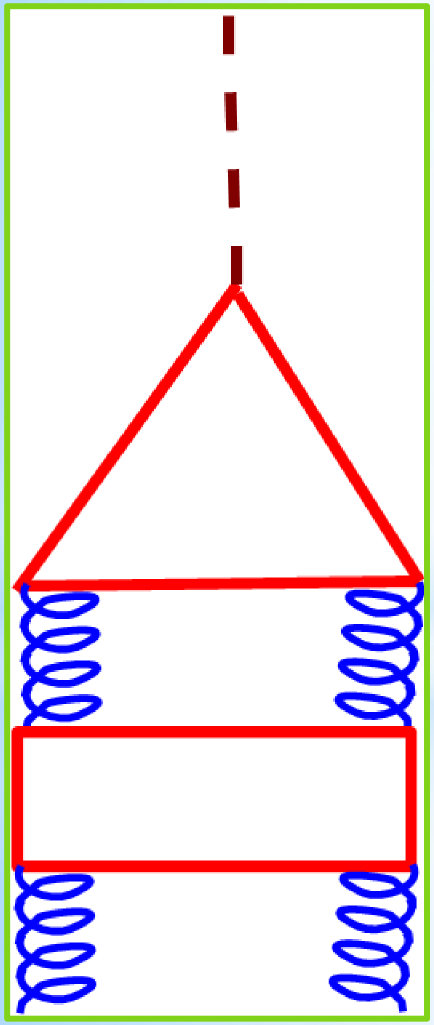
- Dynamics of EM fields can be most likely neglected in HIC, higher Landau levels not
- Chiral instability: duration of HIC too short [Tuchin, 1411.1363]
- As such, might be important in astrophysical context

# Discussion and outlook

- Effects of dynamical gauge fields can be much more important for non-Abelian fields [Akamatsu, Rothkopf, Yamamoto 1512.02374]
- Anomalous transport is essentially real-time ("non-dissipative" might be misleading)

# Back-up

# Effect of interactions in CME and NMR

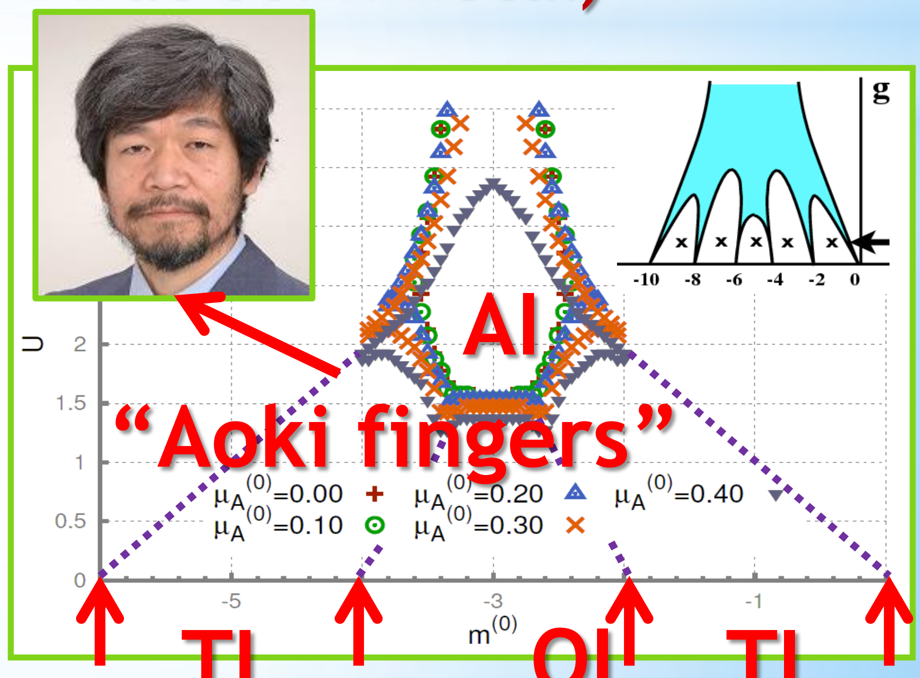


## Condensed matter setup

- EM fields are dynamical
- Short-range four-fermion interactions
- Chiral symmetry is not exact

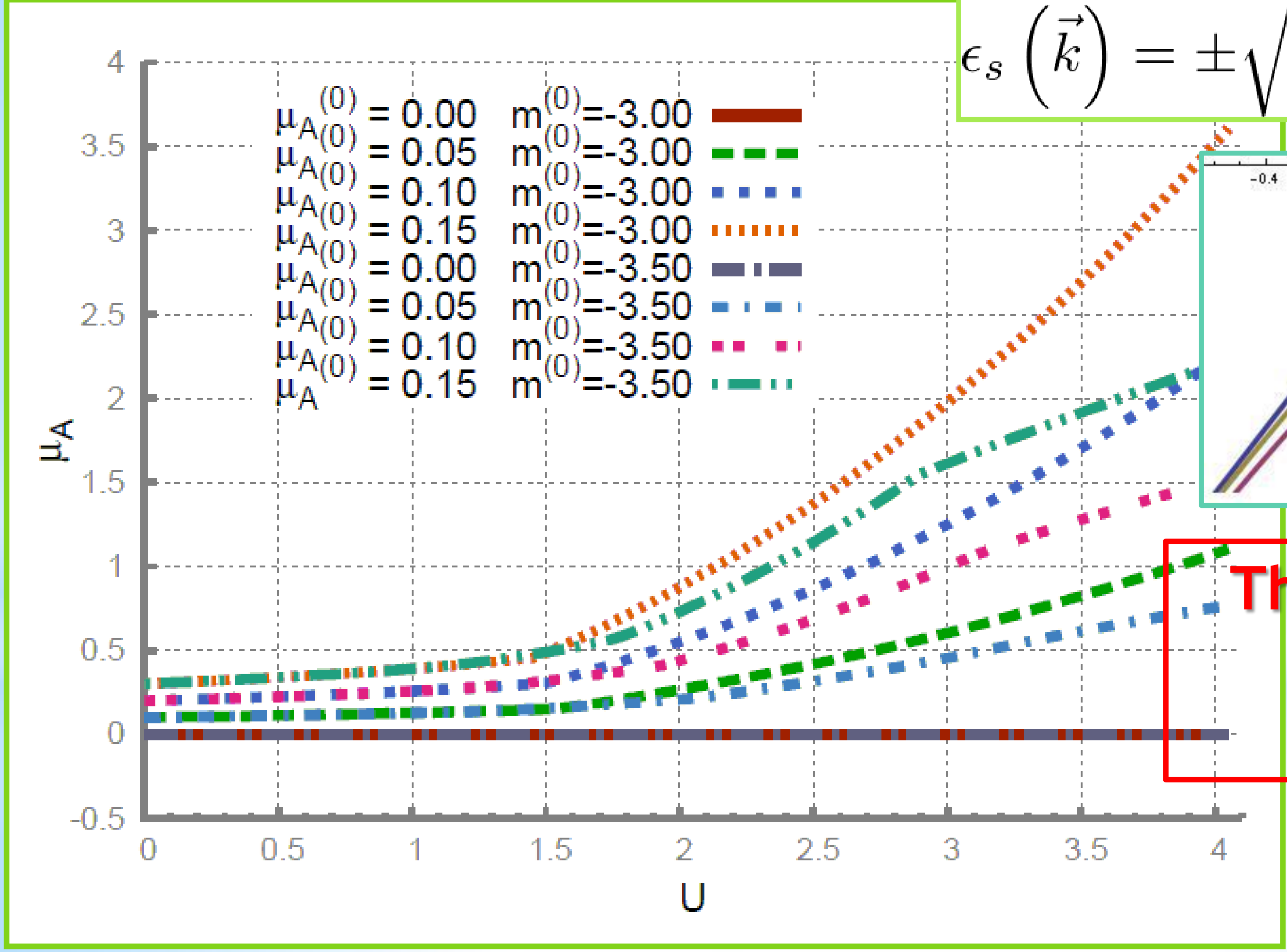
**Mean-field study with Wilson fermions (aka two-band Dirac semimetal)**

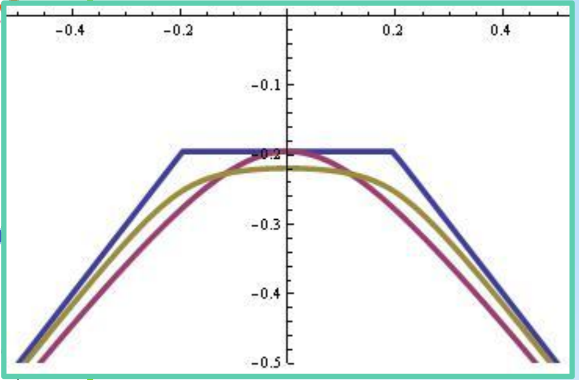
[PB, Puhr, Valgushev 1505.04582]



# Enhancement of chiral chemical potential

$$\epsilon_s\left(\vec{k}\right) = \pm\sqrt{\left(|\vec{k}| \pm \mu_A\right)^2 + m^2}$$



The effect of $\mu_A$ is similar to mass!!!

**Mean-field value of chiral chemical potential is strongly enhanced by interactions in all phases**

**Outlook: real-time calculations with E and B?**