Liquid droplet formula for Polyakov loop geometrical clusters in SU (2) gluodynamics

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Dubna, November 1, 2016

Outline

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4. Size distributions of gaseous (anti)clusters and Liquid Droplet Formula

5. Surface tension of gases in physical units

6. Properties of (anti)cluster liquid and strongly inhomogeneous objects

7. New order parameters

8. Conclusions

A. I. Ivanytskyi, K. A. Bugaev et al., arXiv:1606.047 [hep-lat]

Motivation I

Traditionally, the deconfinement in SU(N) color gluodynamics is described as the break down of Z(N) symmetry

However, such a language is well suited for the phase transitions of solid-liquid and solid-solid types.

Furthermore, i) hadronic matter at low energy densities is a gas!

ii) at high energy densities the QGP is (probably) the most perfect fluid!

=> we need a language which is suited for GAS-to-LIQUID phase transition (PT)

Moreover, i) the language of symmetry breaking does not work for deconfinement PT in presence of quarks

ii) the same is true for the chiral symmetry restoration PT, if one uses non-vanishing quark masses

=> we need a language which can be used in presence of quarks with realistic masses

Motivation II

There are several exactly solvable cluster models for the LIQUID-GAS PT:

Fisher Droplet Model and its successors for ordinary liquid-gas PT

. M. E. Fisher, Physics 3, 255 (1967)

Statistical Multifragmentation Model for nuclear liquid-gas PT

. K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. 62 (2000)

Quark-Gluon Bags with Surface Tension Model of deconfinement PT

. K. A. Bugaev, Phys. Rev. C 76, 014903 (2007)

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. 76 (2013), 341

However, to use this framework we need to know

i) the T-dependence of surface tension of QGP bags

ii) the Fisher exponent of QGP bags

=> Lattice QCD allows us to determine all these quantities and to verify whether the known cluster models are suited to study deconfinement PT

Goals

Using the cluster approach to LQCD we hope

1. **to give a physical meaning to the concept of QGP bags**

2. **to formulate appropriate order parameters of this PT**

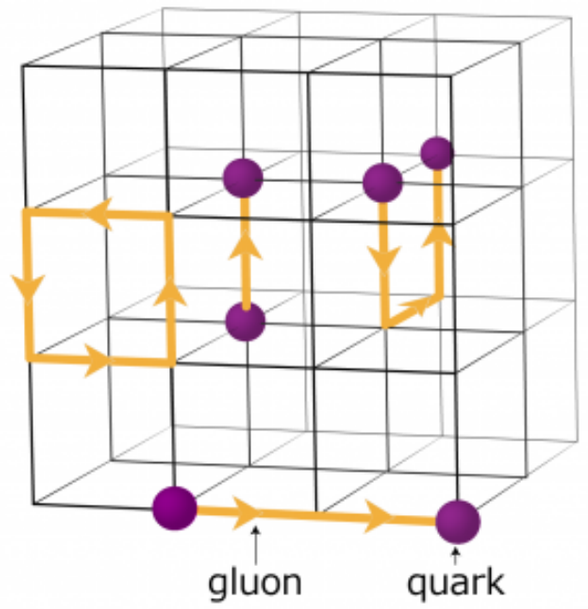
3. **to formulate the signals of 2-nd order LIQUID-GAS PT which maybe observed in the experiments and will help to locate (tri)CEP**

Definition of Polyakov loop

$$L(\tilde{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\tilde{x}, t)$$

$U_4(\tilde{x}, t)$ – temporal gauge link defined by gluon field

$SU(2) \Rightarrow L(\tilde{x}) \in [-1, 1]$, real



SU(2) Polyakov loop L (x) = Continuous Spin

existing at each spatial point of lattice

Similarly to Gattringer we define spins via cut-off L_cut

C. Gattringer, Phys. Lett. B 690, (2010) 179.

C. Gattringer and A. Schmidt, JHEP 1101 (2011) 051.

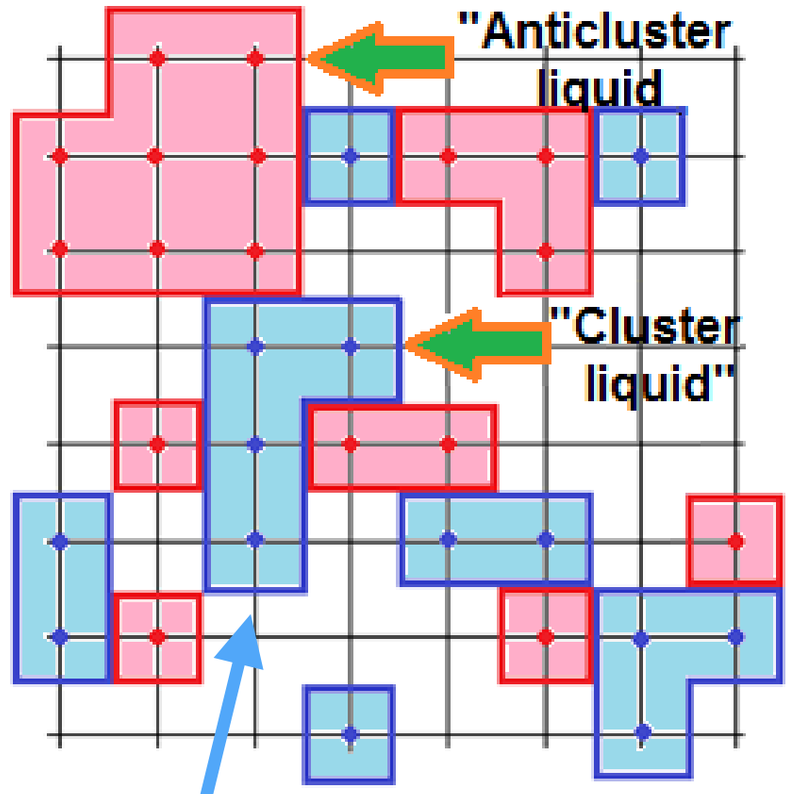
1. If $L > +|L_{cut}|$ it is spin Up,
2. If $L < -|L_{cut}|$ it is spin Down,
3. If $L: -|L_{cut}| < L < |L_{cut}|$ it is aux. Vacuum.

Definition of Polyakov loop Clusters

Monomer Up has all neighbors spin Down or auxVac.

Dimer Up =Two neighboring monomers Up have all other neighbors spin Down or auxVac.

Geometrical cluster of N same sign spin monomers is surrounded by opposite sign spins or aux Vac:



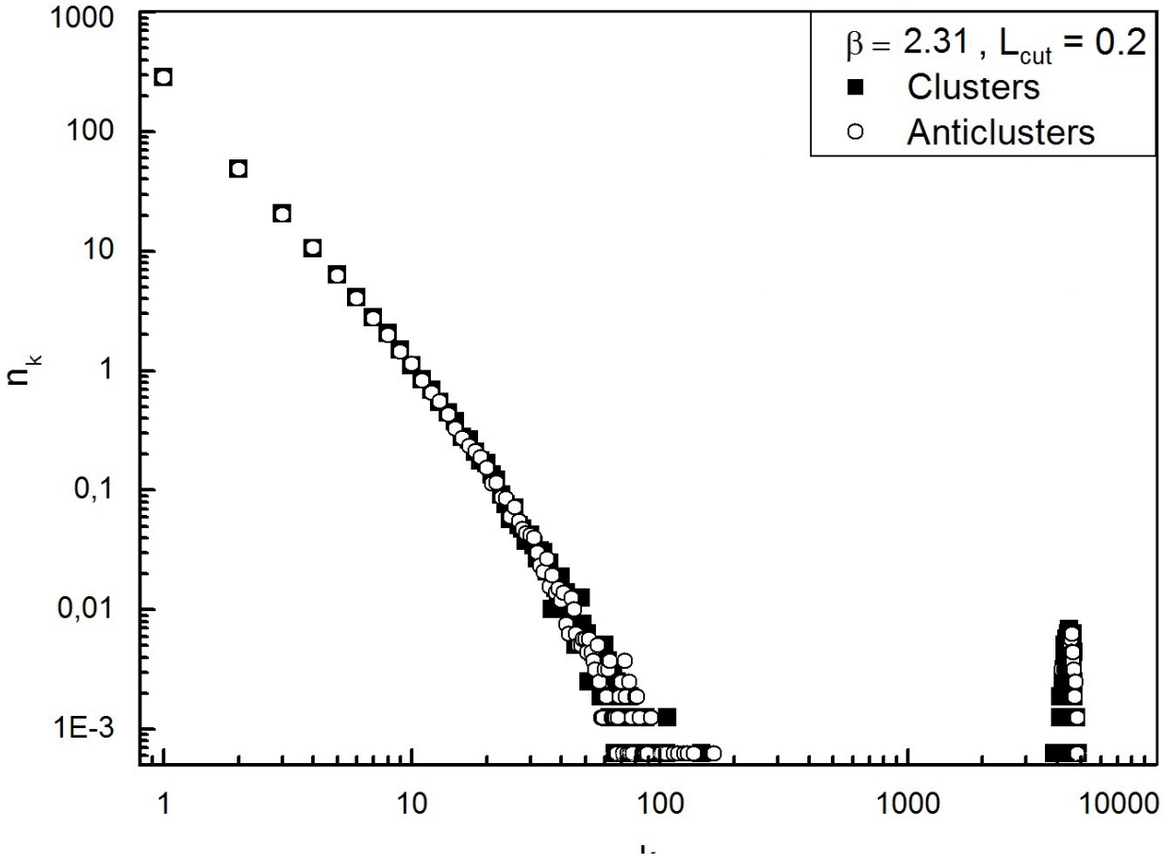
(Anti)clusters can be either “spin up"or “spin down"ones

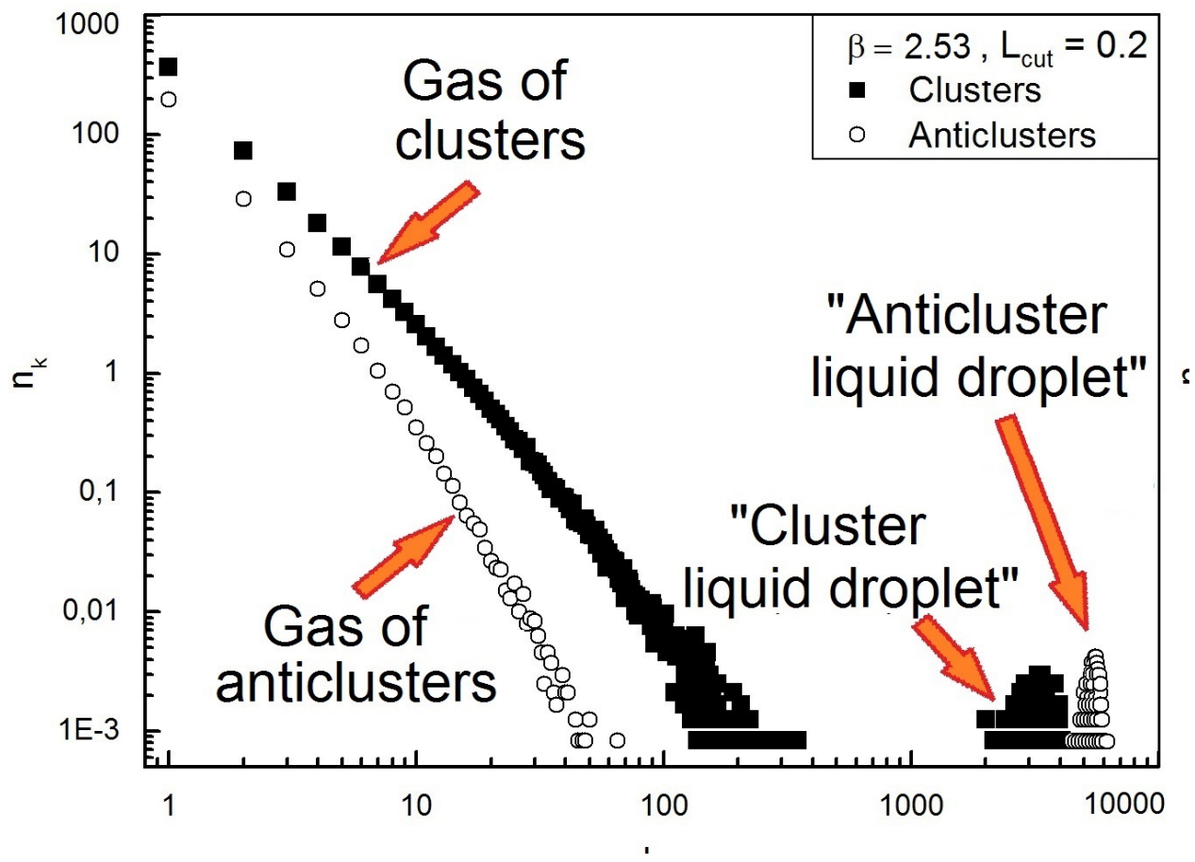
- Largest fragment - “anticluster liquid droplet"
- Next to largest fragment of opposite sign - “cluster liquid droplet"
- Gas of (anti)clusters has the same Polykov loop sign as their “liquids"

Size Distributions of Clusters I

- Numerical simulations on $3+1$ dimensional lattice of size $N_\sigma = 24$, $N_\tau = 8$
- 13 values of inverse coupling $\beta \in [2.31, 3] \Rightarrow$ 13 values of physical temperature
- vacuum cut-off parameter $L_{cut} = 0.1$ and 0.2
- Average over 1600 independent configurations for all β and L_{cut}

In thermodynamic limit the critical value is $\beta c = 2.5115$

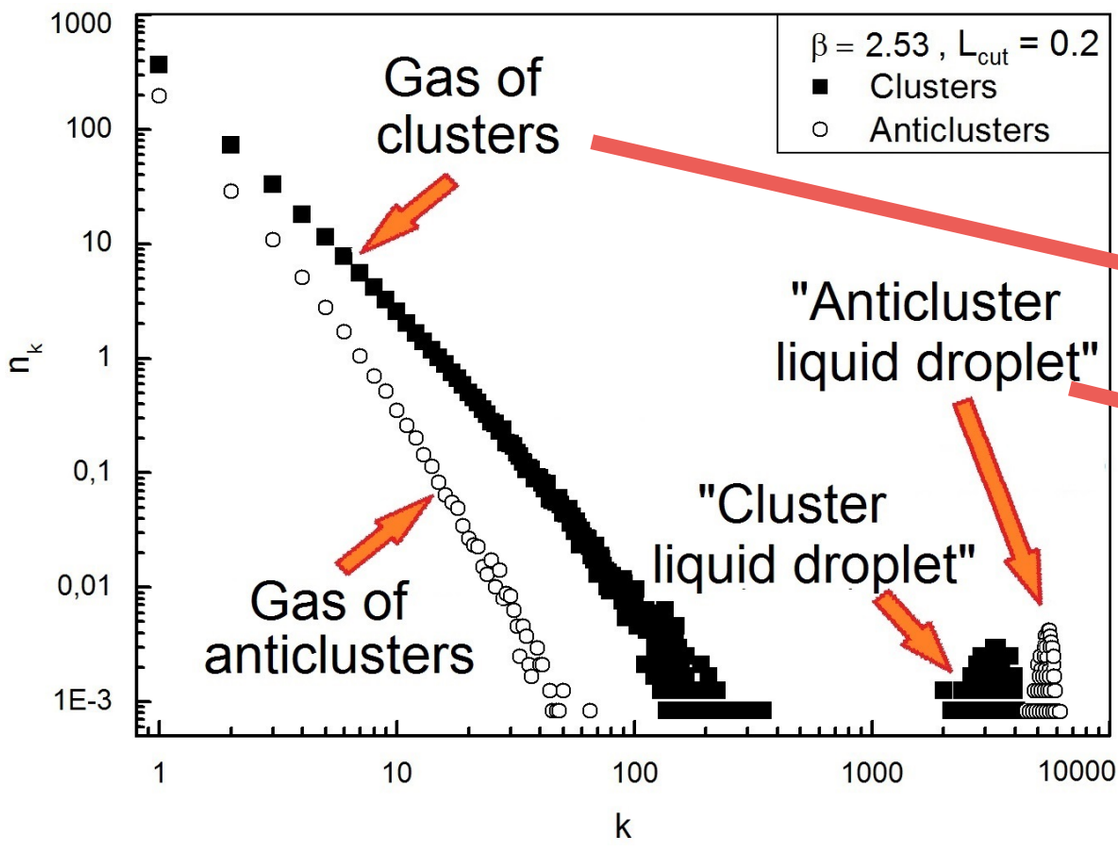


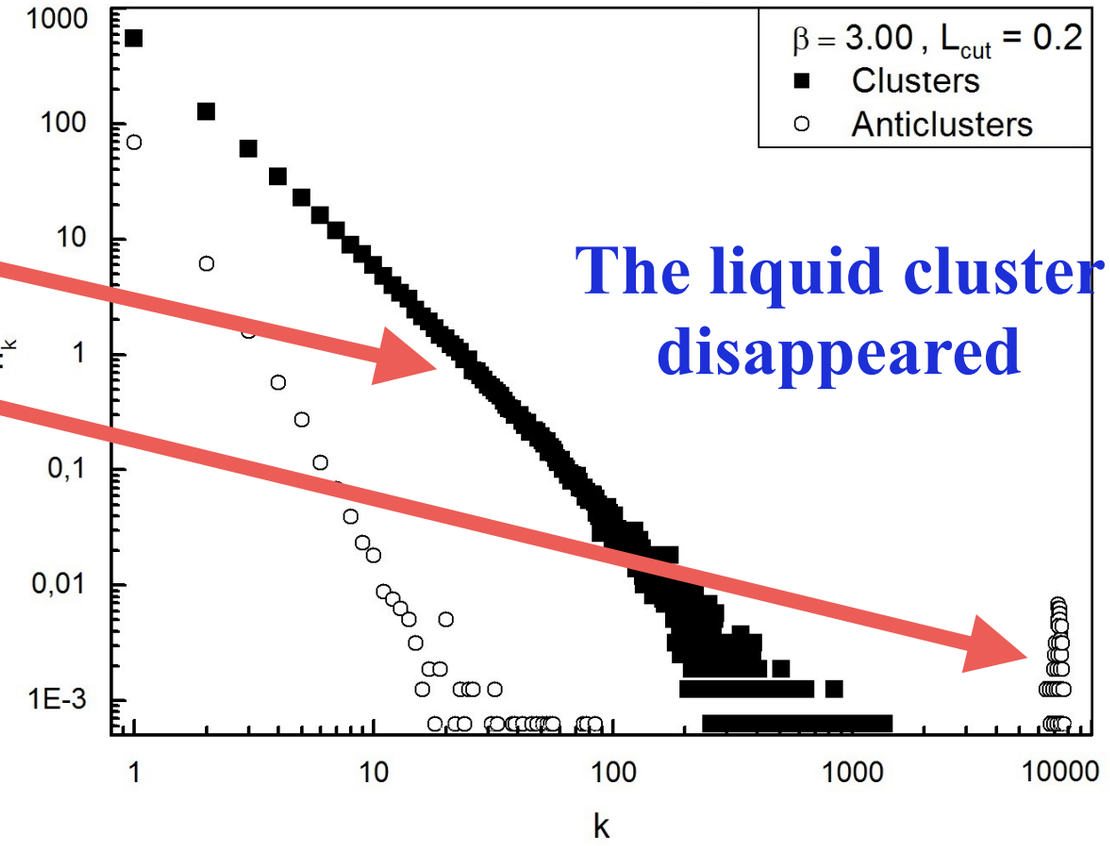


Distributions at low $\beta \leq \beta_c \simeq 2.52$ (phase of restored global Z(2) symmetry)

- symmetry between (anti)cluster distributions
- gas and “liquid"domains are well separated

Size Distributions of Clusters II





The liquid cluster disappeared

Distributions at high $\beta > \beta_c \simeq 2.52$ (phase of broken global Z(2) symmetry)

- no symmetry between (anti)cluster distributions
- “cluster liquid"evaporates to cluster gas
- anticluster gas condensates to “anticluster liquid"

=> Distributions are rather sensitive to value of β!

Can we describe the gas distributions by the liquid droplet formula?

Liquid Droplet Formula

Since a priori k-min is unknown we perform a free fit of

(anti)cluster size distributions for all k-min

according to Liquid Drop Model

$$n_k^{th} = C \exp\left(\mu k - \sigma k^{\kappa} - \tau \ln k\right), \qquad k > k\text{-}min - 1$$

0.667

normalization — bulk — surface — Fisher index

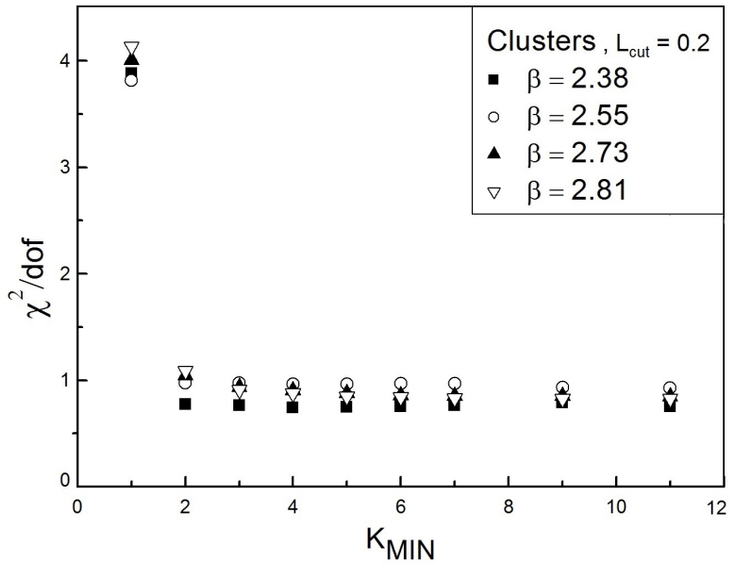
M. E. Fisher, Physics **3**, 255 (1967).

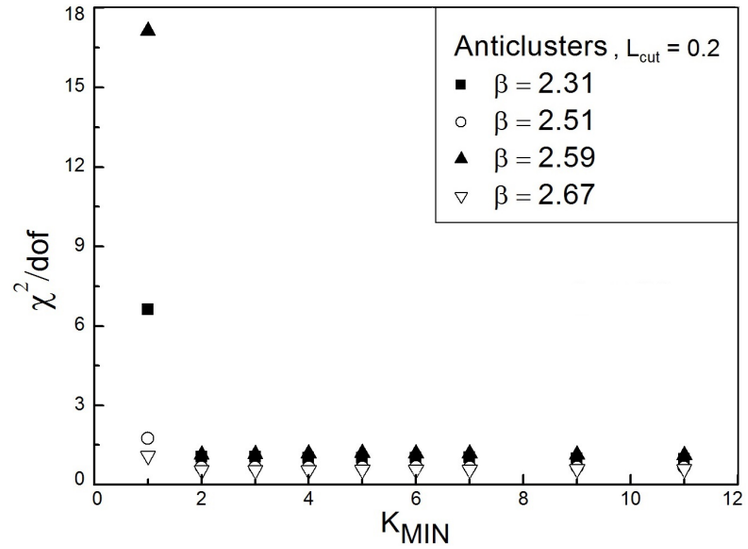
K. A. Bugaev, M. I. Gorenstein, I. N. Mishustin and W. Greiner, Phys. Rev. **C 62**, 044320 (2000)

Free fit parameters: C, μ, σ, τ

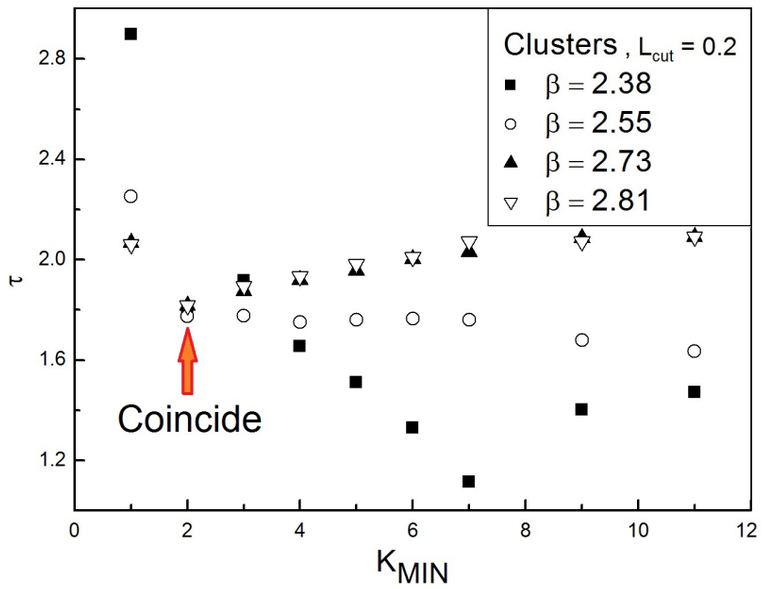
Defining the minimal N-mer and Fisher index τ

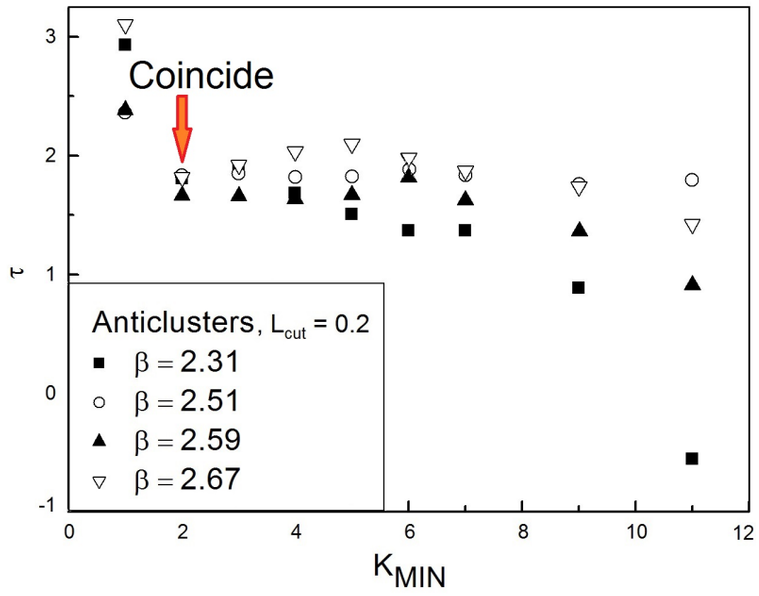
- LDF describes size distributions with almost the same quality for all $k_{min} \geq 2$





- Fisher topological exponent τ is temperature independent at $k_{min} = 2$ in agreement with Fisher droplet model M.E. Fisher, Physics 3, 255 (1967)





$k_{min} = 2, \quad \tau = 1.806 \pm 0.008$

This value of τ = const of β as required by LDM!

This Is Important Finding!

Since in exactly solvable models τ defines the universality class:

Fisher droplet model: for d=2 => τ=2.07; for d=3 => τ=2.209

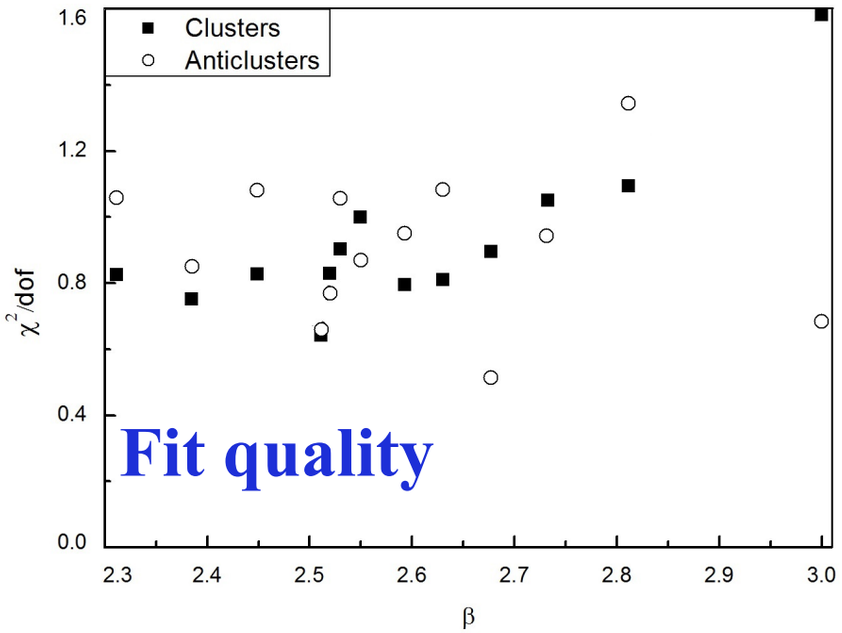
SMM and QGBags with surface tension with 3CEP: τ=1.825±0.025

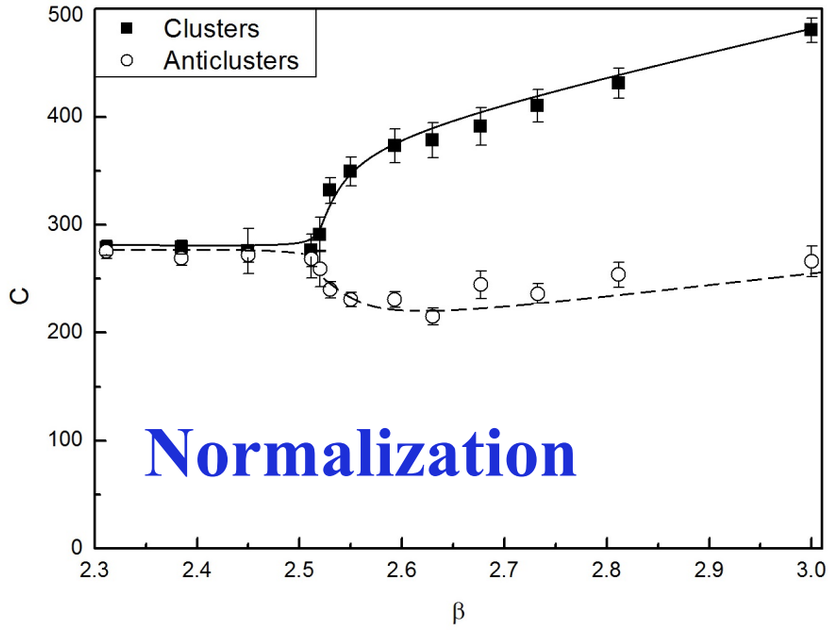
QGBags with surface tension with CEP: $\tau > 2$

However, at the moment we cannot say that QCD has 3CEP!

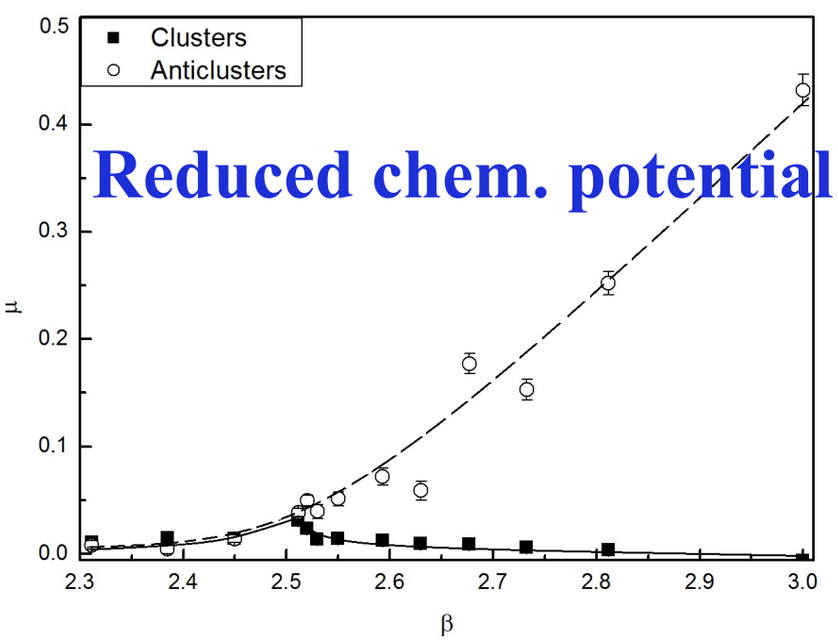
Fixed τ=1.806 Fit Results For Cut-off 0.2

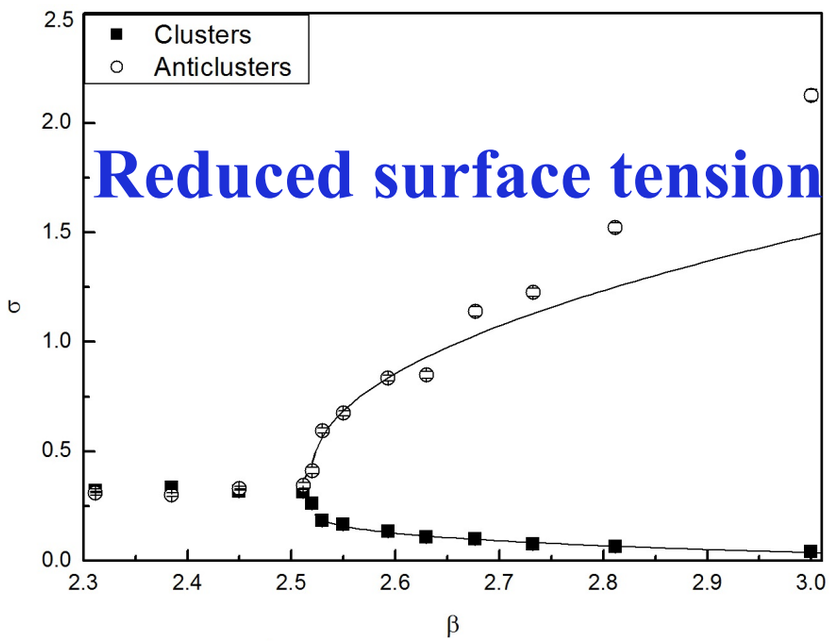
Fitting parameters: C, μ, σ





β	$a_\sigma(\beta)/a_\sigma(\beta_c^\infty)$	T/T_c^∞
2.3115	1.7132	0.5837
2.3850	1.4057	0.7114
2.4500	1.1783	0.8487
2.5115	1.0000	1.0000
2.5200	0.9774	1.0231
2.5300	0.9514	1.0510
2.5500	0.9016	1.1092
2.5930	0.8030	1.2453
2.6300	0.7269	1.3757
2.6770	0.6405	1.5612
2.7325	0.5516	1.8128
2.8115	0.4459	2.2423
3.0000	0.2685	3.7244





At $\beta = 2.52$ global Z(2) symmetry breaks down $\Rightarrow$
chemical nonequilibrium between (anti)clusters ($\mu_{\rm Cl} \neq \mu_{\rm aCl}$)

$\Rightarrow$ Break down of symmetry leads to bifurcations in gas quantities!

Surface Tension in Physical Units for Fixed τ=1.806

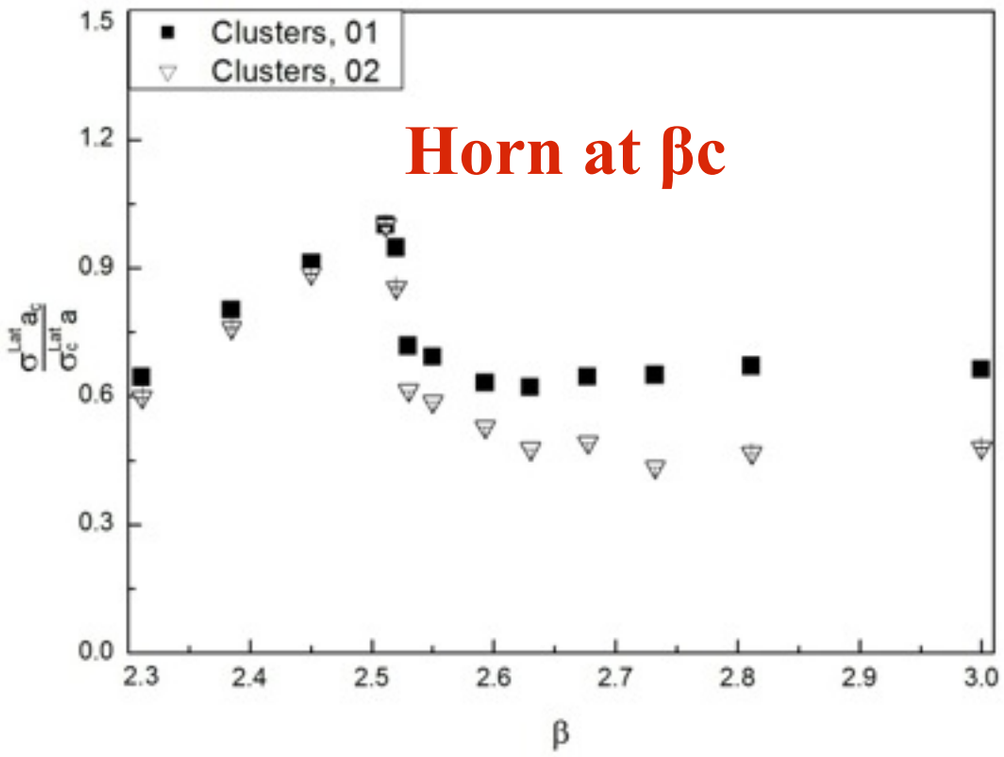
The β-dependence of physical surface tension defined as

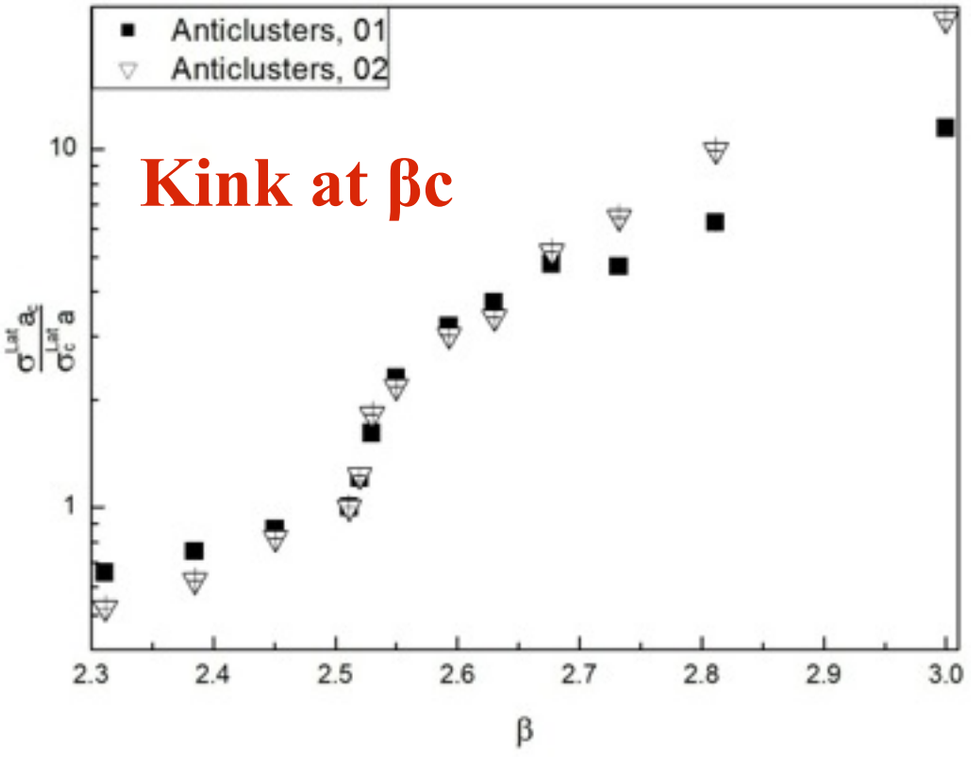
$$\sigma_A^{phys}(\beta) \equiv T\frac{\sigma_A(\beta)}{[\,a_\sigma(\beta)\,]^2} = T_c^\infty \frac{a_\sigma(\beta_c^\infty)}{a_\sigma(\beta)}\frac{\sigma_A(\beta)}{[\,a_\sigma(\beta)\,]^2},$$

But the ratio

$$\sigma_A(\beta)\, a_\sigma(\beta_c^\infty)/\sigma_A(\beta_c^\infty)/a_\sigma(\beta)$$

is more convenient





$$\sigma_{cl}^{phys}(T) = \frac{Const}{[\,a_\sigma(\beta)\,]^2} \sim T^2 \quad \text{for} \quad 1.25\,T_c^\infty < T \le 3.7\,T_c^\infty \qquad \textbf{BUT} \quad \sigma_{acl}^{phys}(T) \sim T^4$$

Other Important Findings

In contrast to existing exactly solvable models of cluster type the physical surface tension of Polyakov loop (anti)clusters DOES NOT VANISH above PT!

=> Hence SU(2) gluodynamics should have a different mechanism for 2-nd order PT!

However, the KINKs in physical surface tension are known from existing cluster models!

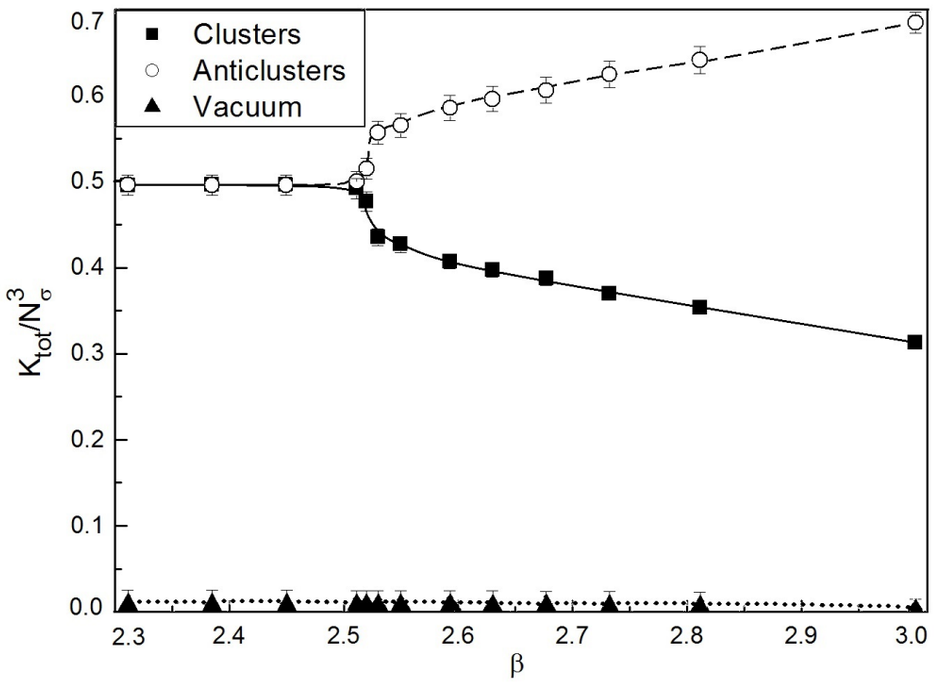
K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. 76 (2013), 341

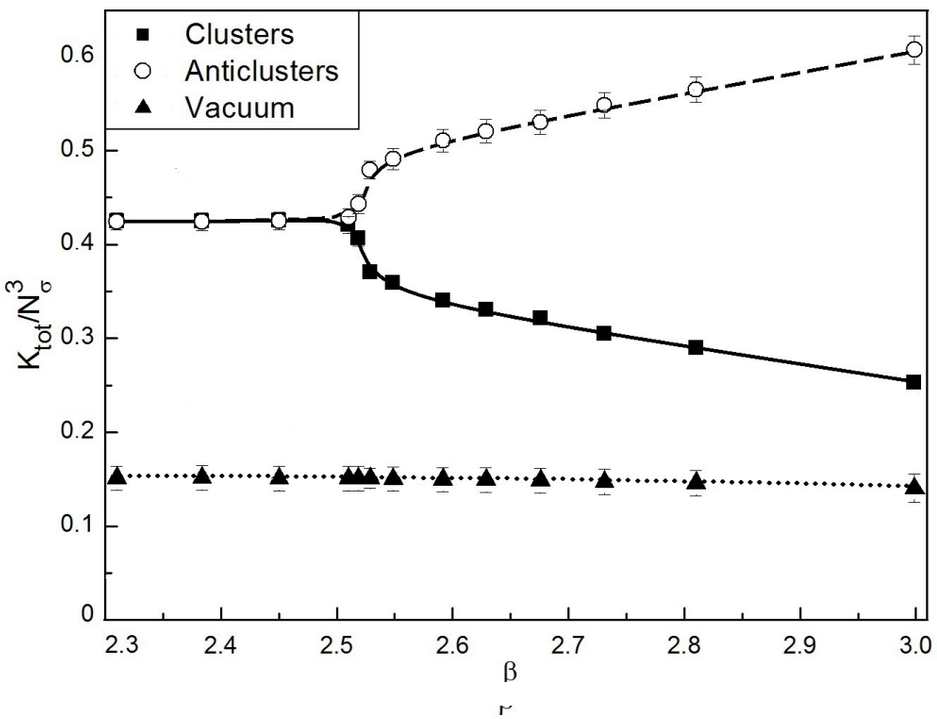
This is the surface tension induced PT which was invented to generate the CEP!

Properties of Auxiliary Vacuum

Both gases and aux. Vac. occupy a small part of the lattice 3-d volume

$$\frac{\mathrm{K_{tot}}}{\mathrm{N_\sigma^3}} = \begin{cases} \sum_{\mathrm{k}} \mathrm{k n_k^{(a)Cl}} \Big/ \mathrm{N_\sigma^3}, & \text{(anti)clusters} \\ 1 - \mathrm{K_{tot}^{aCl}} - \mathrm{K_{tot}^{Cl}}, & \text{auxiliary vacuum} \end{cases}$$





Volume fraction of vacuum is independent on β and/or on temperature

Incompressible auxiliary vacuum?

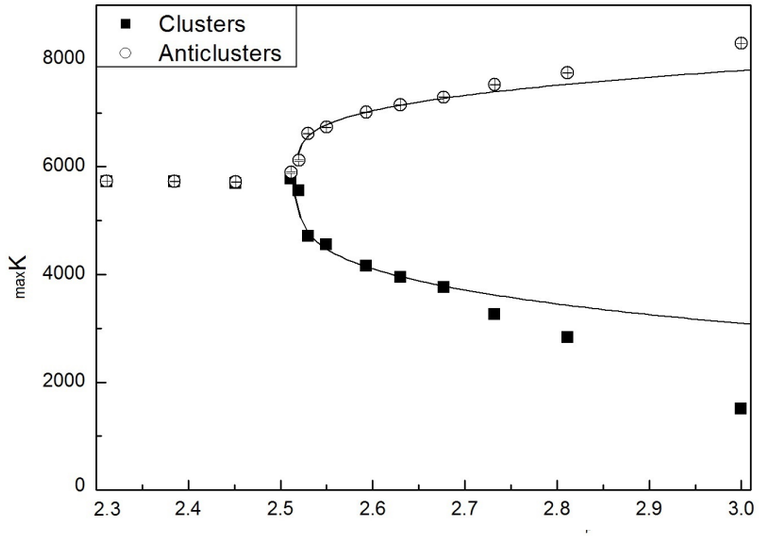
Properties of Liquid (Anti)Cluster

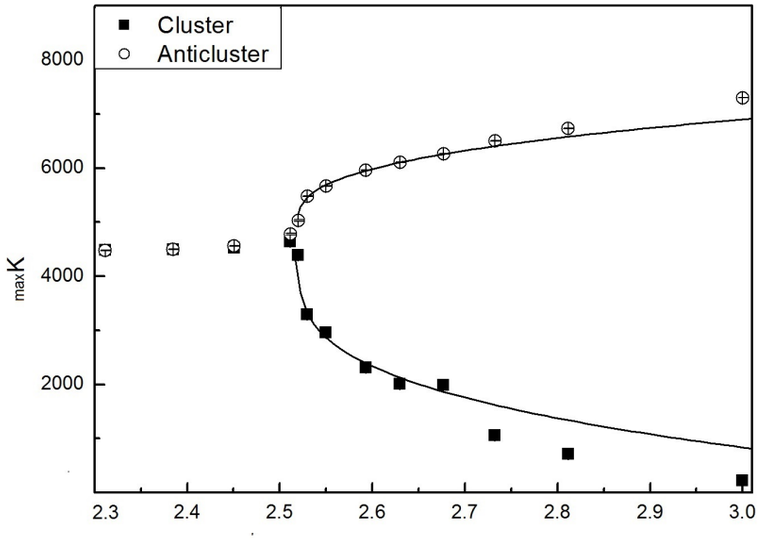
The mean value of Polyakov loop <L> is an order parameter in gluodynamics

One can show that $\quad |\mathrm{L}| \sim \max \mathrm{K}_{\mathrm{aCl}} - \max \mathrm{K}_{\mathrm{Cl}}$

$$\max \mathrm{K} = \sum_{\tilde{\mathrm{x}}} \mathrm{k}^{1+\tau} \mathrm{n}_{\mathrm{k}} \Big/ \sum_{\tilde{\mathrm{x}}} \mathrm{k}^{\tau} \mathrm{n}_{\mathrm{k}}$$ **is the mean liquid (=largest) (anti)cluster**

$$\beta > \beta_{\mathrm{c}}: \ \max \mathrm{K}(\beta) - \max \mathrm{K}(\beta_{\mathrm{c}}) = \mathrm{a} \cdot (\beta_{\mathrm{c}} - \beta)^{\mathrm{b}}$$





L_{cut}	type	a	b	χ^2/dof
0.1	Cl	-3056 ± 246	0.2964 ± 0.0284	$16.32/4 \simeq 4.08$
0.1	aCl	2129 ± 160	0.3315 ± 0.0269	$8.94/4 \simeq 2.235$
0.2	Cl	-4953 ± 443	0.3359 ± 0.0289	$12.3/3 \simeq 4.01$
0.2	aCl	2462 ± 87.7	0.3750 ± 0.0129	$2.068/4 \simeq 0.517$

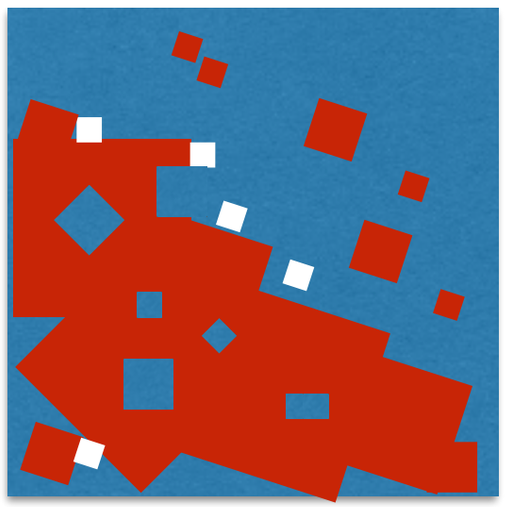
strong fluctuations

Except for aCL with cut-off 0.2 the exponent b corresponds to 3-d Ising model!

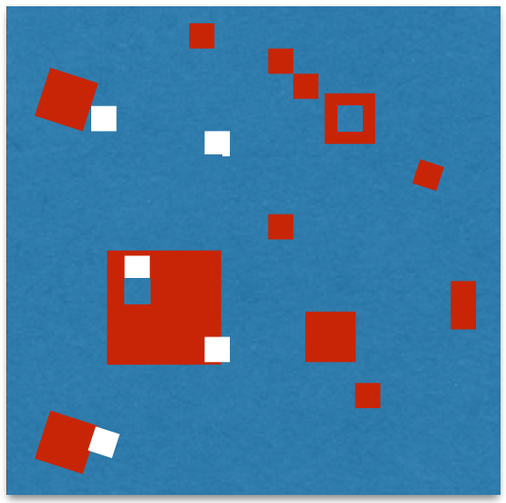
Space Inhomogeneity and Existence of SLOJKAs

Where the gaseous (anti)clusters are located?

T slightly above Tc



T >> Tc

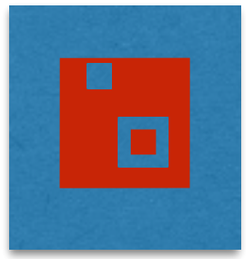


Gaseous clusters (anticlusters) are mainly located INSIDE the anticluster (cluster) LIQUID. Hence each LIQUID looks like Swiss Cheese!

Small portion of gaseous clusters (anticlusters) is located INSIDE other anticlusters (clusters), not the largest one!

At T >> Tc the largest cluster is also located inside the anticluster LIQUID!

Hence at T >> Tc we observe SLOJKAs = highly inhomogeneous structures!



What do they form in thermodynamic limit? Kind of vortexes?
Other topological defects?

Surface Free Energy of Liquid (Anti)Cluster

Can be deduced from locations of gaseous (anti)clusters

total geometrical volume

$$F_A^{surf} = \Sigma_A^{outer} \left[\max K_A + \widetilde{\sum_k} k\, n_{\bar{A}}(k) \right]^{\frac{D_A-1}{D_A}} - T\sigma_{\bar{A}} \widetilde{\sum_k} k^{\frac{2}{3}}\, n_{\bar{A}}(k)$$

surface tension of outer surface

fractal dimension

total free energy of inner non-native gaseous clusters

Due to periodic boundary conditions at T >> Tc there is no outer surface for anticluster LIQUID droplet!

=> at T >> Tc the largest droplet (liquid) has NEGATIVE SURFACE TENSION!

Its existence was predicted in K. A. Bugaev and G. M. Zinovjev, Nucl. Phys. A 848 (2010) 443

New Order Parameters

Above we have seen that $|\mathrm{L}| \sim \max \mathrm{K}_{\mathrm{aCl}} - \max \mathrm{K}_{\mathrm{Cl}}$

The ratio $\dfrac{\langle F_{Acl}\rangle - \langle F_{cl}\rangle}{\langle F_{Acl}\rangle + \langle F_{cl}\rangle}$ **can serve as an order parameter**

Where F is mean volume of gas, total volume of (anti)clusters, or surface tension coefficient of gas

Evidently, one can relate |L| and $\dfrac{\sigma_{Acl} - \sigma_{cl}}{\sigma_{Acl} + \sigma_{cl}}$ **for gases**

Conclusions

- The cluster approach based on Polyakov loop geometrical clusters is suggested.

- In terms of liquid-gas cluster model the PT in SU(2) gluodynamics is an evaporation of smaller liquid droplet into corresponding gas and condensation of another gas onto the largest liquid droplet.

- The size distributions of the gaseous (anti)clusters are analyzed on the basis of the Liquid Droplet Model. It is shown that even the dimers are described within this approach with high accuracy.

- The Fisher topological constant τ is found to be 1.806 ± 0.008 which disagrees with the Fisher Droplet Model value, but agrees with SMM and QGPbags with surface tension model with 3CEP.

- Any quantity which shows bifurcation can be used as the order parameter.

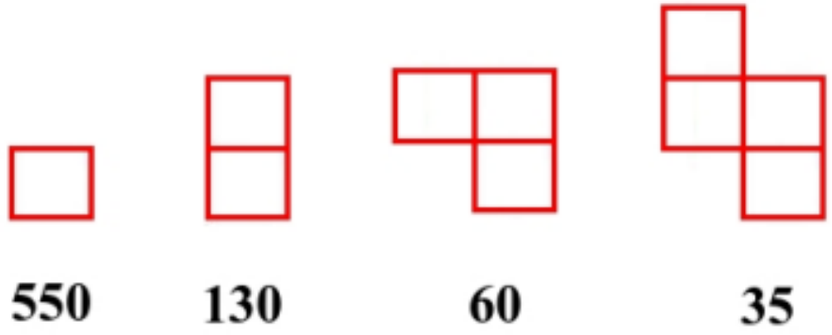
- In contrast to existing cluster models the surface tension of (anti)clusters does not vanish above PT and, hence, we need to search for another mechanism of 2-nd order PT.

Thank you for your attention!

Space Inhomogeneity

Example: β = 3 and cut-off 0.2

$^{max}K_{acl}$ = 7300 - the volume of largest anticluster;
$^{max}K_{cl}$ = 223 - the volume of largest cluster;
V_{acl}^{gas} = 89 - the total volume of the gas of anticlusters;
V_{cl}^{gas} = 2848 - the total volume of the gas of clusters;
V_{vac} = 1707 - the volume of auxiliary vacuum;



Since at hight T the surface tension and chem. potential of clusters is about 0, then size distribution is a power like!

Assuming DENSE PACKING of all clusters one needs at least 3100 surrounding anticlusters or aux. Vacuum, but one can get 1796 only!

=> Gaseous clusters are located inside of anticluster LIQUID droplet!

=> High T is not an exception, hence, the clusters are located inside of anticluster LIQUID droplet and vice versa!