QCD phase structure from the functional RG

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fQCD collaboration - QCD (phase diagram) with fRG:

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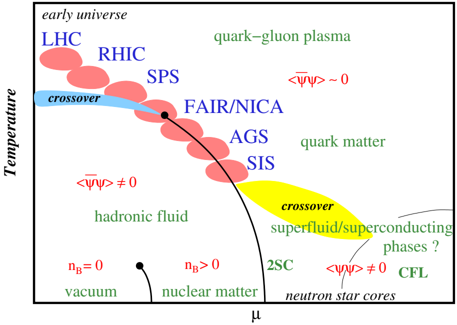

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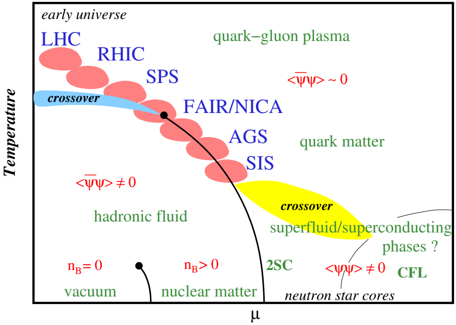
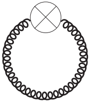
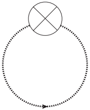
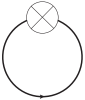
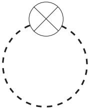


large part of this effort: vacuum YM-theory and QCD - why?

QCD with the fRG

- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$ [Wetterich '93]

$\partial_k \Gamma_k = \frac{1}{2}$  $-$  $-$  $+ \frac{1}{2}$ 

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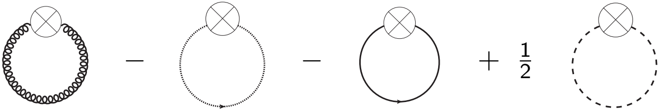

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$

- ∂_k: integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear
- mesons due to bosonization of four-Fermi interaction

Low-energy effective description (PQM model)

- "low enough" momentum-scales $k \leq \Lambda_{\mathrm{QM}}$: glue sector decouples
- remainder: quarks and mesons

$$\Gamma_{k\equiv\Lambda_{\mathrm{QM}}}[\Phi] \propto \int \left[Z_{q,k}\bar{q}\partial\!\!\!/ q + h_k\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)q\right]$$

$$+\int\left[Z_{\sigma,k}\sigma\left(\partial^2+m_\sigma^2\right)\sigma + Z_{\vec{\pi},k}\vec{\pi}\left(\partial^2+m_\pi^2\right)\vec{\pi} + U_k(\sigma,\vec{\pi}) + \mathcal{U}(\Phi,\Phi^\dagger)\right]$$

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- additional approximations necessary: cf. talk B.-J. Schaefer
  - LPA: $Z_{q,k} = Z_{\sigma,k} = Z_{\vec{\pi},k} \equiv 1$
  - LPA': $h_k \equiv h$ [Helmboldt, Pawlowski, Strodthoff, 2014]
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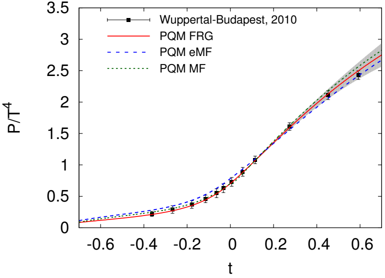
- usual applications $\Lambda_{\mathrm{QM}} \approx 0.7 - 1$ GeV - justified?

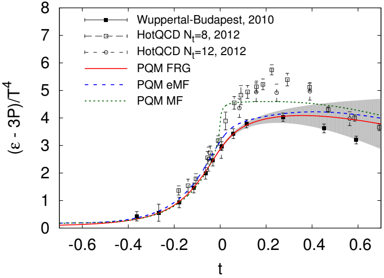
Equation of state (EOS)

- pressure
- $t = \frac{T - T_c}{T_c}$



[Herbst, MM, Pawlowski, Schaefer, Stiele, 2013]

- interactions measure
- correct dofs below $t \lesssim 0.2$



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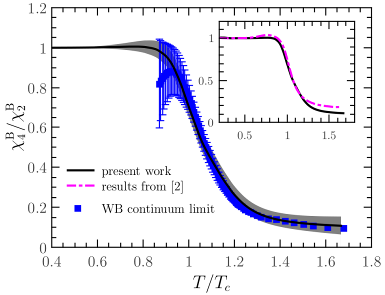
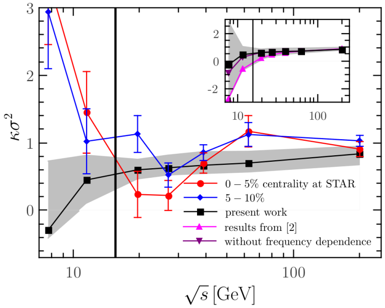
Kurtosis

[Fu, Pawlowski, 2015,2016], [Fu, Pawlowski, Schaefer, 2016]

- $\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}$
- $\mu(\sqrt{s})$ from freeze-out curve





- fluctuations in finite volume: cf. talk B.-J. Schaefer

η'-meson (screening) mass at chiral crossover

- drop in η' mass at chiral crossover? [Csörgo et al., 2010]

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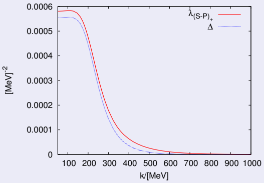

- RG-scale dependence from QCD

  [MM, Pawlowski, Strodthoff, 2014]

- temperature dependence $k(T)$:
  - $\lambda_{(S-P)_+,QCD}(k) \equiv \lambda_{(S-P)_+,PQM}(T)$

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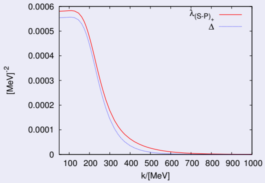

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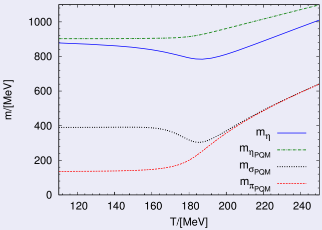

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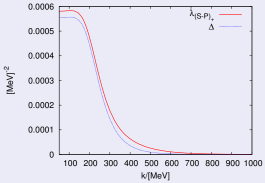

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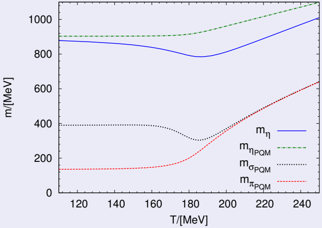

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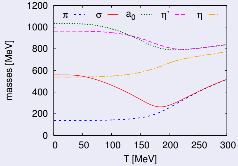
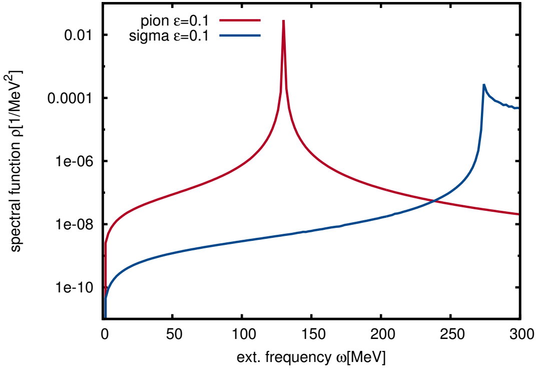
- screening masses
- $N_f = 2 + 1$: [MM, Schaefer, 2013]



Spectral functions

[Pawlowski, Strodthoff, 2015]

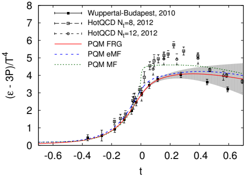
$(O(N)$-model, $T = 0)$



- $\rho(\omega, \vec{p}) = \frac{\mathrm{Im}\Gamma_R^{(2)}(\omega,\vec{p})}{\mathrm{Im}\Gamma_R^{(2)}(\omega,\vec{p})^2 + \mathrm{Re}\Gamma_R^{(2)}(\omega,\vec{p})^2}$, $\quad \Gamma_R^{(2)}(\omega, \vec{p}) = -\lim_{\epsilon\to 0} \Gamma_E^{(2)}(-\mathrm{i}\,(\omega + \mathrm{i}\epsilon)\,, \vec{p})$
- direct calculation, no numerical analytical continuation
- $O(4)$-symmetric regularization

Phase structure with functional methods

- works well at $\mu = 0$: agreement with lattice



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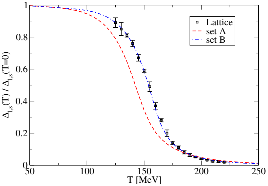

[Luecker, Fischer, Welzbacher, 2014]

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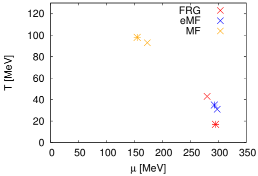

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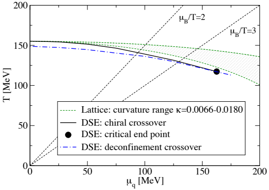

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    - input for Polyakov loop potential
  - quark propagator DSE:
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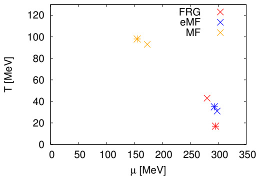

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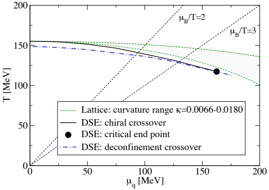

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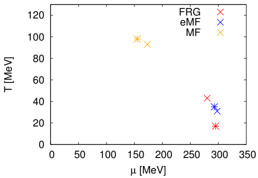
possible explanations for disagreement:

- $\mu \neq 0$: relative importance of diagrams changes
  $\Rightarrow$ summed contributions vs. individual contributions
- μ-dependent initial values necessary in PQM/PNJL approaches



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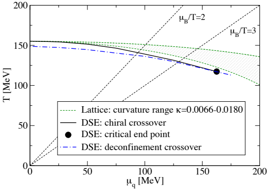

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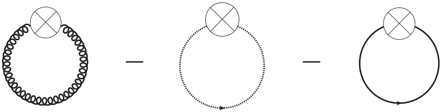

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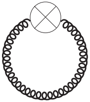
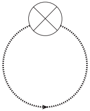
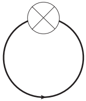
$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2}$  $-$  $-$ 

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Vertex expansion

- approximation necessary - vertex expansion

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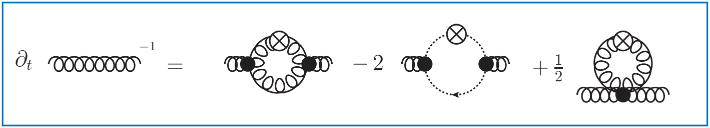
  $\Rightarrow$ equations for 1PI n-point functions, e.g. gluon propagator:



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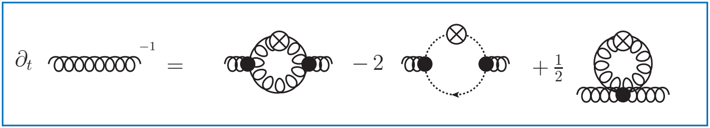

- want "apparent convergence" of $\Gamma[\Phi] = \lim_{k\to 0} \Gamma_k[\Phi]$

Landau gauge QCD

- two crucial phenomena: SχSB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
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- quenched matter part [MM, Strodthoff, Pawlowski, 2014]
- outlook: unquenching [Cyrol, MM, Strodthoff, Pawlowski, in preparation]
- pure YM-theory (cf. talk Anton Cyrol) [Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

Chiral symmetry breaking

- χSB $\Leftrightarrow$ resonance in 4-Fermi interaction λ (pion pole):

Chiral symmetry breaking

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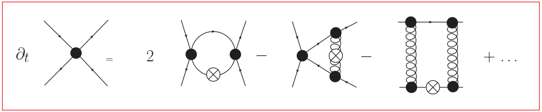
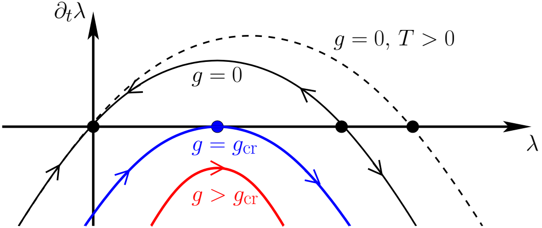
- resonance $\Rightarrow$ singularity without momentum dependency

$$\partial_t \lambda = a\,\lambda^2 + b\,\lambda\,\alpha + c\,\alpha^2, \quad b > 0,\ a, c \leq 0$$

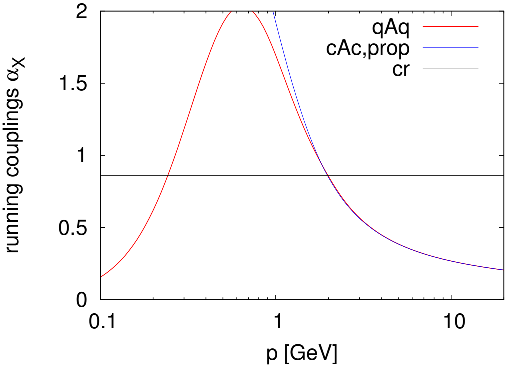




[Braun, 2011]

(transverse) running couplings

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]



- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

4-Fermi vertex via dynamical hadronization [Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels $\rightarrow$ meson exchange
- efficient inclusion of momentum dependence $\Rightarrow$ no singularities

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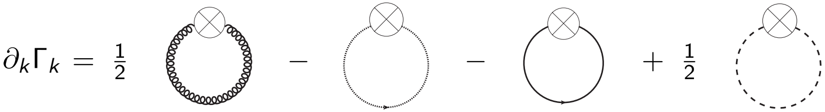
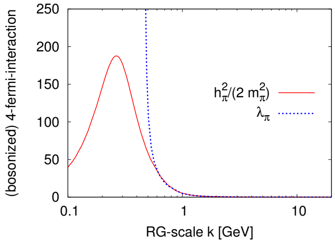
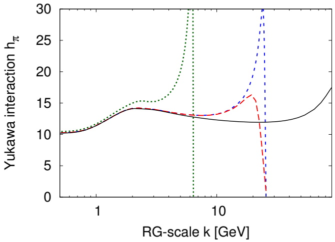
- efficient inclusion of momentum dependence ⇒ no singularities
- identification of $\Lambda_{\mathrm{QM/NJL}} \approx 300$ MeV





[MM, Strodthoff, Pawlowski, 2014]



[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

[MM, Strodthoff, Pawlowski, 2014]

Flow equations

(FormTracer: cf. talk A. K. Cyrol, [Cyrol, MM, Strodthoff, 2016])

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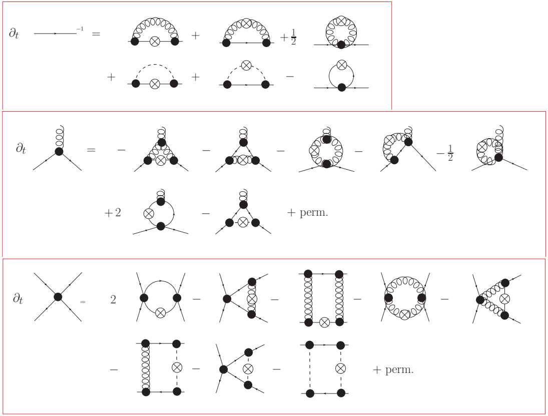

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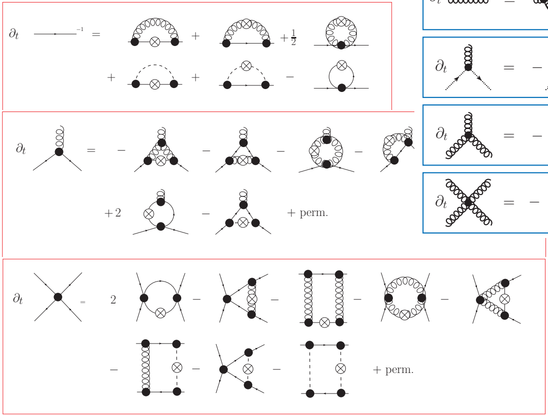

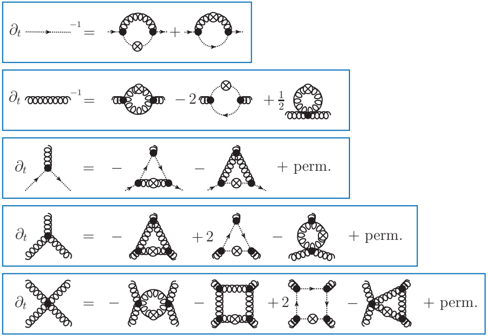

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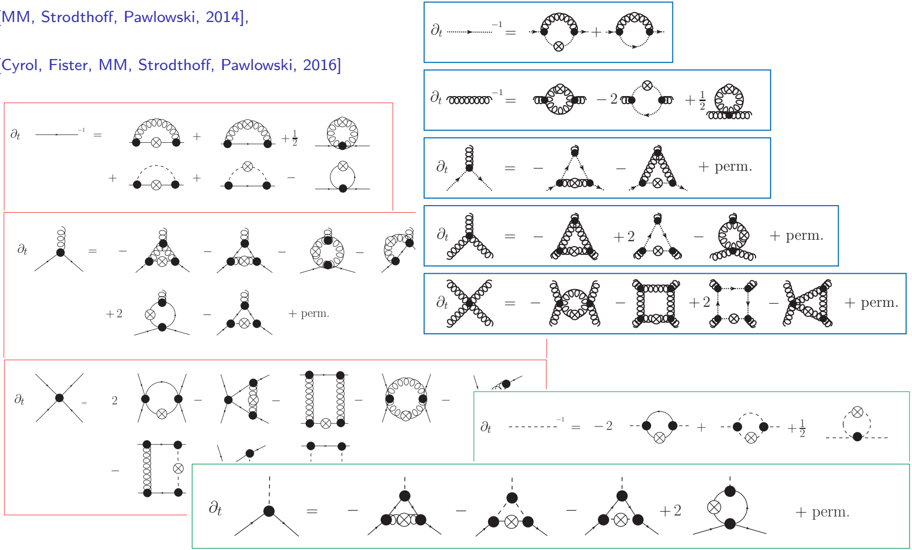
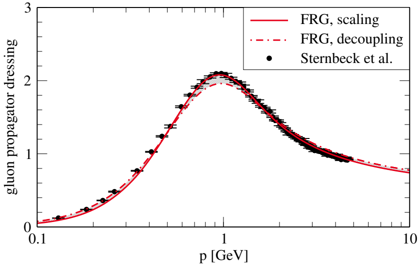


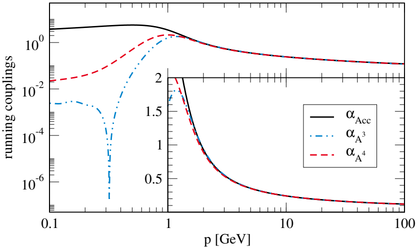
YM theory

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

- $\Gamma^{(2)}_{AA}(p) \propto Z_A(p)\, p^2 \, \left(\delta^{\mu\nu} - p^\mu p^\nu / p^2\right)$



- running couplings



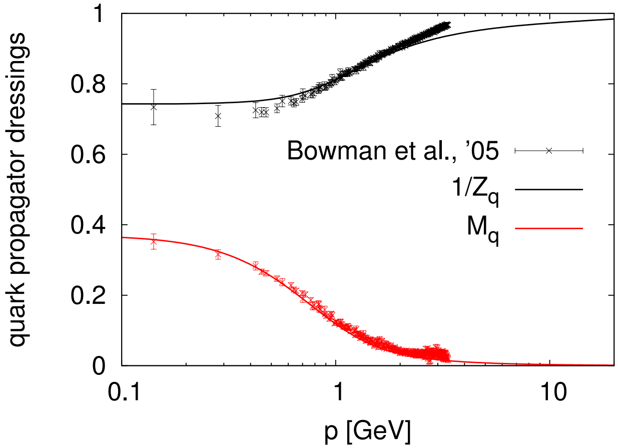
- more details ⇒ Talk Anton K. Cyrol

lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

Quark propagator

[MM, Pawlowski, Strodthoff, 2014]

- $\Gamma^{(2)}_{\bar{q}q}(p) \propto Z_q(p)\,(\not{p} + M(p))$



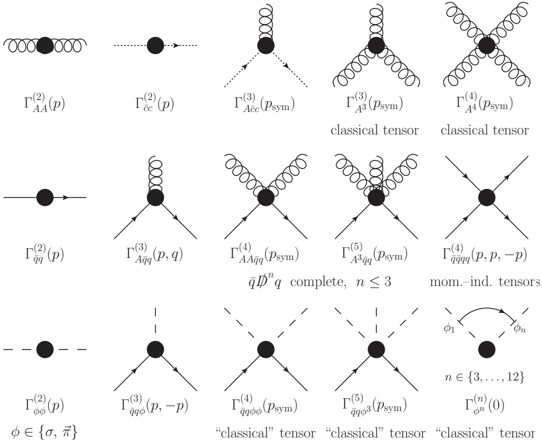
- fRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

Outlook: unquenching

[Cyrol, MM, Pawlowski, Strodthoff, in prep.]

extended truncation:



systematics of improving the truncation?

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[Cyrol, MM, Pawlowski, Strodthoff, in prep.]

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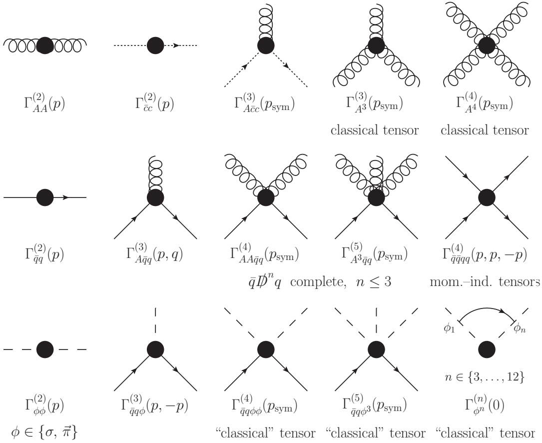


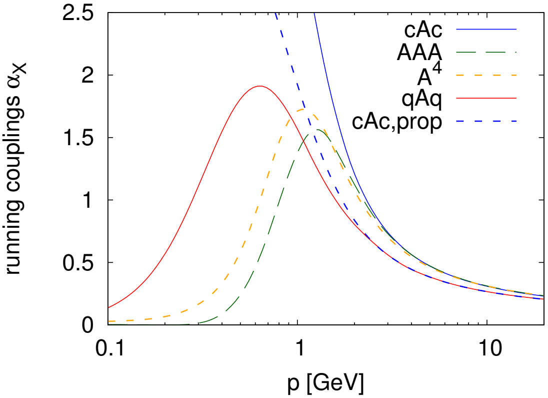
systematics of improving the truncation?

$\Rightarrow$ BRST-invariant operators, e.g. $\bar{\psi}\not{D}^n\psi$

Outlook: running couplings

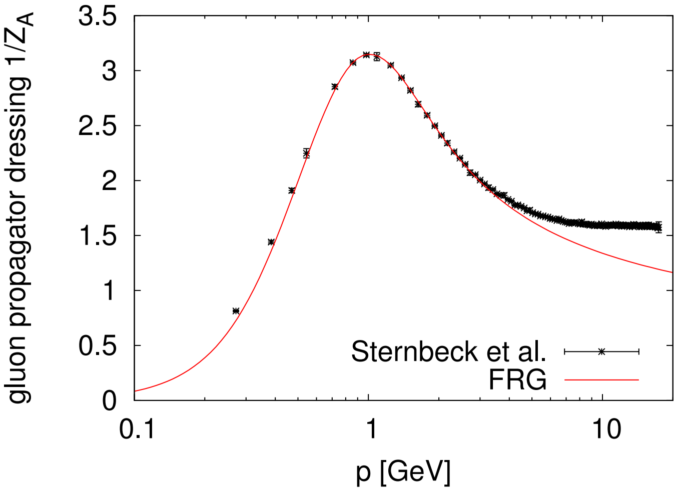
[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

- $\alpha_{cAc} = \dfrac{\left(\Gamma^{(3)}_{cAc}(p)\right)^2}{4\pi\, Z_A(p) Z_c(p)^2}$
- $\alpha_{AAA} = \dfrac{\left(\Gamma^{(3)}_{AAA}(p)\right)^2}{4\pi\, Z_A(p)^3}$
- $\alpha_{A^4} = \dfrac{\left(\Gamma^{(4)}_{A^4}(p)\right)}{4\pi\, Z_A(p)^2}$
- $\alpha_{qAq} = \dfrac{\left(\Gamma^{(3)}_{qAq}(p)\right)^2}{4\pi\, Z_A(p) Z_q(p)^2}$
- $\alpha_{cAc,\mathrm{prop}} = \dfrac{1}{4\pi\, Z_A(p) Z_c(p)^2}$



Outlook: gluon propagator

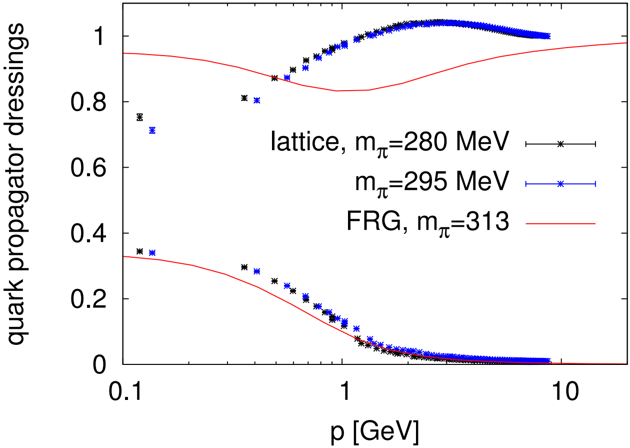
[Cyrol, MM, Pawlowski, Strodthoff, in preparation]



lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.

Outlook: quark propagator

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]



- comparison fRG with lattice: bare mass and scale setting

lattice data: Orlando Oliveira, Kzlersu, Silva, Skullerud, Sternbeck, Williams, arXiv:1605.09632 [hep-lat].

Summary and outlook

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  - EOS, kurtosis, axial anomaly, spectral functions, phase structure, ...
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