

QCD phase structure from the functional RG

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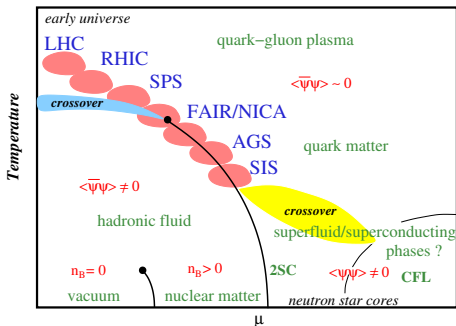


GEFÖRDERT VOM

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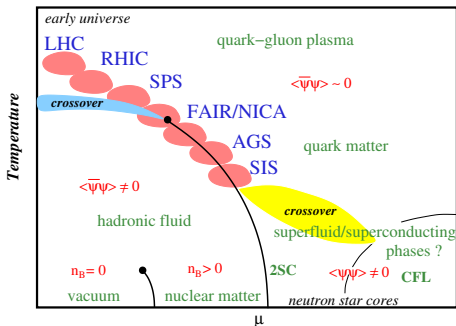
fQCD collaboration - QCD (phase diagram) with fRG:

J. Braun, L. Corell, A. K. Cyrol, L. Fister, W. J. Fu, M. Leonhardt, MM,
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large part of this effort: vacuum YM-theory and QCD - why?

QCD with the fRG

- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

[Wetterich '93]

$$\partial_k \Gamma_k = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right)$$

The diagrammatic equation shows the derivative of the effective action Γ_k with respect to the scale k . It is equal to $\frac{1}{2}$ times the difference of four diagrams. Each diagram consists of a circle with a cross in the center, representing a ghost loop. The first diagram has a solid, wavy line for the loop. The second diagram has a solid, straight line for the loop. The third diagram has a solid, straight line for the loop with an arrow pointing clockwise. The fourth diagram has a dashed, straight line for the loop.

$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

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The diagrammatic equation shows the derivative of the effective action with respect to the regulator k . It consists of four terms:

- Diagram 1: A circle with a wavy internal line and a cross on top.
- Diagram 2: A circle with a dashed internal line and a cross on top.
- Diagram 3: A circle with a solid internal line and a cross on top.
- Diagram 4: A circle with a dashed internal line and a cross on top.

$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

- ∂_k : integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear
- mesons due to bosonization of four-Fermi interaction

Low-energy effective description (PQM model)

- “low enough” momentum-scales $k \leq \Lambda_{\text{QM}}$: glue sector decouples
- remainder: quarks and mesons

$$\Gamma_{k \equiv \Lambda_{\text{QM}}}[\Phi] \propto \int [Z_{q,k} \bar{q} \not{\partial} q + h_k (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) q]$$
$$+ \int [Z_{\sigma,k} \sigma (\partial^2 + m_\sigma^2) \sigma + Z_{\vec{\pi},k} \vec{\pi} (\partial^2 + m_\pi^2) \vec{\pi} + U_k(\sigma, \vec{\pi}) + \mathcal{U}(\Phi, \Phi^\dagger)]$$

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- additional approximations necessary:

cf. talk B.-J. Schaefer

- ▶ LPA: $Z_{q,k} = Z_{\sigma,k} = Z_{\vec{\pi},k} \equiv 1$
- ▶ LPA': $h_k \equiv h$
- ▶ LPA'+Y: all shown(!) operators run

[Helmholtz, Pawłowski, Strodthoff, 2014]

[Rennecke, Pawłowski, 2014]

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- external information:

- ▶ Polyakov-loop potential $\mathcal{U}(\Phi, \Phi^\dagger)$ from lattice input

- ▶ initial $h_{\Lambda_{\text{QM}}}$, $U_{\Lambda_{\text{QM}}}$, $m_{\sigma, \Lambda_{\text{QM}}}$ and $m_{\vec{\pi}, \Lambda_{\text{QM}}}$ from vacuum observables

Low-energy effective description (PQM model)

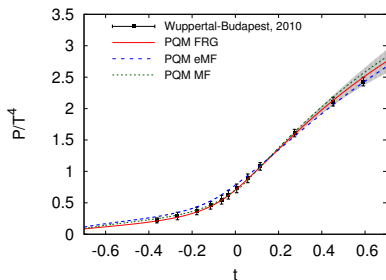
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- usual applications $\Lambda_{\text{QM}} \approx 0.7 - 1 \text{ GeV}$ - justified?

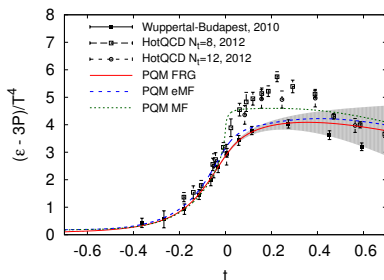
Equation of state (EOS)

- pressure
- $t = \frac{T - T_c}{T_c}$



[Herbst, MM, Pawlowski, Schaefer, Stiele, 2013]

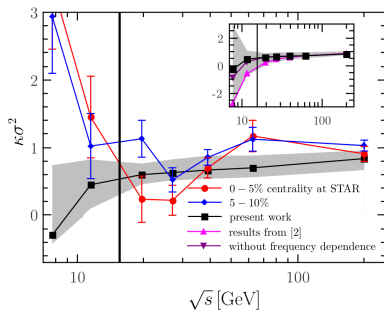
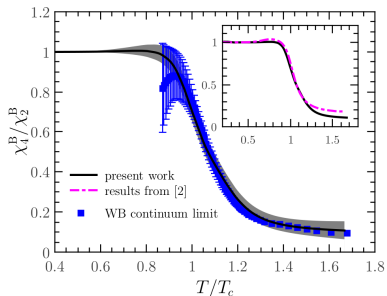
- interactions measure
- correct dofs below $t \leq 0.2$



[Herbst, MM, Pawlowski, Schaefer, Stiele, 2016]

- $\chi_n^B = \frac{\partial^n}{\partial (\mu_B/T)^n} \frac{p}{T^4}$

- $\mu(\sqrt{s})$ from freeze-out curve



- fluctuations in finite volume: cf. talk B.-J. Schaefer

η' -meson (screening) mass at chiral crossover

- drop in η' mass at chiral crossover?

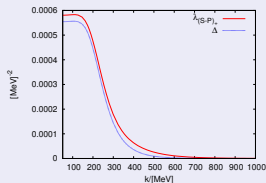
[Csörgo et al., 2010]

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't Hooft determinant



- RG-scale dependence from QCD

[MM, Pawłowski, Strodthoff, 2014]

- temperature dependence $k(T)$:

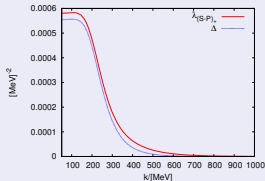
$$\lambda_{(S-P)_+, QCD}(k) \equiv \lambda_{(S-P)_+, PQM}(T)$$

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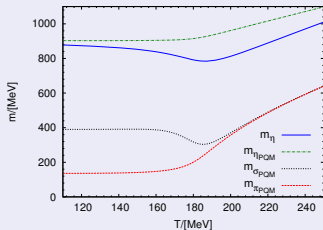


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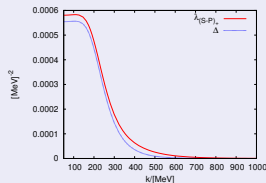
- screening masses

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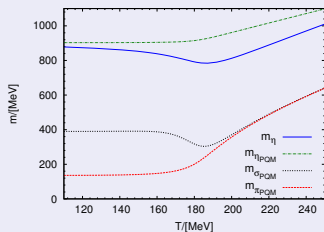


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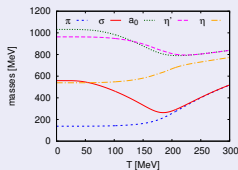


[Heller, MM, 2015]

- screening masses

- $N_f = 2 + 1$:

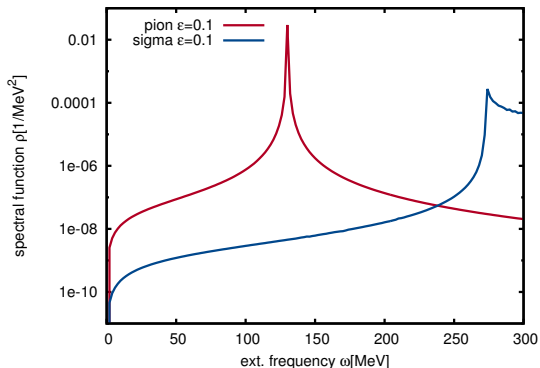
[MM, Schaefer, 2013]



Spectral functions

[Pawlowski, Strodthoff, 2015]

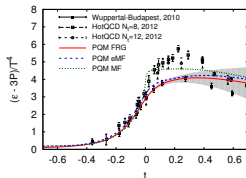
($O(N)$ -model, $T = 0$)



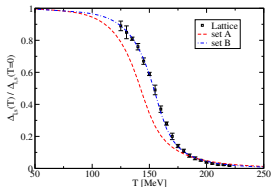
- $\rho(\omega, \vec{p}) = \frac{\text{Im}\Gamma_R^{(2)}(\omega, \vec{p})}{\text{Im}\Gamma_R^{(2)}(\omega, \vec{p})^2 + \text{Re}\Gamma_R^{(2)}(\omega, \vec{p})^2}$, $\Gamma_R^{(2)}(\omega, \vec{p}) = -\lim_{\epsilon \rightarrow 0} \Gamma_E^{(2)}(-i(\omega + i\epsilon), \vec{p})$
- direct calculation, no numerical analytical continuation
- $O(4)$ -symmetric regularization

Phase structure with functional methods

- works well at $\mu = 0$: agreement with lattice



[Herbst, MM, Pawłowski, Schaefer, Stiele, '13]

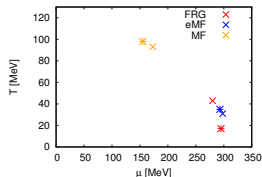


[Luecker, Fischer, Welzbacher, 2014]

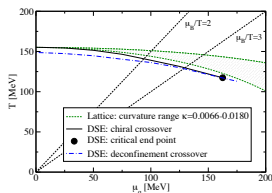
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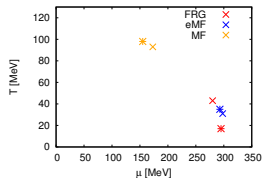
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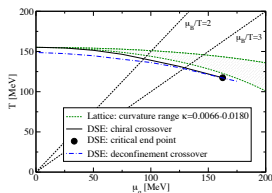
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- calculations need model input:
 - ▶ Polyakov-quark-meson model with fRG:
 - ★ initial values at $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
 - ★ input for Polyakov loop potential
 - ▶ quark propagator DSE:
 - ★ IR quark-gluon vertex



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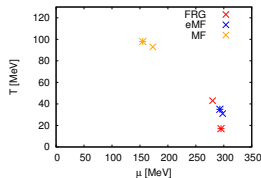
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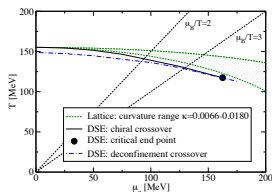
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possible explanations for disagreement:

- $\mu \neq 0$: relative importance of diagrams changes
 \Rightarrow summed contributions vs. individual contributions
- μ -dependent initial values necessary in PQM/PNJL approaches



[MM, Schaefer, 2013]



[Luecker, Fischer, Fister, Pawłowski, '13]

QCD with the fRG ... again

- use only perturbative QCD input
 - ▶ $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$
 - ▶ $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$

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$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} \right)$$


$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

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- ∂_k : integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear

Vertex expansion

- approximation necessary - vertex expansion

$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1})$$

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- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
⇒ equations for 1PI n -point functions, e.g. gluon propagator:

The diagrammatic equation shows the derivative of the inverse gluon propagator with respect to the coupling t . On the left, a wavy line representing the gluon propagator is shown with a superscript -1 . This is equal to the sum of three diagrams: 1) a gluon loop with a cross on the top arc, 2) a ghost loop with a cross on the top arc, multiplied by -2 , and 3) a ghost loop with a cross on the top arc, multiplied by $+\frac{1}{2}$. The diagrams are enclosed in a blue rectangular box.

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- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
 \Rightarrow equations for 1PI n -point functions, e.g. gluon propagator:

The diagrammatic equation is enclosed in a blue box and reads: $\partial_t \text{gluon propagator}^{-1} = \text{gluon loop} - 2 \text{ghost loop} + \frac{1}{2} \text{ghost-gluon loop}$. The first term is a gluon propagator with a small circle containing a cross, representing a derivative. The second term is a gluon loop diagram with two external gluon lines. The third term is a ghost loop diagram with two external gluon lines and a ghost loop. The fourth term is a ghost-gluon loop diagram with two external gluon lines and a loop containing both a ghost and a gluon line.

- want “apparent convergence” of $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

Landau gauge QCD

- two crucial phenomena: S_χ SB and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation:

see e.g. [Williams, Fischer, Heupel, 2015]

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- quenched QCD: allows separate investigation:

- quenched matter part [MM, Strodthoff, Pawłowski, 2014]
- outlook: unquenching [Cyrol, MM, Strodthoff, Pawłowski, in preparation]
- pure YM-theory (cf. talk Anton Cyrol) [Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]

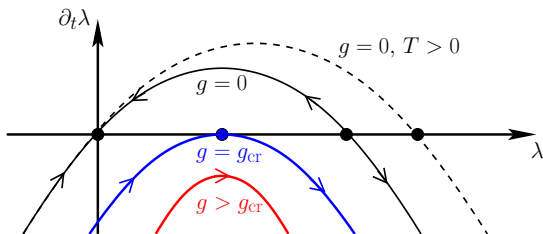
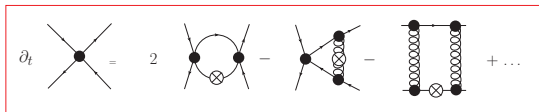
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):

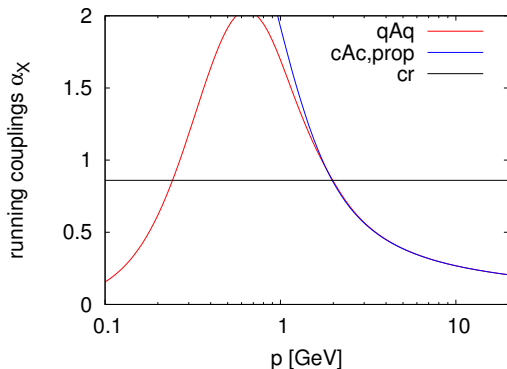
Chiral symmetry breaking

- χ SB \Leftrightarrow resonance in 4-Fermi interaction λ (pion pole):
- resonance \Rightarrow singularity without momentum dependency

$$\partial_t \lambda = a \lambda^2 + b \lambda \alpha + c \alpha^2, \quad b > 0, \quad a, c \leq 0$$



[Braun, 2011]



- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

4-Fermi vertex via dynamical hadronization

[Gies, Wetterich, 2002]

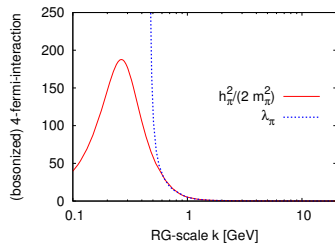
- change of variables: particular 4-Fermi channels \rightarrow meson exchange
- efficient inclusion of momentum dependence \Rightarrow no singularities

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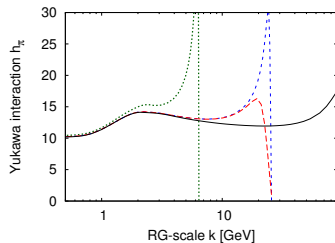
[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels \rightarrow meson exchange
- efficient inclusion of momentum dependence \Rightarrow no singularities
- identification of $\Lambda_{\text{QM/NJL}} \approx 300$ MeV

$$\partial_k \Gamma_k = \frac{1}{2} \text{ (ring with wavy line) } - \text{ (ring with dashed line) } - \text{ (ring with solid line) } + \frac{1}{2} \text{ (ring with dotted line) }$$



[MM, Strodthoff, Pawłowski, 2014]



[Braun, Fister, Haas, Pawłowski, Rennecke, 2014]

[MM, Strodthoff, Pawłowski, 2014]

Flow equations

(FormTracer: cf. talk A. K. Cyrol, [Cyrol, MM, Strodthoff, 2016])

[MM, Strodthoff, Pawłowski, 2014],

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$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---}$$

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$$\partial_t \text{---} \text{---} \text{---} = 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

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$$\partial_t \text{---}^{-1} = \text{---} \circlearrowleft + \text{---} \circlearrowright + \frac{1}{2} \text{---} \circlearrowright \circlearrowleft + \text{---} \circlearrowleft \circlearrowright - \text{---} \circlearrowleft \circlearrowleft$$

$$\partial_t \text{---}^{-1} = \text{---} \circlearrowleft + \text{---} \circlearrowright$$

$$\partial_t \text{---}^{-1} = \text{---} \circlearrowleft - 2 \text{---} \circlearrowright + \frac{1}{2} \text{---} \circlearrowleft \circlearrowleft$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

Flow equations

(FormTracer: cf. talk A. K. Cyrol, [Cyrol, MM, Strodthoff, 2016])

[MM, Strodthoff, Pawłowski, 2014],

[Cyrol, Fister, MM, Strodthoff, Pawłowski, 2016]

$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}$$

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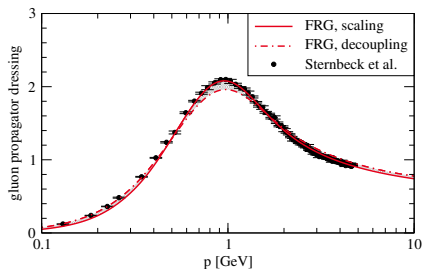
$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} - 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---}$$

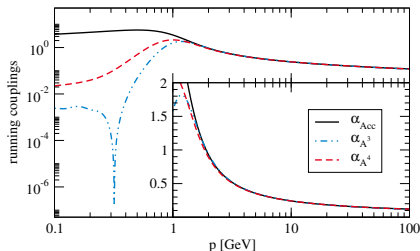
$$\partial_t \text{---}^{-1} = -2 \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---} \text{---}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} + \text{perm.}$$

- $\Gamma_{AA}^{(2)}(p) \propto Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$



- running couplings



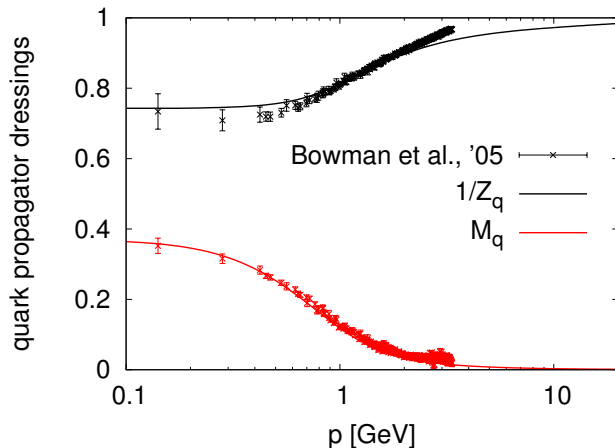
- more details \Rightarrow Talk Anton K. Cyrol

lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Müller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

Quark propagator

[MM, Pawłowski, Strodthoff, 2014]

- $\Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) (\not{p} + M(p))$



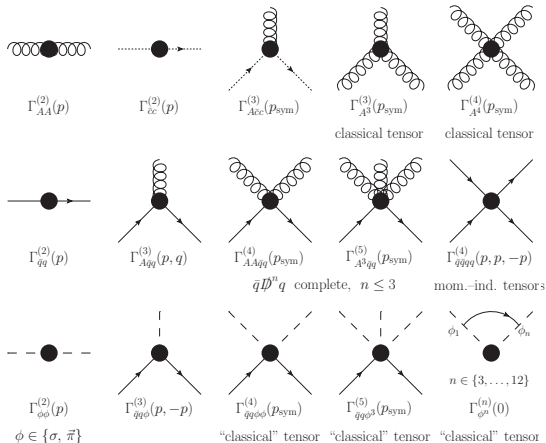
- fRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang, Phys. Rev. D71, 054507 (2005).

Outlook: unquenching

[Cyrol, MM, Pawłowski, Strodthoff, in prep.]

extended truncation:

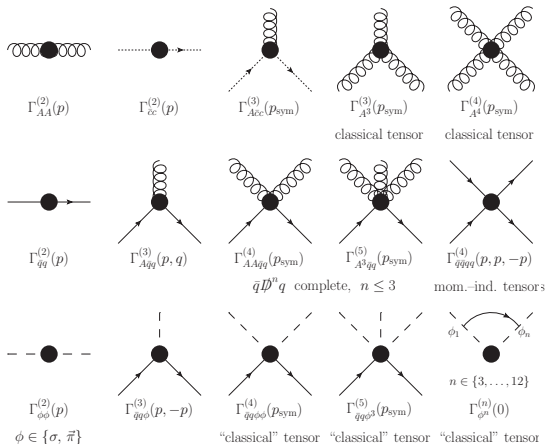


systematics of improving the truncation?

Outlook: unquenching

[Cyrol, MM, Pawłowski, Strodthoff, in prep.]

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\Rightarrow BRST-invariant operators, e.g. $\bar{\psi}\not{D}^n\psi$

Outlook: running couplings

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

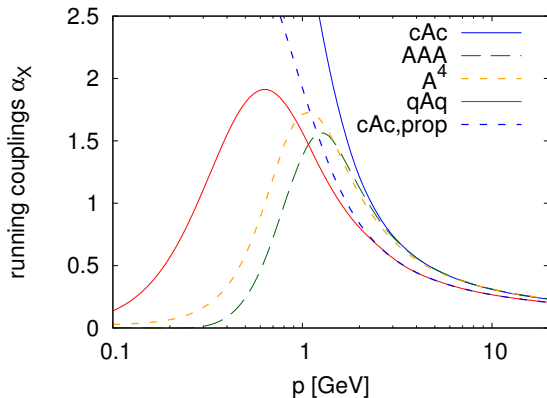
$$\bullet \alpha_{cAc} = \frac{(\Gamma_{cAc}^{(3)}(p))^2}{4\pi Z_A(p)Z_c(p)^2}$$

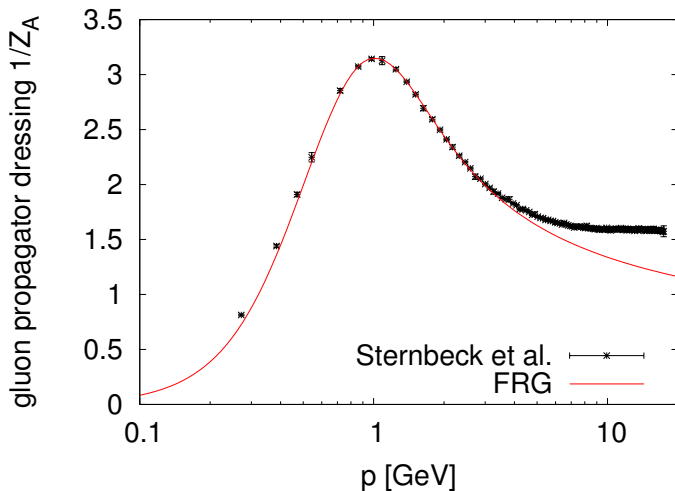
$$\bullet \alpha_{AAA} = \frac{(\Gamma_{AAA}^{(3)}(p))^2}{4\pi Z_A(p)^3}$$

$$\bullet \alpha_{A^4} = \frac{(\Gamma_{A^4}^{(4)}(p))}{4\pi Z_A(p)^2}$$

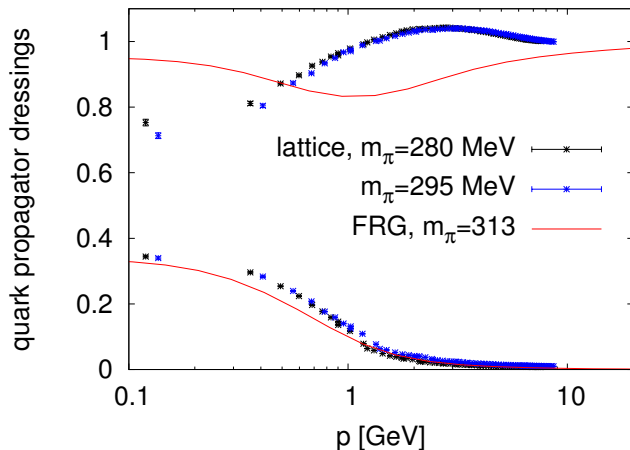
$$\bullet \alpha_{qAq} = \frac{(\Gamma_{qAq}^{(3)}(p))^2}{4\pi Z_A(p)Z_q(p)^2}$$

$$\bullet \alpha_{cAc,prop} = \frac{1}{4\pi Z_A(p)Z_c(p)^2}$$





lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.



- comparison fRG with lattice: bare mass and scale setting

lattice data: Orlando Oliveira, Kzlersu, Silva, Skullerud, Sternbeck, Williams, arXiv:1605.09632 [hep-lat].

Summary and outlook

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