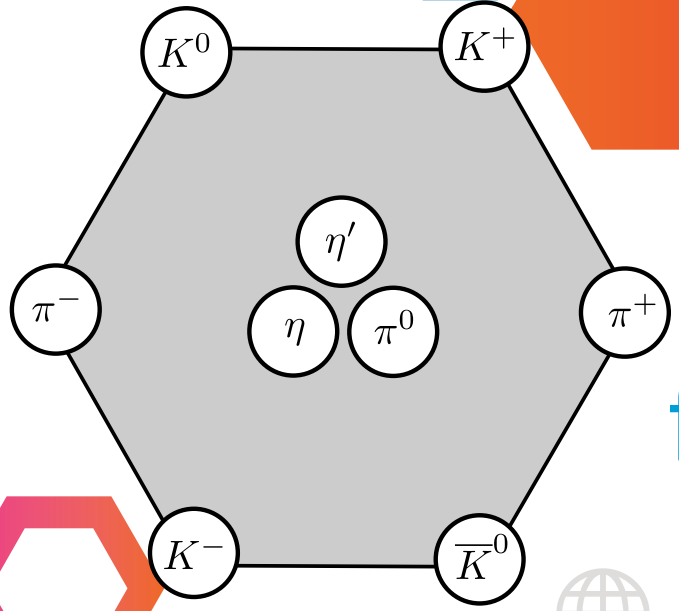


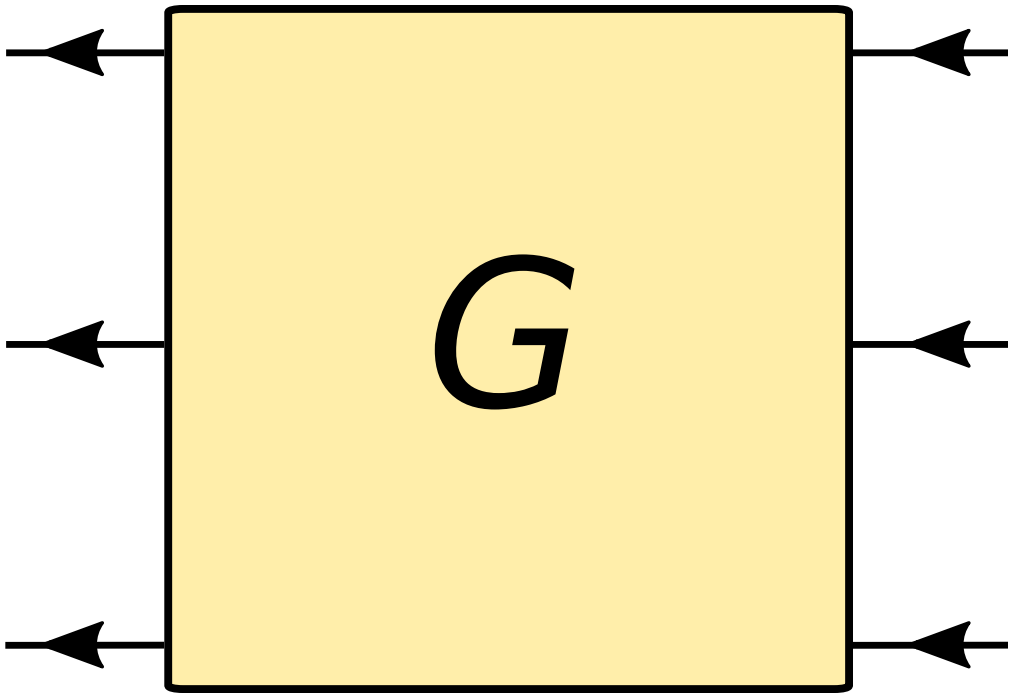
Hadron Properties from Dyson-Schwinger equations

Richard Williams

Together with
C. S. Fischer, G. Eichmann, H. Sanchis-Alepuz

Bundesministerium für Bildung und Forschung

HIC for FAIR
Helmholtz International Center

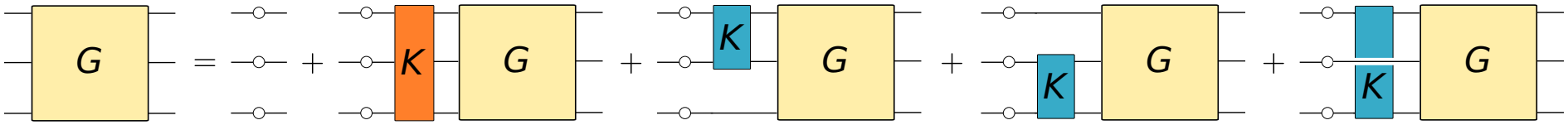


Everything we need to know is encoded in n-point functions.

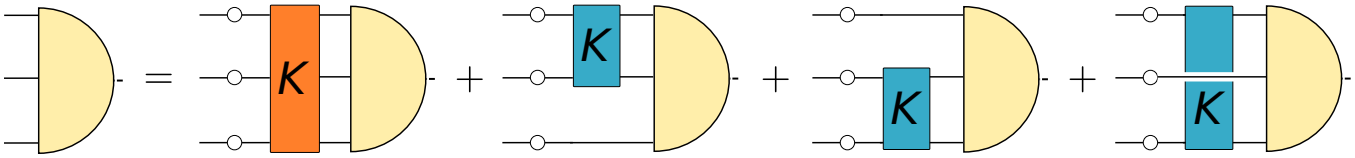
Just have to extract it.

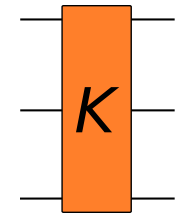
DSE and BSE

Trade one unknown **G**, for another unknown **K**



Solution (on-shell) yields Bethe-Salpeter wavefunction

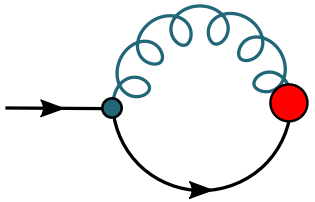




Irreducible 2-, 3-, 4-body kernels **define** equation

Dressed particle constituents: Green's functions

$$\frac{\delta\Gamma[\phi]}{\delta\psi} = \frac{\delta S[\phi]}{\delta\psi} +$$



DSE and BSE

Need to truncate:

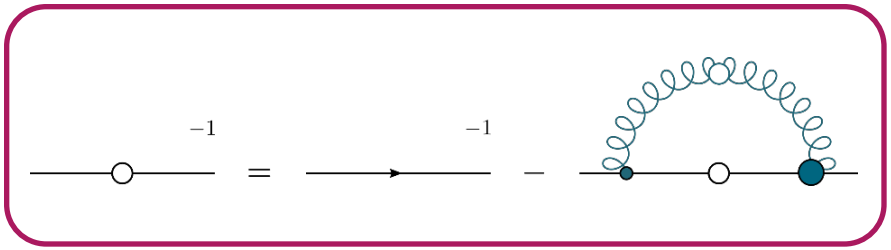
- Leading two-body kernels (must still define)
- Leading 1PI Green's functions (**propagators**, vertices)

Options:

- Systematic "vertex expansion" – see also next two talks
- nPI effective action techniques
- Phenomenological models
- (gauge-fixed) Lattice data **as input**

Simultaneously maintain: (chiral) symmetry

Difficult to ensure Goldstone nature of the pion



Prototype DSE: quark propagator

Input: gluon, quark-gluon vertex

Hierarchical dependence

DSE and BSE

Phenomenological Models

- **Corners** of parameter space constrained (hadronic spectrum, Lattice data)
- **Limited** coupled system
- Numerically **easy** (relatively)
- **Immediately** applicable to calculation of observables

Modelling of Vertices

Systematic Expansions

- **Few parameters** (coupling, quark masses)
- **Extensive** coupled system
- Numerically **hard** (sadly)
- Application to calculation of observables **requires** development

Calculation of Vertices

Examples:

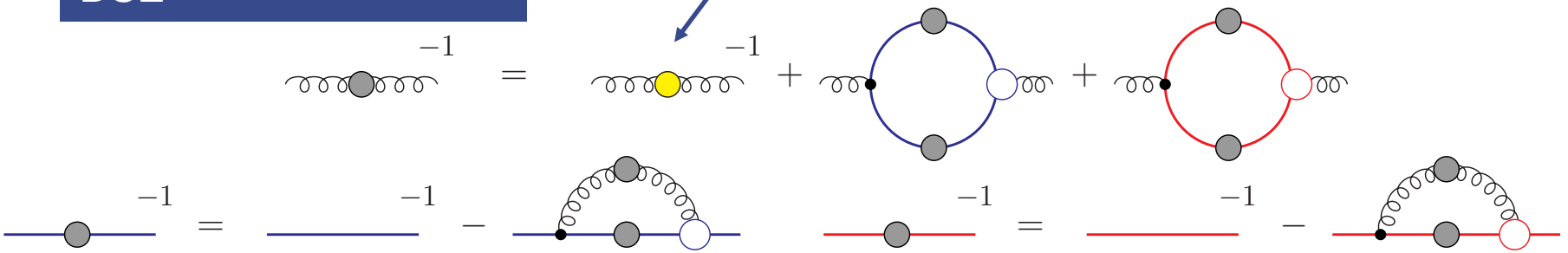
- Bound-states
- QCD phase diagram

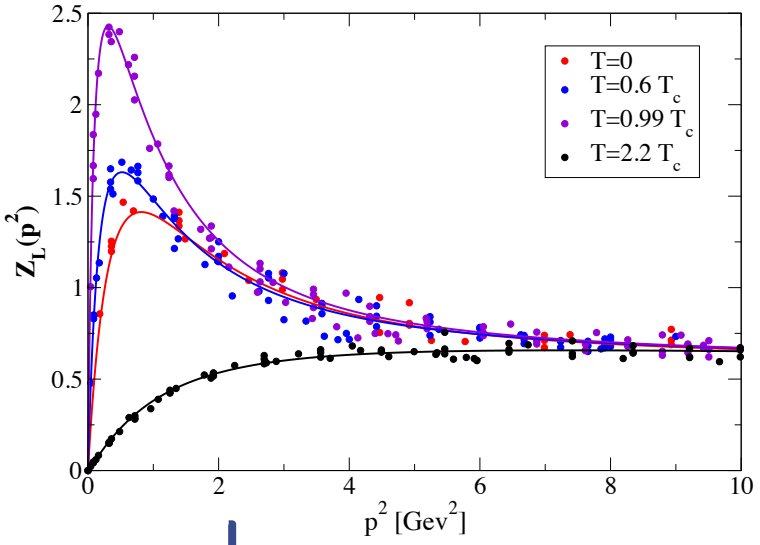
Construct improved phenomenological models

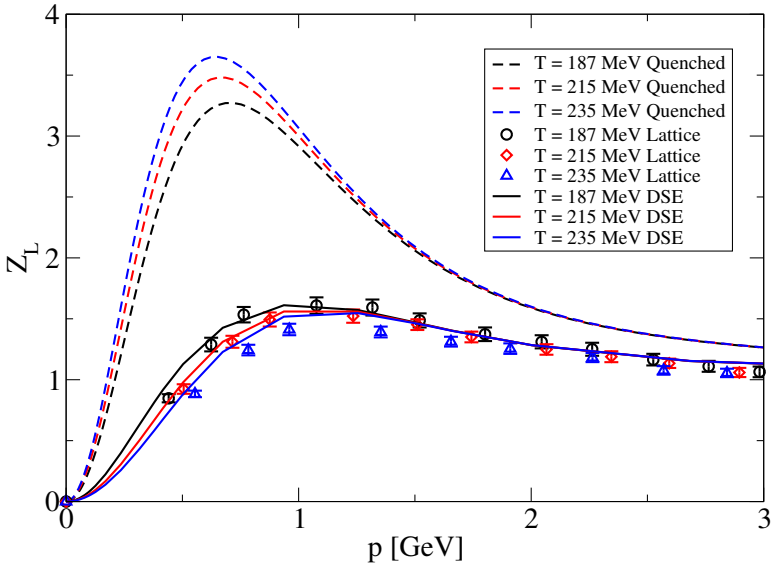
Modelling of Vertices

DSE

Quenched lattice data







Unquenching via DSE

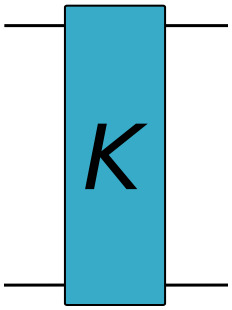
Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: Fischer, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

Model quark-gluon vertex. No sign problem

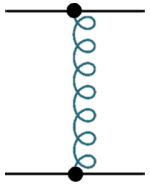
Cucchieri, Maas, Mendes, PRD 75 (2007)
Fischer, Maas, Mueller, EPJC 68 (2010)
Cucchieri, Mendes, PoS FACESQCD 007 (2010)

Aouane et al, PRD 85 (2012) 034501
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503
FRG: Fister, Pawlowski, arXiv:1112.5440

BSE



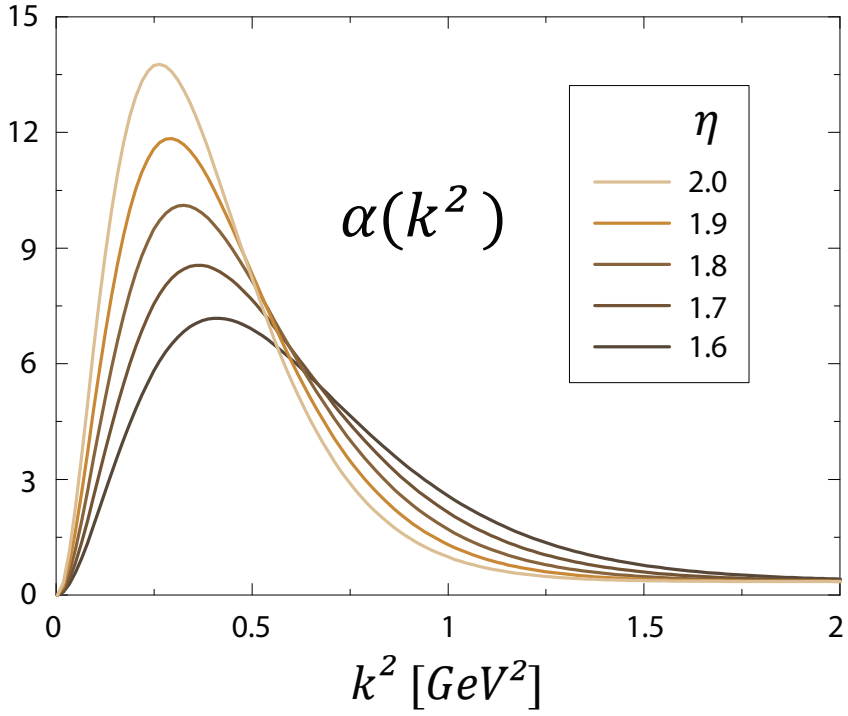
Simplest choice:
One-gluon exchange



Rainbow-ladder

Bare vertices, **dressed** gluon

Compensate with effective interaction



e.g. Maris-Tandy interaction

Good for heavy quarks

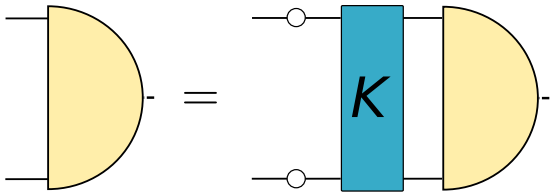
Reliable in DCSB channels

Known shortcomings

[Maris, Tandy PRC 60 (1999) 055214]

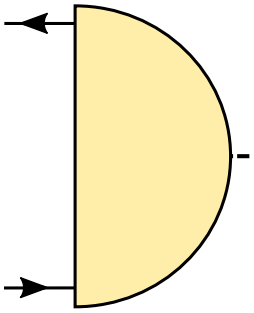
Mesons

Insert **Kernel** and dressed particle constituents into (homogeneous) Bethe-Salpeter equation



State of interest selected by covariant decomposition of the amplitude

(γ_5 toggles parity)



$$J=0:\ \{\mathbb{1},\hat{\not{P}}\}\{\mathbb{1},\ \not{n}\}$$

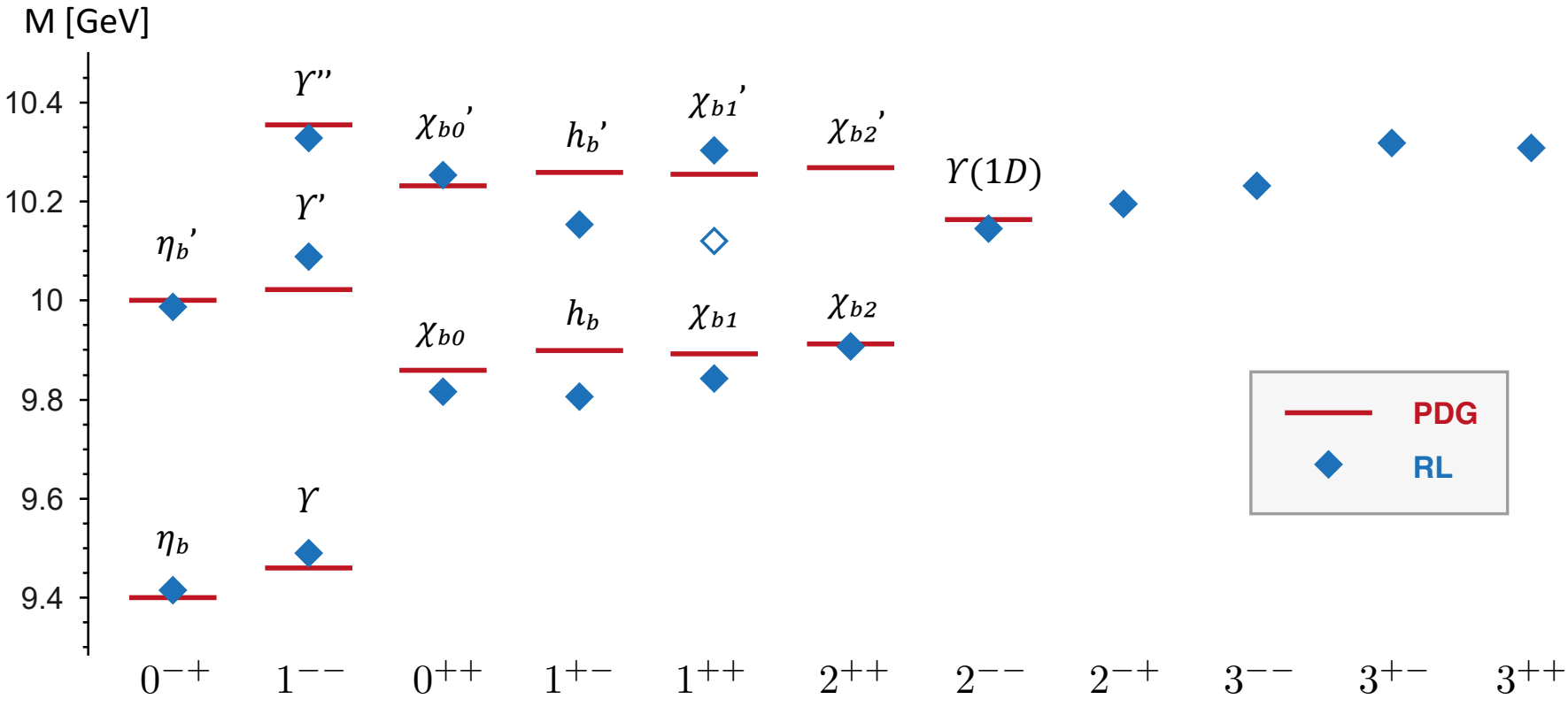
$$J=1:\ \{\mathbb{1},\hat{\not{P}}\}\{\gamma_\perp^\mu,\ n^\mu,\ \gamma_\perp^\mu\not{n}-n^\mu,\ n^\mu\not{n}-\tfrac{1}{3}\gamma_\perp^\mu\}$$

s-waves **p-waves** **d-waves**

Relativistic so "exotic" quantum numbers are allowed as $q\bar{q}$

$$J^{PC}=0^{++},0^{-+},1^{--},0^{+-},0^{--},1^{-+}$$

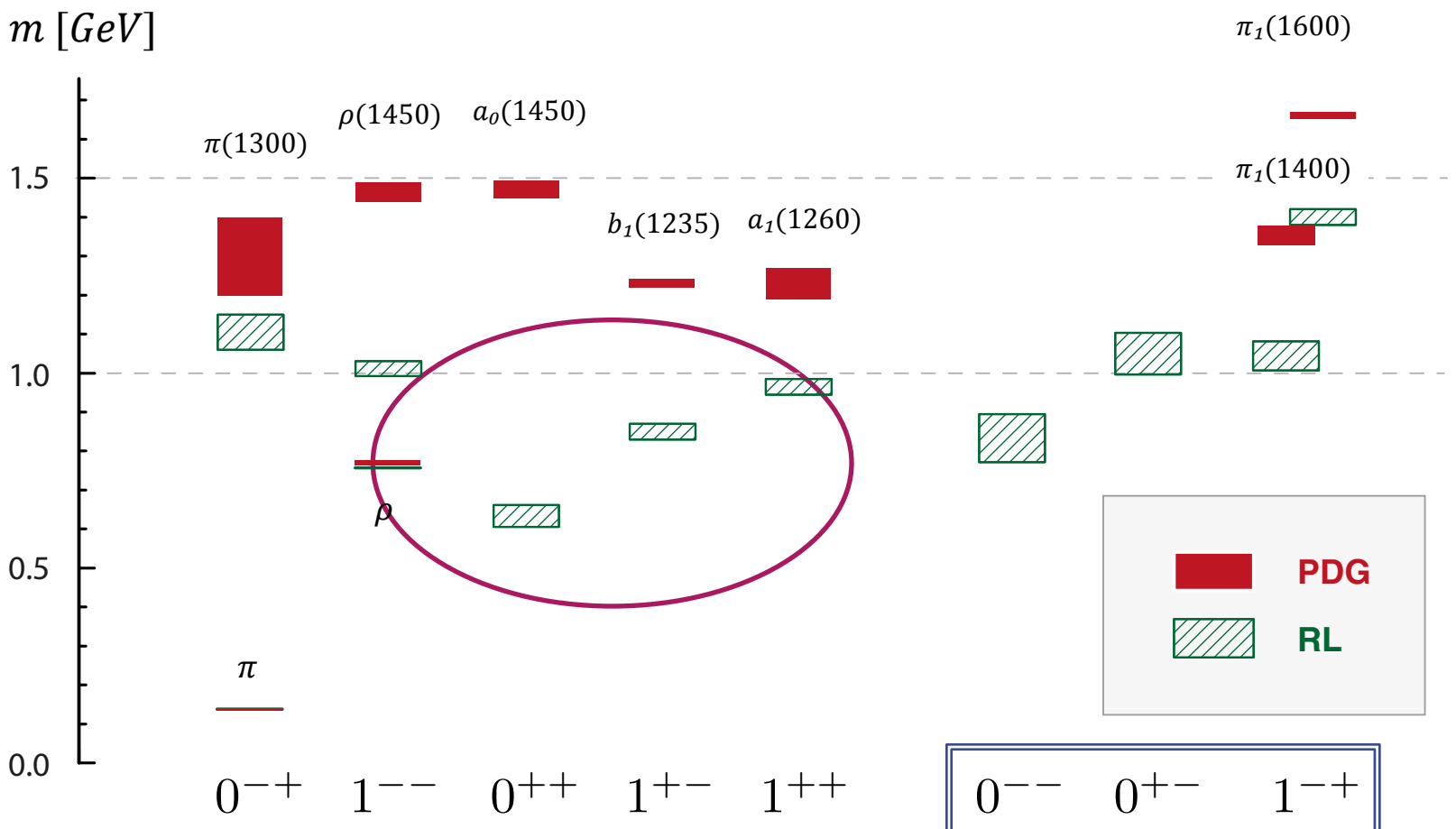
Mesons



Ground/excited state splitting is **good**
Deficiencies in level orderings

[Kubrak, Fischer, RW EPJA 51 (2015) 10]

Mesons

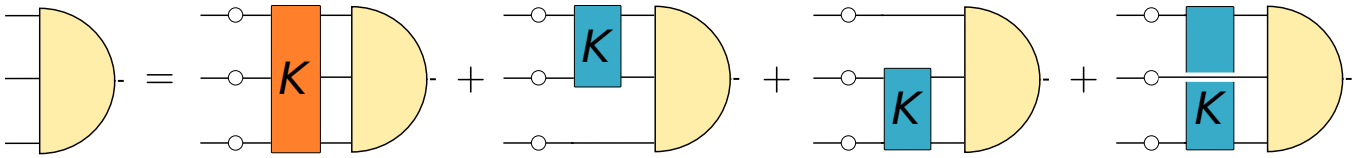


More sensitive to **interaction details**
Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]

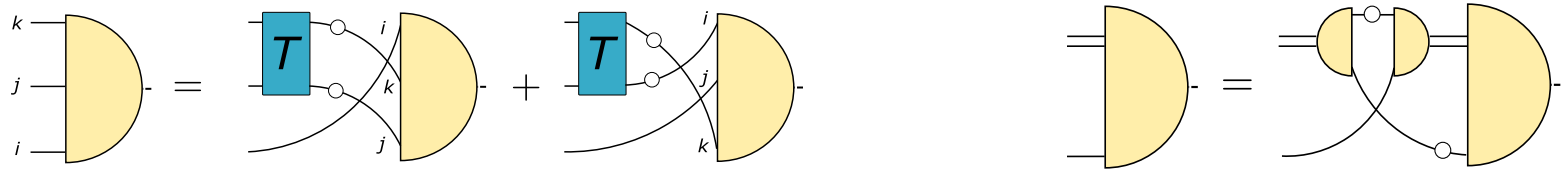
Baryons

Employ the **same interaction kernel**



Can drop the irreducible three-body kernel (calculations suggest small)

Rewrite as **Faddeev equation**

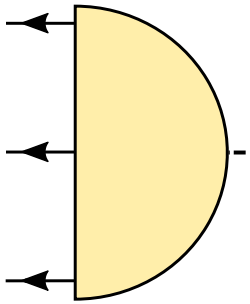


Ansatz of **diquarks** as dynamical correlations (extended objects) yields the **quark-diquark** approximation.

Allows **parallels** to be drawn between meson and diquark spectrum

- Understand the resulting baryon spectrum

Baryons



Generalized **Rarita-Schwinger** projector

$$\mathbb{P}_{(j)}^{\mu_1\cdots\mu_n;\nu_1\cdots\nu_n} = \left(\frac{n+1}{2n+3}\right)\Lambda_+(P)\gamma_T^\mu\gamma_T^\nu P_{(n+1)}^{\mu\mu_1\cdots\mu_n;\nu\nu_1\cdots\nu_n}$$

$$P_S^{\mu_1\cdots\mu_N;\nu_1\cdots\nu_N} = \frac{1}{n!}\sum_P \delta_T^{\mu_1\nu_1}\cdots\delta_T^{\mu_N\nu_N}$$

$$P_{(N)}^{\mu_1\cdots\mu_N;\nu_1\cdots\nu_N} = P_S^{\mu_1\cdots\mu_N;\nu_1\cdots\nu_N} - \frac{1}{2\,(2N-1)!}\sum_P \delta_T^{\mu_1\mu_2} P_S^{\kappa\kappa\mu_3\cdots\mu_N;\nu_1\cdots\nu_N}$$

$$\Gamma^{\alpha\beta\ldots} = \begin{pmatrix} \gamma_5 \mathrm{D}_i\Lambda_\pm\gamma_5 C & \otimes & 1 \\ \mathrm{D}_i\Lambda_\pm\gamma_5 C & \otimes & \gamma_5 \end{pmatrix} \begin{pmatrix} 1 & \otimes & \Gamma_{j,ss\ldots}^{\mu\nu\ldots}\mathbb{P}_+^{\mu\nu..\alpha\beta\ldots} \\ 1 & \otimes & \Gamma_{j,rs\ldots}^{\mu\nu\ldots}\mathbb{P}_+^{\mu\nu..\alpha\beta\ldots} \\ 1 & \otimes & \Gamma_{j,rr\ldots}^{\mu\nu\ldots}\mathbb{P}_+^{\mu\nu..\alpha\beta\ldots} \\ \vdots & \vdots & \vdots \end{pmatrix}$$

Can classify according to **partial-waves**

Baryons

nucleon

	s-wave	p-wave	d-wave
S=1/2	8	24	
S=3/2		12	20

64

delta

	s-wave	p-wave	d-wave	f-wave
S=1/2		24	40	
S=3/2	4	12	20	28

128

spin 5/2

	p-wave	d-wave	f-wave	g-wave
S=1/2		40	56	
S=3/2	12	20	28	36

192

spin 7/2

	d-wave	f-wave	g-wave	h-wave
S=1/2		56	72	
S=3/2	20	28	36	44

256

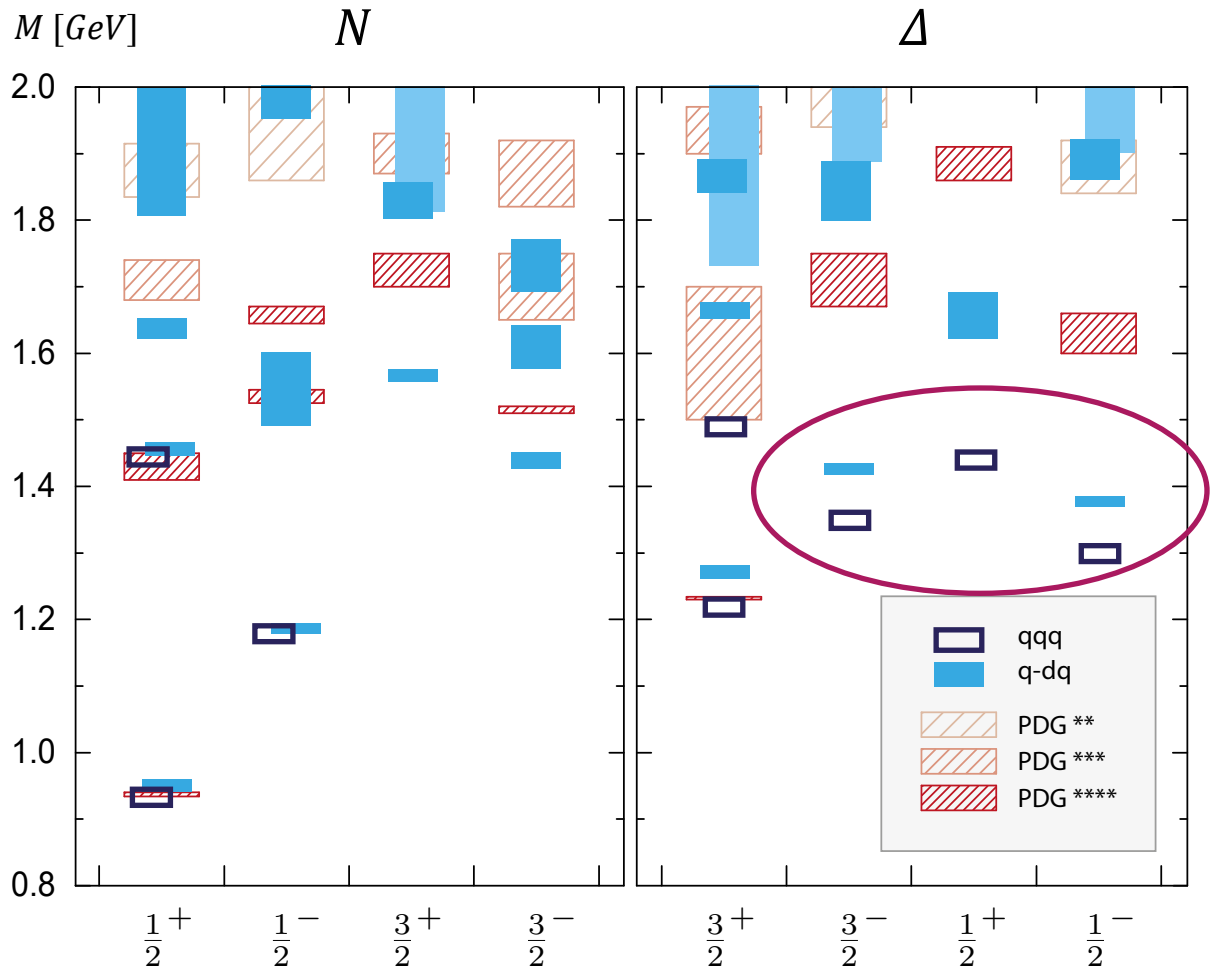
spin 9/2

	f-wave	g-wave	h-wave	i-wave
S=1/2		72	88	
S=3/2	28	36	44	52

320

Baryons

[Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer Prog. Part. Nucl. Phys (in press)]
[Eichmann, Fischer, Sanchis-Alepuz arXiv:1607.05748]



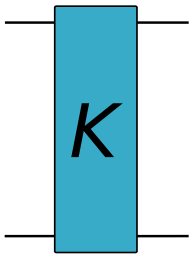
Good **nucleon** and **delta** ground plus excited states

Deficiencies are as expected from **diquark analogues to the mesons**

If we fix mesons, then we fix baryons

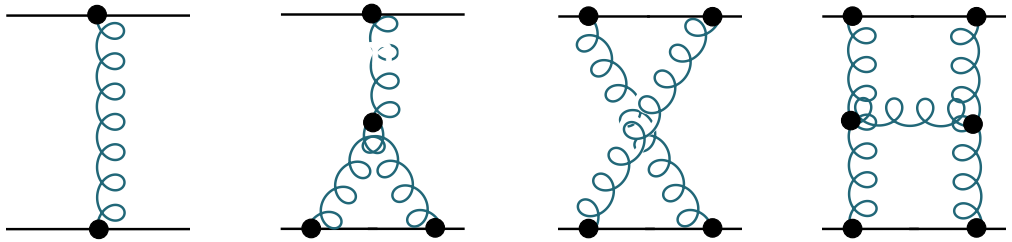
Calculation of Vertices

Beyond rainbow-ladder



Expose additional corrections to the kernel

- use nPI **effective action** techniques

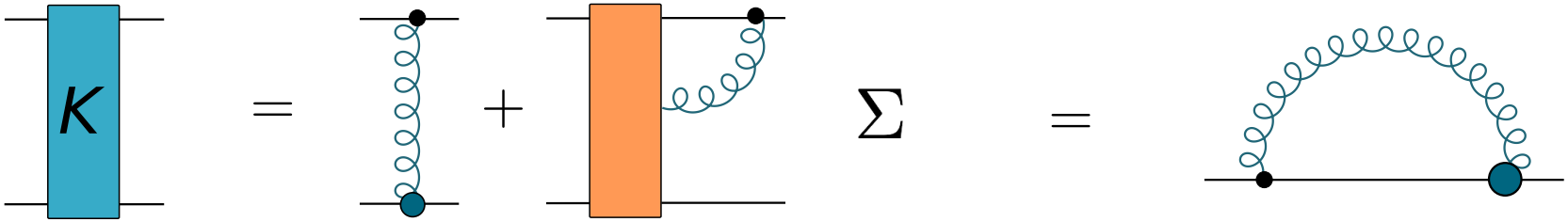


Connected with a vertex expansion (see next two-talks)

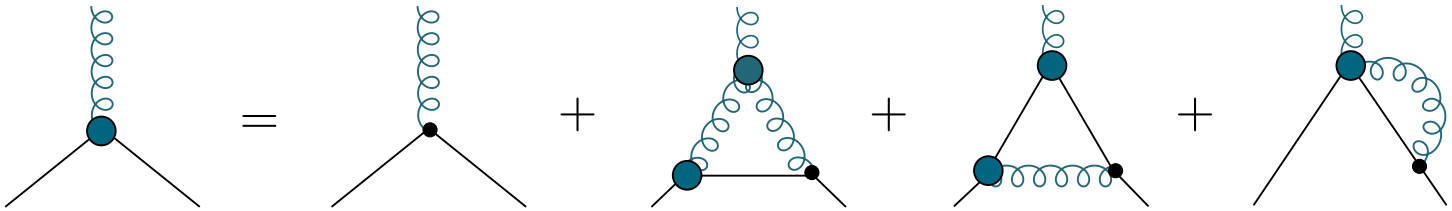
[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]

Closely related to the **2PI effective action:** vertex auxiliarly resummation

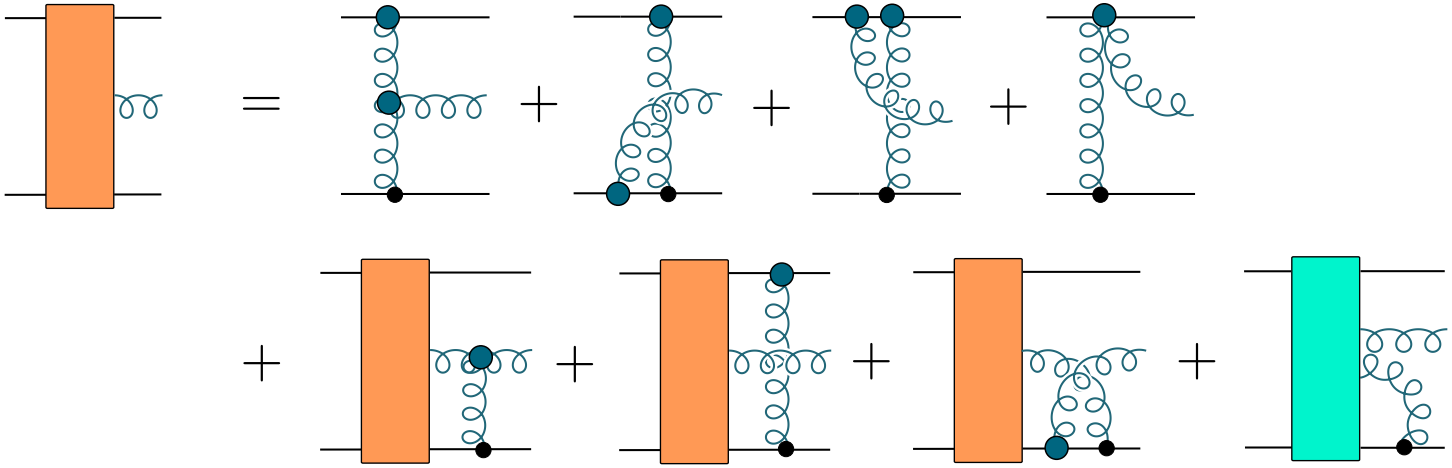
$$K = \left.\frac{\delta^2\Gamma_2[B]}{\delta B\delta B}\right|_{B=S} = \left.\frac{\delta\Sigma[B]}{\delta B}\right|_{B=S} = \frac{\delta\Sigma[S]}{\delta S}$$



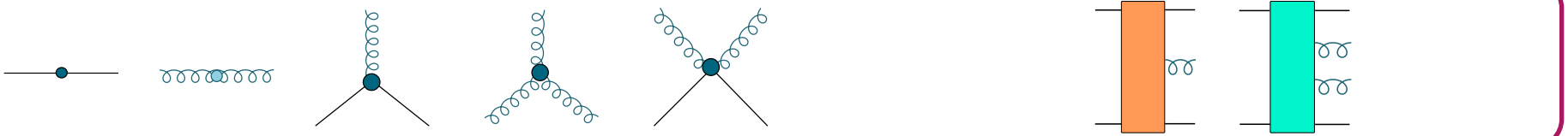
2PI Cutting



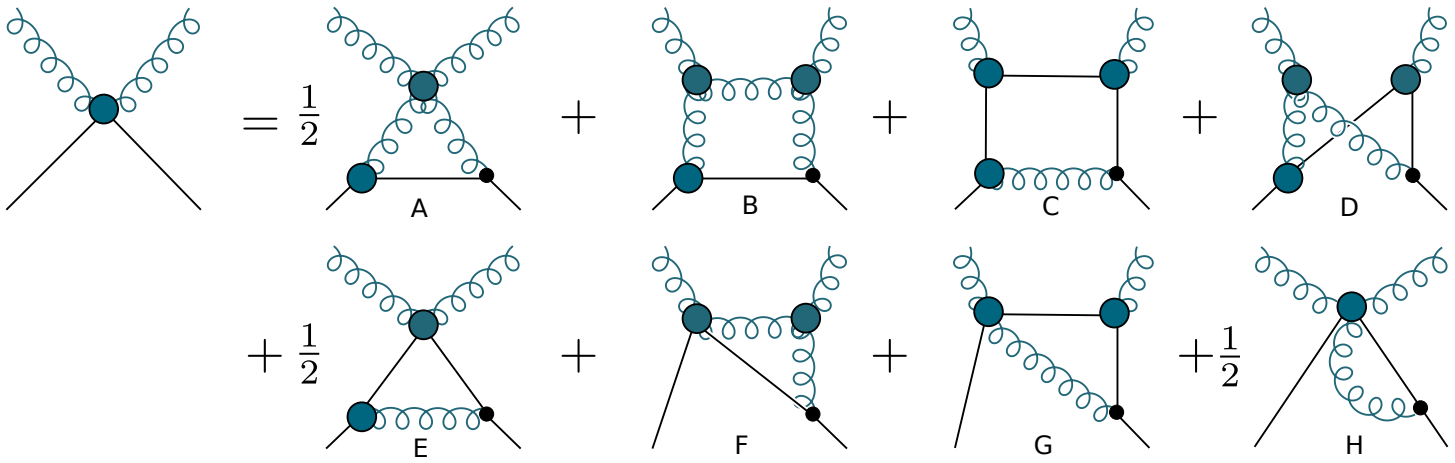
8 functions of 3 variables



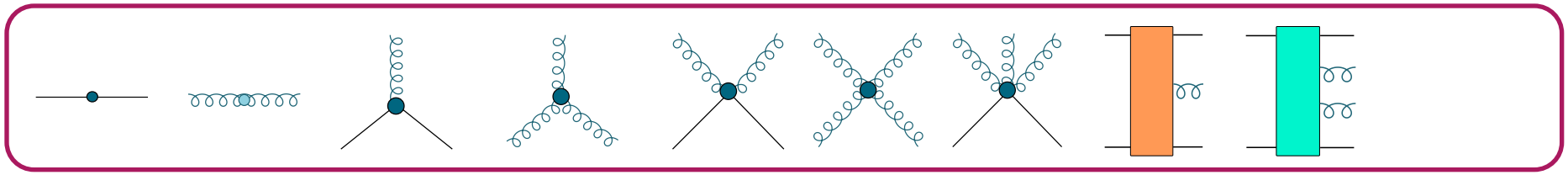
256 functions of 10 variables



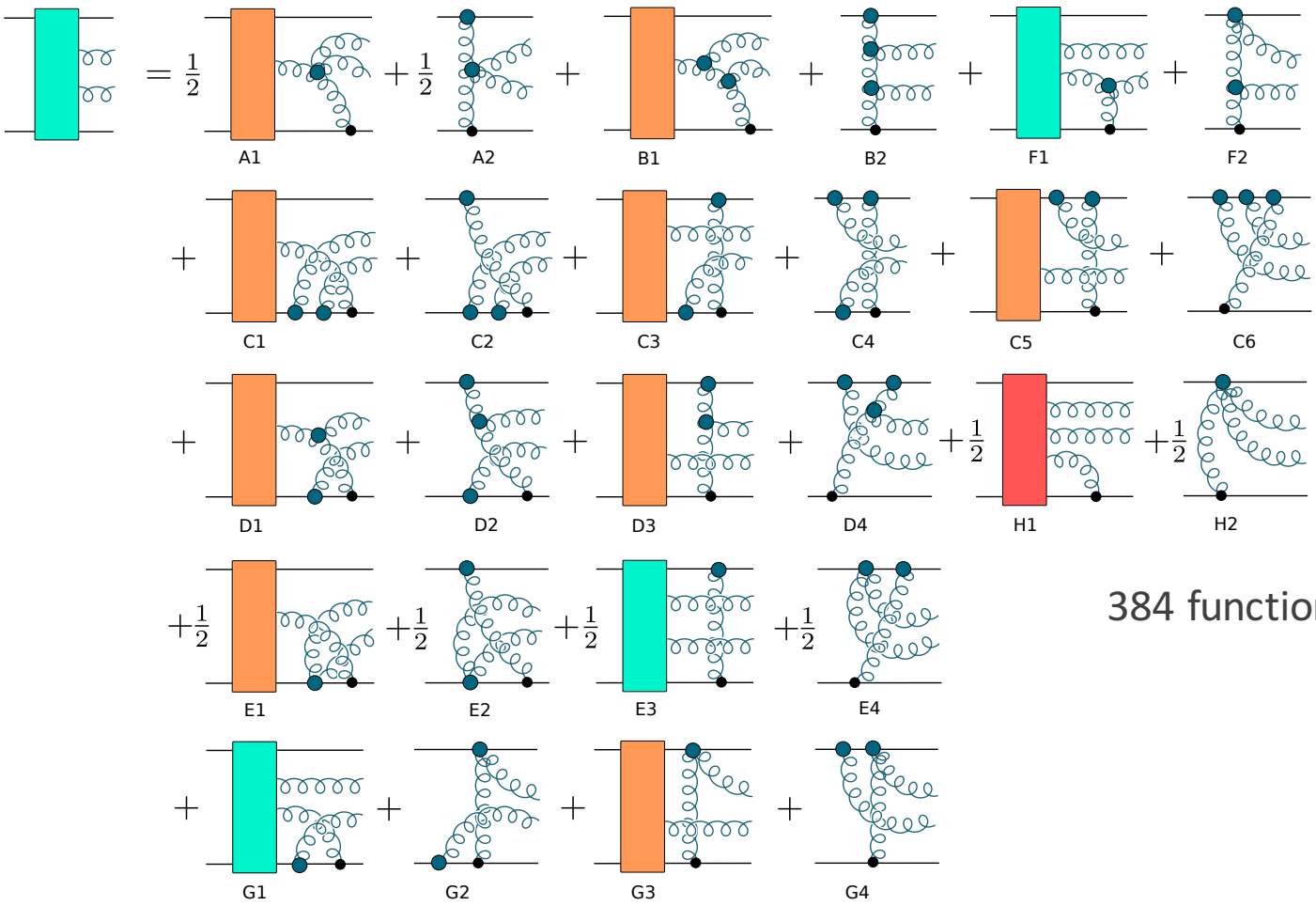
2PI Cutting



72 functions of 6 variables

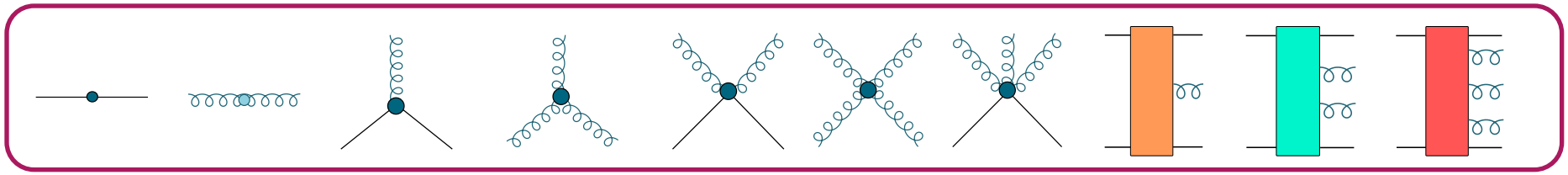


2PI Cutting



384 functions of 15 variables

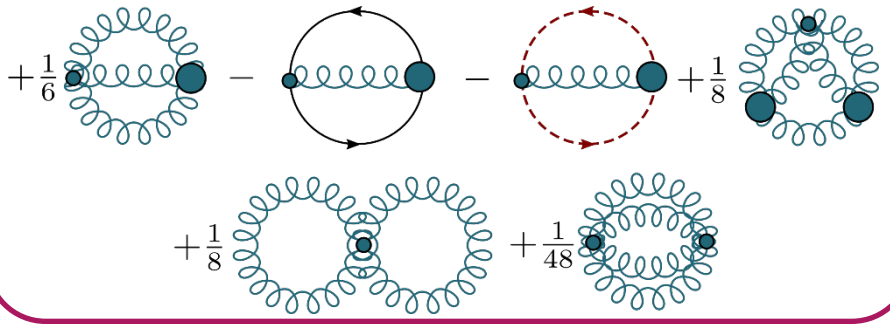
Sigh ☹



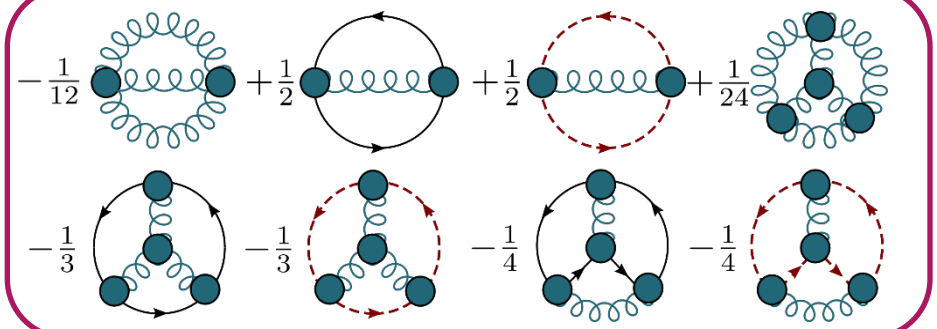
3PI Cutting

$$\Gamma[\phi,D,U] = S_{cl}[\phi] + \frac{i}{2}TrLnD^{-1} + \frac{i}{2}Tr\left[D_{(0)}^{-1}D\right] - i\Phi^{0}[\phi,D,U] - i\Phi^{int}[\phi,D,U] + const.$$

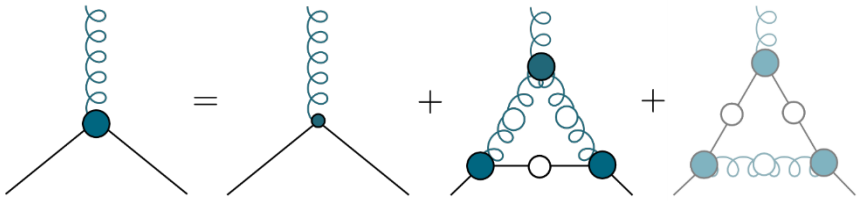
Φ^{0}: non-interacting part

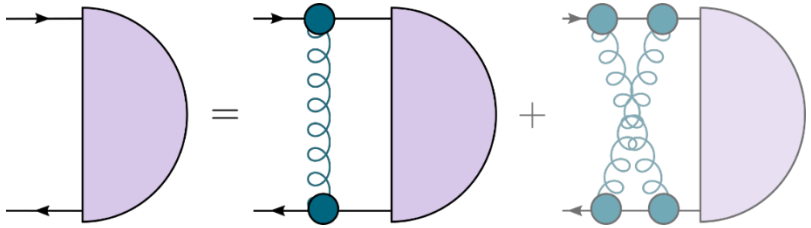


Φ^{int}: interacting part

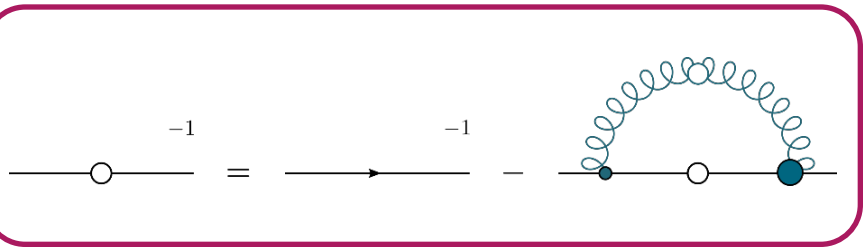


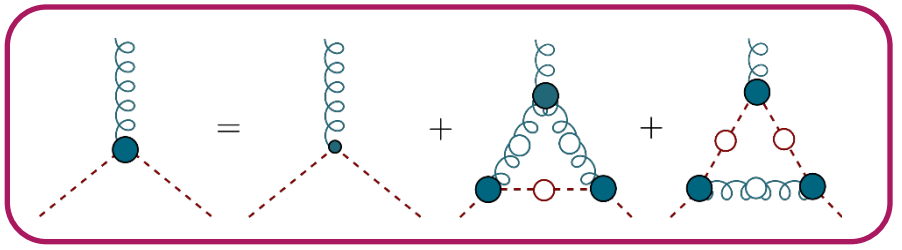
$$K = \left.\frac{\delta^2\Gamma_2[B,U]}{\delta B\delta B}\right|_{B=S,U=V} = \left.\frac{\delta\Sigma[B,U]}{\delta B}\right|_{B=S,U=V} \neq \frac{\delta\Sigma[S,V]}{\delta S}$$

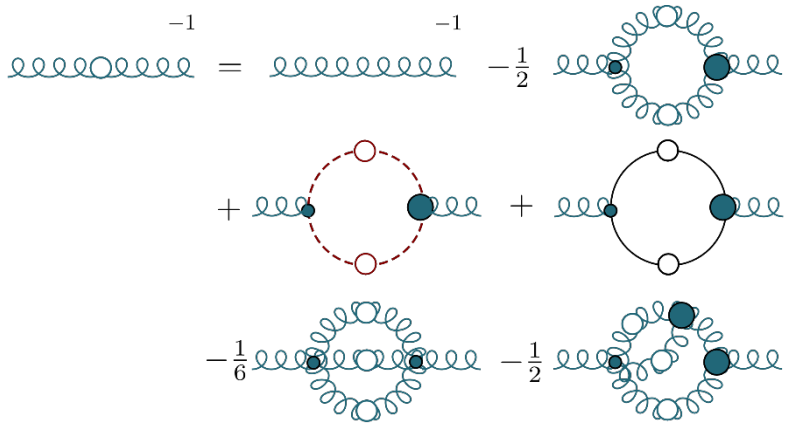


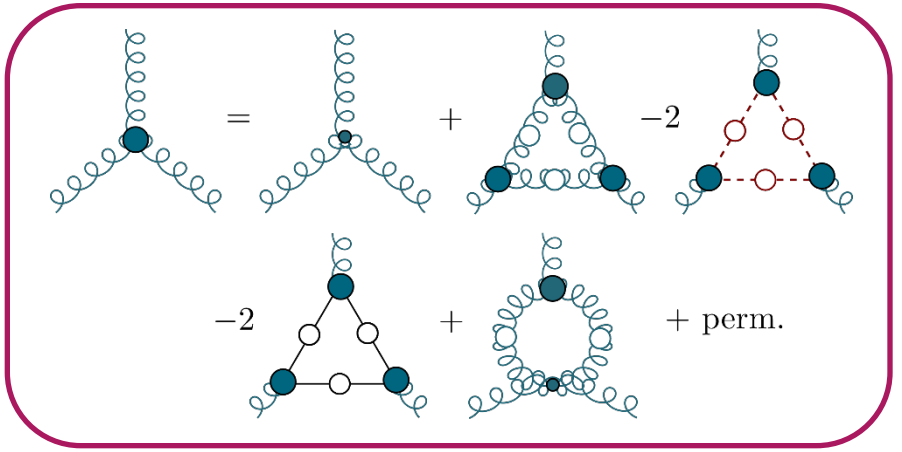


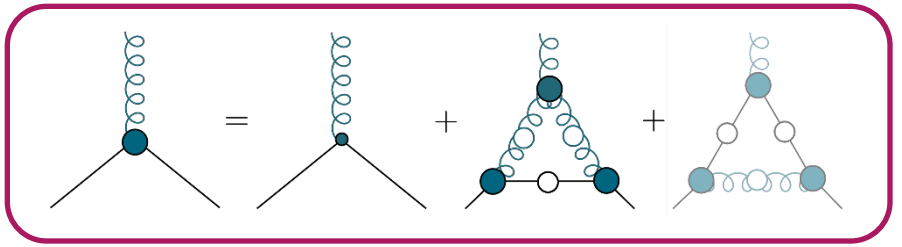
3PI: System of Eqns





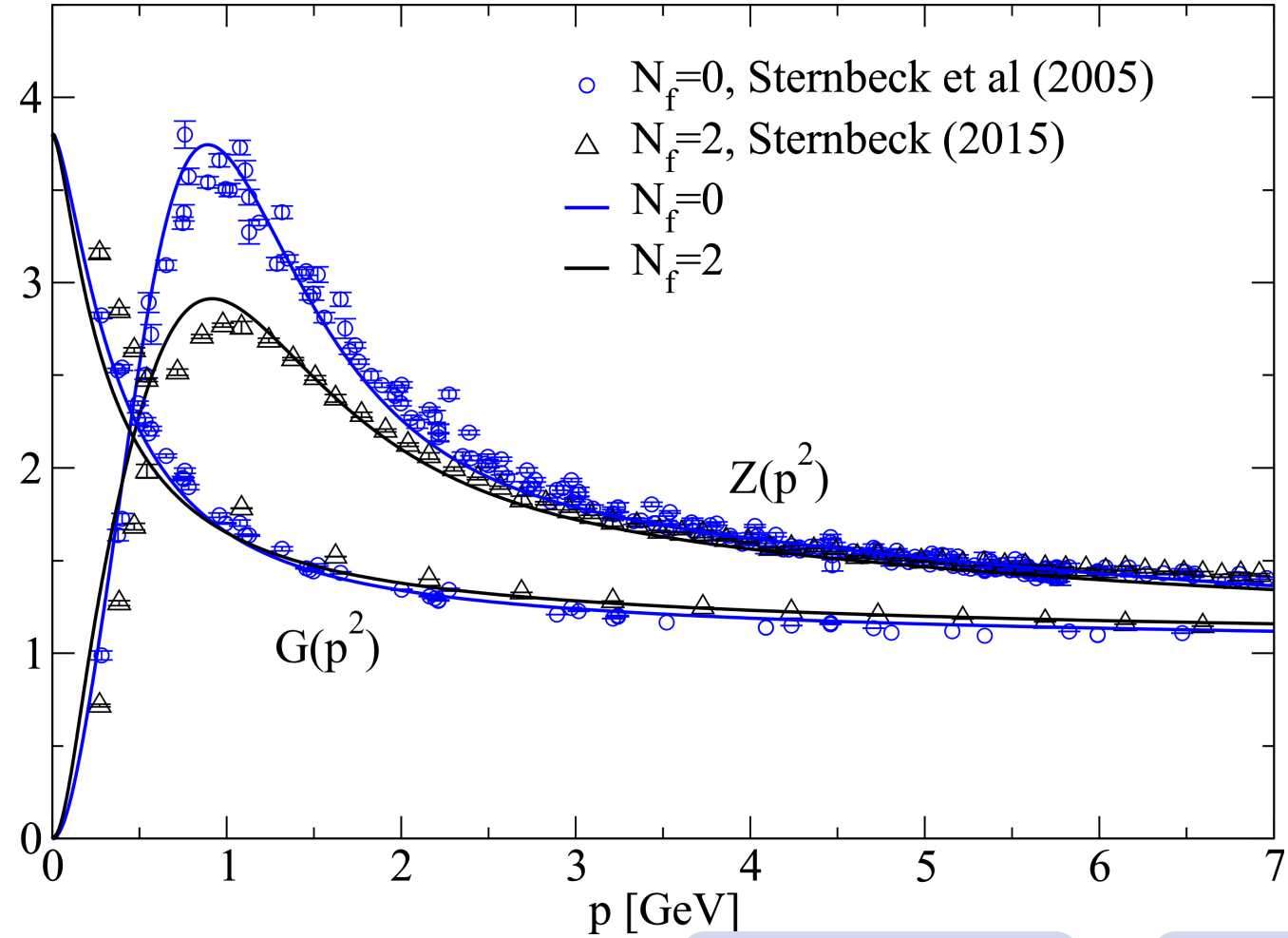






Fixed to gauge-fixed Lattice calculations (thanks: A. Sternbeck)

Ghost and Gluon



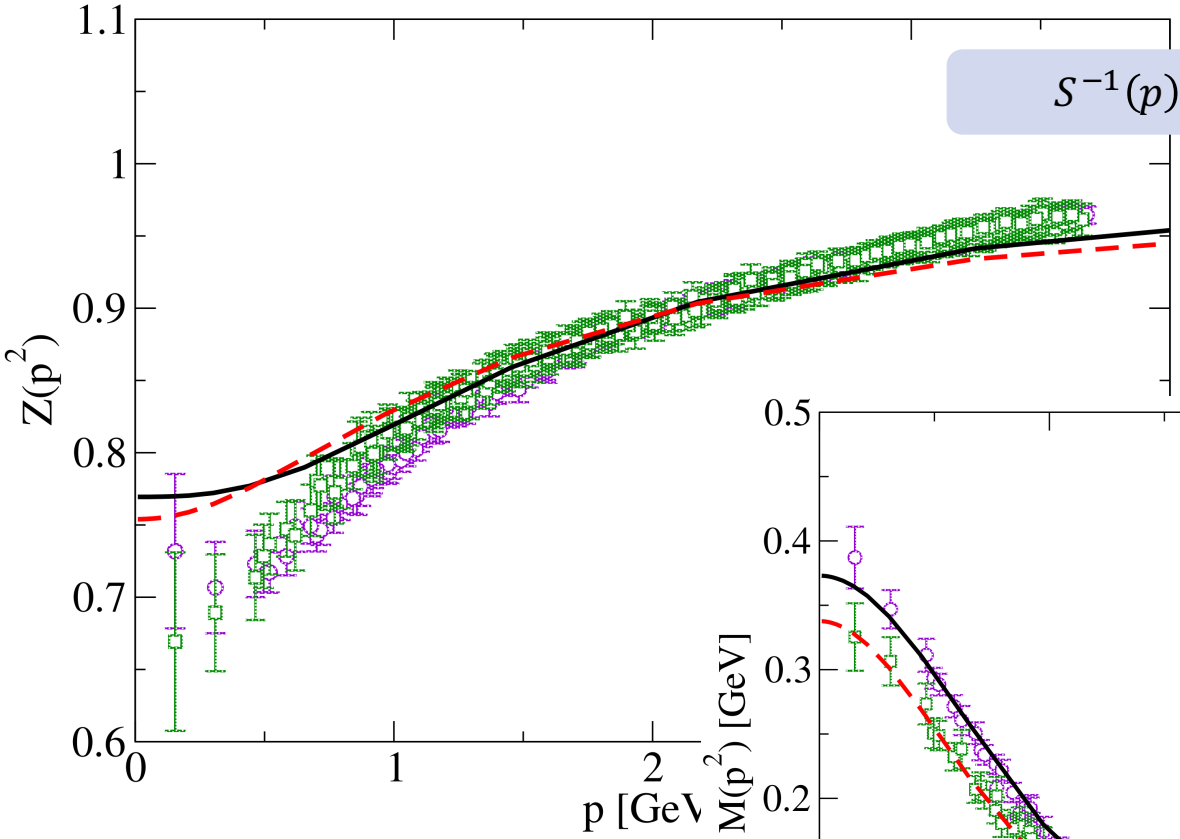
Employ lattice data for ghost/gluon and scale setting

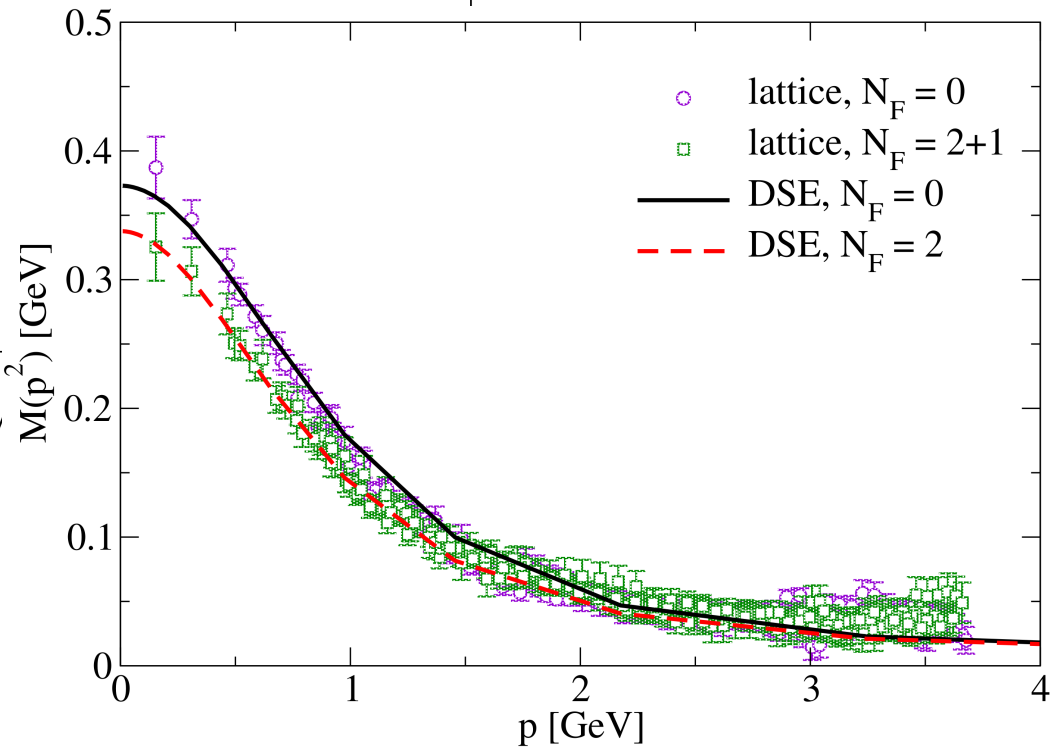
$$D_G(p) = -\frac{G(p^2)}{p^2}$$

$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

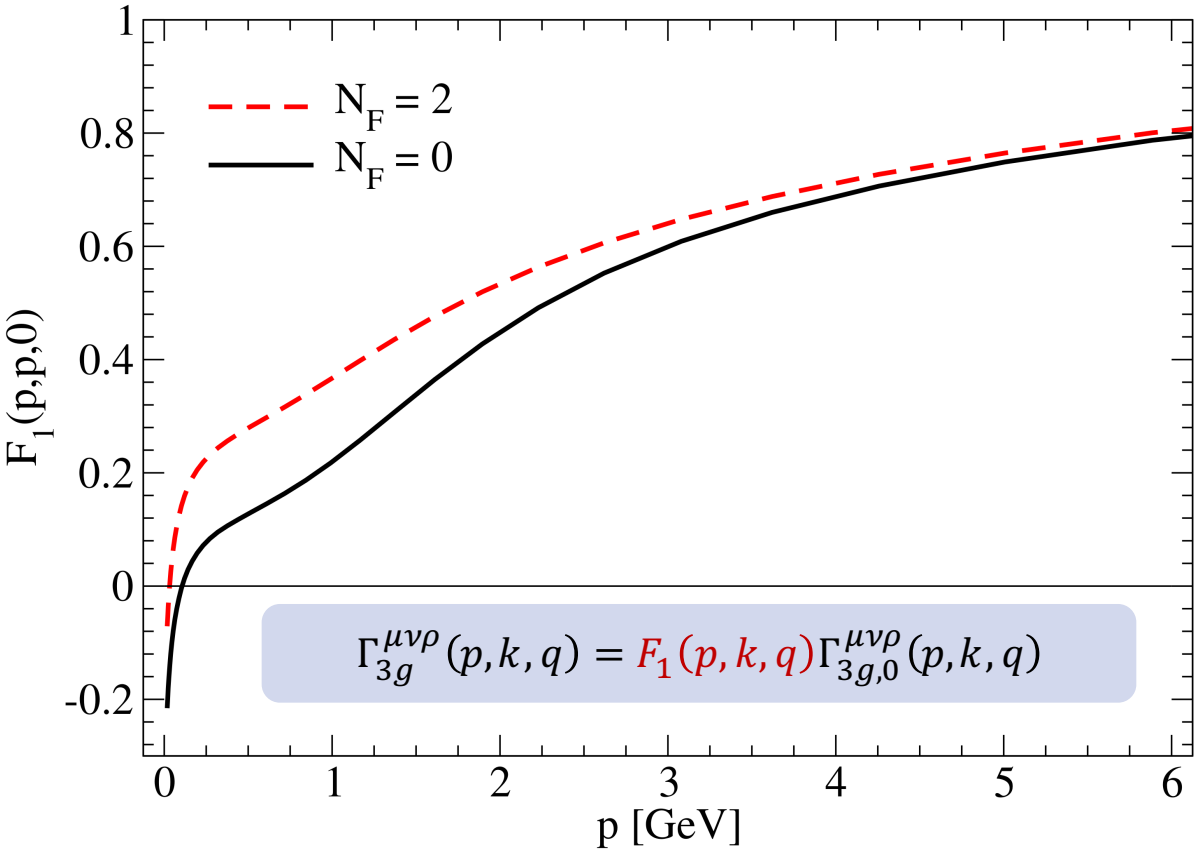
3PI: Quark

$$S^{-1}(p) = A(p^2)(-i\,\gamma\cdot p\ + M(p^2))$$





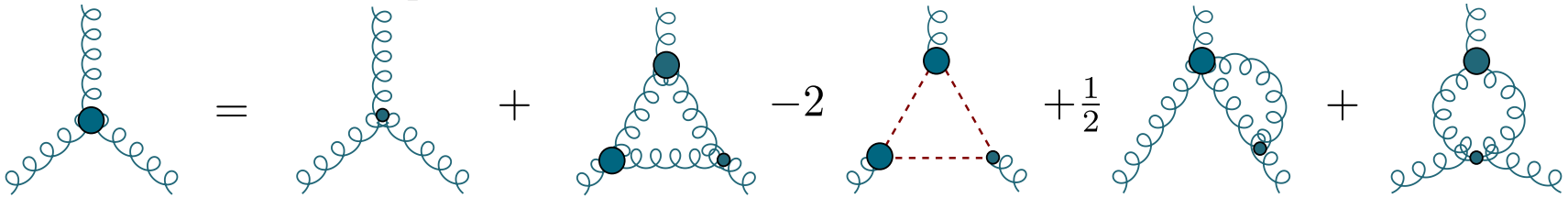
3PI: 3g vertex



unquenching

anti-screening in three-gluon vertex

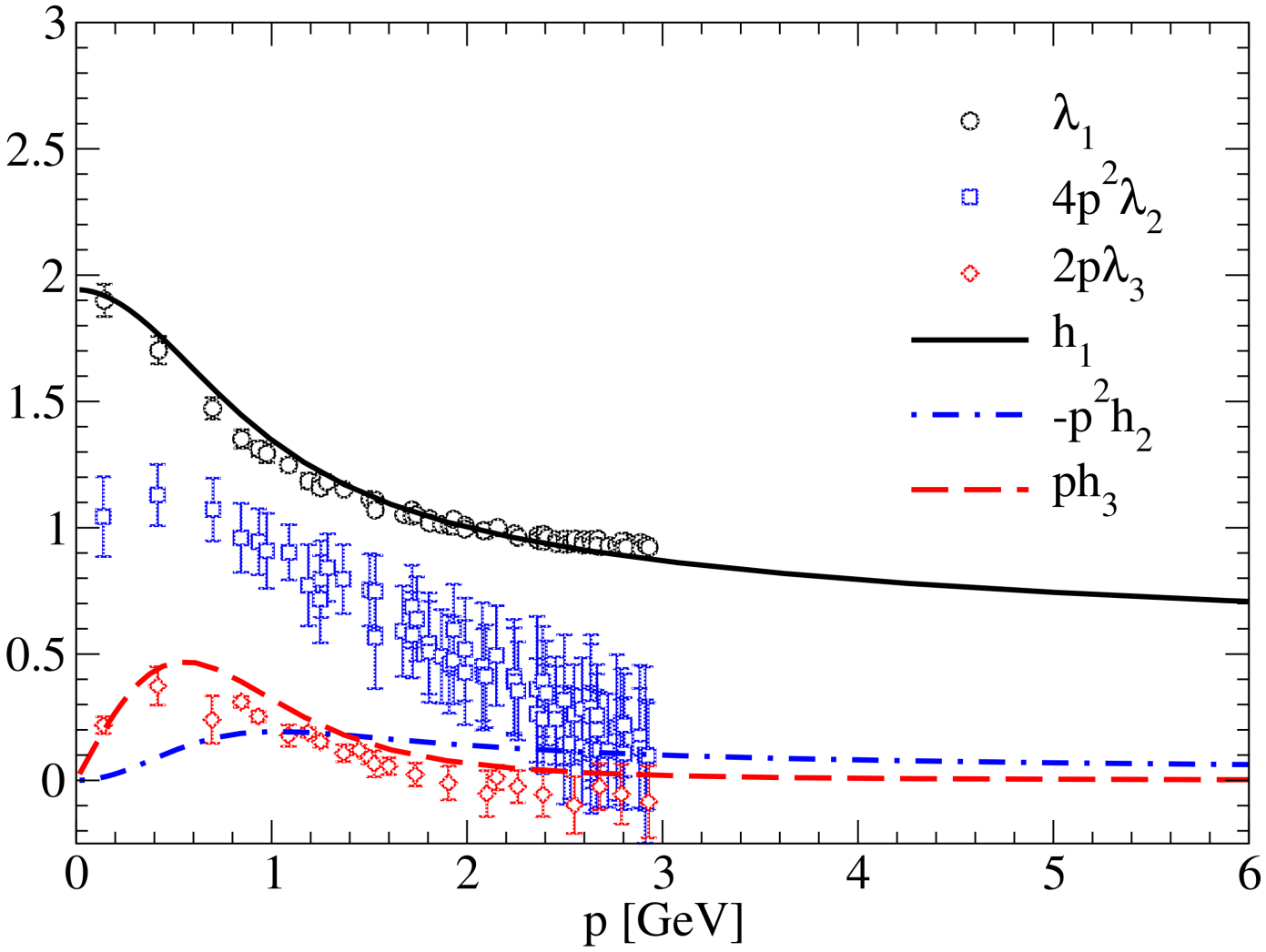
Zero-crossing pushed to deep IR



[Blum, Huber, Mitter, von Smekal PRD 89 (2014) 061703]
[Eichmann, RW, Alkofer, Vujinovic PRD 89 (2014) 105014]

3PI: qg vertex

quenched



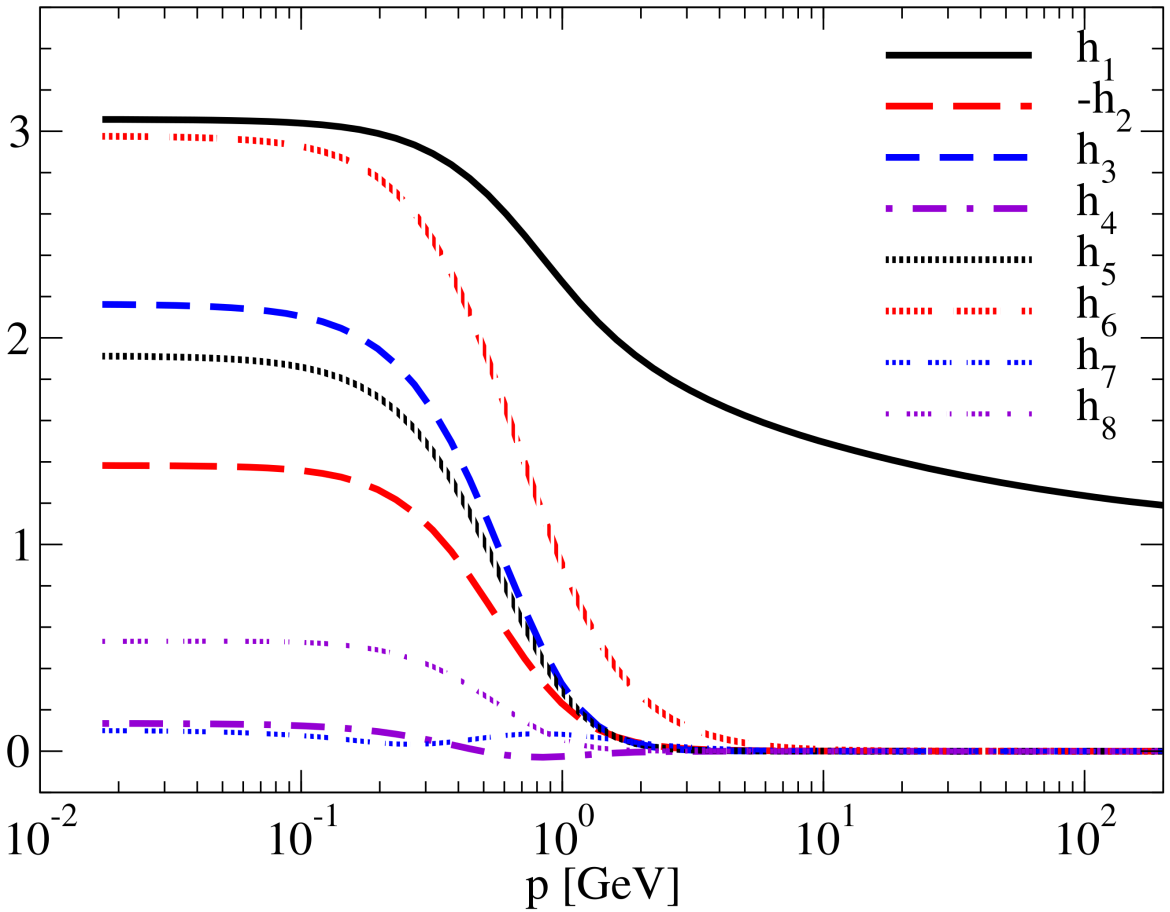
1st and 3rd structures comparable

Remainder hard to extract on lattice

[Skullerud, Bowman, Kizilersu, Leinweber, Williams JHEP 04 (2003) 047]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

3PI: qg vertex

unquenched

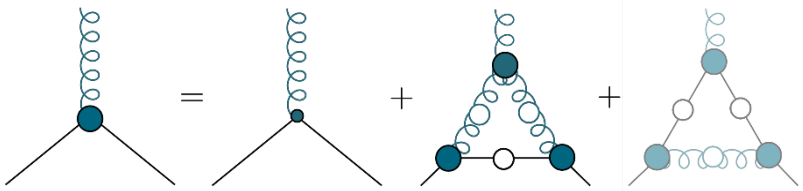


Eight calculated structures

Strong IR enhancements. DCSB present.

Strength/sign dictated by 3PI effective action

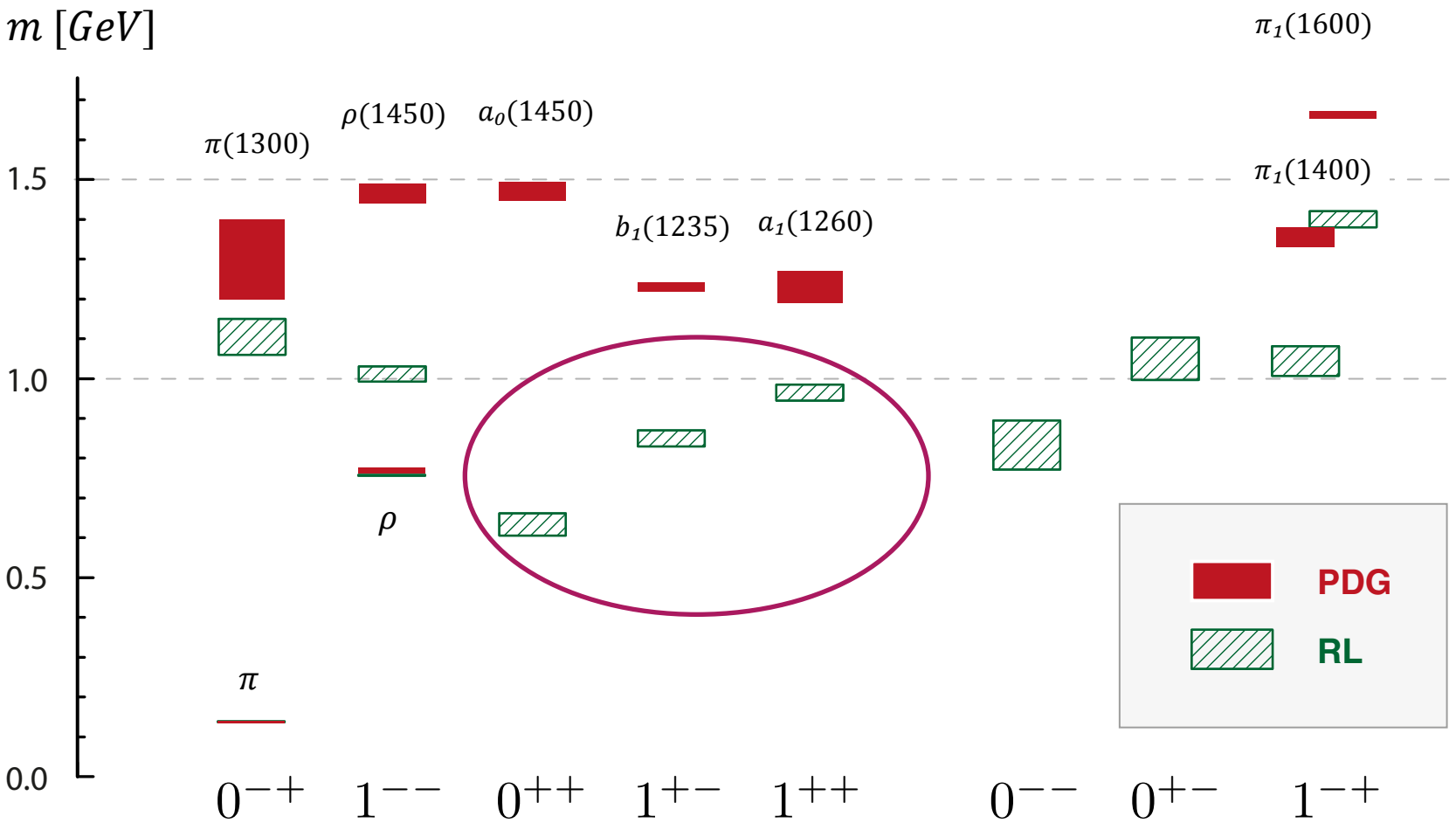
Repulsion/Attraction in different meson/diquark channels



See also: Mitter

[RW, Fischer, Heupel PRD 93 (2016) 034026]

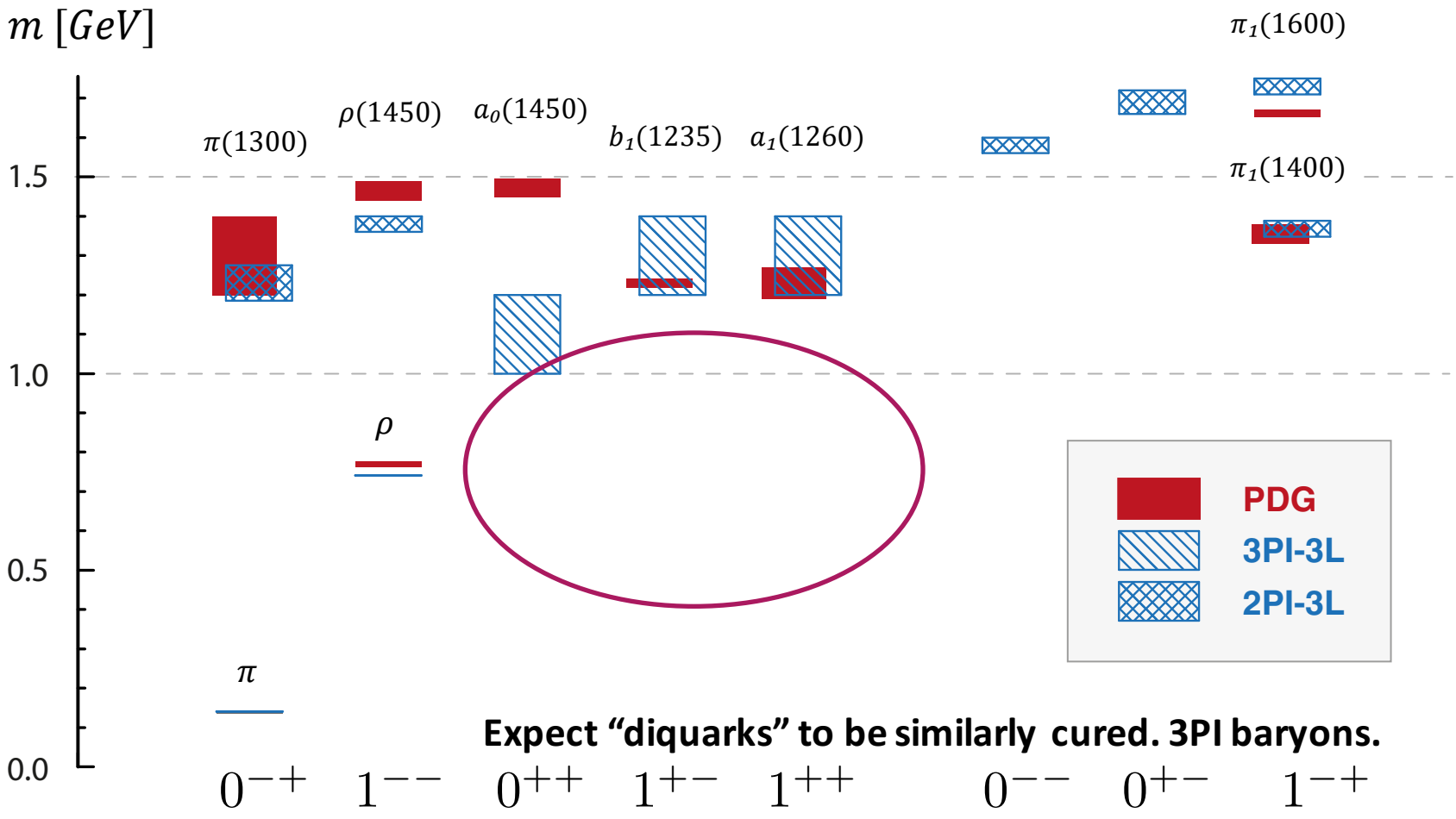
Rainbow-Ladder



- Sensitivity to interaction exasperated in light sector
- Deficiencies in many channels

[Kubrak, Fischer, RW EPJA 50 (2014) 126]

Beyond RL



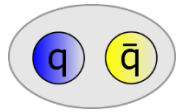
Scalar:	2PI-2L (RL) and 2PI-3L too light	[Chang, Roberts PRC 85 (2012) 052201]
$\rho - a_1$ splitting	2PI-2L (RL) and 2PI-3L too small	[Sanchis-Alepuz, RW PLB 749 (2015) 592]
$a_1 - b_1$ splitting	2PI-2L (RL) and 2PI-3L non-degenerate	[RW, Fischer, Heupel PRD 93 (2016) 034026]

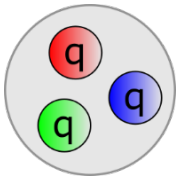
Summary

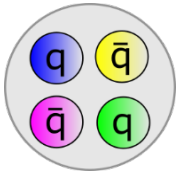
Calculating vertices and constructing Bethe-Salpeter kernels from the 3PI effective action

Improvements in meson/diquark sector

Understand impact on baryon spectrum





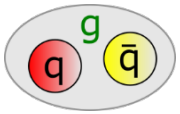


Outlook

Apply 3PI to baryons explicitly

Calculate vertices at finite T/μ_q

Construct new phenomenological model for finite T/μ_q



Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 Prog. Part. Nucl. Phys. (in press)

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Thank you

Outlook

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