# Inverse magnetic catalysis in nonlocal chiral quark models

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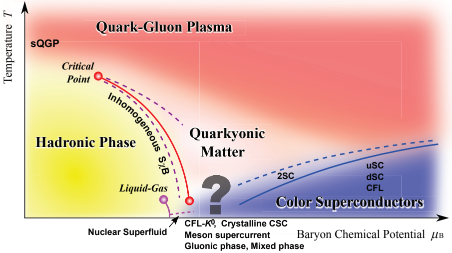
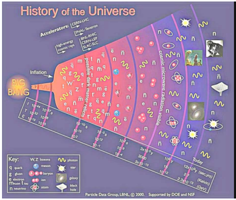
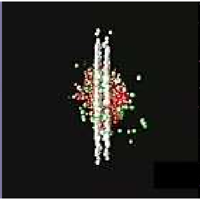
# Motivation

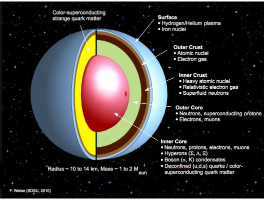


Temperature $T$
Quark-Gluon Plasma
sQGP
Critical Point
Inhomogeneous $S\chi B$
Hadronic Phase
Quarkyonic Matter
Liquid-Gas
2SC
uSC
dSC
CFL
Color Superconductors
Nuclear Superfluid
CFL-$K^0$, Crystalline CSC
Meson supercurrent
Gluonic phase, Mixed phase
Baryon Chemical Potential $\mu_B$

History of the Universe
Accelerators:
Inflation
BIG BANG
Key:
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

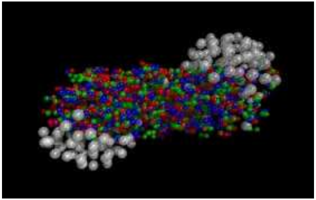



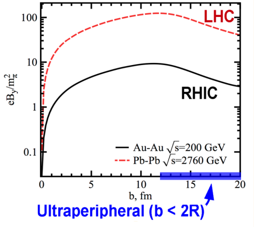


Color-superconducting strange quark matter
Surface
• Hydrogen/Helium plasma
• Iron nuclei
Outer Crust
• Atomic nuclei
• Electron gas
Inner Crust
• Heavy atomic nuclei
• Relativistic electron gas
• Superfluid neutrons
Outer Core
• Neutrons, superconducting protons
• Electrons, muons
Inner Core
• Neutrons, protons, electrons, muons
• Hyperons (Σ, Λ, Ξ)
• Boson (π, K) condensates
• Deconfined (u,d,s) quarks / color-superconducting quark matter
Radius ~ 10 to 14 km, Mass ~ 1 to 2 M sun
F. Weber (SDSU, 2010)

# Motivation: why intense magnetic fields?

Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G

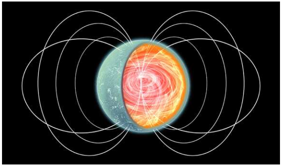





Magnetic field at $T = 0$

(A. Bzdak, V. Skokov (2012))

Magnetars: $B \sim 10^{13} - 10^{15}$ G (Duncan and Thompson (2012))



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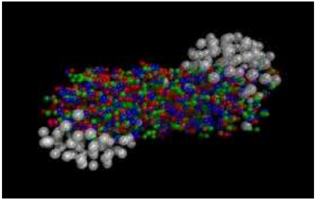
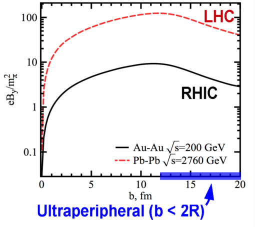
Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G



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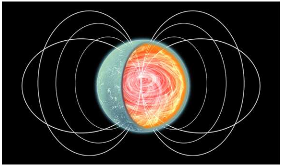


Including a non-zero uniform $B$

- How does QCD phase diagram looks like?
- Are there modifications in the nature of transitions?
- What is the fate of critical endpoints?

From LQCD calculations carried out with physical pion masses

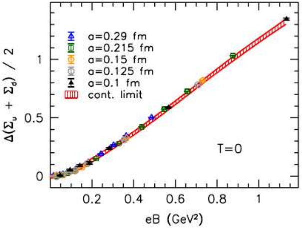
(Bali et al JHEP **1202**, 044 (2012), Phys. Rev. D **86**, 071502 (2012))

- Low temperature: magnetic catalysis

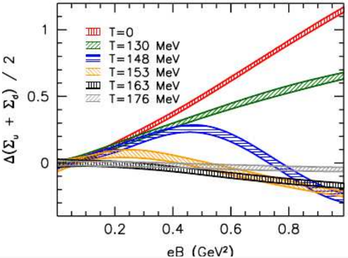
enhancement of the chiral condensate

- Close to the chiral restoration $T_c$:

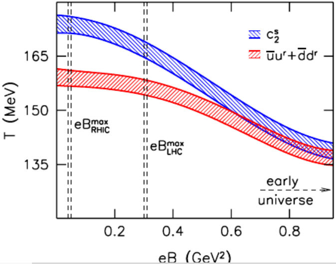
inverse magnetic catalysis




nonmonotonic behavior of light quark condensates




decrease of the $T_c$ with the magnetic field



Many scenarios have been considered to account for the IMC

- LQCD (Bali et al (2013); Bruckmann et al (2013))
- NJL (Kashiwa (2011); Menezes et al(2014); Krein et al (2014); Fayazbakhsh, Sadoogui (2014); Ferrer et al(2015); Mao(2016))
- Linear Sigma Model (Loewe et al (2014/15))
- PQM, QM models (Fraga et al (2014); Andersen, Naylor, Tranberg (2015))
- Holographic approach (Rougemont, Critelli, Noronha (2016)))
- Other effective models (Fraga, Noronha, Palharares (2013); Chao, Chu, Huang (2013); Fukushima, Hidaka (2013); Mueller, Pawlowsky (2015))

However, the mechanism is not yet fully understood.

# NJL Model

Model with chiral quark interaction (Nambu, Jona-Lasinio (1961))
Simplest version: **Scalar** and **pseudoscalar** interactions.

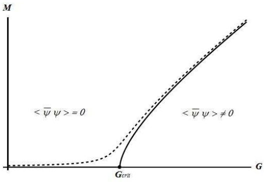
$$\mathcal{L}_{NJL} = \bar{\psi}(i\not{\partial} - m_c)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \qquad \psi^\dagger = (u, d)$$

In the mean field approximation (MFA):

$$(\bar{\psi}\mathcal{O}\psi)^2 \rightarrow 2\langle\bar{\psi}\mathcal{O}\psi\rangle\bar{\psi}\mathcal{O}\psi$$

Dynamical fermion mass generation:

$$M = m_c - 2G\langle\bar{\psi}\psi\rangle \qquad G > G_{crit} \Rightarrow \langle\bar{\psi}\psi\rangle \neq 0$$




Parameters:

- Current quark mass $m_c$
- Coupling constant $G$ (4 fermions)
- Cutoff $\Lambda$

Chosen to reproduce:

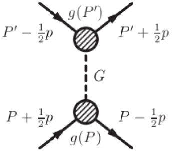
- $m_\pi = 139$ MeV
- $f_\pi = 92.4$ MeV
- Constituent mass $M_0 \sim 300 - 400$ MeV

# non-local NJL Model

$$S_E = \int d^4x \left\{ \bar{\psi}(x) \left( -i\partial\!\!\!/ + m_c \right) \psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\}$$

$$j_a(x) = \int d^4z \, \mathcal{G}(z) \, \bar{\psi}(x + \frac{z}{2}) \, \Gamma_a \, \psi(x - \frac{z}{2})$$

$\Gamma_a = (\mathbb{1}, i\gamma_5\vec{\tau})$, $\mathcal{G}(z)$ nonlocal form factor




- models effective OGE
- leads to momentum dependence in quark propagators
- finite to all orders in the loop expansion $\rightarrow$ no extra cutoffs
- properties of light mesons at zero and finite $T$ and/or $\mu$

# Coupling to the magnetic field

$$\partial_\mu \to \partial_\mu - i\,\hat{Q}\mathcal{A}_\mu(x), \qquad \hat{Q} = \mathrm{diag}(q_u, q_d), \qquad \mathcal{A}_\mu = B\,x_1\,\delta_{\mu 2}$$

The inclusion of gauge interactions implies a change in the nonlocal currents

$$\psi(x-z/2) \to W\,(x, x-z/2)\;\psi(x-z/2) \qquad W(s,t) \;=\; \mathrm{P}\;\exp\left[-i\hat{Q}\int_s^t dr_\mu\;\mathcal{A}_\mu(r)\right]$$

We bosonize the action

$$S_{\mathrm{bos}} = -\ln\det\mathcal{D} + \frac{1}{2G}\int d^4x\Big[\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\Big]$$

$$\mathcal{D}\left(x+\frac{z}{2}, x-\frac{z}{2}\right) \;=\; \gamma_0\,W\left(x+\frac{z}{2}, x\right)\gamma_0\left[\delta^{(4)}(z)\left(-i\partial\!\!\!/ + m_c\right) + \mathcal{G}(z)\left[\sigma\,(x) + i\vec{\tau}\cdot\vec{\pi}\,(x)\right]\right]W\left(x, x-\frac{z}{2}\right)$$

## MFA action per unit volume

$$\frac{S_{\rm bos}^{\sf MFA}}{V^{(4)}} = \frac{\bar\sigma^2}{2G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{d^2\bar p}{(2\pi)^2} \left\{ \ln\left[\bar p^2 + \left(M_{\bar p,0}^{s_f,f}\right)^2\right] + \sum_{k=1}^{\infty} \ln\left[\left(2k|q_f B| + \bar p^2 + M_{\bar p,k}^{-1,f} M_{\bar p,k}^{+1,f}\right)^2 + \bar p^2 \left(M_{\bar p,k}^{+1,f} - M_{\bar p,k}^{-1,f}\right)^2\right] \right\}$$

$$M_{\bar p,k}^{\lambda,f} = (-1)^{k-\frac{1-\lambda s_f}{2}} \int_0^\infty dr\, r\ \exp(-r^2/2) \left[m_c + \bar\sigma\, g\left(\frac{|q_f B|}{2} r^2 + \bar p^2\right)\right] L_{k-\frac{1-\lambda s_f}{2}}(r^2)$$

## Extension to finite temperature

$$\int \frac{d^2\bar p}{(2\pi)^2} F(\bar p^2) \to T \sum_{n=-\infty}^{\infty} \int \frac{dp_3}{2\pi} F(\bar p_n^2)$$

$\bar p_n = (p_3, \omega_n) \qquad \omega_n = (2n+1)\pi T$ Matsubara frequencies

$$\Omega^{MFA}(\sigma; B, T) \quad \rightarrow \quad \text{gap equation:} \quad \frac{\partial \Omega^{MFA}(\sigma; B, T)}{\partial \bar{\sigma}} = 0$$

$$\langle \bar{q}_f q_f \rangle_{B,T} = \frac{\partial \Omega}{\partial m_c} \qquad \text{divergent!}$$

$$\langle \bar{q}_f q_f \rangle^{\text{reg}}_{B,T} = \langle \bar{q}_f q_f \rangle_{B,T} - \langle \bar{q}_f q_f \rangle^{\text{free}}_{B,T} + \langle \bar{q}_f q_f \rangle^{\text{free,reg}}_{B,T}$$

To make contact with LQCD:

$$\Sigma^f_{B,T} = \frac{2\,m_c}{S^4} \left[ \langle \bar{q}_f q_f \rangle^{\text{reg}}_{B,T} - \langle \bar{q}_f q_f \rangle^{\text{reg}}_{0,0} \right] + 1, \quad S = (135 \times 86)^{1/2} \text{MeV}$$

$$\Delta \Sigma^f_{B,T} = \Sigma^f_{B,T} - \Sigma^f_{0,T}$$

$$\Delta \bar{\Sigma}_{B,T} = (\Delta \Sigma^u_{B,T} + \Delta \Sigma^d_{B,T})/2$$

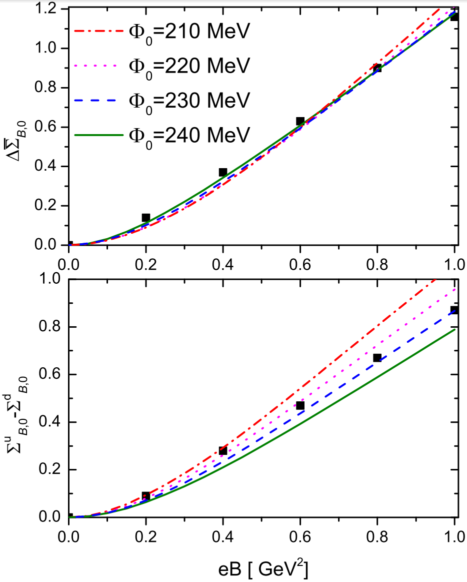
# Condensates at T=0

Gaussian form factor

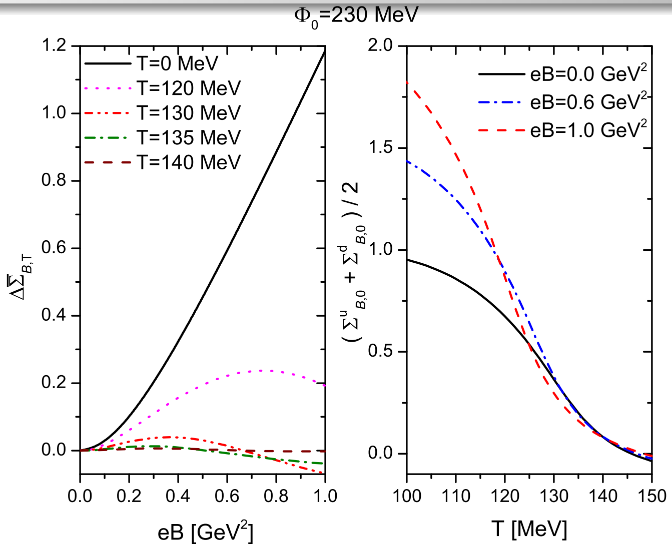
$$g(p^2) = \exp(-p^2/\Lambda^2)$$

$$\Phi_0 = -\langle \bar{q}_f q_f \rangle^{\rm reg}_{0,0}$$

(Squared dots from Bali et al Phys. Rev. D **86**, 071502 (2012))

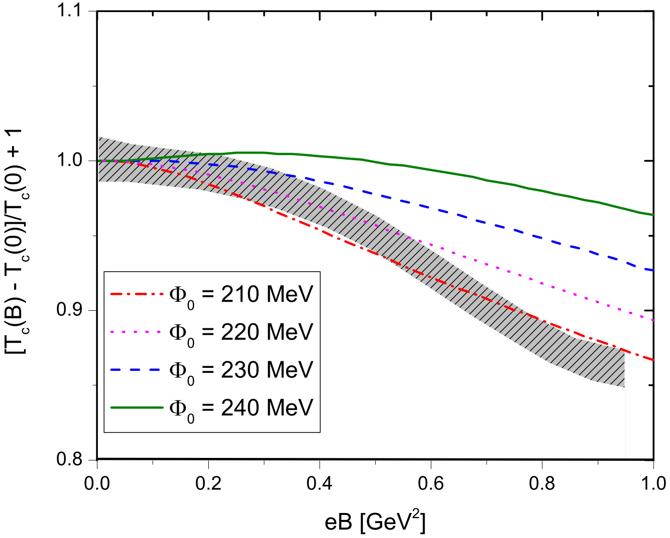


# Condensates at finite T




IMC is naturally displayed!

$Tc(eB = 0) = 129.8$ MeV (higher if coupling to PL is included)




beyond $eB \simeq 0.4$ GeV$^2$ $\rightarrow$ decrease of $T_c$

IMC is observed for all parametrizations!

# Limit of low B: magnetic susceptibility of the vacuum

Tensor polarization operator

$$\langle \bar{\psi}_f \, \sigma_{\mu\nu} \, \psi_f \rangle_A \; = \; q_f F_{\mu\nu} \tau_f \, , \quad \sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$$

$$\tau_f \; = \; 4N_c \int \frac{d^4p}{(2\pi)^4} \, Z(p) \, \frac{M_f(p) - p^2 dM_f(p)/dp^2}{[p^2 + M_f(p)^2]^2}$$

$M_f(p)$ effective mass, $Z(p)$ WFR.

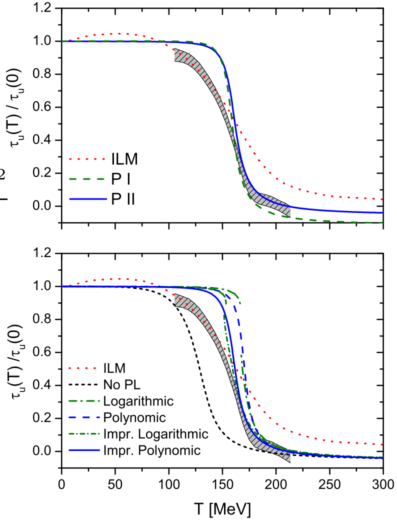
- Form factors

PI: gaussian

PII: Lorenztian (fitted to LQCD)

- Coupling to the PL, with different

PL thermodynamic potentials




V.P. et al Phys. Rev. D **94** (2016), 054038

# Summary and outlook

In the context of a nonlocal extension of the NJL model under an external homogeneous magnetic field...

- We observe at $T = 0$ the expected MC effect, with results in good quantitative agreement with LQCD.
- In contrast to what happens in the standard local NJL model, our results naturally lead to the IMC effect.

Work in progress...

- Extension to other form factors shape.
- Inclusion of WFR.
- Coupling to the Polyakov loop.