

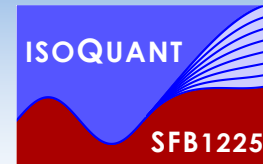
In-medium heavy quarkonium spectral properties from lattice QCD effective field theories

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in collaboration with **S. Kim (Sejong-U.)** and **P. Petreczky (BNL)**

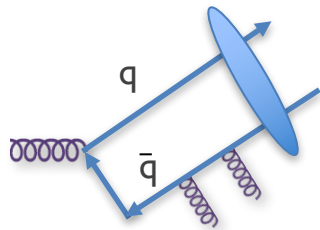
References:

with S.Kim and P. Petreczky PRD91 (2015) 054511, NPA956 (2016) 713
arXiv:1704.05221



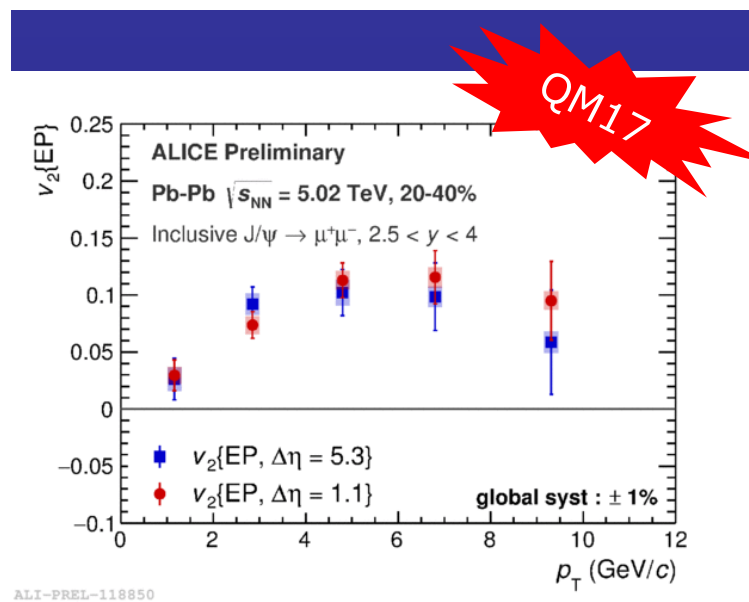
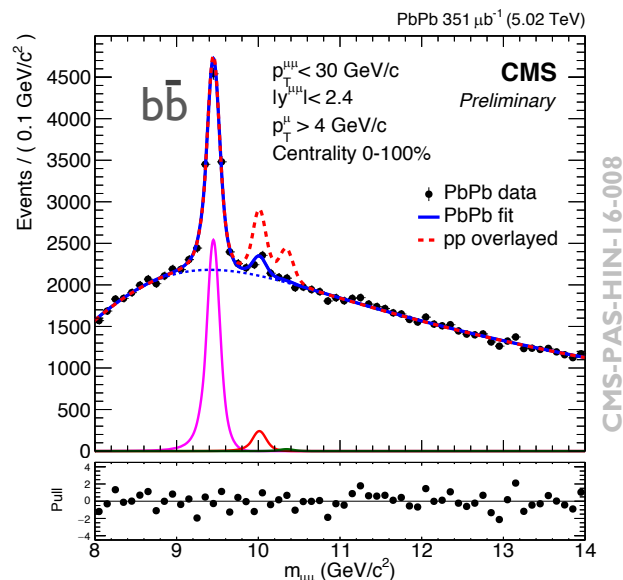
Motivation: Heavy-Ion Collisions

- Hard probes: susceptible to medium but distinguishable from it $Q_{\text{probe}} > T_{\text{med}}$

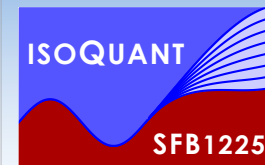


Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q > T_{\text{med}}$

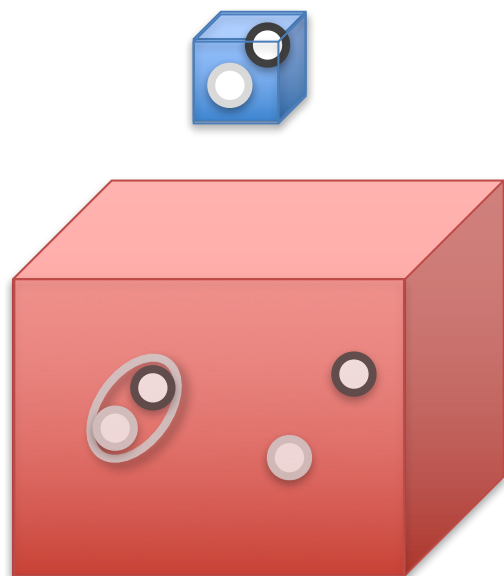
In vacuum: $m^\Upsilon = 9.460 \text{ GeV}$, $\Gamma^\Upsilon = 54(1) \text{ keV}$; $m^{J/\psi} = 3.096 \text{ GeV}$, $\Gamma^{J/\psi} = 93(3) \text{ keV}$



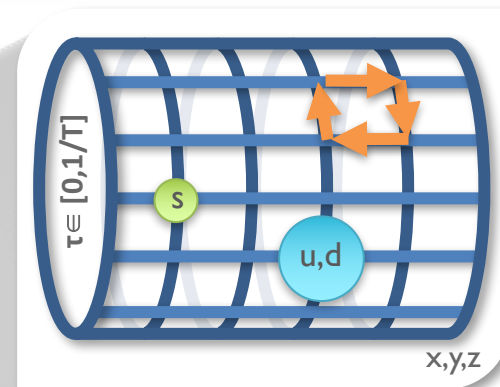
- Goal: First principles insight into heavy quarkonium in heavy-ion collisions



A first-principles scenario



thermal medium
from lattice QCD



Realistic ensembles by HotQCD

$48^3 \times 12$ $m_\pi = 161 \text{ MeV}$ $T_{PC} = 159 \text{ MeV}$

Fixed box: $\beta = 6.664 - 7.825$

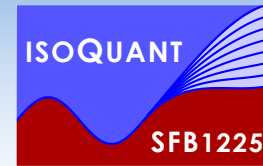
$T = [140 - 407] \text{ MeV}$

$T=0$ configs for calibration $48^3 \times 48, 64$

HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503

T. Matsui and H. Satz: Phys.Lett. B178 (1986) 416

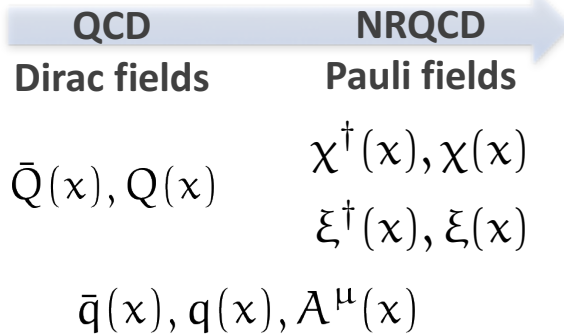
Kinetically equilibrated heavy quarks
presence of in-medium bound eigenstates?
answered by inspecting spectral functions



Heavy Quarks on the Lattice

- Effective field theory from scale separation: $\frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{\mathbf{p}}{m_Q} \ll 1$

Relativistic thermal field theory



$$L_{\text{NRQCD}} =$$

$$\chi^\dagger (iD_t + \frac{D_i^2}{2M_Q} + \dots) \chi + \xi^\dagger (\dots) \xi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{q} (\dots) q$$

- Individual Q or anti-Q in a medium background: Initial value problem $G(\tau) = \langle \chi(\tau) \chi^\dagger(0) \rangle$

$$G(\mathbf{x}, \tau + a) = U_4^\dagger(\mathbf{x}, \tau) \left(1 - \frac{\mathbf{p}_{\text{lat}}^2}{4M_Q a n} + \dots \right)^n G(\mathbf{x}, \tau)$$

Davies, Thacker Phys.Rev. D45 (1992) adaptive discretization in time with Lepage parameter n

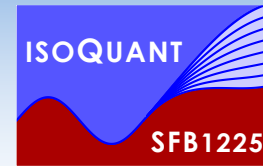
- 3S_1 (Υ) and 3P_1 (χ_{b1}) channel correlators $D(\tau)$ from products of heavy quark propagators $G(\tau)$

$$D(\tau) = \sum_{\mathbf{x}} \langle O(\mathbf{x}, \tau) G_{\mathbf{x}\tau} O^\dagger(\mathbf{x}_0, \tau_0) G_{\mathbf{x}\tau}^\dagger \rangle_{\text{med}} \quad O(^3S_1; \mathbf{x}, \tau) = \sigma_i, \quad O(^3P_1; \mathbf{x}, \tau) = \overleftrightarrow{\Delta}_i \sigma_j - \overleftrightarrow{\Delta}_j \sigma_i$$

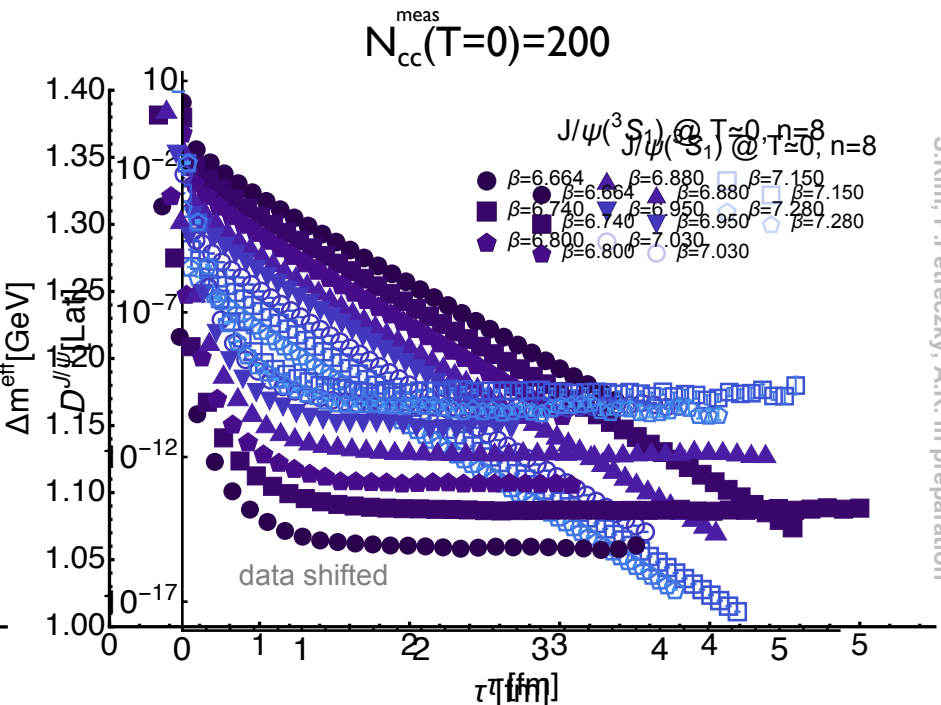
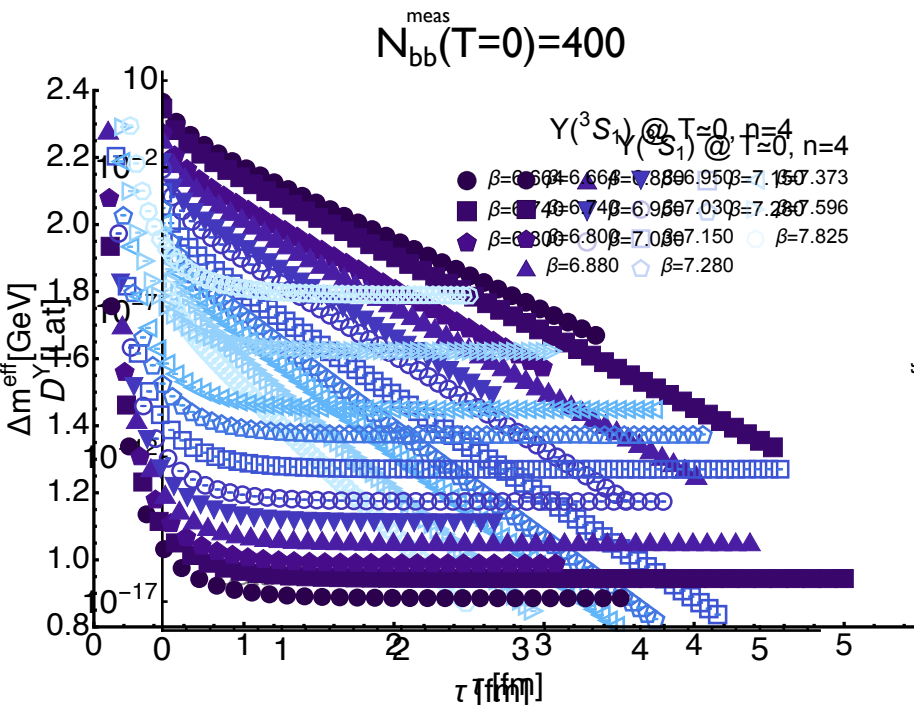
Thacker, Lepage Phys.Rev. D43 (1991)

- Applicability of NRQCD differs for heavier and lighter flavors (bb: n=4, cc:n=8)

$$M_{b,c} a = [2.759 - 0.954] - T = [140 - 407] \text{ MeV} \quad M_c a = [0.757 - 0.42] - T = [140 - 251] \text{ MeV}$$

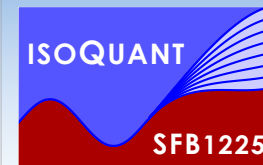


T=0 correlators in NRQCD



For both Bottomonium and Charmonium clear ground state signal at T=0

S.Kim, P.Petreczky, A.R. in preparation



Accessing spectral functions

- Inversion of Laplace transform required to obtain spectra from correlators

$$D(\mathbb{D})_i = \sum_{\omega=1}^{N_\omega} \exp[-\omega \tau] \rho_i^\omega \Delta(\omega)$$

1. N_ω parameters $\rho_i \gg N_\tau$ datapoints
2. data D_i has finite precision

- Give meaning to problem by incorporating prior knowledge: Bayesian approach

M. Jarrell, J. Gubernatis, Physics Reports 269 (3) (1996)

- Bayes theorem: Regularize the naïve χ^2 functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D, I] \propto P[D|\rho] P[\rho|I]$$

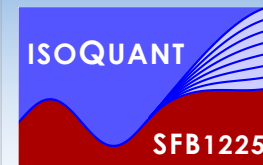
- New prior enforces: ρ positive definite, smoothness of ρ , result independent of units

$$P[\rho|I] \propto e^S \quad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log \left[\frac{\rho_l}{m_l} \right] \right)$$

Y. Burnier, A.R.
PRL 111 (2013) 18, 182003

- **Different from Maximum Entropy Method:** S not entropy, no more flat directions

$$\left. \frac{\delta}{\delta \rho} P[\rho|D, I] \right|_{\rho=\rho^{BR}} = 0$$



An improved Bayesian strategy

- First level: improve the data on which inverse problem is based

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega) \quad \longleftrightarrow \text{Fourier} \quad D(\mu) = \int_0^{\infty} d\omega \frac{2\omega}{\omega^2 + \mu^2} \rho(\omega)$$

- Improvement:** incorporate both Euclidean and imaginary frequency data in unfolding
- Second level: develop improved regulators to better assess systematics

Standard BR method (BRFT)

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log \left[\frac{\rho}{m} \right] \right)$$

- Resolves narrow peaked structures with high accuracy
- Ringing in broad structures if reconstructed from small # of datapoints

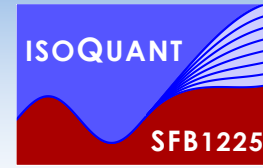
„high gain – high noise“

New low ringing BR method

$$S_{BR}^{lr} = \alpha \int d\omega \left(\left(\frac{\partial \rho}{\partial \omega} \right)^2 + 1 - \frac{\rho}{m} + \log \left[\frac{\rho}{m} \right] \right)$$

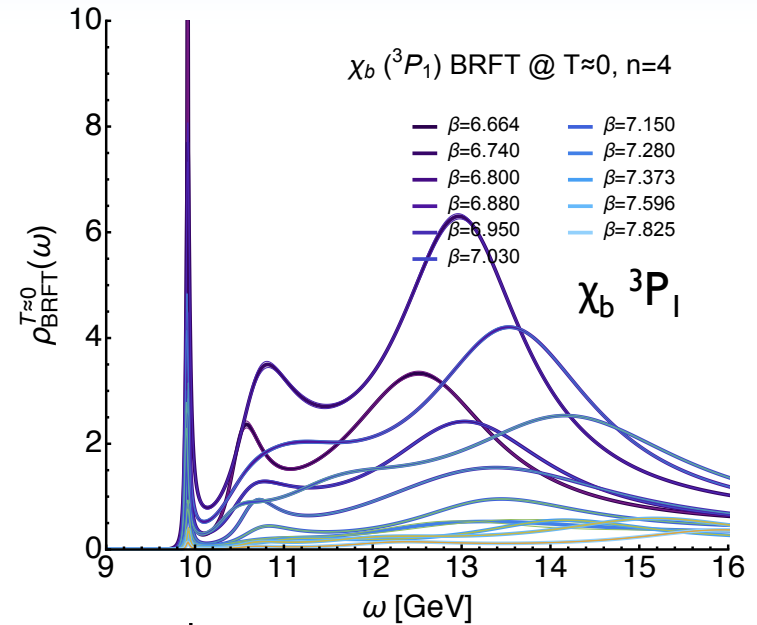
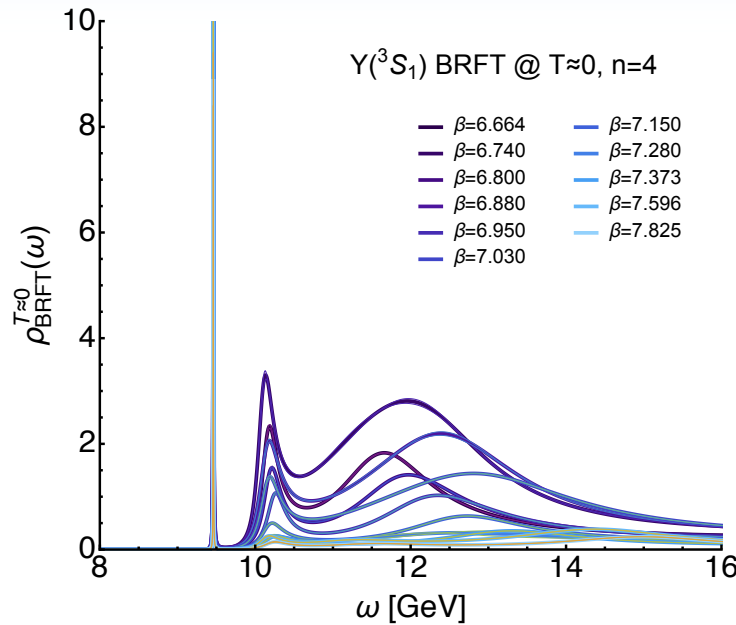
- Introduces penalty on arc length of reconstruction $(dL/d\omega)^2 = 1 + (d\rho/d\omega)^2$
- Efficiently removes ringing but may lead to overestimated peak widths

„low gain – low noise“



Calibrating Bayesian spectra at T=0

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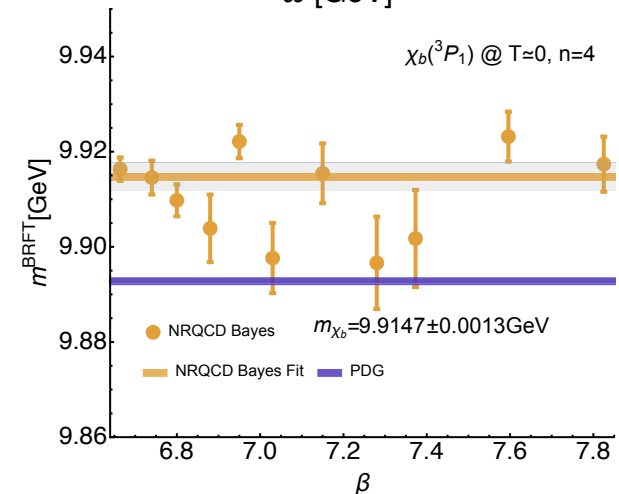


m_Y from PDG to calibrate absolute freq. scale

Check systematic error of lattice computation by postdiction of P-wave ground state mass

$$M_{\chi_{b1}}^{\text{NRQCD}} = 9.9147(13) \text{ GeV} \quad M_{\chi_{b1}}^{\text{PDG}} = 9.89278(3) \text{ GeV}$$

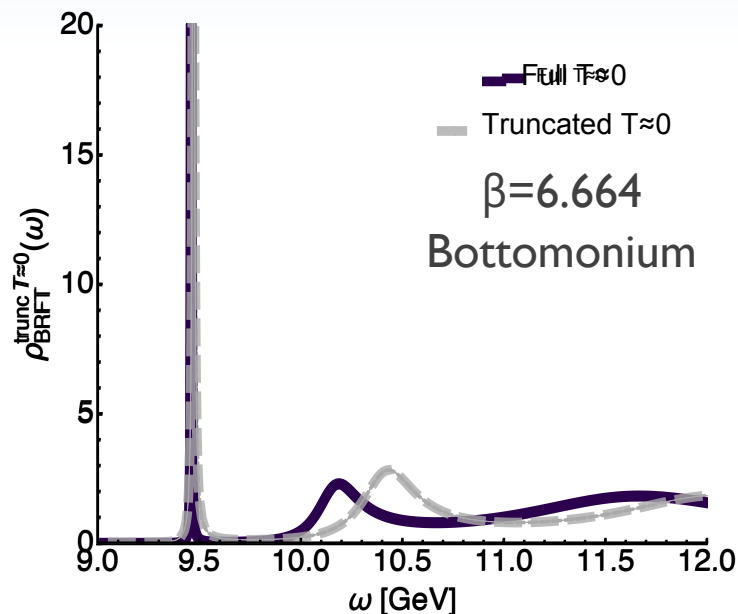
$$M_{\chi_{c1}}^{\text{NRQCD}} = 3.5086(39) \text{ GeV} \quad M_{\chi_{c1}}^{\text{PDG}} = 3.51066(7) \text{ GeV}$$



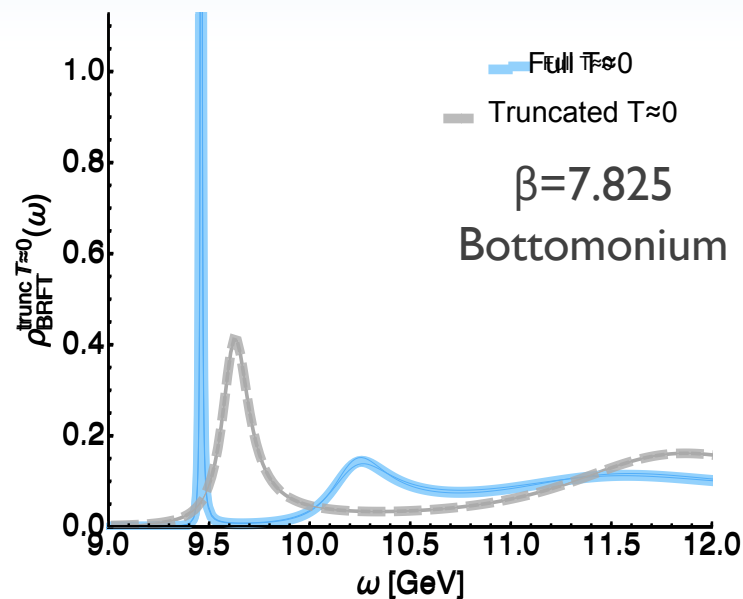


Taking control of systematics I

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$$\Delta M_{6.664} = 9.3(2) \text{ MeV}$$



$$\Delta M_{7.825} = 159(1) \text{ MeV}$$

- The “high-gain” BR method resolves the $T=0$ ground state very well from $N_\tau=48-64$ points
- How does accuracy suffer from limited available information at $T>0$ ($N_\tau=12$) ?

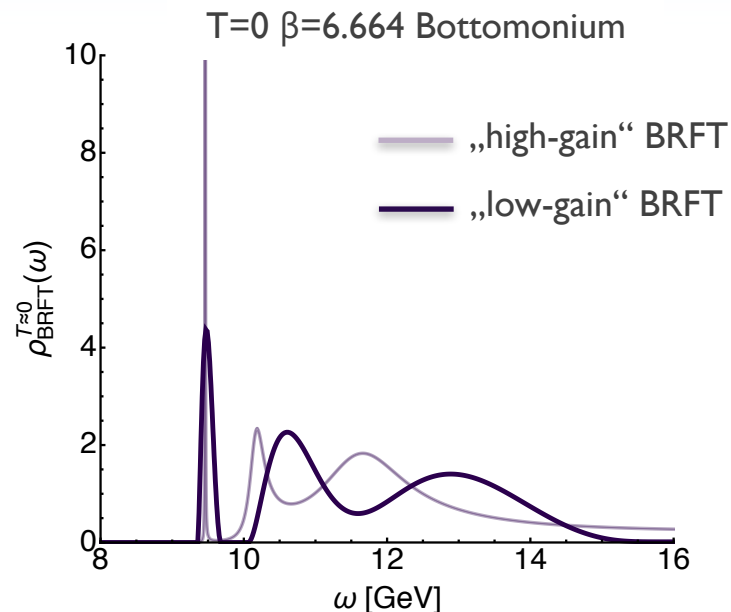
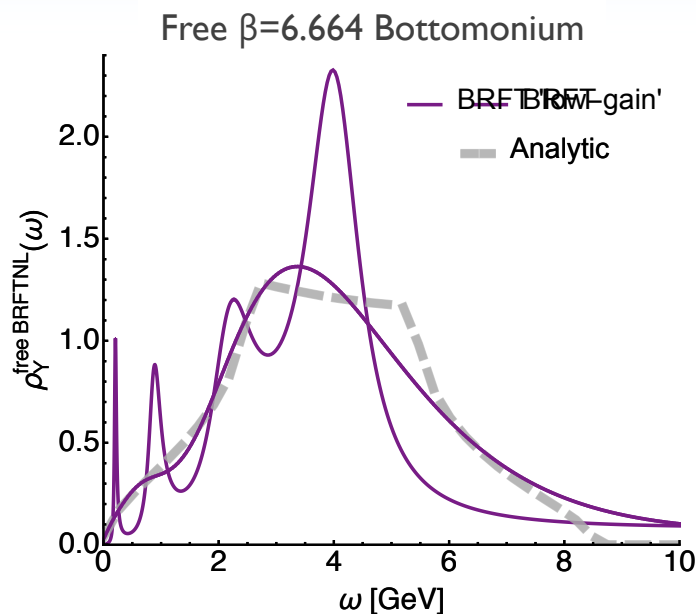


Systematic shift of peaks to higher frequencies, as well as broadening. needs to be accounted for when analyzing $T>0$ spectra

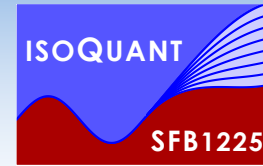


Taking control of systematics II

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- Standard “high-gain” BR on small ($N_\tau=12$) simulation datasets suffers from ringing
- New “low-gain” BR removes ringing from reconstructed analytic free spectra at low ω
- New “low-gain” BR method still identifies presence of peaks encoded in data
- Strategy: - Test with “low-gain” reconstruction whether peaks are genuine
- Use “high-gain” reconstruction to extract peak features, e.g. position

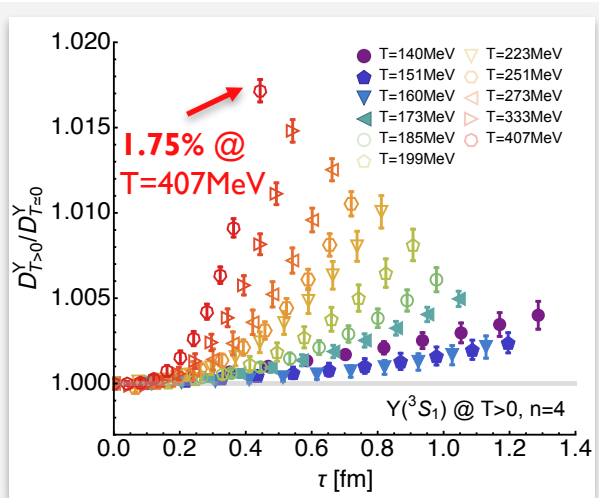


T>0 effects in $Q\bar{Q}$ correlators

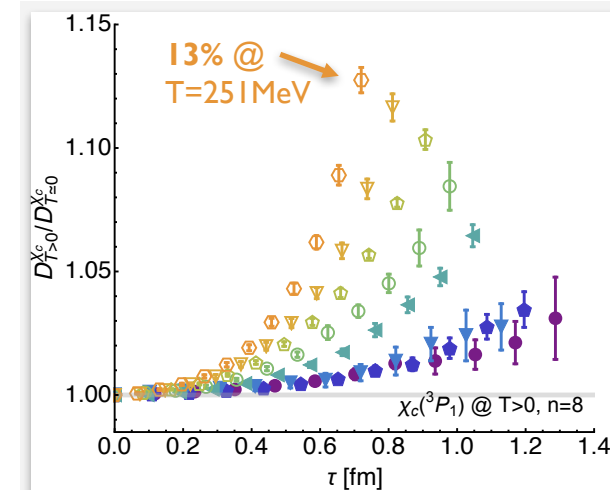
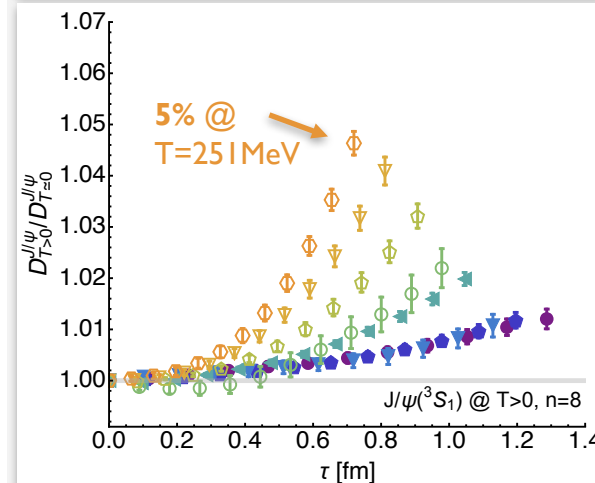
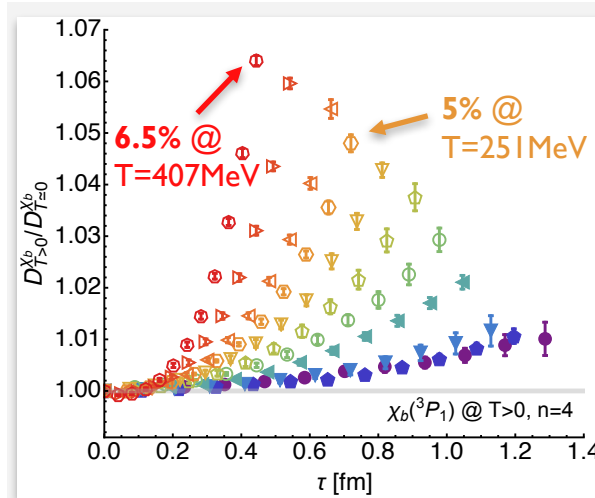
$E_{\text{bind}}(T=0) \sim 1.1 \text{ GeV}$

$E_{\text{bind}}(T=0) \sim 640 \text{ MeV}$

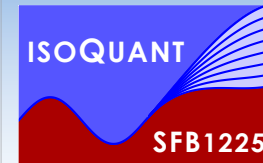
$E_{\text{bind}}(T=0) \sim 200 \text{ MeV}$



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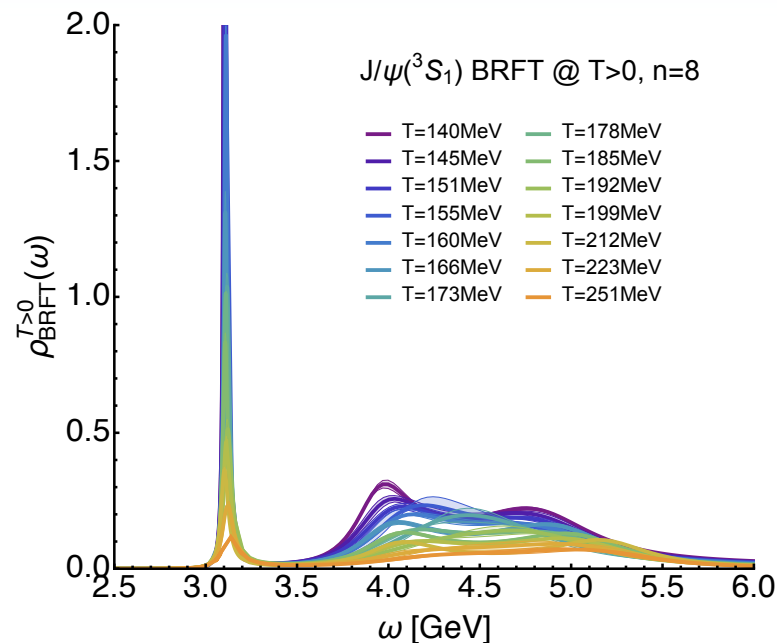
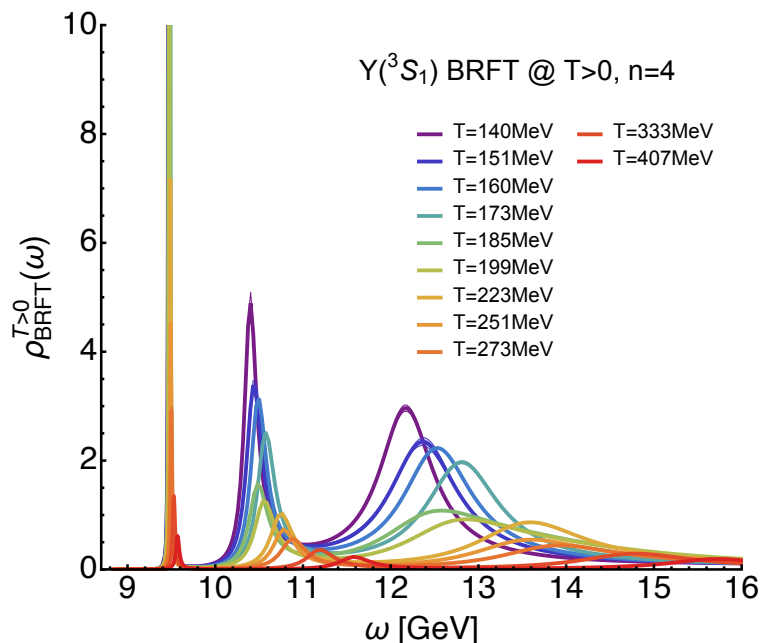


- Upsilon shows non-monotonous behavior around $T \sim T_C$
(bb $3S_1$ channel contains most excited states)
- Hierarchical $T > 0$ modification w.r.t. vacuum binding energy

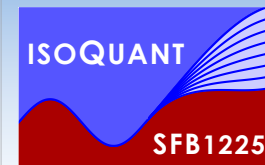


NRQCD S-wave spectra at $T > 0$

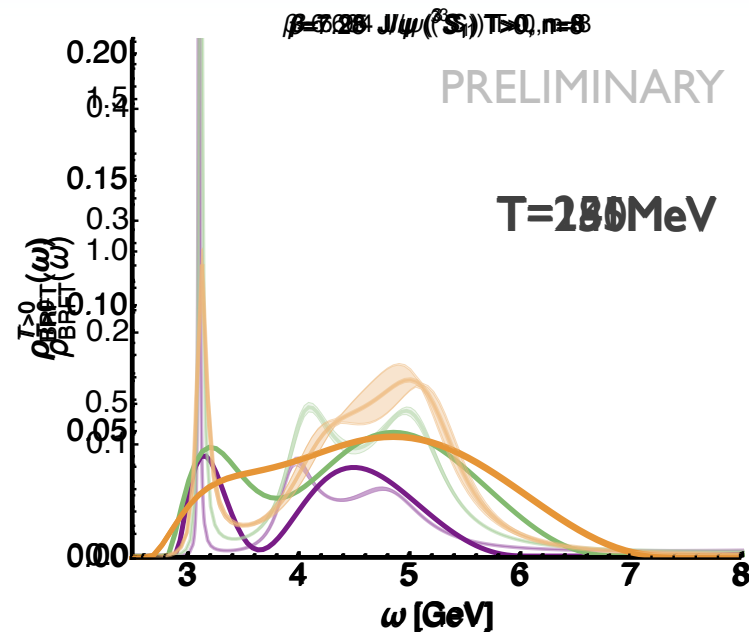
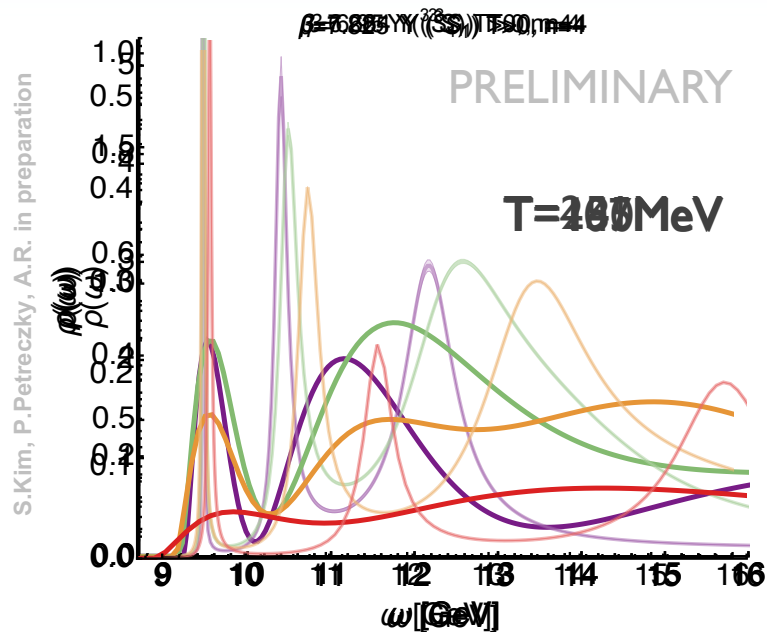
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- Ground state well resolved and well separated from higher lying structures
- Combining Euclidean and imaginary frequency data reduces ringing at large ω
- Gradual broadening and shifting of lowest lying peak visible



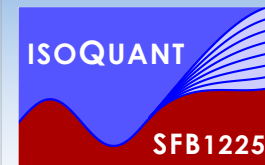
Presence of ground state signals?



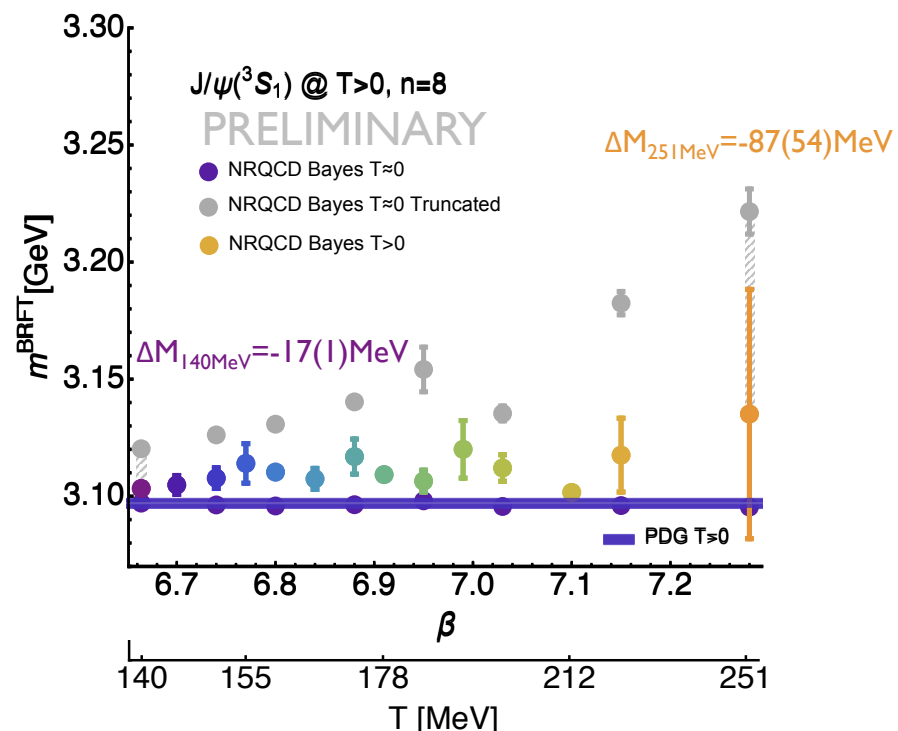
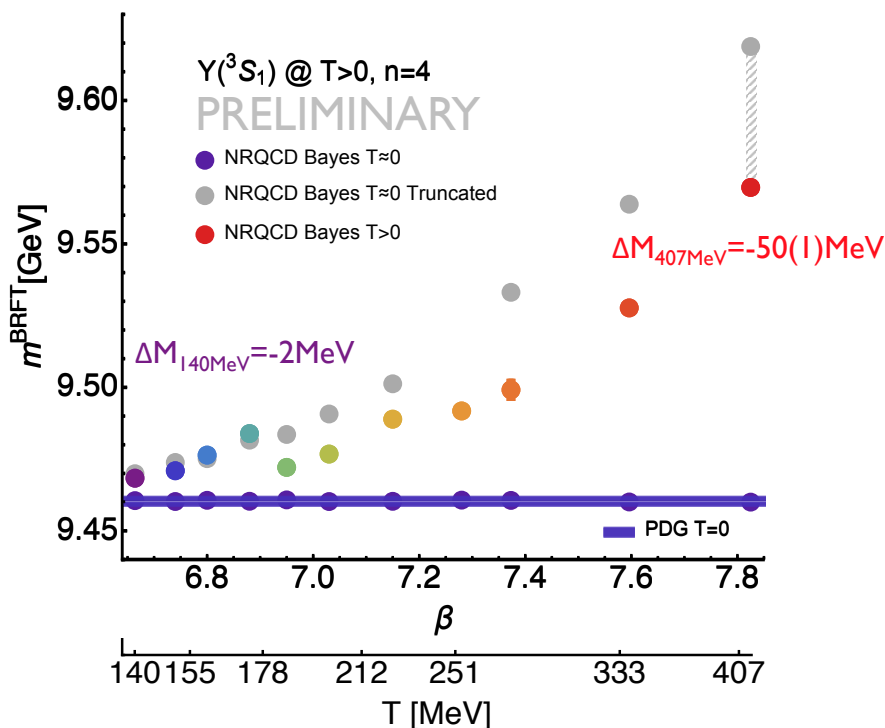
- New “low-gain” BR method shows gradual weakening of ground state signal
- At highest temperature in individual channels: weak ground state remnants remain visible

Upsilon signal up to $T=407\text{MeV}$

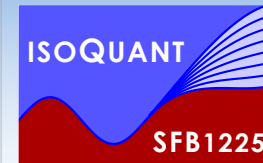
Faint J/ψ signal up to $T=251\text{MeV}$



In-medium S -wave mass shifts

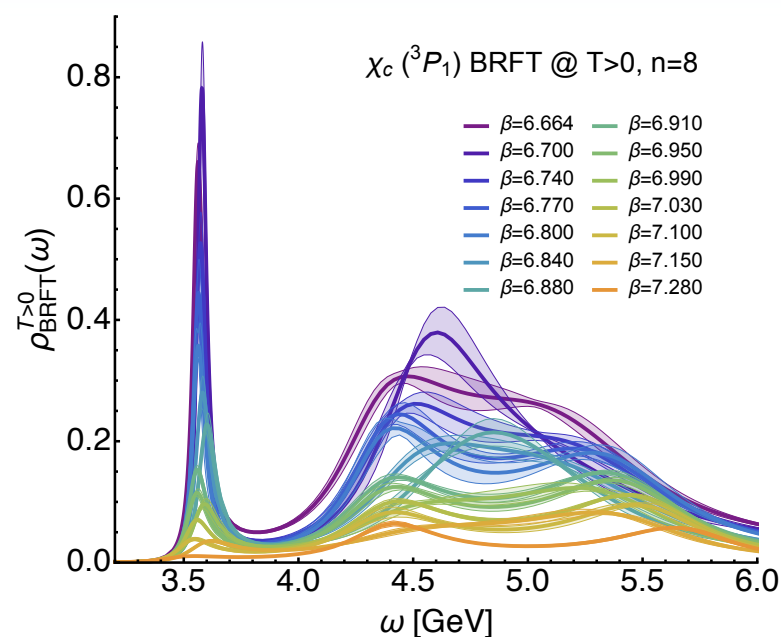
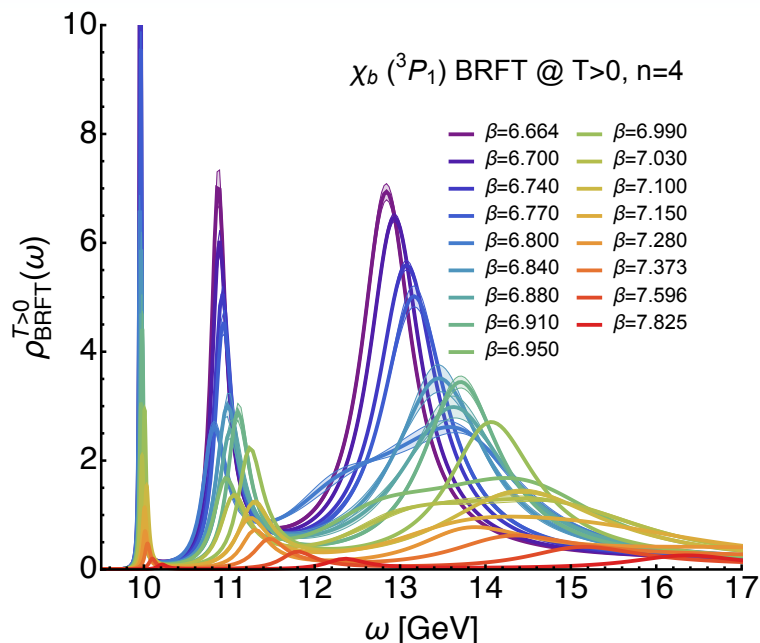


- Naïve inspection of in-medium modification appears to show increasing masses
- BR method systematics: Low number of datapoints introduces shifts to larger masses
- Actual in-medium effect: lowering of bound state mass, consistent with potential studies

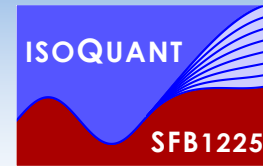


NRQCD P-wave spectra at $T>0$

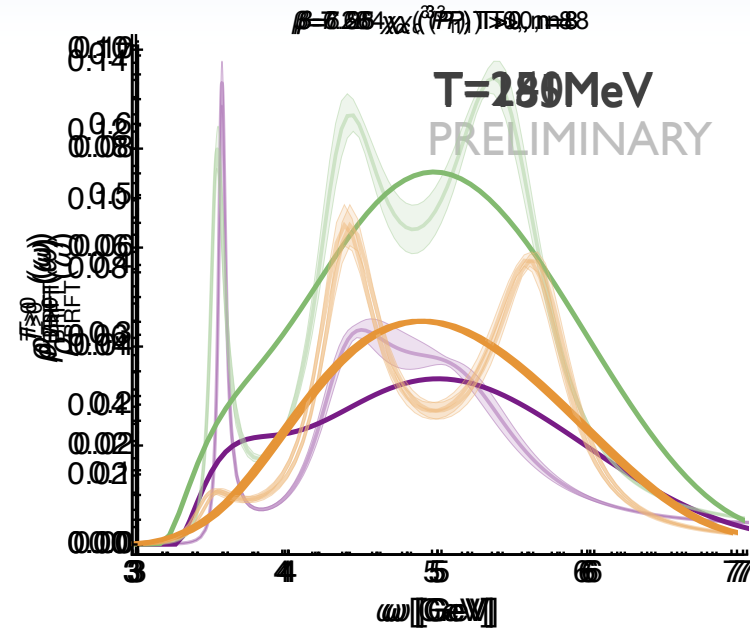
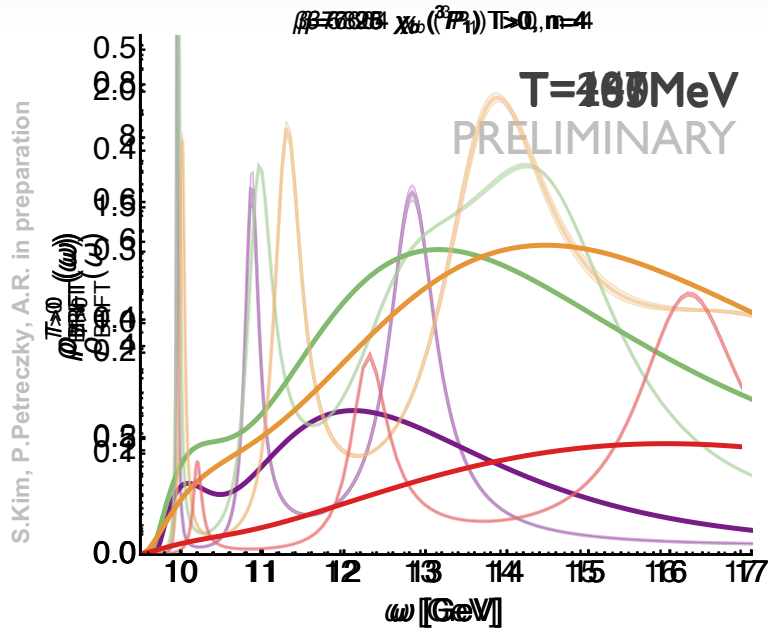
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- ▣ Lower signal to noise ratio in underlying correlators makes reconstruction less precise
- ▣ Ground state well resolved and well separated from higher lying structures
- ▣ Gradual broadening and shifting of lowest lying peak visible



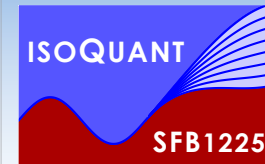
Survival of ground state signals?



- New “low-gain” BR method shows gradual weakening of ground state signal
- Genuine bound state signal lost at intermediate temperatures

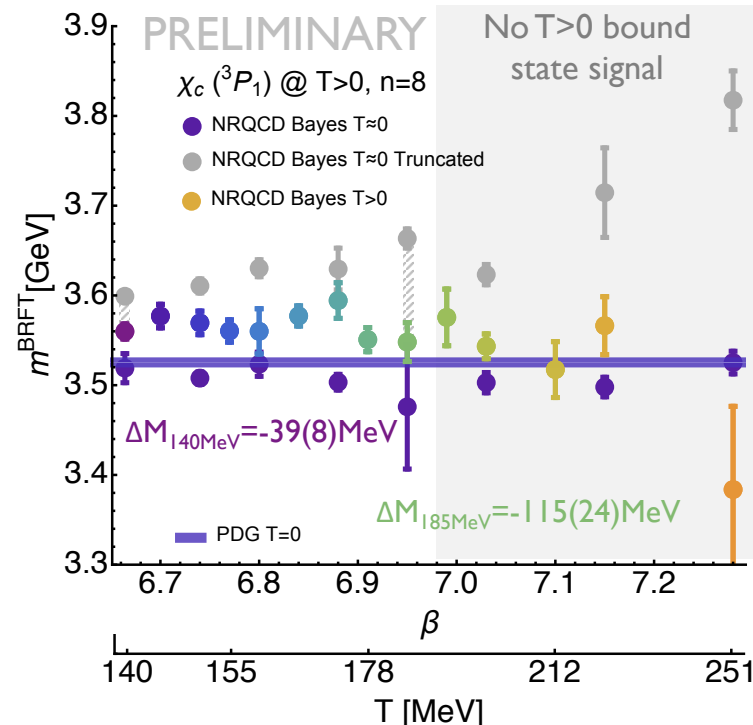
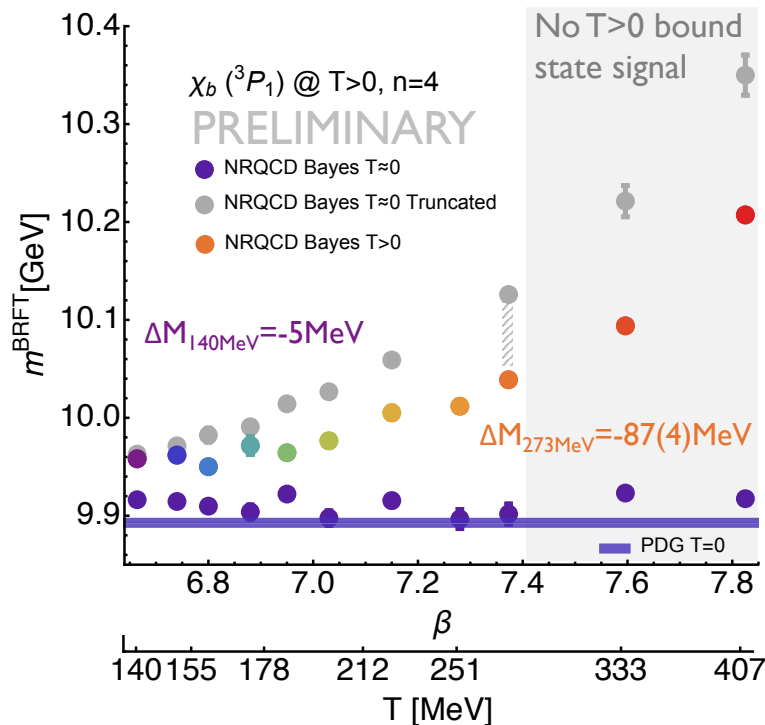
χ_b signal up to $T=273\text{MeV}$

χ_c signal up to $T=185\text{MeV}$

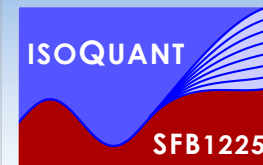


In-medium P-wave mass shifts

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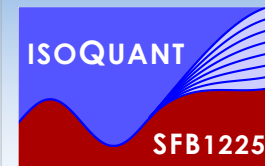
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- Actual in-medium effect: lowering of bound state mass, consistent with potential studies



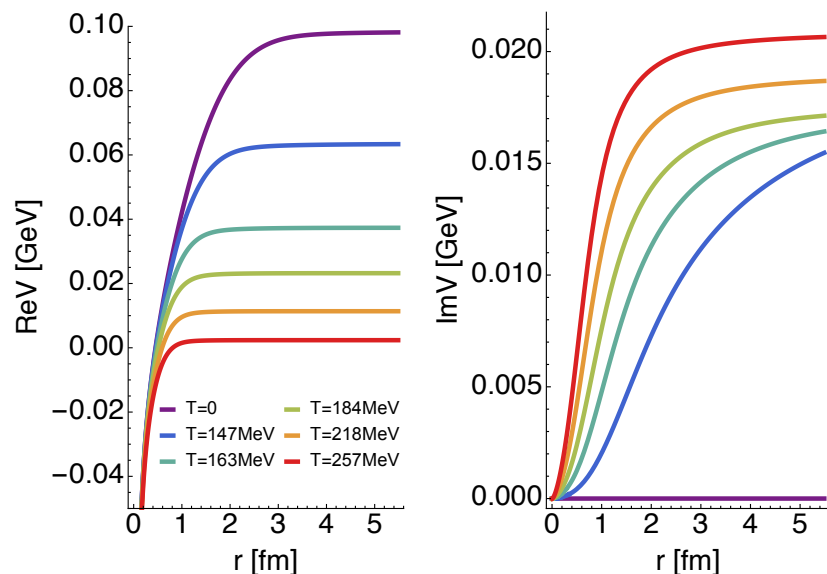
Conclusions

- Heavy-ion experiments: Intricate quarkonium phenomenology
- Direct access to in-medium quarkonium from first principles: lattice NRQCD
 - S.Kim, P. Petreczky, A.R.: Phys.Rev. D91 (2015) 54511, Nucl.Phys. A956 (2016) 713 and arXiv:1704.05221
 - **Progress I:** HotQCD lattices provide higher statistics and larger temperature range
 - **Progress II:** Improved Bayesian spectral reconstruction reduces methods uncertainties
 - **Progress III:** Extension of our previous study to charmonium at finite temperature
- Updated in-medium results for quarkonium in lattice NRQCD:
 - Verified **sequential** in-medium **modification** of correlators according to vacuum E_{bind}
 - First quantitative determination of in-medium **shifts** to **lower masses**

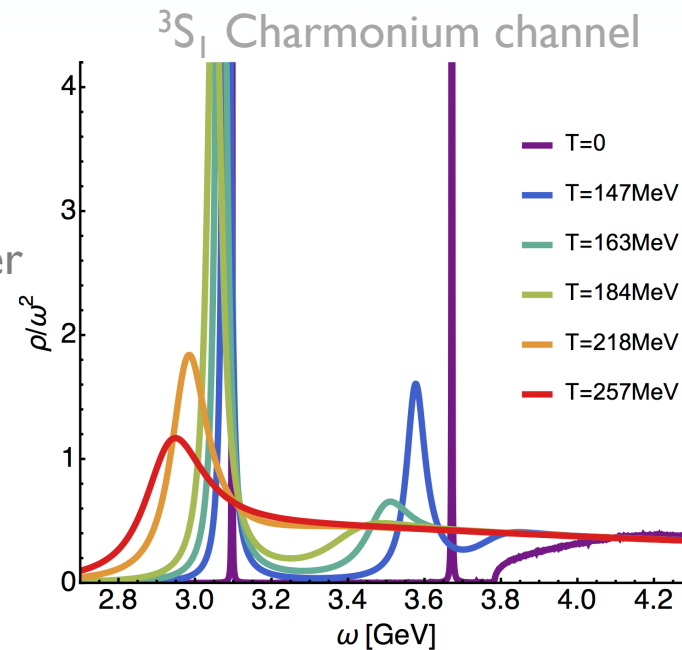
Thank you for your attention - Благодарю вас за внимание



Previous lessons from the lattice $V_{Q\bar{Q}}$



Schrödinger equation

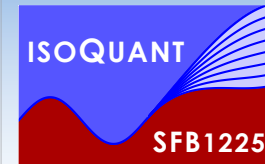


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From full QCD with u,d,s quarks based on asqtad action

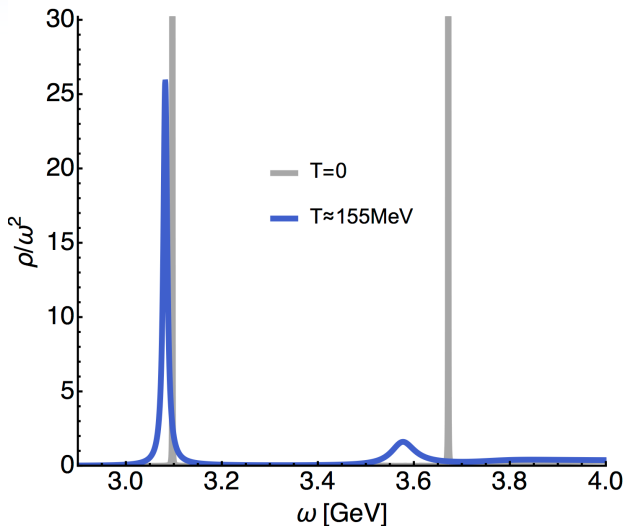
- $Q\bar{Q}$ states suffer from a hierarchical in-medium modification w.r.t. vacuum binding energy
- In-medium states take on *lower* masses and show *increased* thermal widths
- Quarkonium melting is a gradual process: defining T_{melt} is *ambiguous*, popular $E_{\text{bind}} = \Gamma_{\text{therm}}$
- Observables from in-medium spectra: $\psi' / J/\psi$ ratio and P-wave feed-down estimated

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ψ' to J/ψ ratio from $T>0$ spectra

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JHEP 1512 (2015) 101



- Assume instantaneous freezeout: $T>0$ states convert to real vacuum particles at around T_C
- In-medium dilepton emission from area under spectral resonance peaks

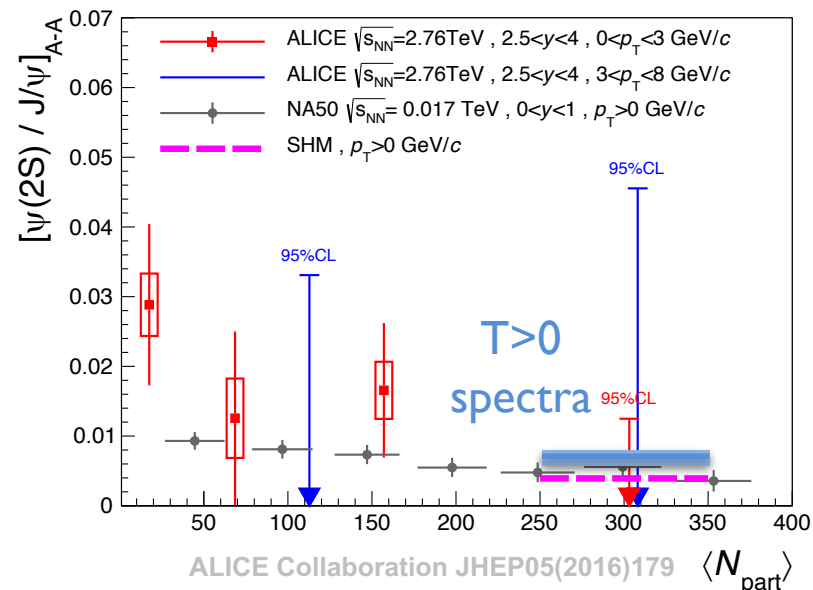
$$R_{\ell\bar{\ell}} \propto \int dp_0 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\rho(\mathbf{P})}{P^2} n_B(p_0)$$

(to leading order $\rho(\mathbf{P}) = \rho(p_0^2 - \mathbf{p}^2)$)

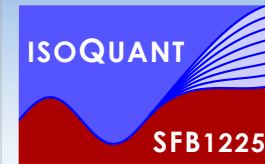
- ”How many vacuum states do the in-medium peaks correspond to?”
- Number density: divide in-medium by $T=0$ dimuon emission rate:

$$\frac{N_{\psi'}}{N_{J/\psi}} = \frac{R_{\ell\bar{\ell}}^{\psi'}}{R_{\ell\bar{\ell}}^{J/\psi}} \frac{M_{\psi'}^2 |\Phi_{J/\psi}(0)|^2}{M_{J/\psi}^2 |\Phi_{\psi'}(0)|^2}$$

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JHEP 1512 (2015) 101



ALICE Collaboration JHEP05(2016)179 $\langle N_{part} \rangle$

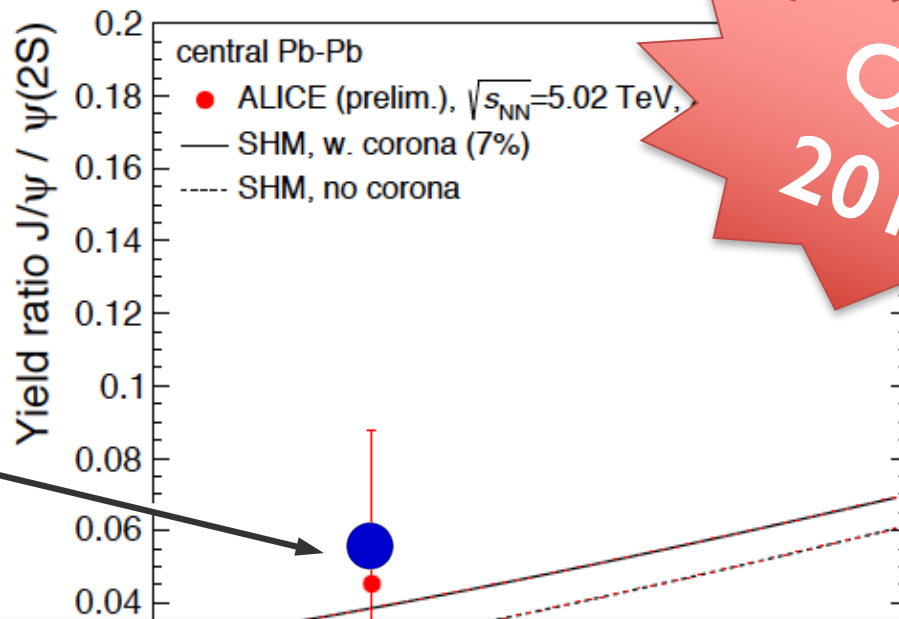


First preliminary ALICE data

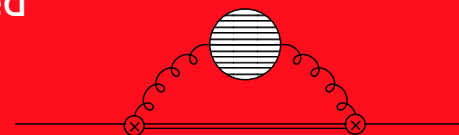
$\psi(2S) / J/\psi$ - Jan. 2017

settle question of hadronic bound states in the QGP

Our $T > 0$ spectral function prediction



Not all in-medium effects on QQ can be captured in a Schrödinger equation with potential: crosscheck results with a *genuine* field theory



ALICE data, Pb-Pb: Quark Matter 2017; pp: [arXiv:1702.00557](https://arxiv.org/abs/1702.00557)

slide from Peter Braun-Munzinger