QCD phase structure
and
Heavy Ion Collisions (2)

V. Bornyakov, D. Boyda, V. Goy, H.Iida, A. Molochkov,
A.Nakamura, A. Nikolaev, M.Wakayama and V. I. Zakharov

Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics

Dubna, 10-14 July 2017

Dmitrij Ivanovich Mendelejev

I am happy to be in Dubna
where 114, 115 and 117 Atoms
are fund.

114 Flerovium

Riken found 113,
Nihonium.

On Monday, we got a very useful relation,

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

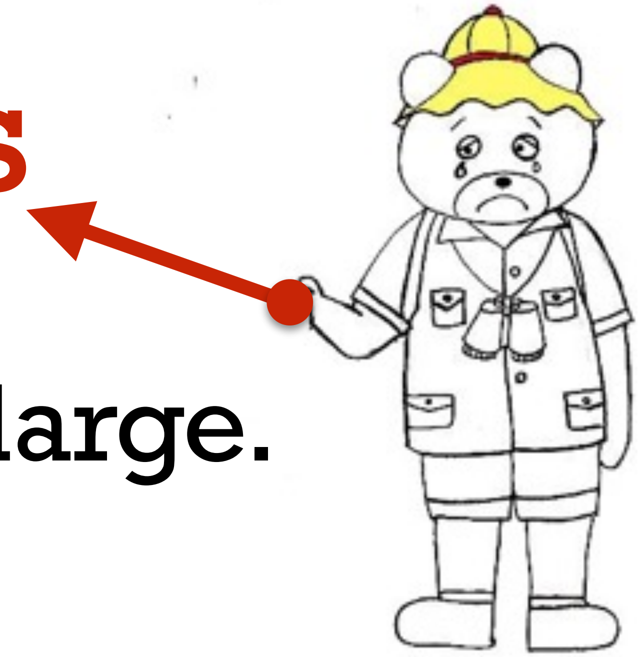
$(\xi \equiv e^{\mu/T}$: Fugacity$)$

We determined Z_n from Experimental data.

Today's Menu,

1. To show this is useful to determine Phase Transition Line especially at NICA and J-PARC
2. Calculate Lee-Yang Zero from these Z_n.
3. Calculate Z_n by Lattice QCD Simulations.

Problems



1) N_{max} is not very large.

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lower estimation of larger density contribution.

2) Measured multiplicities are not Baryon, but Proton.

For Charge or Strangeness Multiplicity, No problem.

I need Baryon Multiplicities !
But there are only Proton Multiplicity data !

What should I do ?

Construct N_B dist. from N_p ?

M.Kitazawa and M.Asakawa,
Proposal of a Method
PhysRevC.86.024904 (arXiv:1205.3292)

NICA White-Paper,
theor0.jinr.ru/twiki/pub/NICA/WebHome/WhitePaper_10.01.pdf

the ratio of the susceptibility can be readily constructed by the high order correlation function for conserved quantities in high-energy nuclear collisions. Baryon numbers are conserved in strong interactions. Simulations have shown that net-proton correlation function is a fair good representation of the net-baryons [16]. Note, at the critical point, the smallness (or disappearance) of the correlation length will lead to a large value of the quark susceptibility ratio. The systematic measurements of the net-charge and net-proton Kurtosis will provide info on the nature of the system.

[16] B. Mohanty, private communication, 2009.

OK, for a while, let us use Proton distribution as a Proxi of the Baryon.

Looking for High Density

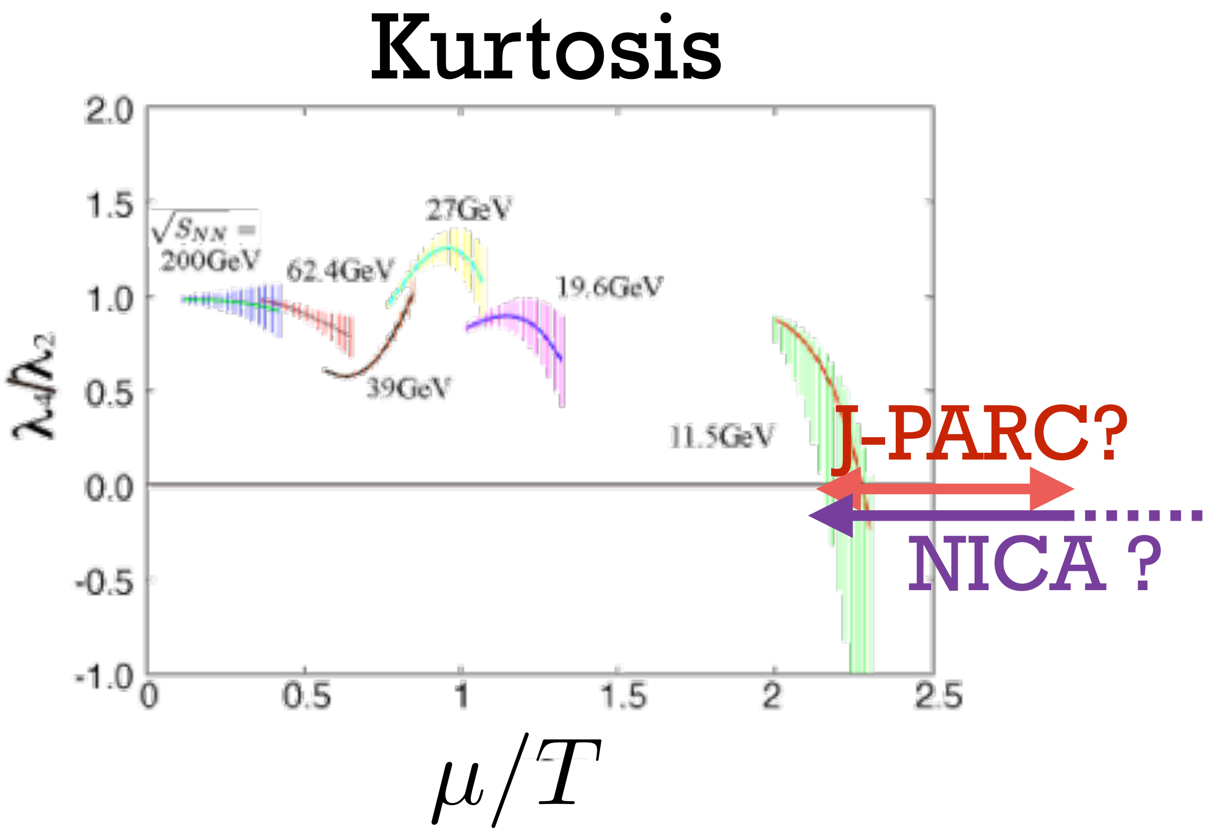
Not too High Energy Regions

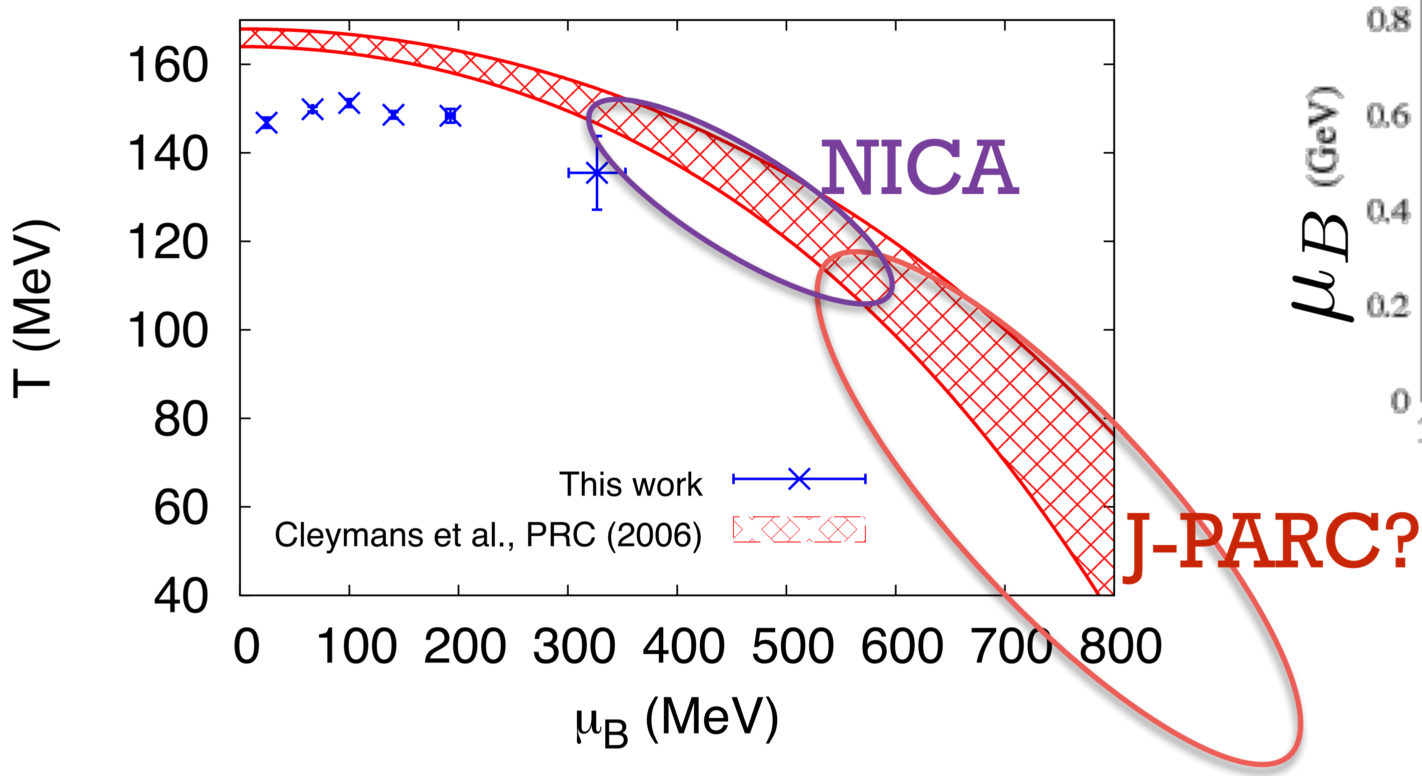
NICA $\sqrt{s_{NN}}$ =4 - 11 GeV

J-PARC $\sqrt{s_{NN}}$ =2 -6.2 GeV

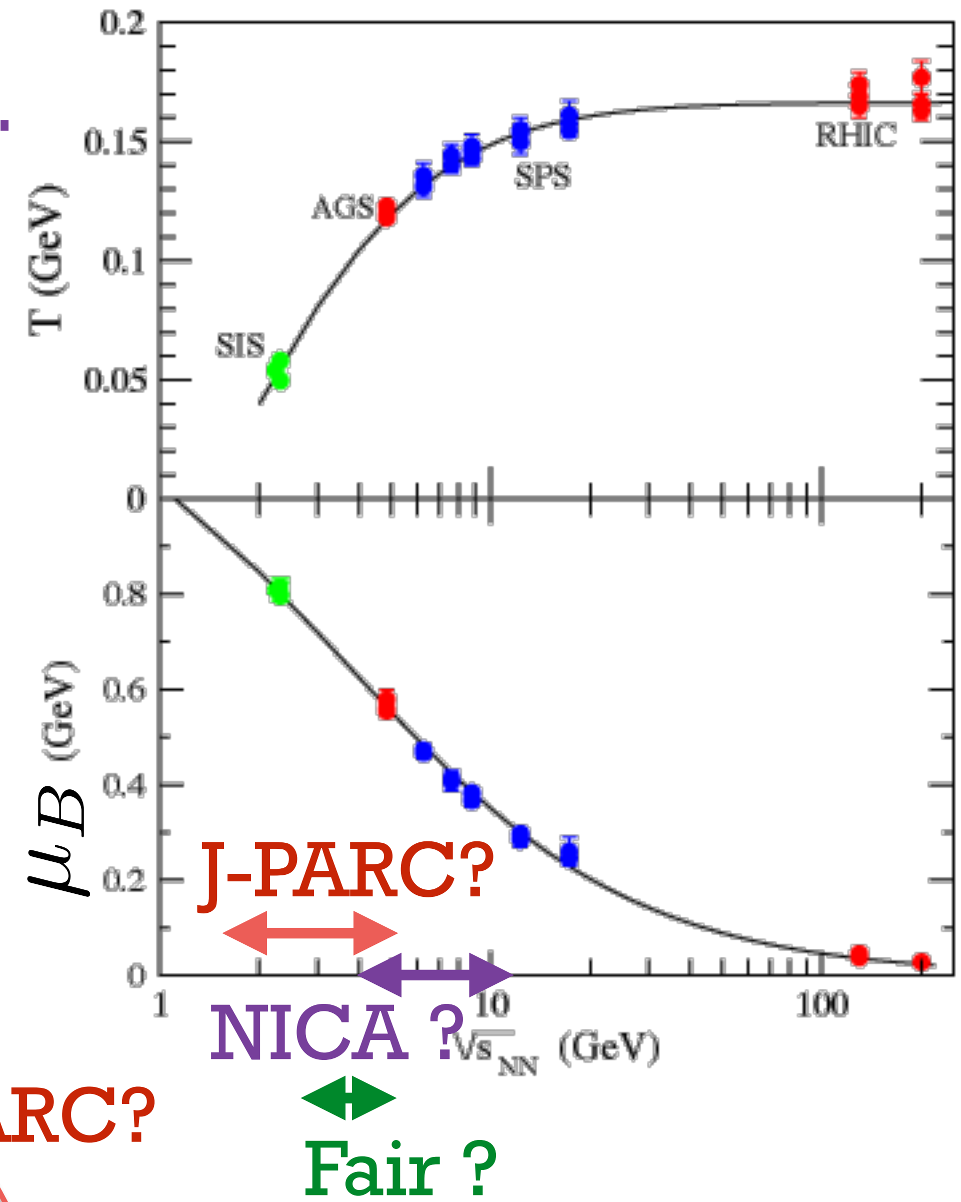
FAIR $\sqrt{s_{NN}}$ =2.7 -4.9 GeV

(CMB@SIS100)



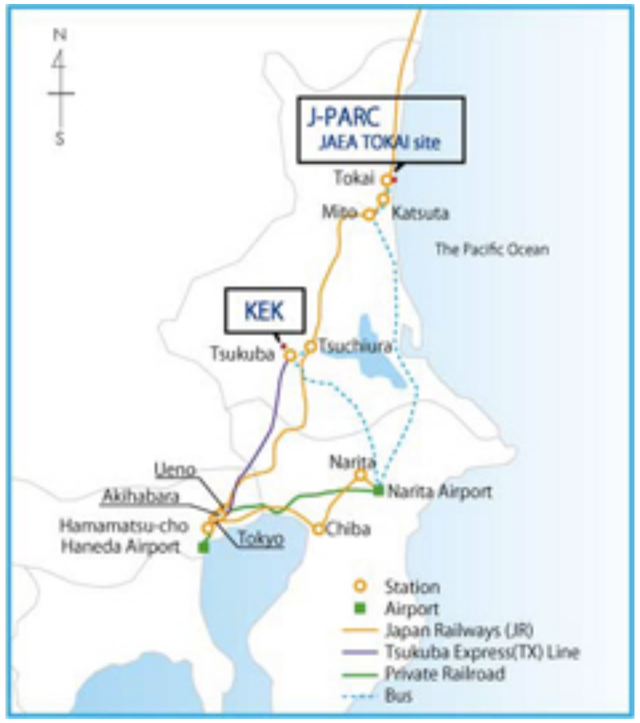


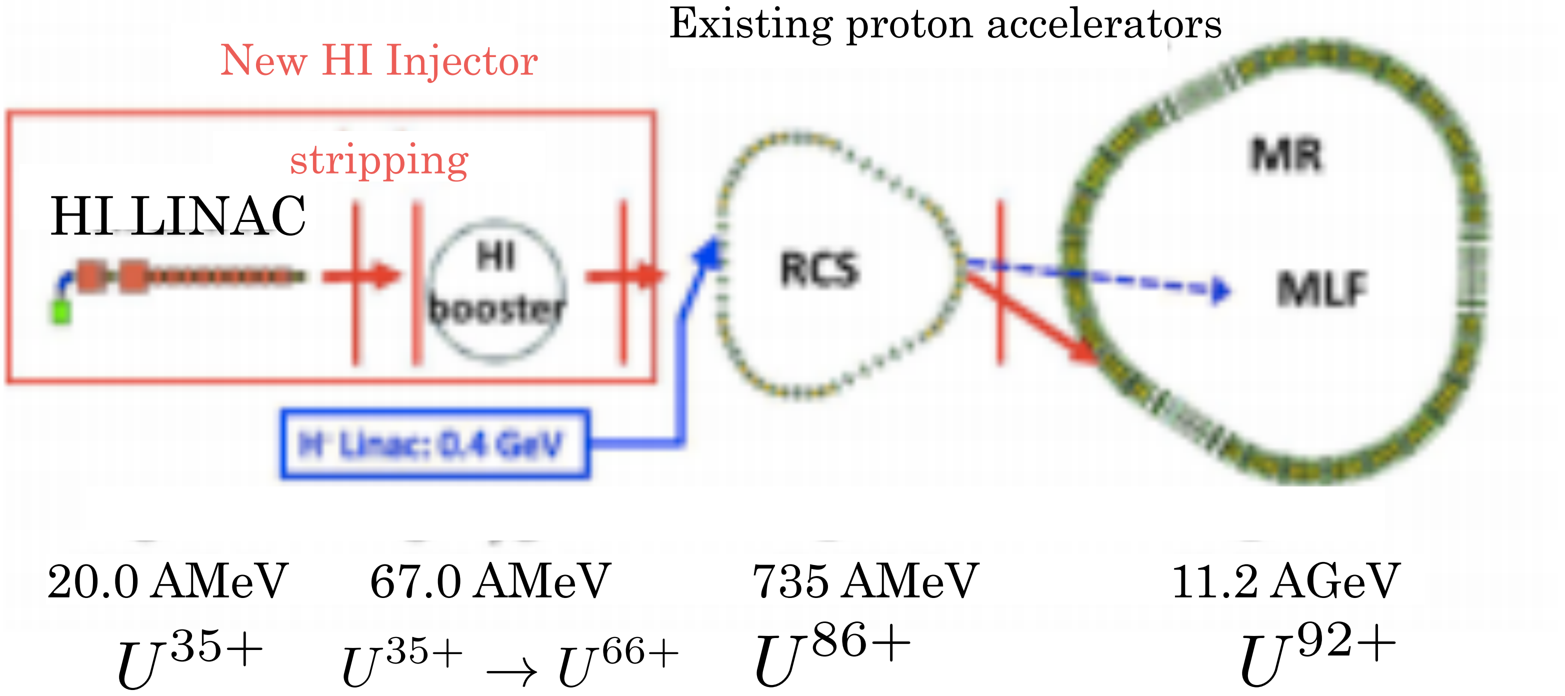
Alba et al., arXiv:1403.4903

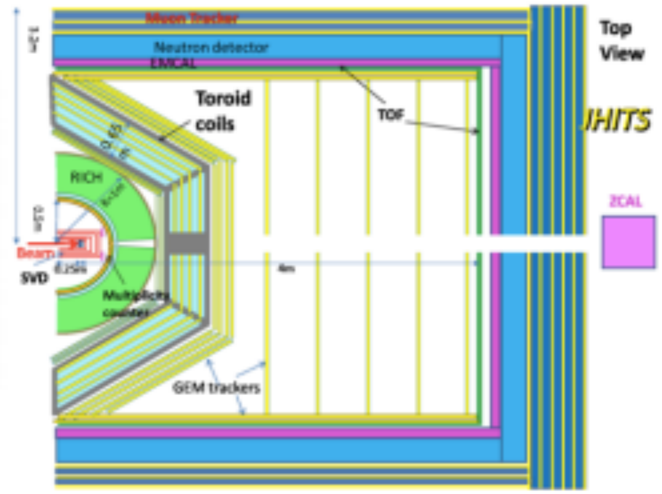


J.Cleymans et al.,
Phys. Rev. C73, (2006) 034905.

J-PARC





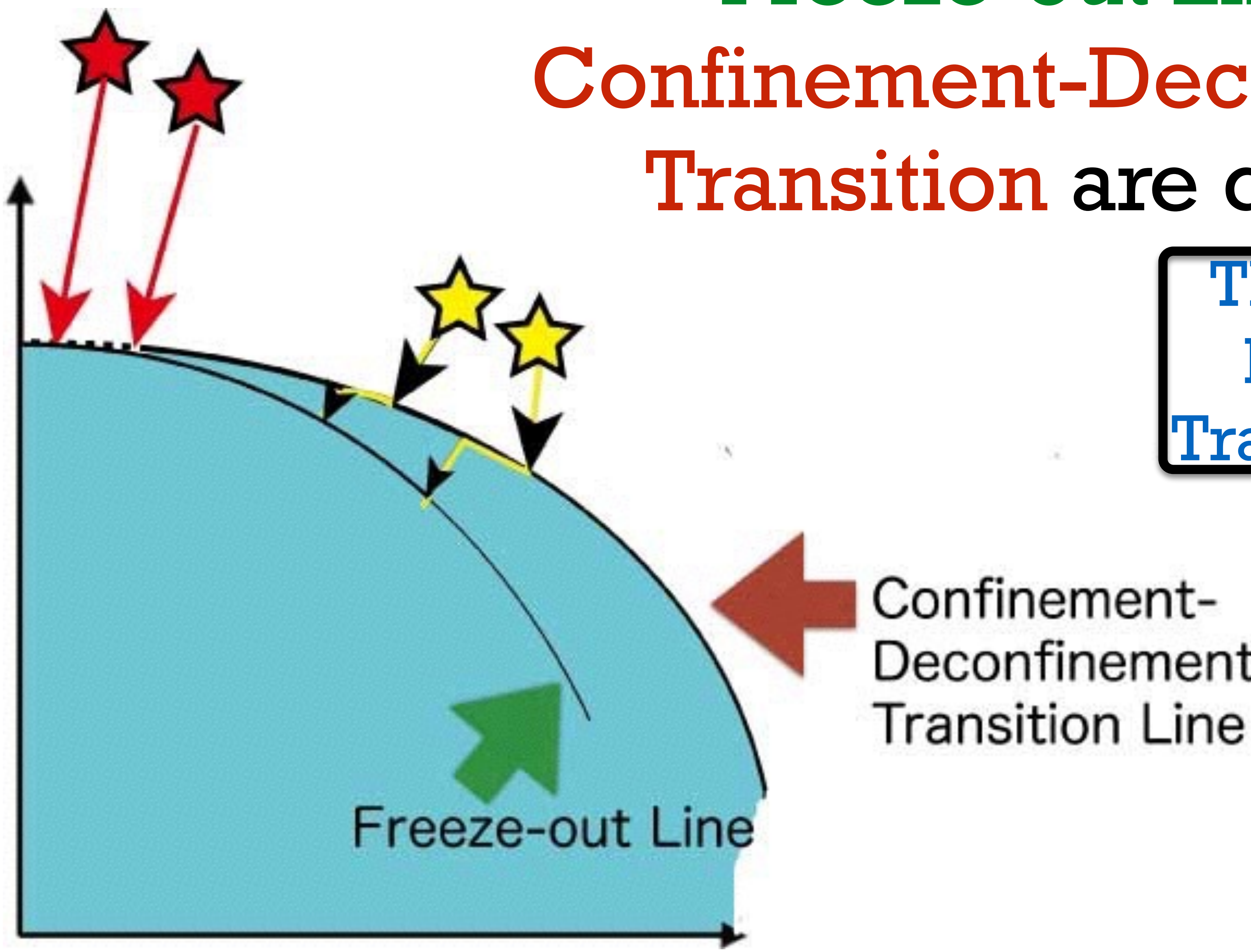


Letter of Intent:
http://j-parc.jp/researcher/Hadron/en/pac_1607/pdf/LoI_2016-16.pdf

July, 2016

Wait !

Freeze-out Line and Confinement-Deconfinement Transition are different.



Then how can
I find Phase
Transition Line ?

Lepton Pairs and/or
Jets Behavior are model dependent.

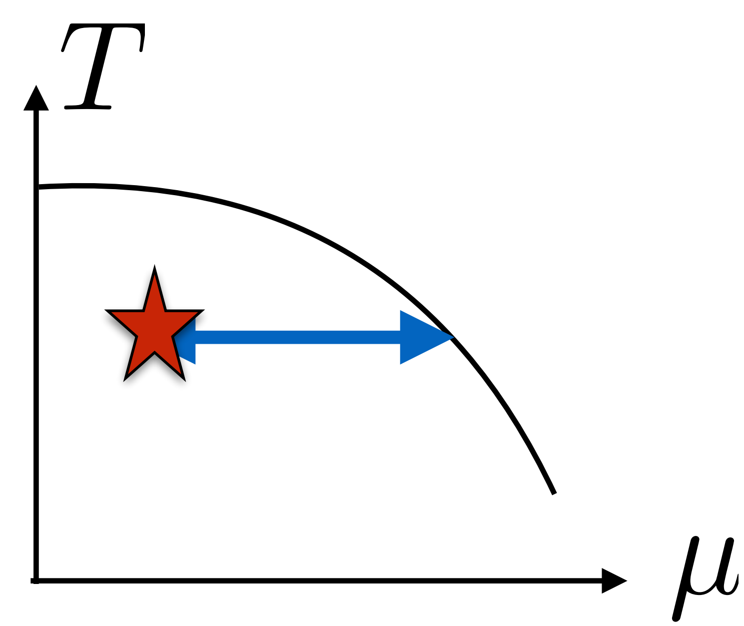
Important Observation

We construct Z_n from Heavy Ion Collision data at Freeze-out T and μ. Then using

$$Z(\mu, T) = \sum_n Z_n(T)(e^{\mu/T})^n$$

for arbitrary values of μ/T at fixed T

We construct Z_n on ★ and calculate moments on ⟷.

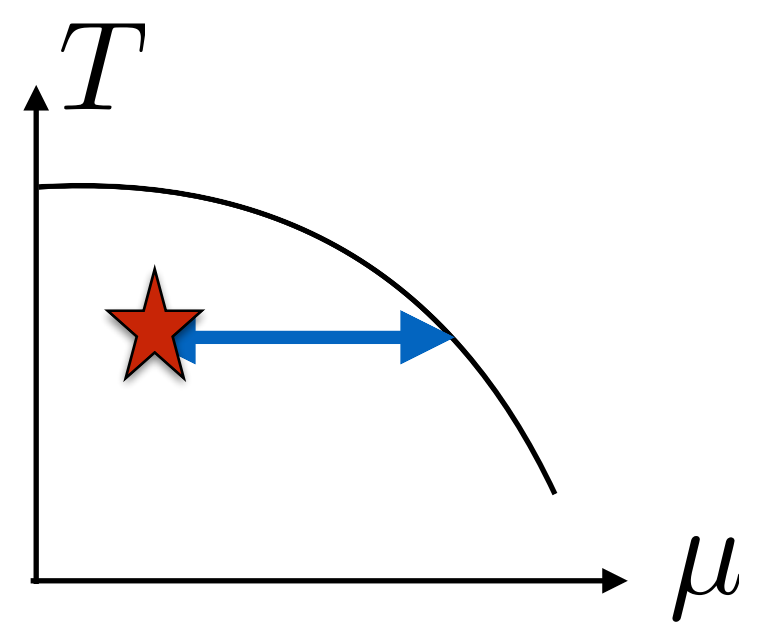


Then how RHIC data look like?

i.e., We construct Z_n from RHIC data and calculate the Moments using

$$Z(\mu, T) = \sum_n Z_n(T)(e^{\mu/T})^n$$

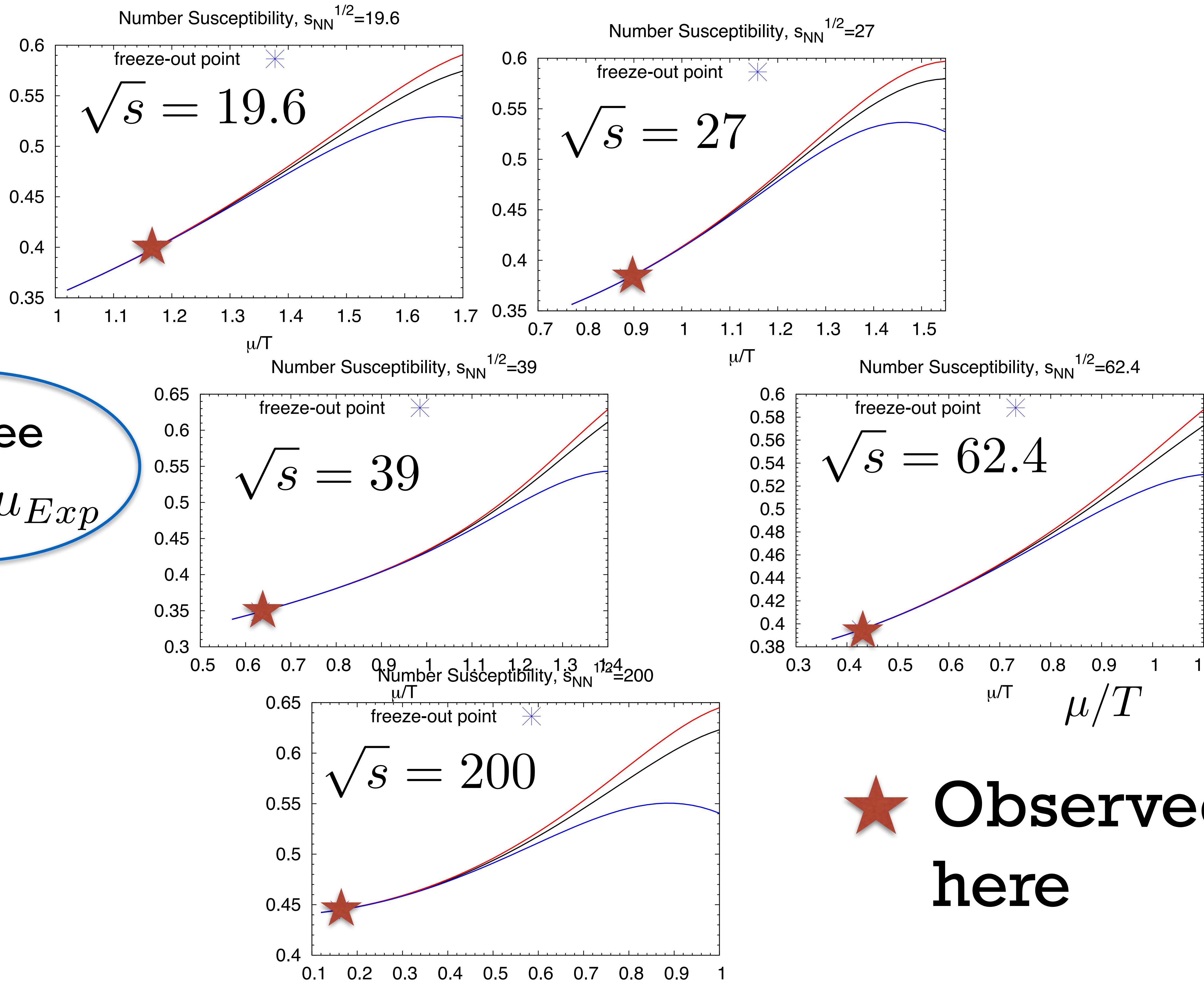
at arbitrary values of μ/T

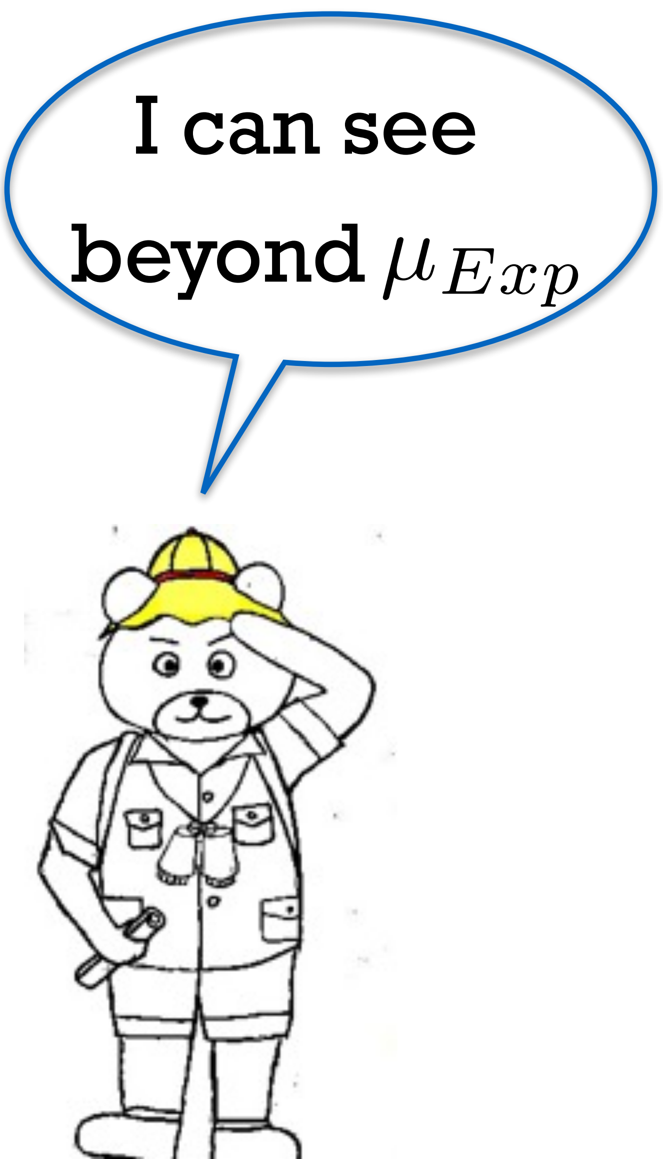


We construct Z_n on 
and calculate moments
on  .

RHIC Data

Susceptivility as a function of μ/T

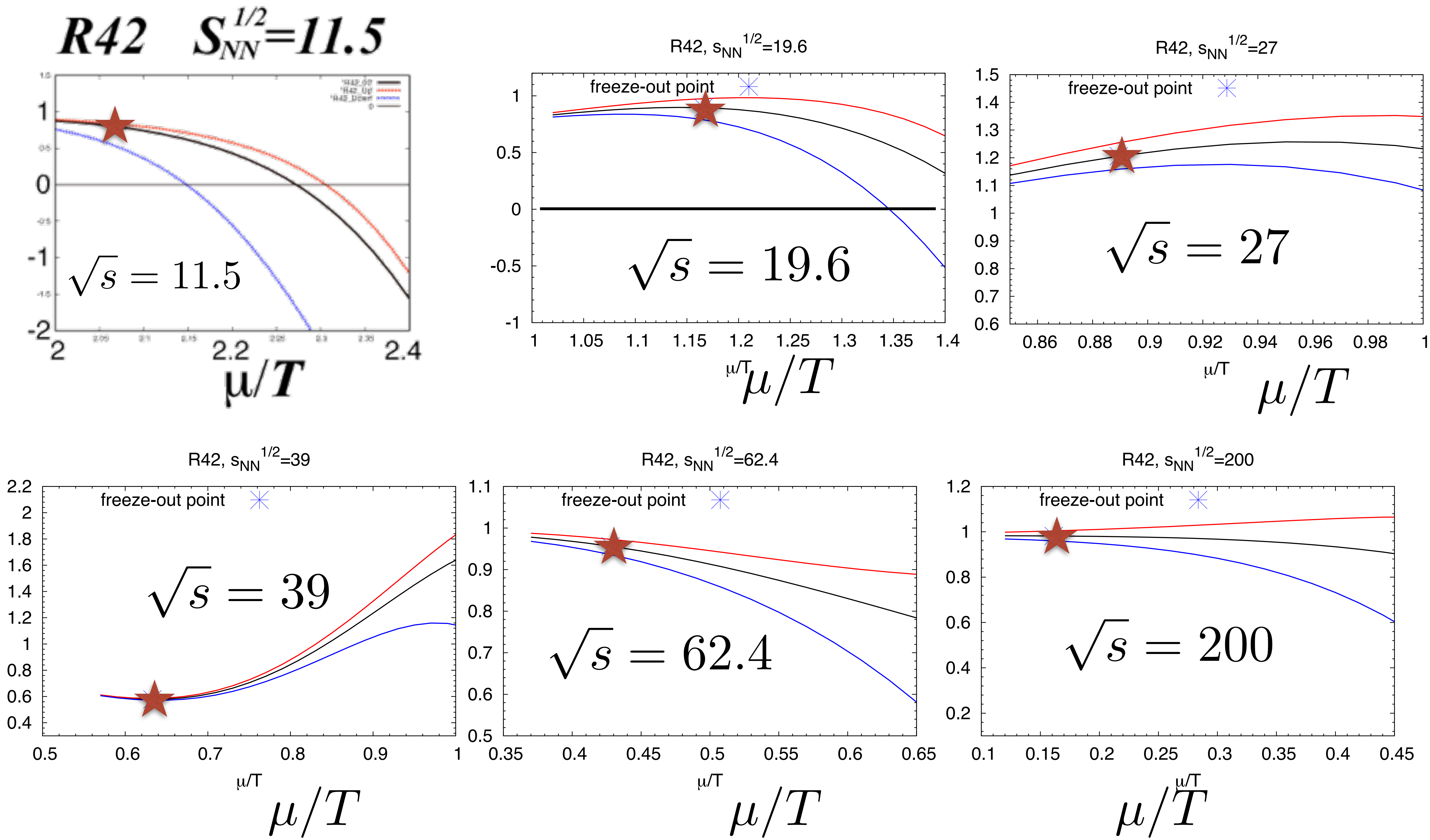




★ Observed here

RHIC Data

Kurtosis $\frac{\lambda_4}{\lambda_2}$ as a function of $\frac{\mu}{T}$



Another Application

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$

This ξ can be complex variables !

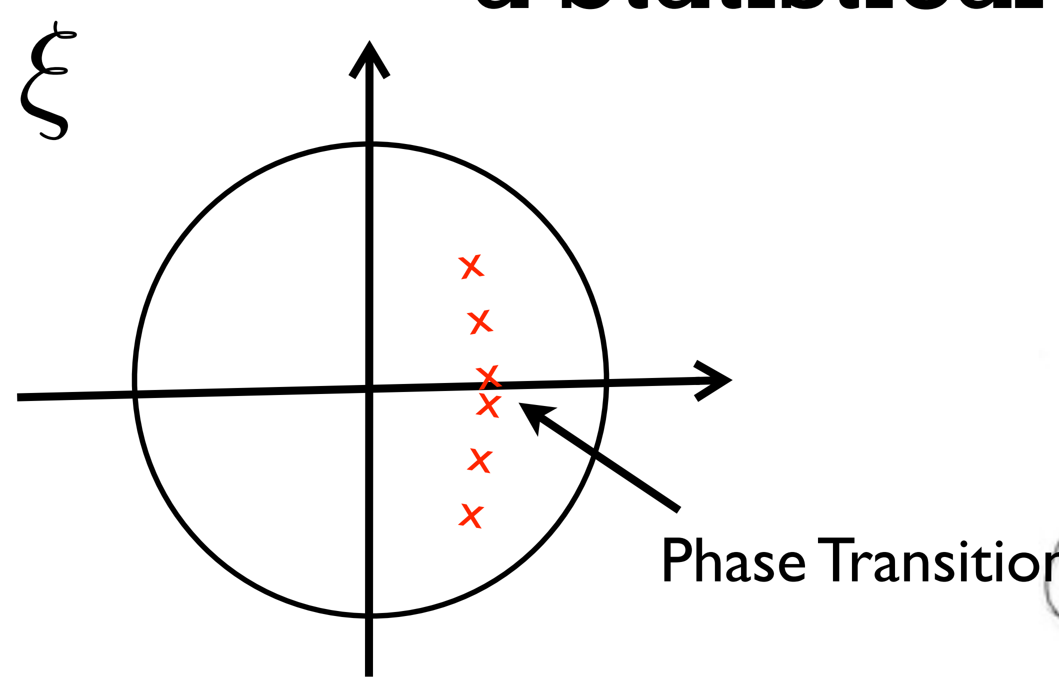
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lee-Yang Zeros (1952)

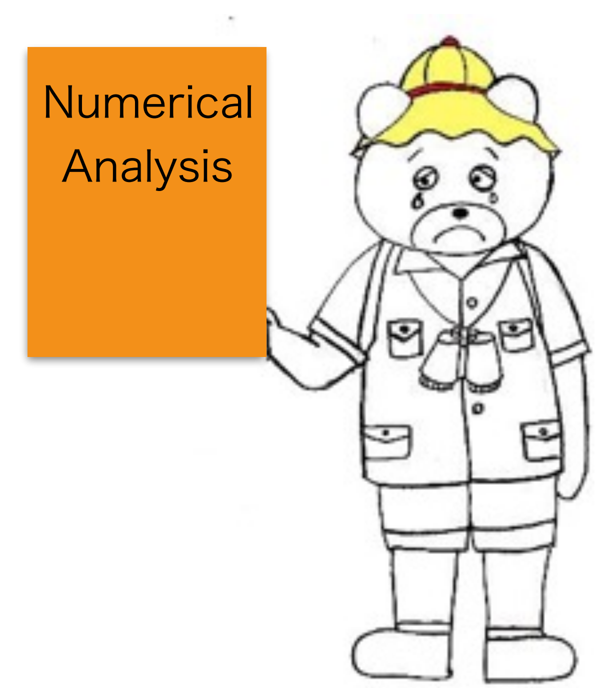
Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System



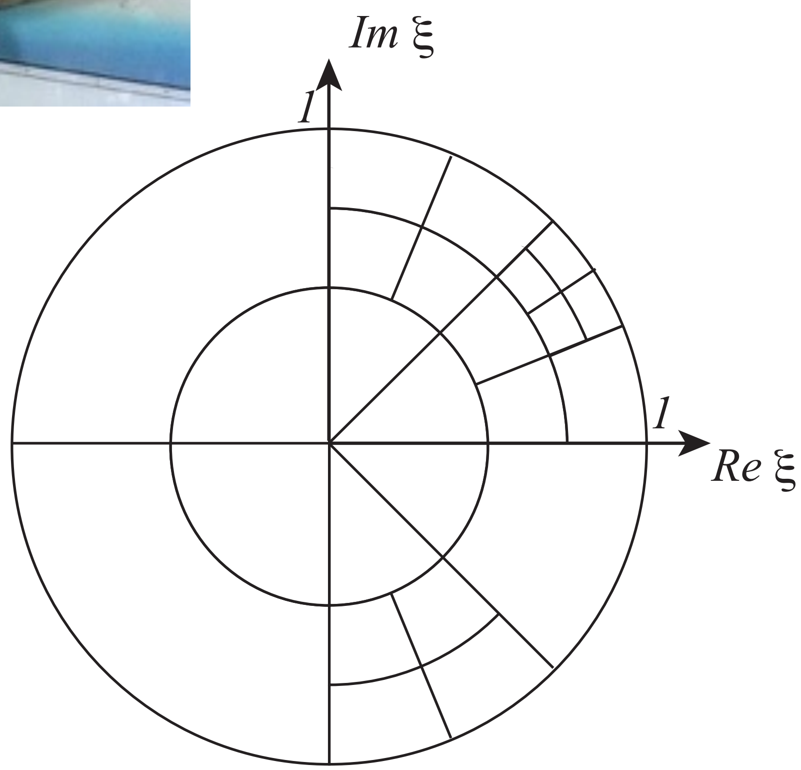
But high-order polynomial zero is a famous ill-posed problem !



Read Text book of Numerical Analysis

cut Baum-Kuchen (cBK) Algorithm

Nakamura-Nagata, Prog.Theor.Phys.2016, 033D01



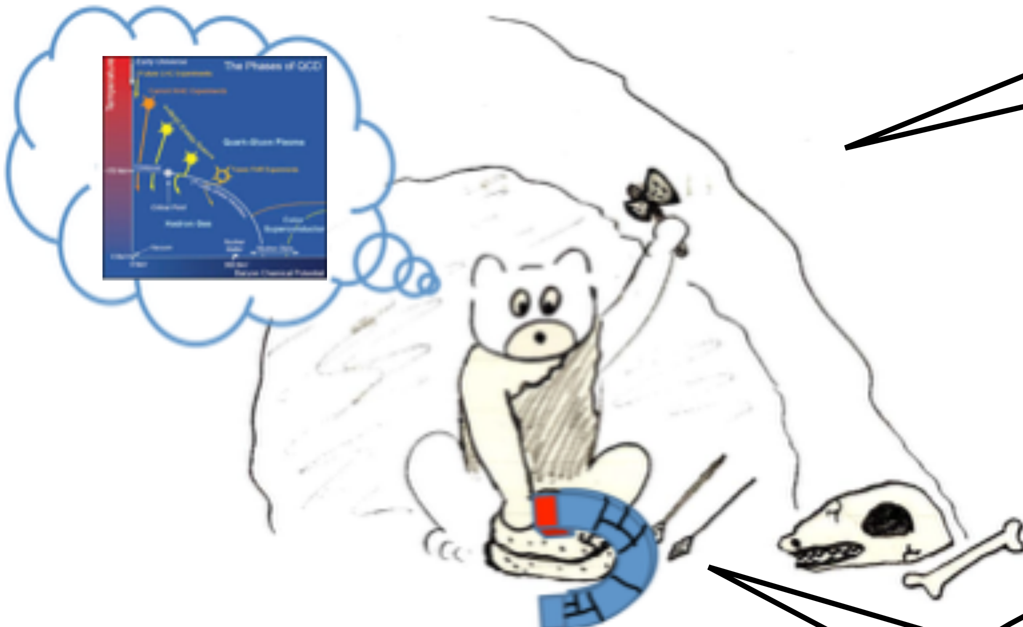
$$f(\xi) = \prod_k (\xi - \alpha_k)$$

$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

$$\frac{1}{2\pi i}\oint_C \frac{f'}{f} d\xi = \left(\begin{array}{c}\text{Number of}\\ \text{Zeros in}\\ \text{Contour } C\end{array}\right)$$

50 - 100 number of significant digits

A Coutour is cut into four pieces if there are zeros inside.



Is this my Original ?

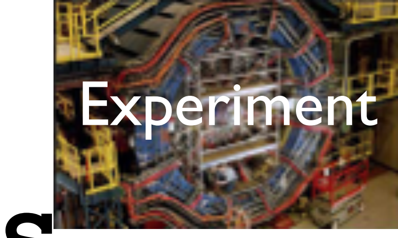
I donot think so.

Let us wait until someone claims.

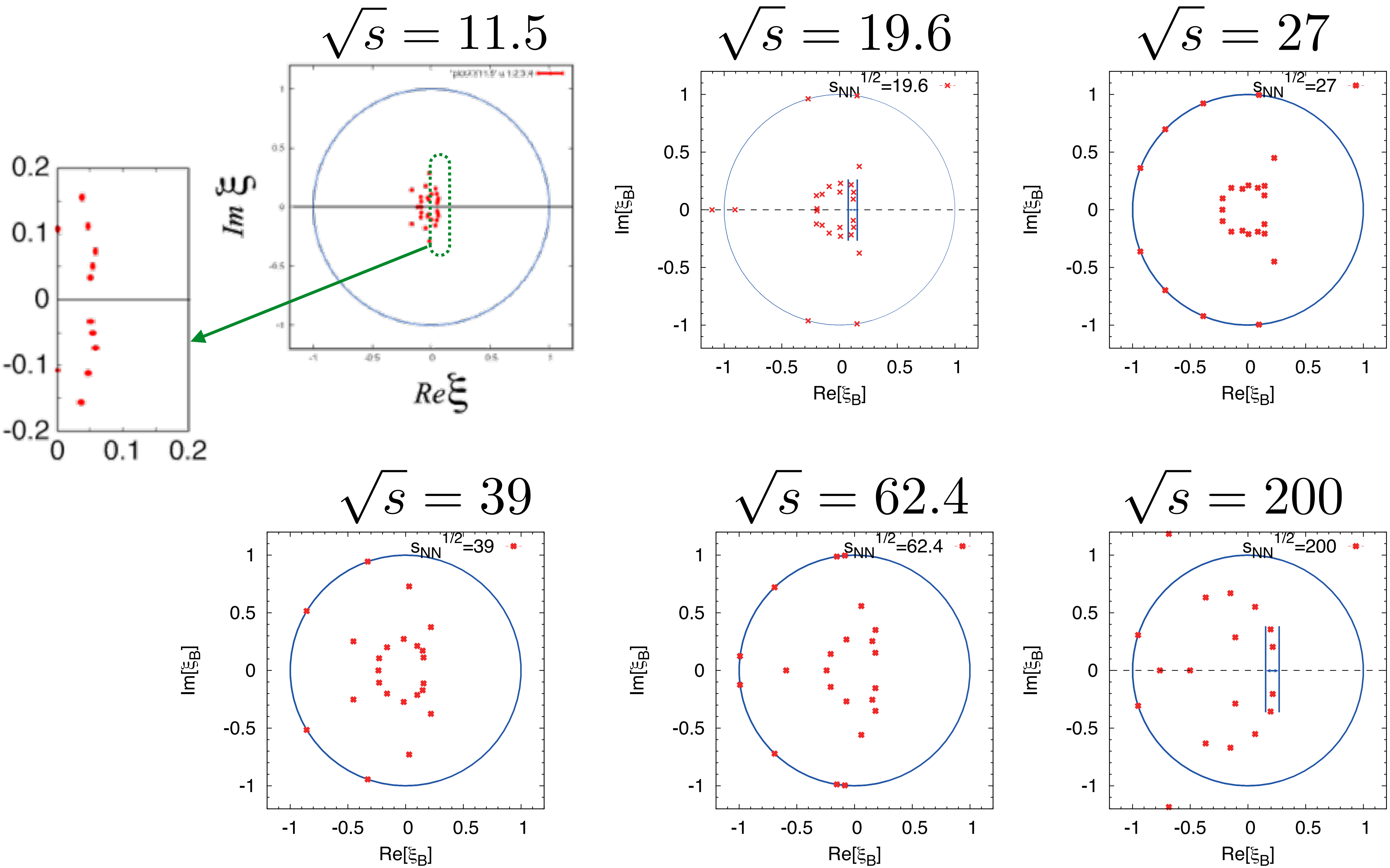
It’s ME

Riemann

Lee-Yang Zeros
from
Experimental Data(RHIC)



Lee-Yang Zeros: RHIC Experiments



We can calculate Z_n also by Lattice QCD

But Sign Problem on Lattice ?

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields}) \times \boxed{\det D(\mu)} \; e^{-(\text{Gluon Action})}$$

Complex if μ is real.

Our Lattice

- Clover improved Wilson action
- Iwasaki gauge action
- Lattice 4×16^3 (L ≈ 3.2fm, a ≈ 0.2fm)
- $m_\pi / m_\rho = 0.8$ ($m_\pi = 0.7$GeV)
  T/T_c = 0.84, 0.93,0.99, 1.08, 1.20, 1.35
- 20 - 40 points Imμ,
  1800 - 3800 configurations at each point
- Parameters were taken from
  S. Ejiri et. al., PRD 82, 014508 (2010)
- Our cluster: Vostok1 (20 GPU K40)

For Pure Imaginary μ ➡ $\det D$ real

. M. D'Elia, M. P. Lombardo, Proceedings of the GISELDA Meeting held in Frascati, Italy, 14-18 January 2002, hep-lat/0205022, 22 May 2002
Ph. de Forcrand, O. Philipsen, Nucl. Phys. B642 290 (2002)16, hep-lat/02050,16 May 2002

A.Hasenfratz and Toussant, 1992

$$Z_n = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

All information is in Imaginary Chemical Potential regions!

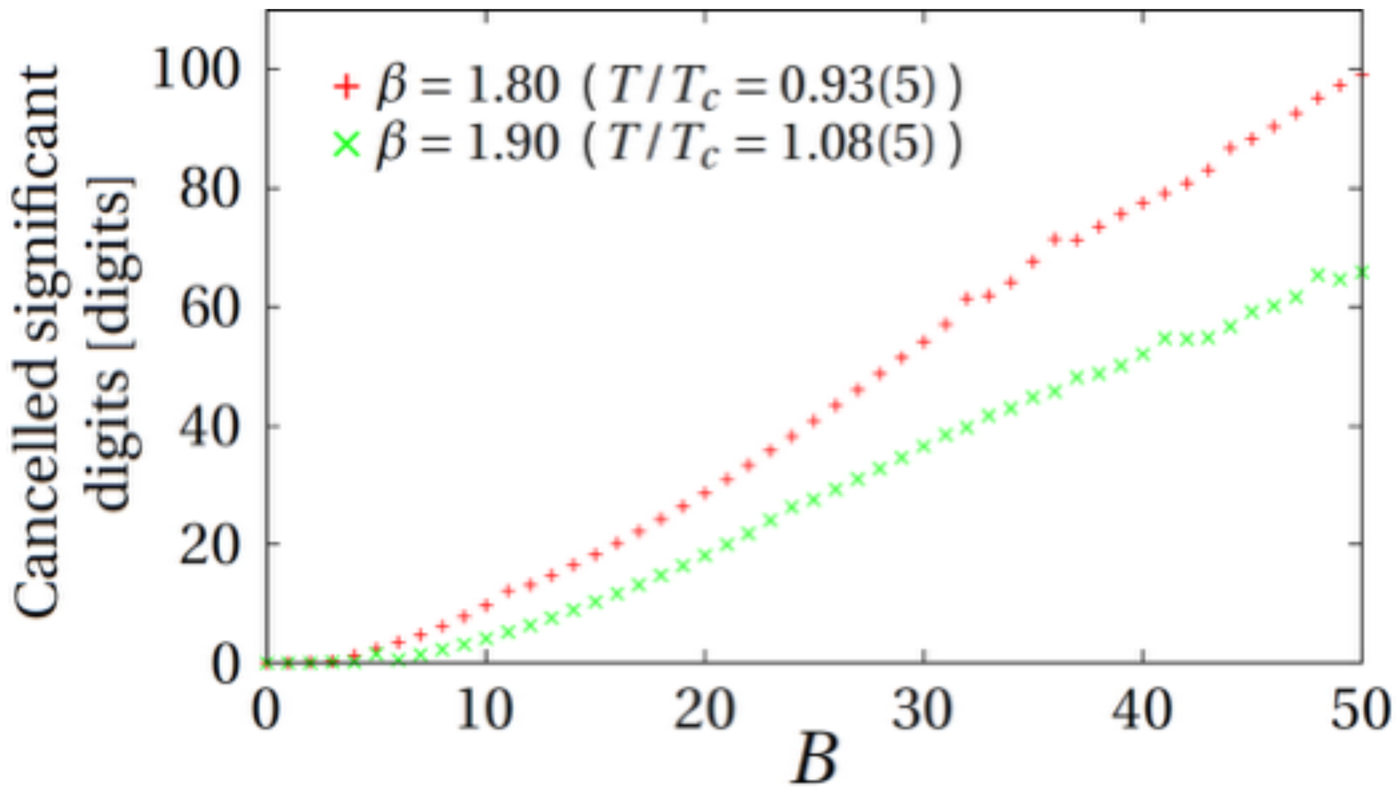
Great Idea ! But practically it did not work.

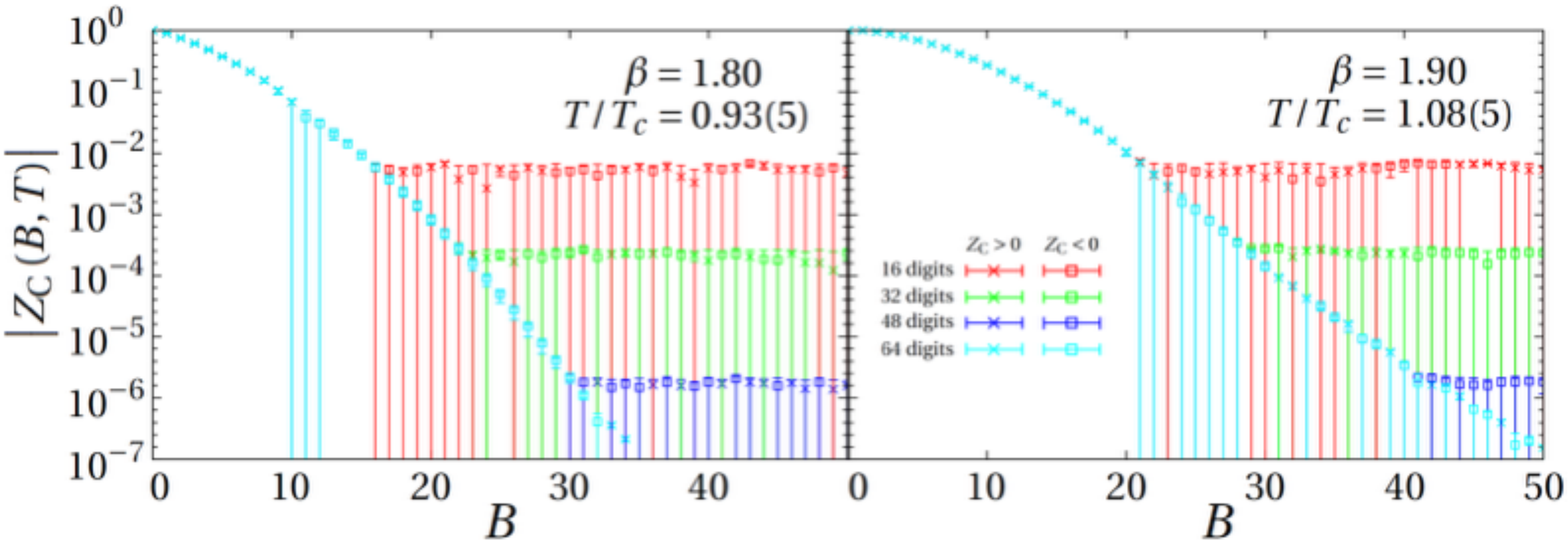
For few years,we must develop several Engineering Methds.

1) Integration method
2) Multi-Precision Calculations

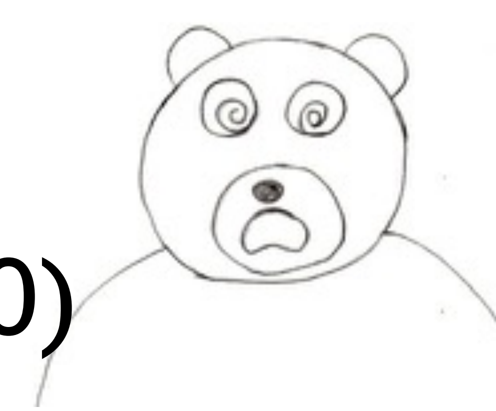
Big Cancellation in FFT !

S.Oka, arXiv:1511.04711





θintegration → Multi-Precision (50 - 100)



Integration Method

$$n_B = \frac{1}{3V} T \frac{\partial}{\partial \mu} \log Z_G$$

$$= \frac{N_f}{3 N_s^3 N_t} \int \mathcal{D}U e^{-S_G} \mathrm{Tr} \Delta^{-1} \frac{\partial \Delta}{\partial \mu} \det \Delta$$

(For pure imaginary μ, n_B is also imaginary)

Then, for fixed T

$$Z(\theta \equiv \frac{\mu}{T}) = \exp(V \int_0^\theta n_B d\theta')$$

$$Z_k = \frac{3}{2\pi} \int_{-\pi/3}^{+\pi/3} d\theta \exp\left(i\, k\theta + \int_0^\theta n_B d\theta' \right)$$

We map Information in Pure Imaginary Chemical Potential to Real ones.

See also, D'Elia, Gagliardi and Sanfilippo, Phys. Rev. D 95, 094503 (2017)

- We measure the number density at many pure imaginary chemical potential $n_B(\mu_I)$.

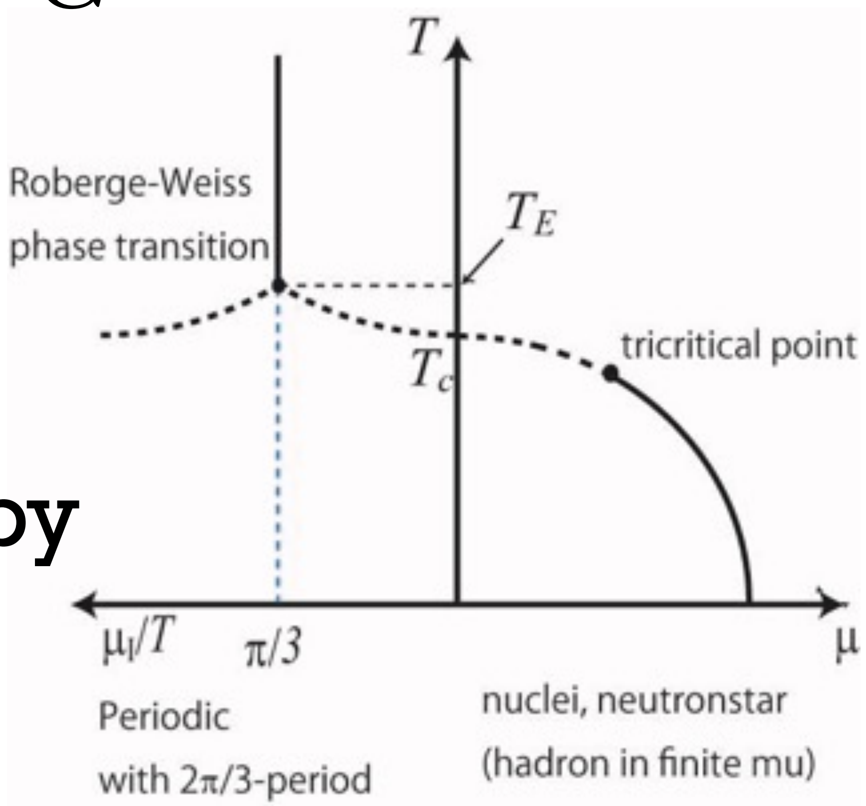
- We construct Grand Partition Function Z_G, by integrating $n_B(\mu_I)$

- By Fourier transformation, we get Z_n

- Then we can calculate Real μ regions by

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

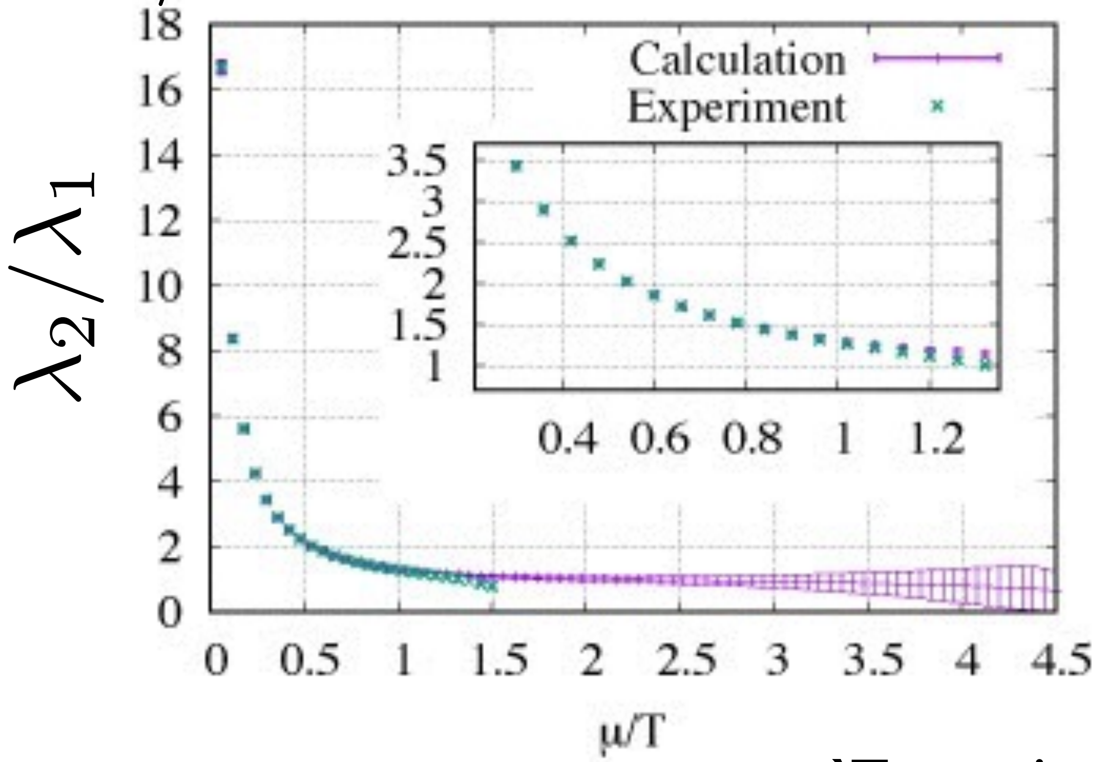
$$\xi \equiv e^{\mu/T}$$ Fugacity

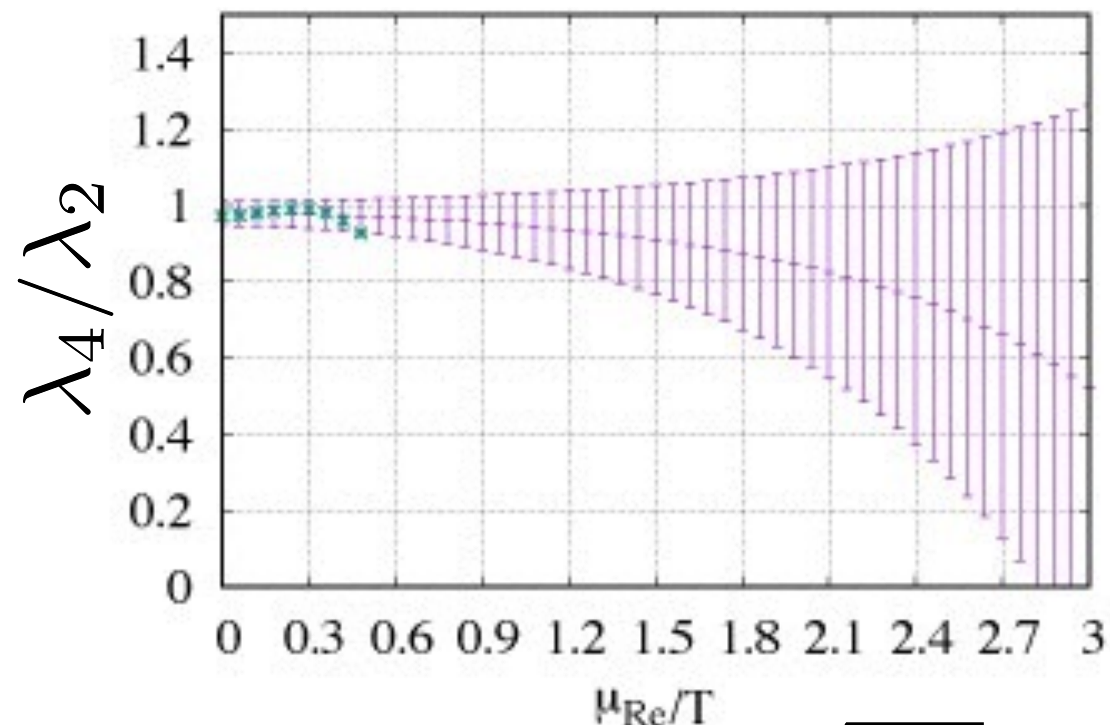


Moments $\lambda_k = (T\frac{\partial}{\partial\mu})^k \log Z$

D.Boyda

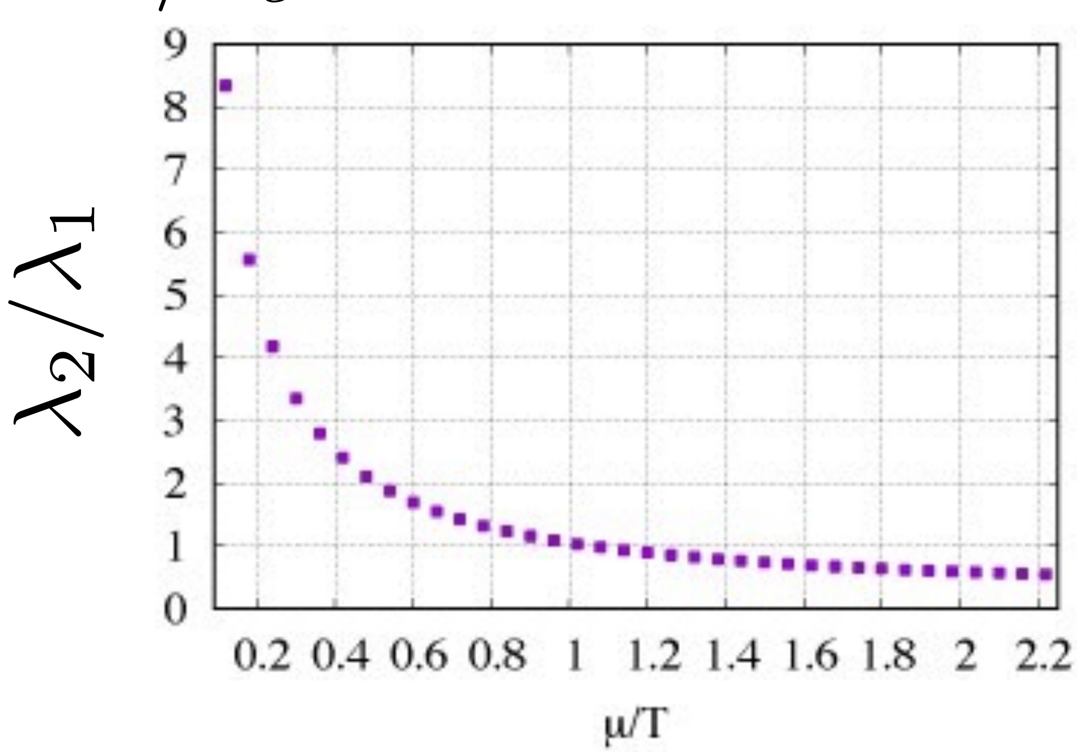
$T/T_c = 0.93$

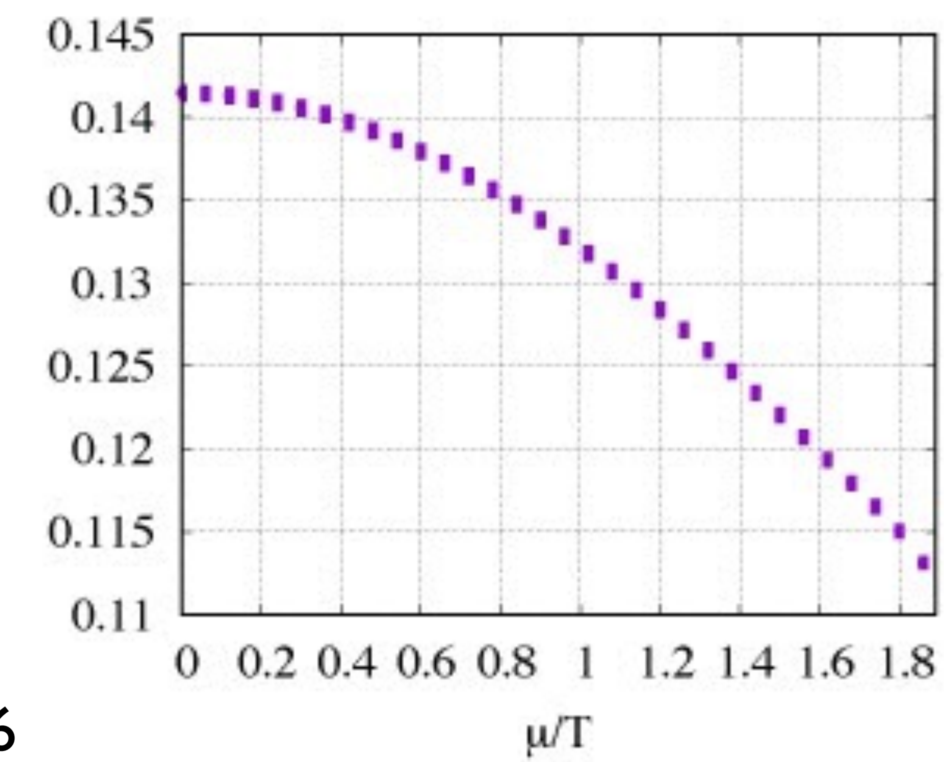




✖ `Experiment' constructed from RHIC Star $\sqrt{s_{NN}} = 39\,(\mathrm{GeV})$

$T/T_c = 1.35$

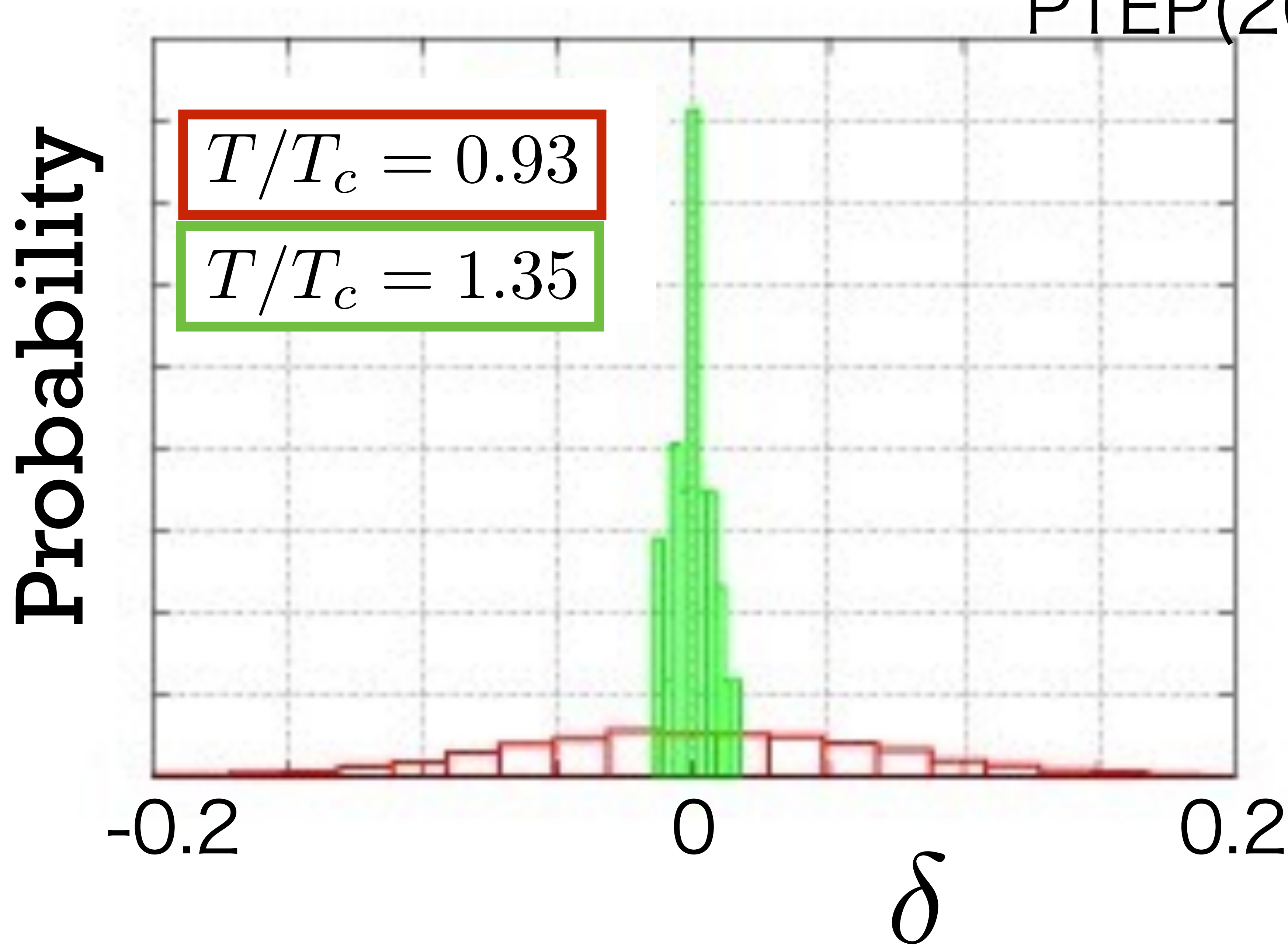




Hidden Sign Problem ?

Z_n **have phase on each configuration !**

V.Goy et al.,
PTEP(2017) 031D01



$z_n \simeq |z_n| e^{in\delta}$

$Z_n = \langle z_n \rangle$
are real positive.

References

A.Li et al.(Kentucky), Phys.Rev.D82:054502,2010, arXiv:1005.4158

A.Suzuki et al.(Zn Collaboration), Lattice 2016 Proceedings,

V.Goy et al.(Vladivostok), Prog Theor Exp Phys (2017) (3): 031D01,arXiv:1611.08093

Where comes the phase of z_n ?

A.Li et al.(Kentucky), Phys.Rev.D82:054502,2010,
arXiv:1005.4158

$$Z = \int \mathcal{D}U \left(\det \Delta(\mu)\right)^{N_f} e^{-S_G} \quad = e^{\log(1-\kappa Q)}$$

$$\begin{aligned} \det \Delta(\mu) &= \det(1 - \kappa Q(\mu)) \\ &= \exp\left(A_0 + \sum_{n>0} [e^{in\phi} W_n + e^{-in\phi} W_n^\dagger] \right) \\ &= \exp\left(A_0 + \sum_n A_n \cos(n\phi + \delta_n) \right) \end{aligned}$$

$$\begin{aligned} A_n &\equiv 2|W_n| \\ \delta_n &\equiv arg(W_n) \end{aligned}$$

We use $W_{-n} = W_n$

Then,

$$z_n \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-ik\phi} e^{A_0 + A_1 \cos(\phi+\delta_1) + A_1 \cos(2\phi+\delta_2) \cdots}$$

In the lowest order,

$$\int_0^{2\pi} \frac{d\phi}{2\pi} e^{-ik\phi} e^{A_0 + A_1 \cos(\phi+\delta_1)} = e^{A_0} \int_{\delta_1}^{2\pi+\delta_1} \frac{d\phi'}{2\pi} e^{-ik(\phi'-\delta_1)} e^{A_1 \cos\phi'}$$

$$= e^{A_0+ik\delta_1} \int_{\delta_1}^{2\pi+\delta_1} \frac{d\phi'}{2\pi} e^{-ik\phi'} e^{A_1 \cos\phi'}$$

$$= e^{A_0+ik\delta_1} \int_0^{2\pi} \frac{d\phi'}{2\pi} e^{-ik\phi'} e^{A_1 \cos\phi'}$$

$$= e^{A_0+ik\delta_1} I_k(A_1)$$

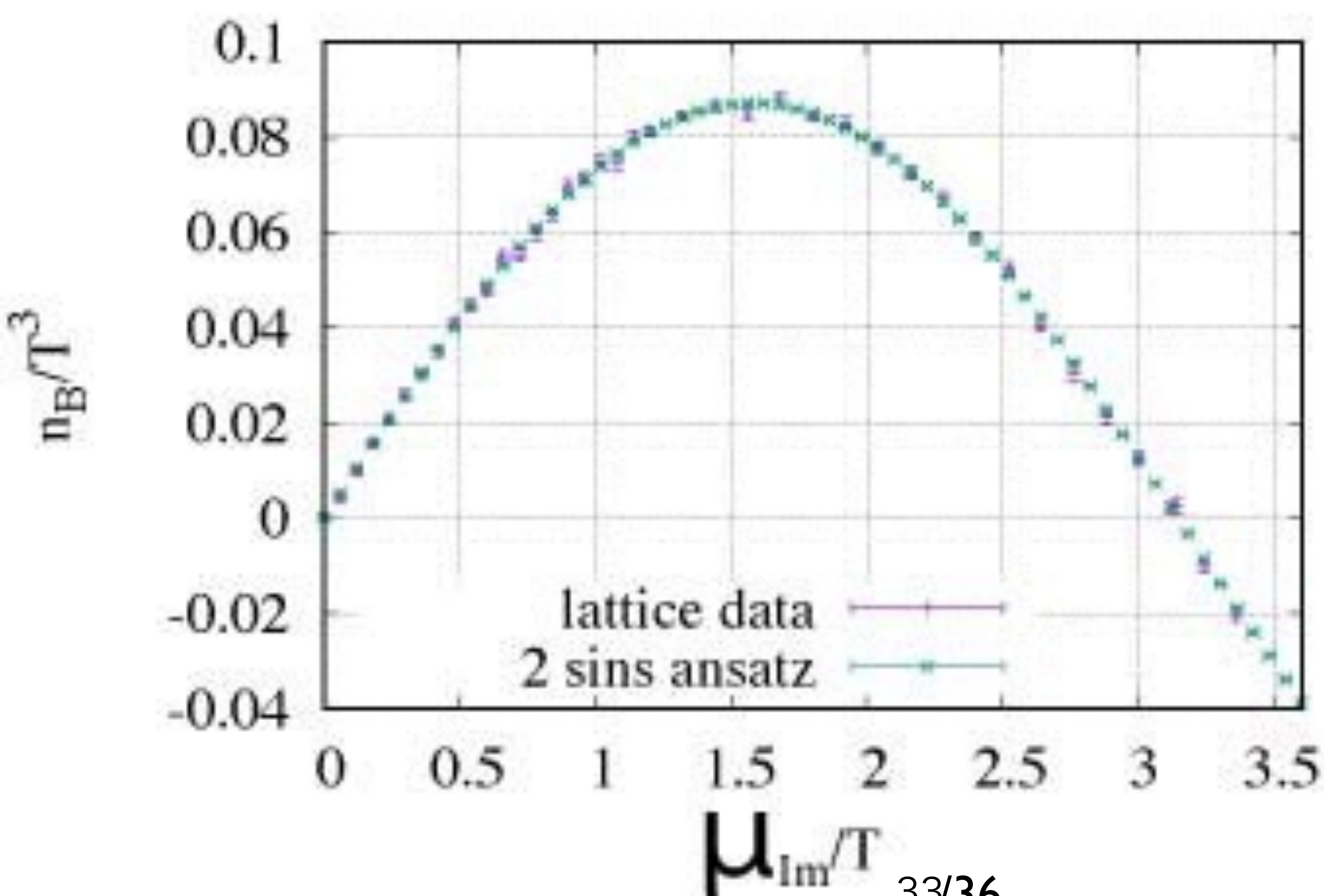
$$\propto z_k$$

where we use

$$I_n(z) = \frac{(-1)^n}{2\pi} \int_0^{2\pi} e^{z\cos t} e^{-int} dt$$

A Remark of Function Form of $n_B(\mu_I)$

Preliminary

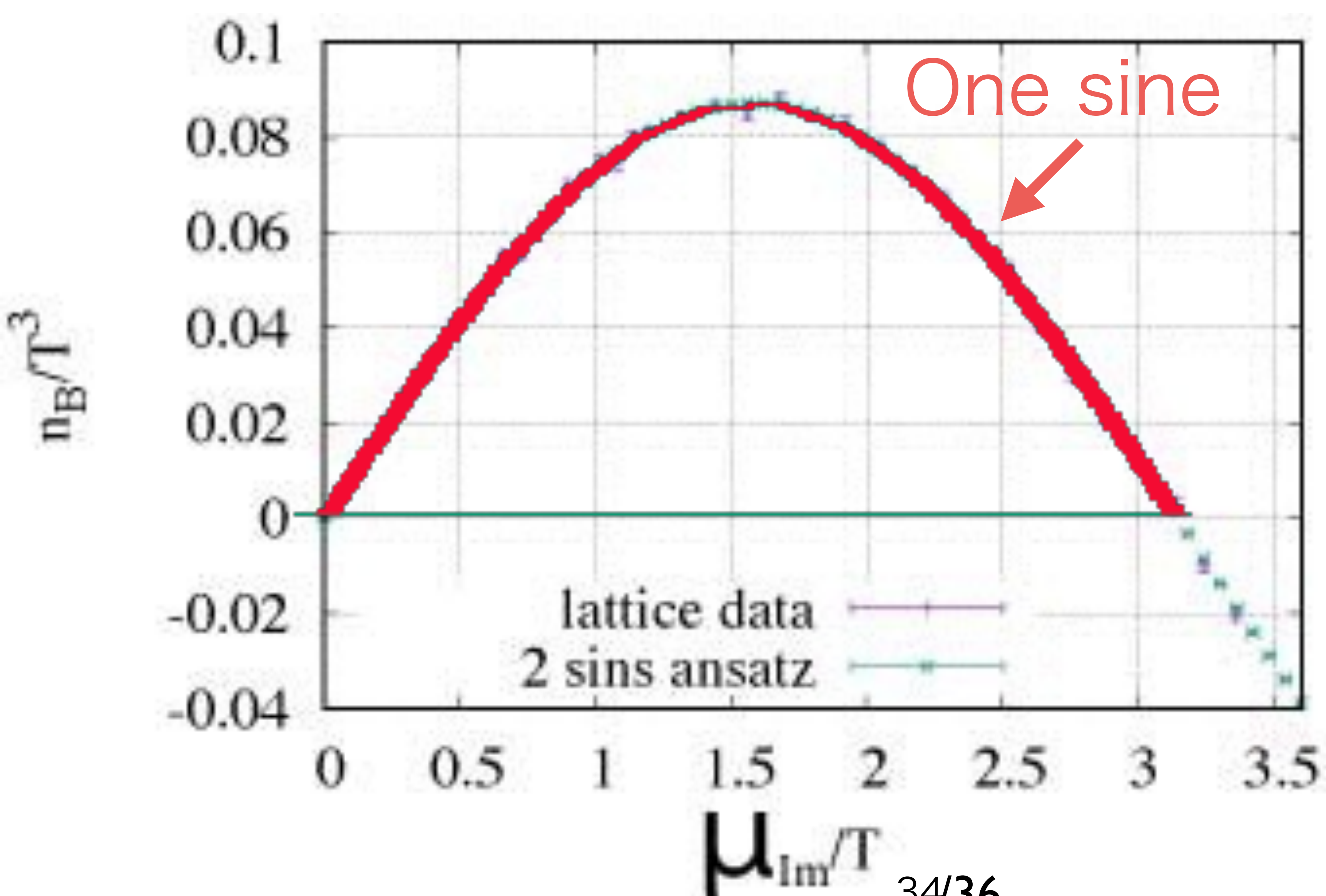


$n_B(\mu_I)$ is well approximated by sine function at $T<Tc$.

Takahashi et al. Phy. Rev. D 91 (1) (2015) 014501.
Bornyakov et al., Phys.Rev. D95, 094506 (2017)

A Remark of Function Form of $n_B(\mu_I)$

Preliminary



$n_B(\mu_I)$ is well approx-imated by sine function at $T<Tc$.

Takahashi et al. Phy. Rev. D 91 (1) (2015) 014501.
Bornyakov et al., Phys.Rev. D95, 094506 (2017)

In general,

$$n_B/T^3 = \sum_k f_{3k} \sin(k\theta_I)$$

$f_3 = 0.0871(3)$, $f_6 = -0.00032(27)$ ($\chi^2/\mathrm{dof} = 0.93$)

Lowest order,

$$n_B/T^3 \sim f_3 \sin(\theta_I)$$

$$\Rightarrow \quad Z_n \propto I_n(f_3)$$

This is Skellam Model, which is used in Heavy Ion Collisions to describe the gross structure.

(Skellam is the difference of two independent Poisson Distributions. f6, f9 … include the dynamics.)

Concluding Remarks

- We have developed the Canonical Approach for revealing QCD Phase Structure.
  The canonical partition functions Z_n drop very rapidly as n goes large, and we need multi-precision calculations.
- The phase of z_n fluctuates rapidly as n goes large in the confinement phase.
  No such problem in the deconfinement phase.
  i.e., we plan to predict thermo-dynamical quantities for LHC.
- Our Quark masses are heavy.
  Now it is time to go towards Physical Parameters.

Coming Soon !