QCD phase structure
and
Heavy Ion Collisions

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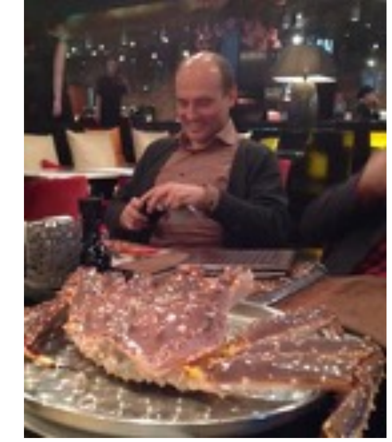
Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics

Dubna, 10-14 July 2017

FEFU
FAR EASTERN FEDERAL UNIVERSITY

A. Molochkov



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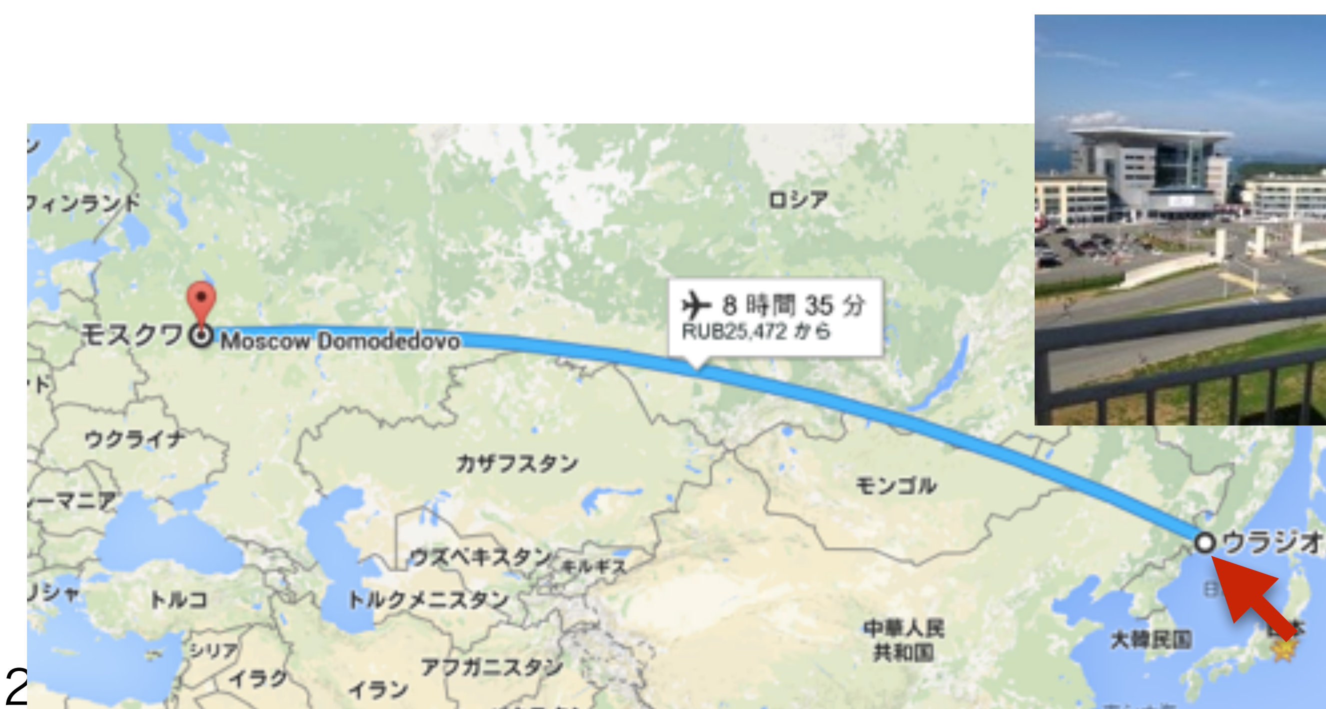


Main Message
that I want to show in this Talk

NICA and J-PARC will
provide us
Important Information
about QCD Phase Diagram

And our
Canonical
Formulation
may be
useful
there.

It takes very
long time until
your idea is understood.

But I use
only Statistical
Mechanics !?

L.M.

Because your
Approach is
different.

Lattice QCD simulations provide the fundamental information as a first principle calculation.

However,

Sign Problem in Finite Density lattice QCD prevents our mission.

QCD at finite density

μ : Chemical Potential

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi\, e^{-\beta S_G - \bar{\psi}\Delta\psi}$$

$$= \int \mathcal{D}U \prod_f \det \Delta(\mu)\, e^{-\beta S_G}$$

$$\Delta(\mu) = D_\nu \gamma_\nu + m + \mu\gamma_0$$

$$\Delta(\mu)^\dagger = -D_\nu \gamma_\nu + m + \mu^* \gamma_0 = \gamma_5 \Delta(-\mu^*)\gamma_5$$

$$\Rightarrow (\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$$

$(\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$

For $\mu = 0$

$$(\det \Delta(0))^* = \det \Delta(0)$$

$\det \Delta$ ➡ *Real*

For $\mu \neq 0$ (in general)

$\det \Delta$ ➡ *Complex*

$$Z = \int \mathcal{D}U \prod_f \det \Delta(m_f, \mu_f) e^{-\beta S_G}$$

⬆

Complex ➡ Sign Problem

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \, O \; \det \Delta \, e^{-\beta S_G}$$

In Monte Carlo simulation, configurations are generated according to the Probability:

$$\det \Delta \, e^{-\beta S_G} / Z$$

$\det \Delta$: *Complex*

⇒ Monte Carlo Simulations very difficult !

$$\langle O \rangle = \frac{\int DU O \det \Delta e^{-S_G}}{\int DU \det \Delta e^{-S_G}}$$

$$\det \Delta = |\det \Delta| e^{i\theta}$$

$$\langle O \rangle = \frac{\int DU O |\det \Delta| e^{i\theta} e^{-S_G}}{\int DU |\det \Delta| e^{-S_G}} \times \frac{\int DU |\det \Delta| e^{-S_G}}{\int DU |\det \Delta| e^{i\theta} e^{-S_G}}$$

$$= \frac{\langle O e^{i\theta} \rangle_{|\det|}}{\langle e^{i\theta} \rangle_{|\det|}}$$

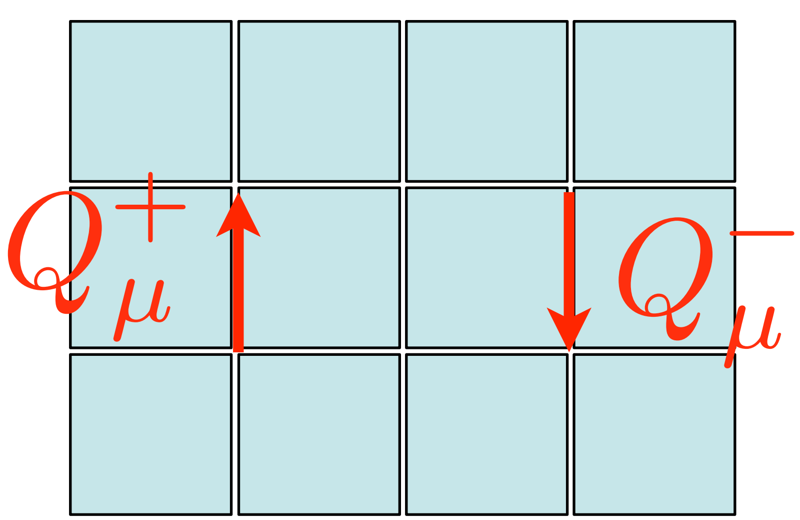
Origin of the Sign Problem

Wilson Fermions $\Delta = I - \kappa Q$

KS(Staggered) Fermions $\Delta = m - Q'$

$$= m(I - \frac{1}{m}Q)$$

$$Q = \sum_{i=1}^{3} (Q_i^+ + Q_i^-) + (e^{+\mu} Q_4^+ + e^{-\mu} Q_4^-)$$



$$Q_\mu^+ = **U_\mu(x)\delta_{x',x+\hat{\mu}}$$

$$Q_\mu^- = **U_\mu^\dagger(x')\delta_{x',x-\hat{\mu}}$$

$$\det \Delta = e^{\mathrm{Tr}\log\Delta} = e^{\mathrm{Tr}\log(I-\kappa Q)}$$

$$= e^{-\sum_n \frac{1}{n}\kappa^n \mathrm{Tr} Q^n}$$

Hopping Parameter expansion or 1/(Large Mass) expansion

Only closed loops remain.

The lowest μ dependent terms

$$\kappa^{N_t} e^{\mu N_t} \mathrm{Tr}(Q^+ \cdots Q^+)$$

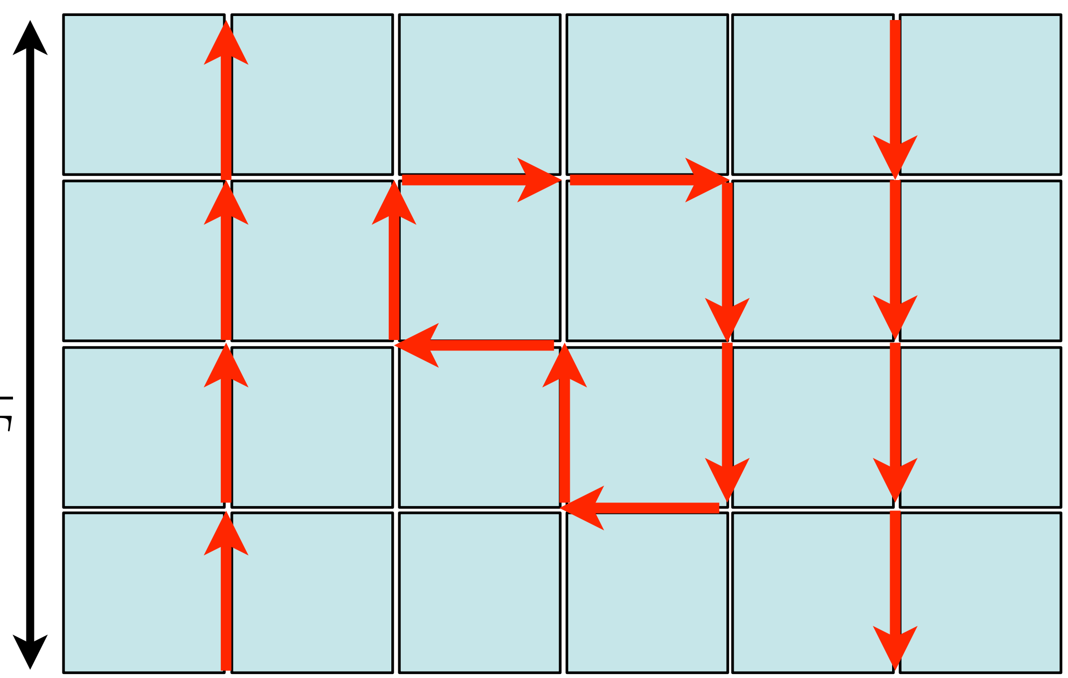
$$= * * \kappa^{N_t} e^{\mu/T} \mathrm{Tr} L$$

$$\kappa^{N_t} e^{-\mu N_t} \mathrm{Tr}(Q^- \cdots Q^-)$$

$$= * * \kappa^{N_t} e^{-\mu/T} \mathrm{Tr} L^\dagger$$

$\mathrm{Tr}L$: Polyakov Loop

$N_t = \frac{1}{T}$



Add the both

$$* * \kappa^{N_t} \left(\cosh \frac{\mu}{T} \Re \mathrm{Tr} L + i \sinh \frac{\mu}{T} \Im \mathrm{Tr} L \right)$$

There are several cases where no Sign Problem occurs

- **Pure Imaginal chemical potential**
  - $(\det\Delta(\mu))^* = \det\Delta(-\mu^*)$

$$\mu = i\mu_I \quad \Rightarrow \quad (\det\Delta(\mu_I))^* = \det\Delta(\mu_I)$$

- **Color SU(2)**
  - $U_\mu^* = \sigma_2 U_\mu \sigma_2$

$$\det\Delta(U,\gamma_\mu)^* = \det\Delta(U^*,\gamma_\mu^*) = \det\sigma_2\Delta(U,\gamma_\mu^*)\sigma_2$$
$$= \det\Delta(U,\gamma_\mu)$$

- **Finite iso-spin**

$\mu_d = -\mu_u$

$$\det\Delta(\mu_u)\det\Delta(\mu_d) = \det\Delta(\mu_u)\det\Delta(-\mu_u)$$
$$= \det\Delta(\mu_u)\det\Delta(\mu_u)^* = |\det\Delta(\mu_u)|^2$$

(Phase Quench)

Pion-Condensation Problem

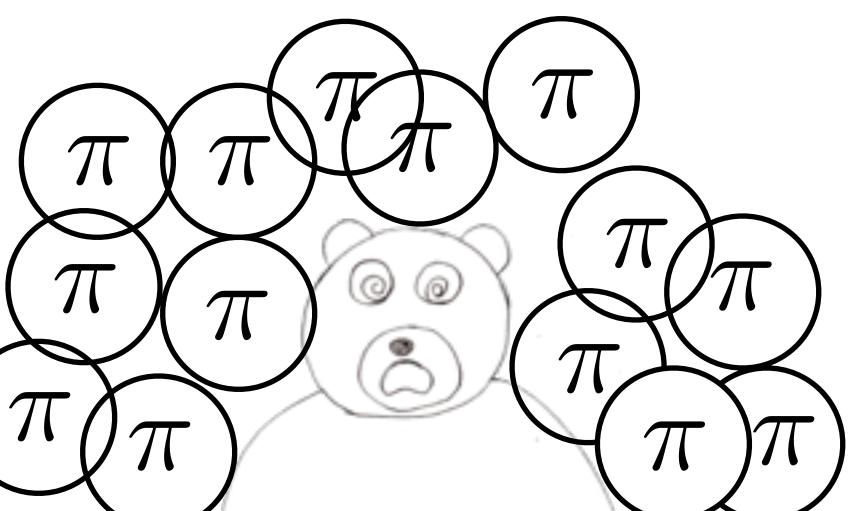
Phase Quench = Finite-Isospin

$$\int |\det\Delta(\mu)|^2 e^{-S_G} = \int \det\Delta(\mu)\det\Delta(\mu)^* e^{-S_G}$$

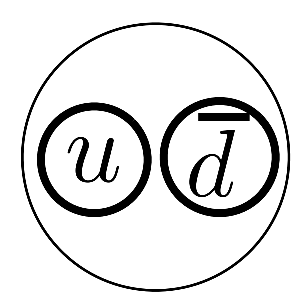
$$= \int \det\Delta(\mu)\det\Delta(-\mu) e^{-S_G}$$

$$= \int \det\Delta(\mu_u)\det\Delta(\mu_d) e^{-S_G}$$

$$\mu_u = \mu, \quad \mu_d = -\mu$$



For $\mu > \dfrac{m_\pi}{2}$

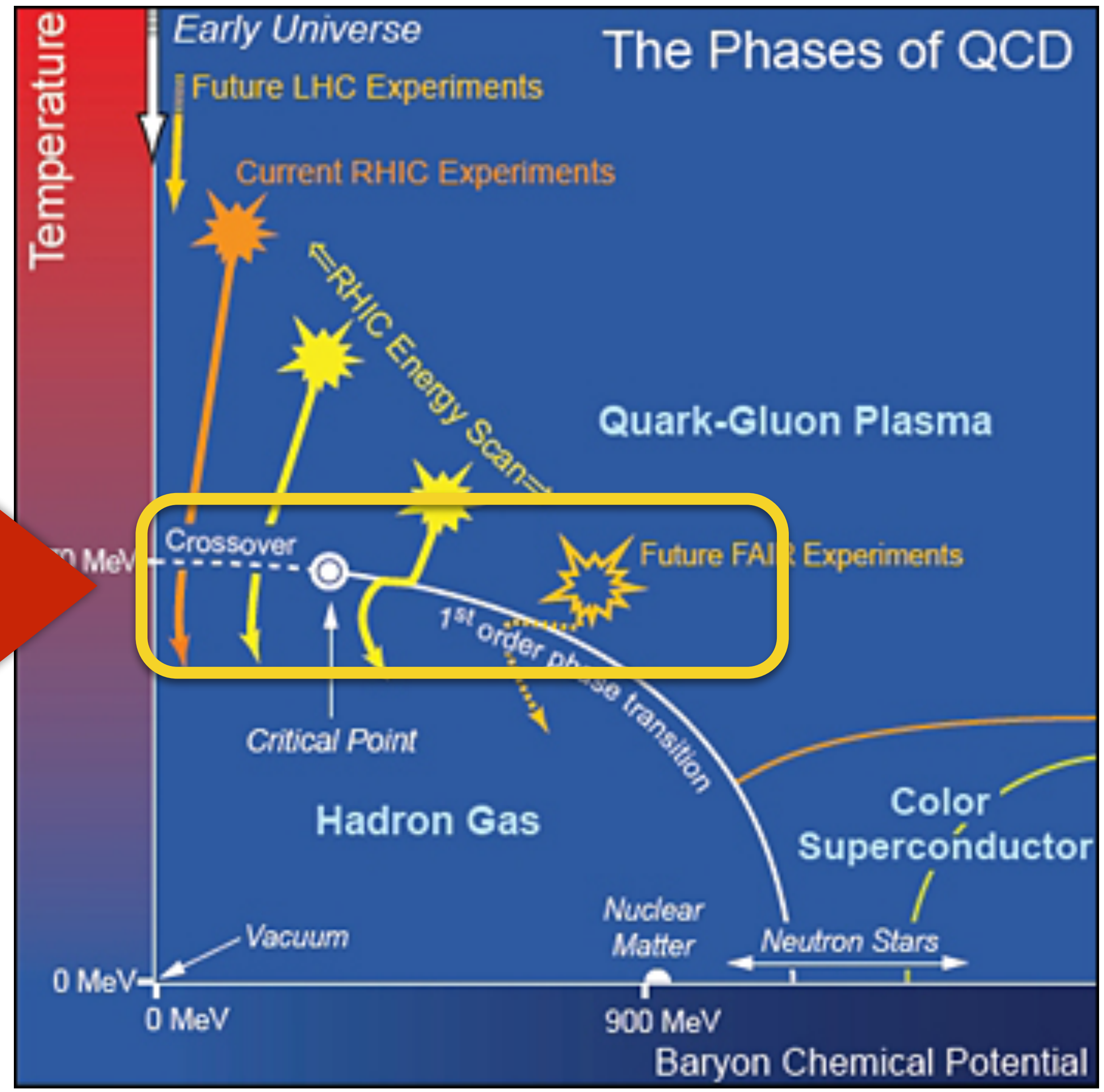


π^+ is created by μ

Objective of Vladivostok Group

Study here

including
LHC(ALICE),RHIC,FAIR,
NICA, J-PARC

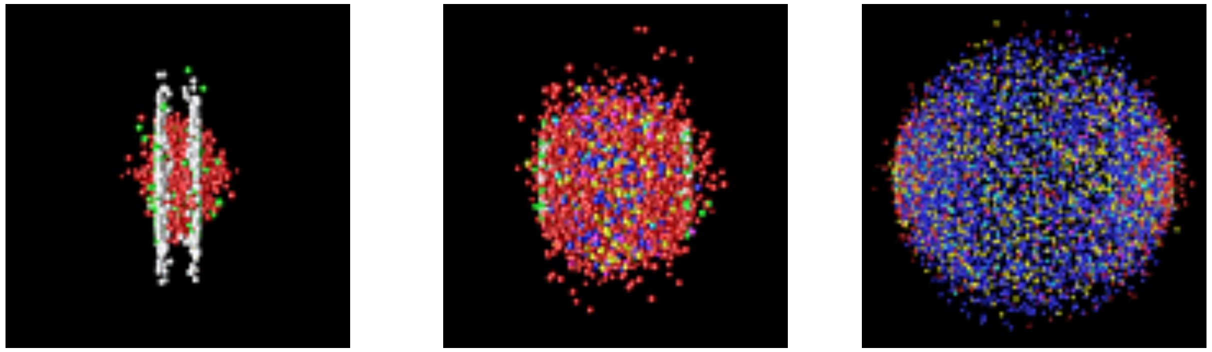


http://www.bnl.gov/rhic/news2/news.asp?a=1870&t=today

We assume

the Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

with μ (chemical Potential)
and T (Temperature)



$$Z(\mu, T)$$

Grand Canonical Partition Function

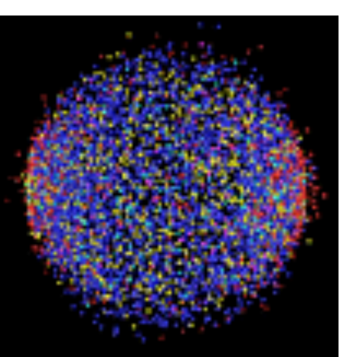
This Statistical Description is good at least as a first approximation

with Two Parameters Chemical Potential, μ
and Temperature, T

$Z_{GC}(\mu, T)$ Grand Canonical Partition Function

Alternative: Number, n and Temperature, T

$Z_C(n, T)$ Canonical Partition Function



or Z_N

Our Tool

Canonical Approach

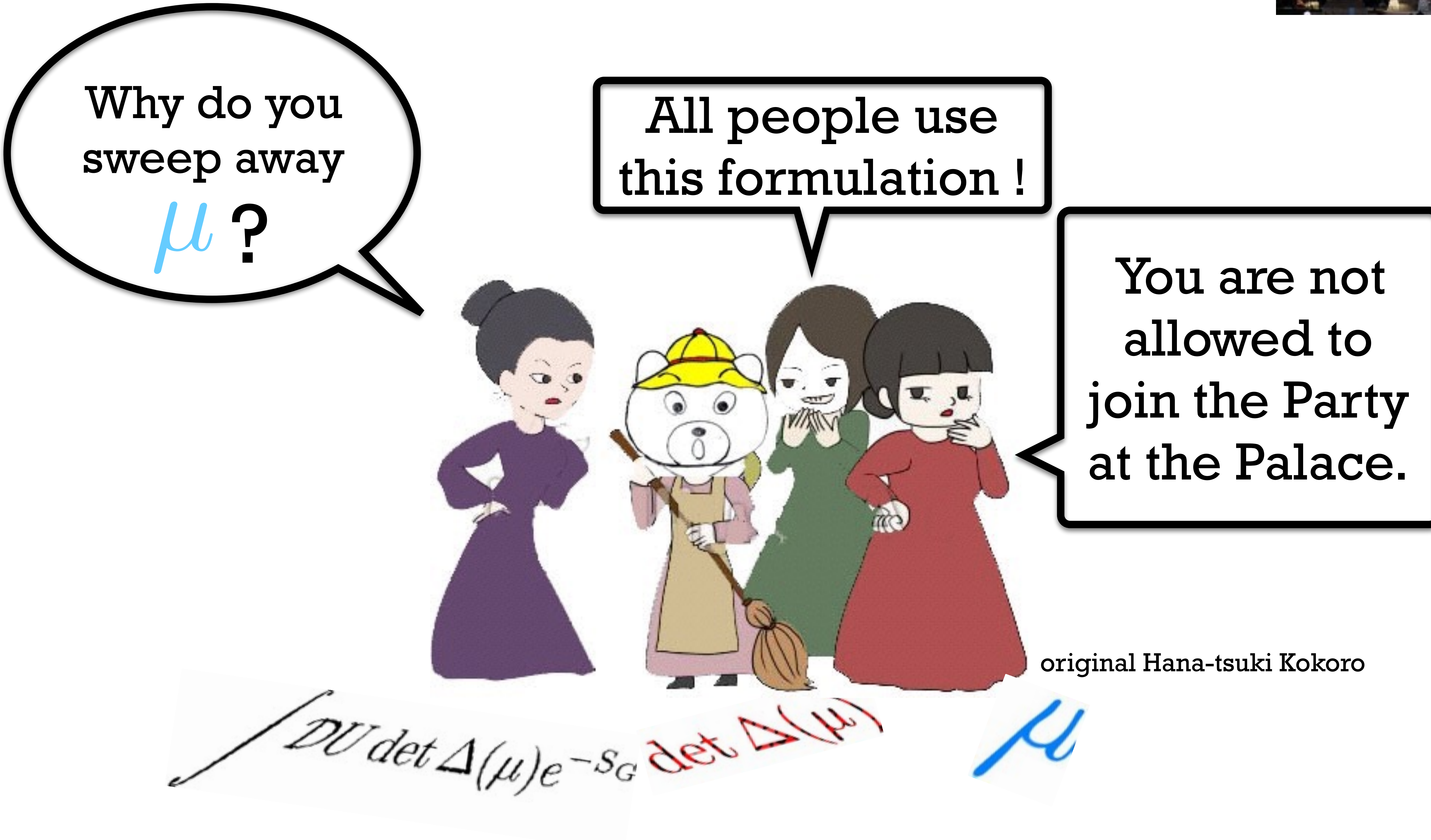
Not so well-known

From Experiments

Z_N

From Lattice

Canonical Partition Functions



original Hana-tsuki Kokoro

Advantage to use Z_n

μ

- We can construct (approximate) Z_n from experimental Baryon number and Charge Distributions.
- We can circumvent the sign problem in Lattice QCD.
- We can construct Grand Partition Function $Z(\mu, T)$ from Z_n.
- New approach, i.e., Challenging !

They are equivalent and related as

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$\xi \equiv e^{\mu/T}$ Fugacity

Quick Proof of
Fugacity Expansion

$$Z(\mu, T) = \sum_n Z_n(T)(e^{\mu/T})^n$$

(Left Hand Side)= $\mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n|e^{-(H-\mu\hat{N})/T}|n\rangle$$

$$= \sum_n \langle n|e^{-H/T}|n\rangle\, e^{\mu n/T}$$

$Z_n(T)$

This is a very useful relation.

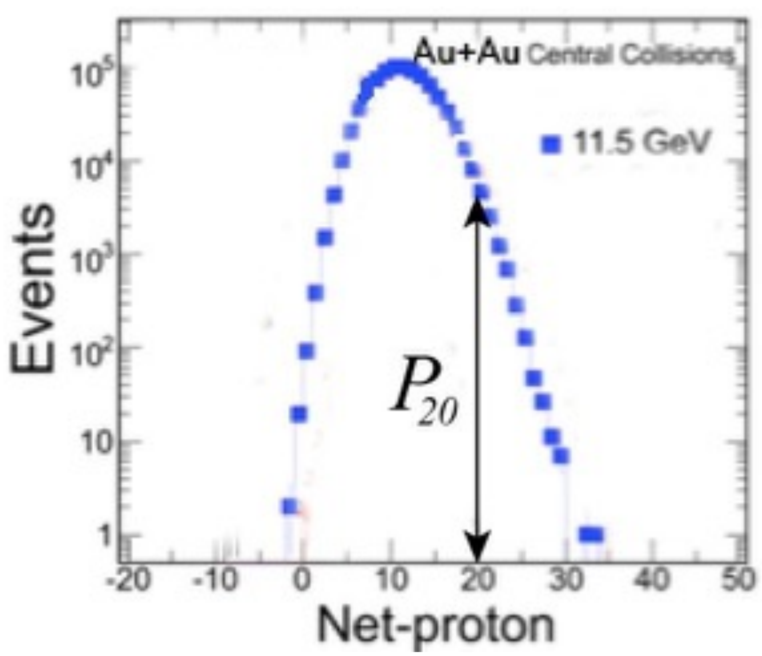
The partition function stands for the Probability

$$Z_{GC}(\mu, T) = \sum_n Z_n(T)\xi^n$$

System with μ and T

Probability to find (net-)baryon number=n

We extract Z_n from experimental multiplicity at RHIC



$$P_n = Z_n \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

ξ unknown

$$Z_n = P_n / \xi^n$$

Z_n satisfies

$$Z_{+n} = Z_{-n}$$

RHIC tells us Z_n

(Particle-AntiParticle Symmetry)

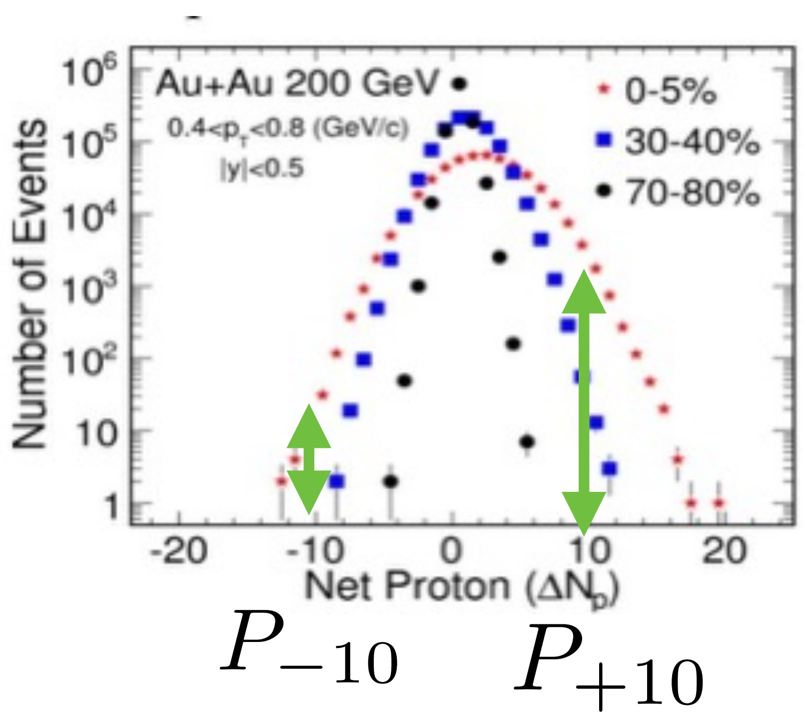
$$P_n = cZ_n\xi^n$$
$$P_{-n} = cZ_{-n}\xi^{-n}$$

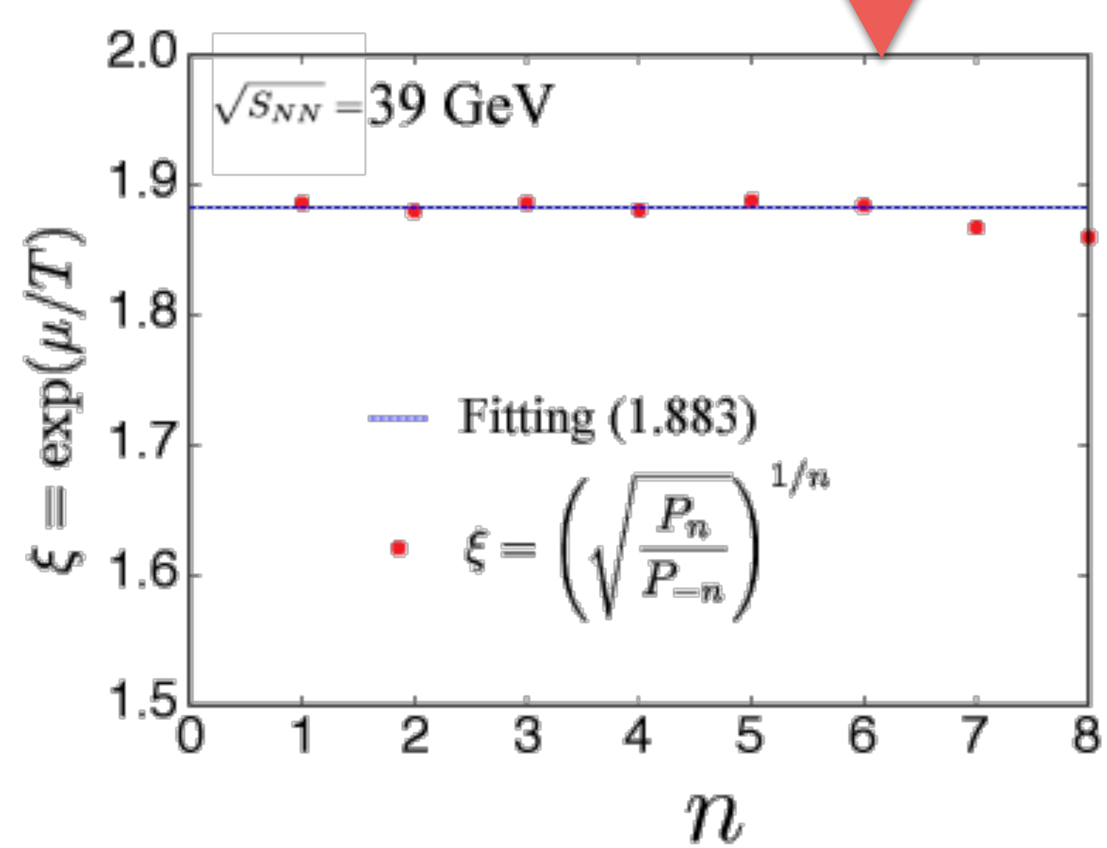
$$Z_{+n} = Z_{-n}$$

$$P_nP_{-n} = c^2Z_nZ_{-n} = c^2Z_n^2$$

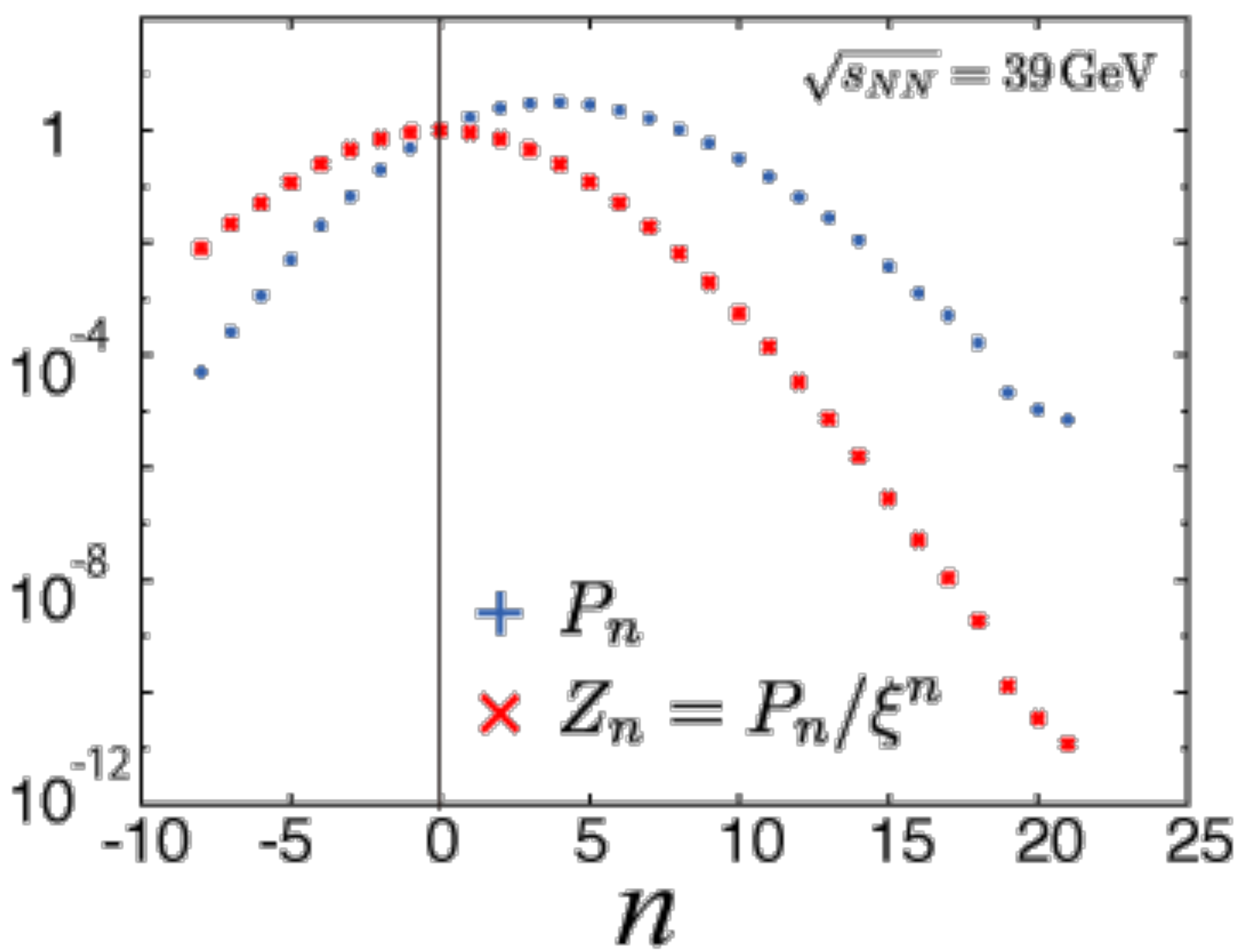
or $\sqrt{P_nP_{-n}} = cZ_n$

$$\xi^n = \frac{P_n}{cZ_n} = \frac{P_n}{\sqrt{P_nP_{-n}}} \quad \Rightarrow \quad \xi = \left(\sqrt{\frac{P_n}{P_{-n}}}\right)^{1/n}$$

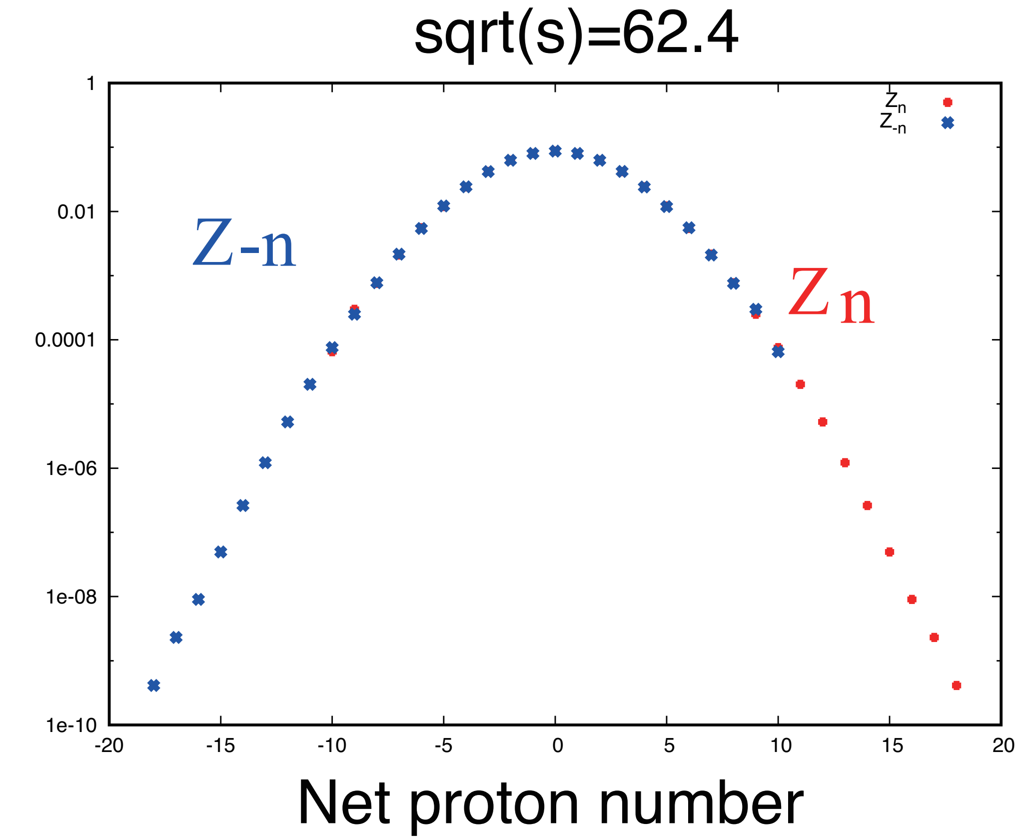




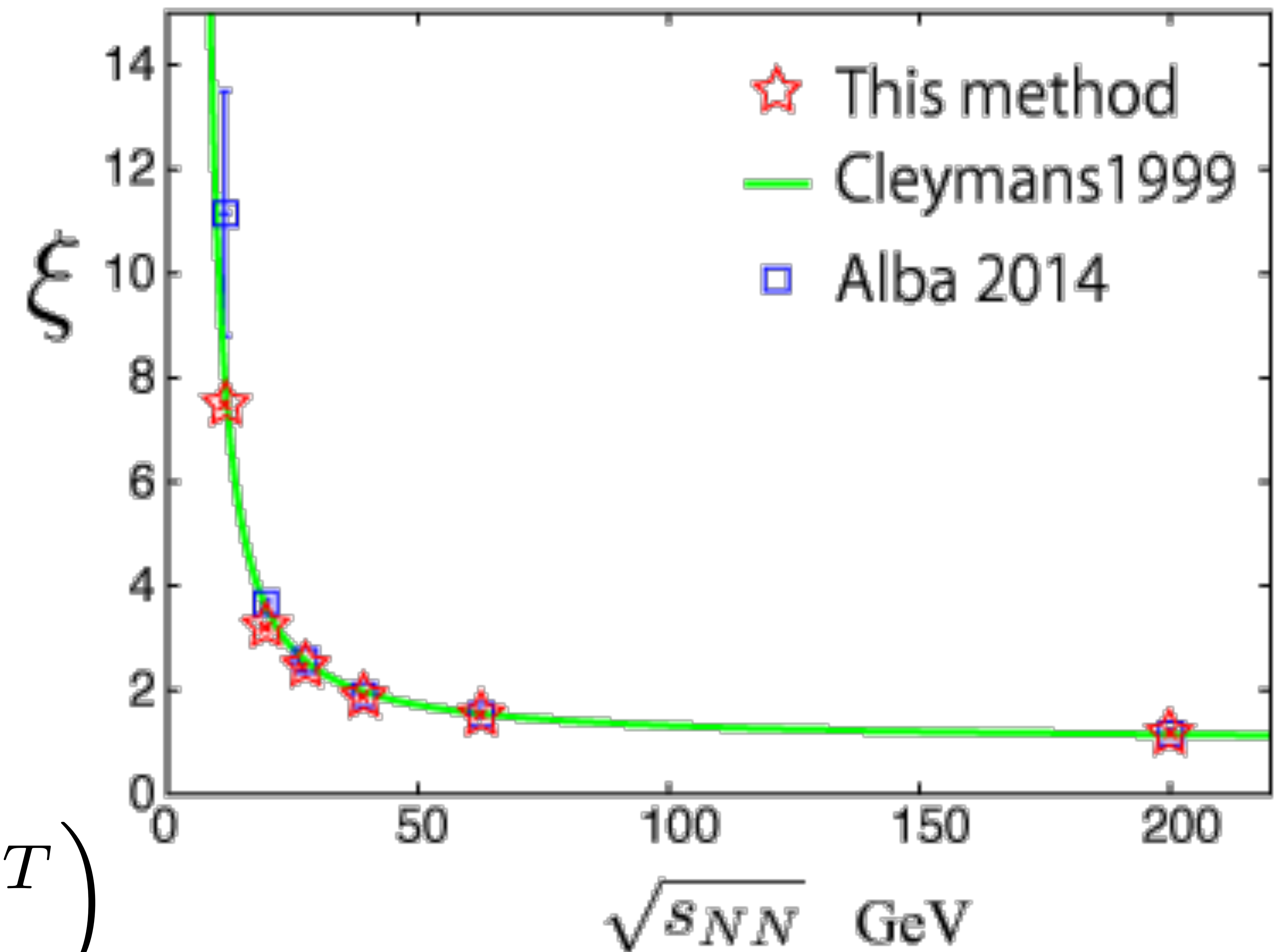
Here we demand $Z_{+n} = Z_{-n}$



$$\xi = 1.88336$$



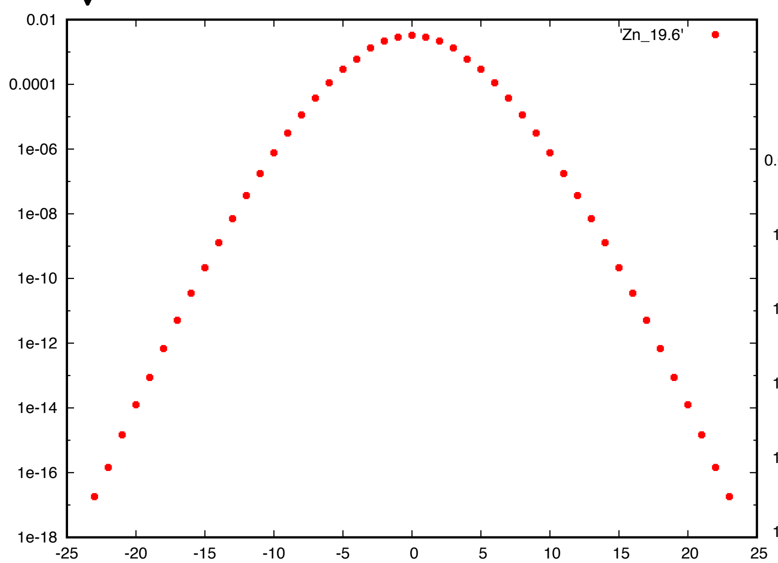
Fitted ξ are very consistent with those by Freeze-out Analysis.



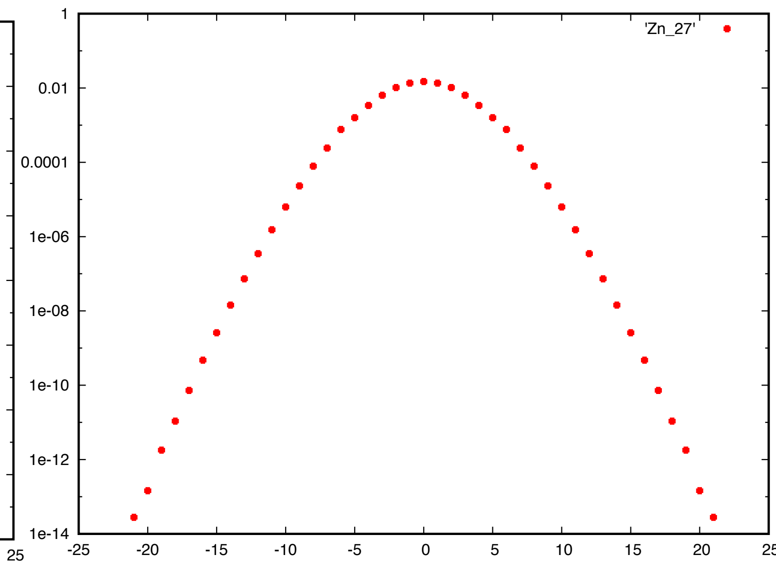
$$\left(\xi \equiv e^{\mu/T}\right)$$

Z_n from RHIC data

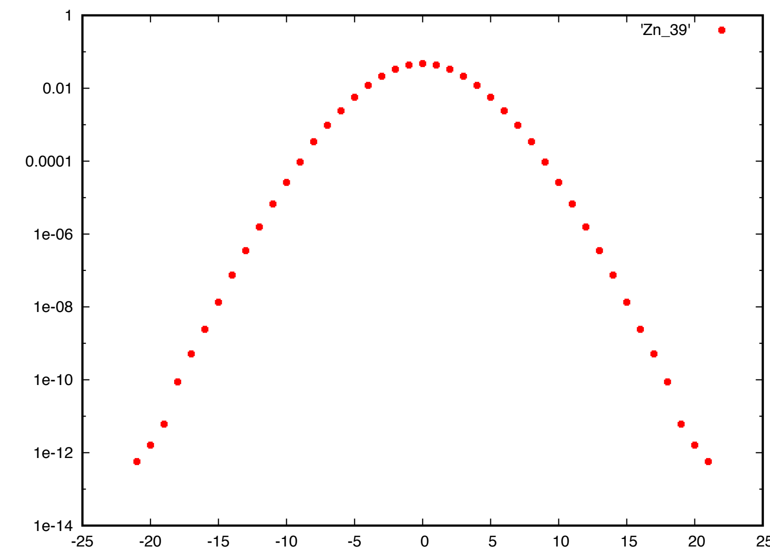
$\sqrt{s} = 19.6\mathrm{GeV}$



$\sqrt{s} = 27\mathrm{GeV}$

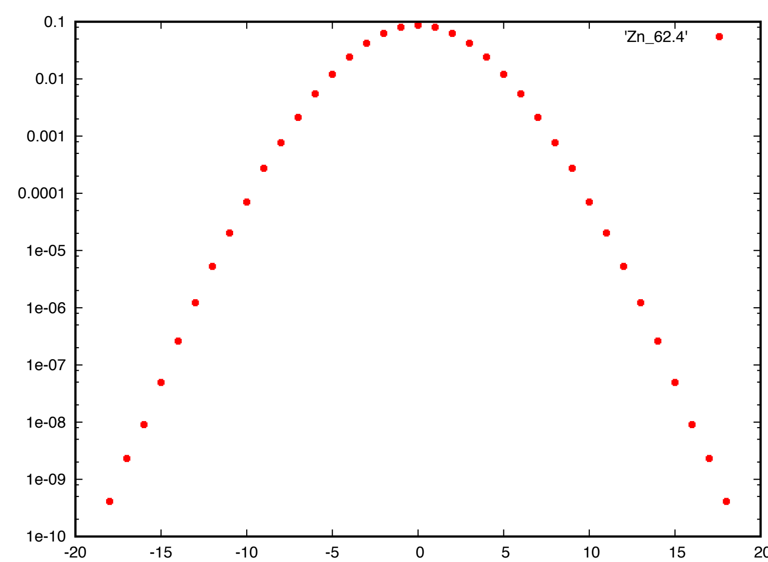


$\sqrt{s} = 39\mathrm{GeV}$

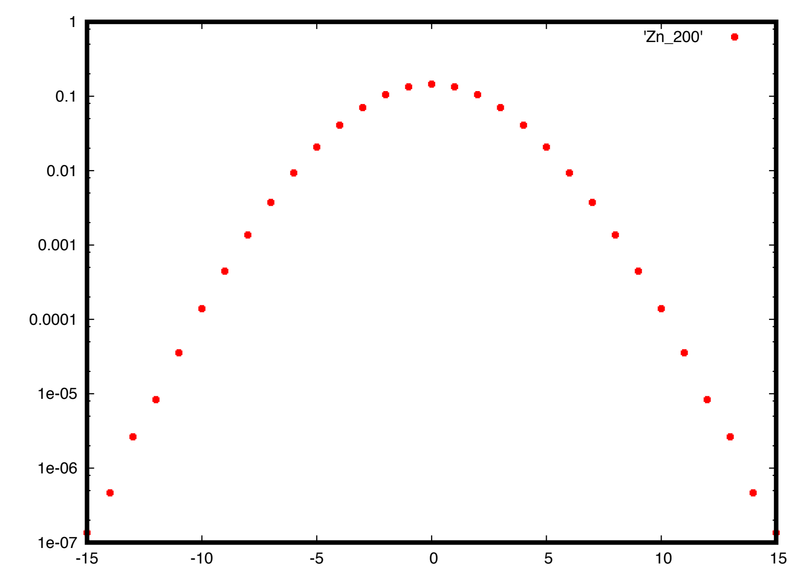


Can I see Difference?

$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



Yes, You Can !
We will see it.

Very useful relation, because

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$(\xi \equiv e^{\mu/T}$: Fugacity$)$

$Z_n(T)$ ← $Z(\xi, T)$ at some ξ and T

$Z_n(T)$ → $Z(\xi, T)$ at ANY ξ

for both Experiments and Lattice