# Lattice QCD for Beyond the Standard Model physics

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Beyond the Standard Model:
HUGE territory!

This talk: Two examples of applications of topics discussed here which have a close connections to BSM physics

I Dark Matter: limit on the mass for post-inflationary Axions from QCD topology

II Higgs mass generation via QCD-like dynamics

The two faces of the axion

Dark Matter Candidate

QCD Topology,
Strong CP Problem

Inspiring paper:
Berkowitz, Buchoff, Rinaldi **Phys.Rev. D92 (2015) no.3, 034507**

# $\theta$ term, strong CP problem and topology

$$\mathcal{L}_{QCD}(\theta) = \mathcal{L}_{QCD} + \frac{g^2\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

Ammitted but $\theta < 10^{-9}$

$$Q = \int d^4x \, \frac{g^2}{32\pi^2}\mathrm{tr}F\tilde{F}$$

$$Z_{QCD}(\theta, T) = \int [dA][d\psi][d\bar{\psi}] \exp\left(-T\sum_t d^3x \, \mathcal{L}_{QCD}(\theta)\right) = \exp[-VF(\theta, T)]$$

$$\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\bigg|_{\theta=0} \equiv \chi(T) = (< Q^2 > - < Q >^2)/V$$

# Axions 'must' be there: solution to the strong CP problem

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Postulate axions, coupled to Q:

$$\mathcal{L}_{\rm axions} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\frac{a}{f_a} + \theta\right)\,\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

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Axion potential

$$m_a^2(T)f_a^2 = \left.\frac{\partial^2F(\theta,T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi(T),$$

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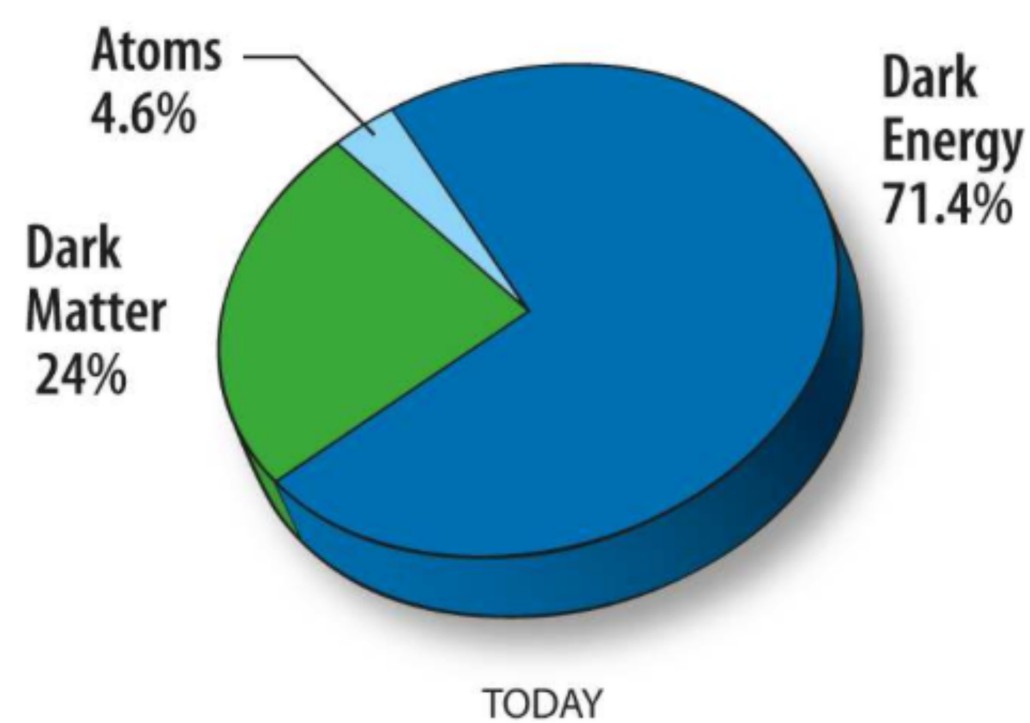
Axion:

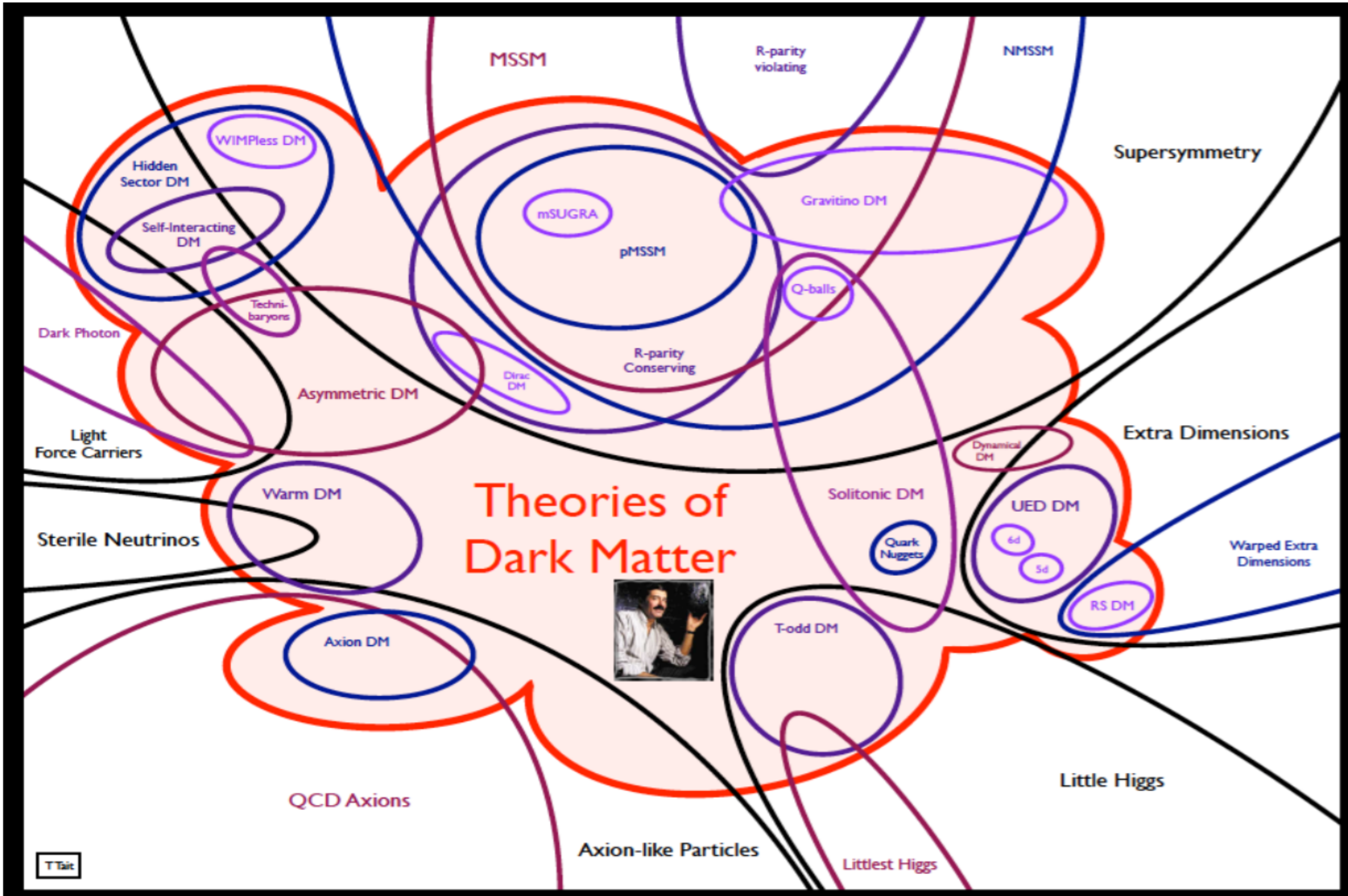
theoretically well motivated

searched and not found in experiments ->weakly coupled

-> Dark Matter Candidate



# Theory landscape (From Tim Tait, Snowmass)



Axion:

theoretically well motivated

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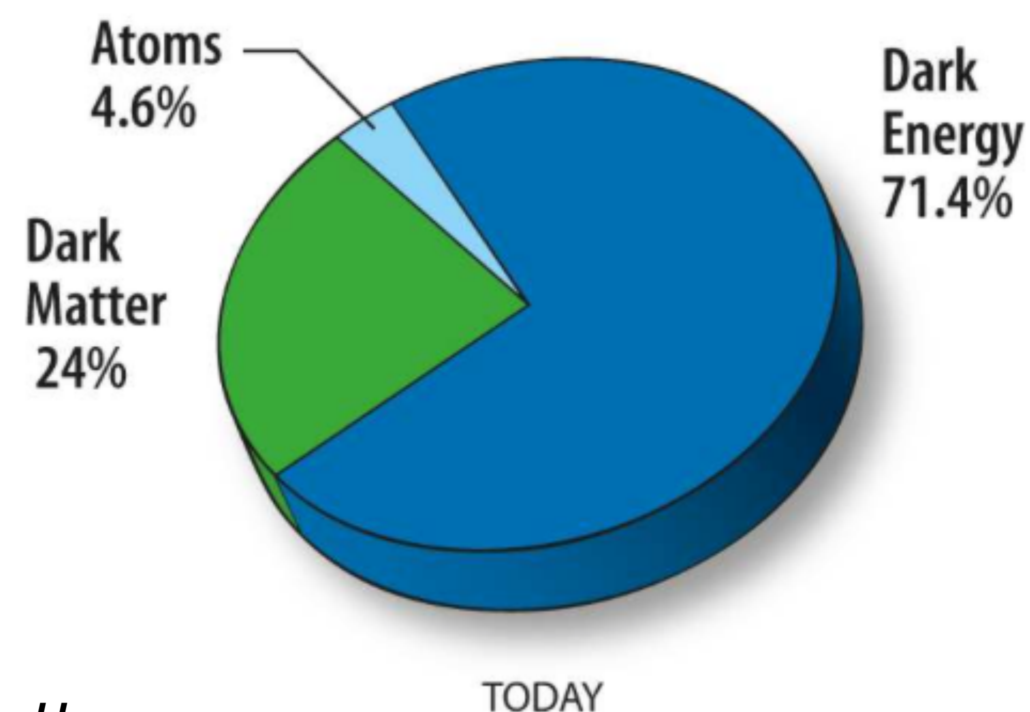
Rough estimate for axion contribution to DM

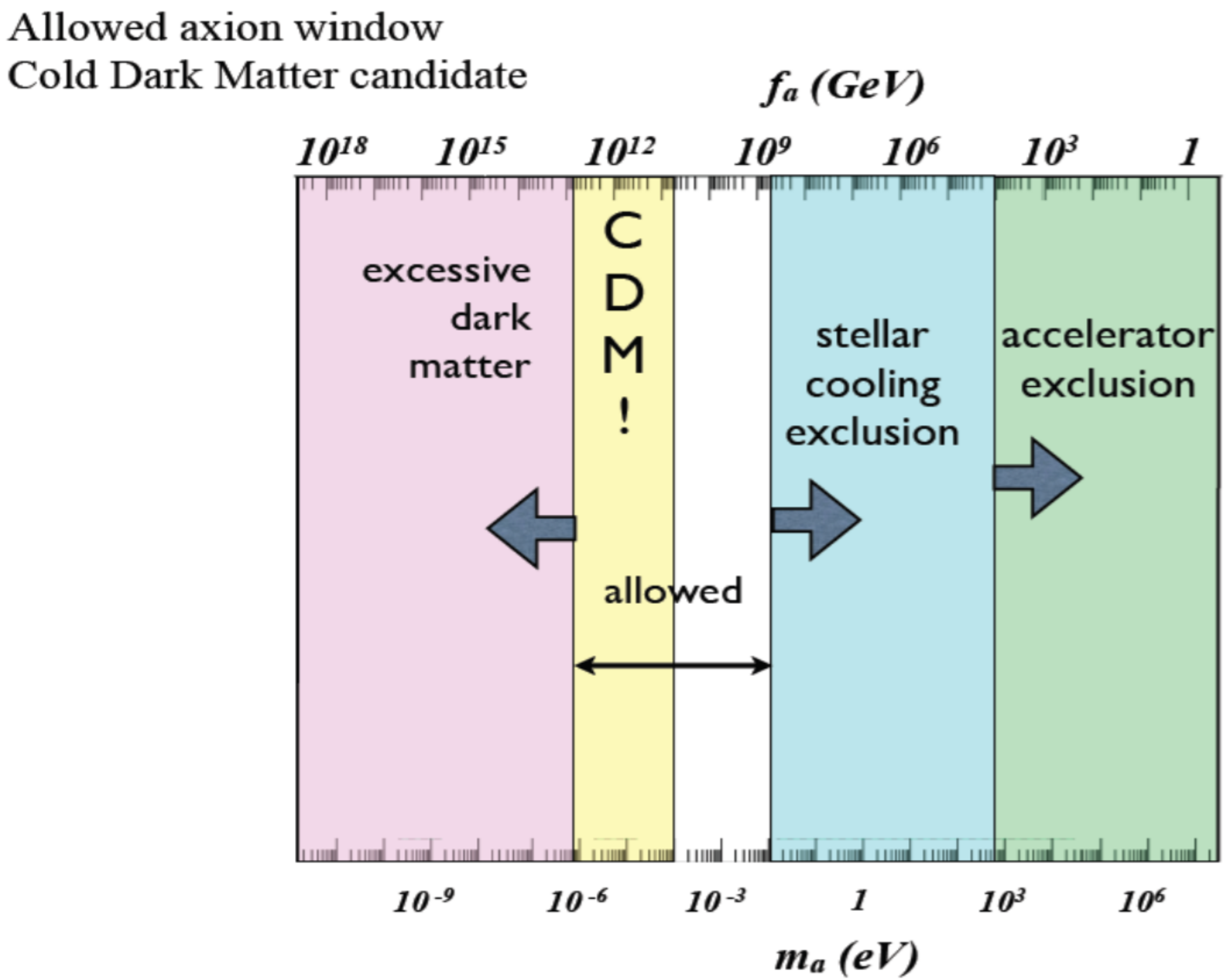
$$\frac{\rho_{\rm axion}}{\rho_{DM}} \sim \left(\frac{f_a}{10^{11}{\rm GeV}}\right)^{\frac{3}{2}}$$

*from ‘almost’ 1 till ‘very small*



# Allowed axion window

Slide by Ann Nelson



Time from Big Bang

Temperature

Hubble parameter

$$H(T) \simeq T^2/M_P$$

Details from the QCD eos

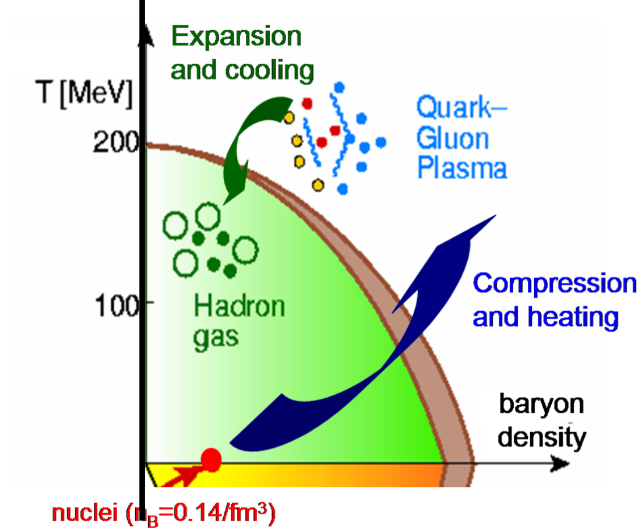
Axions's freezout

$$3H(T) = m_a(T)$$

$$m_a(T) = \sqrt{\chi(T)}/f_a$$

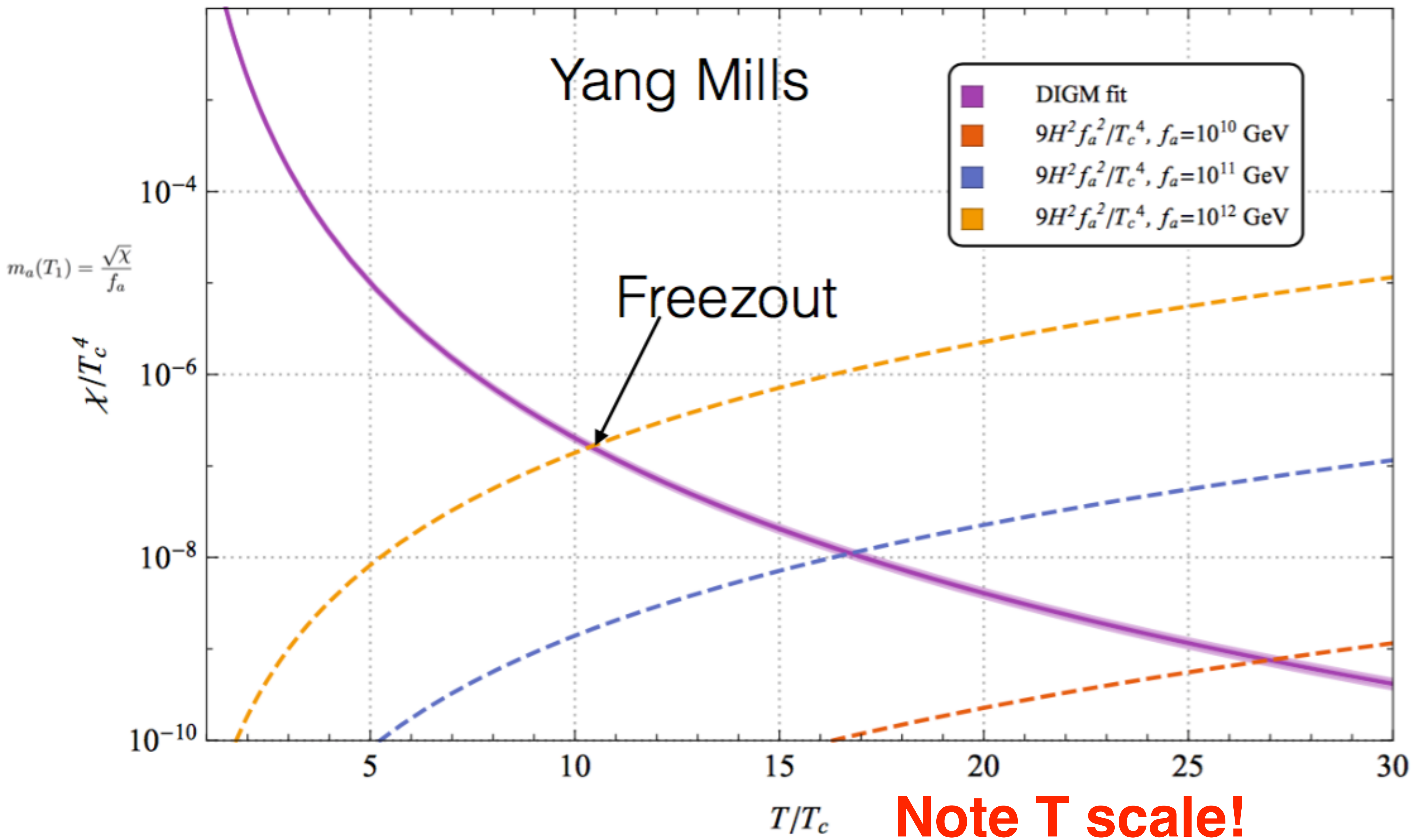
**Axions' mass and density today**

**Quark Gluon Plasma: Topology**



Axion freezout : 3H(T) = $m_a$(T) $=\sqrt{\chi(T)}/f_a$

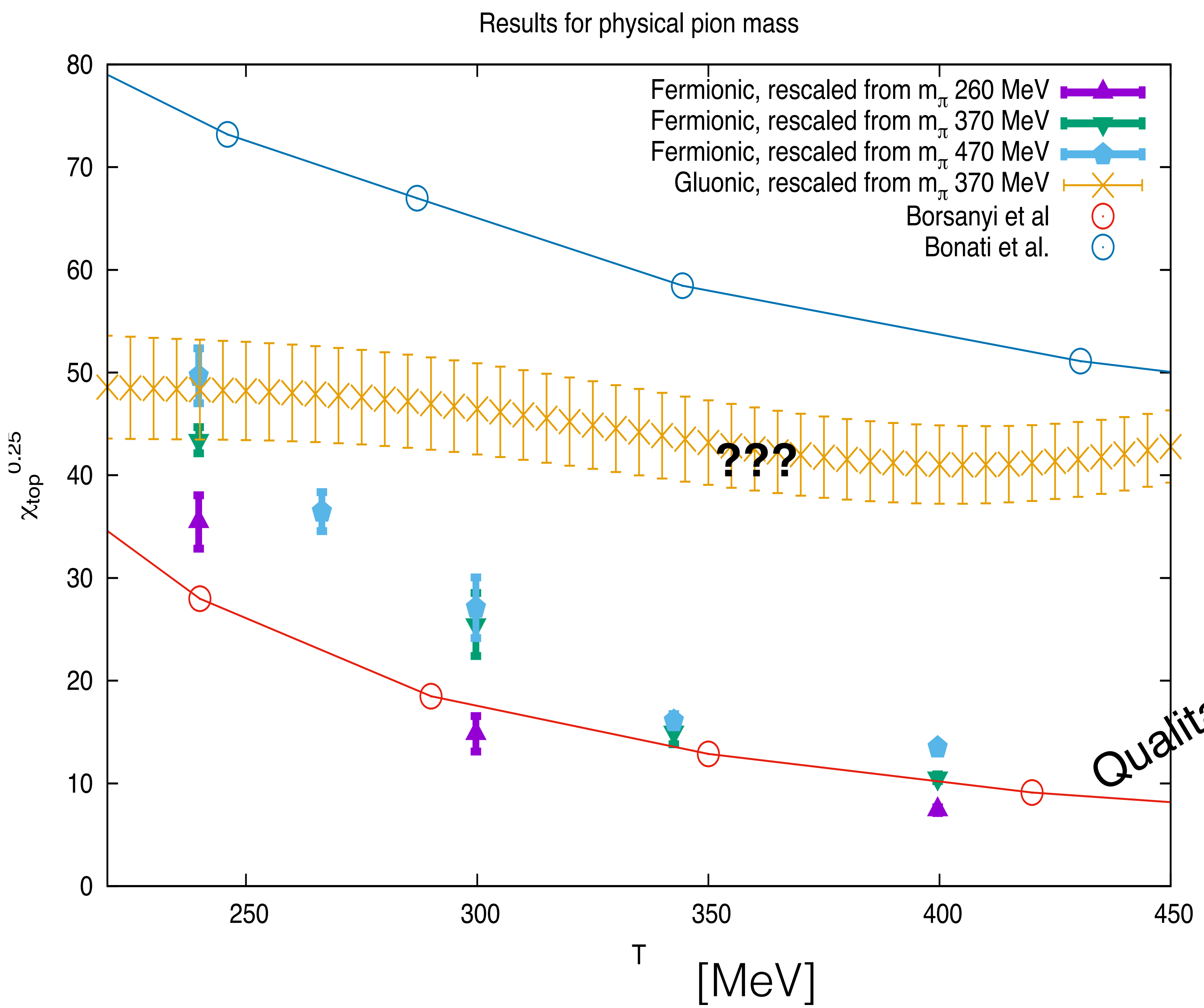
Berkowitz Buchoff Rinaldi 2015

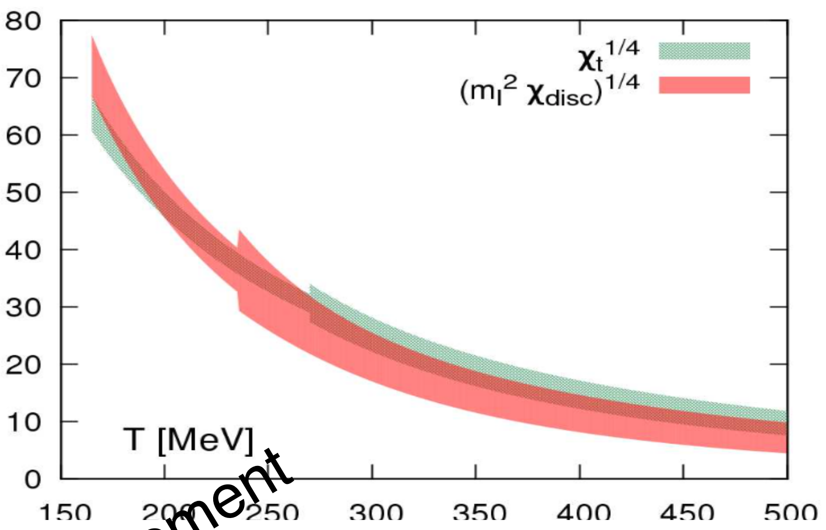



**Note T scale!**

Axion density at freezout controls axion density today

# Needed: Topological susceptibility at ‘nearly unreachable’ temperatures!




Qualitative agreement

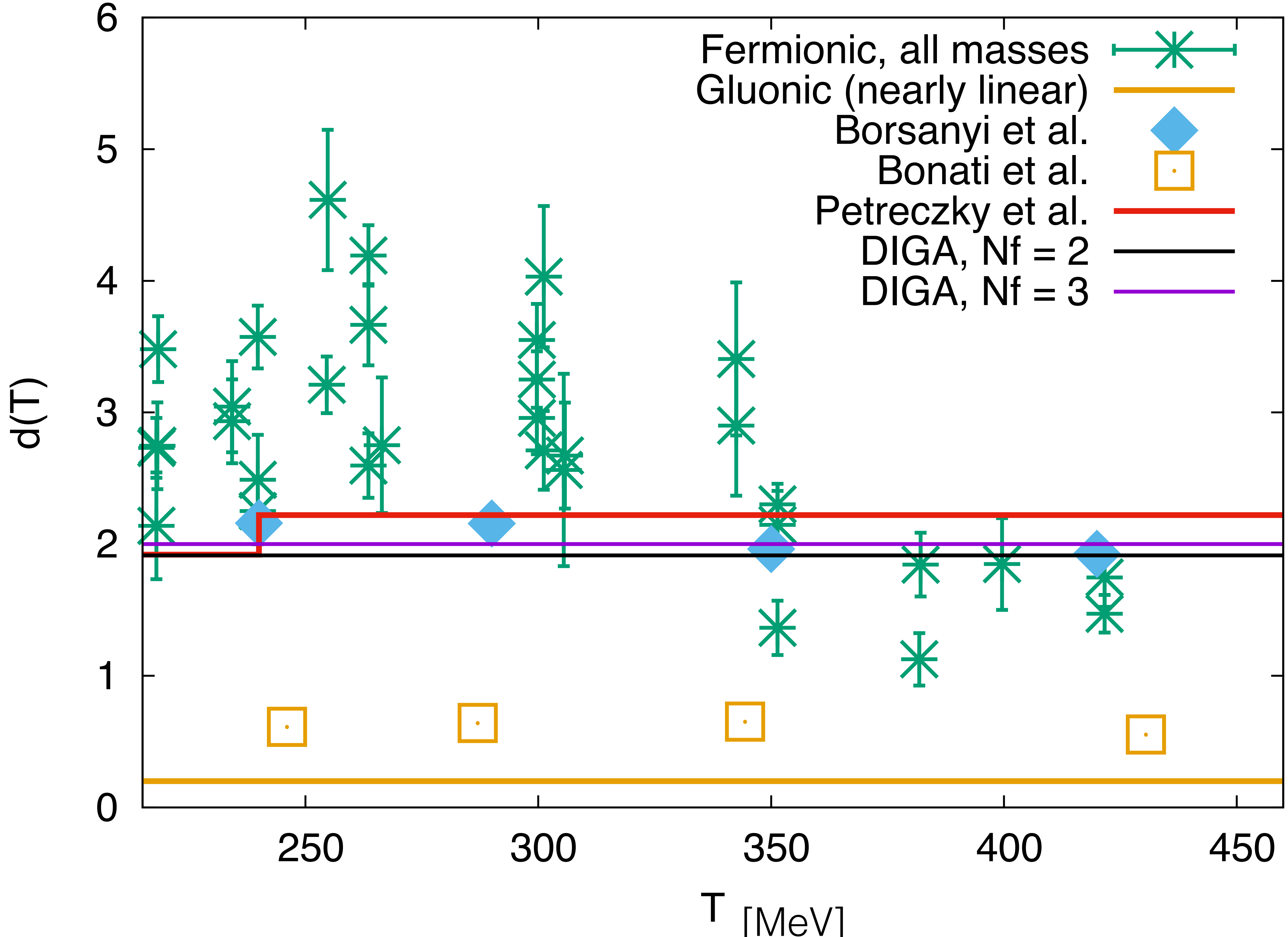
Petreczky et al.

Current results

Effective exponent d(T):

$$\chi_{top}^{1/4} = aT^{-d(T)}$$

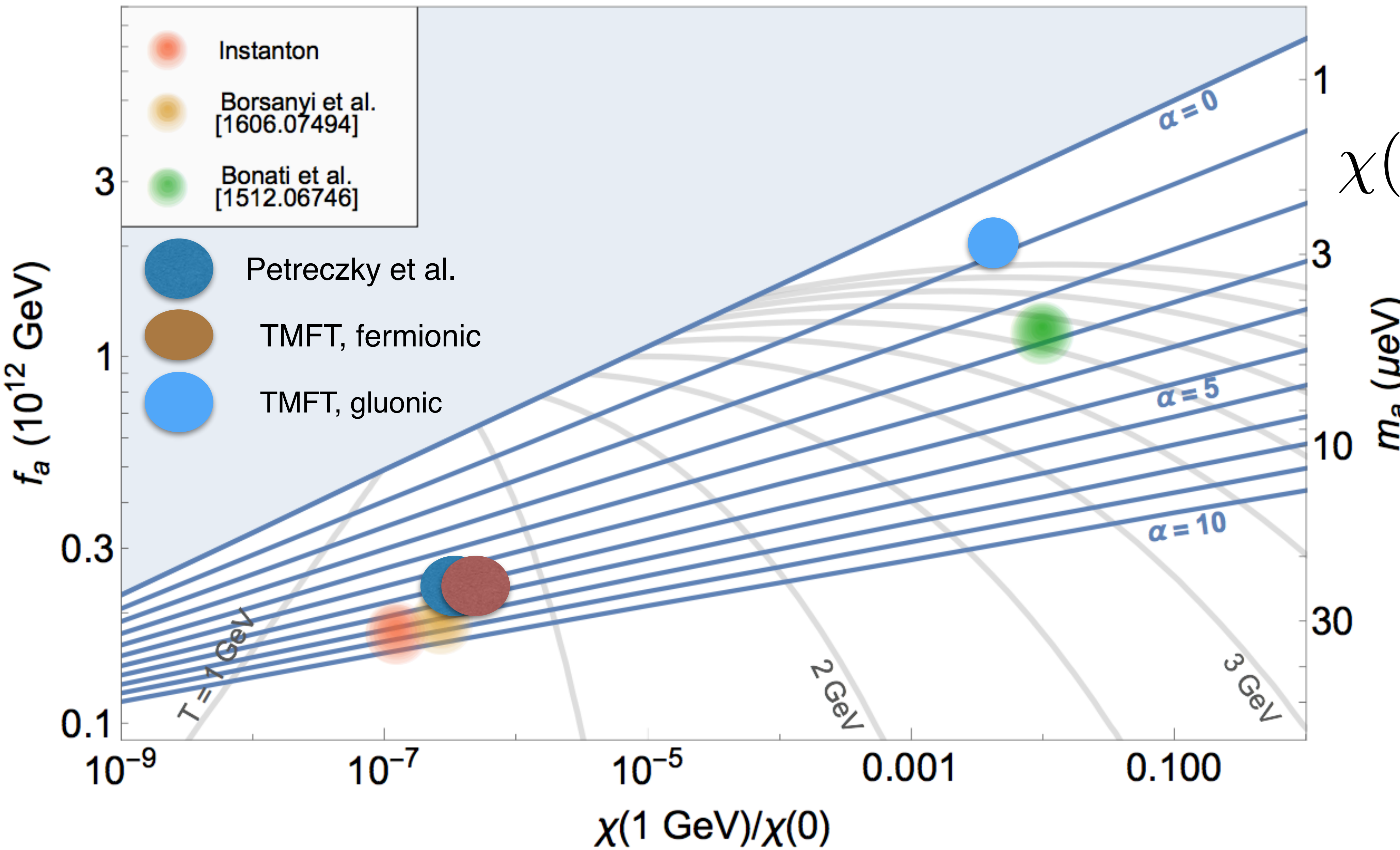
Current results




Extrapolation justifiable??

Needed assumption on fraction of DM made of axions

*Assume:* Axions make all of Dark Matter

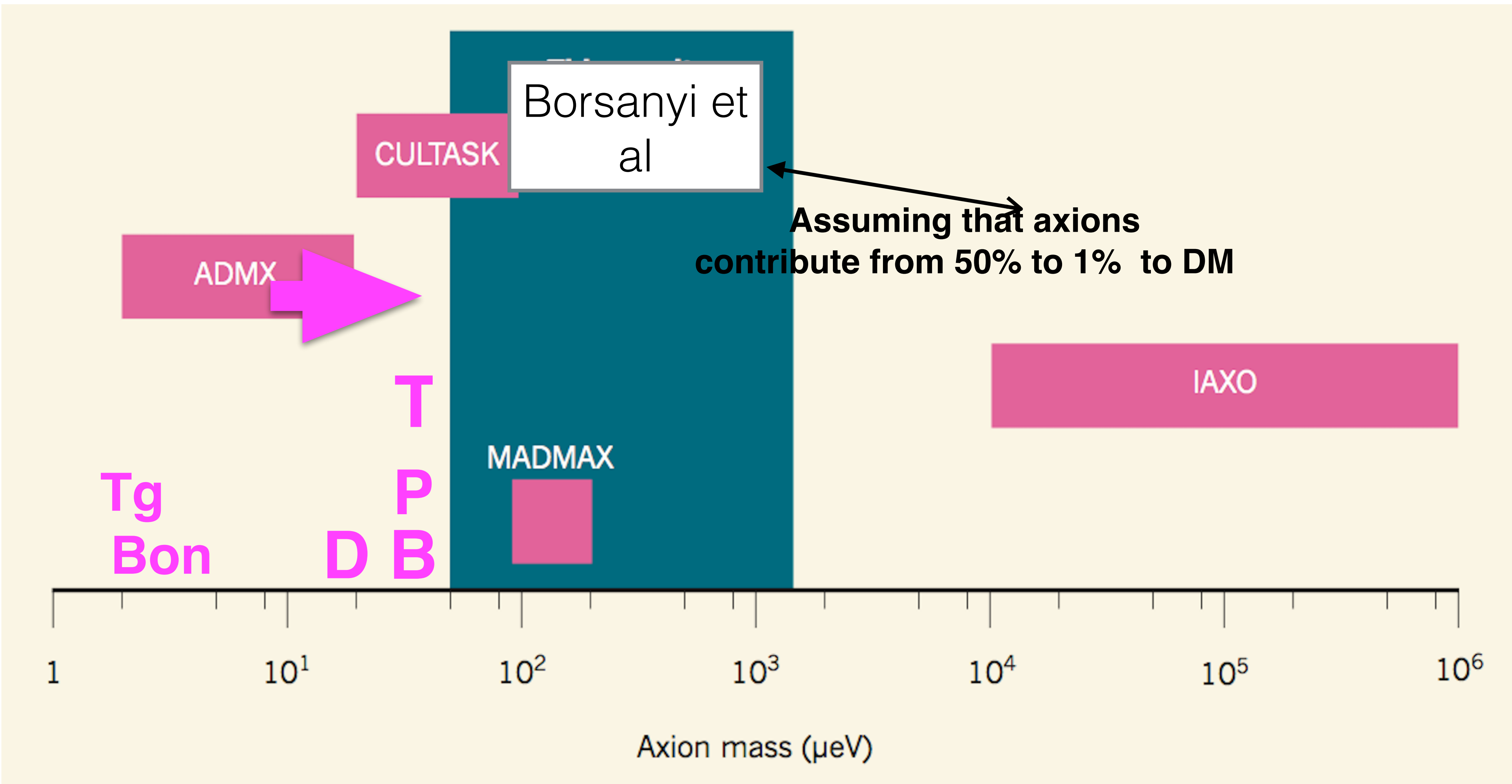



$$\chi(T) \propto 1/T^{\alpha}$$
$$T > 1 \text{ GeV}$$

**PhD Thesis, G. Grilli di Cortona, Sissa 2016 (advisor G. Villadoro)**

# Lower limits on the axion mass assuming that axions make 100% of DM:

Tg: Tmft, gluonic; Bon: Bonati et al.; D: DIGA, B: Borsanyi et al.,
P: Petreczky et al., T: Tmft, fermionic



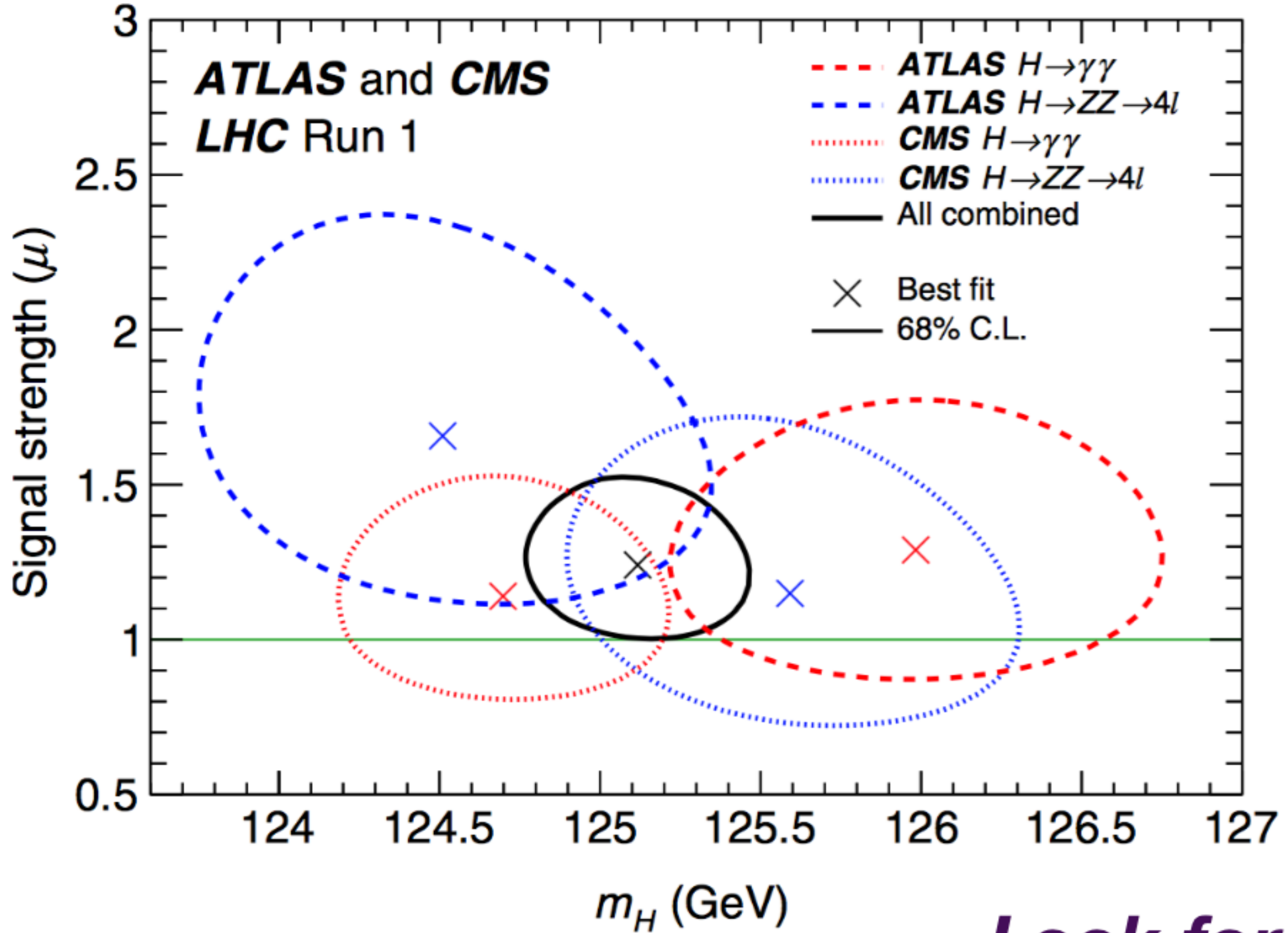

Updated from Nature N&V

Phases of QCD

and Higgs physics

Beyond the Standard Model:

Can we find a theory which produces a narrow Higgs-like status?




*Maybe in the preconformal region?*

It should be a composite scalar particle, lighter then so-far unobserved composite vector states

***Look for theories with scale separation***

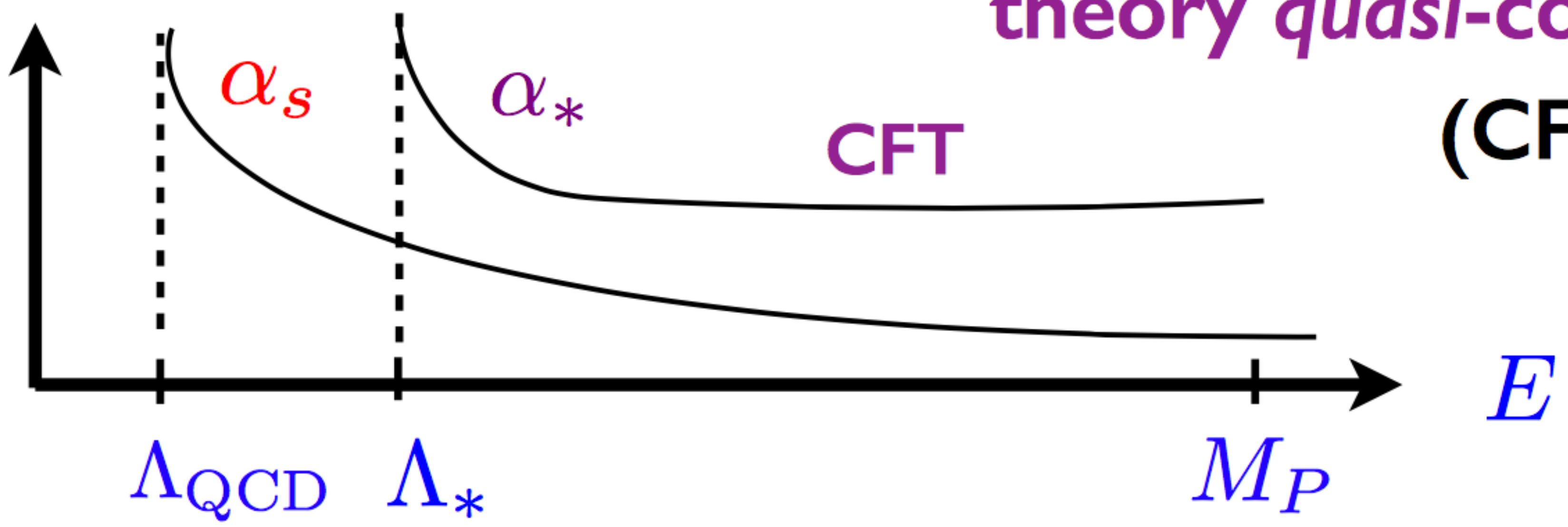
***..as possible BSM candidates***

# Explanation for the smallness of the EW-scale

**QCD as an inspiration:**

**More generically: we need a theory *quasi*-conformal**

**($CFT_4 \leftrightarrow AdS_5$)**




Slide from A. Pomerol

**New strong dynamics at TeV**

It could explain why $m_H \lesssim \Lambda_* \sim \mathrm{TeV} \ll M_P$

**Composite Higgs**

**Solves the problem in one shot!**

(in supersymmetry we still need strong dynamics to break susy at some low-scale)

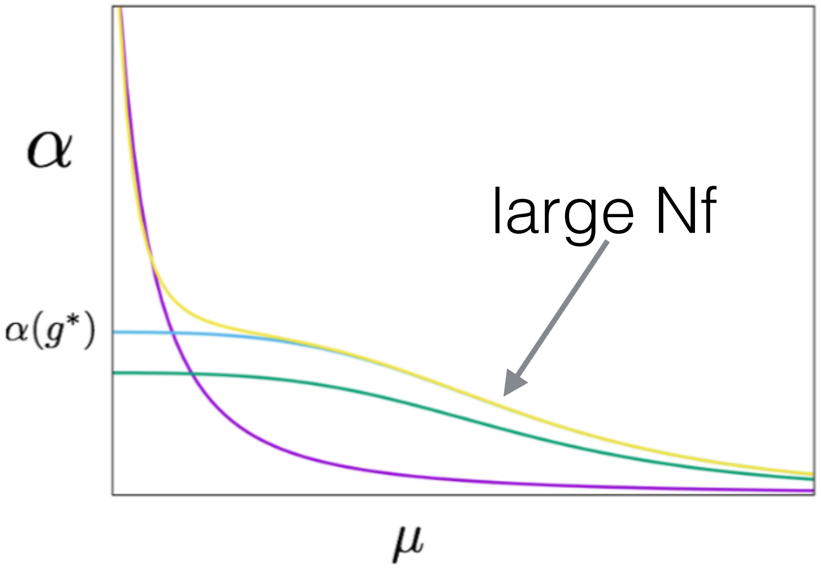
# Adding flavours to strong interactions

# Adding Flavours to QCD:

***Another world***

Symmetry is preserved

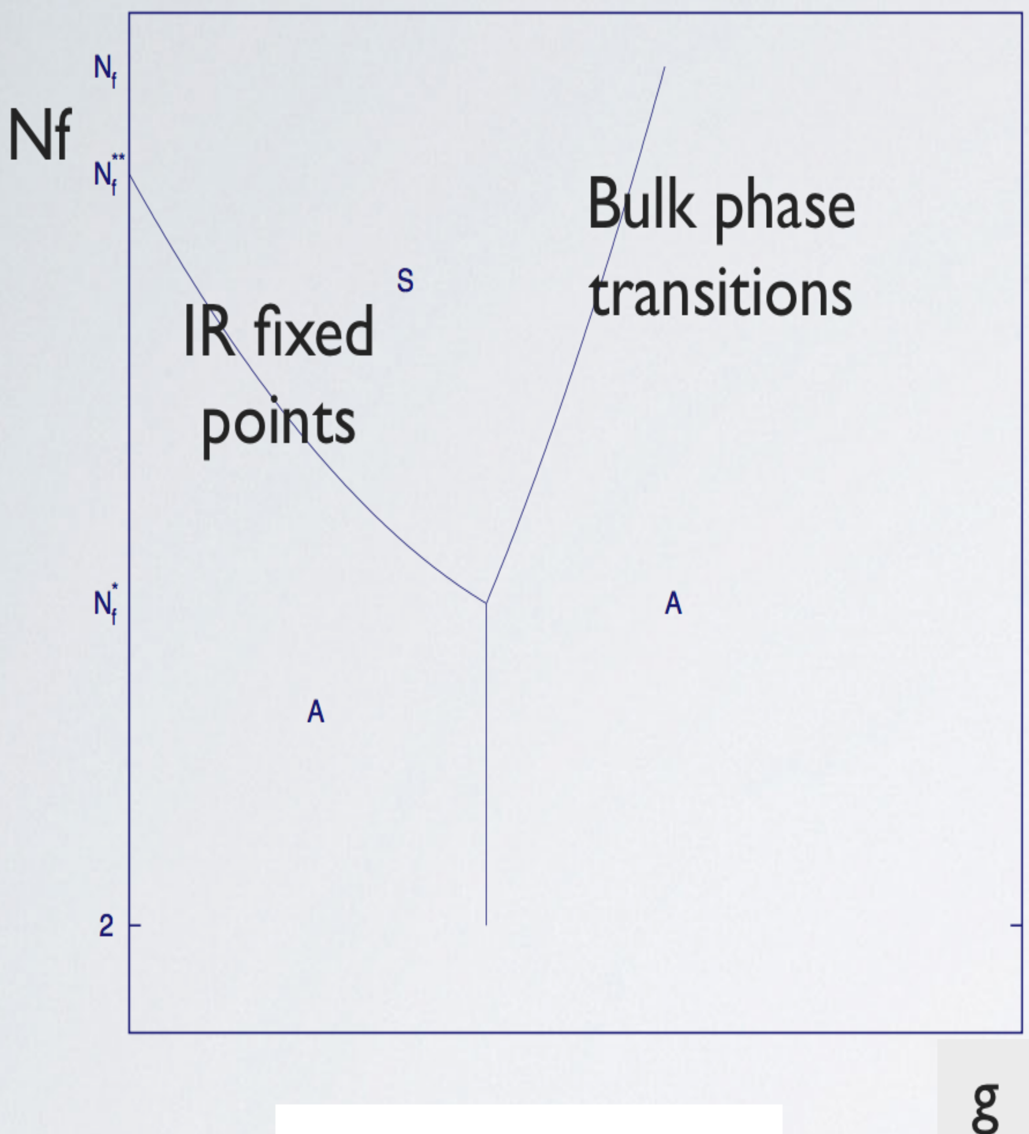
$$SU(N_f) X SU(N_f) \rightarrow SU(N_f)$$




***Interplay between chiral symmetry and running of the coupling which depends on the flavor content***

For large $N_f$ $\alpha_s$ is too small for $\chi$ breaking

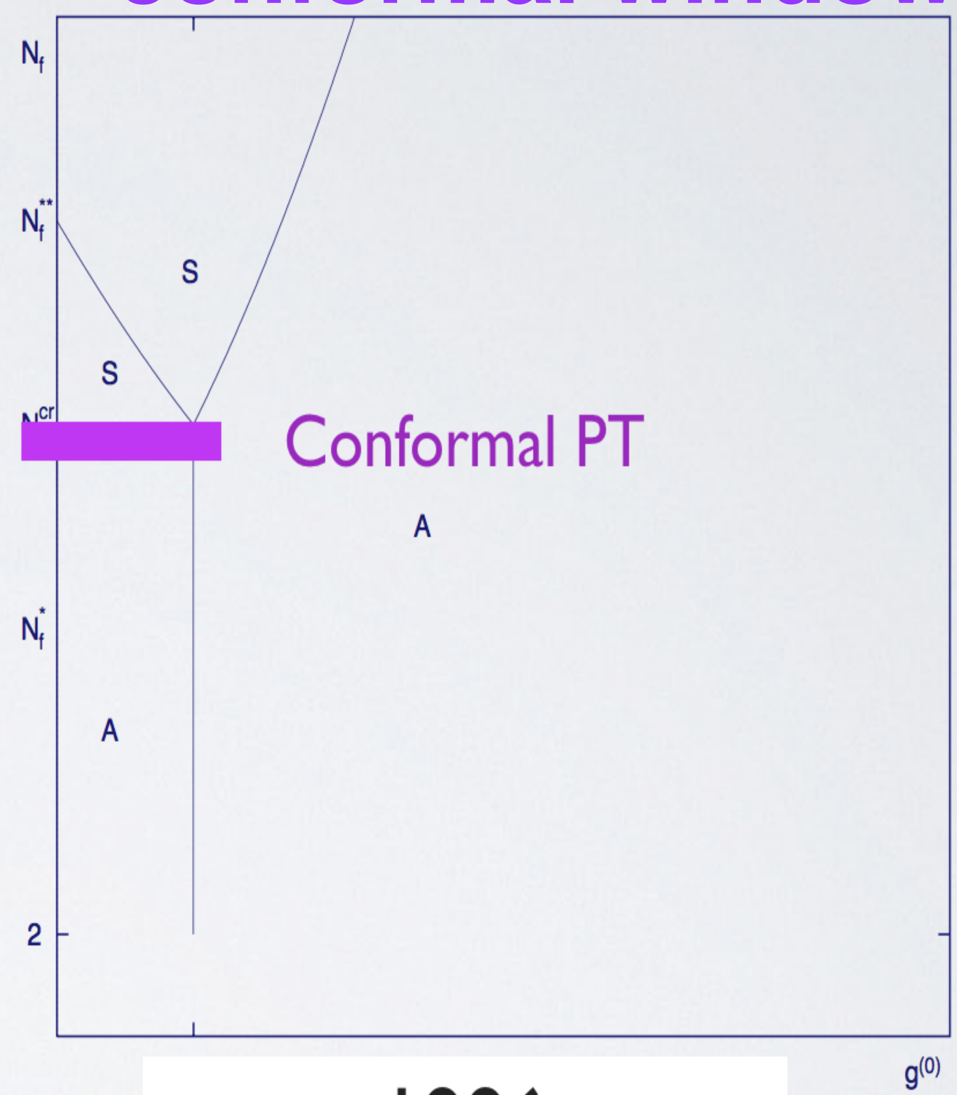
# *Hystorical overview*




1974

Banks Zacs

**Discovery of the QCD conformal window**



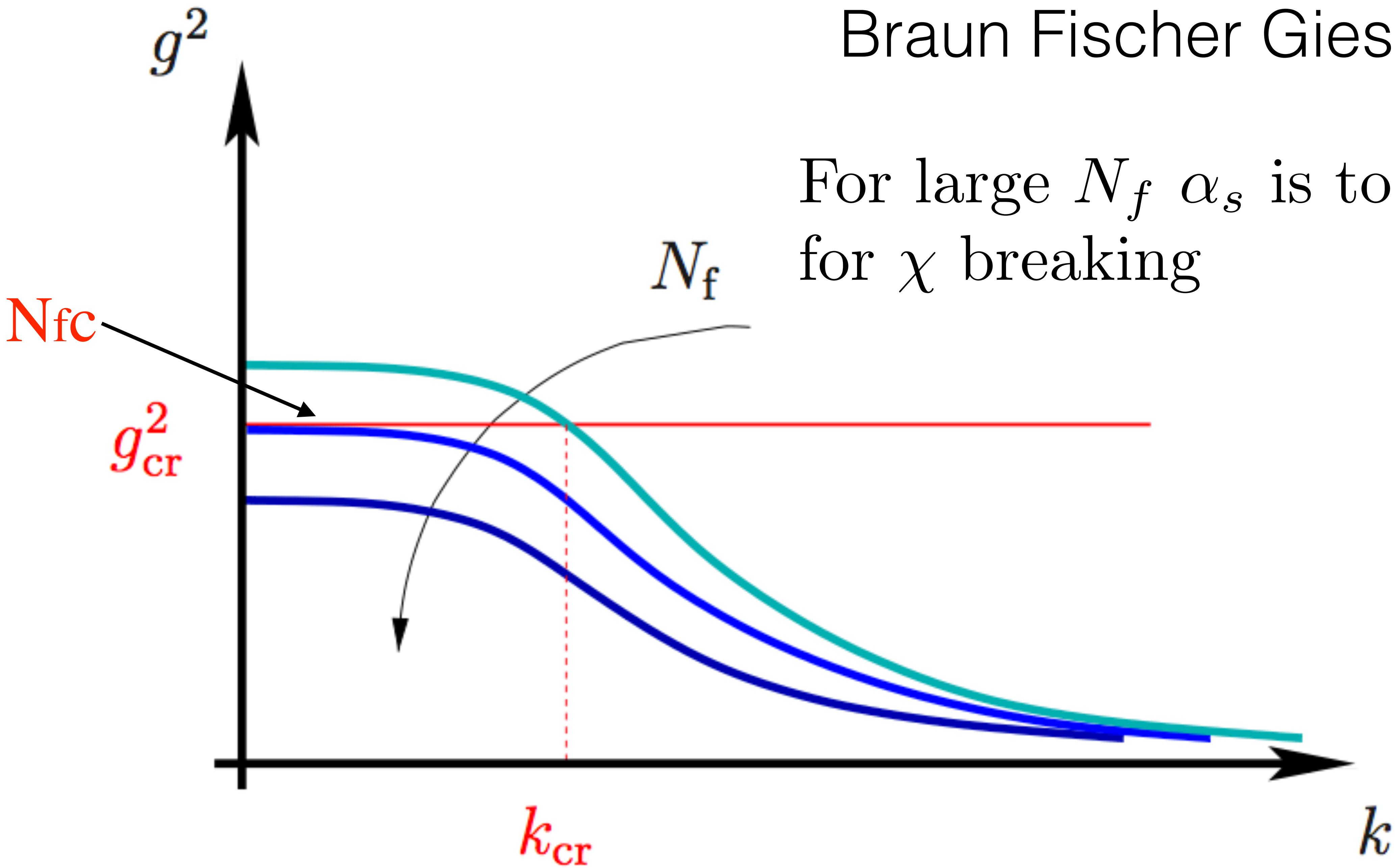

1996

Miransky Yamawaki

# IR running of the gauge coupling for different Nf's

Braun Fischer Gies 2010

For large $N_f$ $\alpha_s$ is too small for $\chi$ breaking



# Phases of Strong Interactions


Which are its features?

Can we observe it?

# Establishing the conformal window

# Similarities and differences between a conformal PT and a 2nd order one

**Conformal transition**

IR IR IR IR IR.....

(X power-law)

Conformal scaling

*Analogies in the broken phase*

*Differences in the symmetric phase*

**EoS**

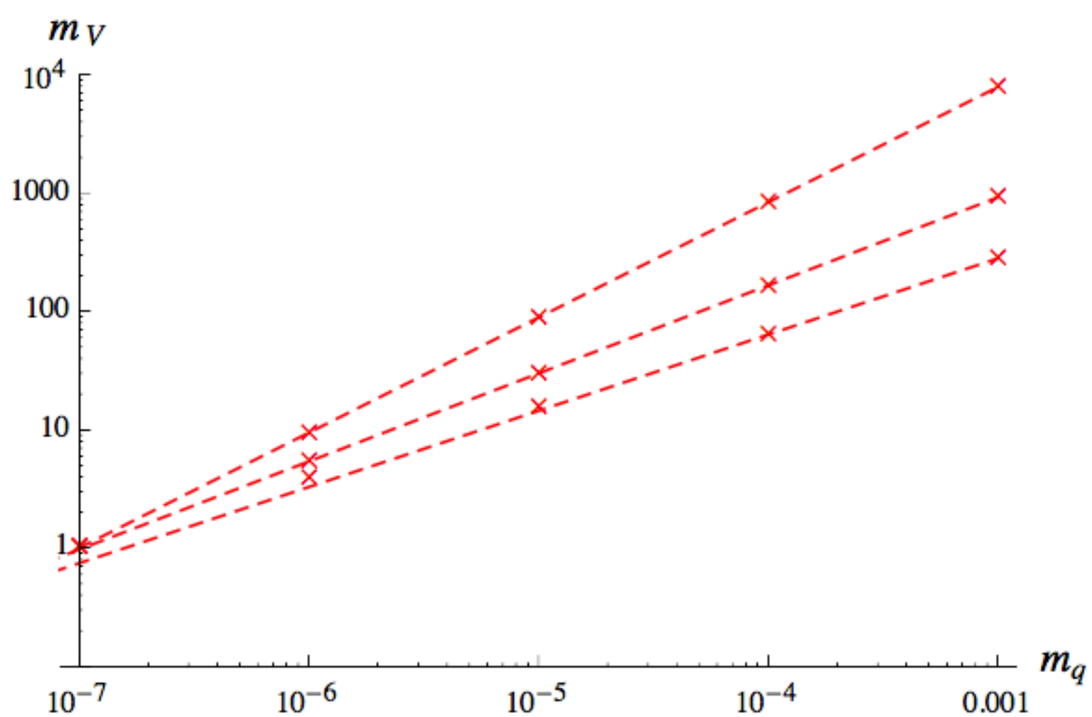
IR

**2nd order transition**

Griffith's analyticity

# Conformal scaling

$$M_H = c_H m^{1/y_h}$$





Power law Scaling with
anomalous dimension

Alho Evans Tuominen 2014

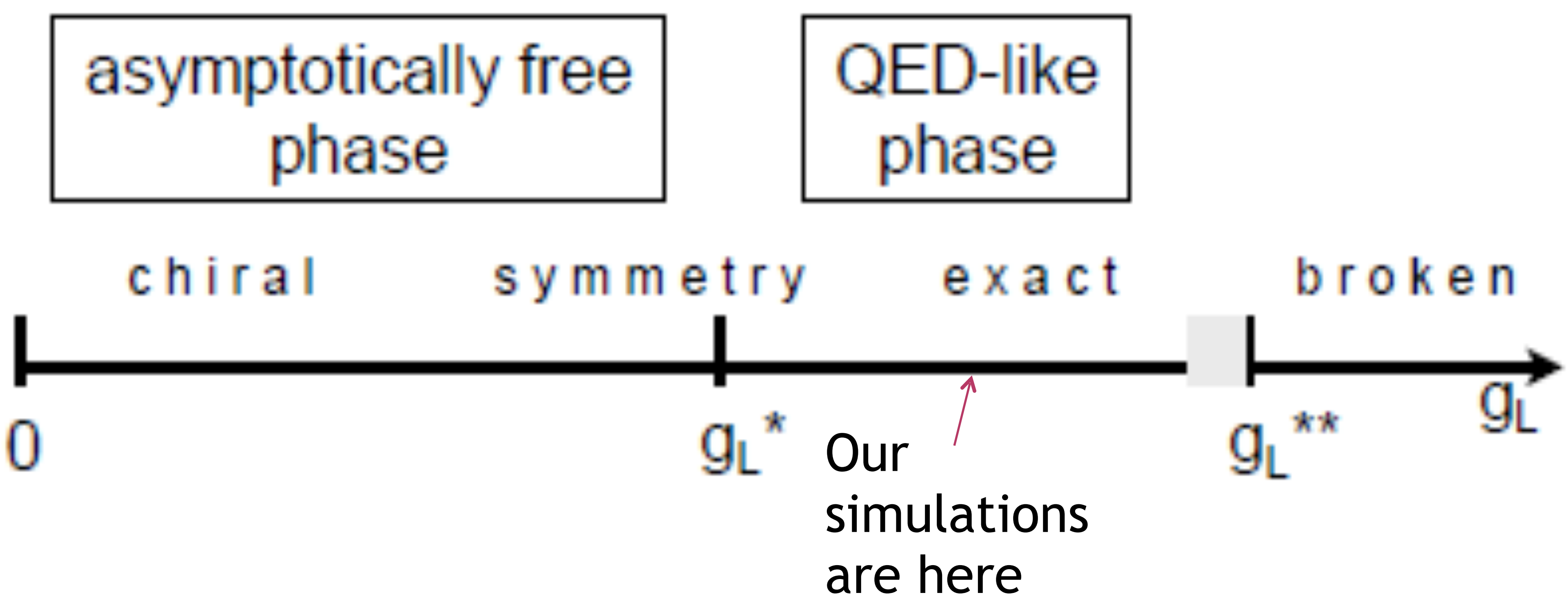
# Lattice corrections to conformal scaling

1: Size $M_H = L^{-1} f_H(x)$ $\quad x \equiv Lm^{1/y_m}$

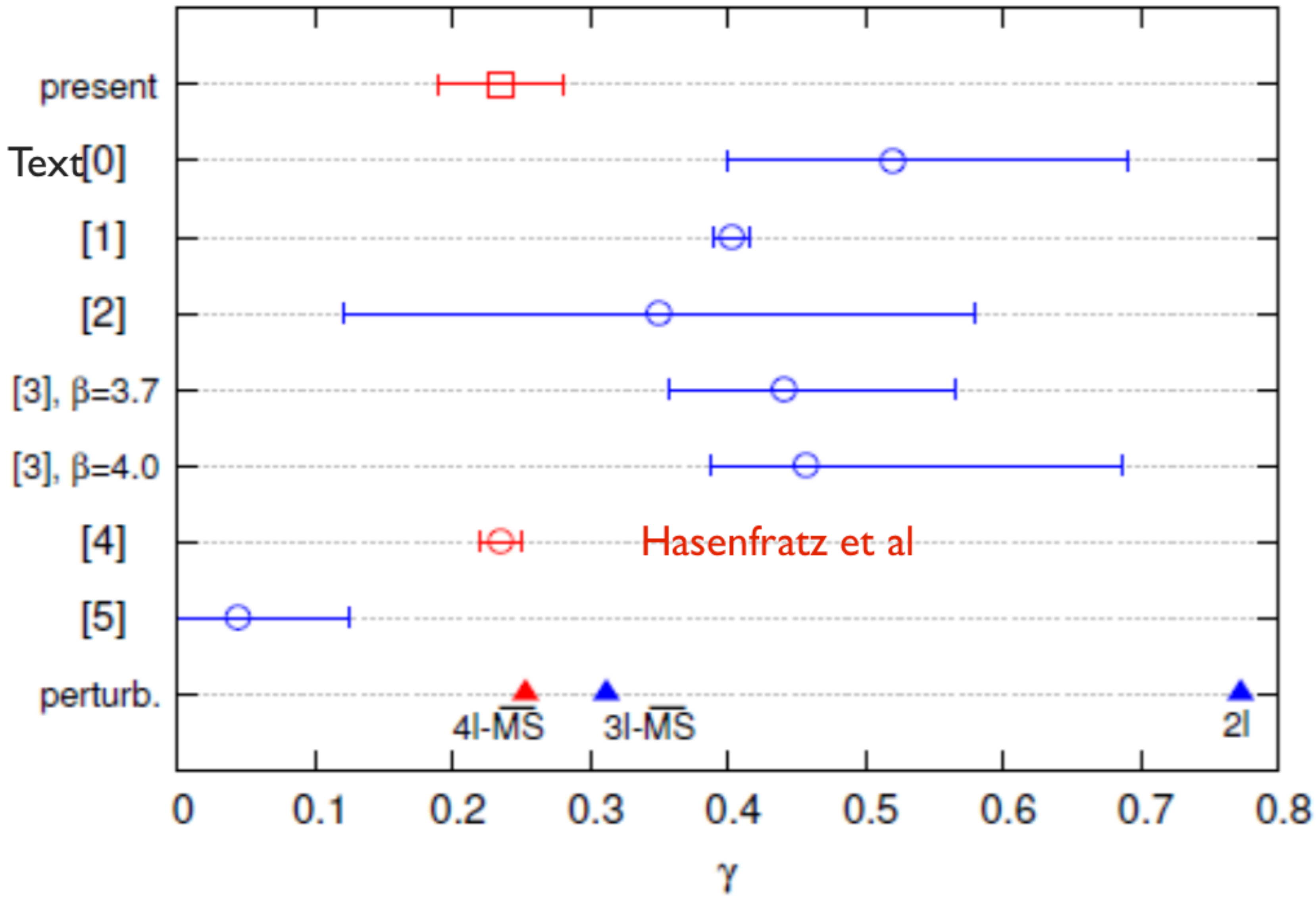
2: Coupling $M_H = L^{-1} f_H(x, g_0 m^{\omega})$

**Del Debbio, Zwicky; Hasenfratz et al; MpL, da Silva, Miura, Pallante**

$$LM_H = F_H(x)\left\{1 + g_0 m^{\omega} G_H(x) + \mathcal{O}\left(g_0^2 m^{2\omega}\right)\right\}$$



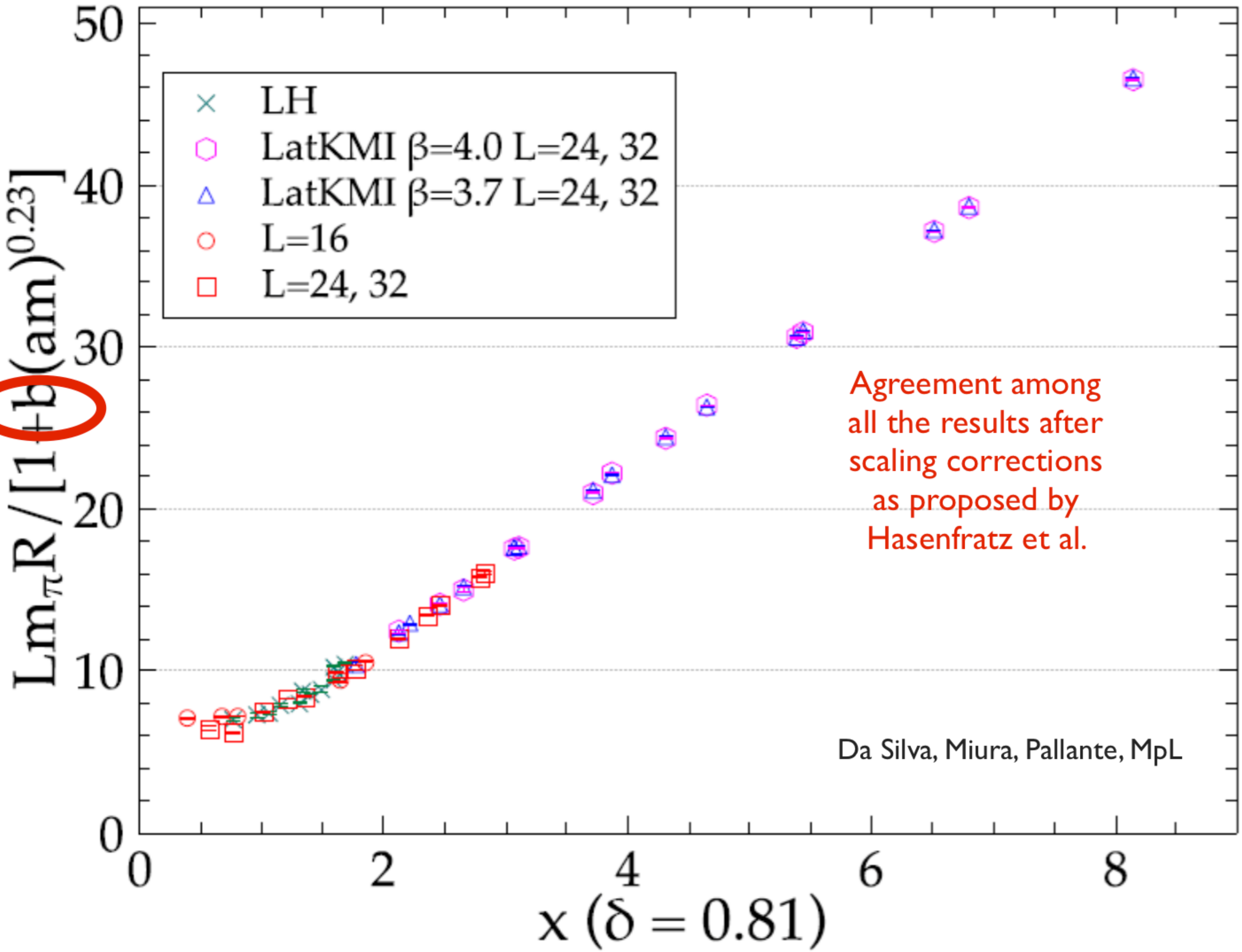
# •Anomalous dimension results with and without scaling corrections •




Corrected

Uncorrected

da Silva, Miura, Pallante, MpL




Agreement among all the results after scaling corrections as proposed by Hasenfratz et al.

Da Silva, Miura, Pallante, MpL

OK

# Next

# Standard picture of scale separation

$$\Lambda_{IR}/\Lambda_{UV} = \mathcal{O}(1).$$

$\Lambda_{UV}$


In the conformal phase IR scales vanish but UV ones survive

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

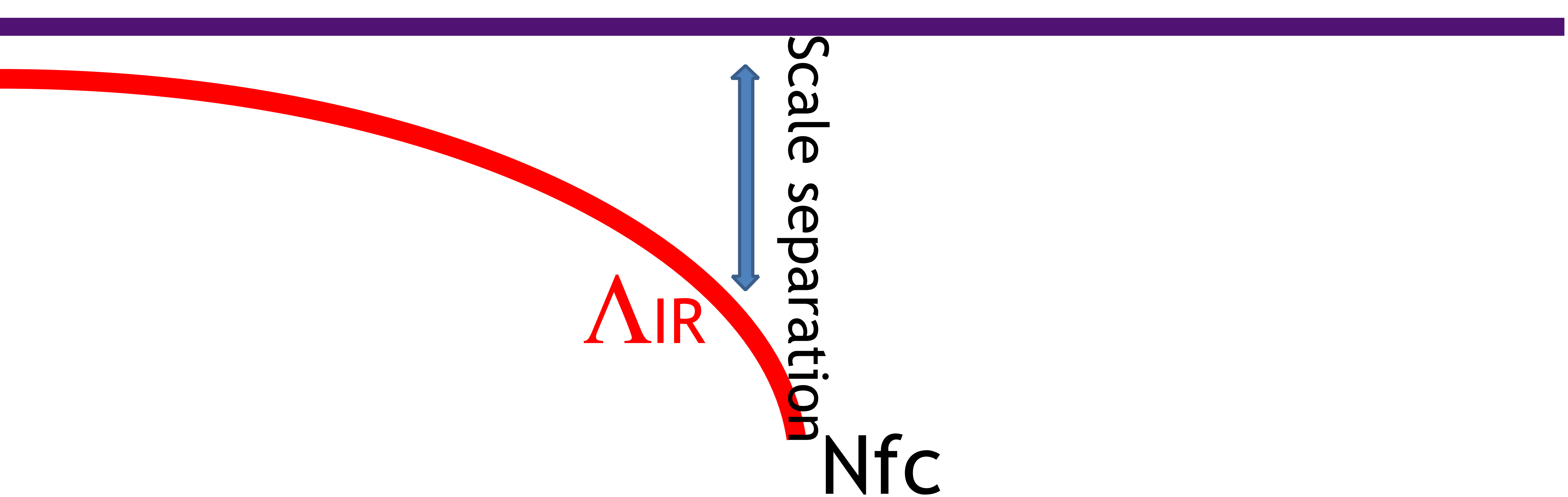
$$x = N_f/N_c$$

The coupling walks for

$$\Lambda_{UV}^{-1} \ll r \ll \Lambda_{IR}^{-1}$$

# Standard picture of scale separation

$$\Lambda_{\rm IR}/\Lambda_{\rm UV} = \mathcal{O}(1).$$

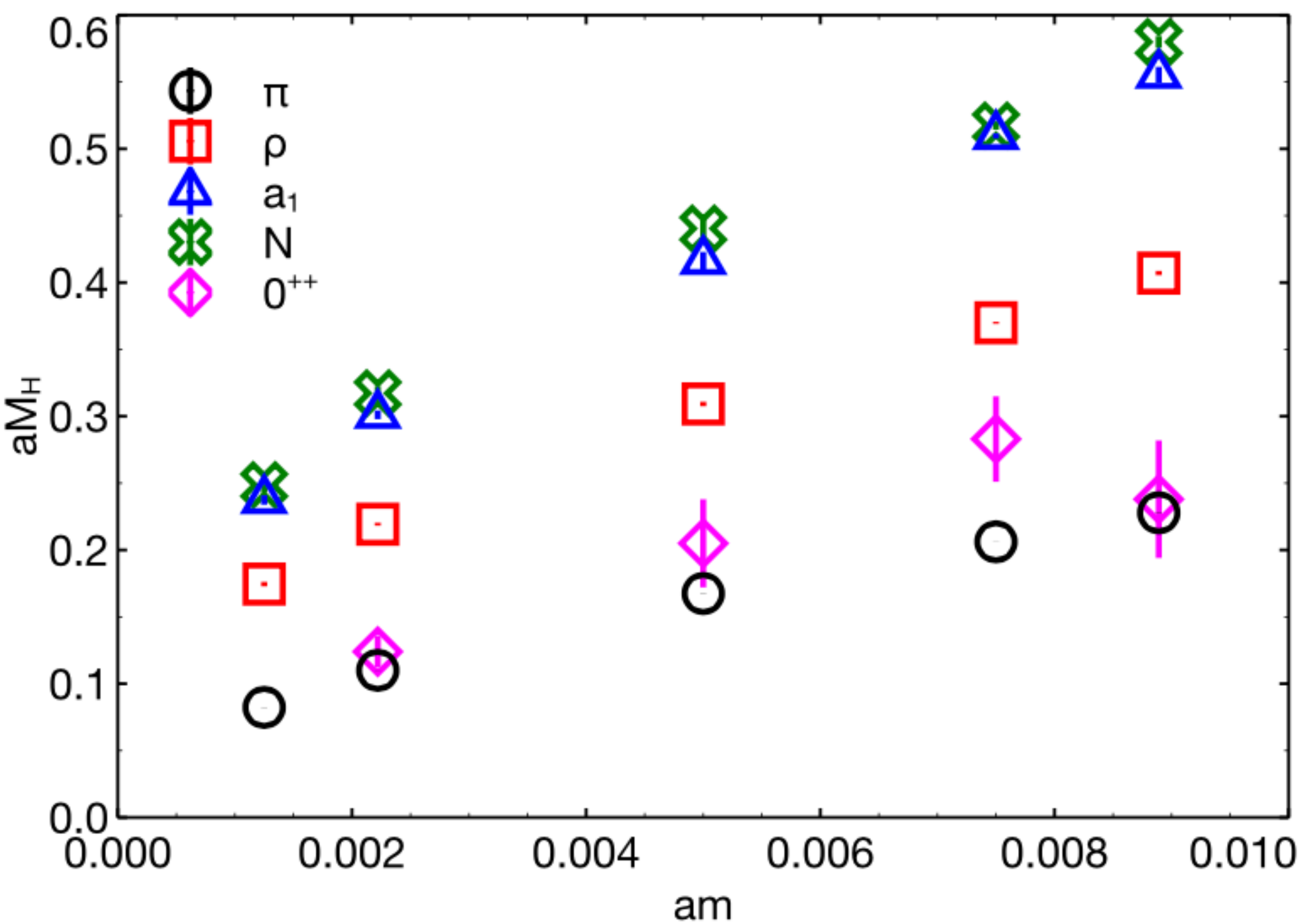



$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

**Strongly interacting dynamics and the search for new physics at the LHC**

T. Appelquist,[1] R. C. Brower,[2,3] G. T. Fleming,[1,3] A. Hasenfratz,[4,3] X. Y. Jin,[5] J. Kiskis,[6] E. T. Neil,[4,7,3] J. C. Osborn,[5,3] C. Rebbi,[2] E. Rinaldi,[8,3] D. Schaich,[9,3,10] P. Vranas,[8] E. Weinberg,[11] and O. Witzel[12,3]
(Lattice Strong Dynamics (LSD) Collaboration)

# Beyond scale separation:

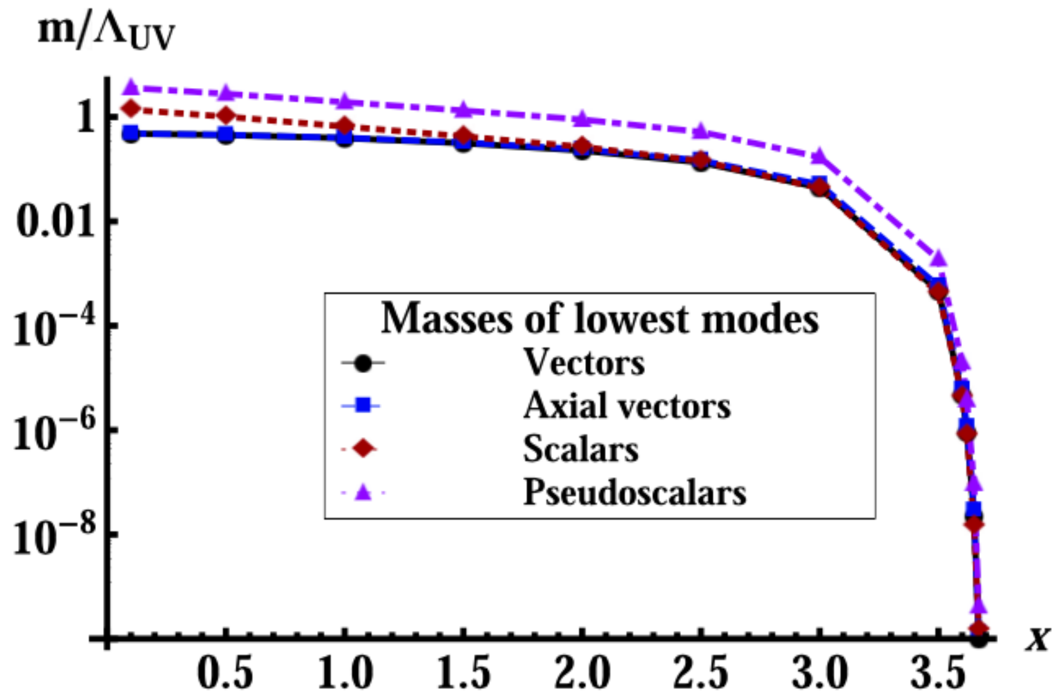


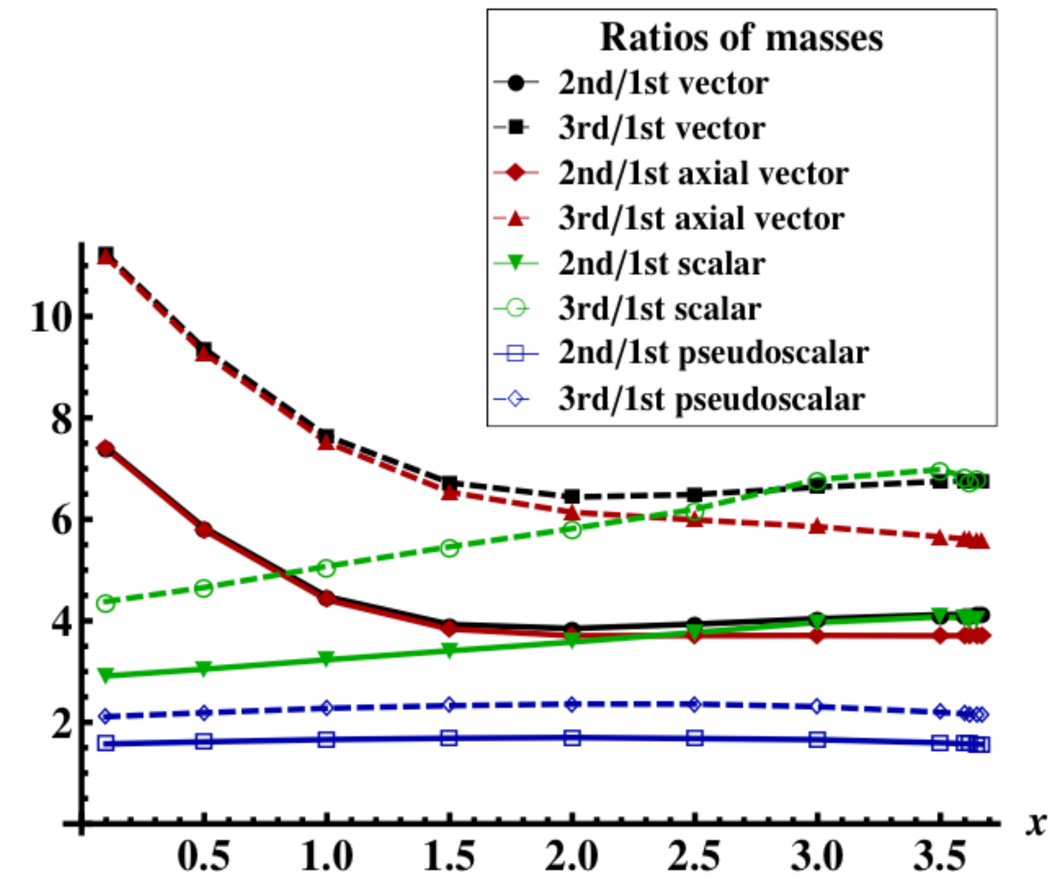

hierarchy of scales in the spectrum: light scalar needed for phenomenology

2016

# (Essential) singularity in the chiral limit and mass ratios: example from holographic V-QCD

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}(n+1)}{\sqrt{x_c - x}}\right)$$



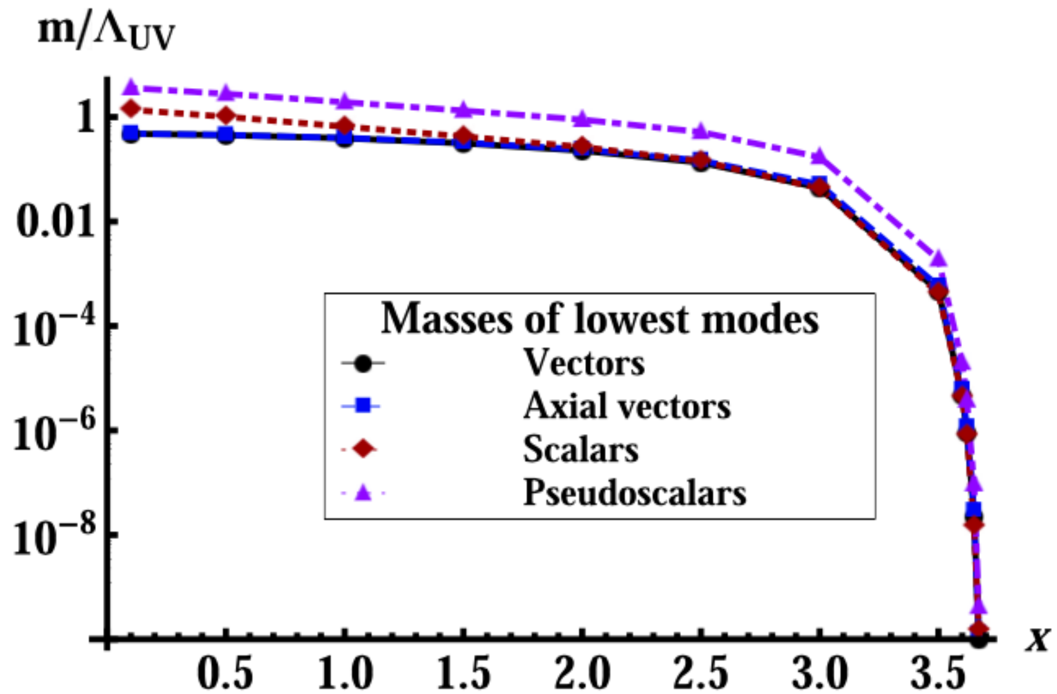



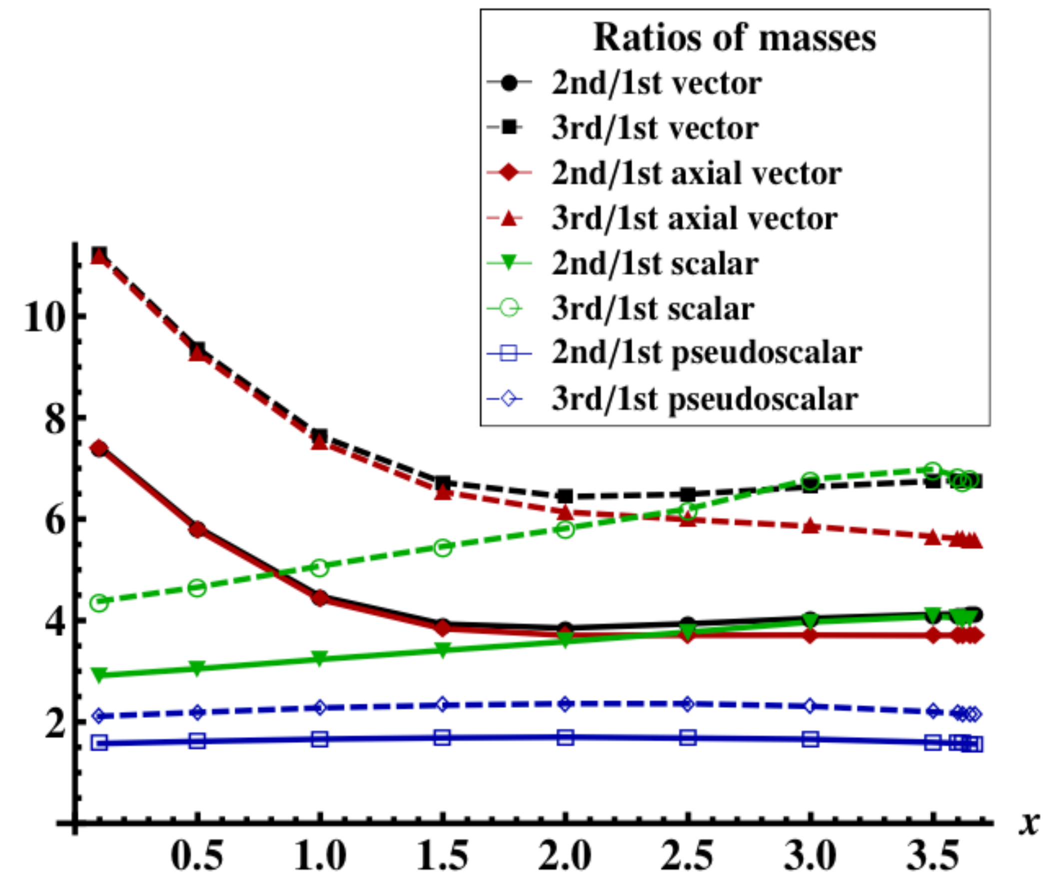

Arean, Iatrakis, Jarvinen, Kiritsis 2013

(Essential) singularity in the chiral limit and mass ratios: example from holographic V-QCD

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}(n+1)}{\sqrt{x_c - x}}\right)$$

***Not unique***




Arean, Iatrakis, Jarvinen, Kiritsis 2013

$\Lambda_{IR}$ not unique:

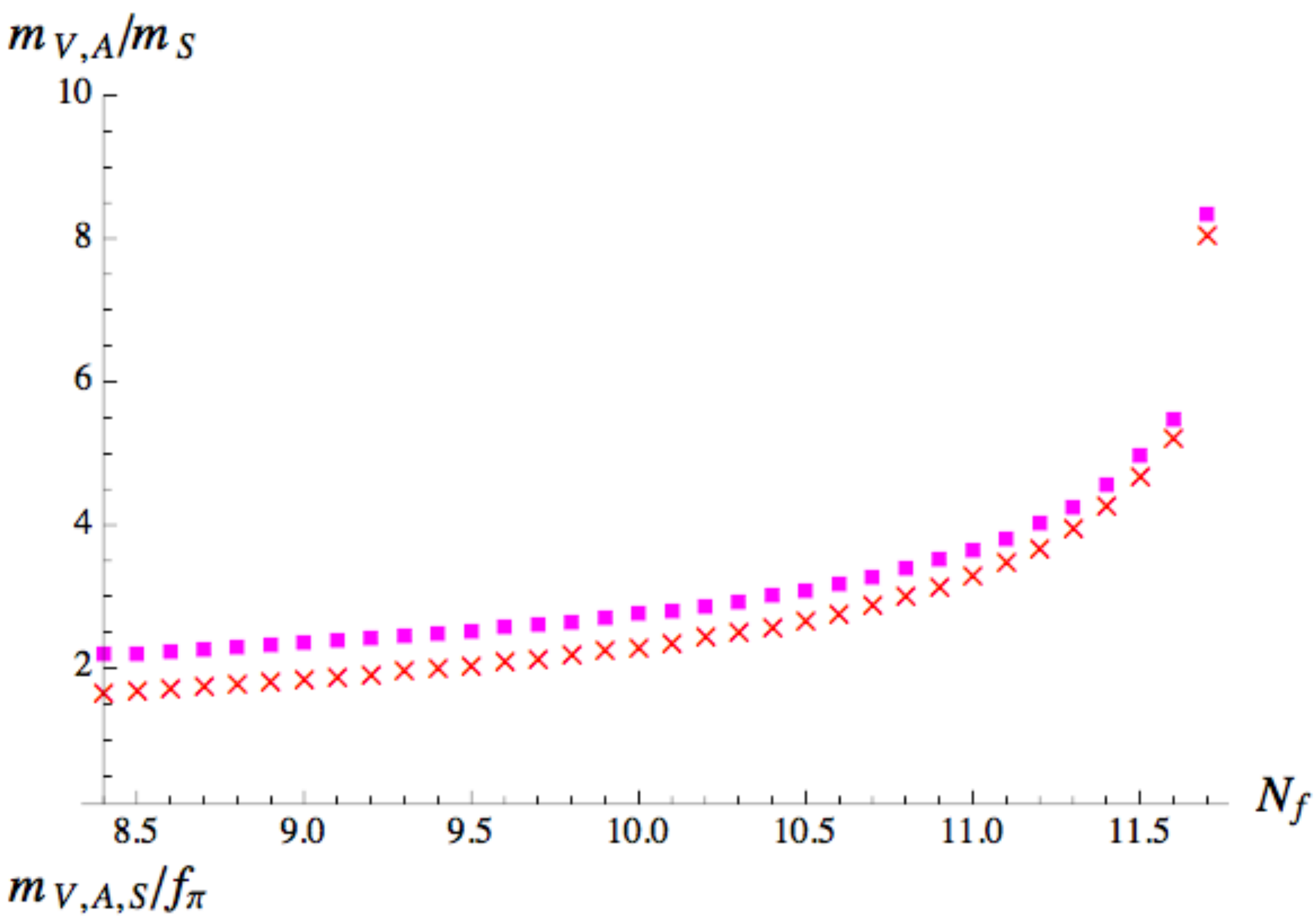
# Power-law corrections to essential singularity

Gies et al. 2013
Alho, Evans, Tuominen 2013

$$O_i = A_i (N_f^c - N_f)^{p_i} \langle \bar{q} q \rangle^{1/3}$$

Power-law X Miranski scaling

May account for <u>hierarchy</u> of scales



# Mass deformed theory I: EoS approach for IR quantities

$$y = f(x)$$
$$y = m / < \bar\psi\psi >^{\delta} \qquad \delta = \frac{6-\eta}{2-\eta}$$

Second order transition:

$$x = ({N_f}^c - N_f) / < \bar\psi\psi >^{\frac{1}{\beta}} \qquad < \bar\psi\psi > = ({N_f}^c - N_f)^{\beta}$$

Essential singularity: Nogawa, Hasegawa, Nemoto, 2012

$$x = e^{\sqrt{({N_f}^c - N_f)}} / < \bar\psi\psi > \qquad < \bar\psi\psi > = e^{\sqrt{({N_f}^c - N_f)}}$$

Continuity of $f(x)$ plus asymptotic forms for $m \to 0$ and $N_f \to {N_f}^c$ imply

$< \bar\psi\psi > \propto e^{\sqrt{({N_f}^c - N_f)}}$ for m *smallish* and $({N_f}^c - N_f)$ *largish*

$< \bar\psi\psi > \propto m^{1/\delta}$ for m *largish* and $({N_f}^c - N_f)$ *smallish*

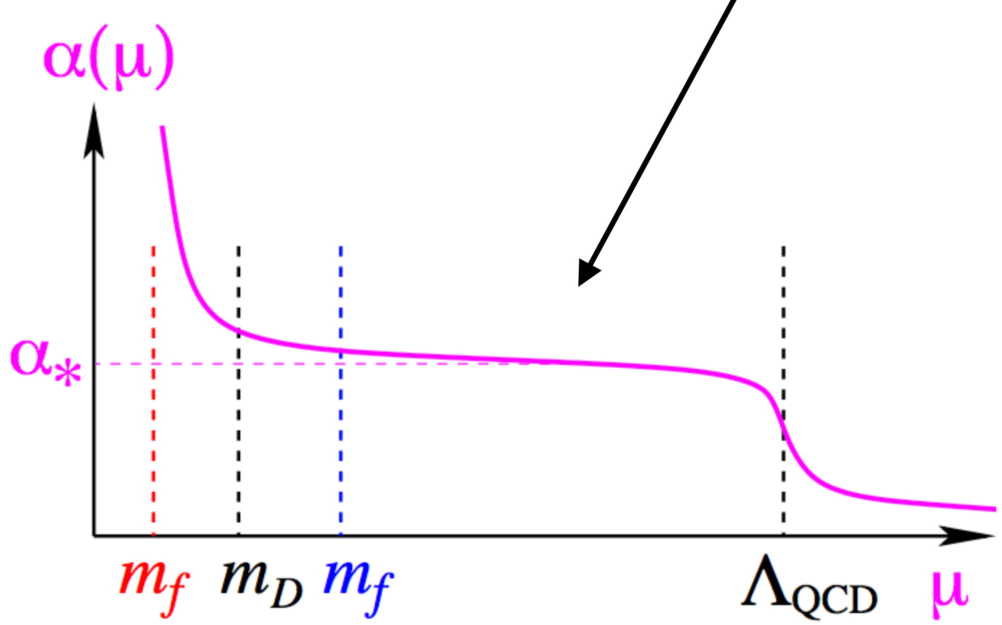
Anomalous dimension appears naturally below Nfc

Scaling limited by Goldstone singularities in the chiral limit (Wallace Zia

# Mass deformed theory II: KMI discussion

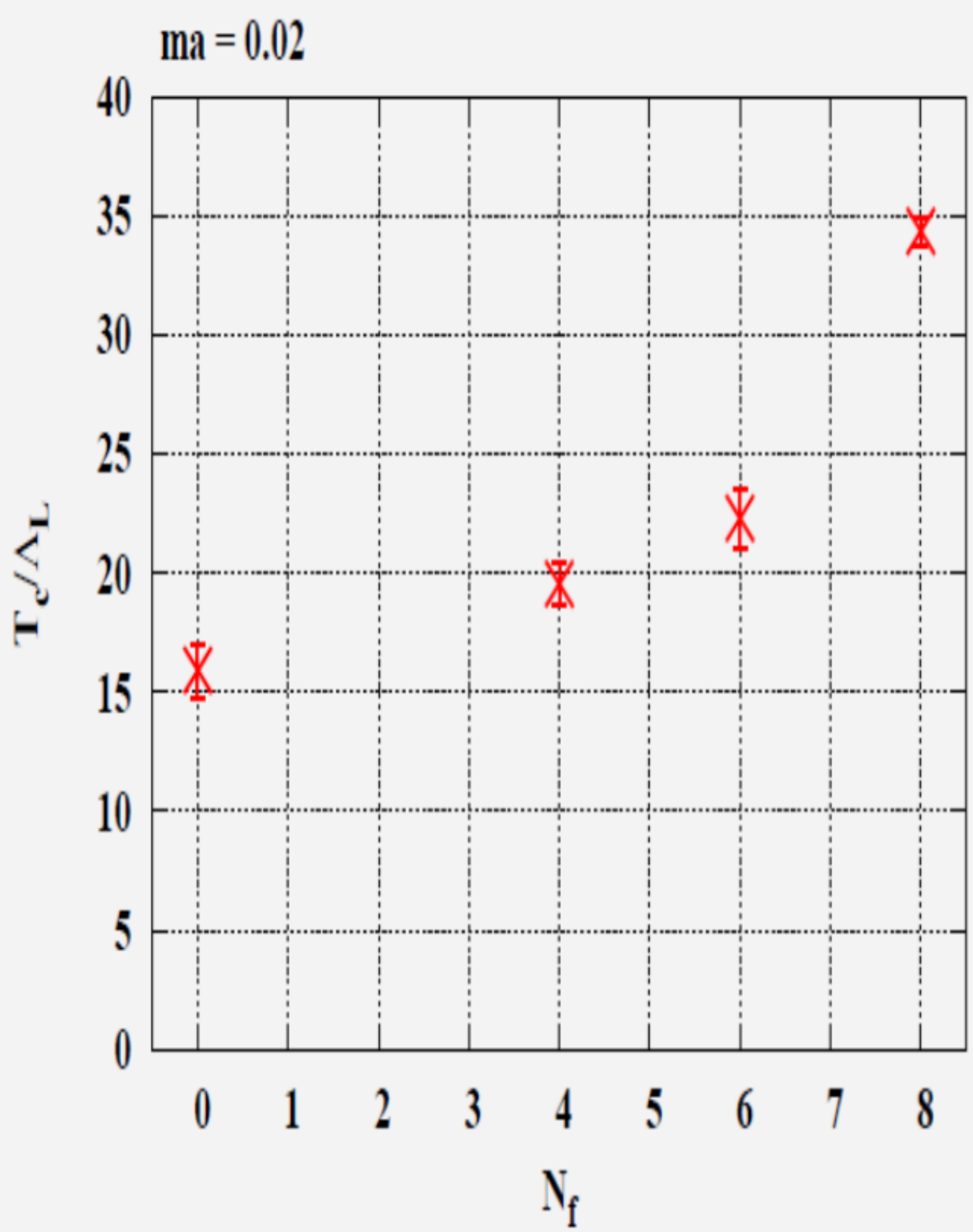
Mutatis mutandis,
Eos approach reproduces  KMI scenario:

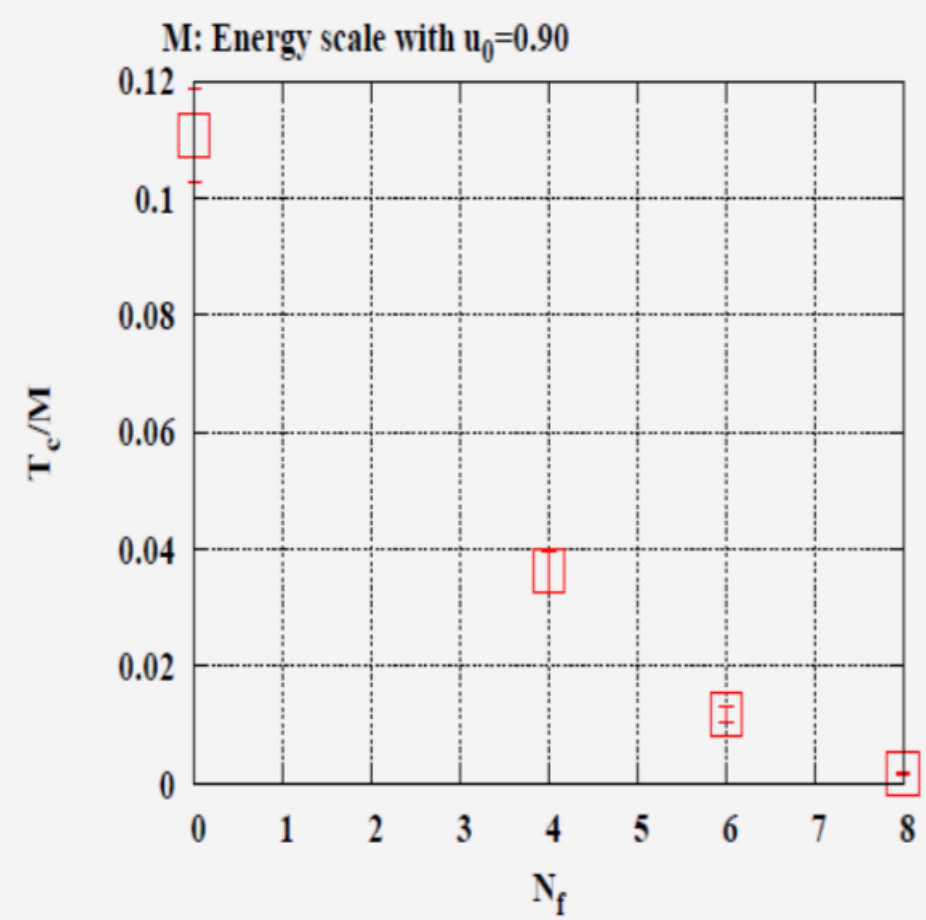
Scaling  with anomalous dimension




KMI 2013

# Search for scale hierarchy - Kohtaroh Miura, MpL, Tiago Nunes da Silva, E Pallante



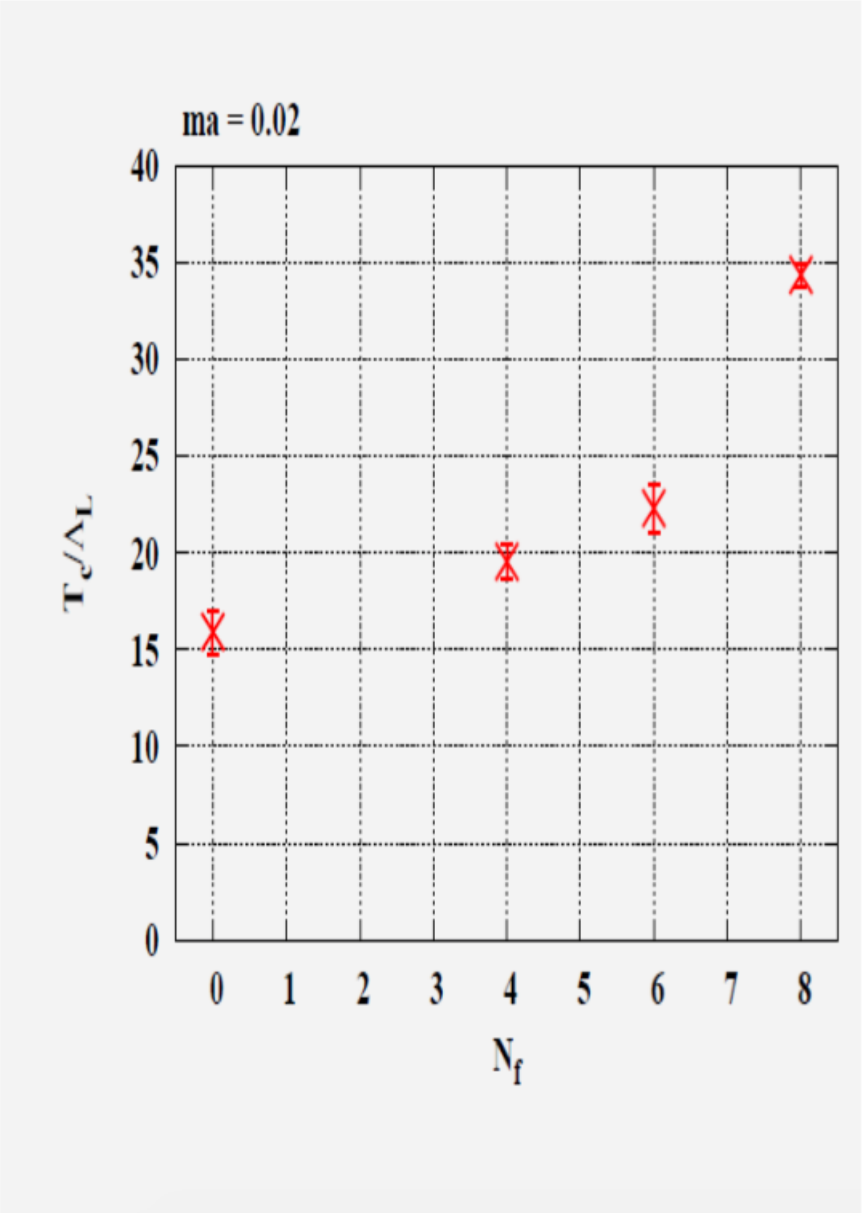

$$\frac{T_c}{M} = \frac{1}{N_t} \exp\left[\int_{g_{\rm ref}}^{g_c} \frac{dg}{B(g)}\right] .$$

K. Miura, MpL, E. Pallante 2012

# Towards a quantitive comparison with holography

K. Miura, MpL, E. Pallante, in progress

$$\frac{2\pi T_c}{M_{KK}} = 1 - \frac{1}{126\pi^3}\lambda_4^2\frac{N_f}{N_c}\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right)$$

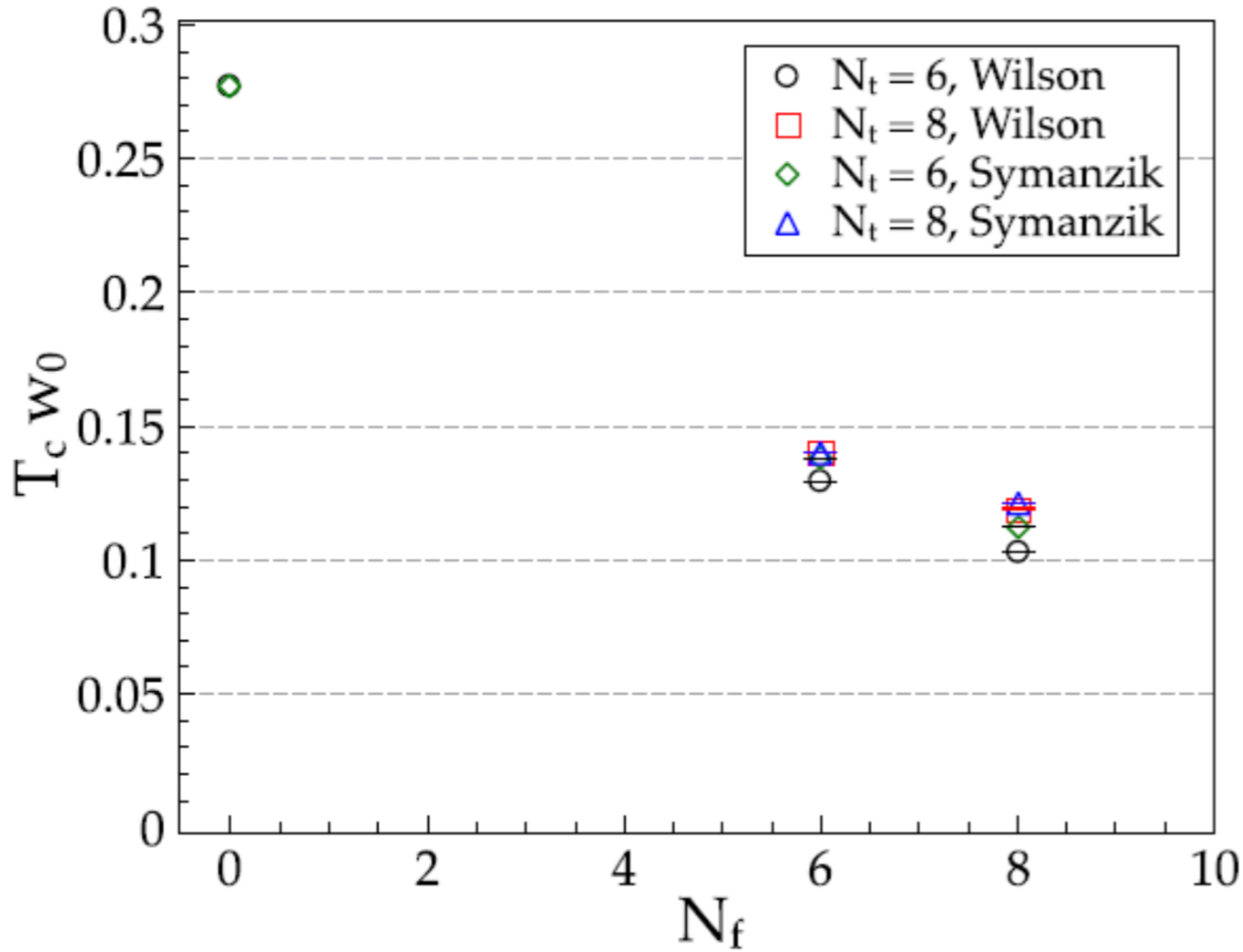



Bigazzi and Cotrone, JHEP 2015

$$\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right) \approx -1.987$$

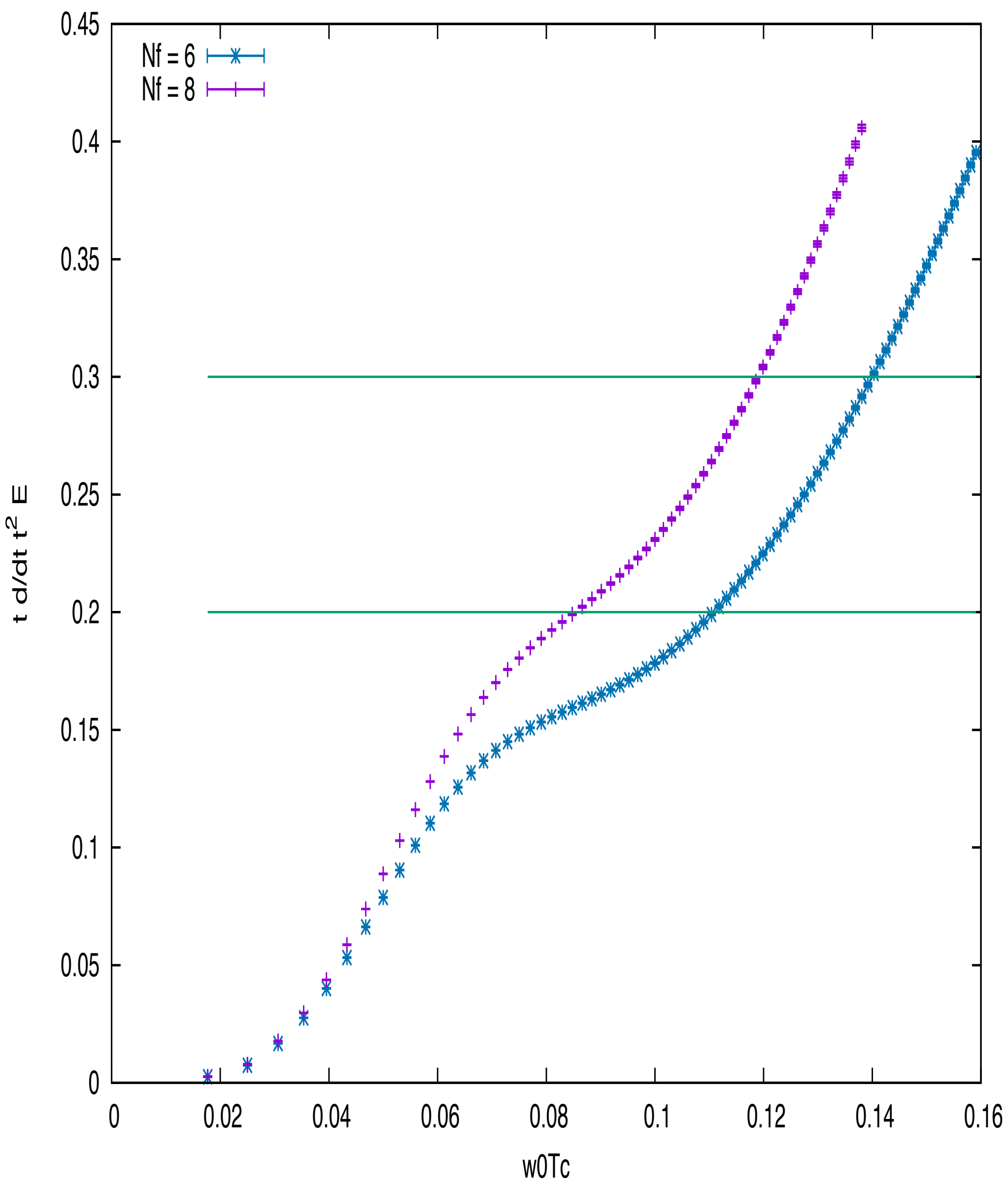
T increases with Nf on the scales
used in these two studies

# Tc on the 1/w0 scale

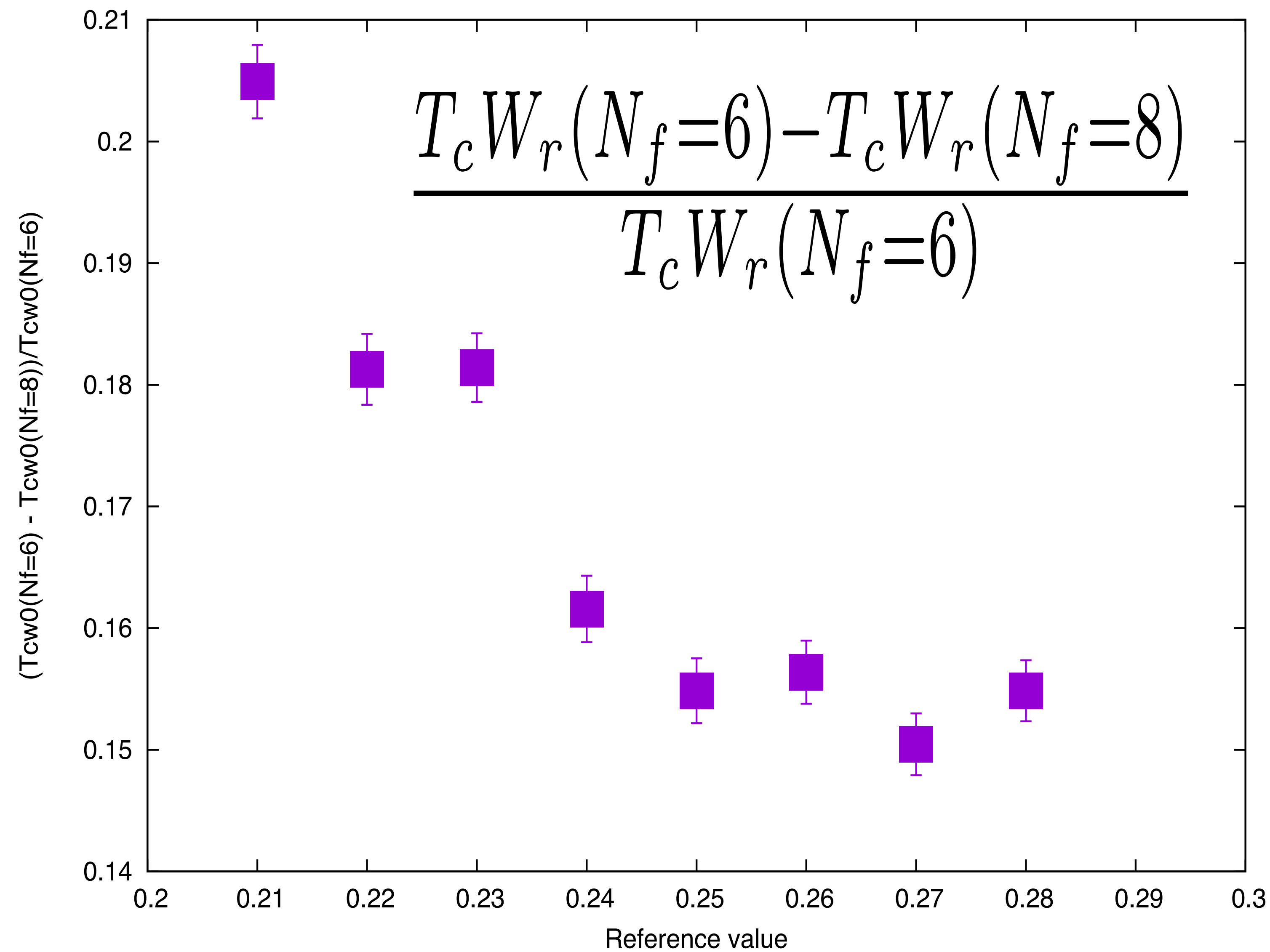



K. Miura, MpL, E.Pallante, in progress

# Moving the scale with Wilson flow



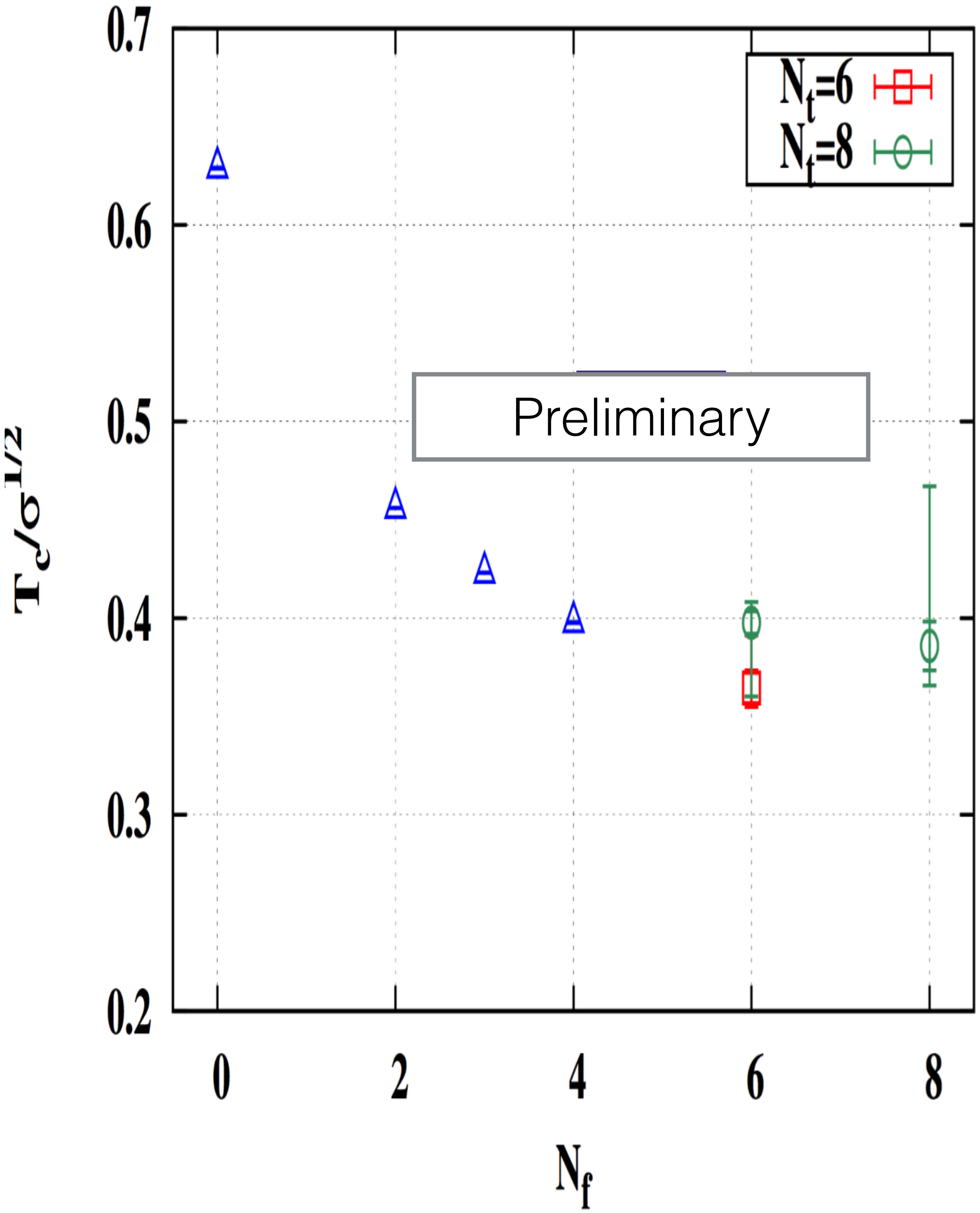

Qualitatively as expected,
limited by lattice artifacts




UV

# Tc and the string tension

KM, MpL, EP, in progress



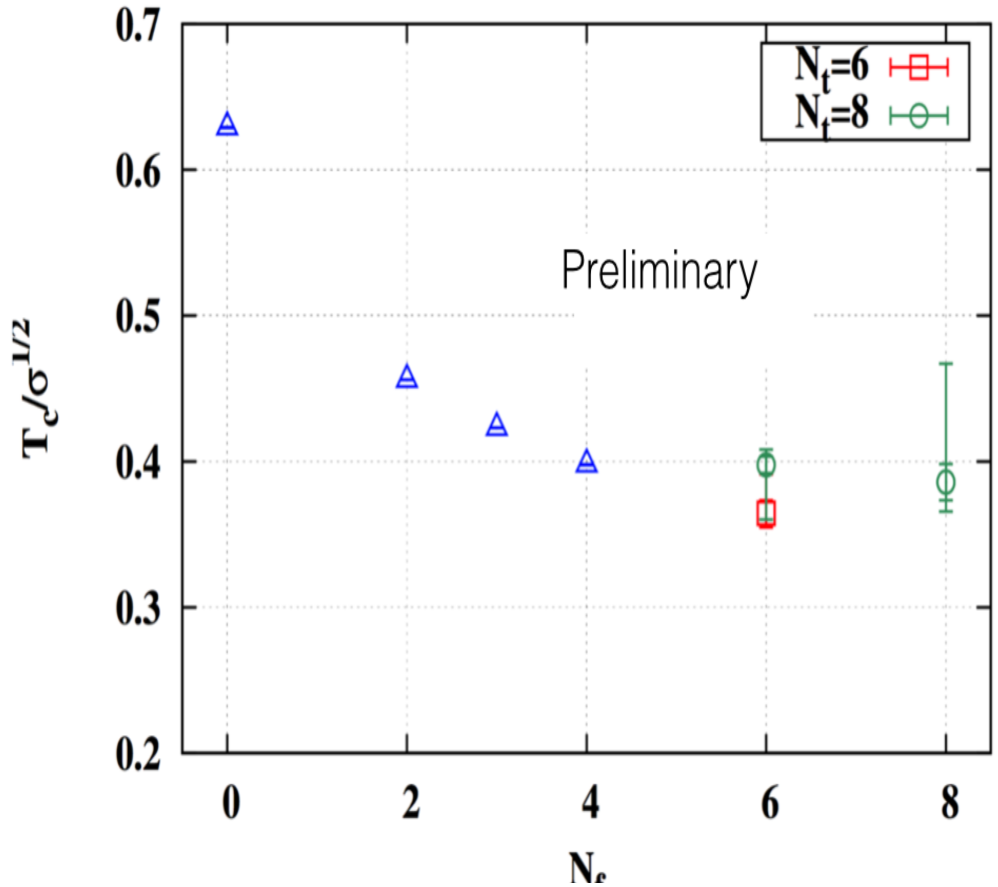

Mild decrease, possibly constant as $N_f \to N_f^c$

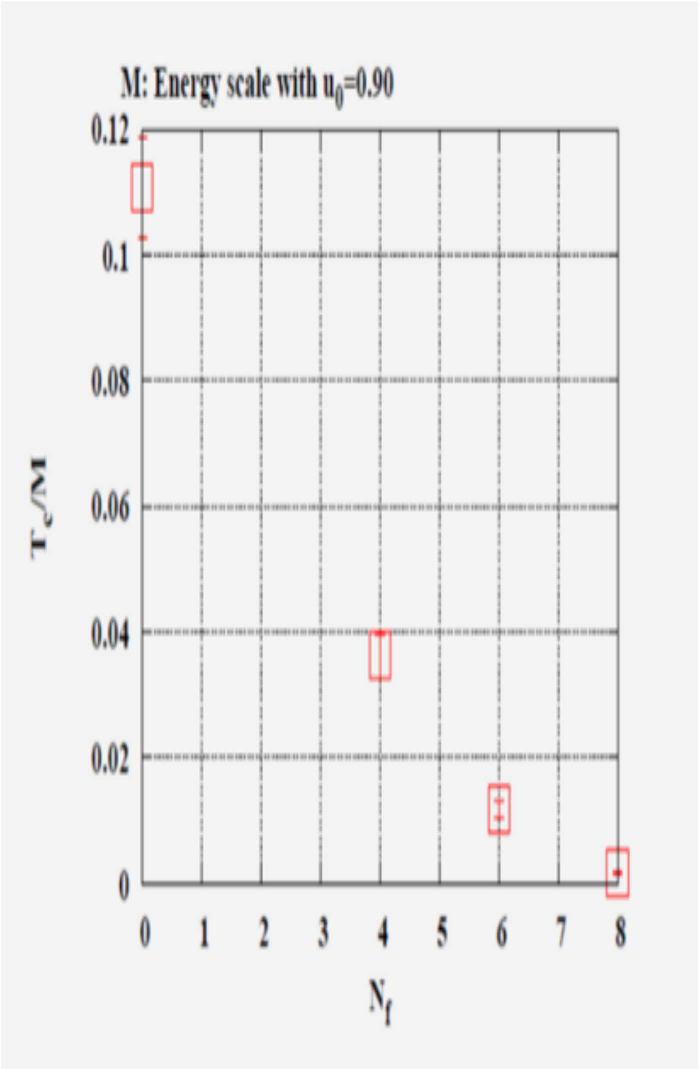
Again similar to the prediction of the WSS model:

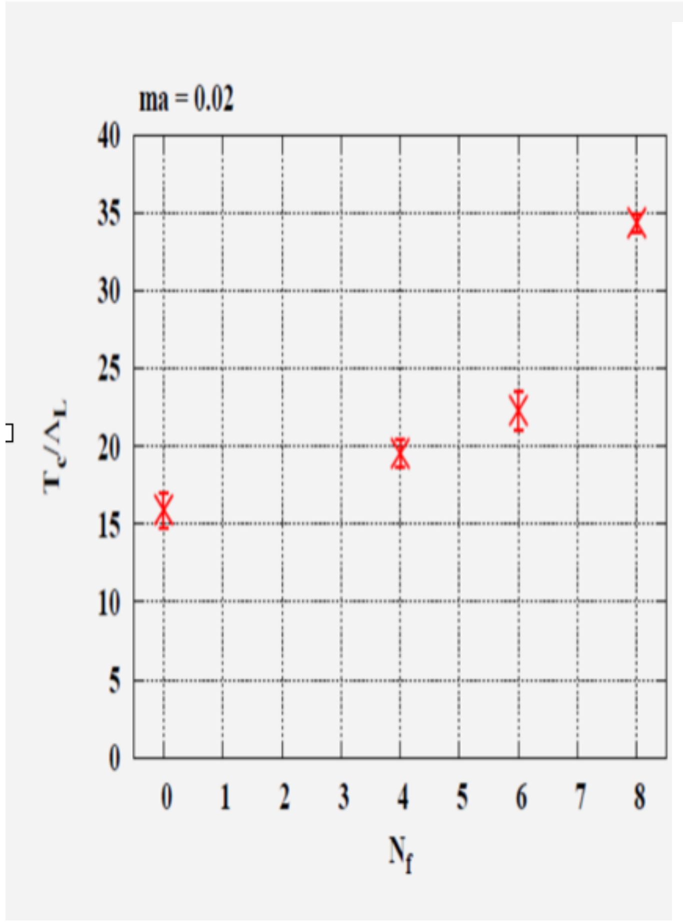
$$\frac{T_c}{\sqrt{\sigma}} \propto (1 - \epsilon N_f / N_c)$$

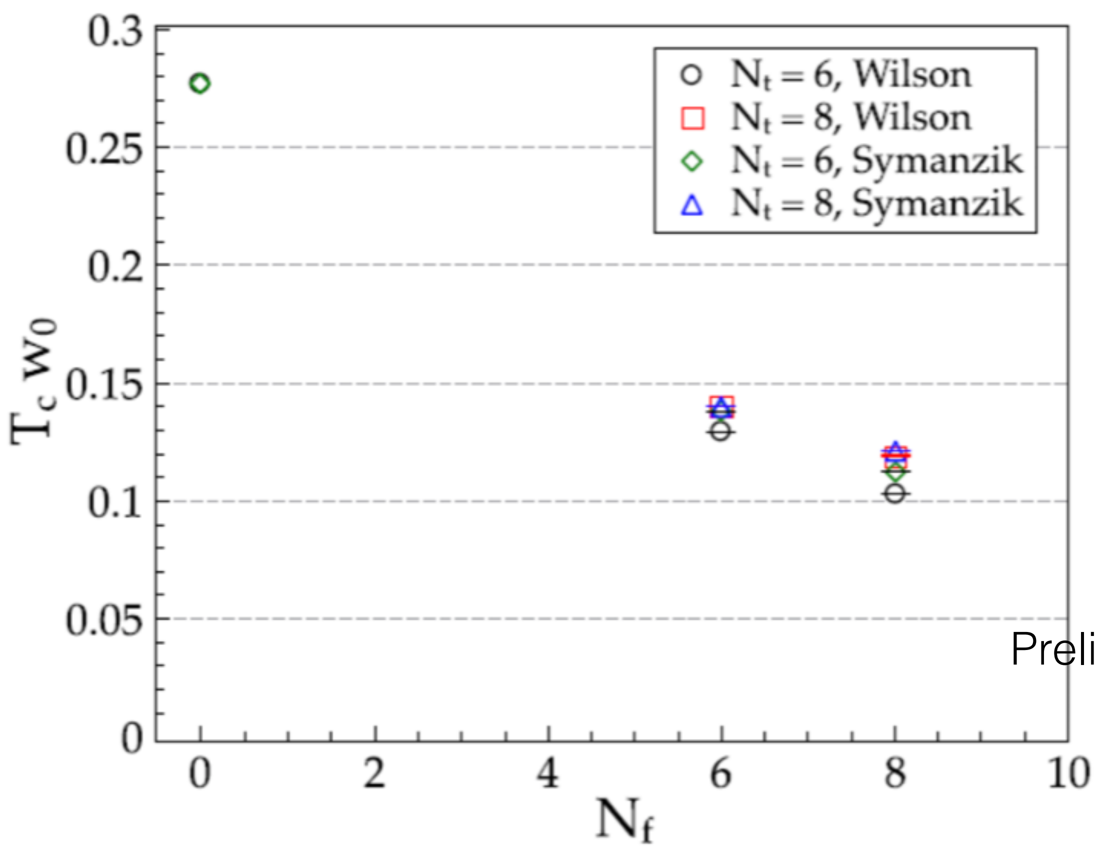
communicated by F. Bigazzi

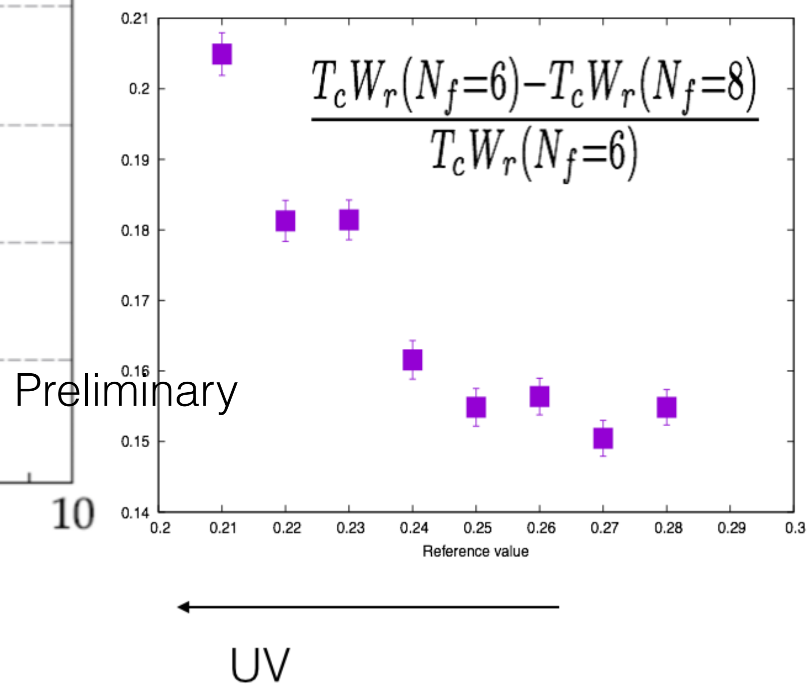
# Hierarchy of scales in the near conformal phase



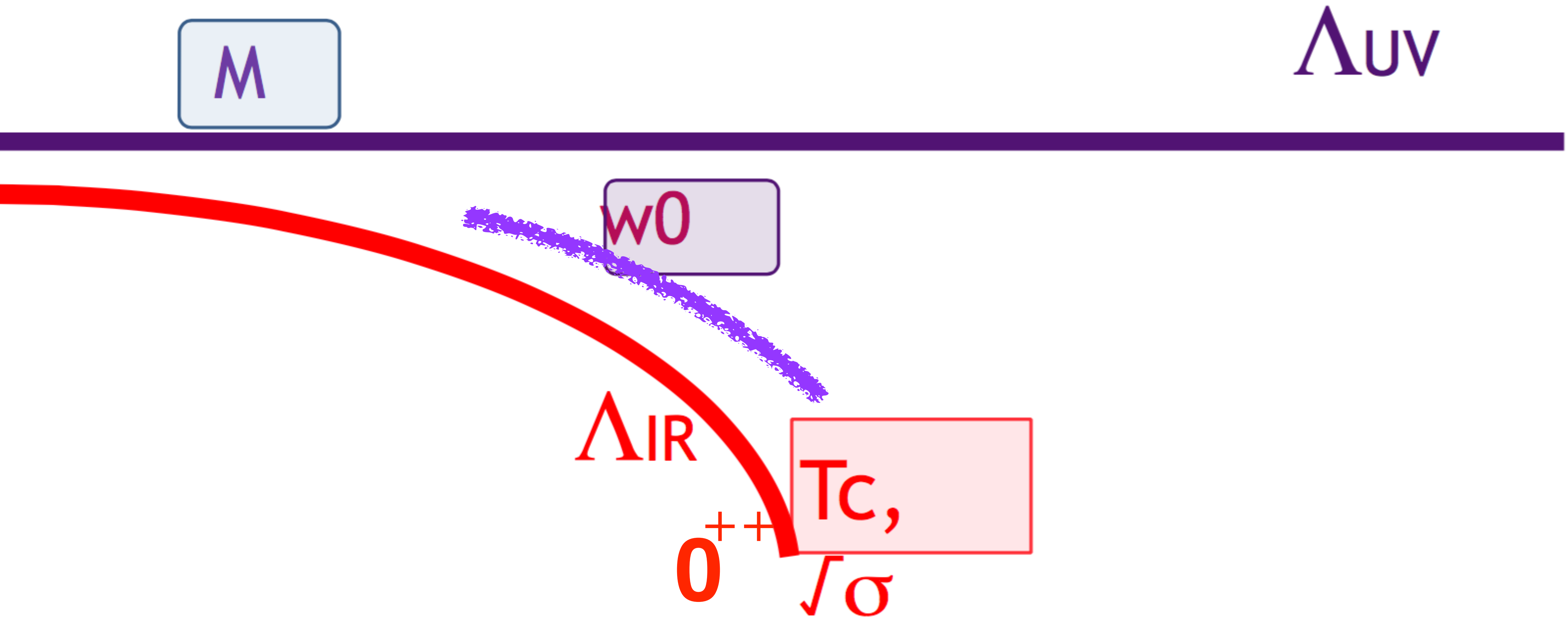



# Hierarchy of scales



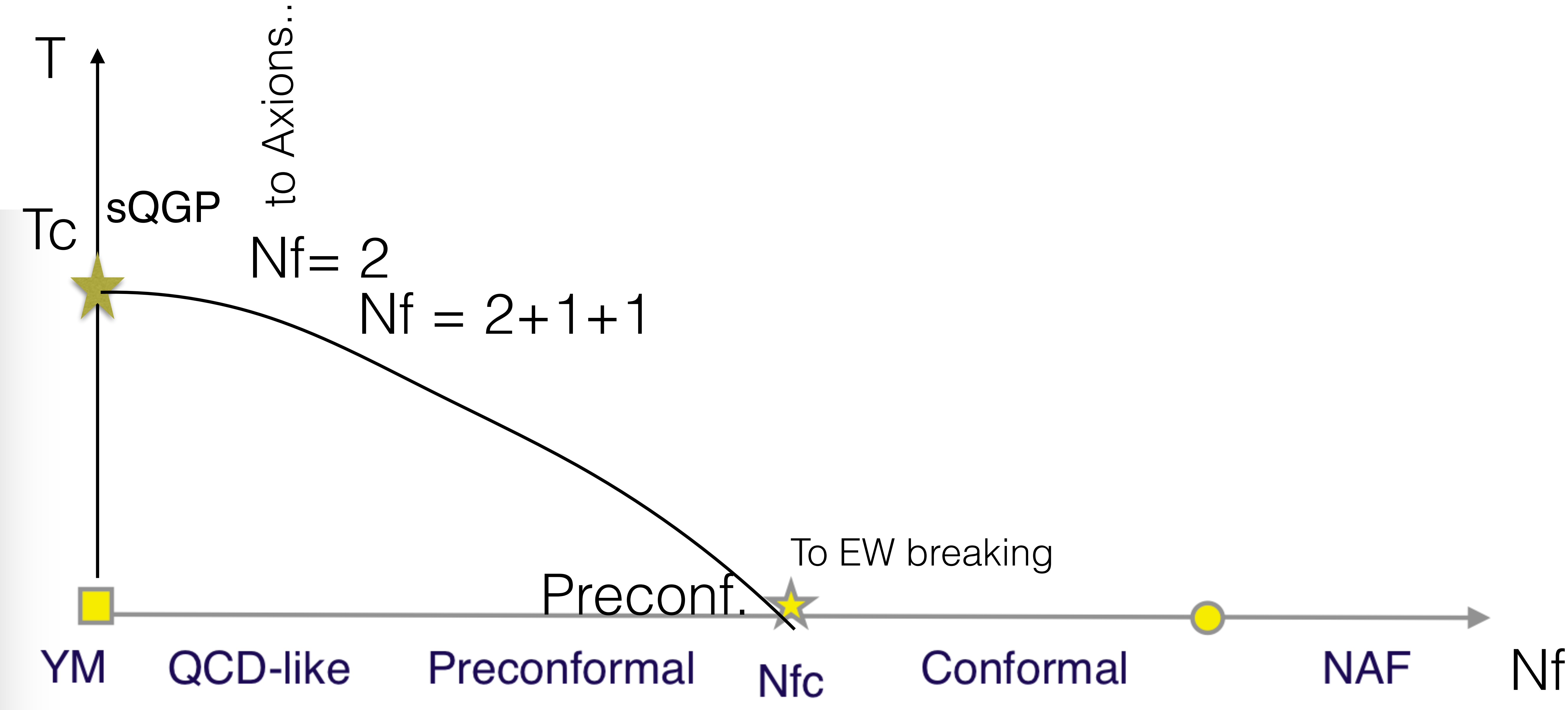
Summary: *Two examples of applications of topics discussed here which have a close connections to BSM physics*

I *Dark Matter: limit of the mass for post-inflationary Axions from QCD topology* —->
calls for more controlled results from topology

II *Higgs mass generation via QCD-like dynamics*

—>observed hierarchy of scales,
<u>special role of the scalar</u> —> further insight from lattice / FRG?? connection with topology

(as advocated by E. Znithiski)




**Comment: interesting to study the critical line in the T, Nf plane… as a way to strongly coupled near conformal QGP..**

Thank You !