QCD-like theories at finite density

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Dubna, 13 July 2017

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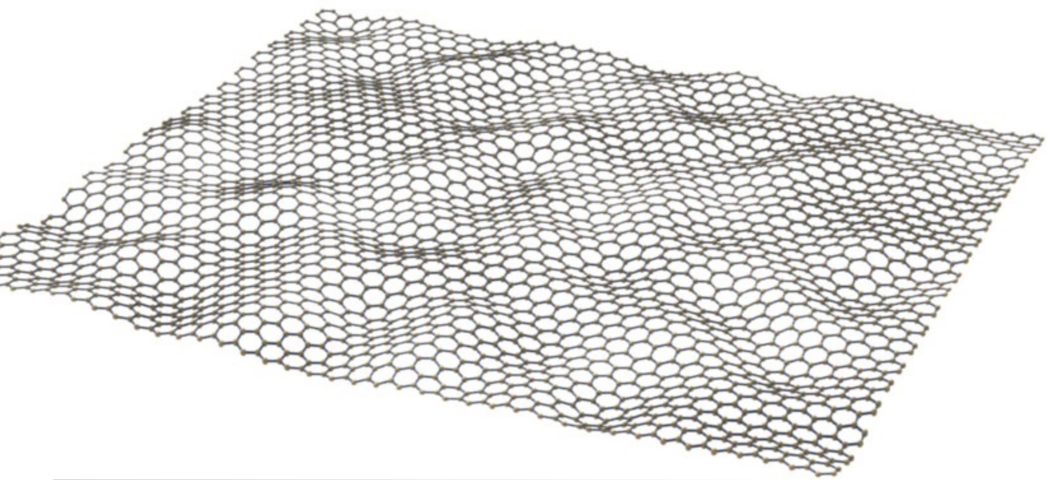
Outline

- Intro & Motivation
- Two-Color QCD with 2 Flavors of Staggered Quarks
- Effective Lattice Theory for Heavy Quarks
- Two-Color QCD in Two Dimensions
- G_2-QCD
- Conclusion

Introduction

[courtesy L. Holicki]

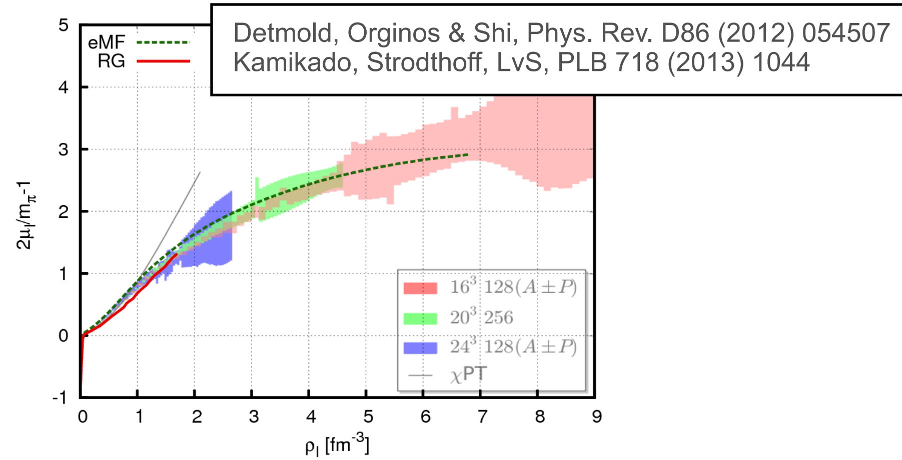


QCD-like Theories

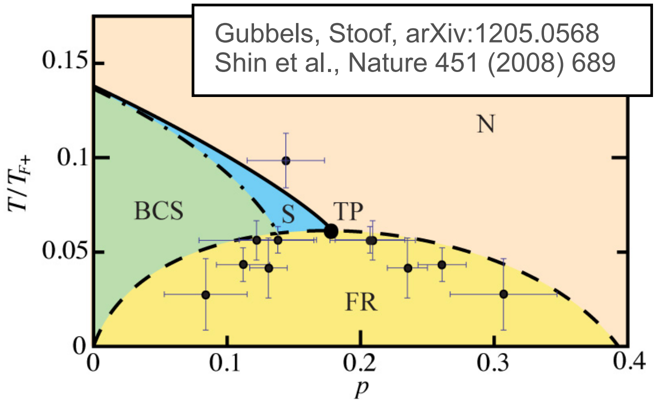
- **compare lattice simulations with functional methods and effective models**
  where there's no sign problem

- **apply to ultracold fermi gases**
  exploit analogies and
  more experimental data

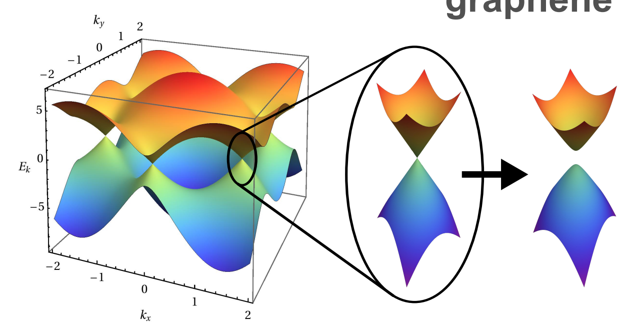
QCD at finite isospin density



polarised fermi gas at unitarity



graphene



- **strongly correlated fermions in 2+1 dimensions**
  electronic properties of graphene

DFG Deutsche Forschungsgemeinschaft

Fermion-Sign Problem

sign problem:

$$\big(\mathrm{Det}\, D(\mu_f)\big)^* = \mathrm{Det}\, D(-\mu_f)$$

- **except if:**

(a) two degenerate flavors with isospin chemical potential

Dyson index:

fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$ $\qquad \beta = 2$

QCD at finite isospin density

(b) anti-unitary symmetry $\quad TD(\mu)T^{-1} = D(\mu)^* \qquad T^2 = \pm 1$

fermion color representation:

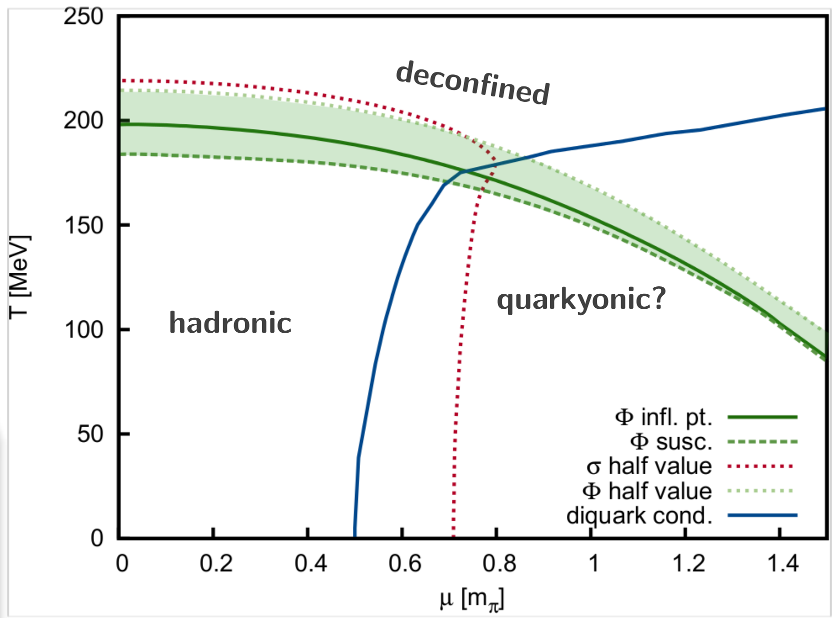
(i) pseudo-real $\quad T^2 = 1 \quad$ **two-color QCD** $\qquad \beta = 1$

(ii) real $\quad T^2 = -1 \quad$ **adjoint QCD, or G_2-QCD** $\qquad \beta = 4$

Phase Diagram of QC_2D

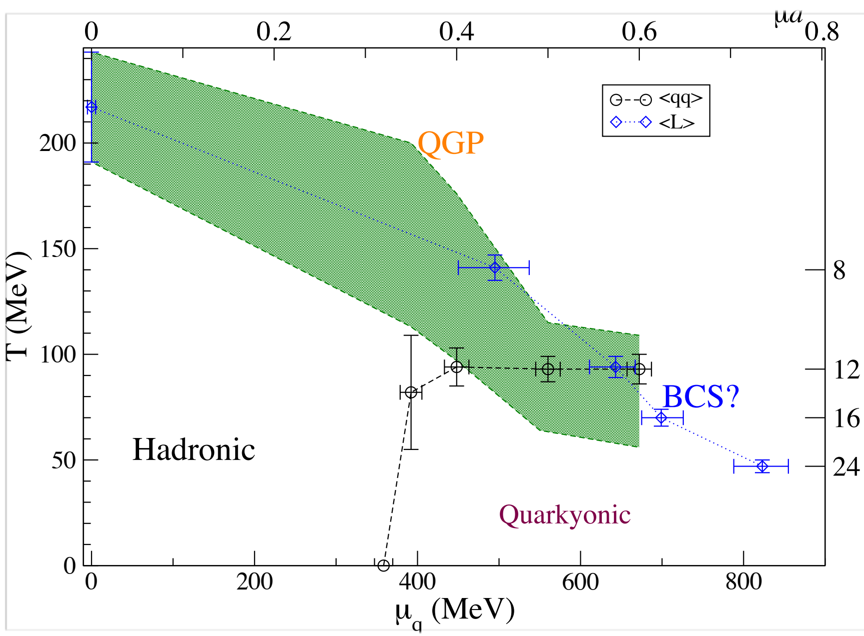
- **Quark-Meson-Diquark model:**

  Strodthoff, Schaefer & LvS,
  Phys. Rev. D85 (2012) 074007



Strodthoff & LvS, PLB 731 (2014) 350

- **Lattice simulations:**



Cotter, Giudice, Hands & Skullerud,
PRD 87 (2013) 034507

Goldstone Spectrum - QC_2D

- **extended flavor symmetry (Pauli-Gürsey), at $\mu = 0$**

  $SU(N_f) \times SU(N_f) \times U(1)$ becomes $SU(2N_f)$

  $N_f = 2$: connects pions and σ-meson with scalar (anti)diquarks.

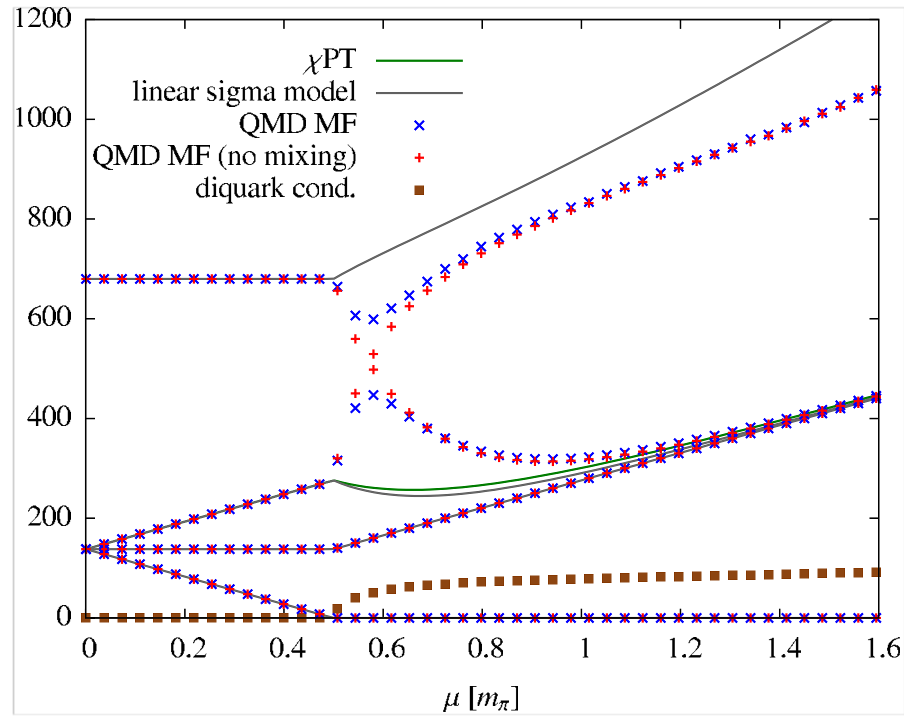
- **Dirac mass (quark condensate)**

  $$SU(4) \to Sp(2)$$

  or $$SO(6) \to SO(5)$$

  Coset: S^5 5 Goldstone bosons: pions and scalar (anti)diquarks

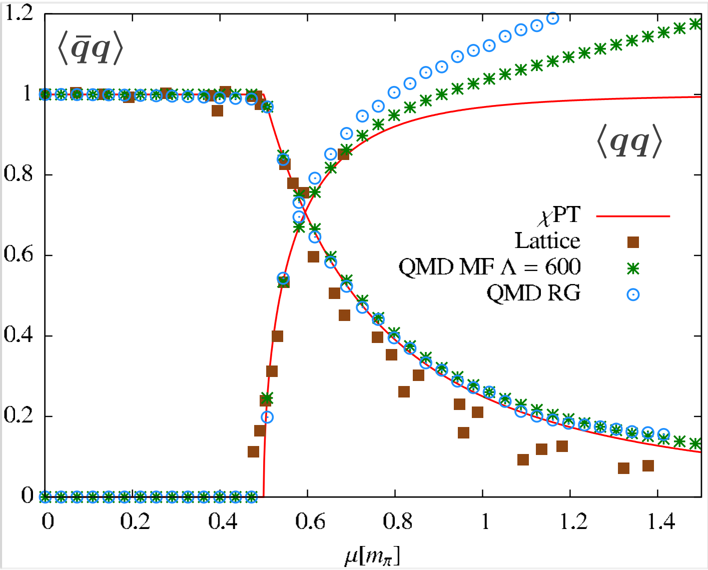
- **color-singlet diquarks (bosonic baryons)**



Strodthoff, Schaefer & LvS, PRD 85 (2012) 074007

Vacuum Realignment

zero temperature condensates



Hands *et al.*, EPJC 17 (2000) 285; EPJC 22 (2001) 451

χPT:

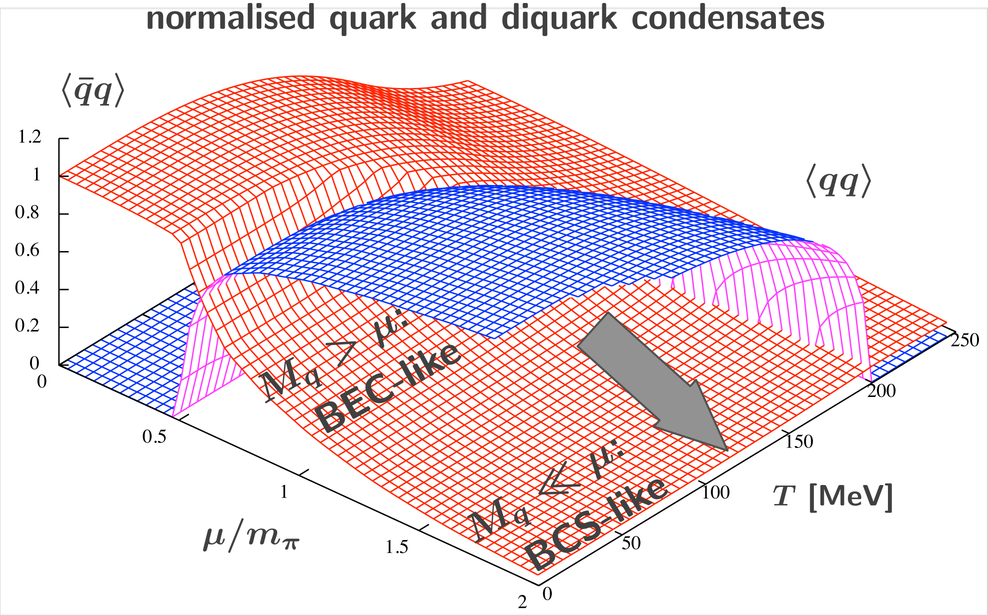
$$\langle \bar{q}q \rangle = 2N_f G \cos\alpha$$

$$\langle qq \rangle = 2N_f G \sin\alpha$$

$$n_B = 8N_f F^2 \mu \sin^2\alpha$$

Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl. Phys. B 582 (2000) 477

- QMD model phase diagram



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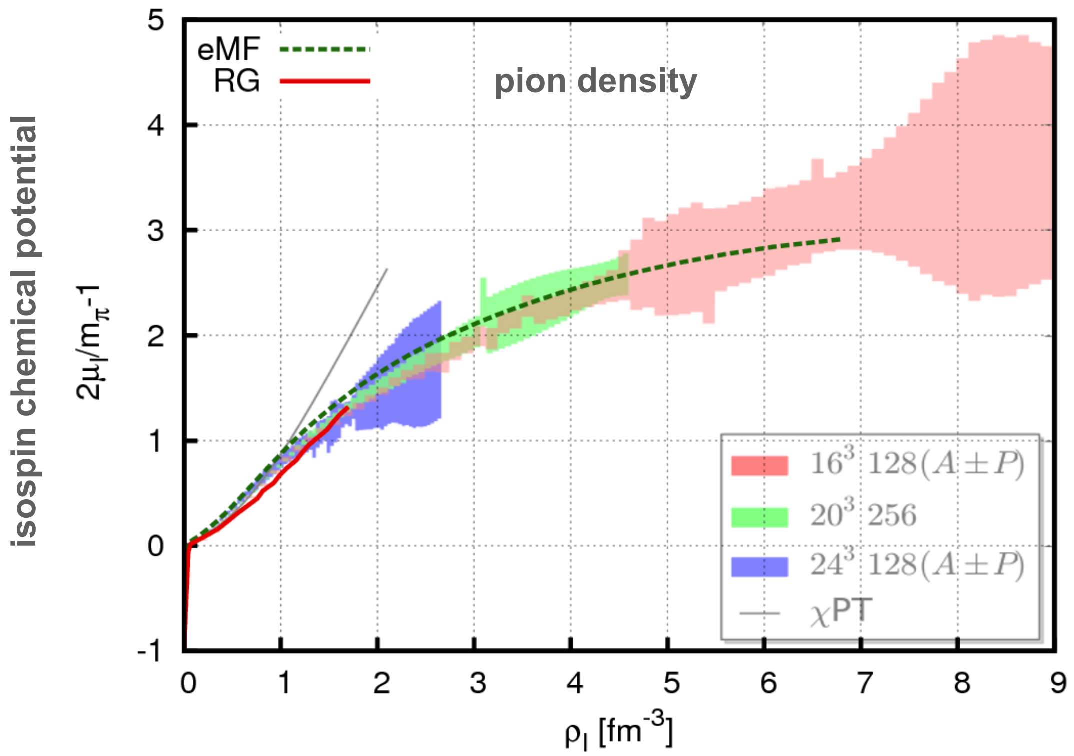
QCD with Isospin Chemical Potential

- *T* = 0 isospin density - FRG vs. lattice QCD:

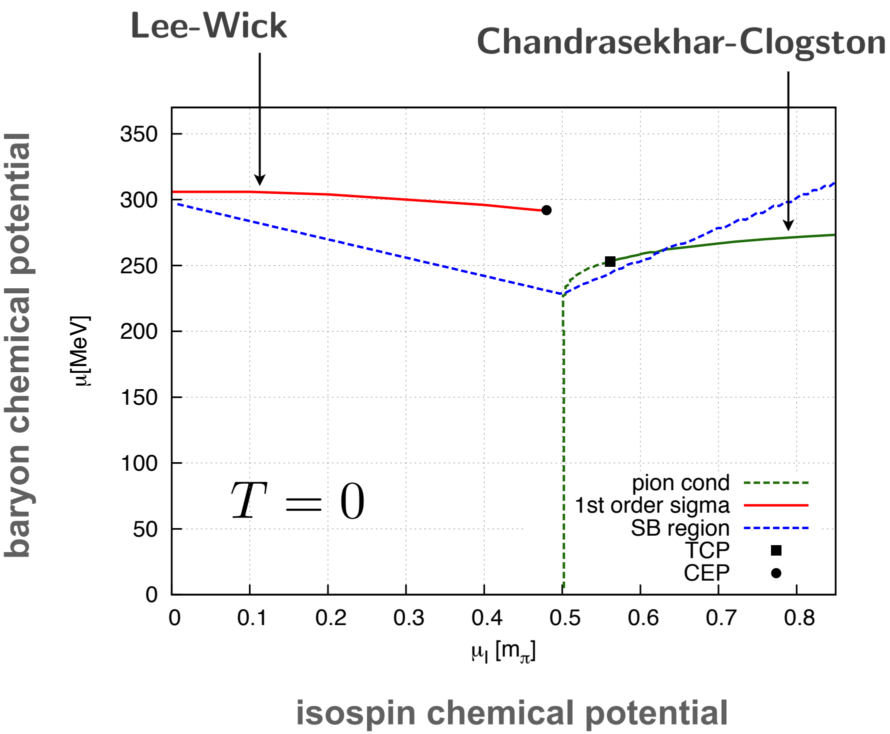


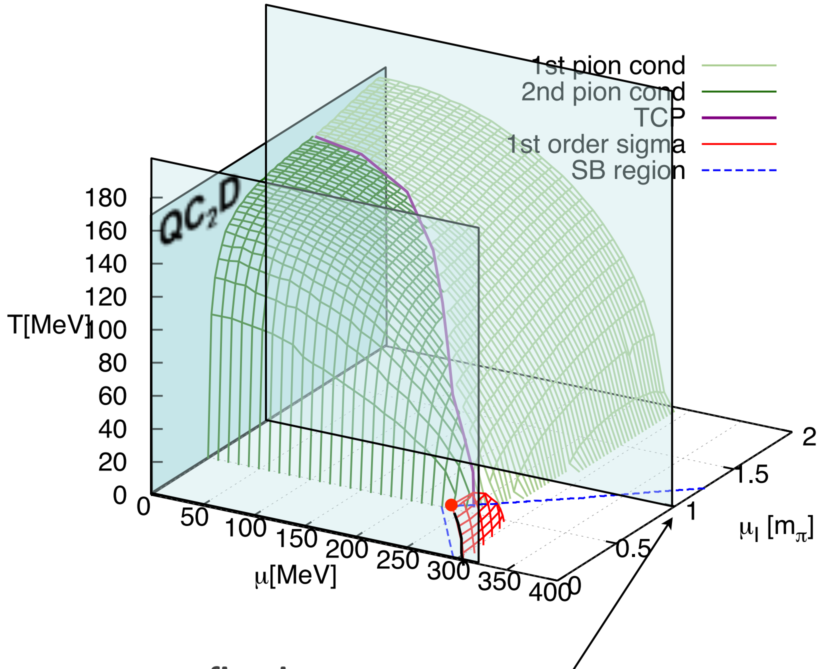
Kamikado, Strodthoff, LvS, PLB 718 (2013) 1044

Detmold, Orginos & Shi, Phys. Rev. D86 (2012) 054507

Baryon & Isospin Chemical Potential

- **Quark Meson Model**





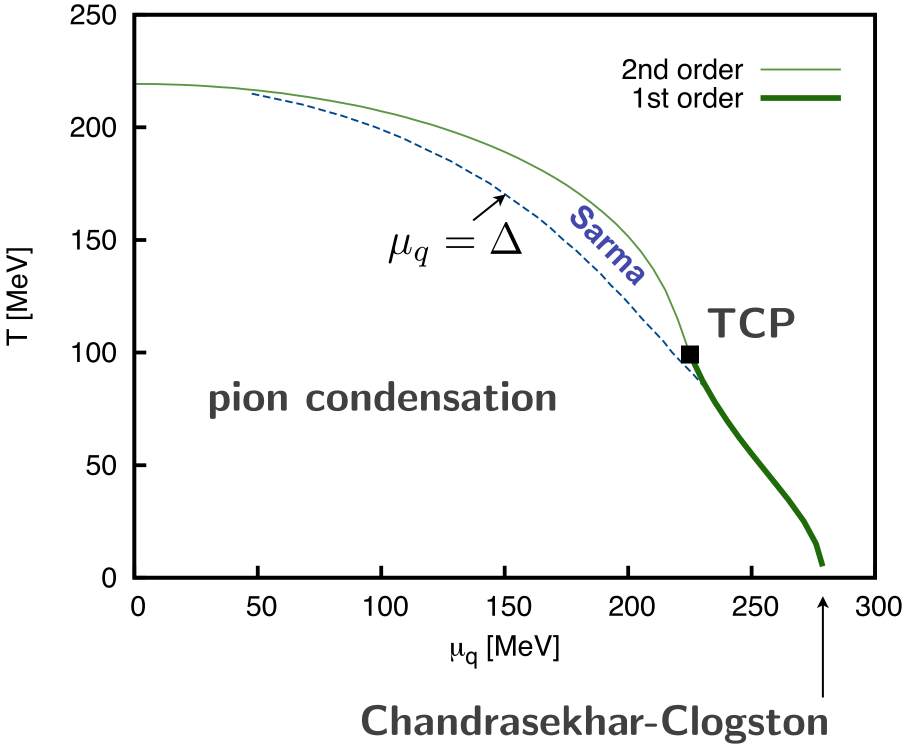
fixed $\mu_I = m_\pi$

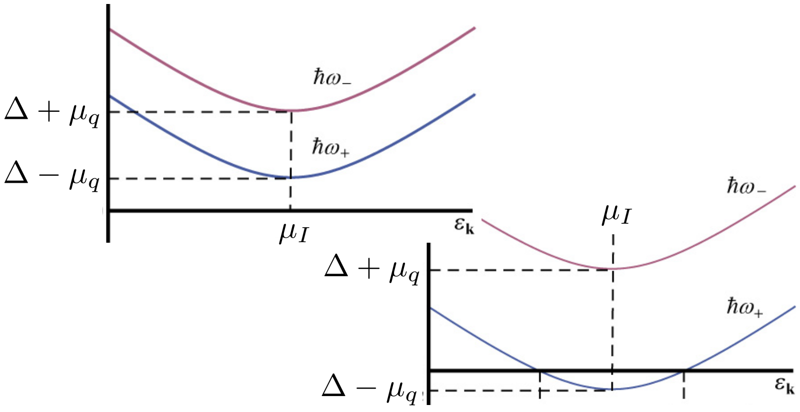
μ for (up/anti-down) imbalance

Up-Antidown Imbalance

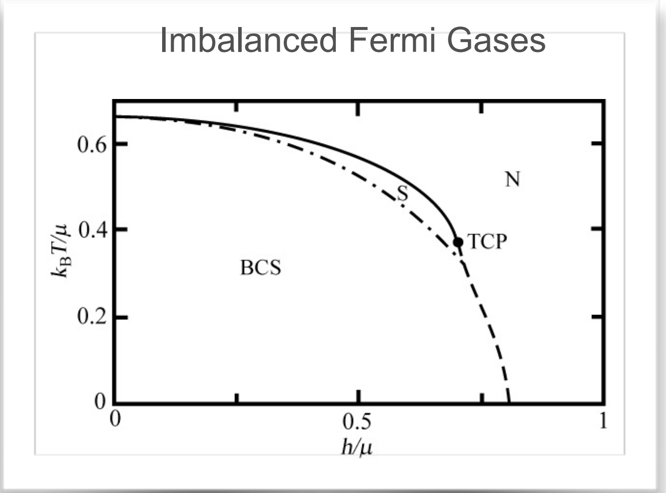
Boettcher, Herbst, Pawlowski, Strodthoff, LvS & Wetterich, PLB 742 (2015) 86

$\mu_I = m_\pi$





- **compare:**



Gubbels, Stoof, 2012

Lattice MC Simulations

Hands, Montvay, Scorzato & Skullerud,
Eur. Phys. J. C 22 (2001) 451

Hands, Kenny, Kim & Skullerud,
Eur. Phys. J. A 47 (2011) 60, …

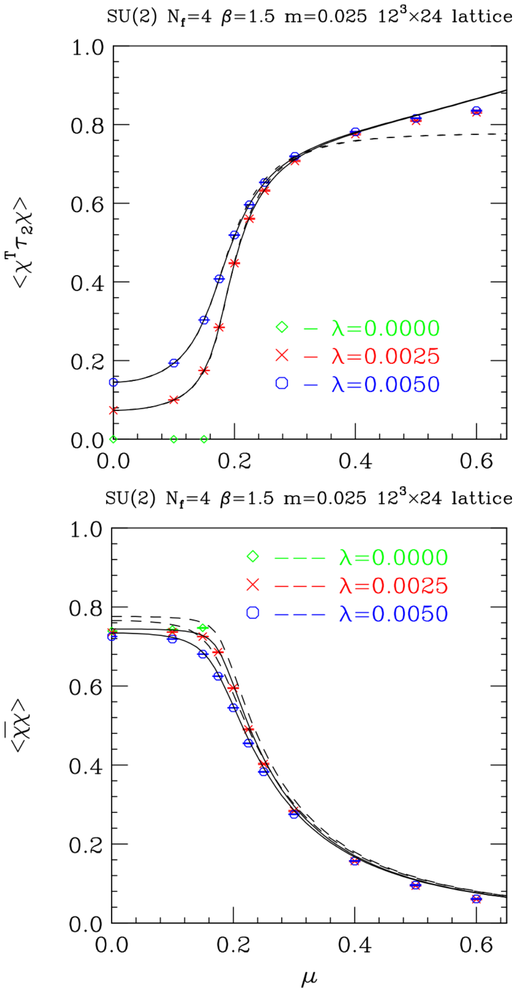
Kogut, Toublan, Sinclair,
Phys. Rev. D 68 (2003) 054507

Braguta, Ilgenfritz, Kotov, Molochkov &
Nicolaev, Phys. Rev. D 94 (2016) 114510

$N_f = 2$ Flavors of Staggered Quarks

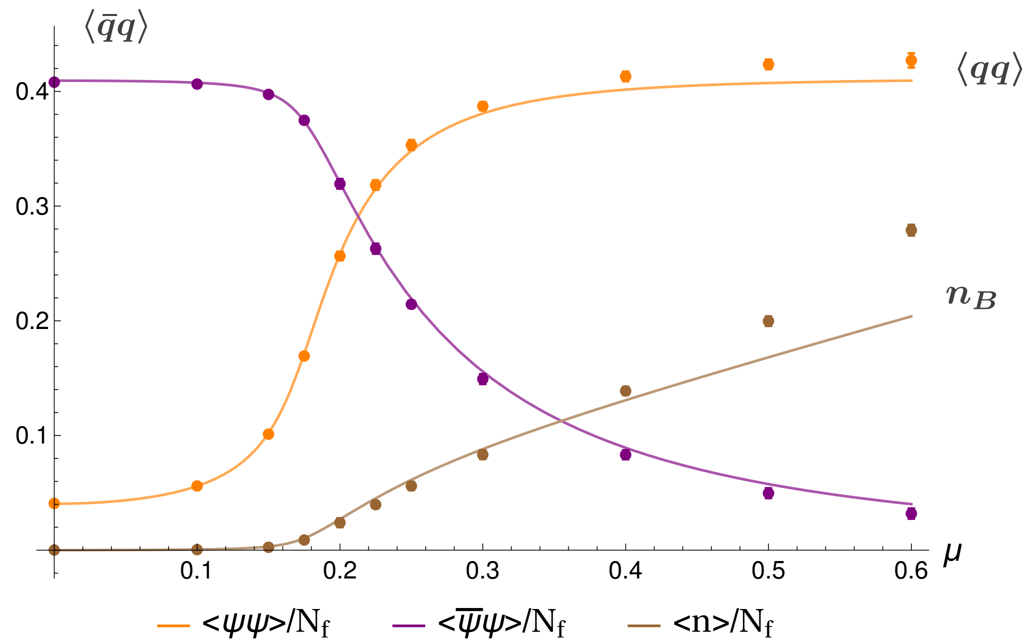
Validate - Previous Results

Kogut, Toublan & Sinclair,
PRD 68 (2003) 054507



- **diquark source**

$$S_f = \overline{\psi} D(\mu)\psi + \frac{\lambda}{2}\left(\psi^T (C\gamma_5)\tau_2\psi + \overline{\psi}(C\gamma_5)\tau_2\overline{\psi}^T\right)$$

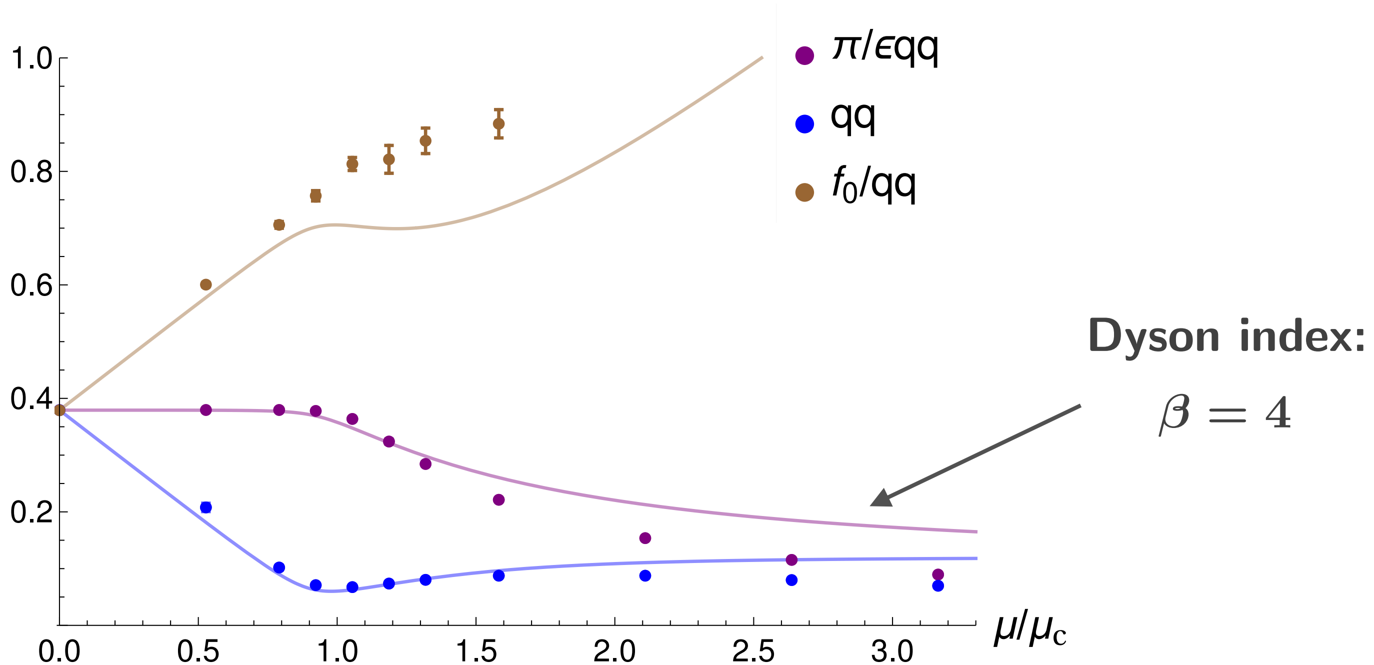


$N_f = 2,\ \beta = 1.5,\ m = 0.025,\ \lambda = 0.0025,\ 12^3 \times 24$ lattice

Goldstone Spectrum - QC_2D

- **mixing at finit density:** f_0/qq: $\frac{1}{2}\left(\chi^T\tau_2\chi + \bar{\chi}\tau_2\bar{\chi}^T\right)\cos\alpha + \bar{\chi}\chi\sin\alpha$

  $\pi/\epsilon qq$: $\bar{\chi}\epsilon\chi\cos\alpha + \frac{1}{2}\left(\chi^T\tau_2\epsilon\chi + \bar{\chi}\tau_2\epsilon\bar{\chi}^T\right)\sin\alpha$



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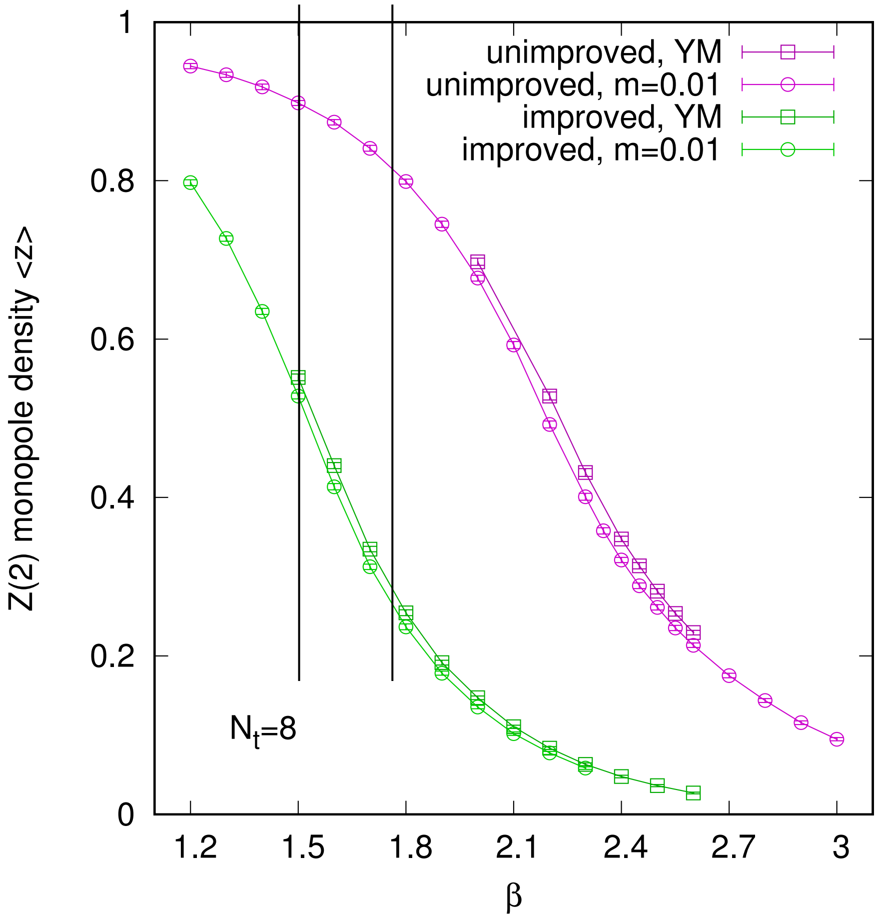
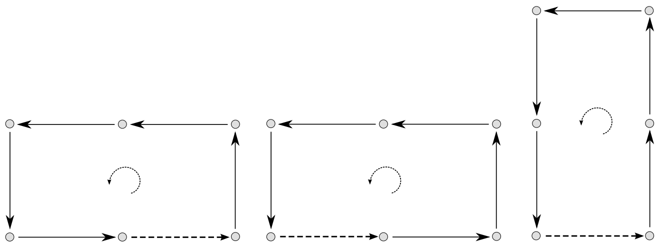
Bulk Phase of SU(2)

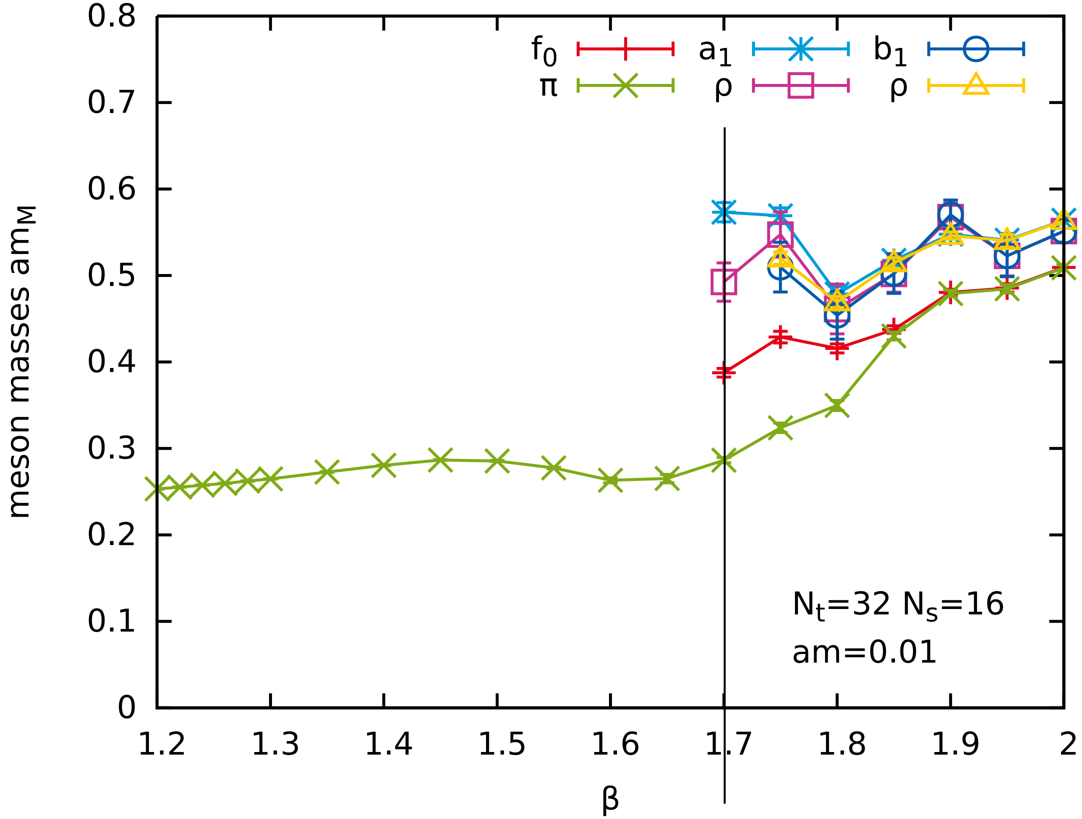


$\langle z \rangle = 1 - \frac{1}{N_C} \sum_C \prod_{P \in \partial C} \mathrm{sgn}\,\mathrm{tr}\, P$

- $\beta = 1.5 \to \langle z \rangle \approx 0.95$
- gauge action Symanzik improvement



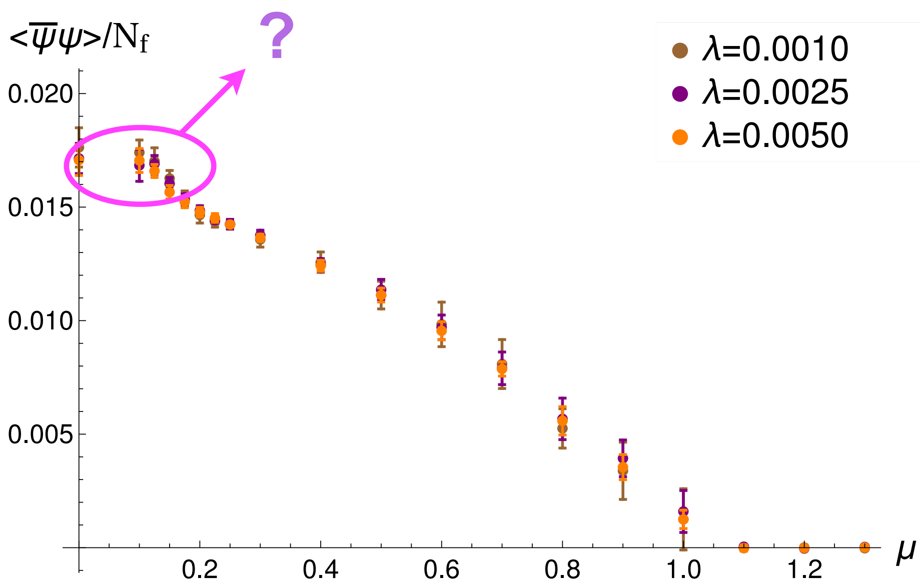
Improved Action - Simulation Parameters



- Compromise: $\beta = 1.7,\ \frac{m_\pi}{m_\rho} = 0.5816(27)$
- $N_s = 16,\ N_t = 32$
- standard rooted staggered quarks ($N_f = 2$), improved gauge action

D. Scheffler, PhD thesis, TU Darmstadt (2015)

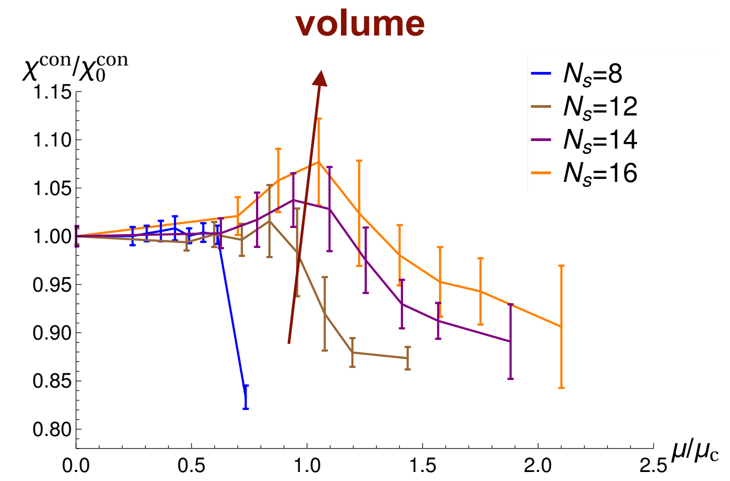
Quark Condensate



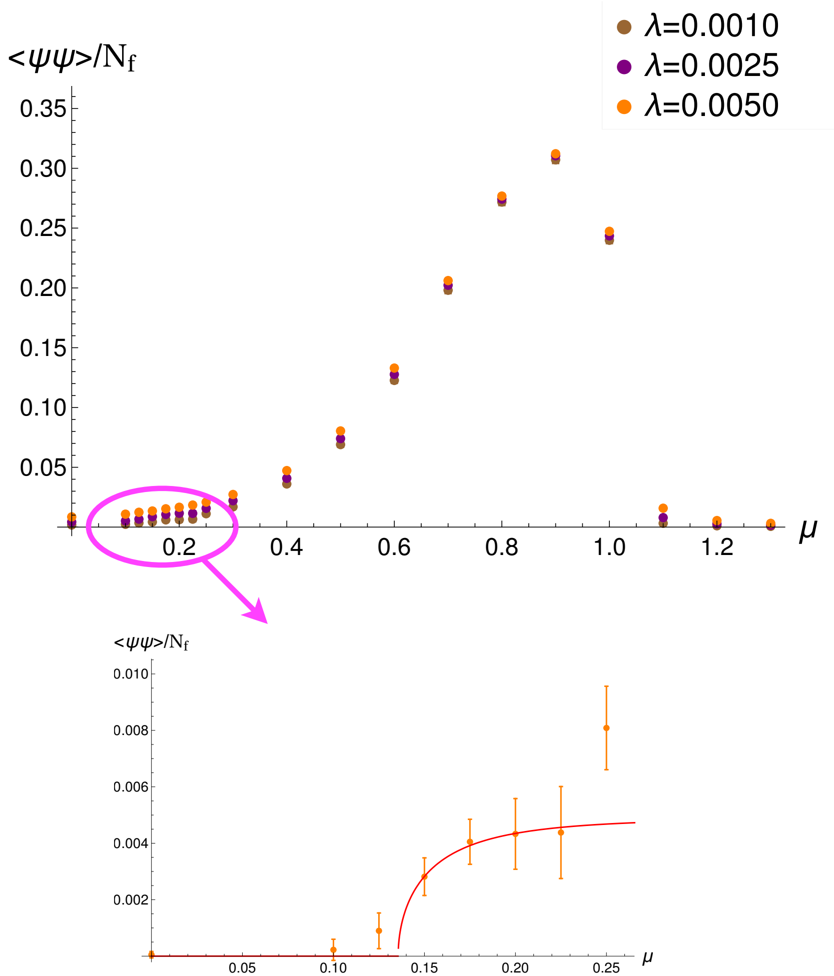
- **additive renormalization:**

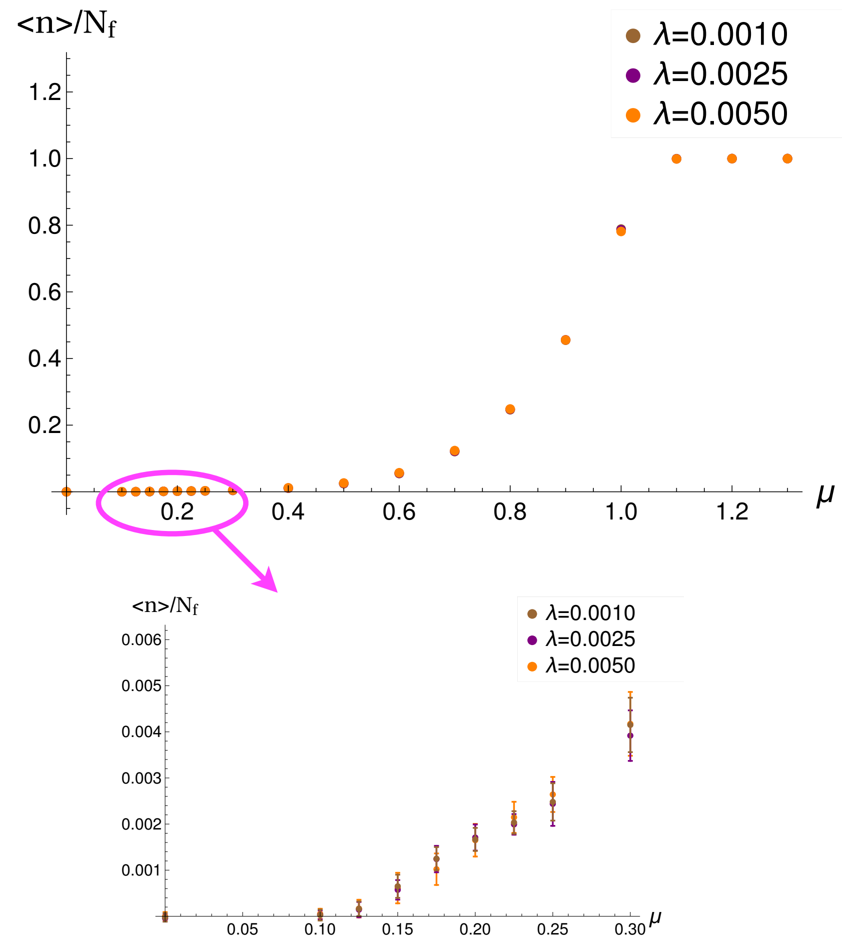
$$\langle\bar\psi\psi\rangle_{m_q} = \langle\bar\psi\psi\rangle_0 + c_2 m_q + \frac{c_{UV}}{a^2} m_q + O(m_q^3)$$

$$\chi_{m_q} = c_2 + \frac{c_{UV}}{a^2} + O(m_q^2)\ .$$

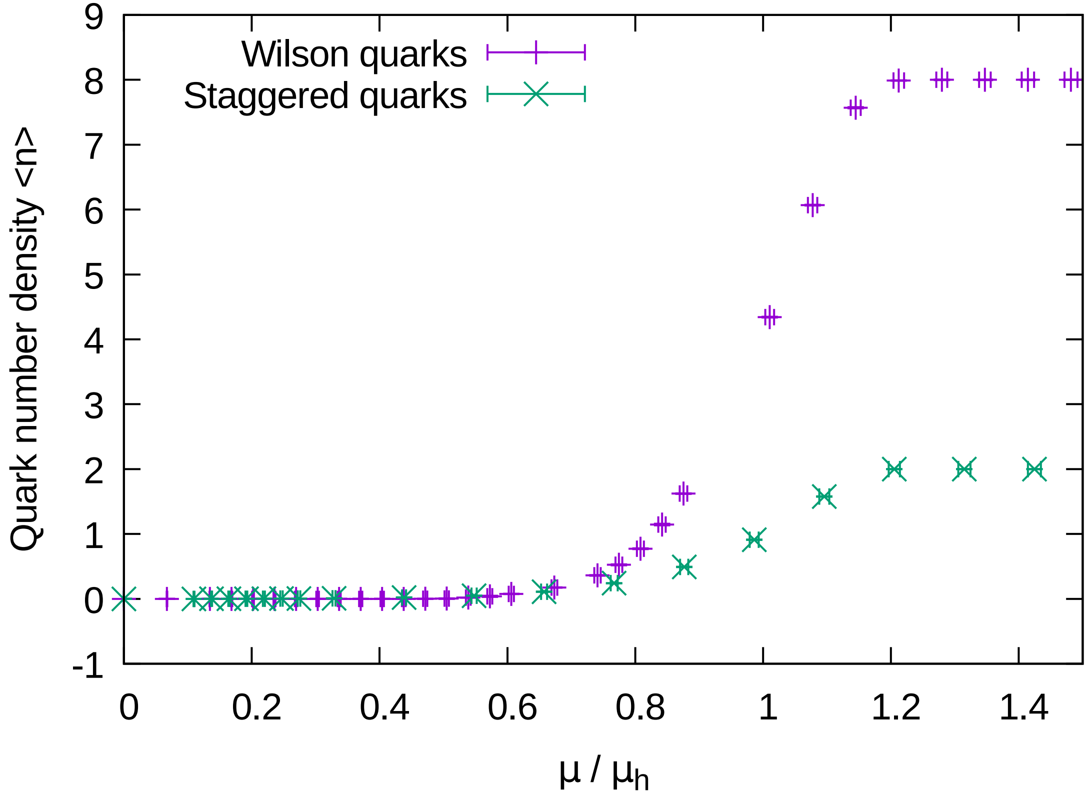


Diquark Condensate & Density

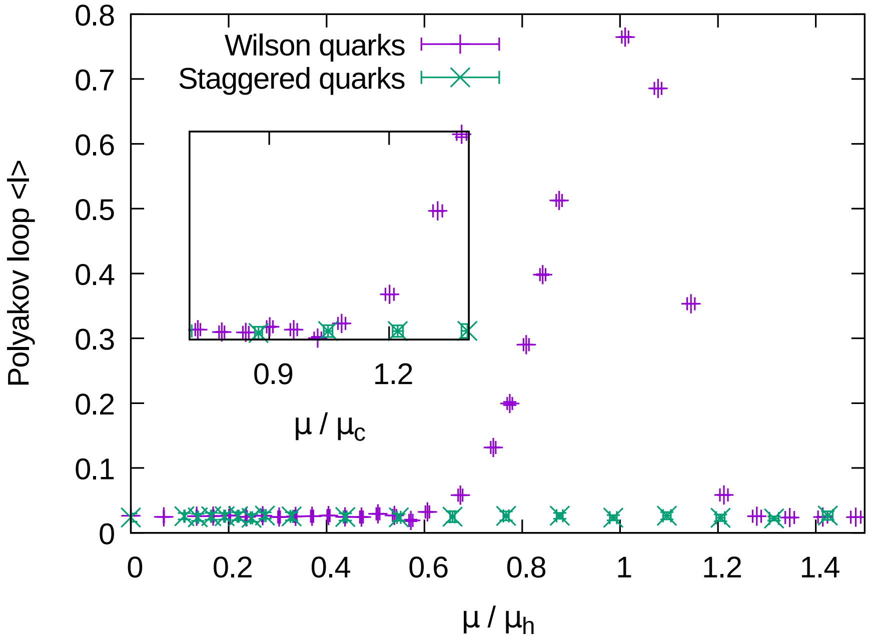


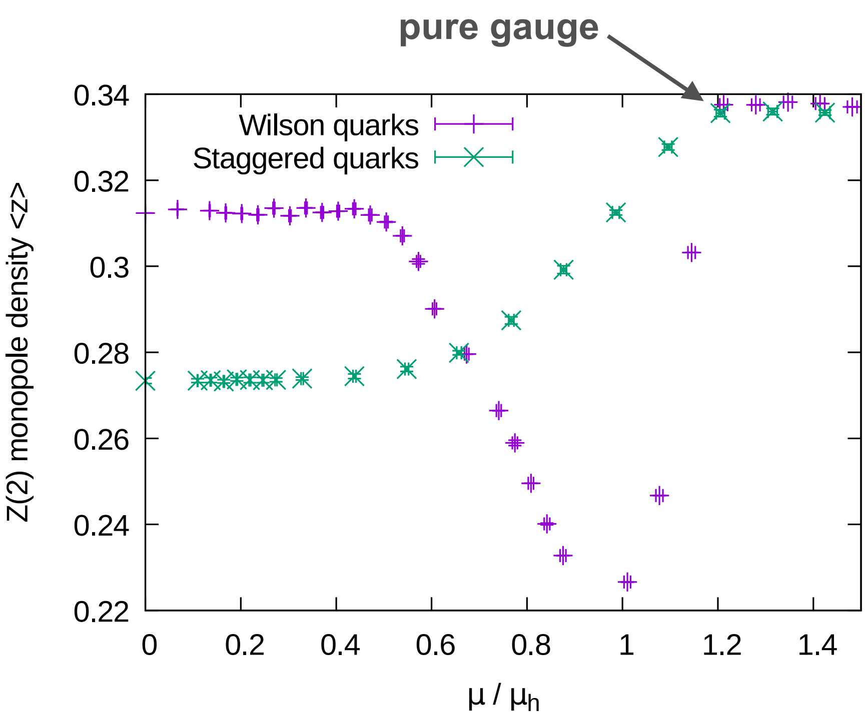


Staggered vs. Wilson



Polyakov Loop & Monopole Density

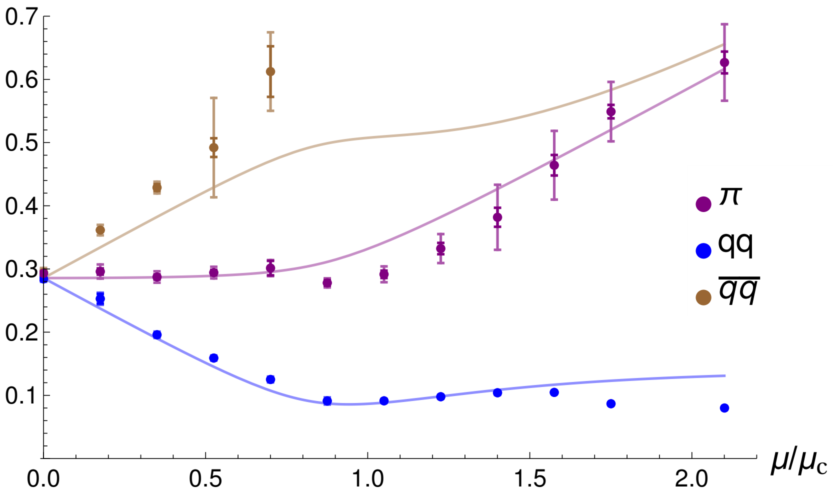




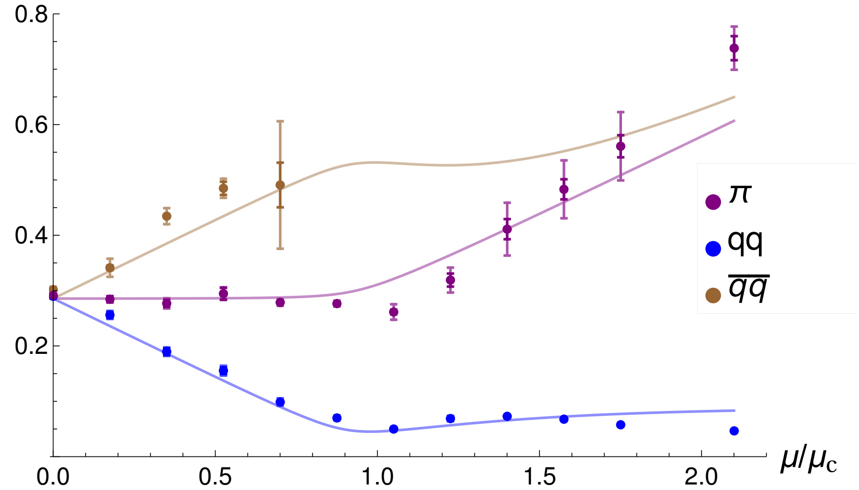
also seen by Braguta *et al.*, arXiv:1605.04090

Goldstone Spectrum

$\lambda = 0.0025$



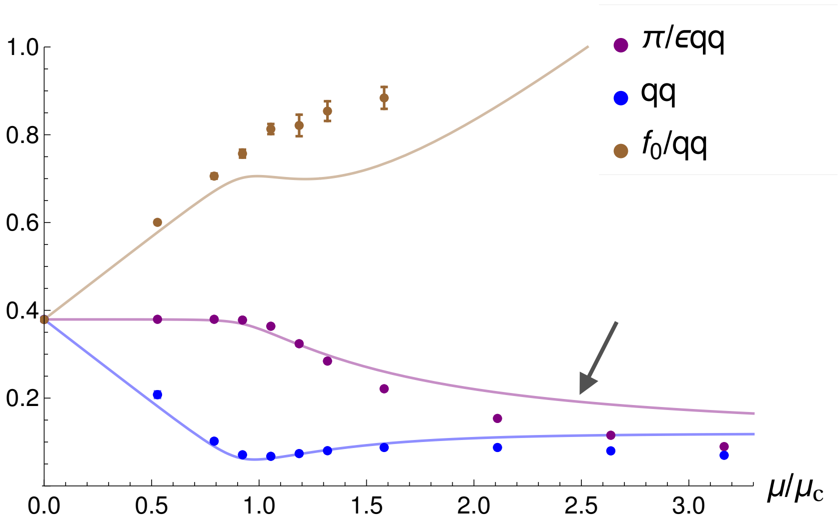
$\lambda = 0.0001$



J. Wilhelm, MSc thesis, JLU Giessen (2016)

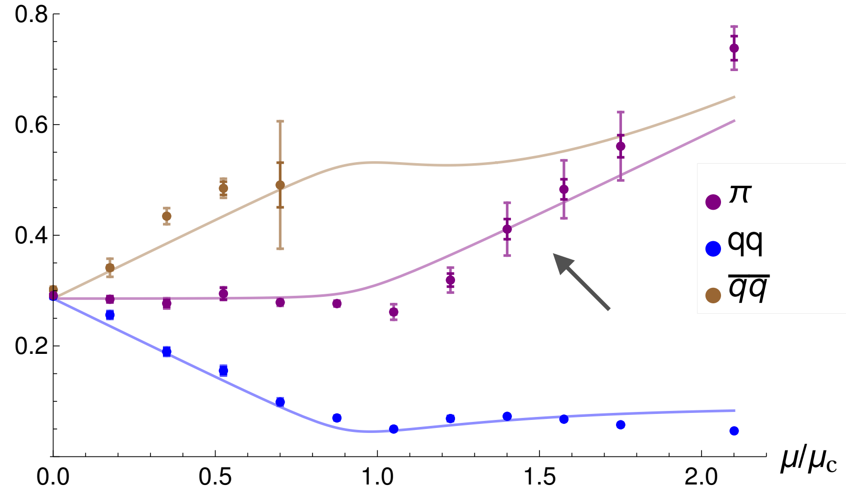
Goldstone Spectrum

unimproved - bulk phase



Dyson: $\beta = 4$

improved - continuum

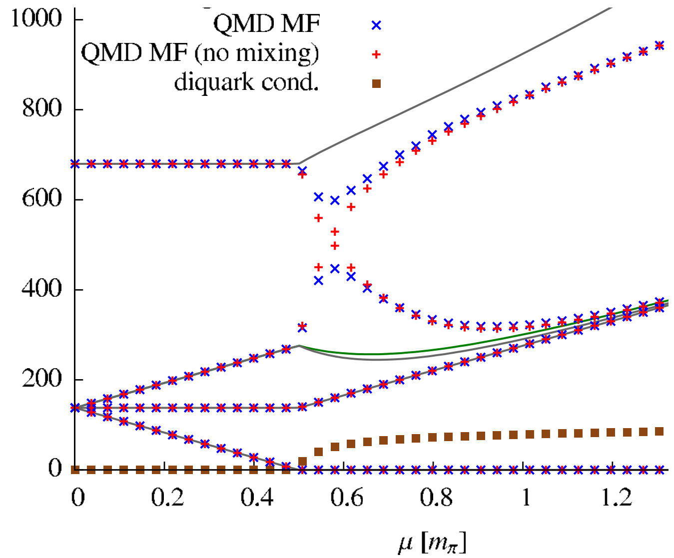


$\beta = 1$

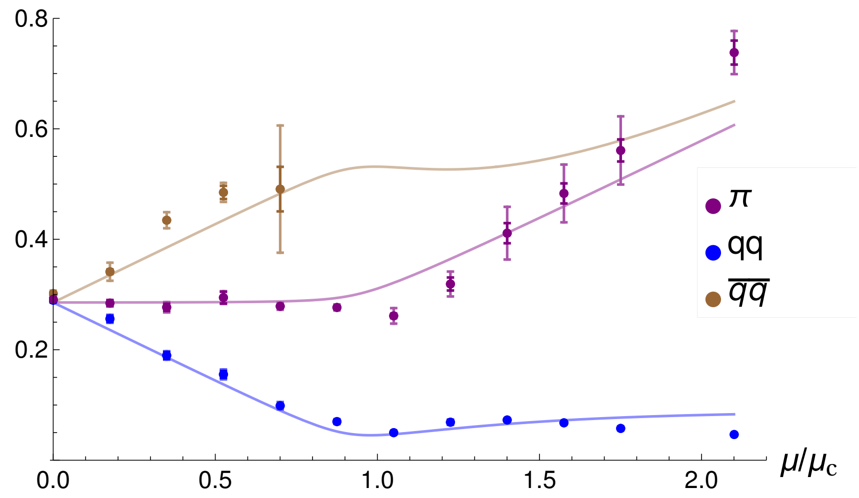
J. Wilhelm, MSc thesis, JLU Giessen (2016)

Goldstone Spectrum

effective chiral model



improved staggered



Heavy Quarks

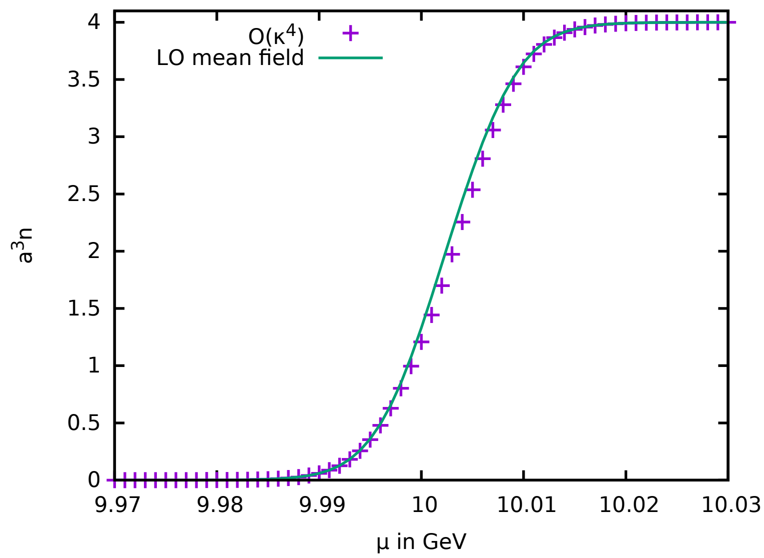
- **effective lattice theory:** systematic expansion in inverse coupling and inverse quark mass

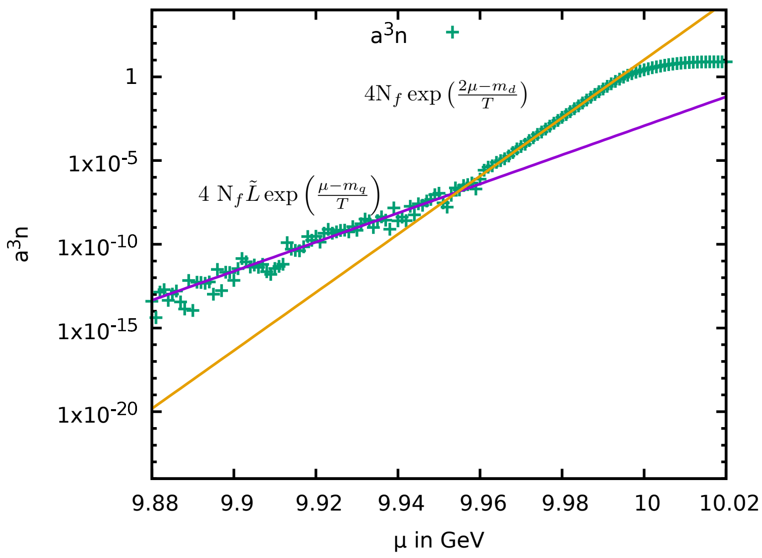
  QCD, simulate despite mild sign problem
  → evidence of liquid-gas transition to nuclear matter

  Fromm, Langelage, Lottini, Neumann, Philipsen, PRL 110 (2013) 12

- **characteristic differences, 2 ↔ 3 colors?**

quark density in QC_2D





Ph. Scior & LvS, PRD 92 (2015) 094504

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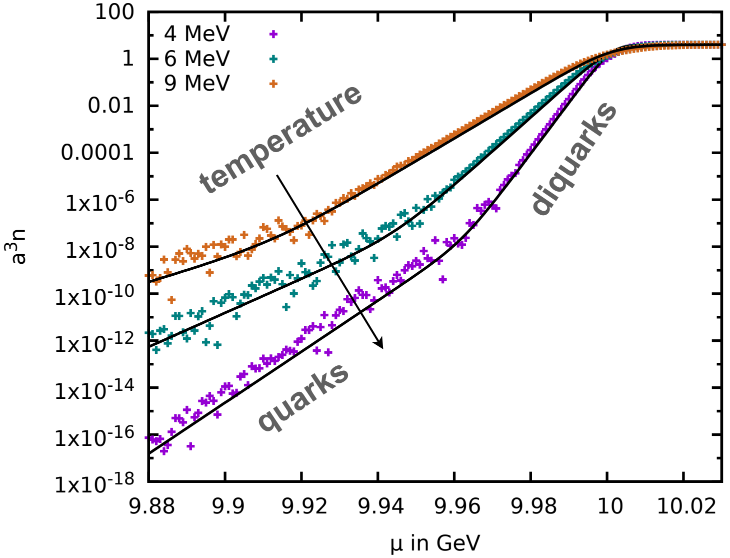

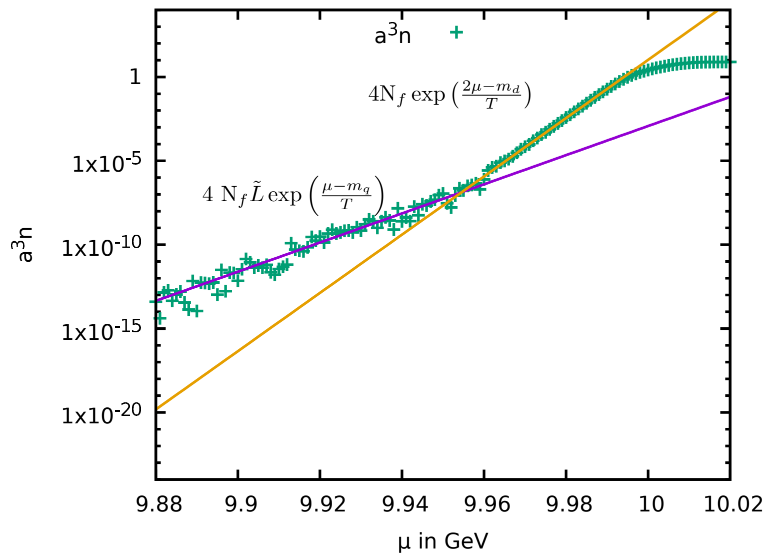

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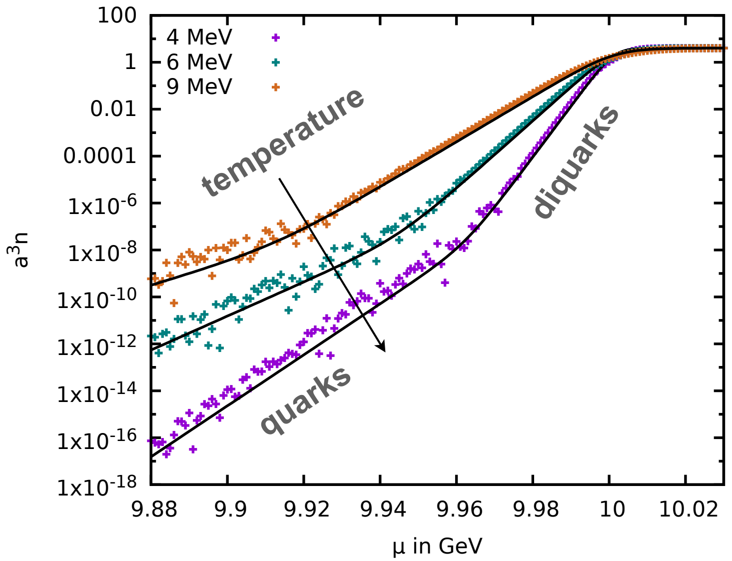
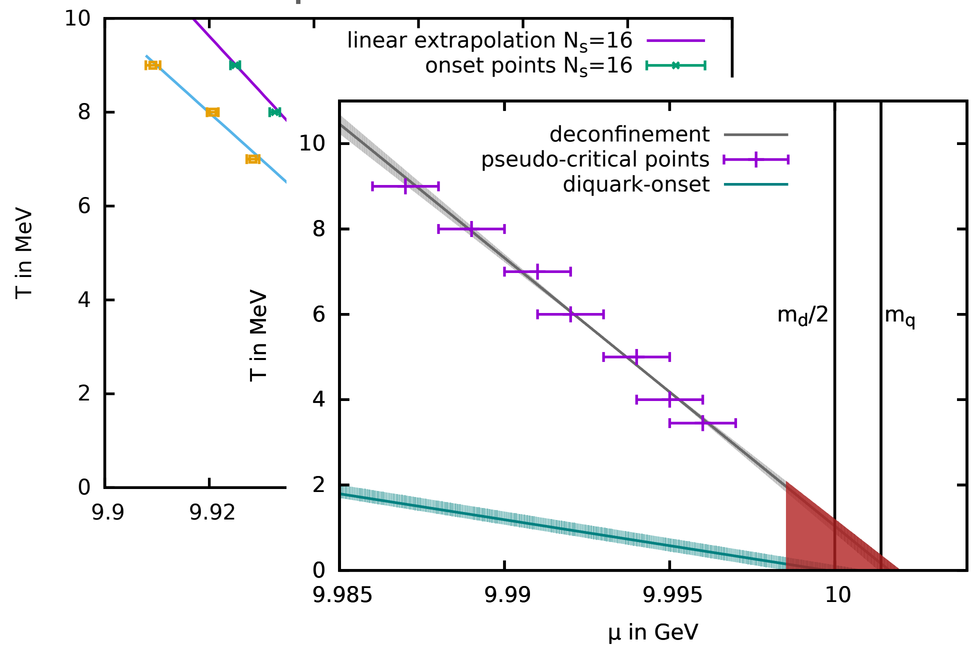
quark density in QC_2D



diquark transition



Ph. Scior & LvS, PRD 92 (2015) 094504

QC_2D in Two Dimensions

Setup

- Two flavour $SU(2)$-QCD in $2d$
- $N_t \times 16$ lattice with $N_t = 2 \ldots 128$ at fixed β and κ
- Physical scale set by pion mass $m_\pi = 200$ MeV at $N_t = 32$

$\Rightarrow$ $a = 0.26(4)\,\text{fm} \sim 0.0013\,\text{MeV}^{-1}$

$\Rightarrow$ $T = 6 \ldots 385\,\text{MeV}$

$\Rightarrow$ $\mu = 0 \ldots 885\,\text{MeV}$

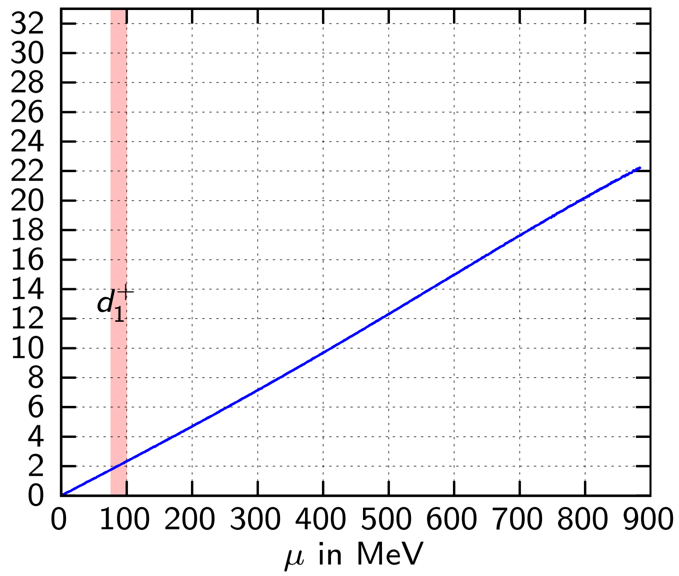
$\Rightarrow$ diquark mass $m_{d_0^+} = 200\,\text{MeV}$

$\Rightarrow$ vector diquark mass $m_{d_1^+} = 177\,\text{MeV}$

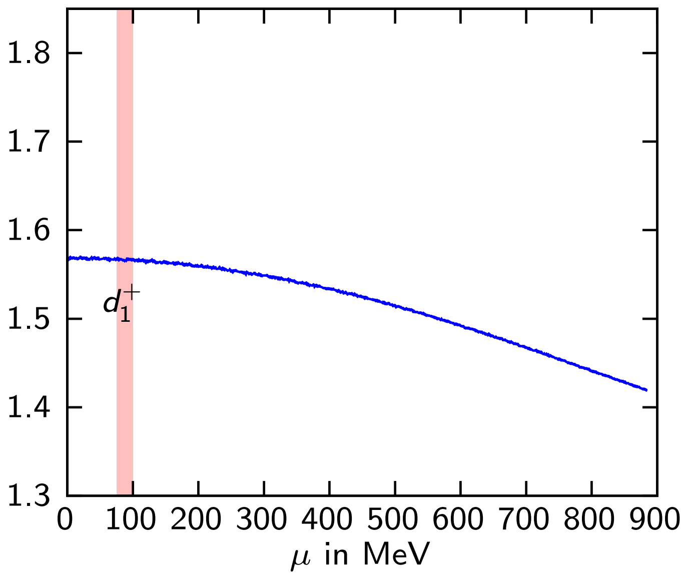
$\Rightarrow$ a meson mass $m_a = 254\,\text{MeV}$

QC$_2$D in Two Dimensions

Quark Number



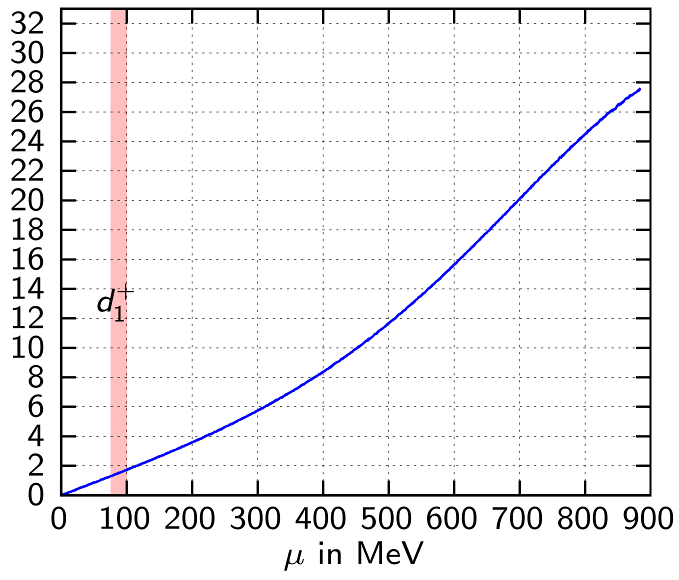
Chiral condensate



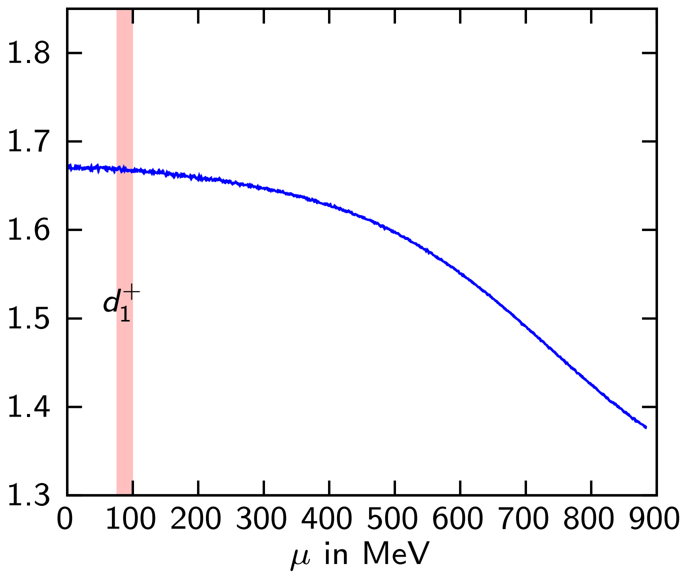
$T = 385$ MeV

QC₂D in Two Dimensions

Quark Number



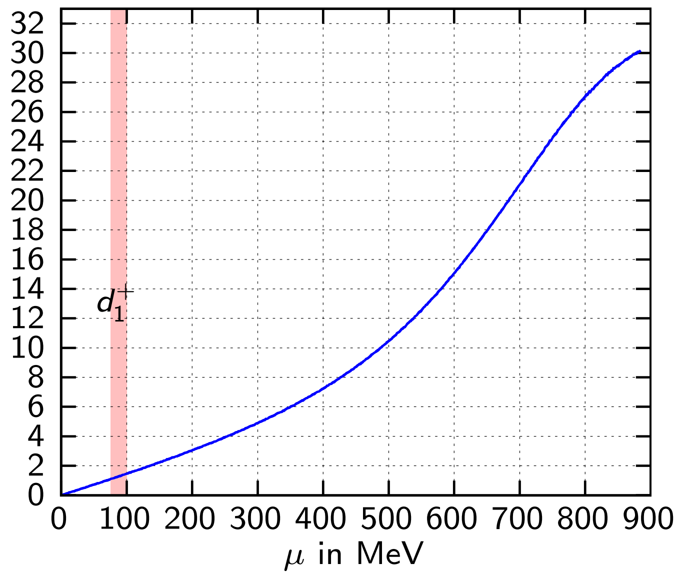
Chiral condensate



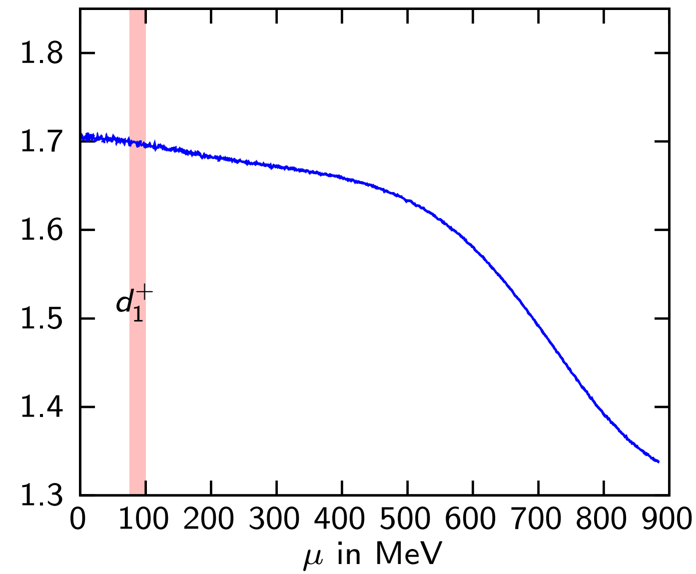
$T = 192$ MeV

QC$_2$D in Two Dimensions

Quark Number



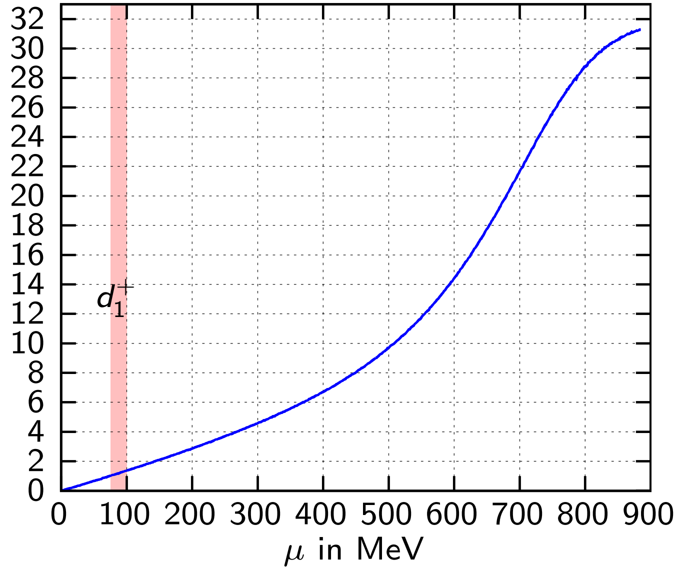
Chiral condensate



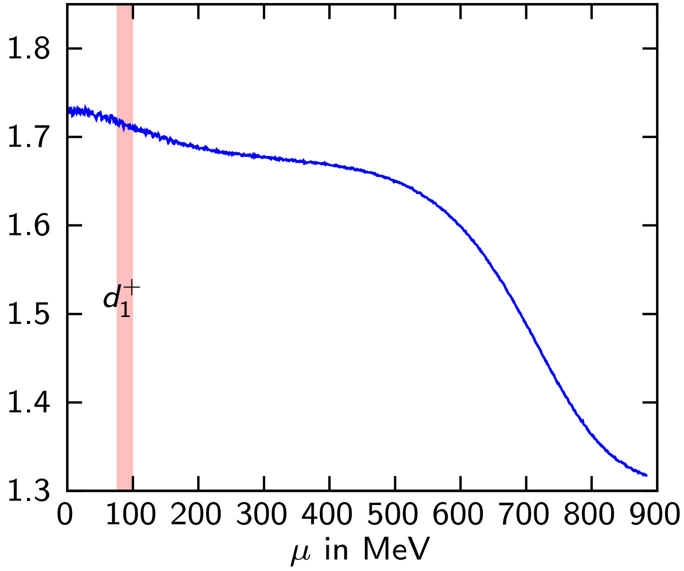
$T = 128$ MeV

QC₂D in Two Dimensions

Quark Number

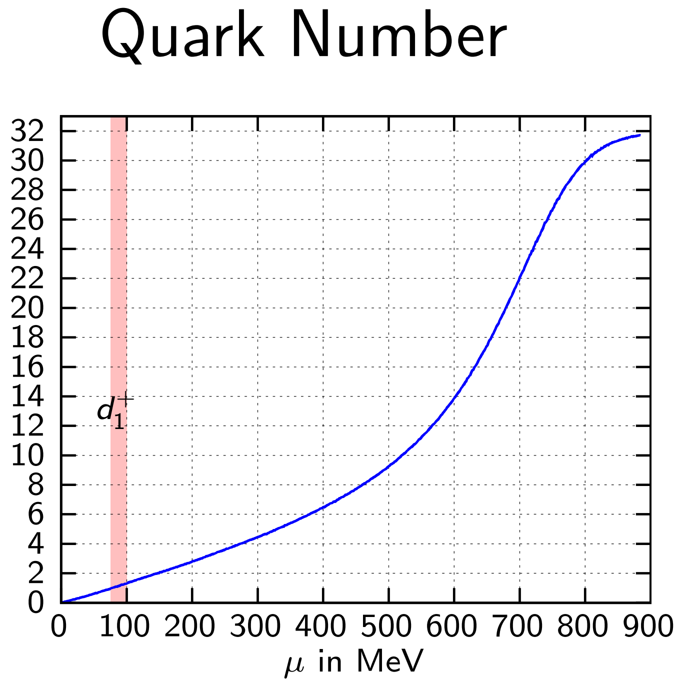


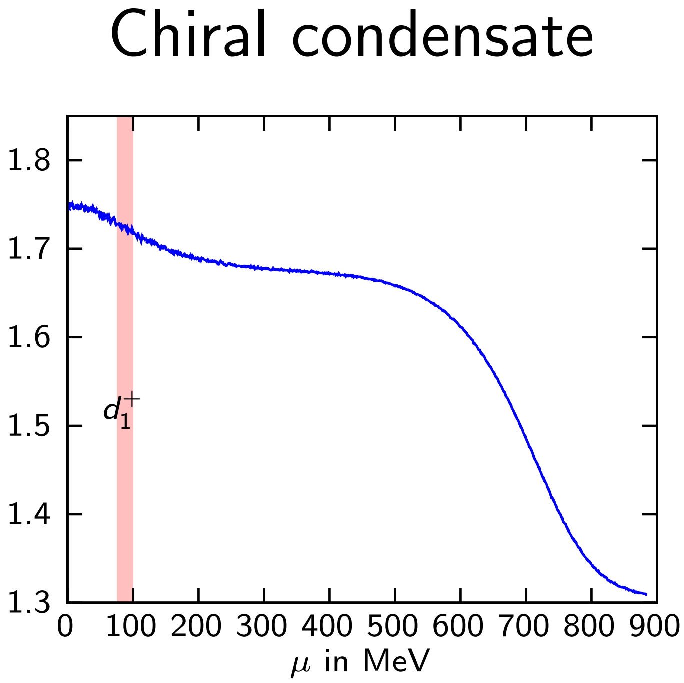
Chiral condensate



$T = 96$ MeV

QC_2D in Two Dimensions

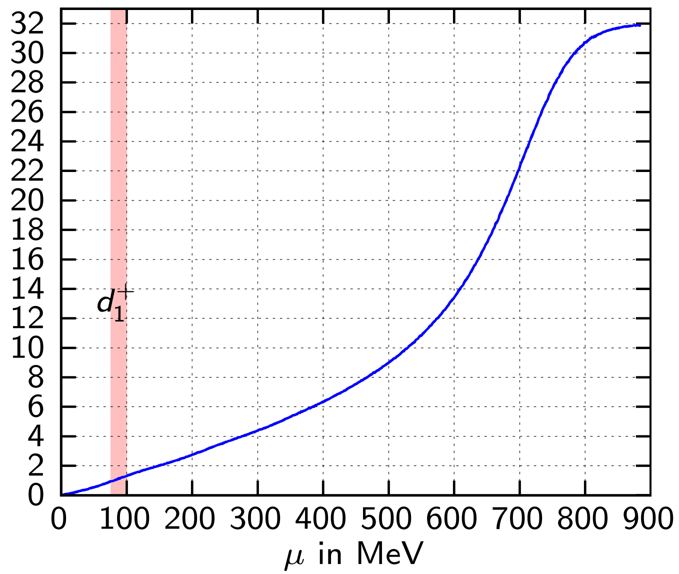




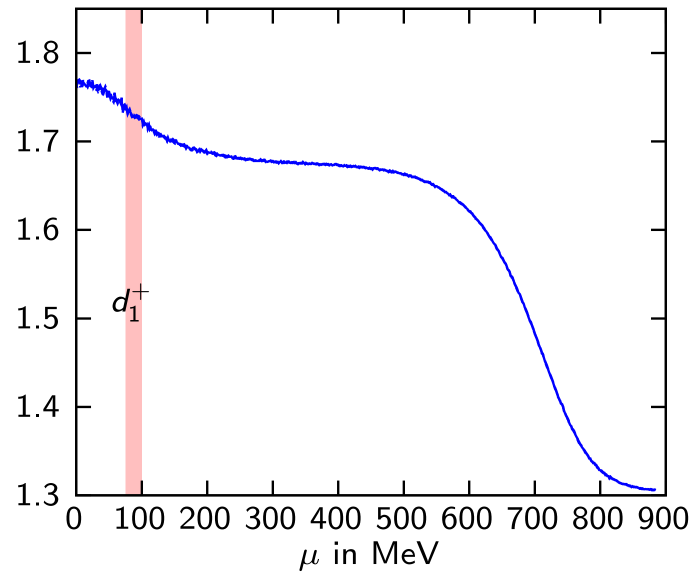
$T = 77$ MeV

QC_2D in Two Dimensions

Quark Number

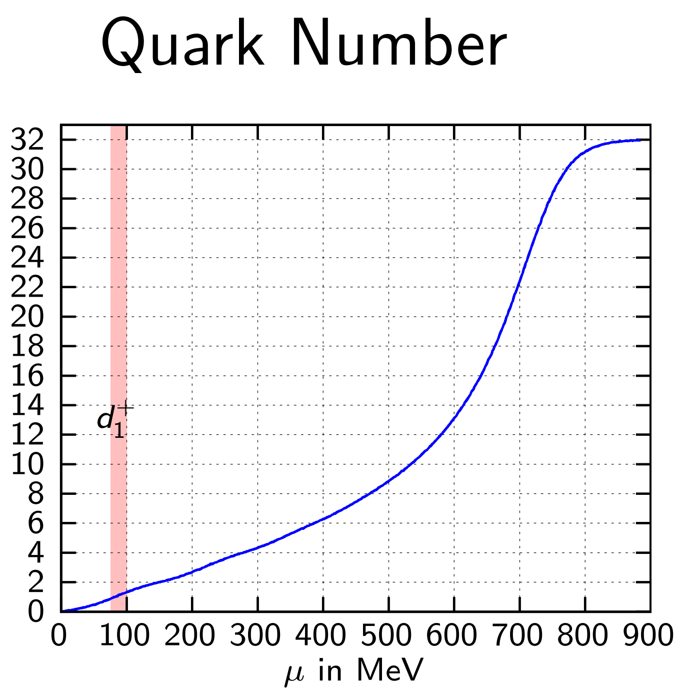


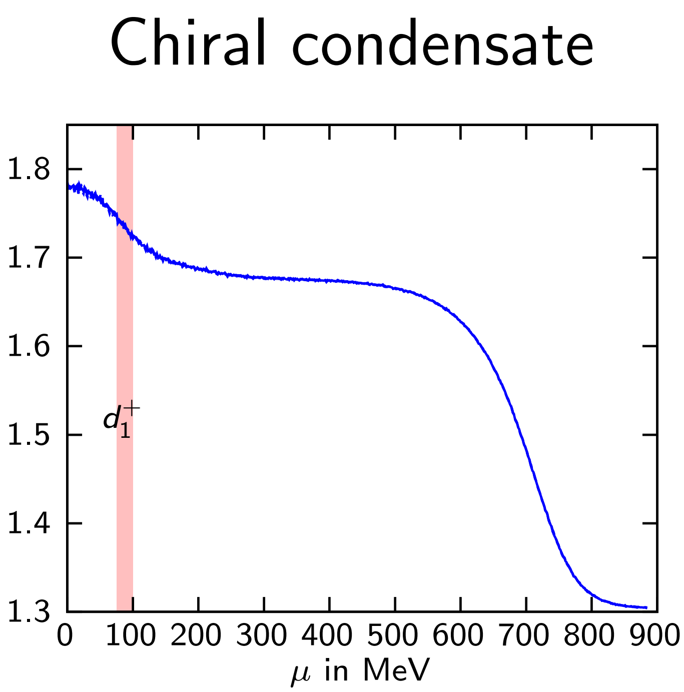
Chiral condensate



$T = 64$ MeV

QC₂D in Two Dimensions

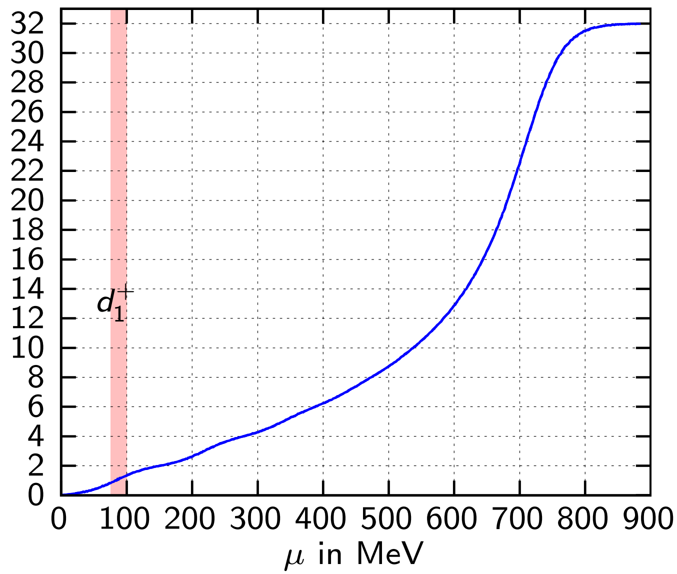




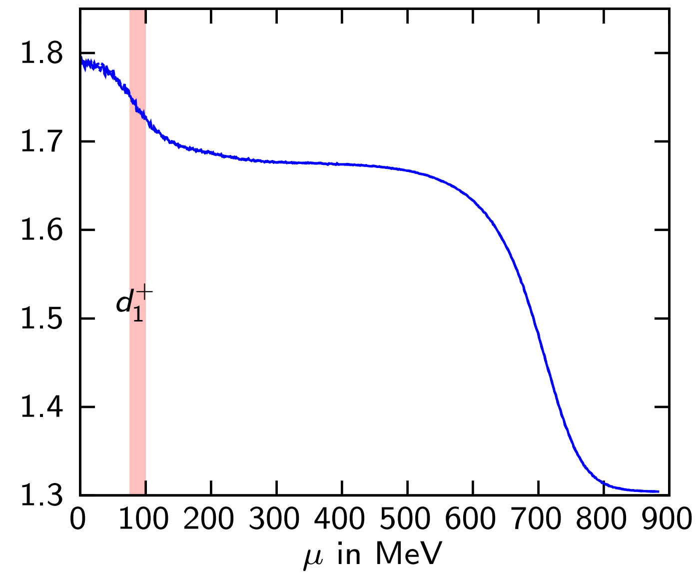
$T = 55$ MeV

QC$_2$D in Two Dimensions

Quark Number



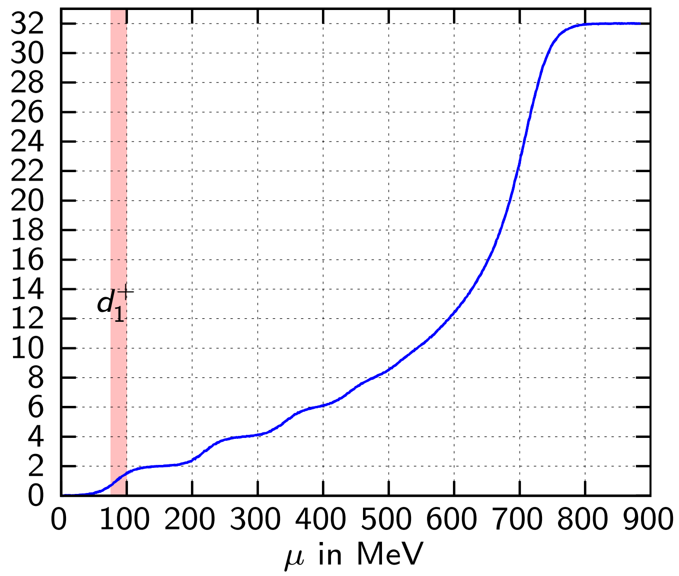
Chiral condensate



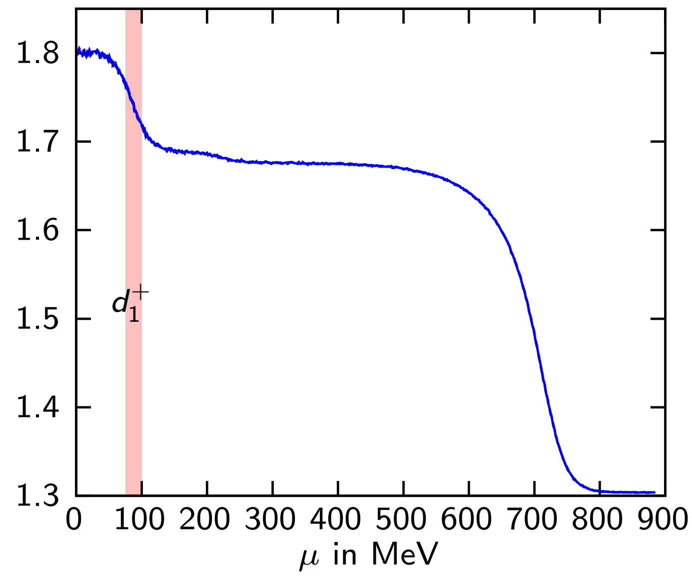
$T = 48$ MeV

QC_2D in Two Dimensions

Quark Number

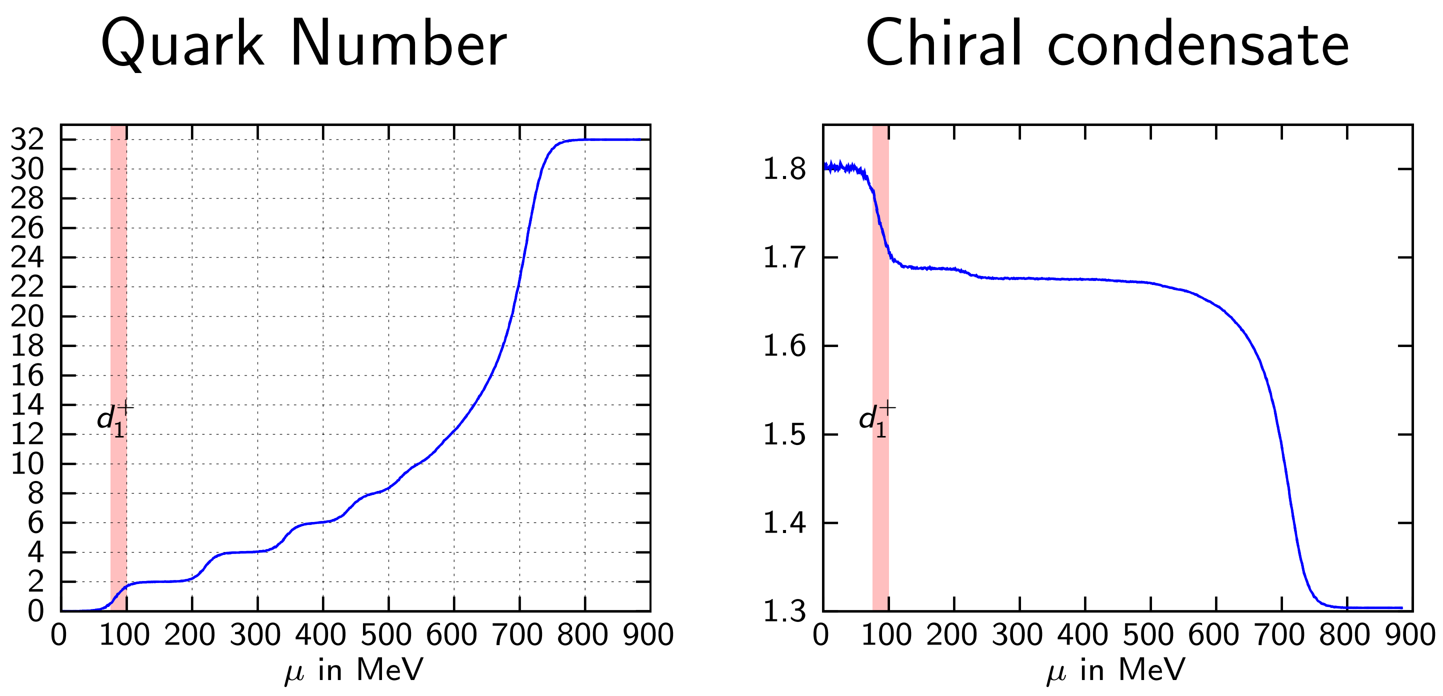


Chiral condensate



$T = 32$ MeV

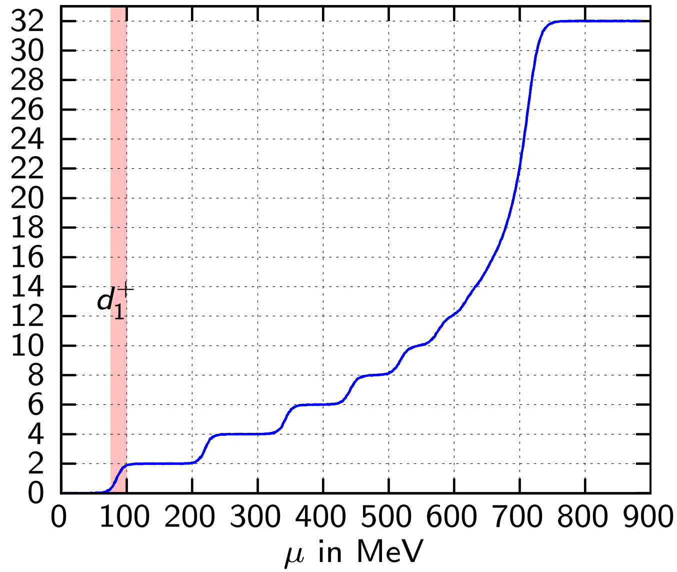
QC₂D in Two Dimensions



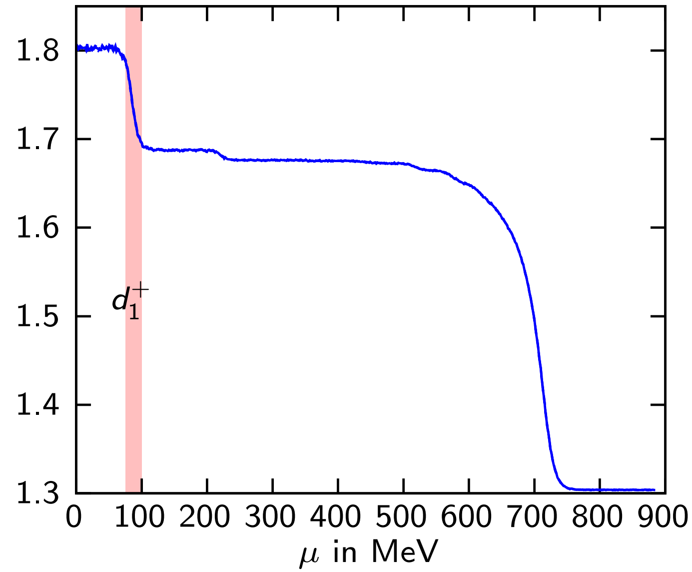
$T = 24$ MeV

QC₂D in Two Dimensions

Quark Number



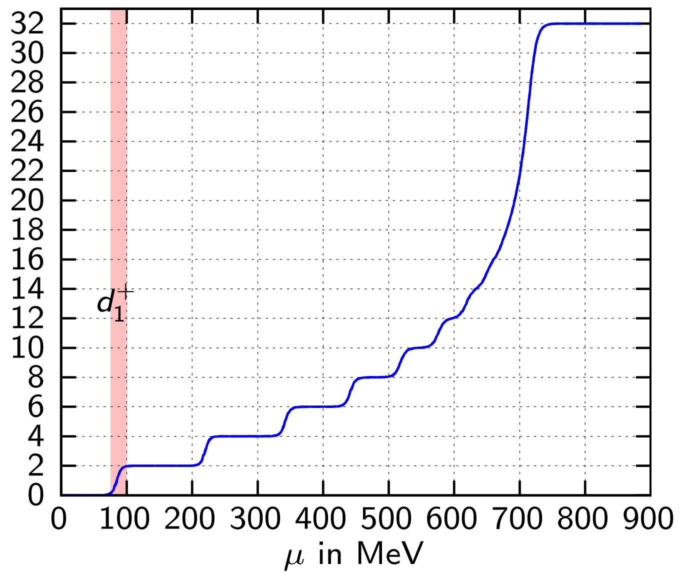
Chiral condensate



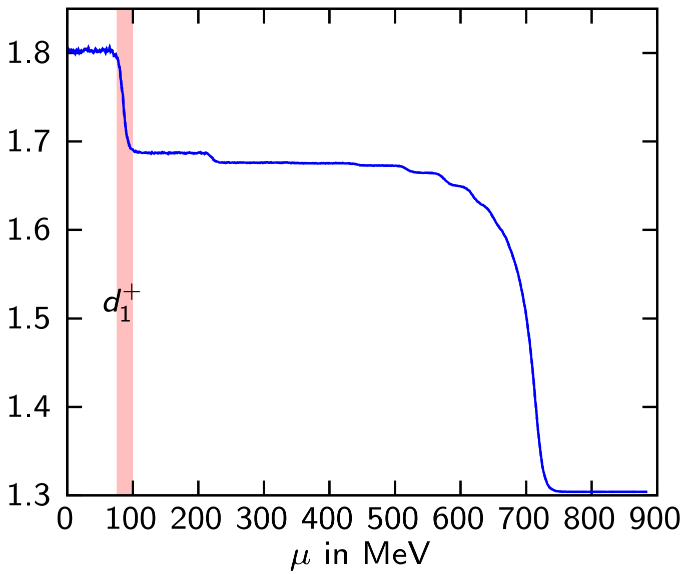
$T = 16$ MeV

QC_2D in Two Dimensions

Quark Number



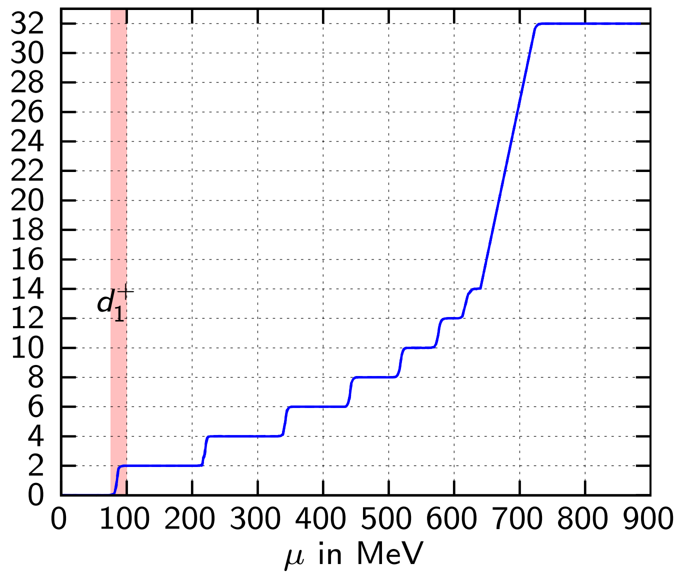
Chiral condensate



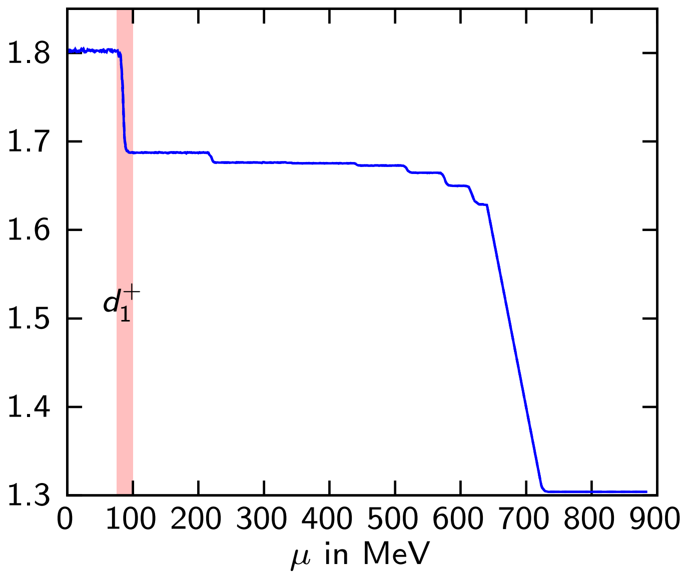
$T = 12$ MeV

QC$_2$D in Two Dimensions

Quark Number



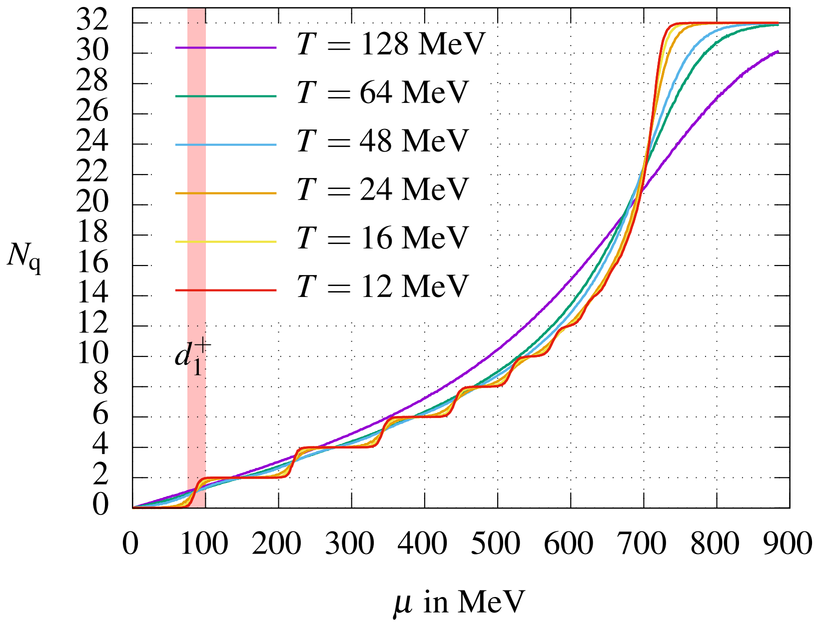
Chiral condensate



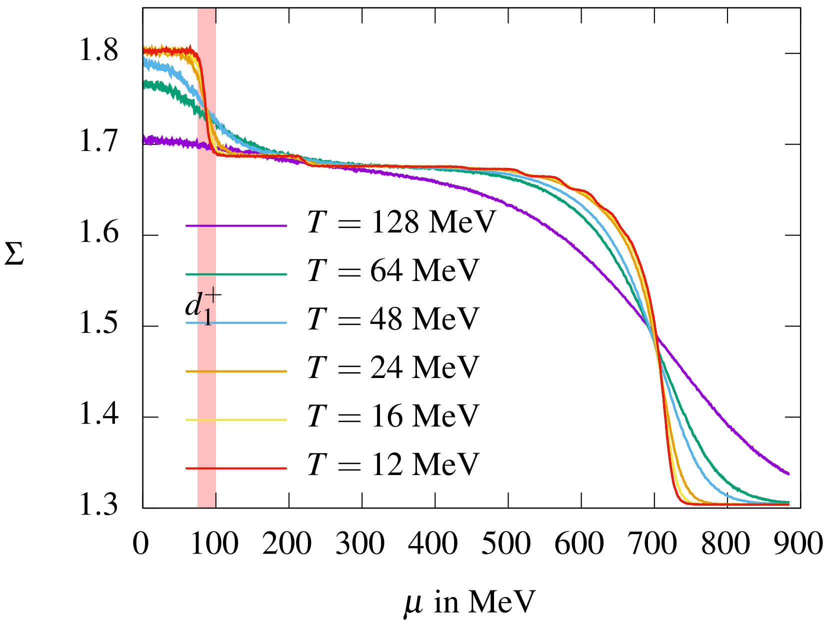
$T = 6$ MeV

QC$_2$D in Two Dimensions

quark number



quark condensate



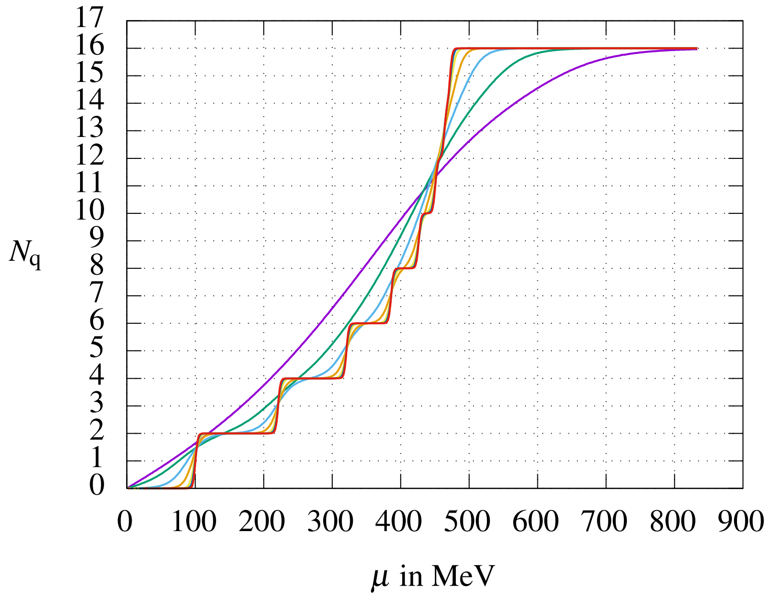
Free Lattice Fermions

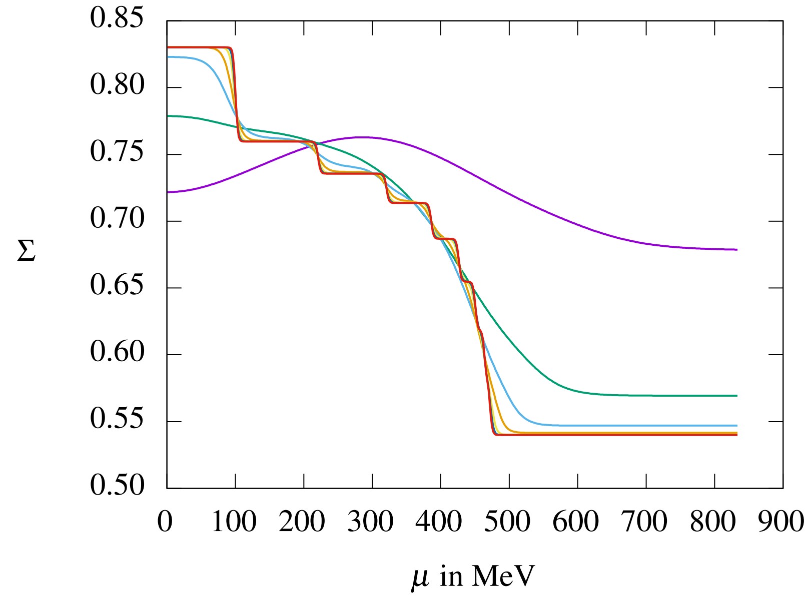
- **ensembles with fixed particle number *k* mod *N*:**

$$Z_N(k) \;=\; \frac{1}{N}\sum_{n=0}^{N-1} e^{\frac{2\pi i}{N}kn}\, Z\Big(\mu - \frac{2\pi i}{N}Tn\Big)$$

- ***N* = 2, *k* = 0:**

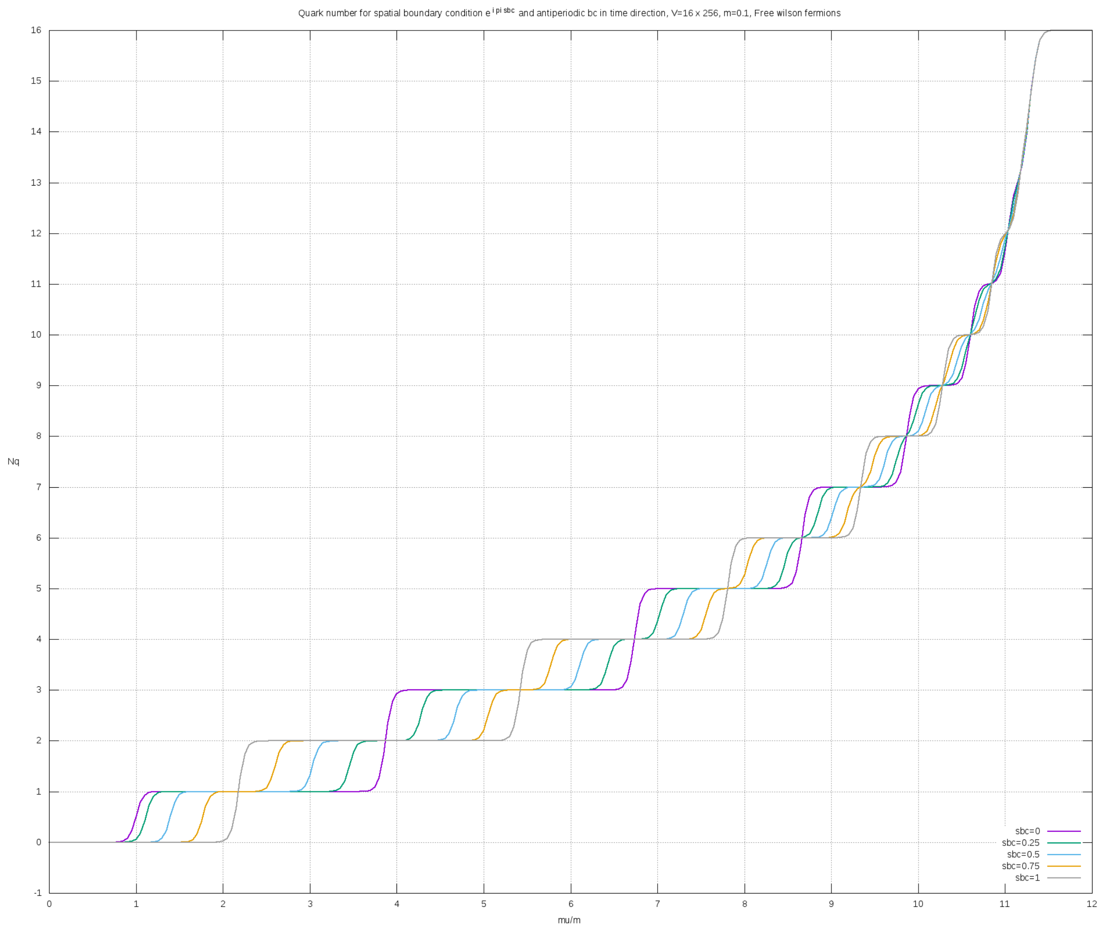
$$Z_{\rm even}(k) \;=\; \frac{1}{2}\Big(Z(\mu) + Z(\mu - i\pi T)\Big)$$





Free Lattice Fermions

- change spatial b.c.'s to probe momenta



G_2-QCD

G_2 gauge theory with fundamental fermions, $T = C\gamma_5 \otimes \mathbb{1}$

- 7 colors, 14 gluons
- bound states with integer quark number (fermionic and bosonic baryons)

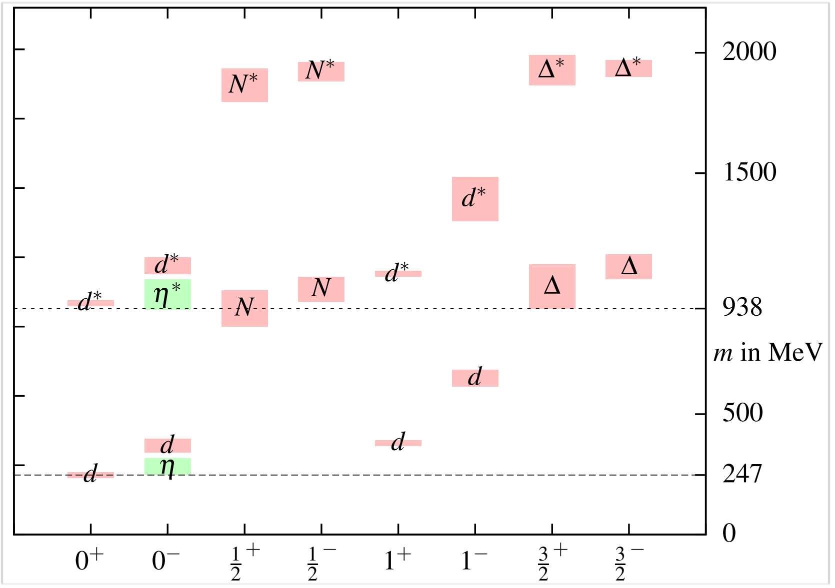
$$
\begin{aligned}
n_{\rm q} &= 1 \sim \text{Hybrid}(H) \sim qggg \\
n_{\rm q} &= 1 \sim \tilde{\Delta}\,, \tilde{N} \sim (\bar{q}q)q \\
n_{\rm q} &= 2 \sim \text{diquarks}(d) \sim q^{\rm T}q \\
n_{\rm q} &= 3 \sim \Delta\,, N \sim (q^{\rm T}q)q
\end{aligned}
$$

- gluodynamic very similar to $SU(3)$ (first order deconfinement transition)

G_2-QCD

- **simulate af finite density**
  250 M core hours →

spectrum G_2-QCD (1 flavor)



Wellegehausen, Maas, Wipf & LvS, Phys. Rev. D 89 (2014) 056007

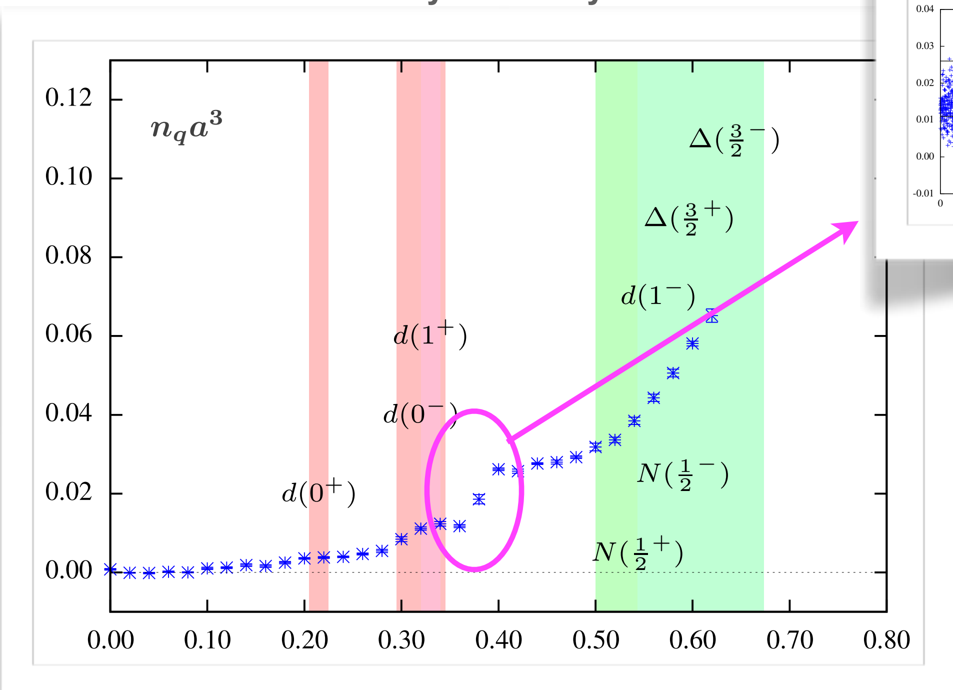
Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901(R)

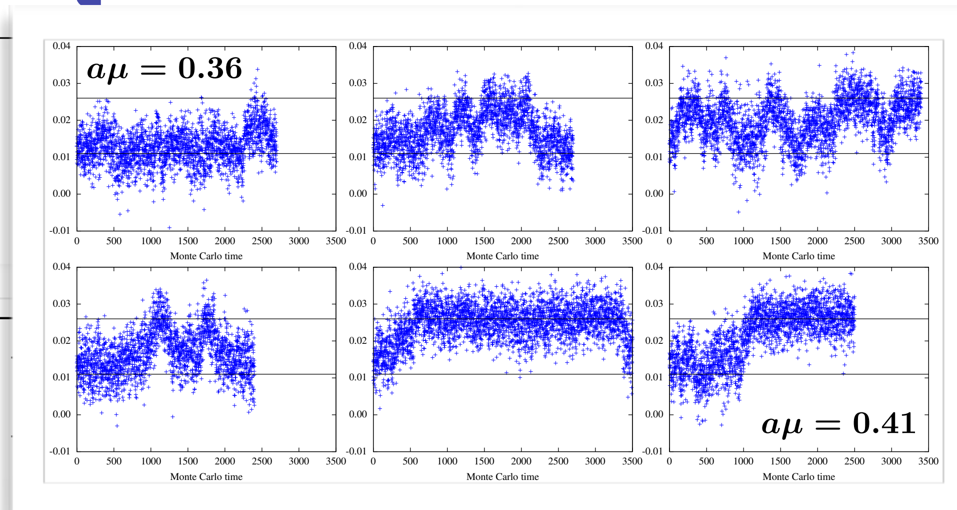
G_2-QCD

- **simulate af finite density**

250 M core hours →

baryon density





first order, G_2-nuclear matter

Wellegehausen, Maas, Wipf & LvS, Phys. Rev. D 89 (2014) 056007

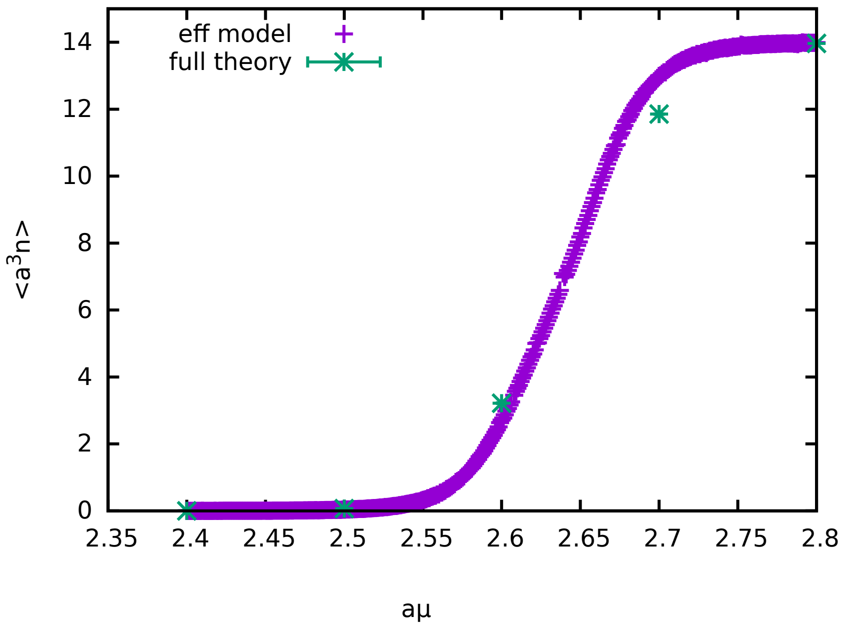
Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901(R)

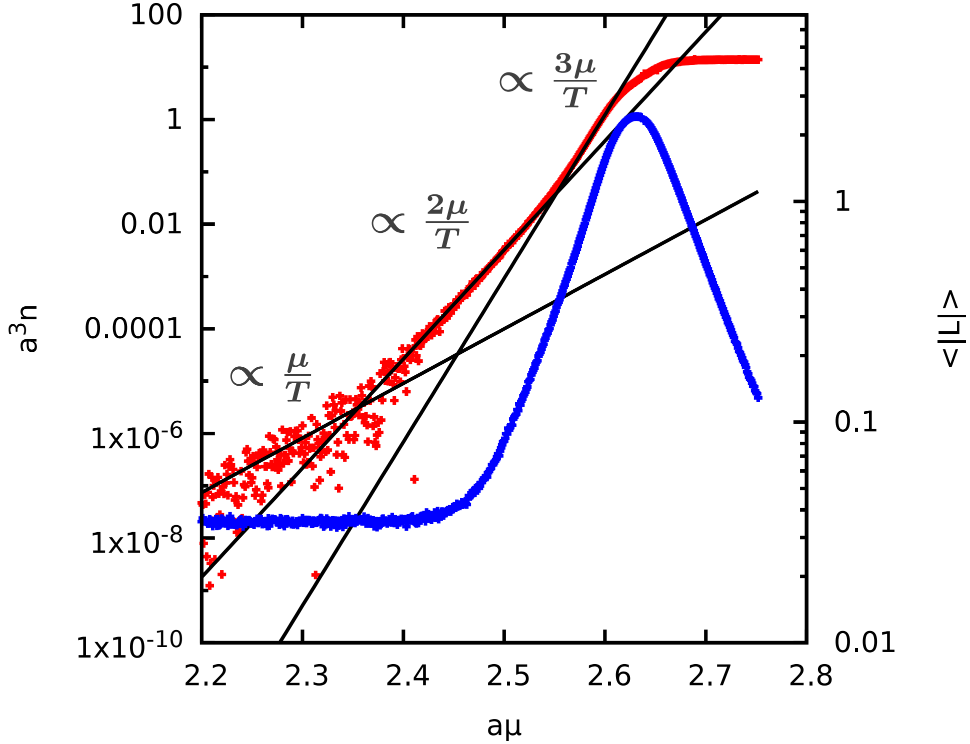
G_2-QCD

- **effective theory for heavy quarks**

$$N_f = 1$$

compare with HMC simulations

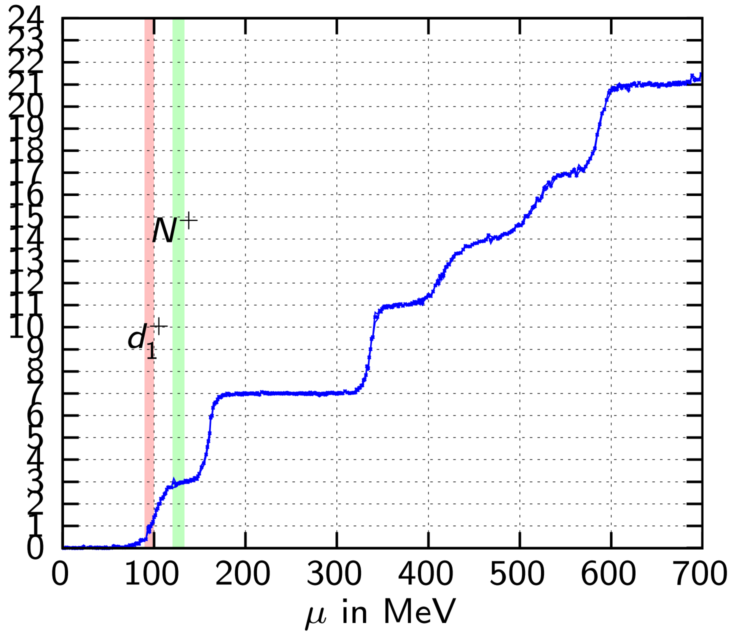




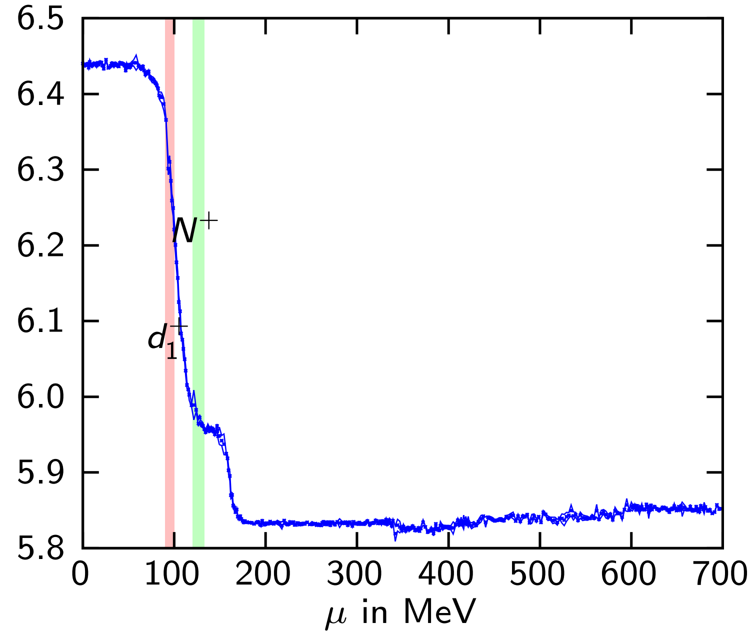
Scior, Wellegehausen & LvS, in preparation (Lattice 2016)

G$_2$-QCD in Two Dimensions

Quark Number



Chiral condensate



$N_t = 64$

$T = 20$ MeV

- **nucleon / delta mass decreases above diquark onset (preliminary)**

Wellegehausen & LvS, in preparation (Lattice 2016)

Conclusions

- **Two-Color QCD with Two Flavors of Staggered Quarks**

  improved action, away from bulk phase ➟ continuum Goldstone spectrum

- **Effective Lattice Theory for Heavy Quarks**

  strong-coupling / hopping expansion ➟ continuous transition to finite diquark density

- **Two-Color QCD in Two Dimensions**

  qualitative understanding from statistical confinement

- **G_2-QCD**

  G_2-nuclear matter, effective theory for heavy quarks with nucleons, understand generic features in two dimensions (way cheaper to simulate)

Thank you for your attention!