Meson mass at high temperatures and densities from lattice QCD

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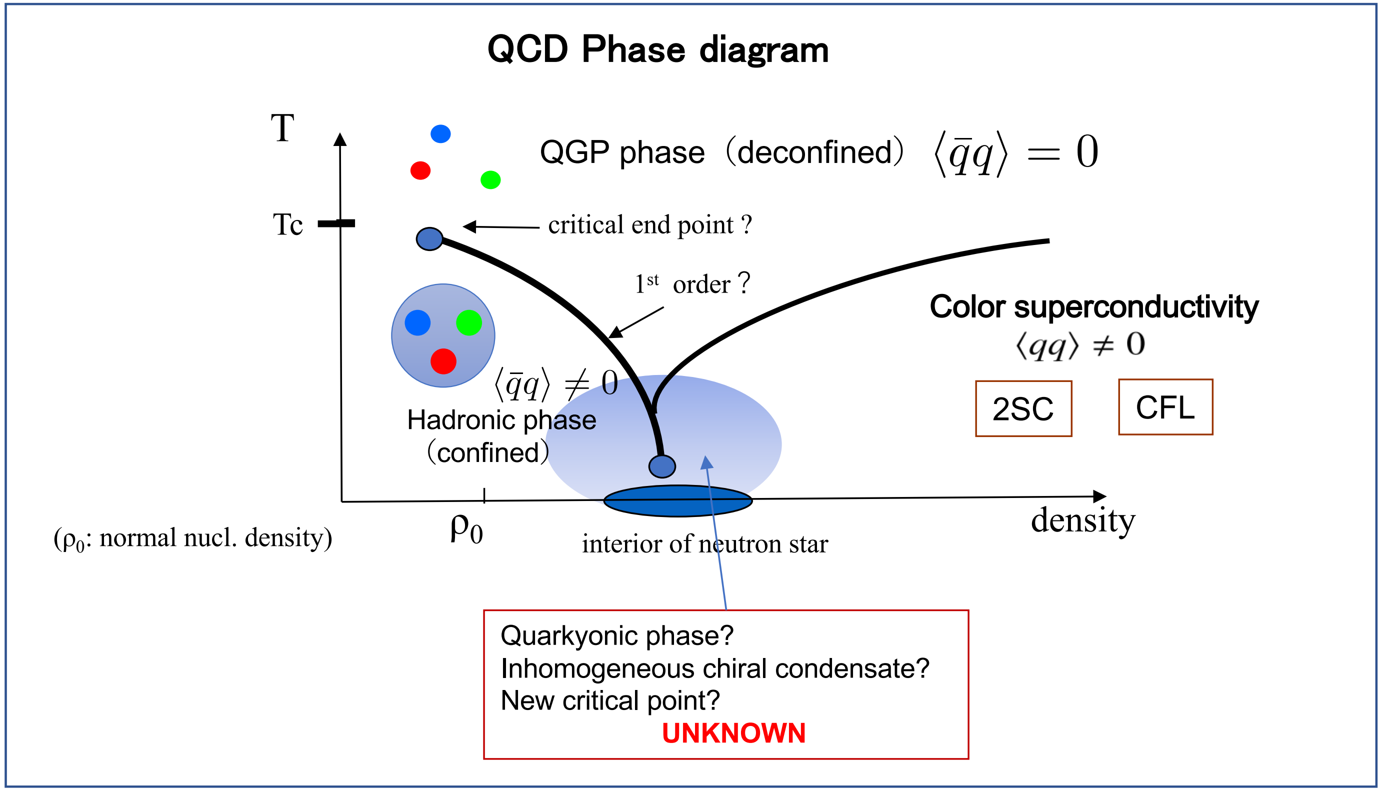
Mini-Workshop on “Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics”, Dubna, 10-14 July, 2017

Explore the phase diagram of QCD

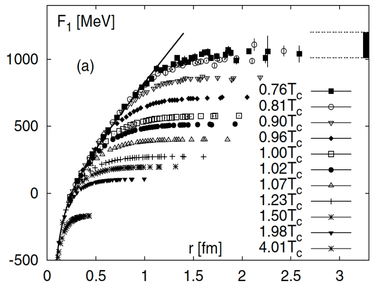


- The properties of the phases
  … reflected in the excitations
  of the system.

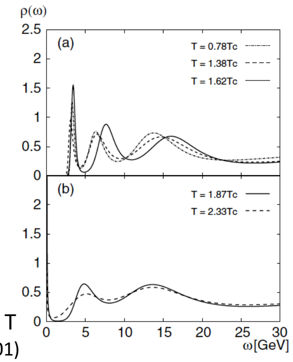
Seek for the properties of hadrons
@ finite T & μ

Mesonic (or Baryonic) excitations of QCD in extreme systems

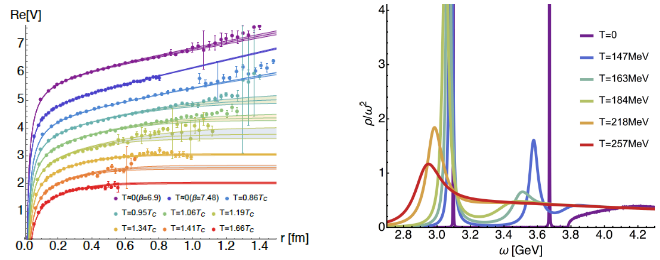
- **Heavy quark sector (e.g., suppression of J/ψ, Υ…)**



Temperature dependence of $q\bar{q}$ potential
(O.Kaczmarek, F.Zantow, PRD71, 114510 (2005))

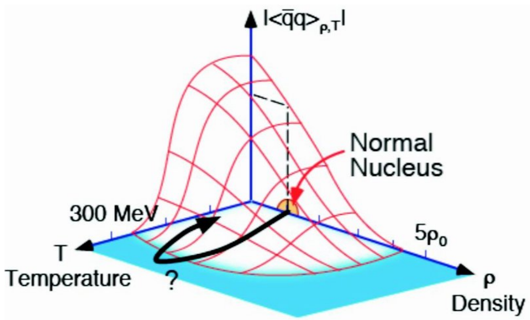


Spectral func. of J/ψ @ finite T
(Asakawa, Hatsuda, Nakahara 2001)



Real part of static potential from lattice QCD (left)
Spectral func. of J/ψ from NR-QCD
(A.Rothkopf, lattice 2016)

- **Light quark sector** (π,σ,**vector mesons**):



Taken from http://niham.nipne.ro/rp9/

- detector for chiral symmetry restoration
  reflected in mass modification, width broadening

cf) Brown-Rho scaling, QCD sum rules for vector mesons @ finite μ

Search for the properties of mesons @ finite μ…Today's topic

Experimental status of properties of vector mesons @ finite density

…related to NICA

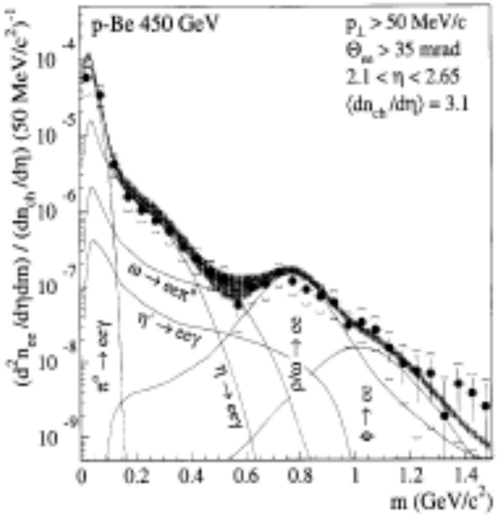
※ lepton-pair (e^+e^- & $\mu^+\mu^-$) observation

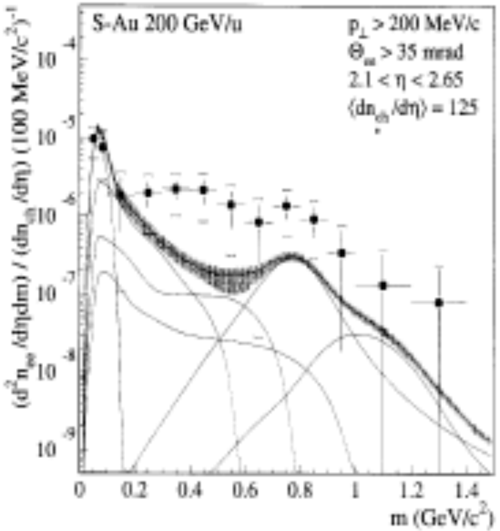
… appropriate because they have negligible final state int.

- **High-energy heavy ion**
  - Bevalac, DLS
  - HADES, SIS
  - CERES, HELIOS/3, NA60, CERN SPS
  - STAR, PHENIX, RHIC
  - LHC
- **Mesons produced in nuclei ("put mesons softly in nuclei")**
  - TAGX, INS (University of Tokyo)
  - E325, KEK
  - CLAS, JLAB
  - CBELSA/TAPS, electron stretcher accelerator in Bonn
  - LEPS @ Spring-8
  - ANKE-COSY

Dilepton spectrum @ low energy

CERN SPS





CERES experiment：p-Be 450GeV（low density）
S-Au 200GeV（high density）

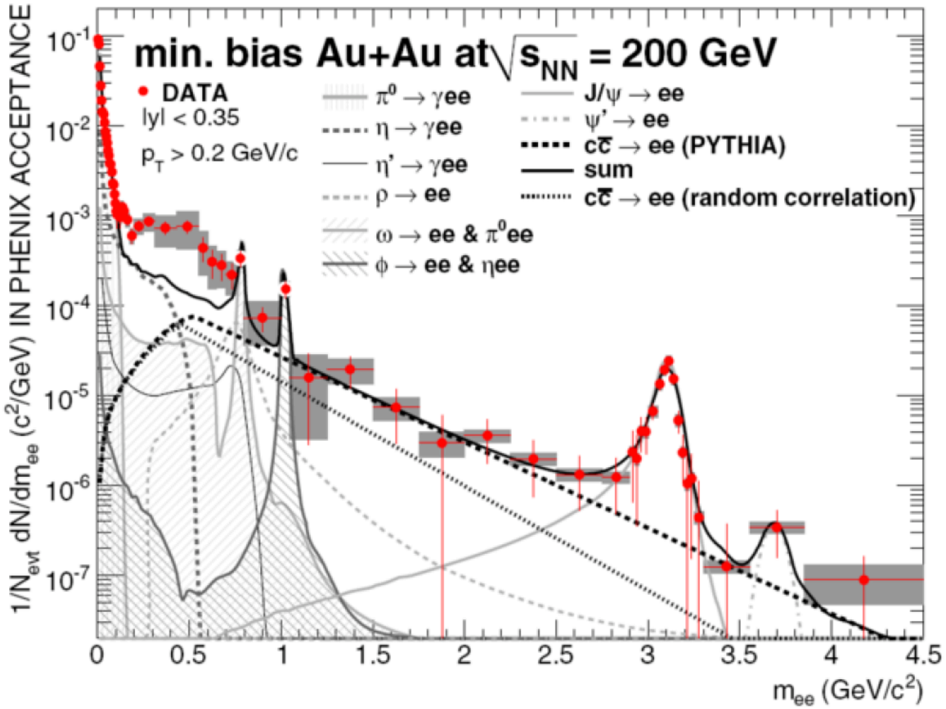
… shows the modification of vector mesons in medium (width broadening)

further analysis by NA60

Dilepton spectrum @ low energy

PHENIX, RHIC:

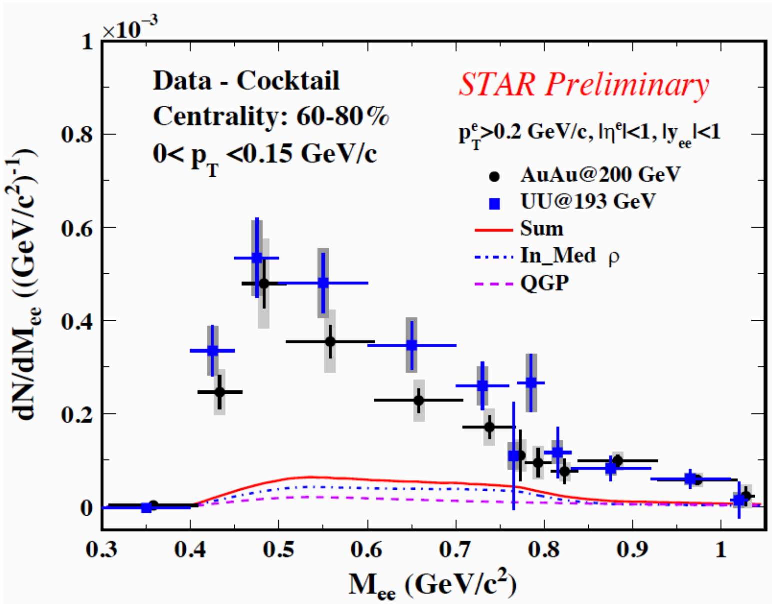
Axel Drees, NPA

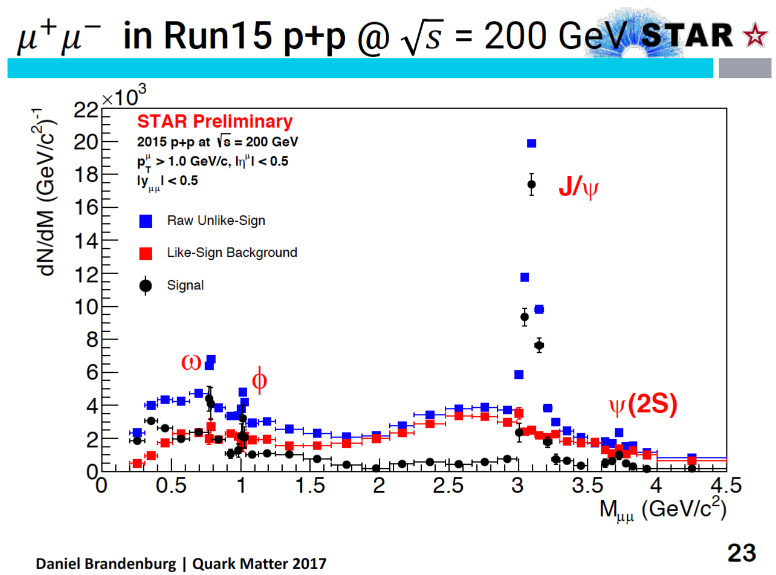


Dilepton spectrum @ low energy

STAR, RHIC

QuarkMatter2017





$\mu^+\mu^-$

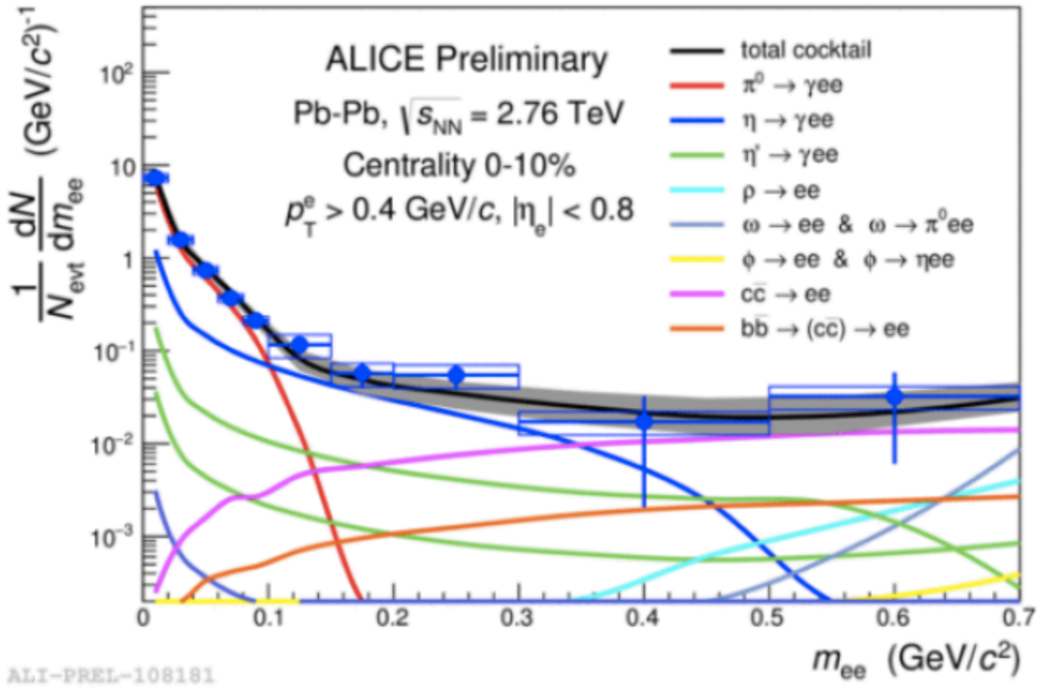
There will be new results

Not due to the hadronic cont. , which is modified in medium

Coherent photoproduction?

Dilepton spectrum @ low energy

ALICE LHC
SQM2016



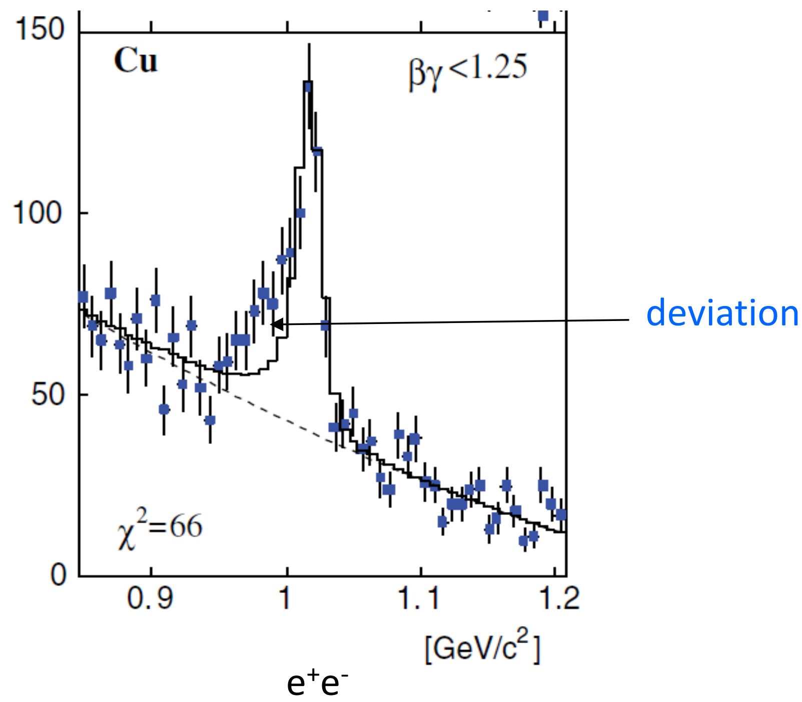
No significant enhancement??
…still uncertainty is large…

Dilepton spectrum @ low energy

- KEK PS-375

ρ, ω: 9% mass reduction, no width broadening

Φ: 3.4% mass reduction, 3.6 times width broadening



Dilepton spectrum @ low energy

- **CLAS, J-LAB**
  No modification of mesons due to finite density effect is seen
  "Brown & Rho (scaling analysis),
  Hatsuda & Lee (QCD sum rule) are ruled out…"

Target	M_ρ	Γ_ρ
^{2}H	773.0 ± 3.2	185.2 ± 8.6
C	726.5 ± 3.7	176.4 ± 9.5
Fe, Ti	779.0 ± 5.7	217.7 ± 14.5

※The table is taken from Hayano & Hatsuda (2010)

Summary table of experiments for mesons @ finite μ (V. Metak, API Conf. Proc.1322, 73 (2010))

experiment	momentum acceptance	ρ	ω	ϕ
KEK-E325 pA 12 GeV	$p > 0.6$ GeV/c	$\frac{\Delta m}{m} = -9\%$ $\Delta\Gamma \approx 0$	$\frac{\Delta m}{m} = -9\%$ $\Delta\Gamma \approx 0$	$\frac{\Delta m}{m} = -3.4\%$ $\frac{\Gamma_\phi(\rho_0)}{\Gamma_\phi} = 3.6$
CLAS γA 0.6-3.8 GeV	$p > 0.8$ GeV/c	$\Delta m \approx 0$ $\Delta\Gamma \approx 70$ MeV $(\rho \approx \rho_0/2)$		
CBELSA /TAPS γA 0.9-2.2 GeV	$p > 0$ MeV/c		Δm insensitive $p_\omega < 0.5$ GeV/c $\Delta\Gamma(\rho_0) \approx 130$ MeV $\langle p_\omega \rangle = 1.1$ GeV/c	
SPring8 γA 1.5-2.4 GeV	$p > 1.0$ GeV/c			$\Delta\Gamma(\rho_0) \approx 70$ MeV $\langle p_\phi \rangle = 1.8$ GeV/c
CERES Pb+Au 158 AGeV	$p_t > 0$ GeV/c	broadening favoured over mass shift		
NA60 In+In 158 AGeV	$p_t > 0$ GeV/c	$\Delta m \approx 0$ strong broadening		

So far, for light meson sector, it seems that no clear answer is obtained by experiments.

Our study:

Hadron properties at finite temperature and density with two-flavorWilson fermions

In collab. with Y.Maezawa and K.Yazaki
(H.Iida, Y.Maezawa and K.Yazaki, PoS LATTICE2010)

Lattice QCD at finite density

- **Approaches to overcome sign problem**
  - Imaginary chemical potential
  - Isospin chemical potential
  - N_c=2 “QCD”
  - **Taylor expansion by quark chemical potential…**expansion by μ/T
  - Reweighting
  - Density-of-state method
  - Complex Langevin
  - Lifschetz thimble
  - Canonical approach
  - Histogram method
  - …

Method

- QCD Taro (2002)

  ... **second response of meson masses to the chemical potential**

  in **Staggered fermion** **by Taylor expansion method**

Ref.) S.Choe et al., PRD65, 054501 (2002)

$$\langle\mathcal{O}\rangle = \frac{\int \mathcal{D}U e^{-S}(\det D(\mu))^2\mathcal{O}}{\int \mathcal{D}e^{-S}(\det D(\mu))^2} = \frac{\langle(\mathcal{O}+\dot{\mathcal{O}}\mu+\frac{1}{2}\ddot{\mathcal{O}}\mu^2+O(\mu^3))(1+\frac{\dot{\Delta}}{\Delta}\mu+\frac{\ddot{\Delta}}{\Delta}\mu^2+O(\mu^3))\rangle}{1+\langle\frac{\dot{\Delta}}{\Delta}\rangle\mu+\frac{1}{2}\langle\frac{\ddot{\Delta}}{\Delta}\rangle\mu^2+O(\mu^3)}$$

($\Delta \equiv (\mathrm{detD}(\mu))^2|_{\mu=0}$)

– taking $\mathcal{O}$ the meson correlator G at finite density:

$$G \equiv \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma D^{-1}_{0x}(\mu)\Gamma^\dagger) = \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma\gamma_5(D^{-1}(-\mu))^\dagger_{x0}\gamma_5\Gamma^\dagger)$$

$\rightarrow$ $\mathrm{2nd\ order}: \langle\dot{G}\frac{\dot{\Delta}}{\Delta}\rangle+\frac{1}{2}\langle\ddot{G}\rangle+\frac{1}{2}\langle G\frac{\ddot{\Delta}}{\Delta}\rangle-\frac{1}{2}\langle G\rangle\langle\frac{\ddot{\Delta}}{\Delta}\rangle$ (Note: $\langle\frac{\dot{\Delta}}{\Delta}\rangle=0,\ \langle G\frac{\dot{\Delta}}{\Delta}\rangle=0$)

※ 1st order vanishes for mesons

Method

- **Leading:**

$$\langle G\rangle|_{\mu=0} = \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle$$

- **Second derivative:**

$$\begin{aligned}
\frac{d^2}{d\mu^2}\mathrm{Re}\langle G\rangle|_{\mu=0} =& \left.\begin{aligned} &4\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&- 2\langle \mathrm{Retr}[(D^{-1}\ddot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&- 2\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1}\dot{D}D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \end{aligned}\right\} \frac{1}{2}\langle\ddot{G}\rangle \\
&+ 8\langle \mathrm{Imtr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\cdot \mathrm{ImTr}(D^{-1}\dot{D})\rangle \\
&+ 2\mathrm{Re}\{\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}](2(\mathrm{Tr}(D^{-1}\dot{D}))^2 - \mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + \mathrm{Tr}(D^{-1}\ddot{D}))\rangle \\
&\quad - \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle\langle 2(\mathrm{Tr}(D^{-1}\dot{D}))^2 - \mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D}) + \mathrm{Tr}(D^{-1}\ddot{D})\rangle\}
\end{aligned}$$

$$\langle \dot{G}\frac{\dot{\Delta}}{\Delta}\rangle$$

$$\frac{1}{2}\left(\langle G\frac{\ddot{\Delta}}{\Delta}\rangle - \langle G\rangle\langle\frac{\ddot{\Delta}}{\Delta}\rangle\right)$$

Noise method:

$$\mathrm{Tr}(A) \simeq \frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\sum_{it,a,\alpha}^{N_t,3,4} \eta^{\dagger}_{i,it,a,\alpha} A\, \eta_{i,it,a,\alpha}$$

$$\frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\eta(i,x)\eta^{*}(i,y) \simeq \delta_{x,y}$$

Our study

- Calculation using
  RG improved gauge action
  & clover-improved Wilson quark
  action with Nf=2

…using configurations
generated by WHOT QCD

In collab. with Y.Maezawa and K.Yazaki
(H.Iida, Y.Maezawa and K.Yazaki, PoS LATTICE2010)

Setup of lattice calculation

- Action:
  RG improved gauge action
  & clover-improved Wilson quark action
- Lattice size & quark masses:
  $16^3 \times 4$, m_{PS}/m_V=0.65, 0.80
- Temperature:
  0.82-4.02 (m_{PS}/m_V=0.65)
  0.76-3.01 (m_{PS}/m_V=0.80)
- Number of configurations:
  100 confs.

m_{PS}/m_V=0.65

β	K	T/Tpc	Traj.
1.50	0.150290	0.82(3)	5000
1.60	0.150030	0.86(3)	5000
1.70	0.148086	0.94(3)	5000
1.75	0.146763	1.00(4)	5000
1.80	0.145127	1.07(4)	5000
1.85	0.143502	1.18(4)	5000
1.90	0.141849	1.32(5)	5000
1.95	0.140472	1.48(5)	5000
2.00	0.139411	1.67(6)	5000
2.10	0.137833	2.09(7)	5000
2.20	0.136596	2.59(9)	5000
2.30	0.135492	3.22(12)	5000
2.40	0.134453	4.02(15)	5000

m_{PS}/m_V=0.80

β	K	T/Tpc	Traj.
1.50	0.143480	0.76(4)	5500
1.60	0.143749	0.80(4)	6000
1.70	0.142871	0.84(4)	6000
1.80	0.141139	0.93(5)	6000
1.85	0.140070	0.99(5)	6000
1.90	0.138817	1.08(5)	6000
1.95	0.137716	1.20(6)	6000
2.00	0.136931	1.35(7)	5000
2.10	0.135860	1.69(8)	5000
2.20	0.135010	2.07(10)	5000
2.30	0.134194	2.51(13)	5000
2.40	0.133395	3.01(15)	5000

Method

- **Leading:**

$$\langle G\rangle|_{\mu=0} = \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle$$

- **Second derivative:**

$$\begin{aligned}
\frac{d^2}{d\mu^2}\mathrm{Re}\langle G\rangle|_{\mu=0} =& 4\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&-2\langle \mathrm{Retr}[(D^{-1}\ddot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&-2\langle \mathrm{Retr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1}\dot{D}D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle \\
&+8\langle \mathrm{Imtr}[(D^{-1}\dot{D}D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\cdot \mathrm{ImTr}(D^{-1}\dot{D})\rangle \\
&+2\mathrm{Re}\{\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}](2(\mathrm{Tr}(D^{-1}\dot{D}))^2-\mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D})+\mathrm{Tr}(D^{-1}\ddot{D}))\rangle \\
&\quad -\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^{\dagger}_{x0}\gamma_5\Gamma^{\dagger}]\rangle\langle 2(\mathrm{Tr}(D^{-1}\dot{D}))^2-\mathrm{Tr}(D^{-1}\dot{D}D^{-1}\dot{D})+\mathrm{Tr}(D^{-1}\ddot{D})\rangle\}
\end{aligned}$$

$$\frac{1}{2}\langle\ddot{G}\rangle \qquad \langle\dot{G}\frac{\dot{\Delta}}{\Delta}\rangle$$

$$\frac{1}{2}\left(\langle G\frac{\ddot{\Delta}}{\Delta}\rangle - \langle G\rangle\langle\frac{\ddot{\Delta}}{\Delta}\rangle\right)$$

Noise method:

$$\mathrm{Tr}(A) \simeq \frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\sum_{it,a,\alpha}^{N_t,3,4}\eta^{\dagger}_{i,it,a,\alpha}A\;\eta_{i,it,a,\alpha}$$

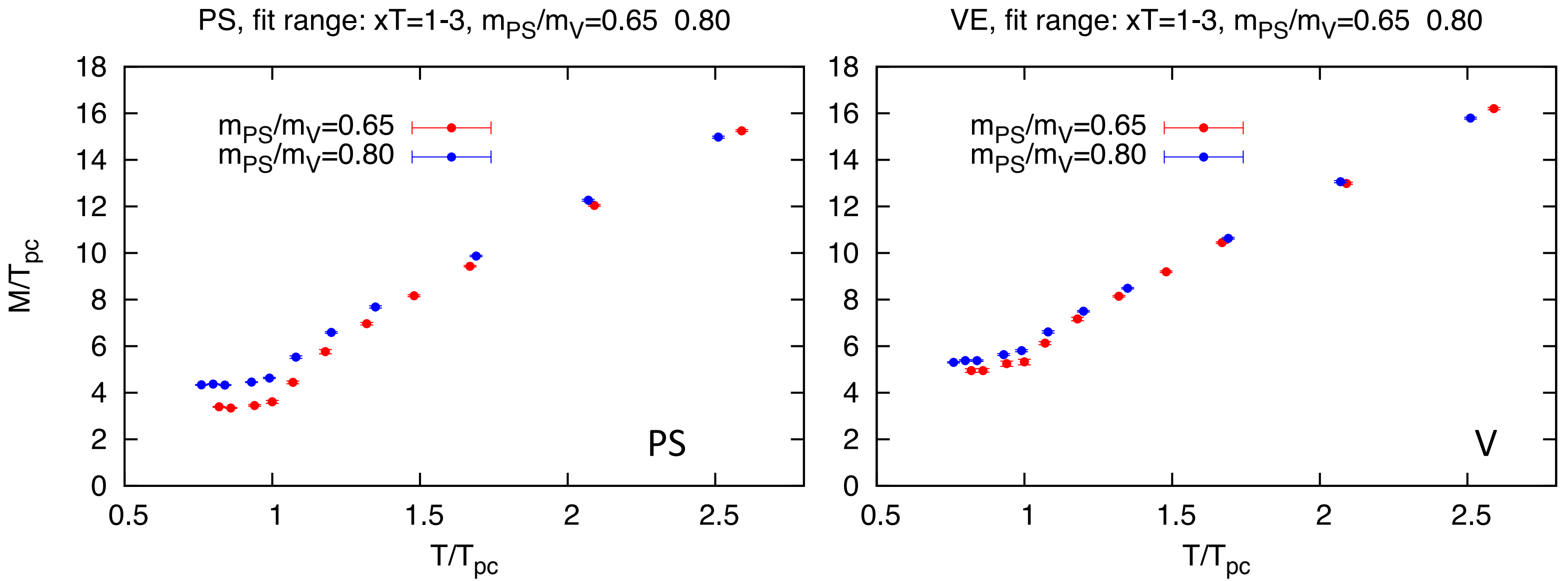
$$\frac{1}{N_{\mathrm{noise}}}\sum_{i=1}^{N_{\mathrm{noise}}}\eta(i,x)\eta^*(i,y)\simeq\delta_{x,y}$$ …100 U(1) noises

Results

Leading order

Results: leading order

- Temperature dependence (M/Tpc).

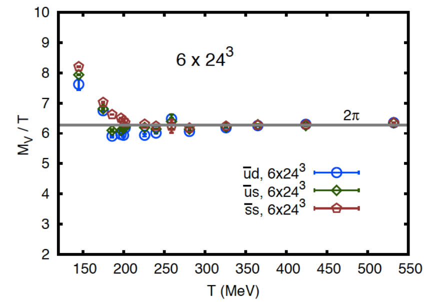


- Meson screening mass increases very slowly below Tc, and rapidly above Tc.

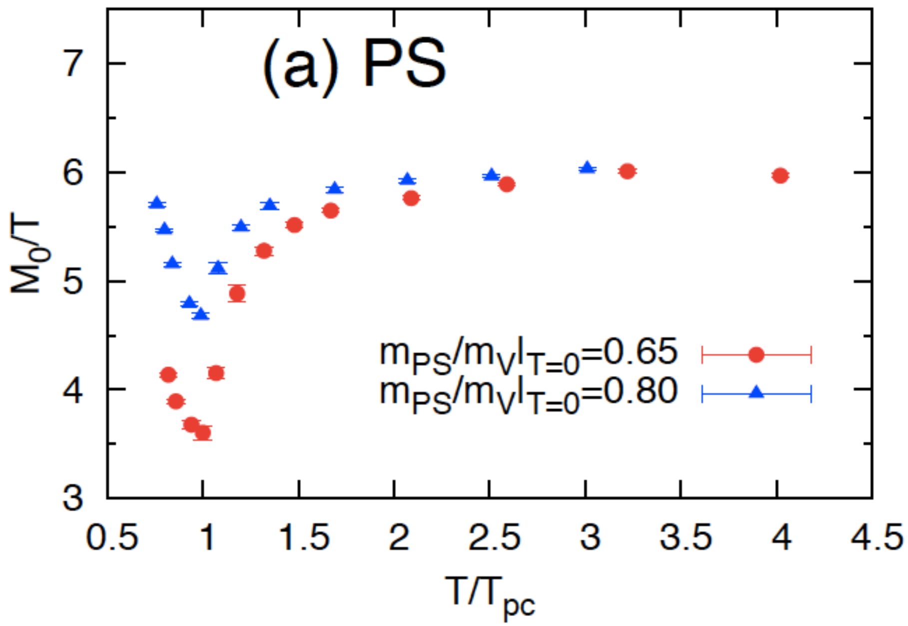
Results: leading order

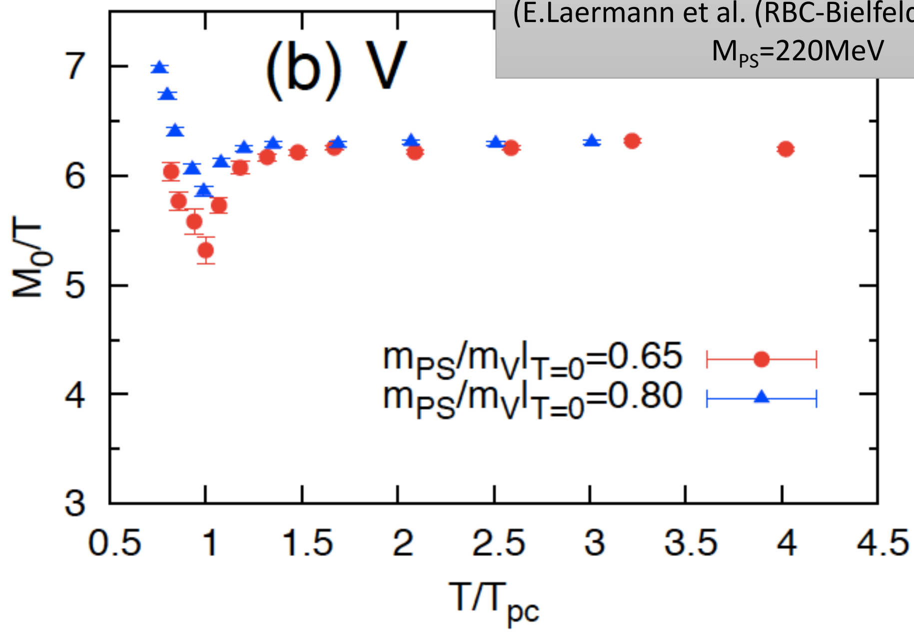
- Temperature dependence.

$$M(\mu,T) = M_0 + M_2(\frac{\mu}{T})^2 + O((\frac{\mu}{T})^4)$$



Results from staggered fermion
(E.Laermann et al. (RBC-Bielfeld Coll.) 2010)
M_{PS}=220MeV



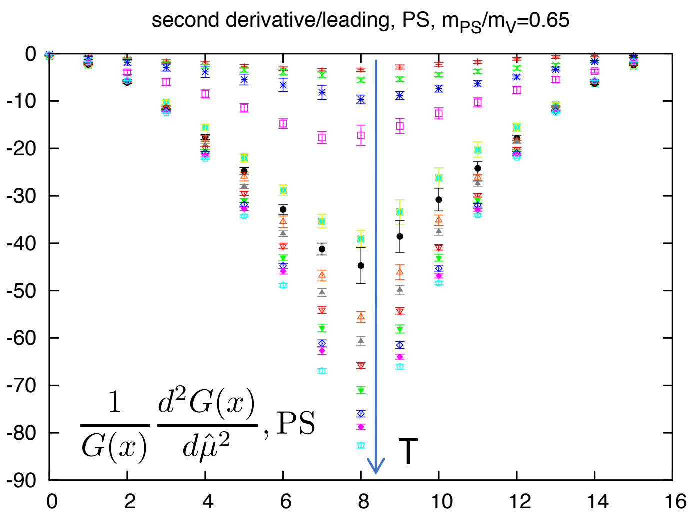


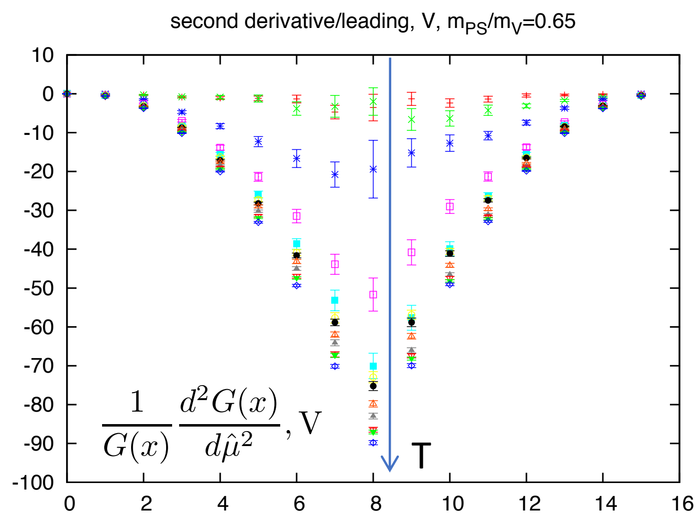
- There is a specific structure around Tc (in the plot of M/T)
- Meson masses become 2πT at high temperature
- Quark mass dependence of meson masses is larger in PS channel than V channel

Results

2^{nd} order

Results: second order





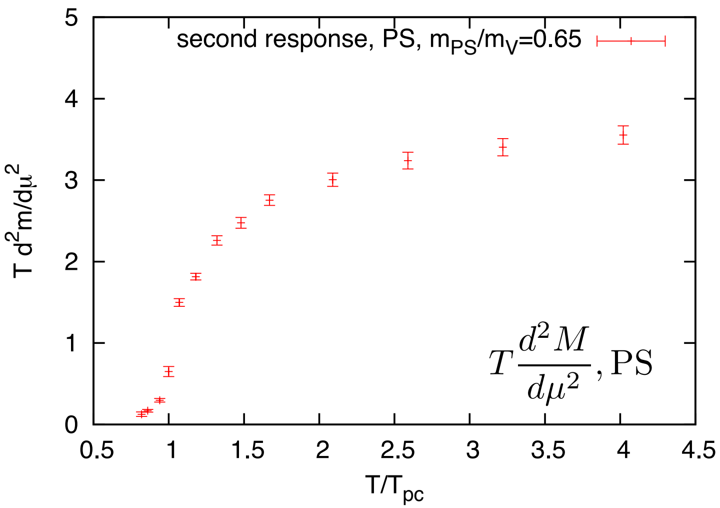
We fit the correlators by the following functional form:

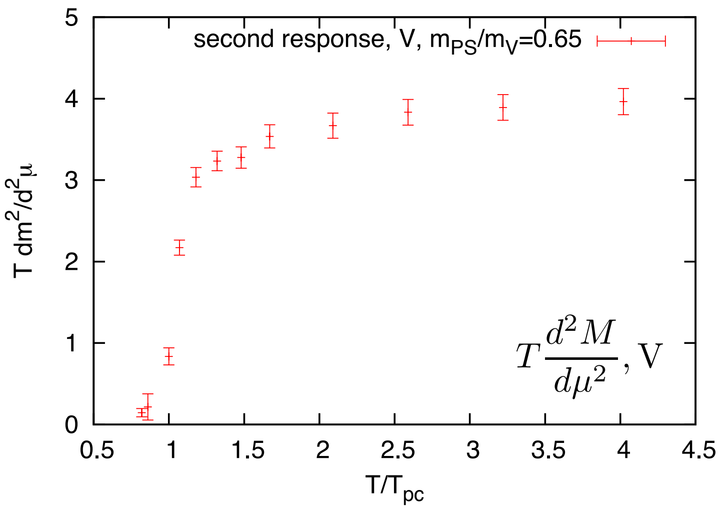
Leading order: $G(x) = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x-\hat{x})})$

2nd order: $\frac{1}{G(x)}\frac{d^2G(x)}{d\hat{\mu}^2} = \frac{1}{A}\frac{d^2A}{d\hat{\mu}^2} + \frac{d^2\hat{M}}{d\hat{\mu}^2}\left\{\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right] - \frac{L_x}{2}\right\}$

→ Second derivative of meson masses, $\frac{d^2\hat{M}}{d\hat{\mu}^2}(= N_tT\frac{d^2M}{d\mu^2})$,is obtained.

Second derivative of meson mass



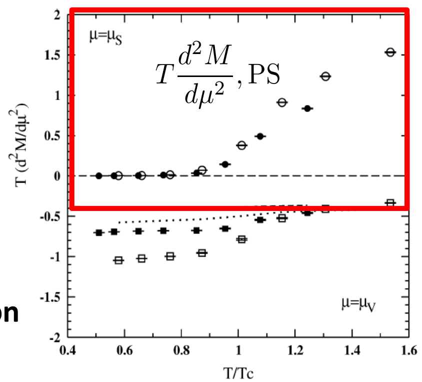


- **Second response is positive**
- **Becomes large after phase transition**
- **Jump around Tc**

… mostly due to operator part

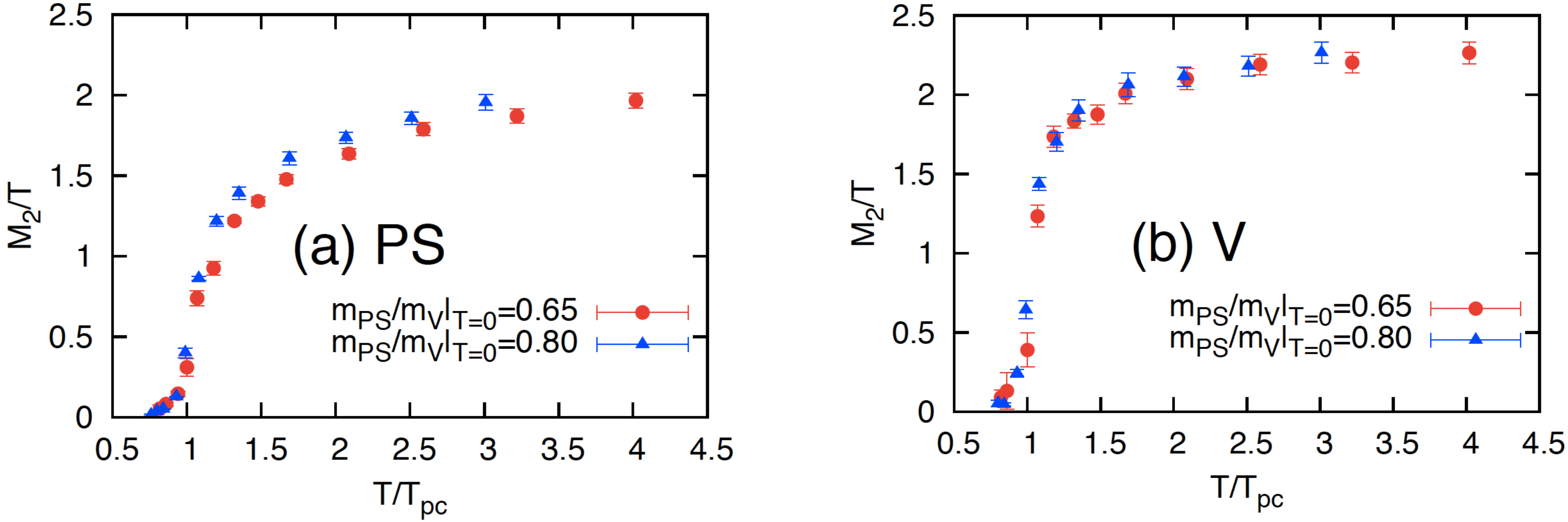
cf) staggered fermion

Results by staggered fermion
I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)



m_{PS}/m_V: 0.55(filled), 0.43(open)

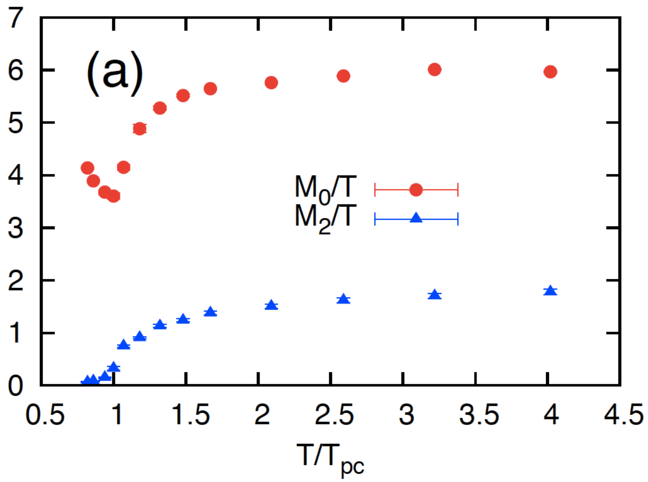
Second derivative of meson mass



$$M(\mu, T) = M_0 + M_2(\frac{\mu}{T})^2 + O((\frac{\mu}{T})^4)$$

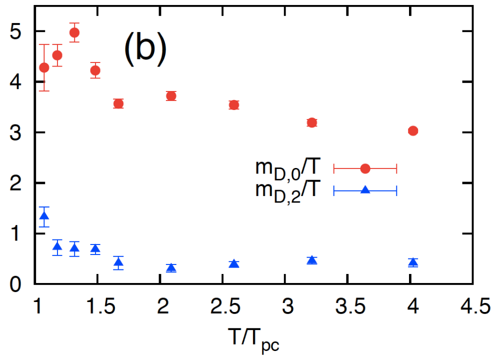
… The response becomes slightly large as quark mass becomes large

Comparison with gluon screening mass



$$\frac{M(\mu)}{T} = \frac{M_0}{T} + \frac{M_2}{T}(\frac{\mu}{T})^2 + O(\mu^4)$$

cf) Second derivative of gluon screening mass
(Y.Maezawa et al., PRD**75**, 074501 (2007))



$$\frac{m_D(\mu)}{T} = \frac{m_{D,0}}{T} + \frac{m_{D,2}}{T}(\frac{\mu}{T})^2 + O(\mu^4)$$

- **Behavior of screening masses are opposite between mesons and gluons:**

$\frac{M_0}{T}$ and $\frac{M_2}{T}$ → large, for T → large ⇔ $\frac{m_{D,0}}{T}$ and $\frac{m_{D,2}}{T}$ → small, for T → large

- The ratio of 2nd order to 0th order is larger for mesons than gluons above Tc:

$\frac{M_2}{M_0} > \frac{m_{D,2}}{m_{D,0}}$ above Tc

mesons…20-30% above Tc
gluons…about 10% above Tc

(Note: quarks couple to μ directly ⇔ gluons couple to μ only through quark loops)

Summary

- We have studied meson screening masses (PS and V) at finite temperature and density in lattice QCD with two-flavor Wilson fermion generated by WHOT-QCD collaboration.

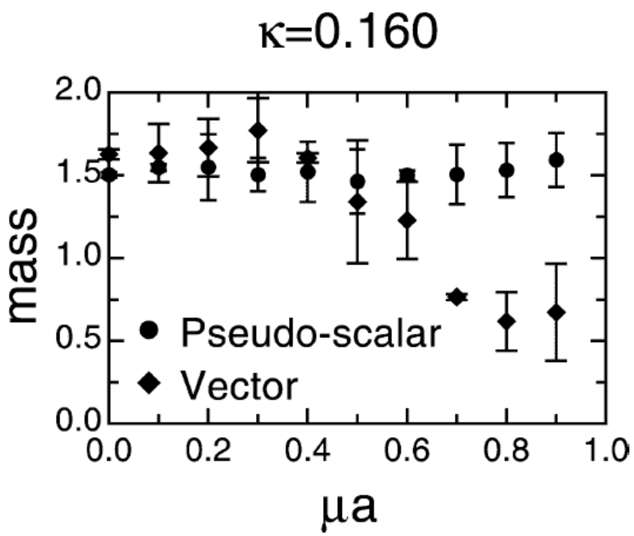
- Finite temperature, μ=0:
  - Below and around Tc : meson masses increase very slowly.
  - Above Tc: increase rapidly and approach to 2πT, where the mesons may become two free quarks.

- Finite μ :
  - $T\frac{d^2M}{d\mu^2}$ is very small below Tc
  - $T\frac{d^2M}{d\mu^2}$ is positive and increases above Tc
  - Meson screening masses have qualitatively different behavior compared to gluon screening mass, which feels μ effect only through quark loops.

Other studies of mesons @ finite μ

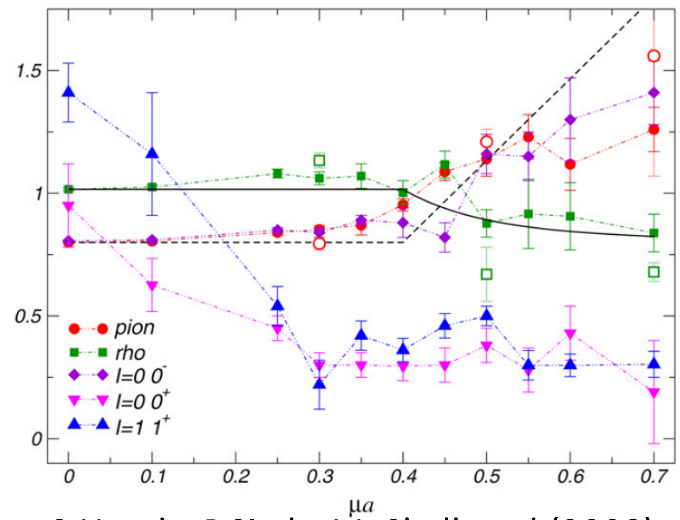
- Direct measurement of mesons @ finite μ in SU(2) gauge theory S.Hands et al., Muroya et al.

S.Muroya, A.Nakamura, C.Nonaka (2003)



Strong modification of vector meson

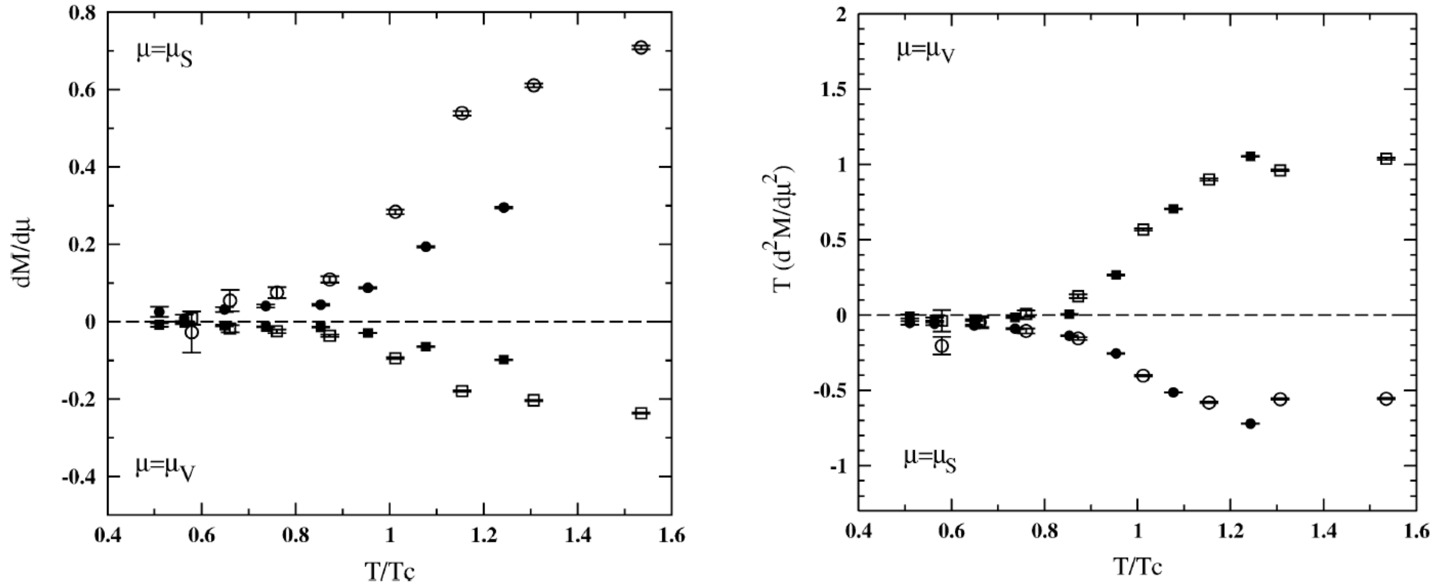
S.Hands, J.B.Kogut, M-P. Lombardo, S.E.Morriso (1999);
S.Hands, P.Sitch, J-I. Skullerud (2008)



S.Hands, P.Sitch, J-I. Skullerud (2008)

Baryons @ finite μ

- I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)



・ 1st order exists for baryons … positive (for μ_s)
・ 2nd order is negative (for μ_s)

Comment on unsatisfactory things

- Taylor exp. … not applicable to zero temperature systems

  direct comp. w/ experiment would not be appropriate

- Quark mass is still large

  it is doubtful that the calculations respect chiral symmetry restoration

- It is desired to extract spectral func.
- Low energy excitation @ finite μ is not investigated

  for the search of CEP, soft mode around it is important

Soft mode around CEP

→ NOT $q\bar{q}$, but the linear combination of
$q\bar{q}$ and ρ_B, s (entropy density) when quark mass is finite (Z_2CP)

Fujii (2003); Fujii & Ohtani (2004)
FRG in quark-meson model: Yokota, Kunihiro, Morita (2016)

If we can overcome these things, that's good.

Thank you for your attention!