Deutsche Forschungsgemeinschaft

ISOQUANT SFB1225

In-medium Landau gauge gluon spectral functions from LQCD with Nf=2+1+1 dynamical quarks

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References:

A.R.: PRD95 (2017) 056016

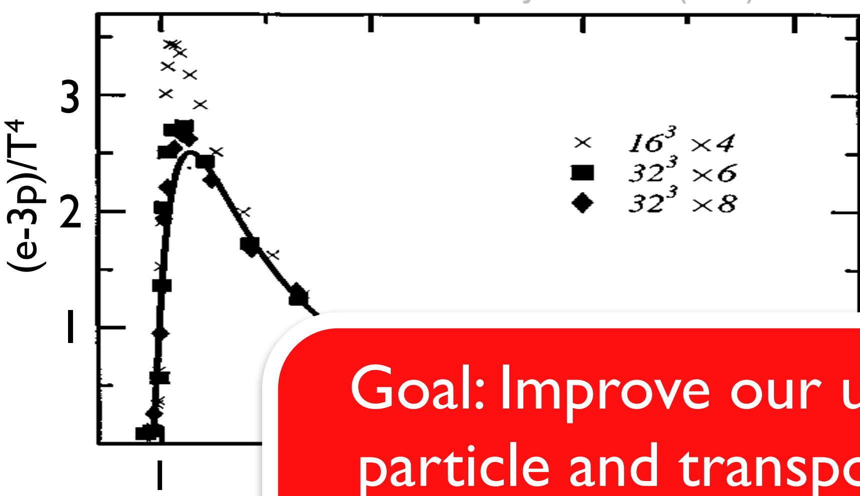
with E.M. Ilgenfritz, J.M. Pawlowski and A. Trunin: arXiv:1701.08610

International Mini-Workshop on "Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics" – JINR, Dubna, Russian Federation – July 13th 2017

Physics questions

Quasi-particle descriptions of the (Q)GP

- Early on: ideal gas of massive gluons
  A. Peshier et. al. Phys.Rev. D54 (1996)



$m_g^2(T$ …

$g^2($ …

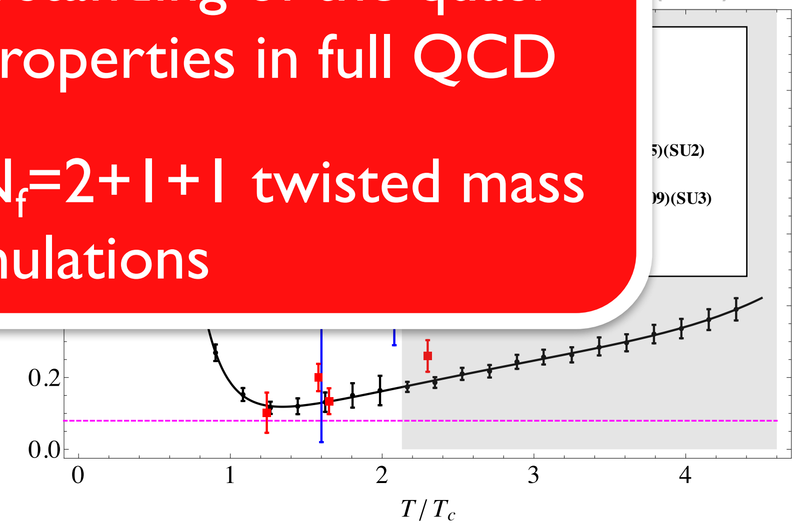
Transport properties of QCD

- Viscosity from zero momentum spectrum
  Meyer, Nakamura et. al., Borsanyi et. al., Braguta, Kotov

$$\eta = \frac{1}{20} \lim_{\mathbf{p}\to 0} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x\, e^{ipx} \langle [T_{12}(0), T^{12}(x)] \rangle$$

$$\eta(T) = \pi \lim_{\omega\to 0} \frac{\rho^{12,12}(\omega)}{\omega}$$

- Use functional methods to express EMT … functions
  … (2014) 091501
  … (2015) 112002



Goal: Improve our understanding of the quasi-particle and transport properties in full QCD

➔ Gluon spectra from N_f=2+1+1 twisted mass LQCD simulations

- Current quasiparticle models e.g. PHSD
  W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919

$$\rho_{q/g}(\omega, T) = \frac{4\omega\Gamma_{q/g}(T)}{\left(\omega^2 - \mathbf{p}^2 - M_{q/g}^2(T)\right)^2 + 4\omega^2\Gamma_{q/g}^2(T)}$$

Lattice QCD setup

- N_f=2+1+1 flavors of twisted Mass Wilson fermions in the thermal QCD medium

R. Baron et al. PoS LAT2010, 123 (2010) and F. Burger et al. (tmft) PoS LAT2013 (2013) 153

ETMC ens. $(T=0)$	D45.32
tmfT ens. $(T\neq 0)$	D370
β	2.10
a [fm]	0.0646
m_π [MeV]	369(15)
$T_{\rm deconf}$ [MeV]	193(13)(2)
$N_\tau = N_{q_4}$ range	4-20

D370 N_τ	4	6	8	10	11	12	14	16	18	20
T MeV	762	508	381	305	277	254	218	191	170	152
N_s	32	32	32	32	32	32	32	32	40	48
$N_{\rm meas}$	310	400	120	410	420	380	790	610	590	280

- Gluon correlator $D^{ab}_{\mu\nu}(\mathbf{q}) = \langle A^a_\mu(-\mathbf{q})A^b_\nu(\mathbf{q})\rangle$ requires gauge fixing (e.g. $\partial_\mu A^\mu = 0$)
  - Minimize $F_U[g] = \frac{1}{3}\sum_{x,\mu} \mathrm{ReTr}\left(g^\dagger_x U_{x\mu} g_{x+\mu}\right)$ via gauge transf. $U_{x\mu} \overset{g}{\mapsto} U^g_{x\mu} = g^\dagger_x U_{x\mu} g_{x+\mu}$
- At T>0 separation into longitudinal (electric) & transversal (magnetic) mode

$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right),$$

$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P^T_{\mu\nu}\,.$$

$$D_T(q) = \frac{1}{2N_g}\left\langle \sum_{i=1}^{3} A^a_i(q)A^a_i(-q) - \frac{q_4^2}{\vec{q}^{\,2}} A^a_4(q)A^a_4(-q)\right\rangle$$

$$D_L(q) = \frac{1}{N_g}\left(1 + \frac{q_4^2}{\vec{q}^{\,2}}\right)\langle A^a_4(q)A^a_4(-q)\rangle$$

The challenge of gluon spectra

- Already perturbation theory predicts Landau gauge spectra non-positive definite

Alkofer, von Smekal, Phys. Rept. 353 (2001), Cornwall MPL A28 (2013)

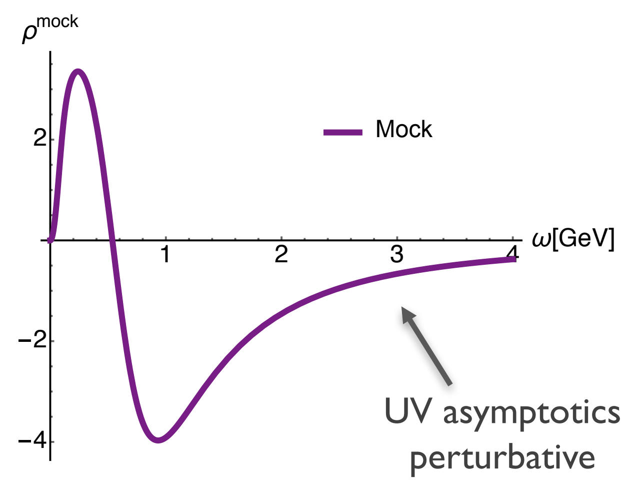
$$D^{ab}_{\mu\nu} \sim \frac{1}{p^2}\left[\mathrm{Log}\left(\frac{p^2}{\mu^2}\right)\right]^{-\frac{13}{22}}$$

→ decays *faster* than p^{-2}

$$D(p) \propto \int_0^\infty d\omega \, \frac{2\omega\, \rho(\omega)}{p^2 + \omega^2}$$

→ denominator contains only p^{-2}

→ $$\lim_{p\to\infty} D(p)p^2 = \int_0^\infty d\omega\, 2\omega\, \rho(\omega) = 0$$



Unfolding of real-time information

- Inversion of integral transform required to obtain spectra from correlators

$$D(\tau_i) = \sum_{l=1}^{N_\omega} \Delta\omega_l \, K(\tau_i, \omega_l) \, \rho(\omega_l)$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. data D_i has finite precision

- Going to imaginary frequencies improves the inverse problem (see also Backus-Gilbert/Sumudu)

see e.g. B. B. Brandt et al. PRD92, 094510 (2015) and F. Pederiva et al. J. Phys. Conf. Ser. 527 (2014)

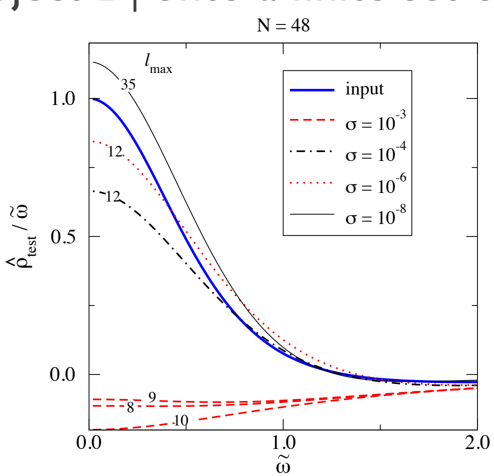
$$D(\tau) = \int_0^\infty d\omega \, \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh[\omega\beta/2]} \rho(\omega) \quad \xrightarrow[\tau \to \mu]{\text{Fourier}} \quad D(\mu) = \int_0^\infty d\omega \frac{2\omega}{\mu^2 + \omega^2} \rho(\omega)$$

- One possibility: direct projection methods (Pade, **Cuniberti**, …)

e.g. Cuniberti, Michelli, Viano Commun.Math.Phys. 216 (2001)

  - Project D_i onto a finite set of basis functions: analytically continue the basis functions

Y. Burnier, M.Laine Eur.Phys.J. C71 (2011) 1619



- cancellation in basis function coefficients requires very high precision of data ($D_j = D_j^{ideal} + \delta D_j$)
- divergent structures in the correlator D must be subtracted
- in practice with real-word lattice data only qualitatively satisfactory results achieved

The Bayesian strategy

- **Bayes Theorem:** Systematic inclusion of additional prior knowledge (I)
  C.M. Bishop, Pattern Recognition and Machine Learning, Springer (2007), Jarrell, Gubernatis, Phys. Rep. 269 (1996)

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

$$P[D|\rho,I] = e^{-L},\ L = \frac{1}{2}\sum_i (D_i - D_i^{\rho})^2/\sigma_i^2$$

Likelihood: How is the data measured

$$\frac{\delta P[\rho|D,I]}{\delta \rho_l} \stackrel{!}{=} 0$$

$$P[\rho|I] = e^S,\ S = S[\rho(\omega), m(\omega)]$$

Prior: What else is known about ρ
(functional form of S and default model m: $\delta S/\delta\rho|_{\rho=m}=0$)

- Previously BR prior: ρ positive definite, smoothness of ρ, result independent of units

$$P[\rho|I] \propto e^S \qquad S = \alpha \sum_{l=1}^{N_\omega} \Delta\omega_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)$$

Y.Burnier, A.R.
PRL 111 (2013) 18, 182003

Generalized BR method

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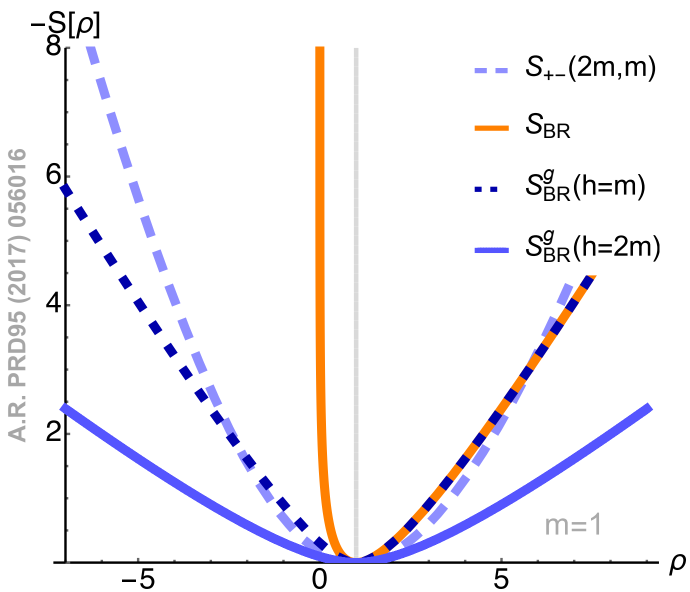
- Bayesian proposals to treat non-positive definite spectra in the literature:
  - use quadratic prior $S=(\rho-m)^2$: too strong imprinting of m on end results
    Dudal, Oliveira, Silva PRD89 (2014) 014010
  - decompose ρ into $\rho^+>0$ and $\rho^-<0$ apriori: beyond prior information
    Hobson, Lasenby, Mon. Not. Roy. Astron. Soc. 298, 905 (1998); Qin, Rischke PRD88 (2013) 056007
  - add shift onto the data & use standard methods: remnant dependence on shift?
    see e.g. Haas, Fister, Pawlowski PRD90 (2014) 091501



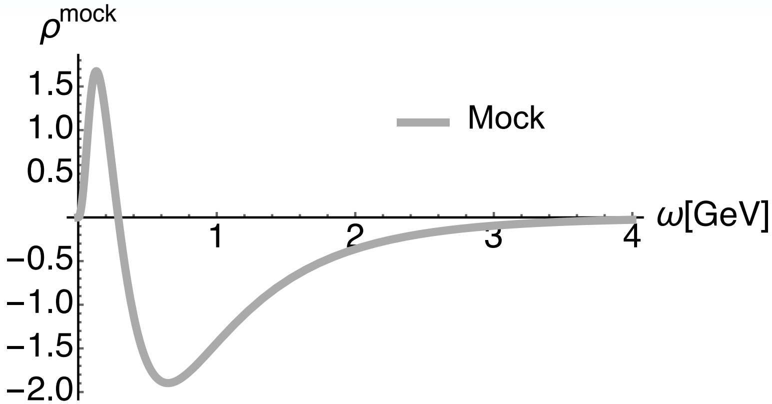
- Here instead generalized BR prior:

A.R. PRD95 (2017) 056016

$$S^g_{BR} = \alpha \int d\omega \Big(-\frac{|\rho - m|}{h} + \log\Big[\frac{|\rho - m|}{h} + 1\Big]\Big)$$

  - absolute deviation |ρ-m| vs. previously ρ/m
  - new default model function h: confidence in m
  - weakest amongst different priors: *let the data speak*

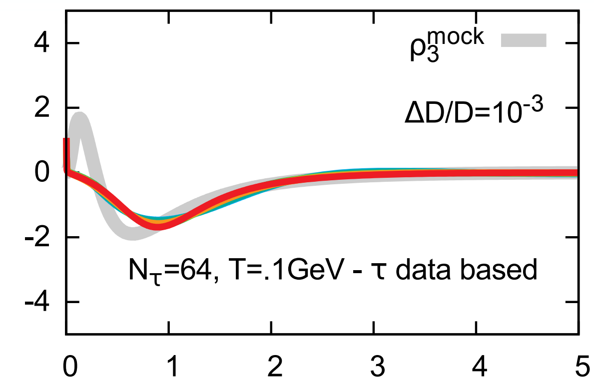
Mock data tests



A.R. PRD95 (2017) 056016

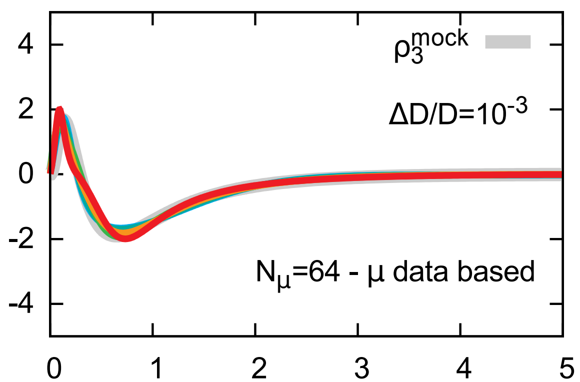
$$\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh[\omega\beta/2]}$$

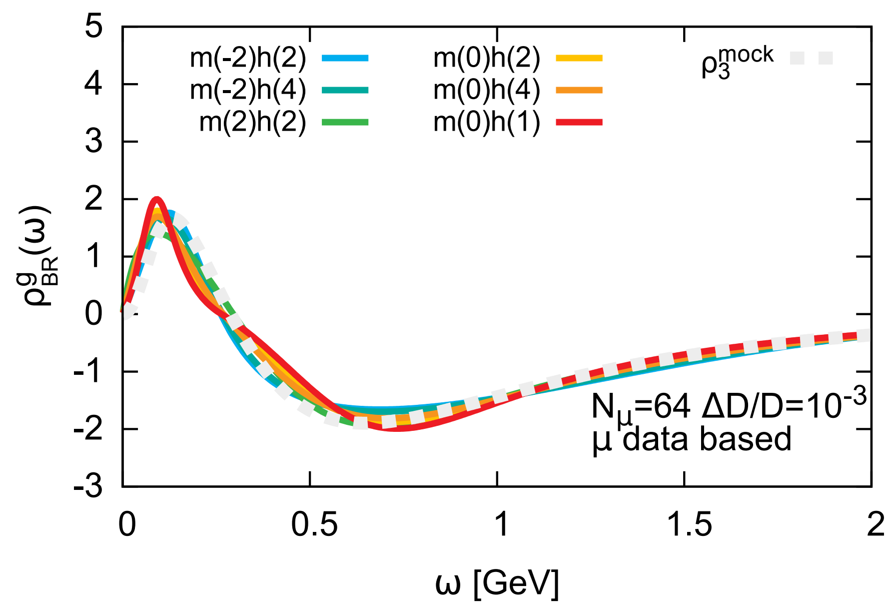
Euclidean based recon-
struction exp. hard!



$$\frac{2\omega}{\mu^2+\omega^2}$$

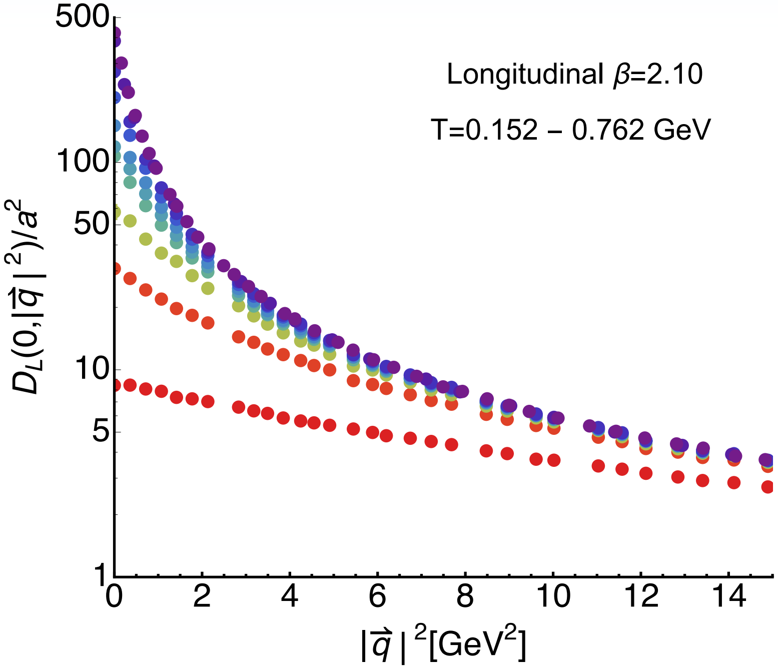
Imaginary frequency
based rec. improves

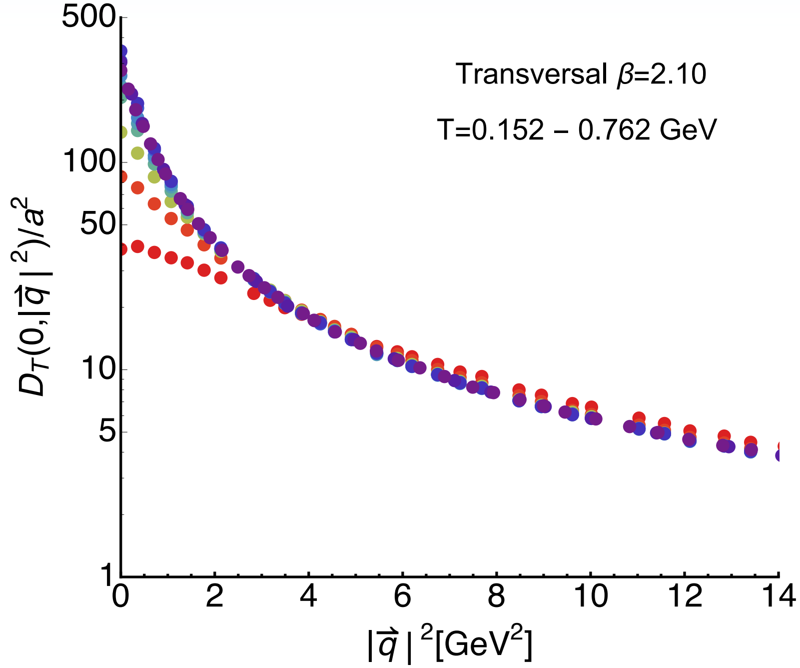




Gluon correlators in Landau gauge

Ilgenfritz, Pawlowski, A.R., Trunin
arXiv:1701.08610





- Conventional separation into longitudinal (electric) & transversal (magnetic) mode

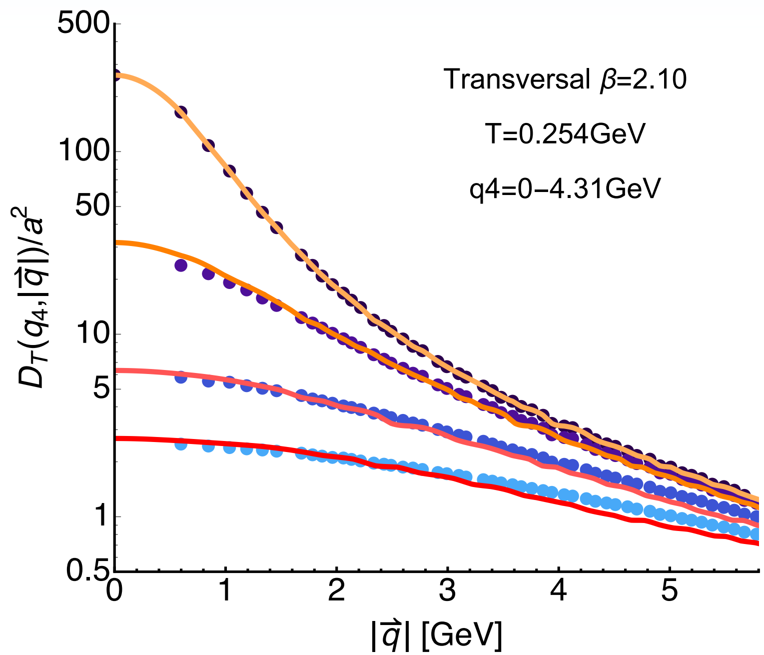
$$P^T_{\mu\nu} = (1-\delta_{\mu 4})(1-\delta_{\nu 4})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}\right), \qquad D_T(q) = \frac{1}{2N_g}\left\langle \sum_{i=1}^{3} A_i^a(q)A_i^a(-q) - \frac{q_4^2}{\vec{q}^{\,2}} A_4^a(q)A_4^a(-q)\right\rangle$$

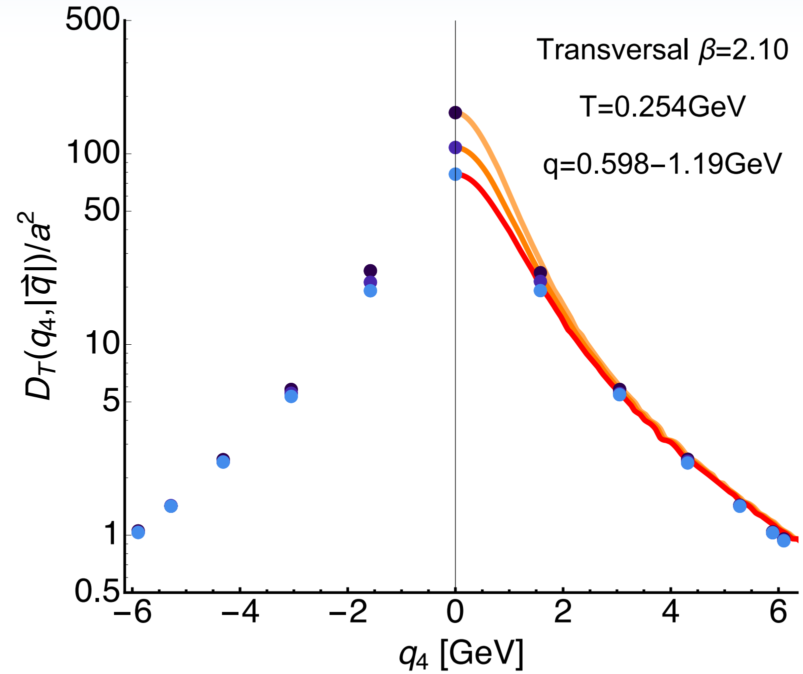
$$P^L_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P^T_{\mu\nu}\,. \qquad D_L(q) = \frac{1}{N_g}\left(1 + \frac{q_4^2}{\vec{q}^{\,2}}\right)\langle A_4^a(q)A_4^a(-q)\rangle$$

- An electric and magnetic mass visible at q_4=0 **q**=0 : D(0,0)=1/M^2 increase with T

The O(4) assumption

Ilgenfritz, Pawlowski, A.R., Trunin
arXiv:1701.08610





- Previous studies used O(4) invariance assumption to generate correlators for $q_4>0$

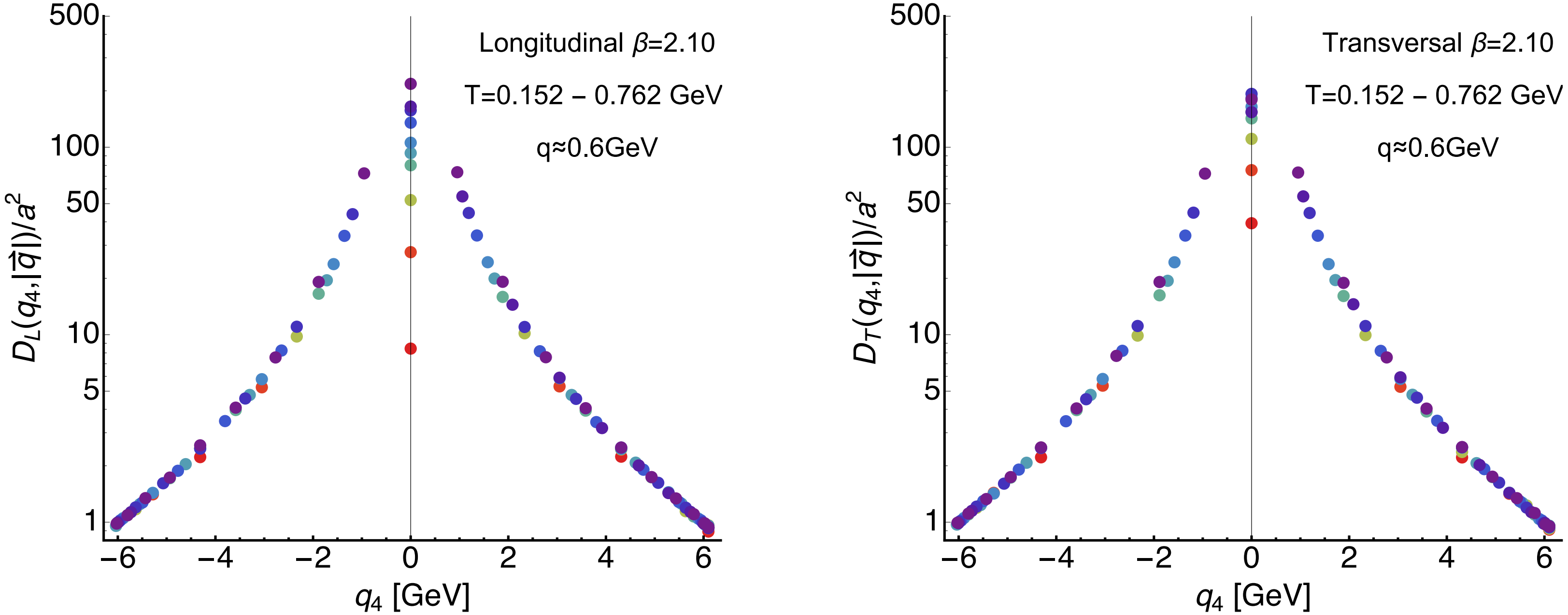
  see e.g. Dudal, Oliveira, Silva PRD89 (2014) 014010

$$D(q_4, \mathbf{q}) \approx D(0, \sqrt{q_4^2 + \mathbf{q}^2})$$

- Here: explicitly compute finite q_4 and observe range of validity of assumption
  - Close to $q_4=0$ ok but already deviations at $q_4 \sim 2\pi T$ and end of Brillouin zone problematic

Raw reconstruction datasets

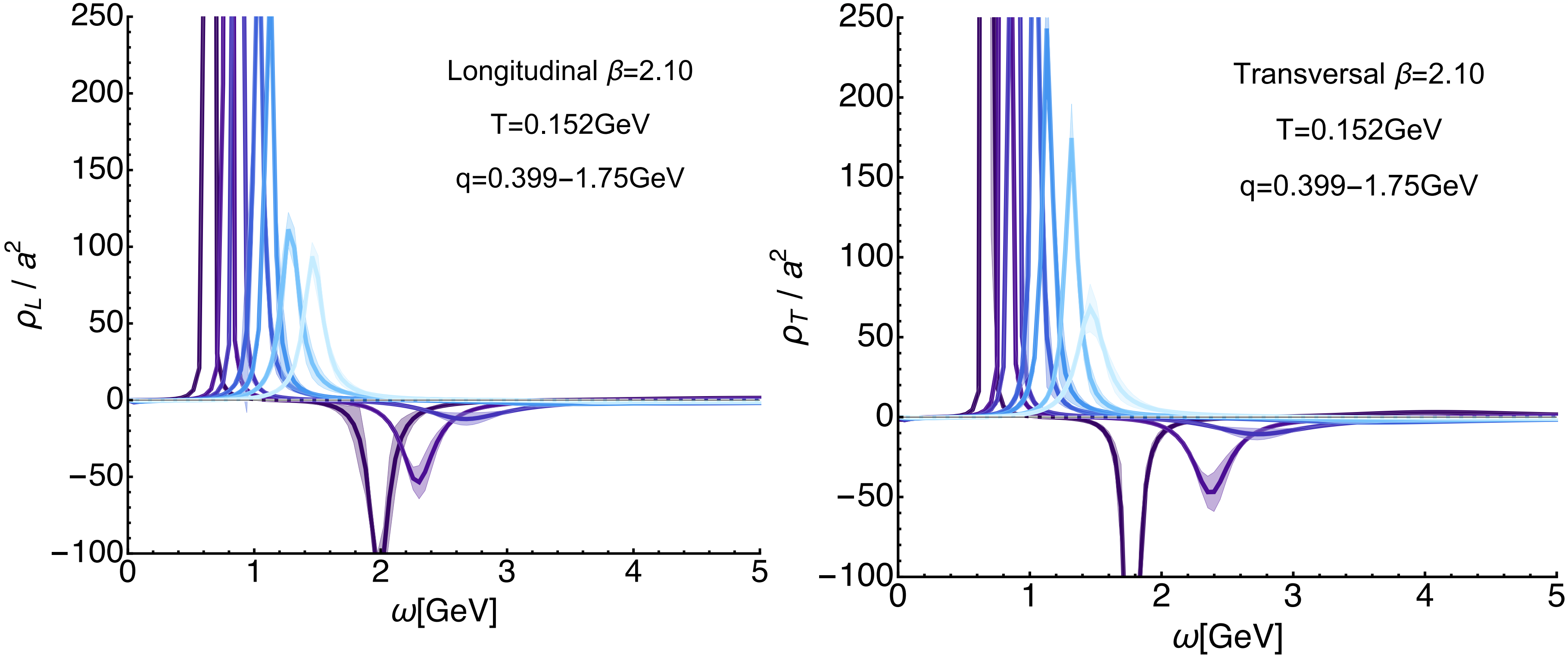
Ilgenfritz, Pawlowski, A.R., Trunin
arXiv:1701.08610



- Challenge: while at $q_4=0$ strong difference visible already at $q_4 \approx 2\pi T$ very similar
- See Jan's talk on how to resolve the correlator beyond the Matsubara frequencies

Reconstructed Spectra I

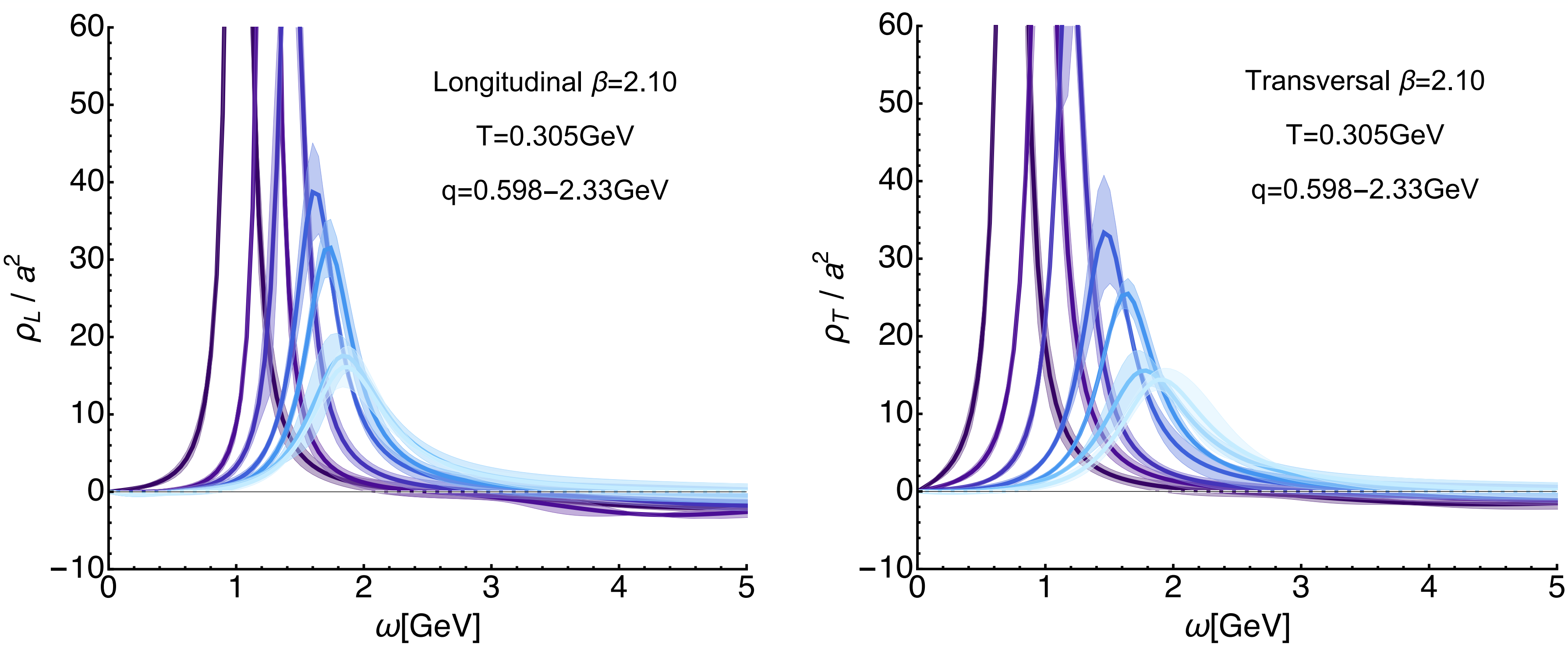
Ilgenfritz, Pawlowski, A.R., Trunin arXiv:1701.08610



- Clear observation of a peak-trough structure in both channels at low T
- Negative contribution appears slightly stronger in transversal sector

Reconstructed Spectra II

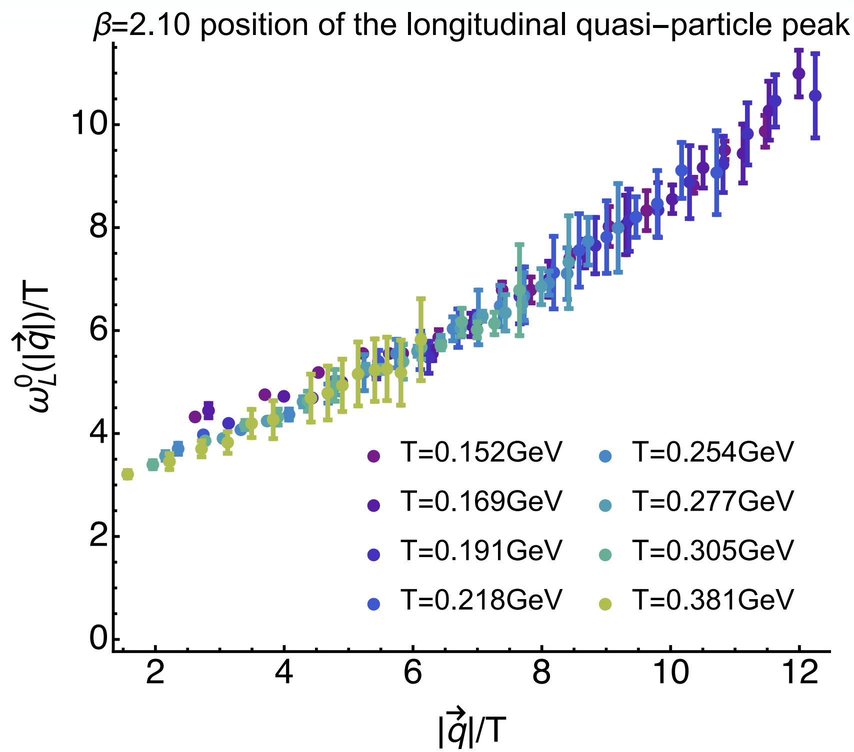
Ilgenfritz, Pawlowski, A.R., Trunin arXiv:1701.08610

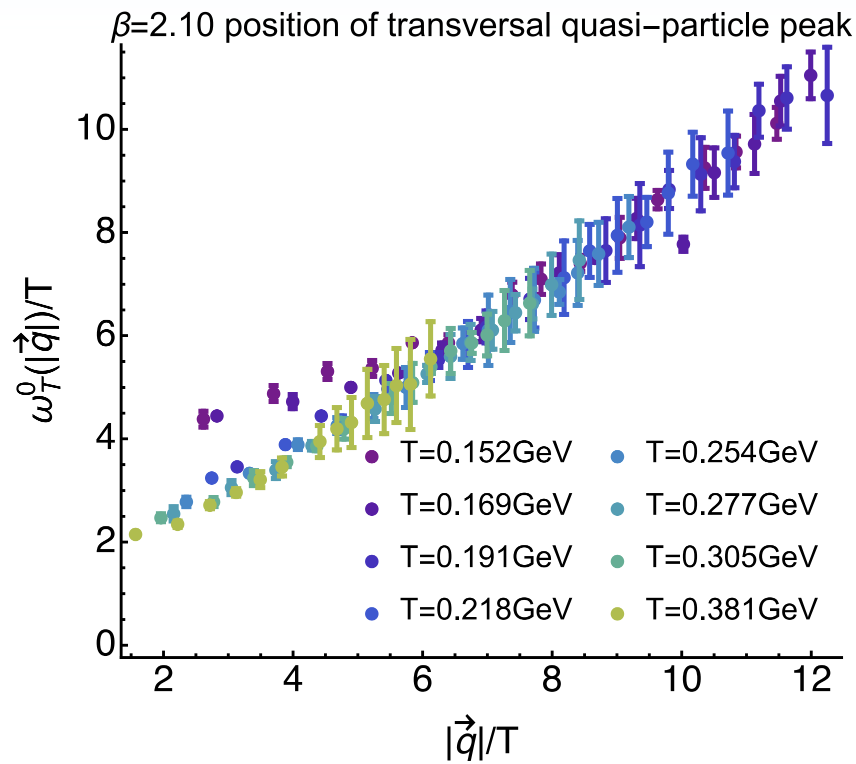


- Negative trough significantly reduced at $T>T_C$

Gluon dispersion relation I

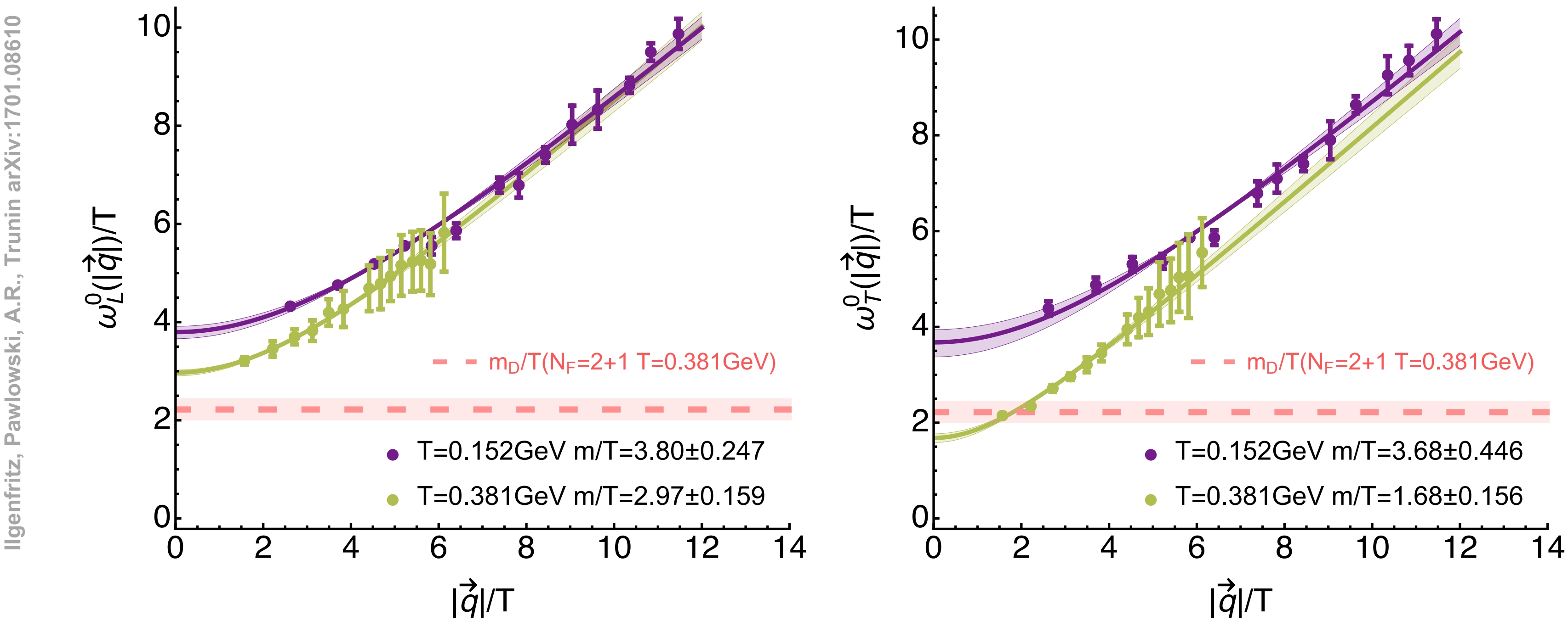
Ilgenfritz, Pawlowski, A.R., Trunin arXiv:1701.08610





- Use the peak position of the lowest lying structure to define dispersion relation
- |**q**| dependence same for large values of spatial momenta, differences at small |**q**|

Gluon dispersion relation II



- Quantitative fit with modified free theory ansatz $\omega^0(\mathbf{q}) = A\sqrt{B^2 + |\mathbf{q}|^2}$
- Resulting masses in qualitative agreement with weak coupling expectations

$$m_{el} \sim gT, \quad m_{mag} \sim g^2 T$$

Summary

- Investigating gluon properties provides complementary insight into QGP physics
- Lattice QCD simulations with gauge fixing are an appropriate non-pert. tool
- Extracting spectral properties from the lattice as ill-posed inverse problem
  - Positivity violation precludes application of standard approaches
  - Novel Bayesian approaches (BR) available for positive definite and general spectra
- Investigation of gluon properties in Nf=2+1+1twisted mass lattice QCD
  - First study not to rely on assumption of O(4) invariance for correlators
  - Clear observation of quasi-particle structure at small frequencies
  - Dispersion relation with masses in qualitative agreement with weak coupling

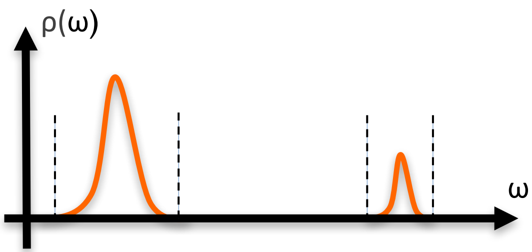
Thank you for your attention - Благодарю вас за внимание

Deriving the BR prior

- Both functional form of prior distribution and supplied m(ω) encode prior info
  - Anticipate the situation where no prior estimation of ρ exists m(ω)=const.
- Wish to axiomatically derive **P[ρ|I]=Exp[S]** to encode: **ρ>0** & **ρ smooth** (if N_τ=0)
  - **Axiom I**: Subset independence (same as in Maximum Entropy Method)

$$S[\Omega_1, m(\Omega_1)] + S[\Omega_2, m(\Omega_2)] = S[\Omega_1 \cup \Omega_2, m(\Omega_1 \cup \Omega_2)]$$

$$S \propto \int d\omega \; s(\rho(\omega), m(\omega), \omega)$$



  - **Axiom II**: Scale invariance (new)
    - ρ itself does not have to be probability distribution: scales differently from 1/ω

$$S = \tilde{\alpha} \int d\omega \; s\Big(\rho(\omega)/m(\omega)\Big)$$

to make dimensionless: hyperparameter α

The Bayesian strategy

- **Axiom III**: Smoothness of the reconstructed spectrum (new)
  - Goal: in the case of $m(\omega)=m_0$, prior shall choose a smooth spectrum **independent** of m_0
  - Penalty for deviation of $r_l=\rho_l/m_l$ between adjacent values ω_1 and ω_2
  - If changing r_1 and r_2 does not move D^ρ beyond errorbars of the data: $r_1=r_2$

$$r_1 = r_2 \qquad \text{vs.} \qquad r_1 = r(1+\epsilon), \quad r_2 = r(1-\epsilon)$$

  - Penalty independent of r and symmetric in $r_1 \gtrless r_2$

$$2s(r) - s(r(1+\epsilon)) - s(r(1-\epsilon)) = \epsilon^2 C_2 \quad \Rightarrow \quad -r^2 s''(r) = C_2$$

  - Solution of differential equation:

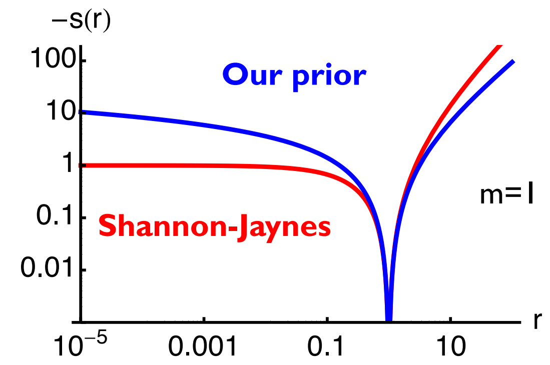
$$S = \tilde{\alpha} \int d\omega \left(C_0 - C_1 \frac{\rho}{m} + C_2 \ln\left(\frac{\rho}{m}\right) \right)$$

The Bayesian strategy

$$S = \tilde{\alpha} \int d\omega \left(C_0 - C_1 \frac{\rho}{m} + C_2 \ln\left(\frac{\rho}{m}\right) \right)$$

- **Axiom IV**: Maximum at the prior (Bayesian meaning of m(ω))
  - In the absence of data S must be maximal at ρ = m, i.e. r = 1

$$S(r=1)=0, \quad S'(r=1)=0, \quad S''(r=1)<0$$



  - The strictly concave result (α>0, S≤0):

$$S = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \ln\left(\frac{\rho}{m}\right) \right)$$

- The prior probability hence is related to an inverse γ-distribution:

$$P[\rho|\alpha, m] = e^S \Big/ \prod_{i=1}^{N_\omega} e^{\alpha\Delta\omega_i} (\alpha\Delta\omega_i)^{-\alpha\Delta\omega_i} m_i \Gamma(\alpha\Delta\omega_i)$$