Exploring the phase structure and dynamics of QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Dubna, July 12th & 14th 2017

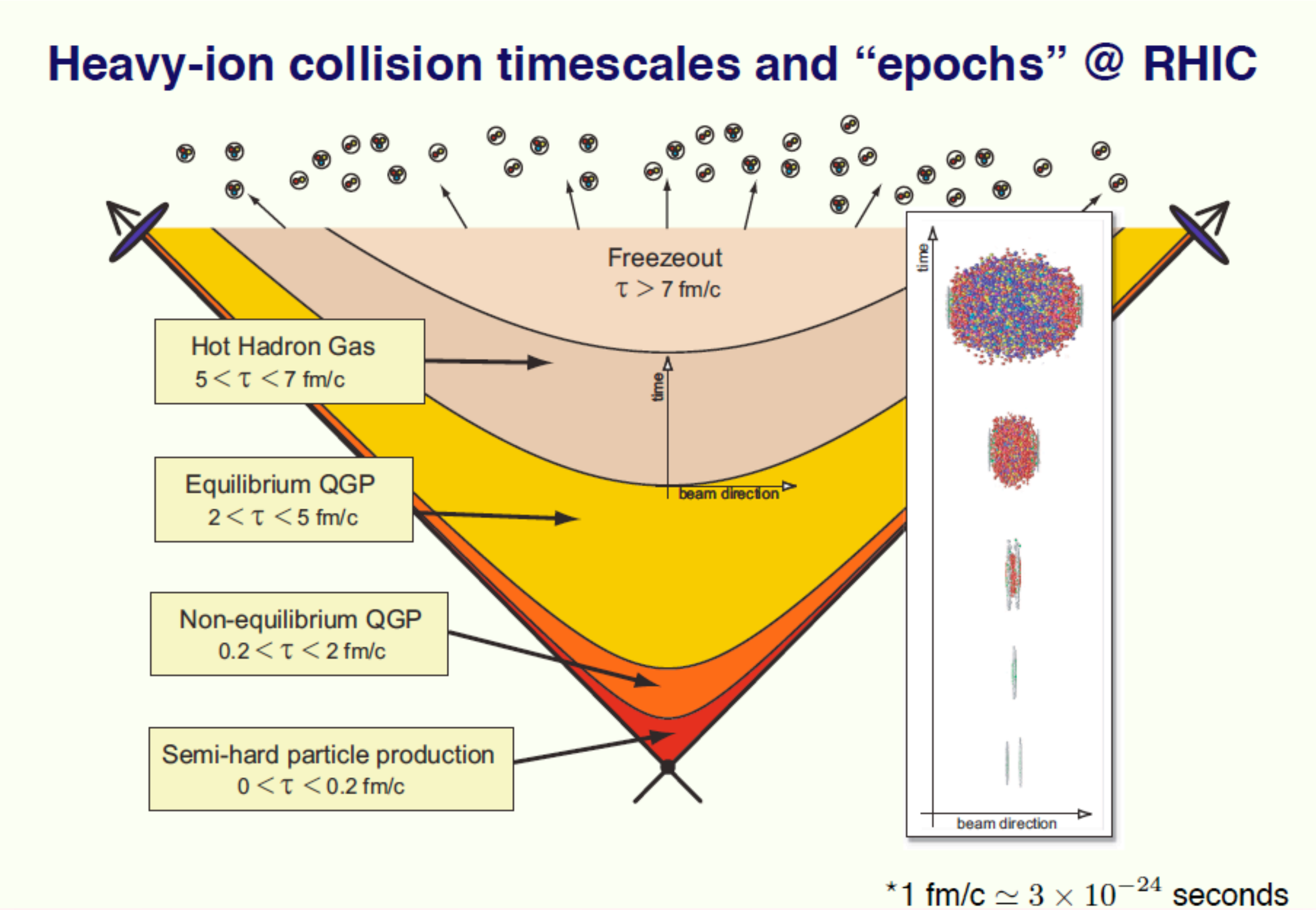
GEFÖRDERT VOM

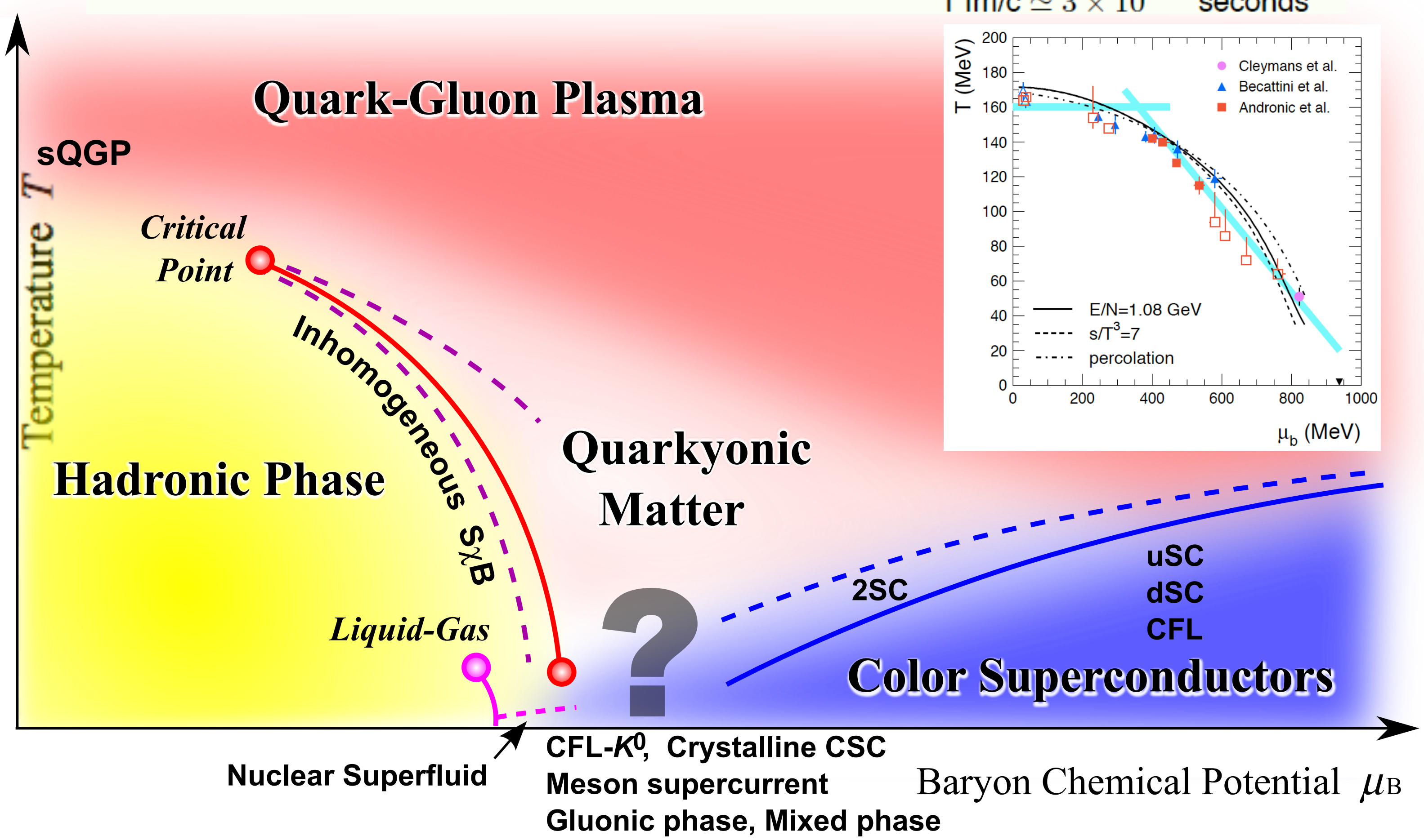
Bundesministerium
für Bildung
und Forschung

ISOQUANT
SFB1225

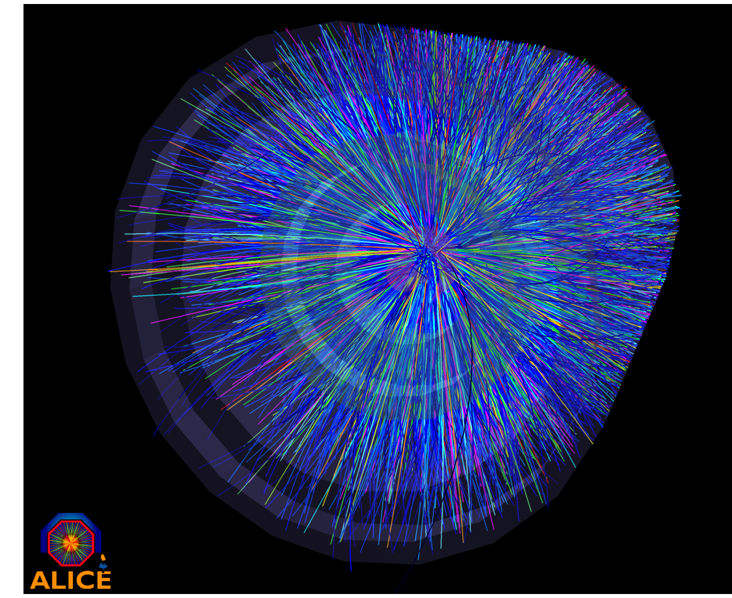
EMMI

Heavy ion collisions

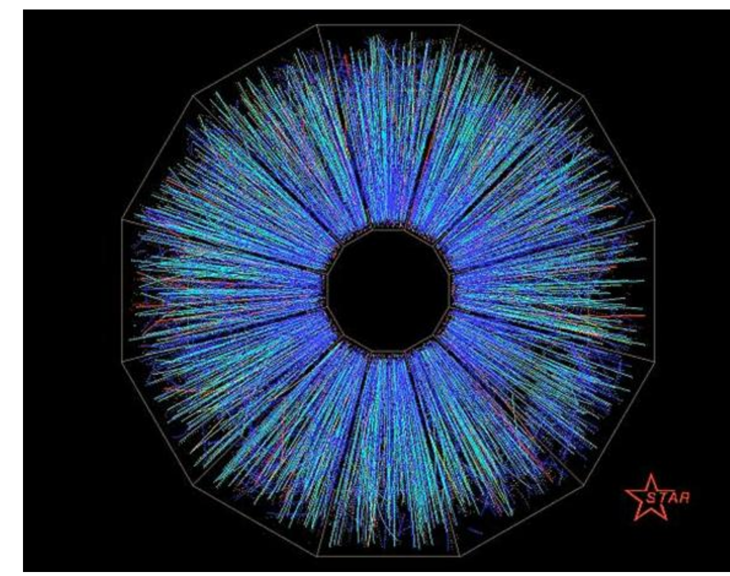




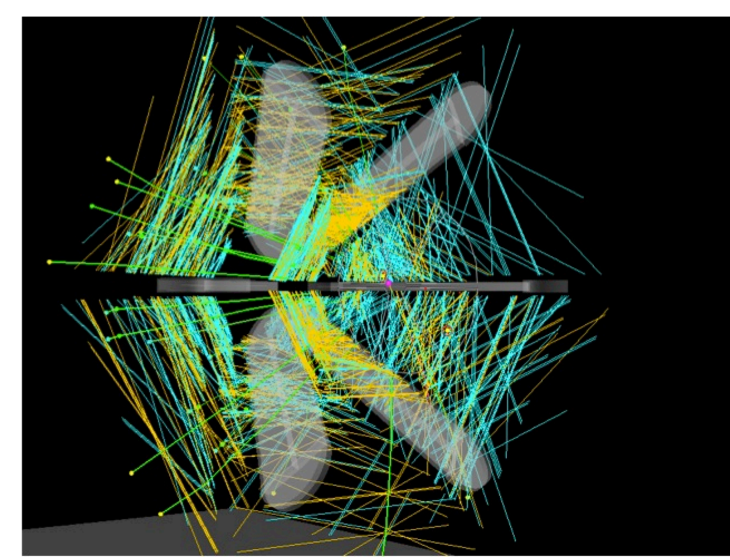
LHC



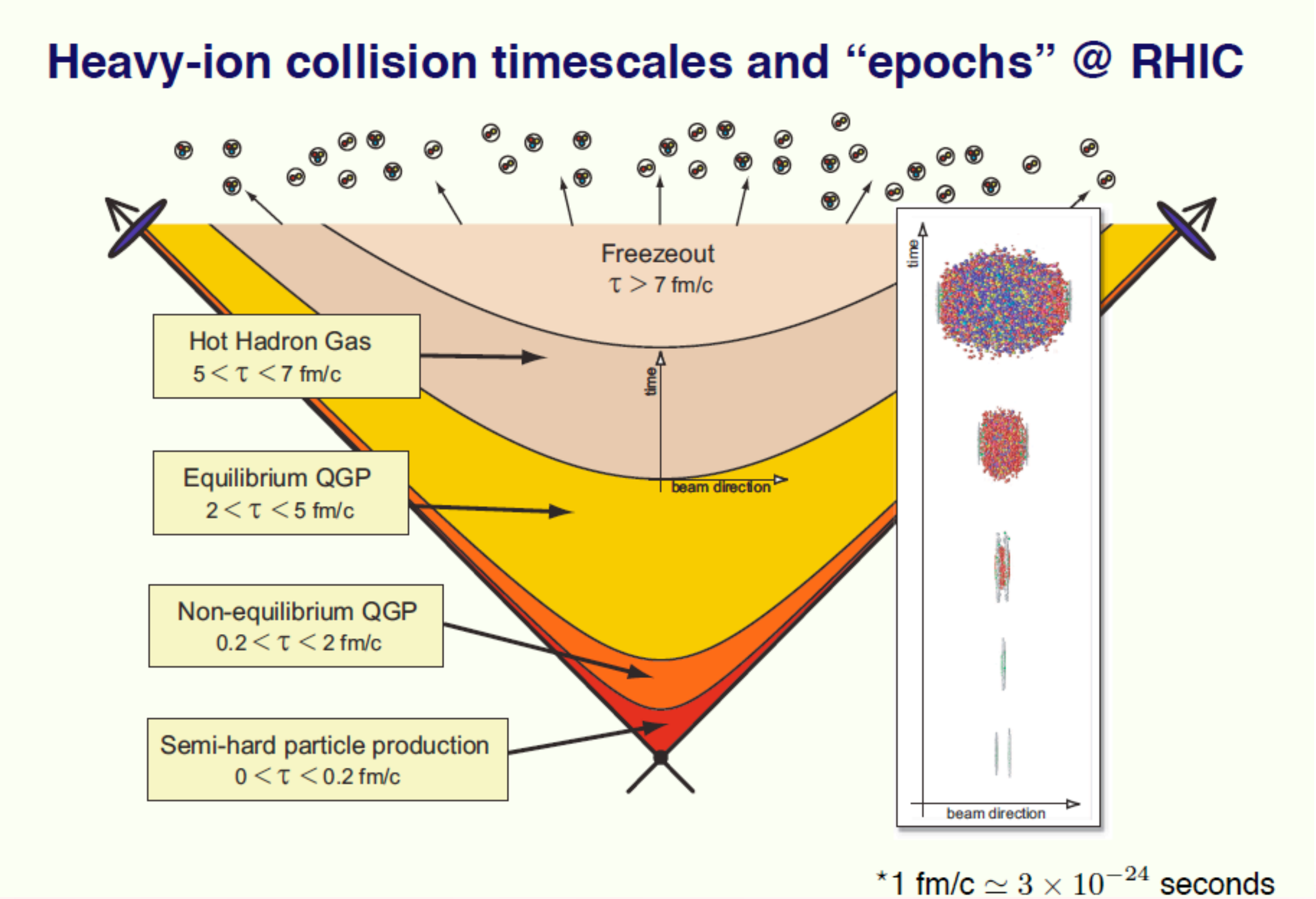
RHIC

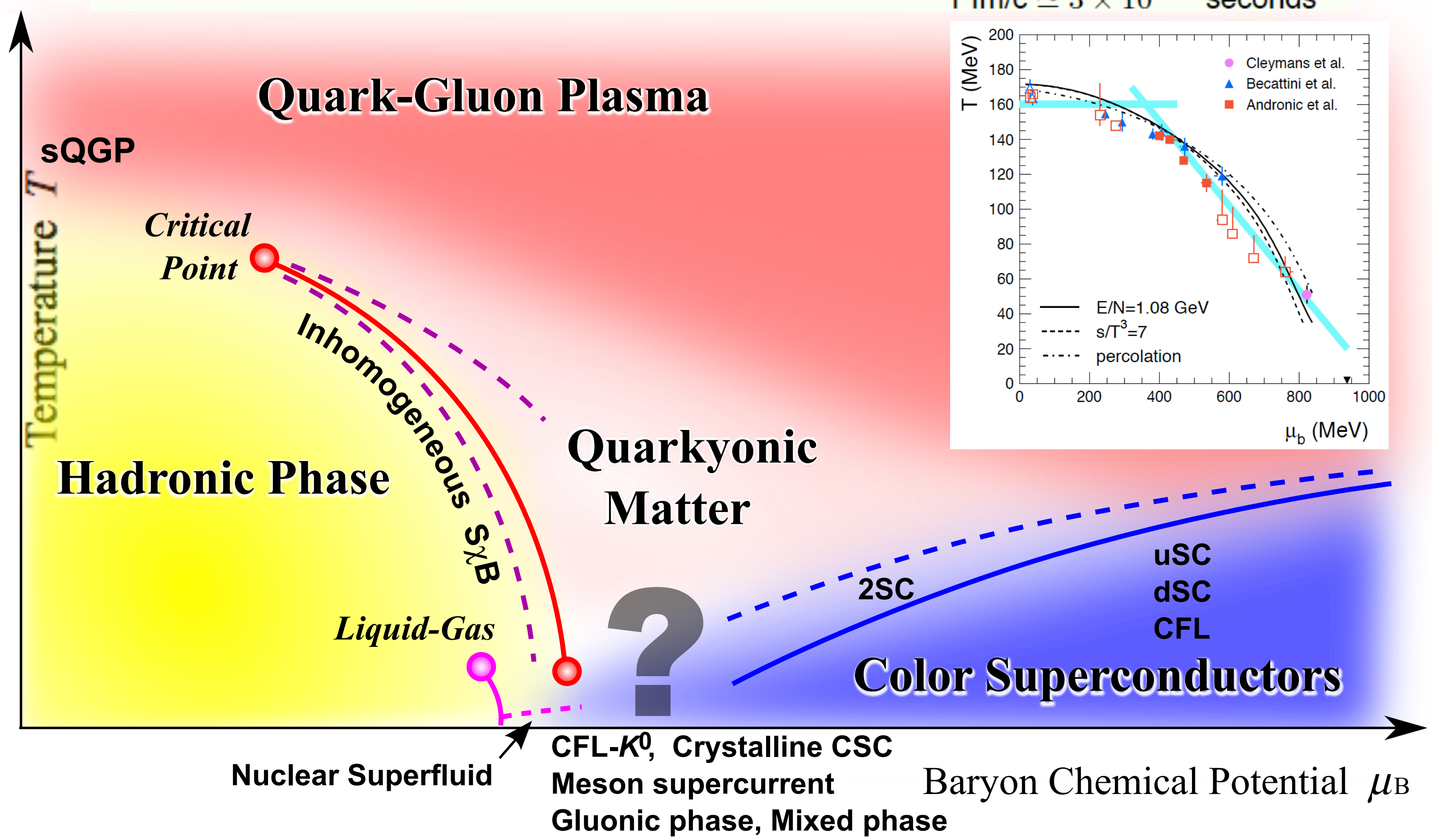


HADES



Heavy ion collisions





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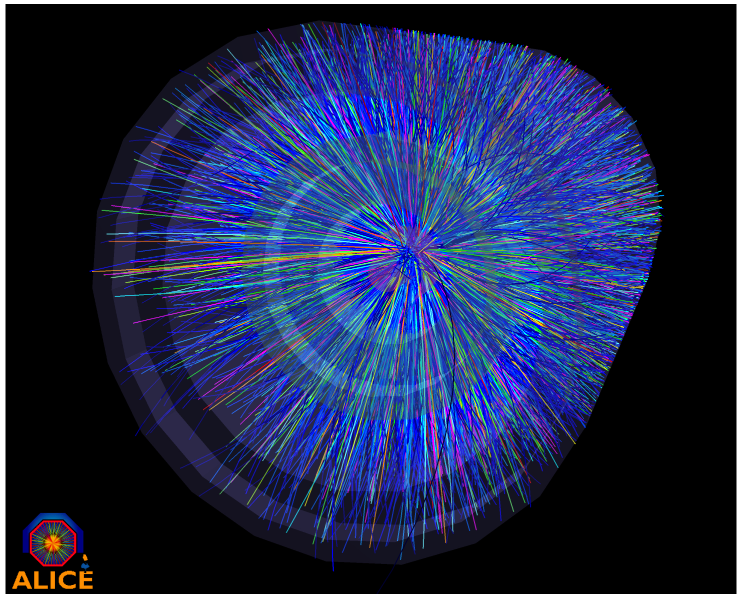
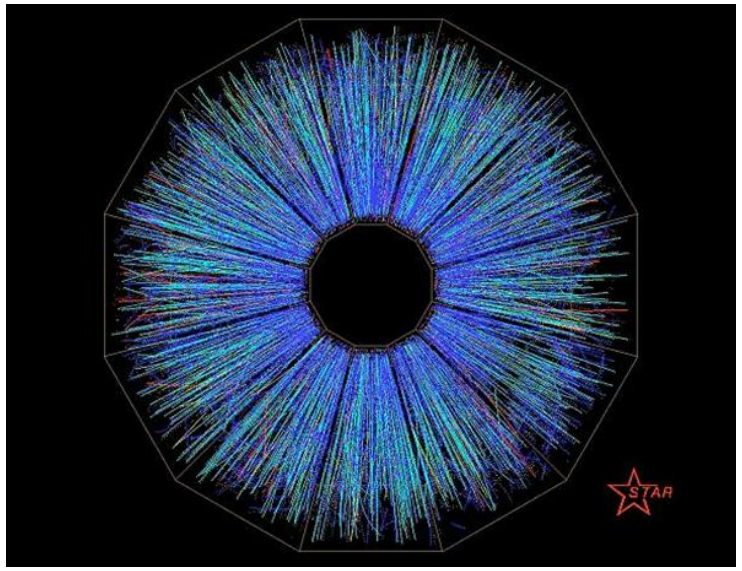
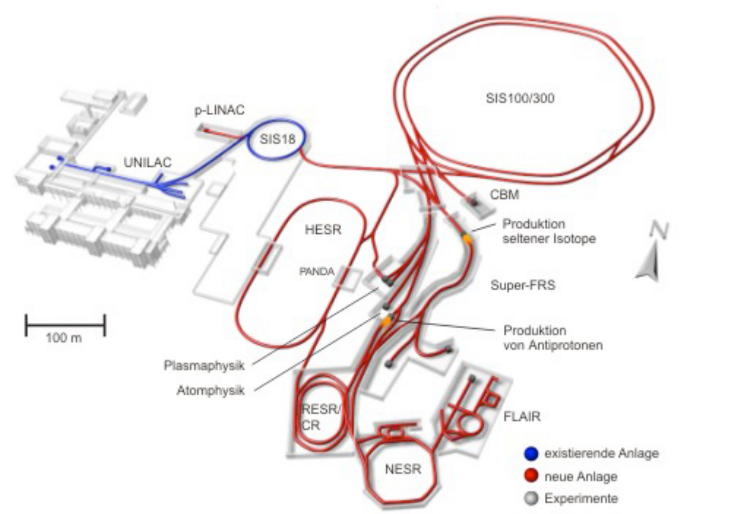


RHIC



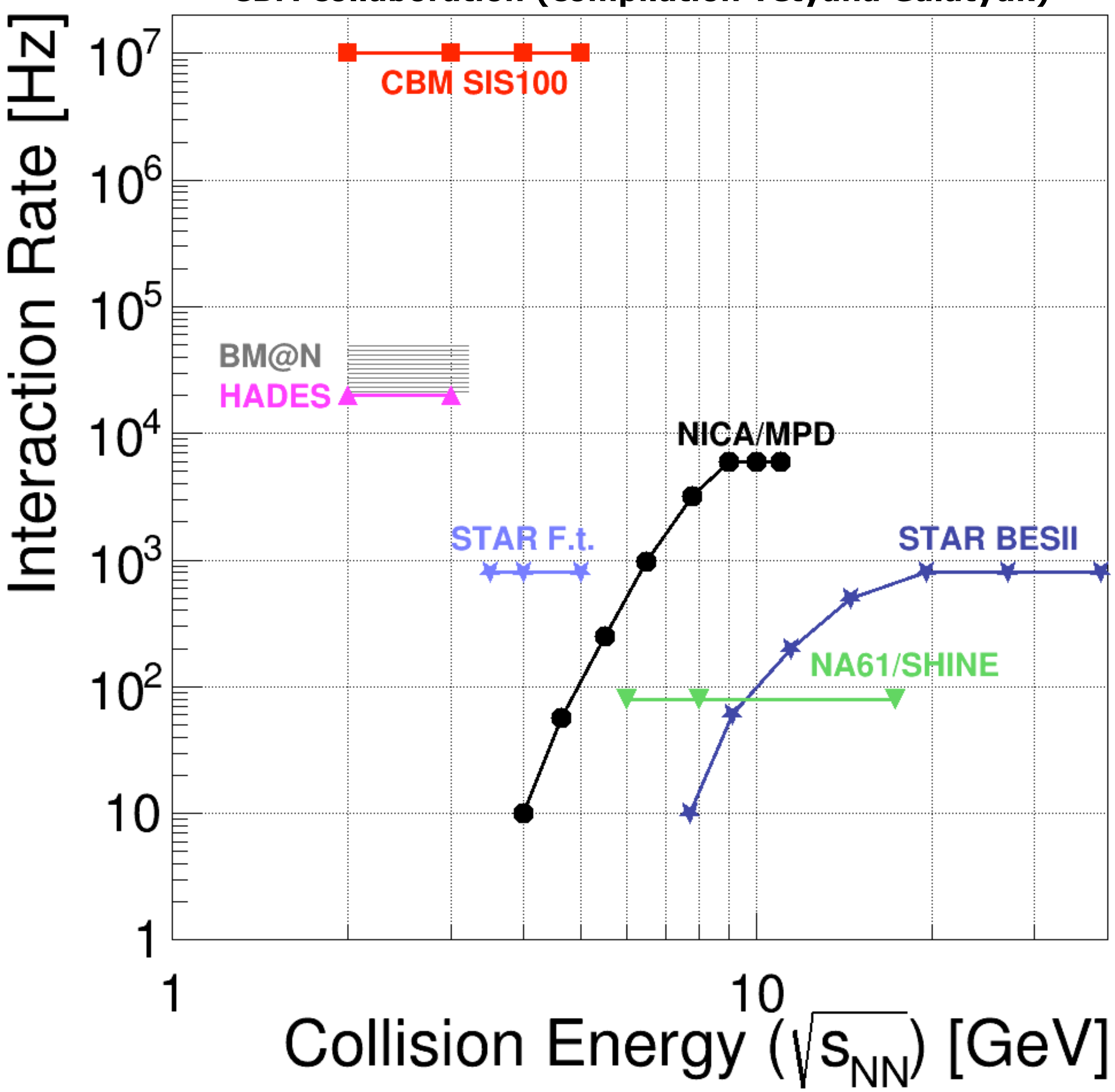
FAIR



NICA

Heavy ion collisions

CBM collaboration (compilation Tetyana Galatyuk)



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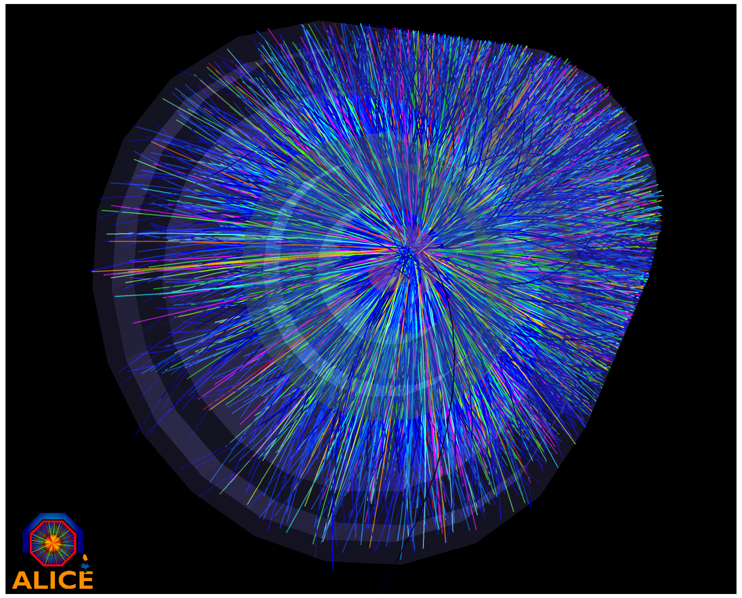

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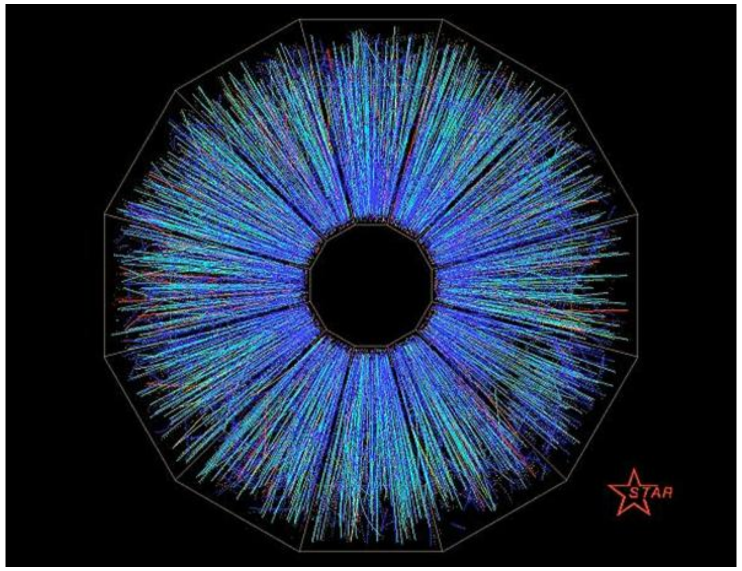

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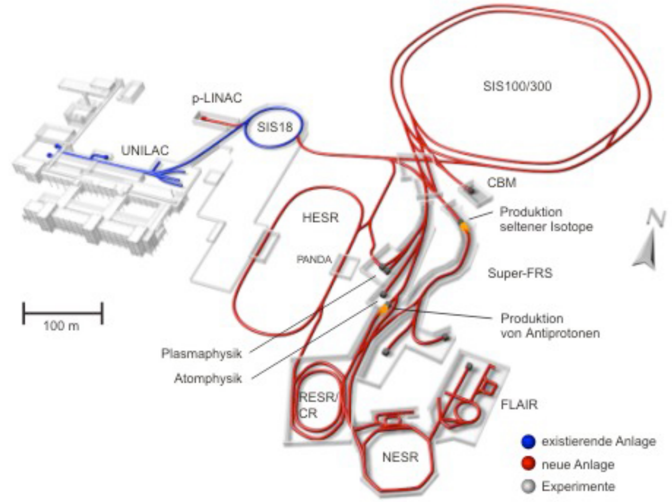
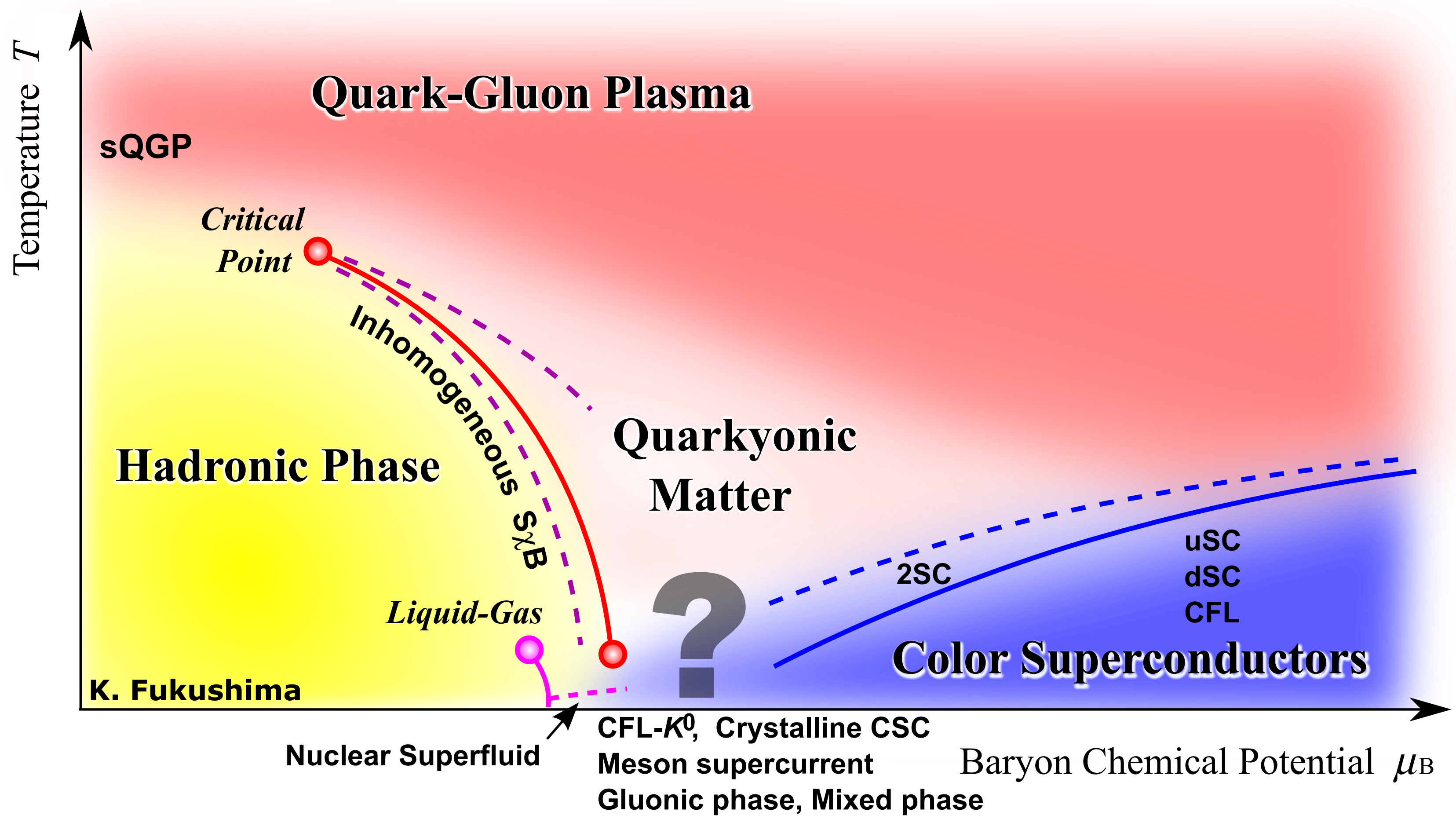


NICA

Phase diagram & order parameters

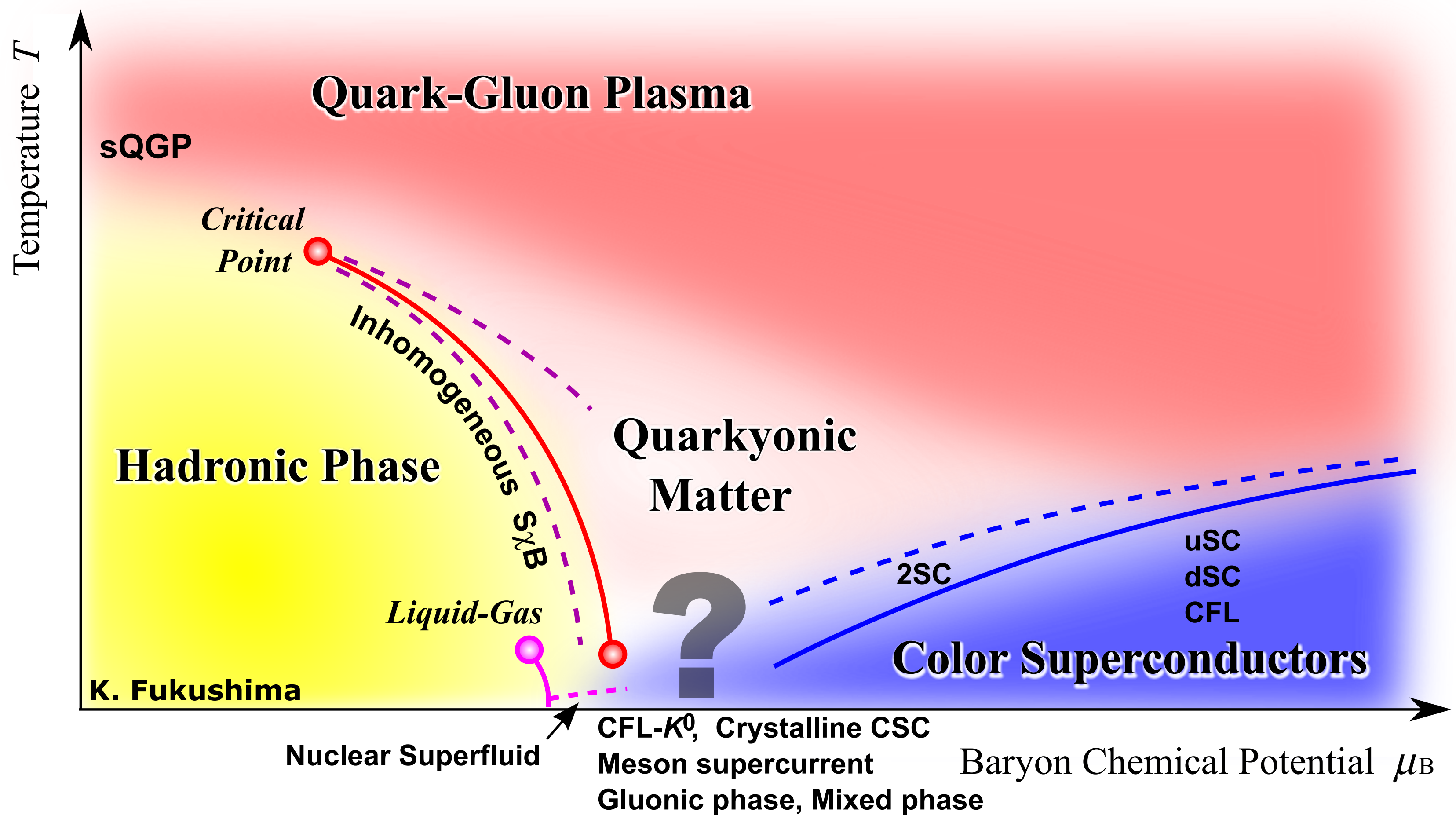


Phases in QCD

quarks massless - massive

chiral condensate $\int_{\vec{\mathbf{x}}} < \bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{x}) >$

Phase diagram & order parameters



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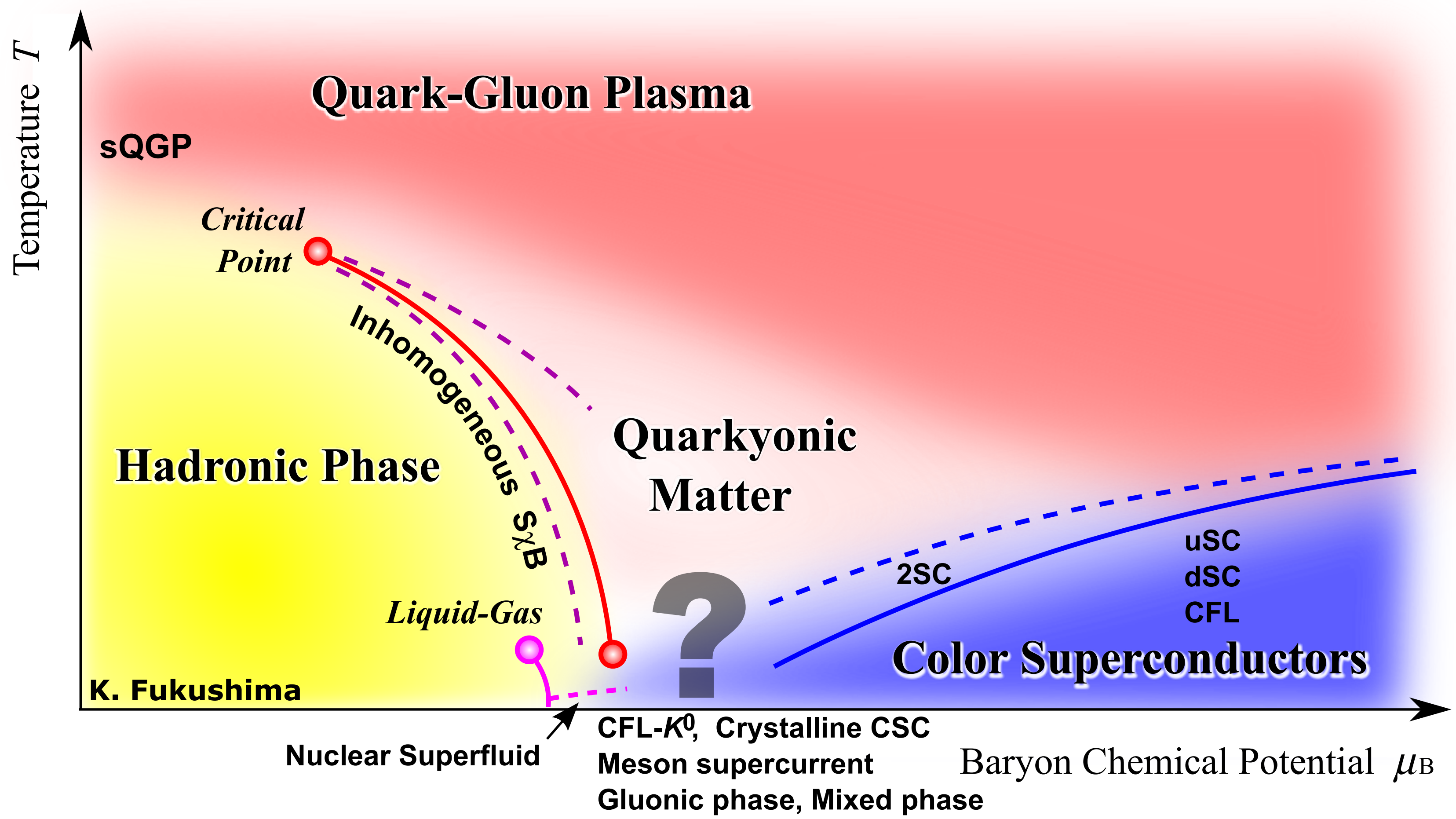
quarks confined - deconfined

Polyakov loop $\mathbf{\Phi} \sim \mathbf{e}^{-\frac{1}{2}\mathbf{F}_{\bar{\mathbf{q}}\mathbf{q}}}$

free energy $\mathbf{F}_{\bar{\mathbf{q}}\mathbf{q}} = \lim_{|\vec{\mathbf{x}}-\vec{\mathbf{y}}|\to\infty} \mathbf{F}_{\bar{\mathbf{q}}(\mathbf{x})\mathbf{q}(\mathbf{y})}$

Phase diagram & order parameters



Phases in QCD

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Polyakov loop $\mathbf{\Phi} = \frac{\mathbf{1}}{\mathbf{N_c}} \langle \mathbf{tr}\, \mathcal{P} \mathbf{e}^{\mathbf{i\,g} \int_0^\beta \mathbf{A_0(x)}} \rangle$

free energy $\mathbf{F_{\bar{q}q}} = \lim_{|\vec{\mathbf{x}}-\vec{\mathbf{y}}|\to\infty} \mathbf{F_{\bar{q}(x)q(y)}}$

Outline

- Introduction

- Phase structure of QCD
  - Confinement & chiral symmetry breaking
  - Finite temperature correlation functions
  - QCD at finite density & fluctuations

- QCD transport
  - Real time correlation functions
  - Single particle spectral functions
  - transport coefficients

- Summary & outlook

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Functional RG for QCD

fQCD collaboration: **J. Braun, L. Corell, A. Cyrol, W.-j. Fu, M. Leonhardt, M. Mitter, JMP, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink**

Heidelberg, Dailan, Darmstadt

Agenda

QCD at finite T & mu

- *Phase structure*
- *Fluctuations*
- *Phenomenology*

Real time correlation functions

- *Hadron spectrum & decays*
- *Transport coefficients*
- *Dynamics*

Selection of papers

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016
Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.XXXXX
Rennecke, PRD 92 (2015) 076012

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035
Cyrol, Mitter, JMP, Strodthoff, in preparation

Fu, JMP, Schaefer, Rennecke, Phys.Rev. D94 (2016)

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Selection of papers

quenched QCD: Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

unquenched QCD: Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.XXXXX

vector mesons: Rennecke, PRD 92 (2015) 076012

pure glue: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

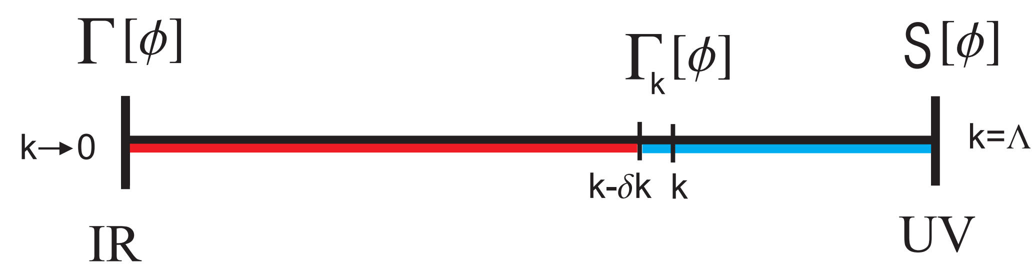
finite T: Cyrol, Mitter, JMP, Strodthoff, in preparation

Fluctuations: Fu, JMP, Schaefer, Rennecke, Phys.Rev. D94 (2016)

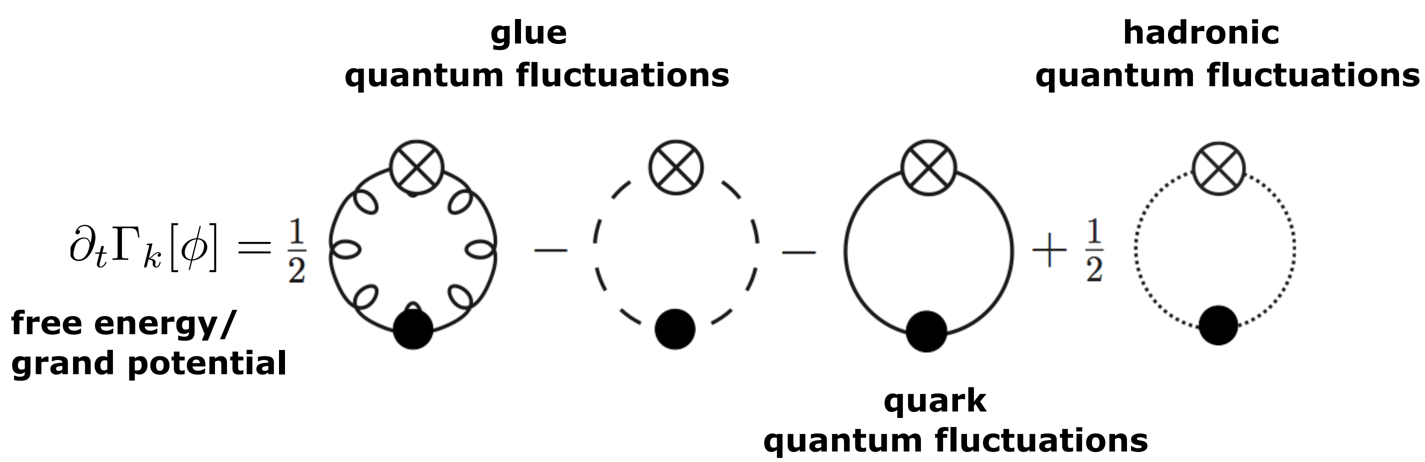
Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

free energy at momentum scale k



ab initio



RG-scale k: $t = \ln k$

closed form

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
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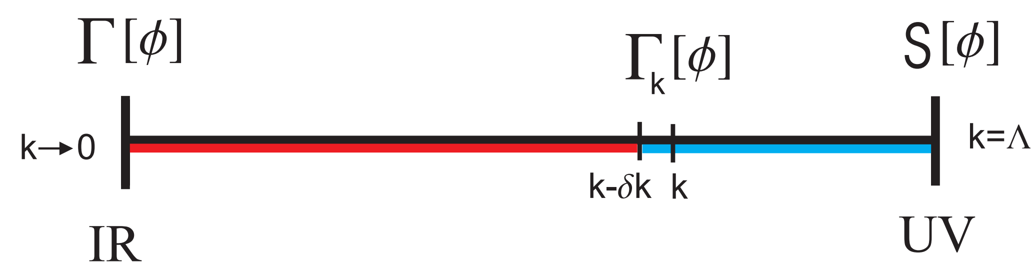
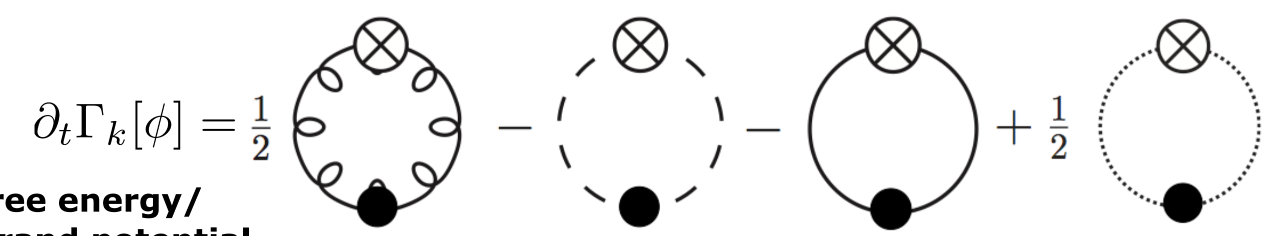
free energy at momentum scale k



ab initio

glue
quantum fluctuations

hadronic
quantum fluctuations



free energy/
grand potential

quark
quantum fluctuations

$\boxed{\text{RG-scale k: } t = \ln k}$

properties

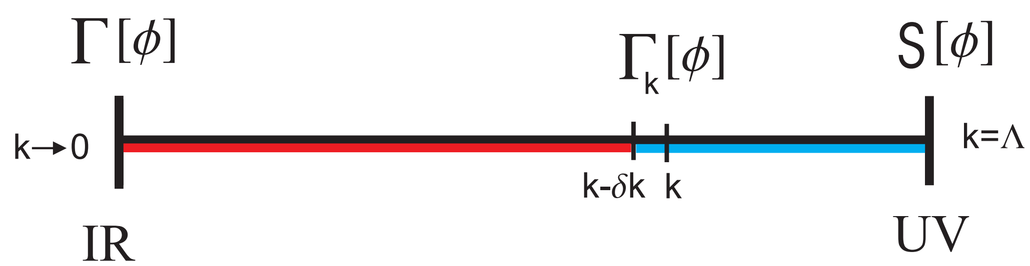
closed form

- access to physics mechanisms
- numerically tractable, also at real time
  no sign problem
  systematic error control via closed form
- low energy models naturally encorporated

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

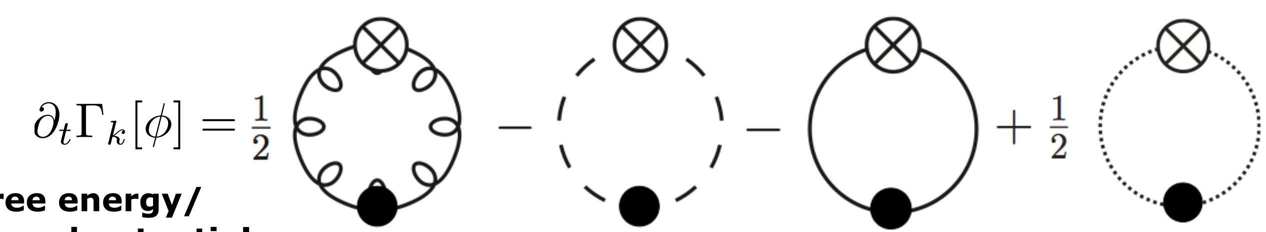
free energy at momentum scale k



ab initio

glue quantum fluctuations

hadronic quantum fluctuations



free energy/ grand potential

quark quantum fluctuations

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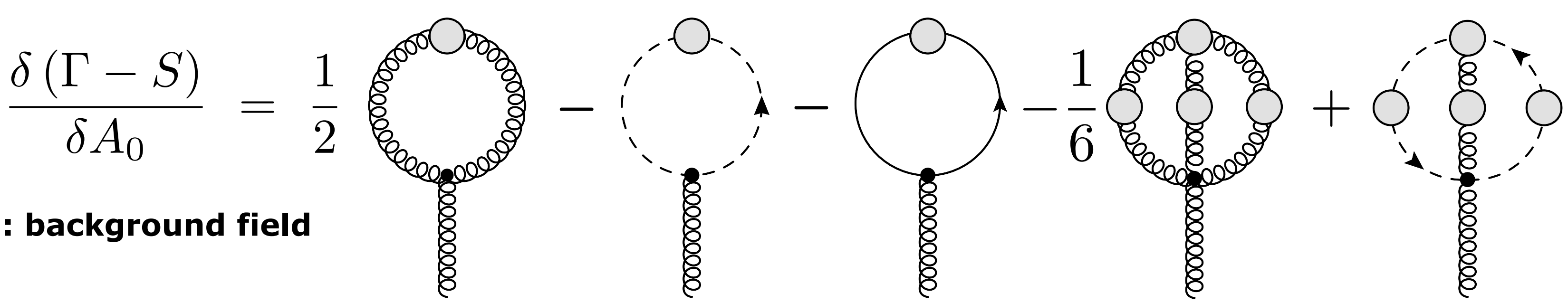

functional DSE :

A_0 **: background field**

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
Denz, Held, Rodigast; unpub.

DoFun
Mathematica package for the derivation of functional equations
Braun, Huber; Comput.Phys. Commun. 183 (2012) 1290-1320

Action

DoFun
VertEXpand

DoFun
ERGE

Vertices/
Feynman Rules

Symbolic
Flow Equations

FormTracer
High-performance, general purpose, easy-to-use Mathematica tracing tool using FORM
Cyrol, Mitter, Strodthoff; arXiv: 1610.09331

Algebraic
Flow Equations

CreateKernels
Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

Compilable Kernels

frgsolver
Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
Cyrol, Mitter, Pawlowski, Strodthoff; unpub.

Numerical solution

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Bundesministerium für Bildung und Forschung
FWF Der Wissenschaftsfonds.
erc
European Research Council
Established by the European Commission

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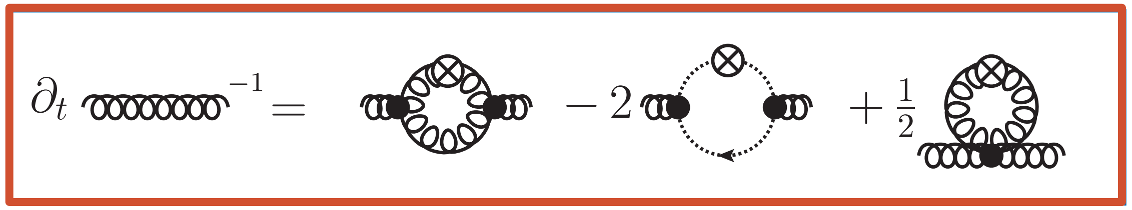
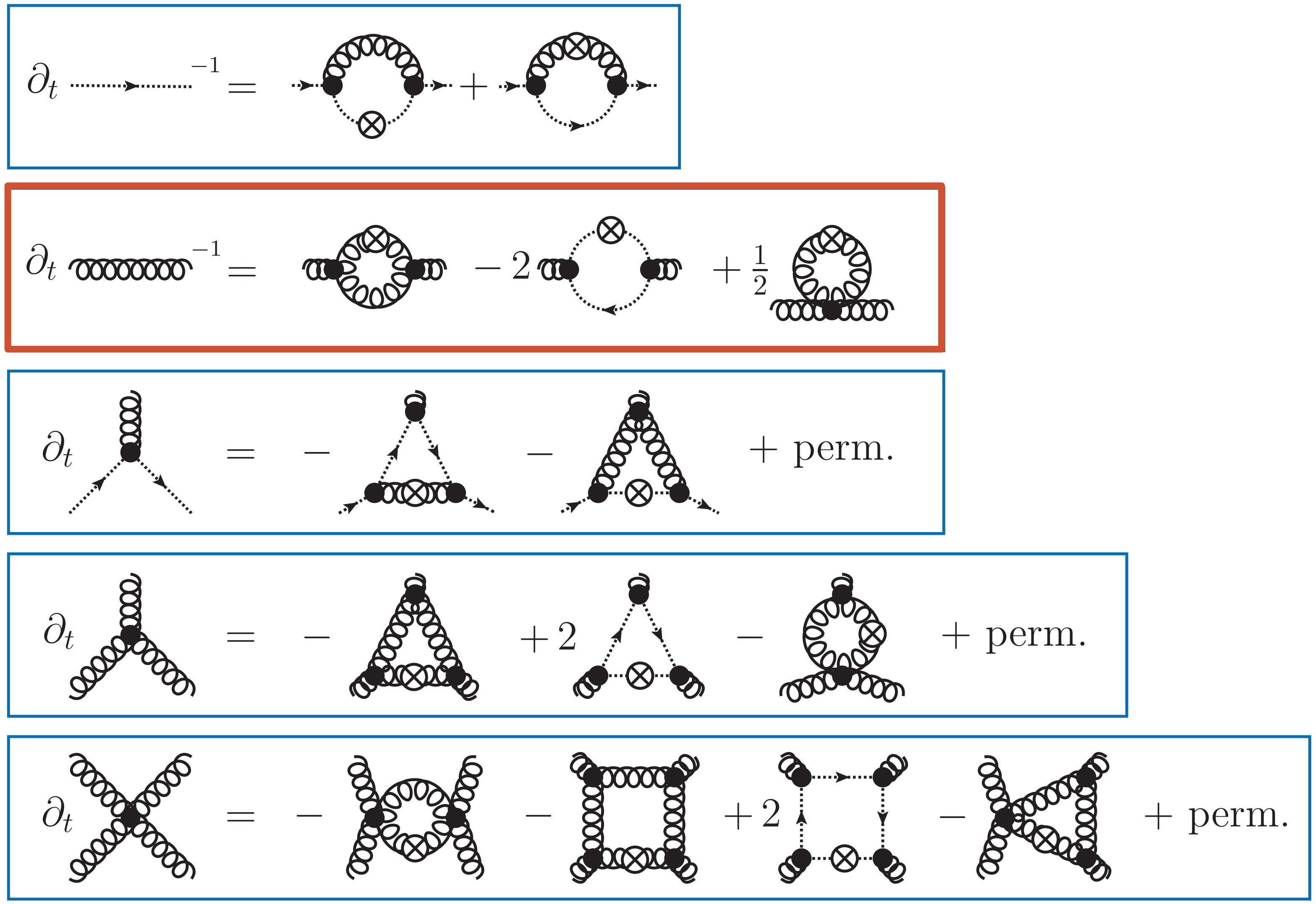
YM-theory: gluonic correlation functions

$$\langle A\, A\rangle(p^2)$$



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions



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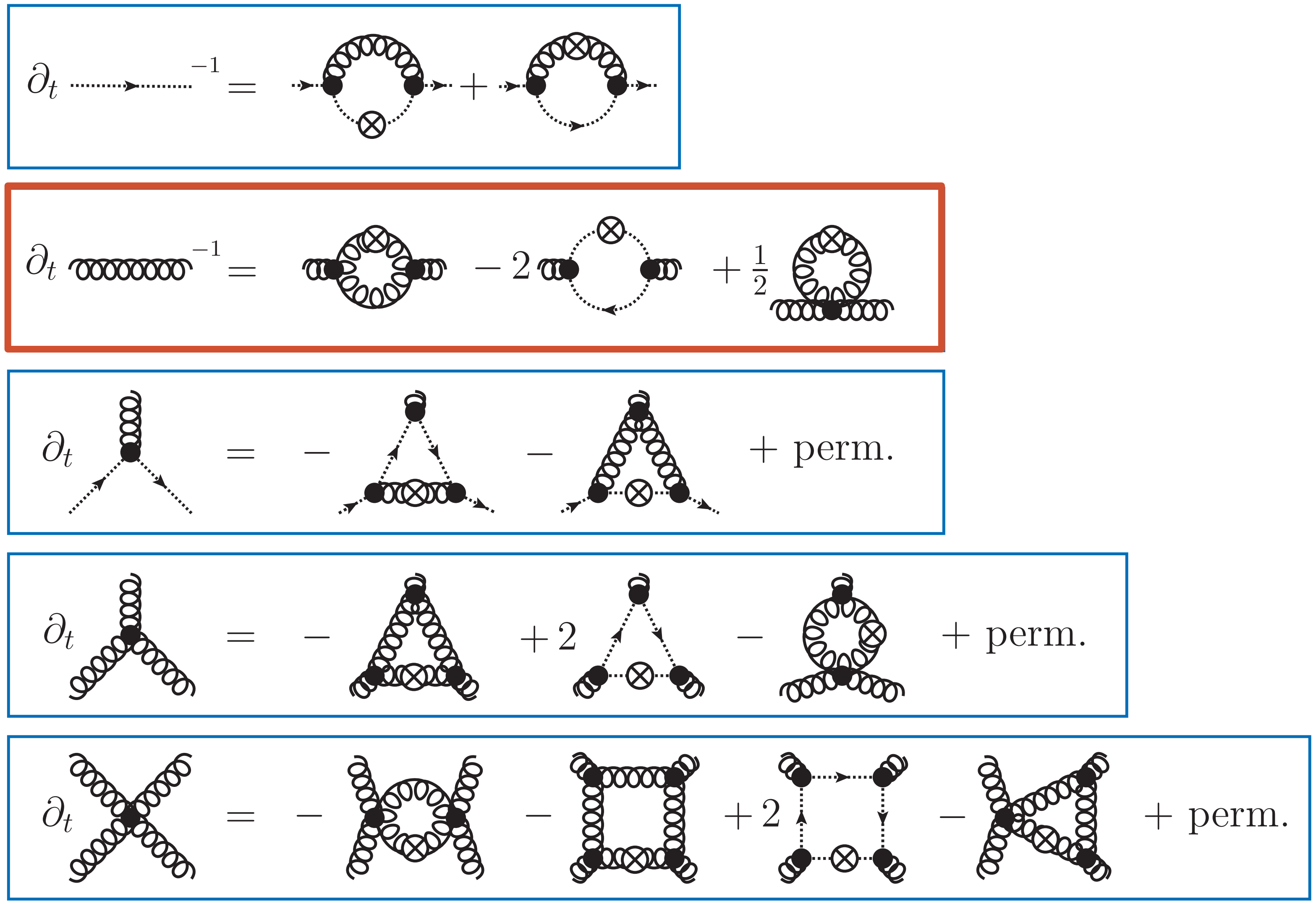
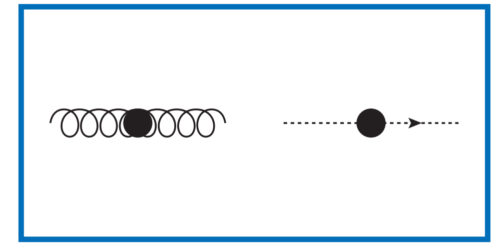
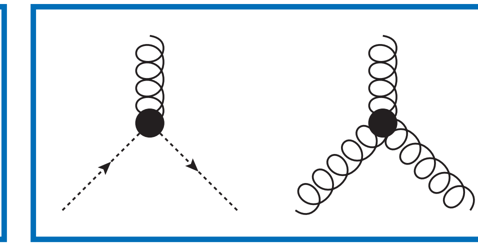
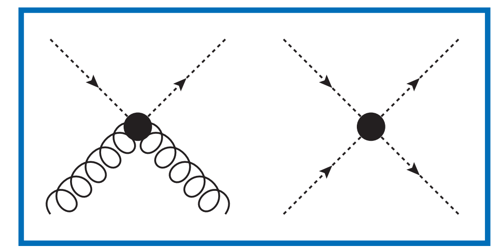




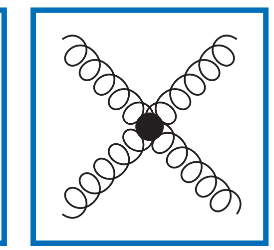
full. mom. dep.



full. mom. dep.
classical tensor structures



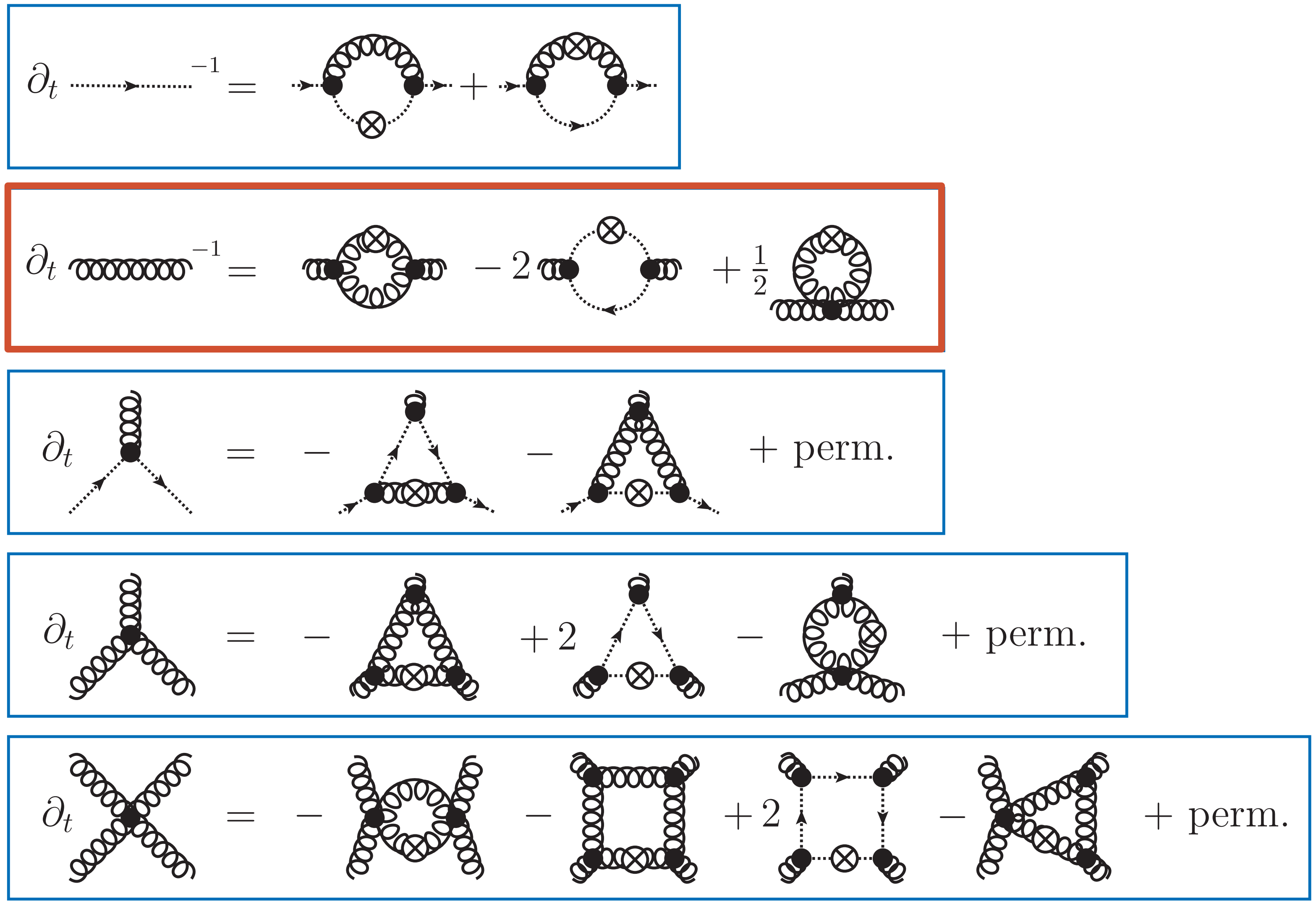
mom. dep. needed by tadpoles
full tensor basis

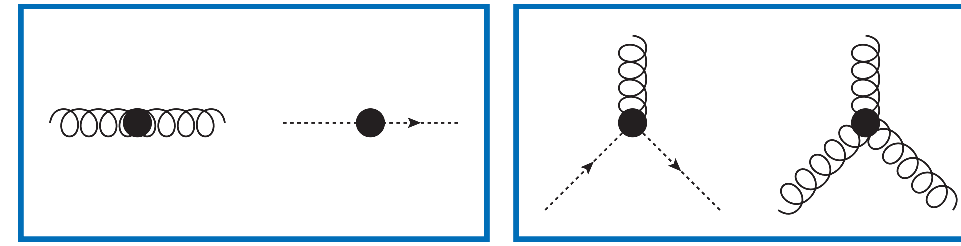


sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions





full. mom. dep.

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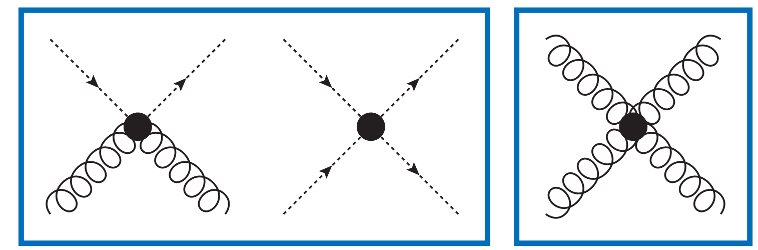


mom. dep. needed by tadpoles
full tensor basis

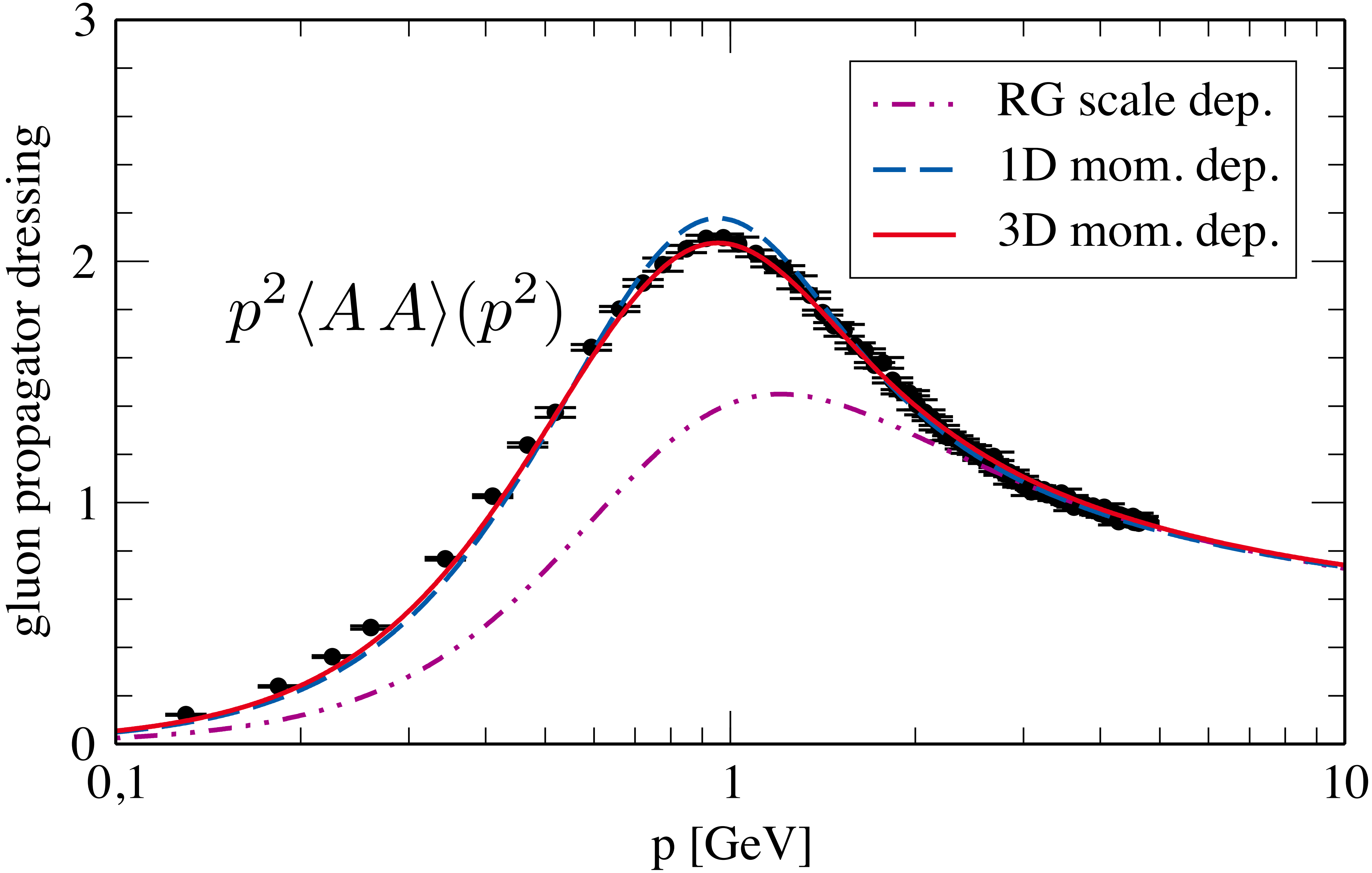
sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: Euclidean gluon propagator

Functional Renormalisation Group



Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

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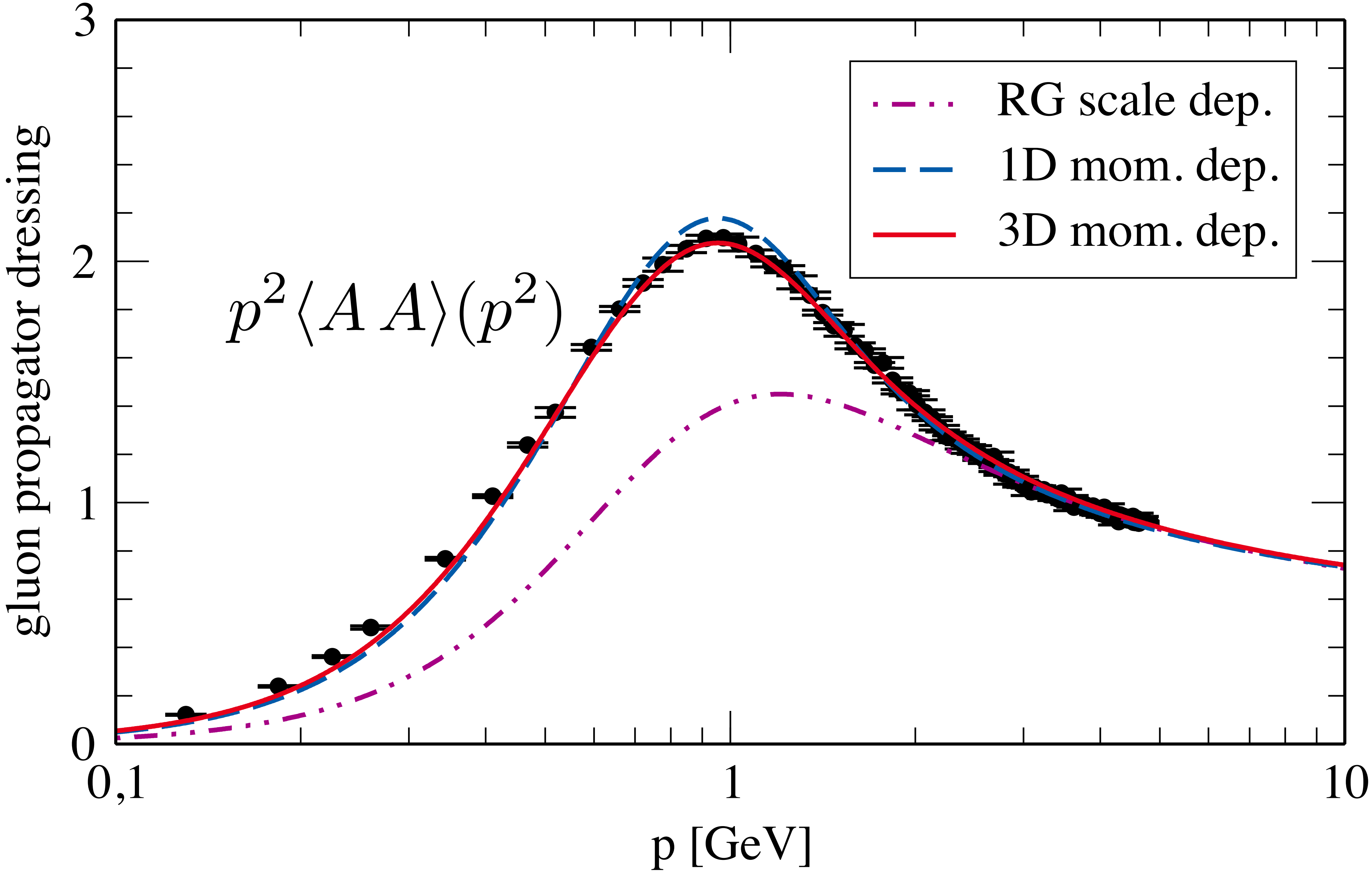

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Aiming at apparent convergence

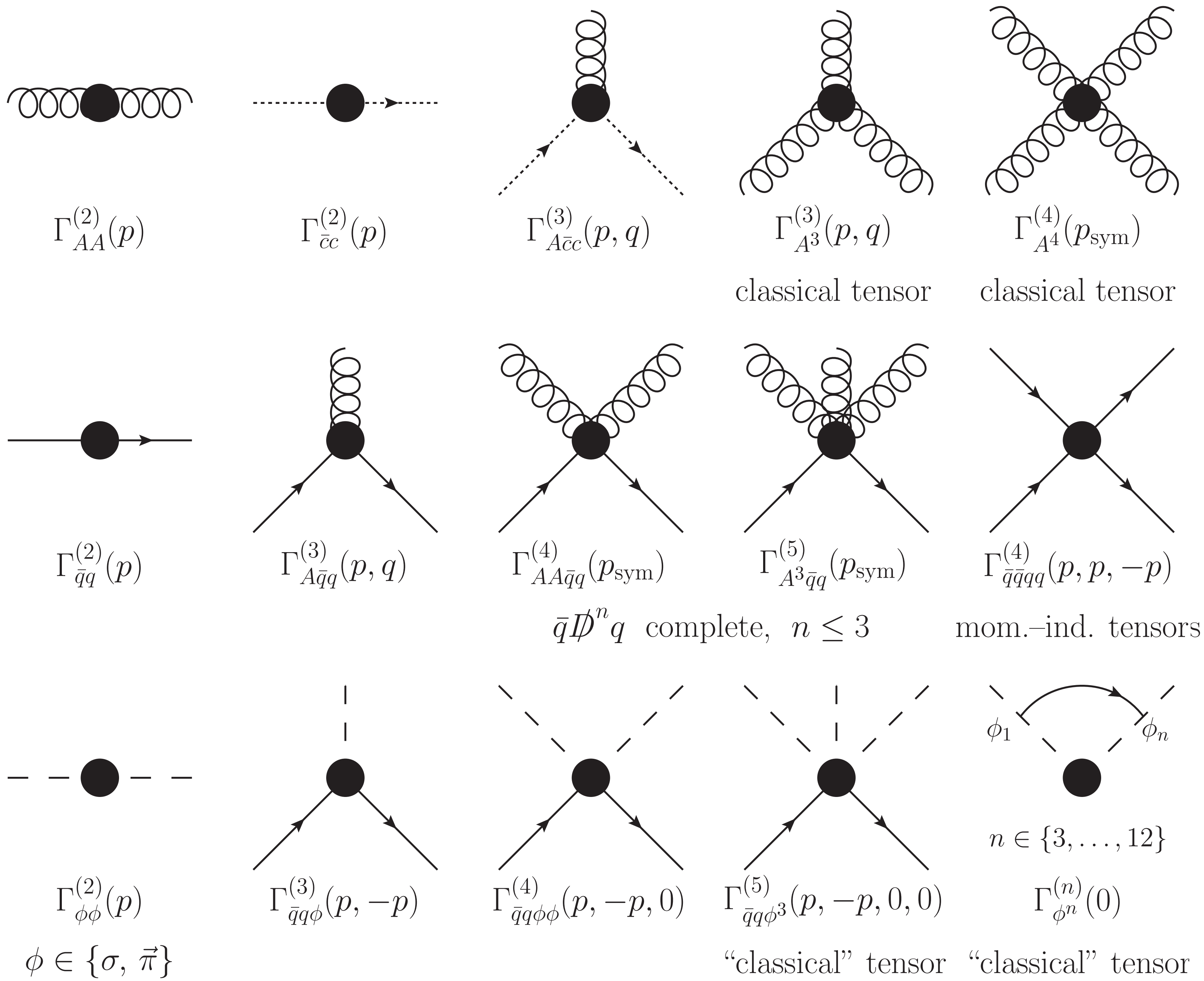
up to date pinch technique:
Aguilar, Binosi, Papavassiliou, PRD 89 (2014) 085032

up to date DSE:
Cyrol, Huber, Smekal, EPJ C75 (2015) 102

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Aiming at apparent convergence

QCD: current set of correlation functions



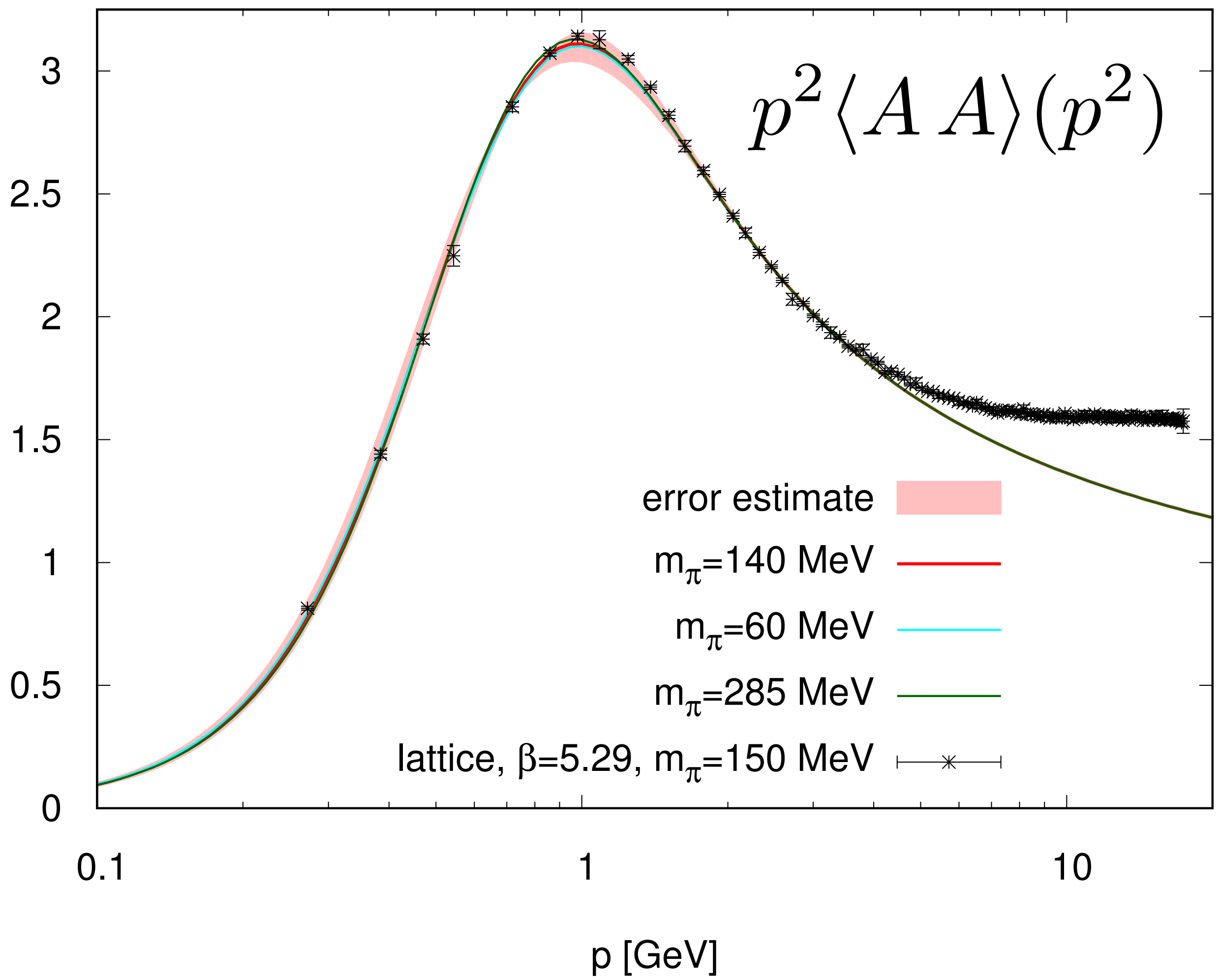
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

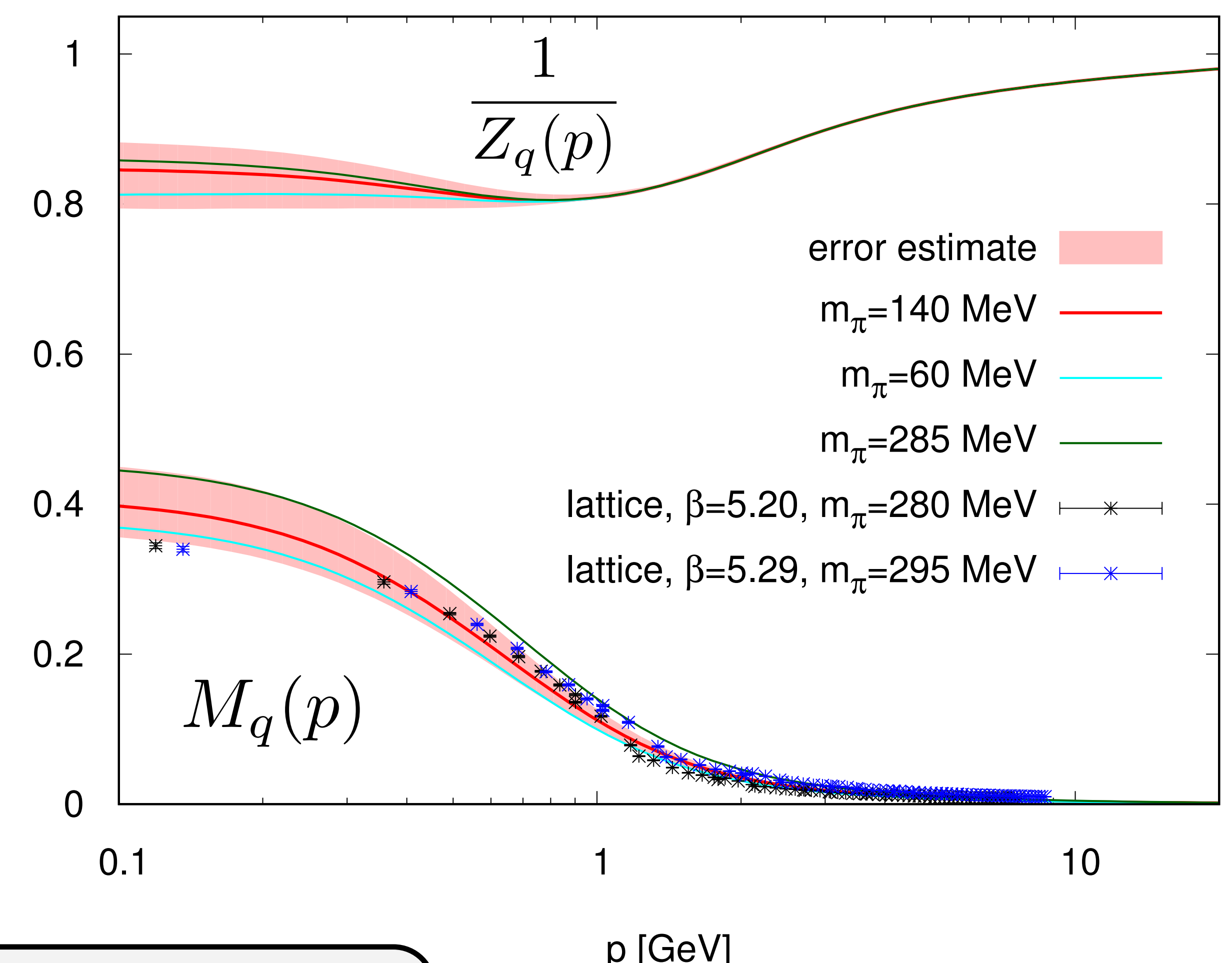
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

QCD: Euclidean propagators



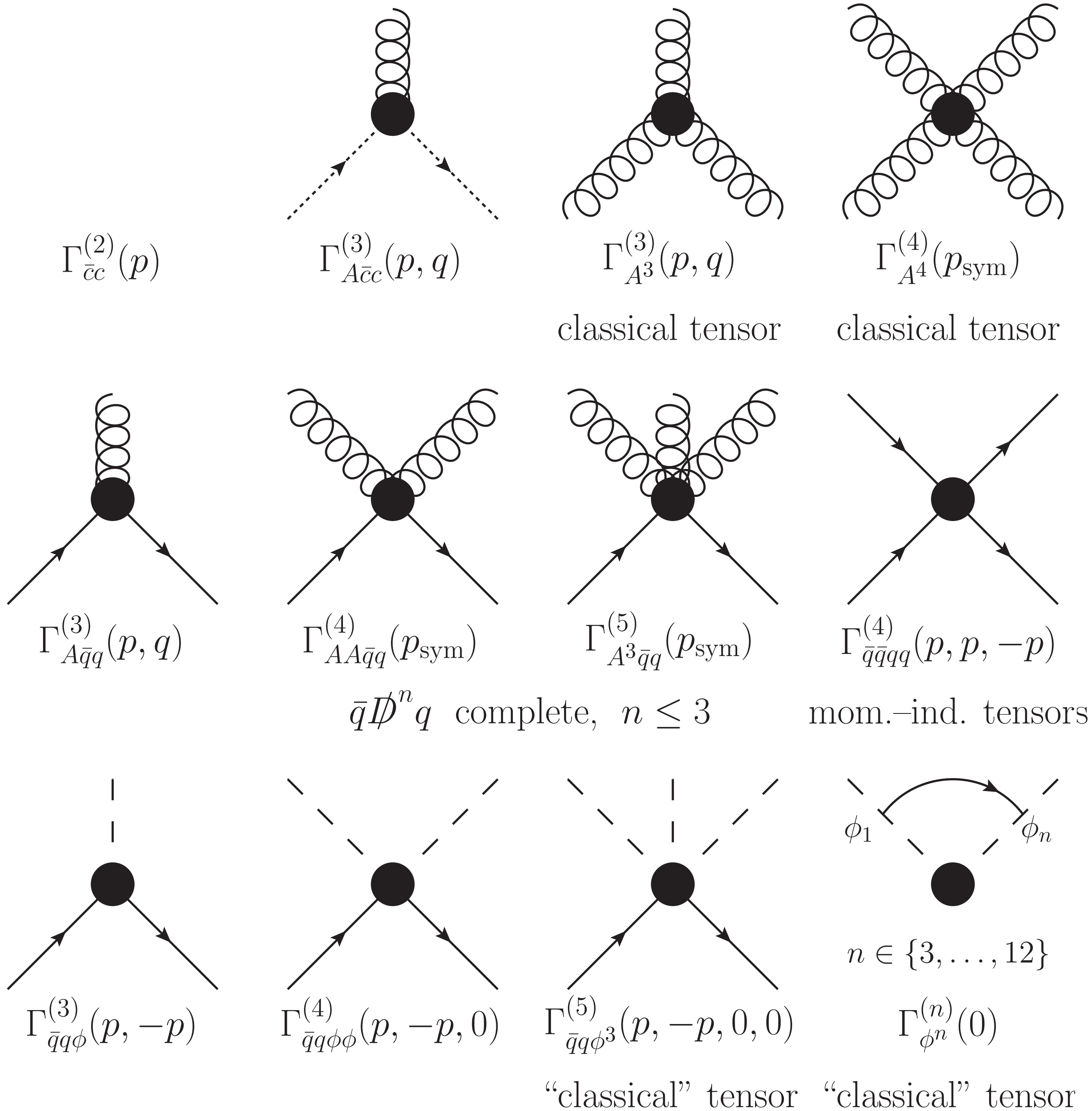
$$\frac{1}{Z_q(p)}\,\frac{1}{i\,\not{p} + M_q(p)}$$



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

QCD: Vertices



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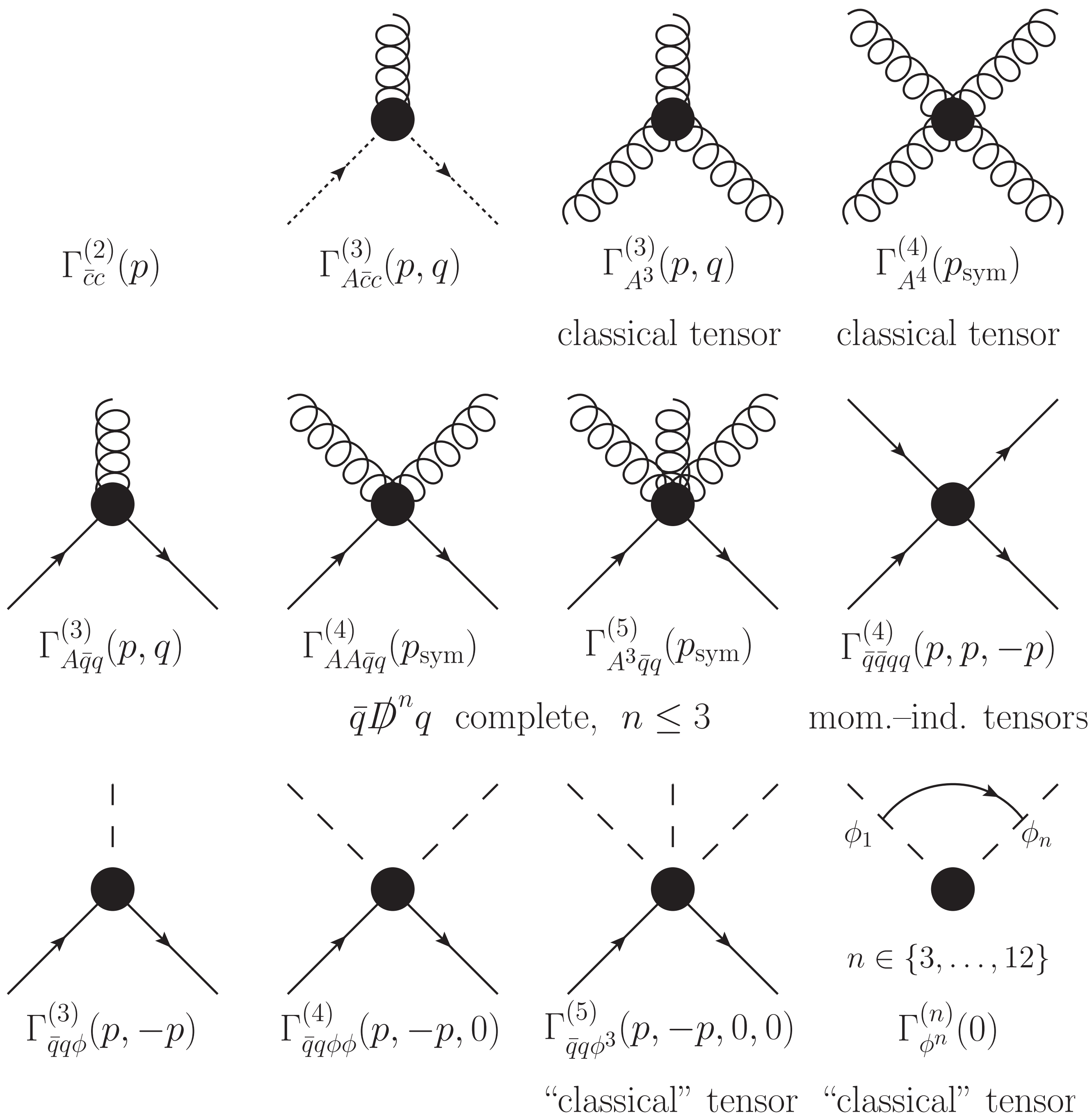
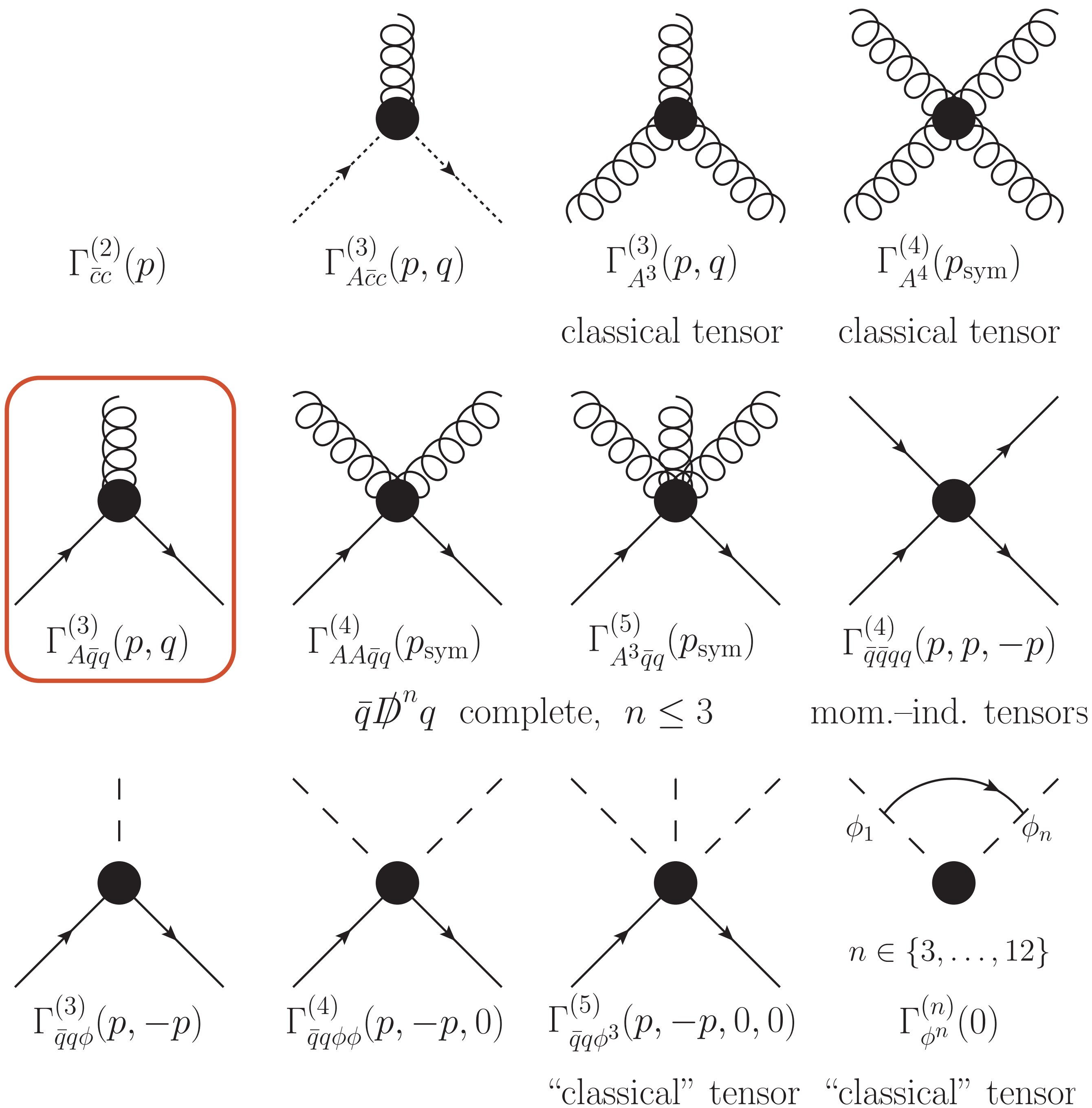


Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

QCD: Vertices



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Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q:$ $\quad \left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$

$\bar{q}\not{D}^2 q:$ $\quad \left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, 1_{4\times 4}$

$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}-\not{q})\gamma_\mu$

$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}+\not{q})\gamma_\mu$

$\bar{q}\not{D}^3 q:$ $\quad \left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$

$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$

$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$

Aiming at apparent convergence

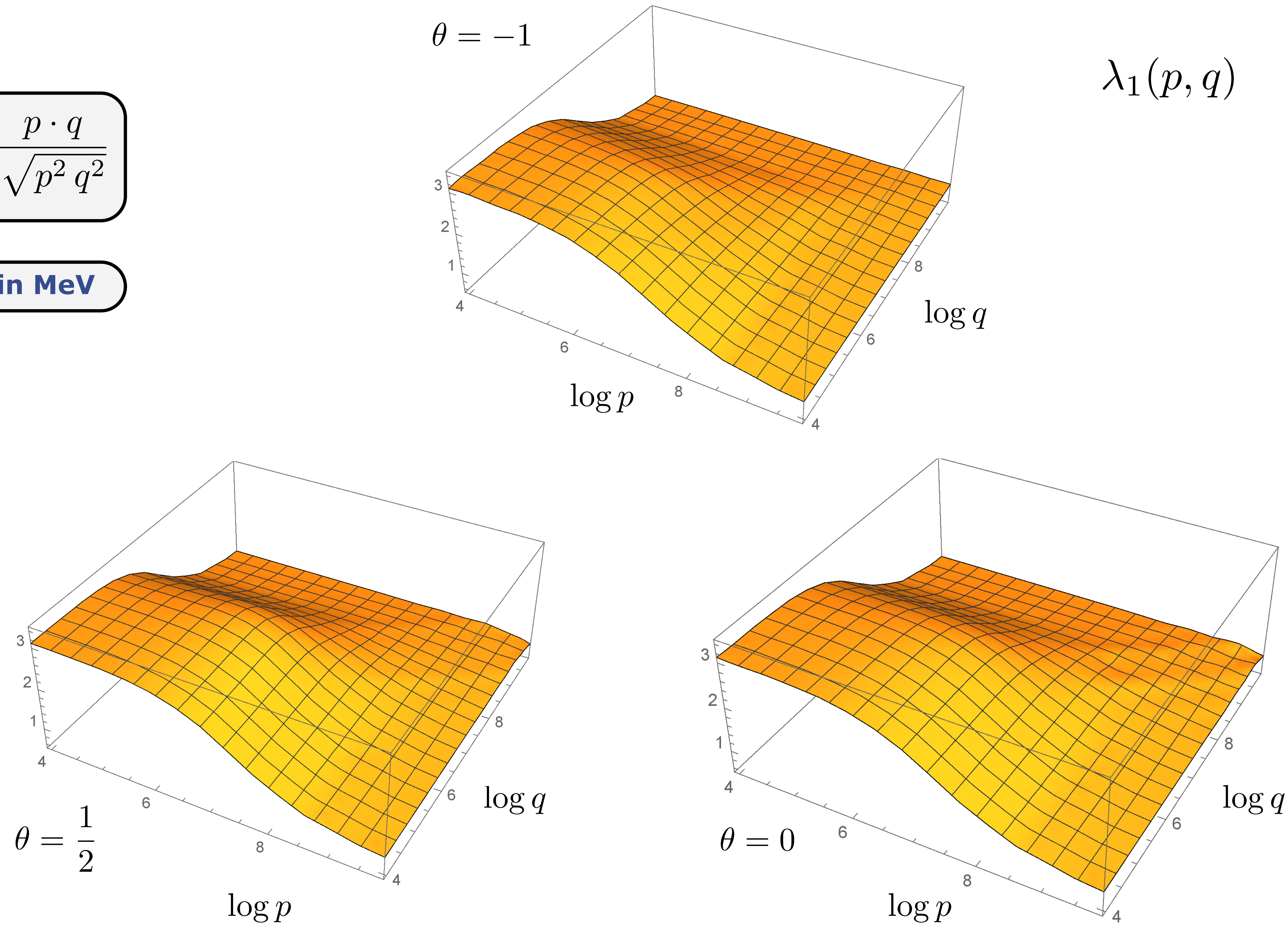
quenched: **Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



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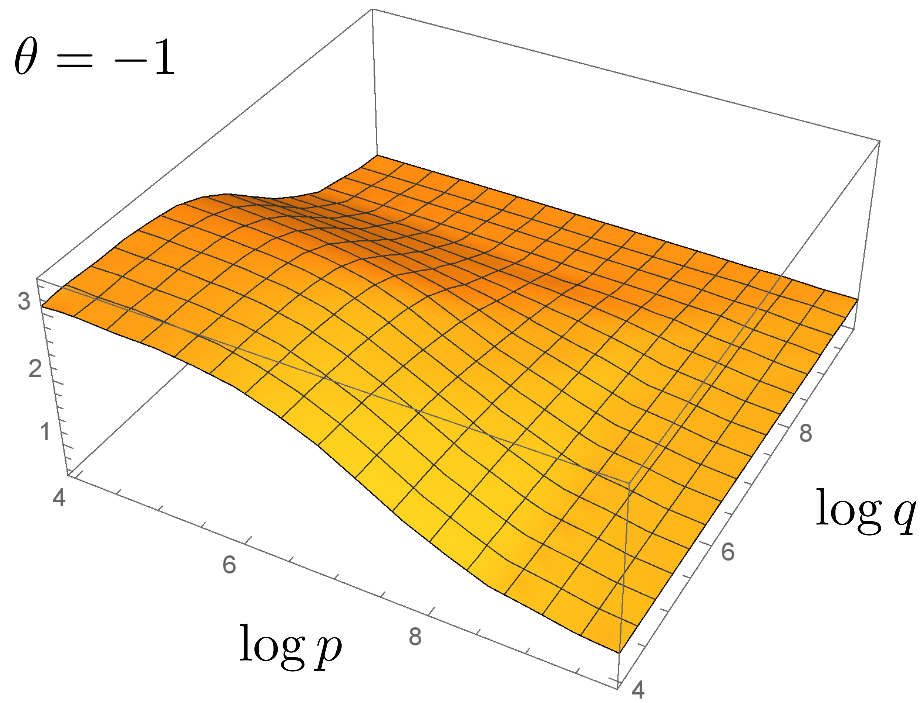
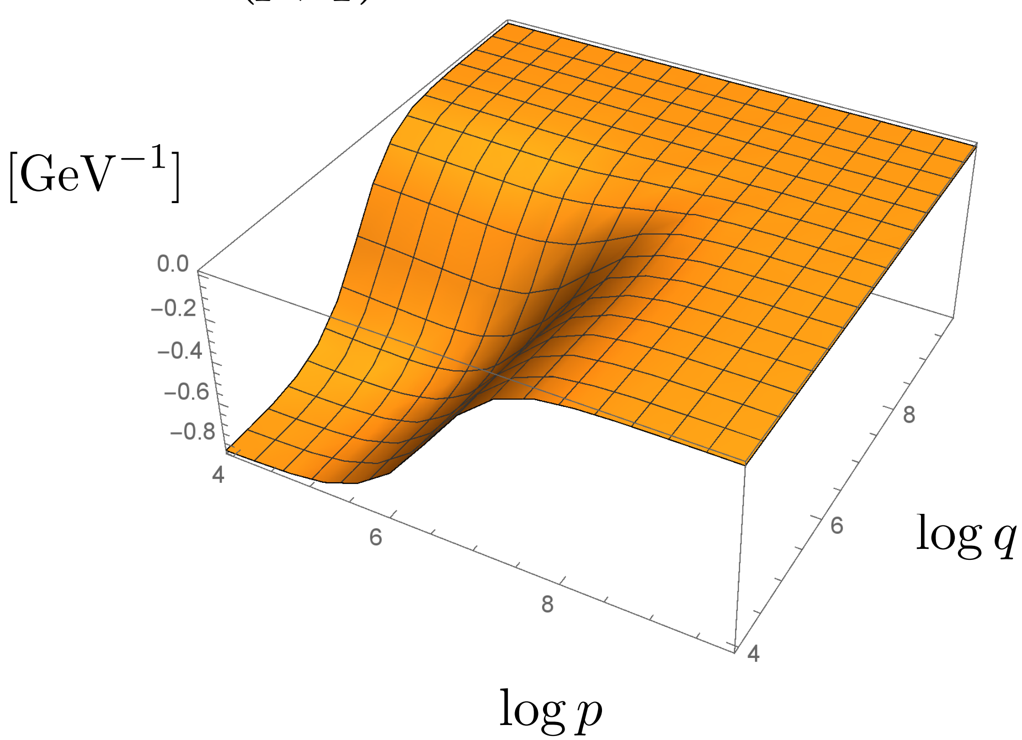
p,q in MeV

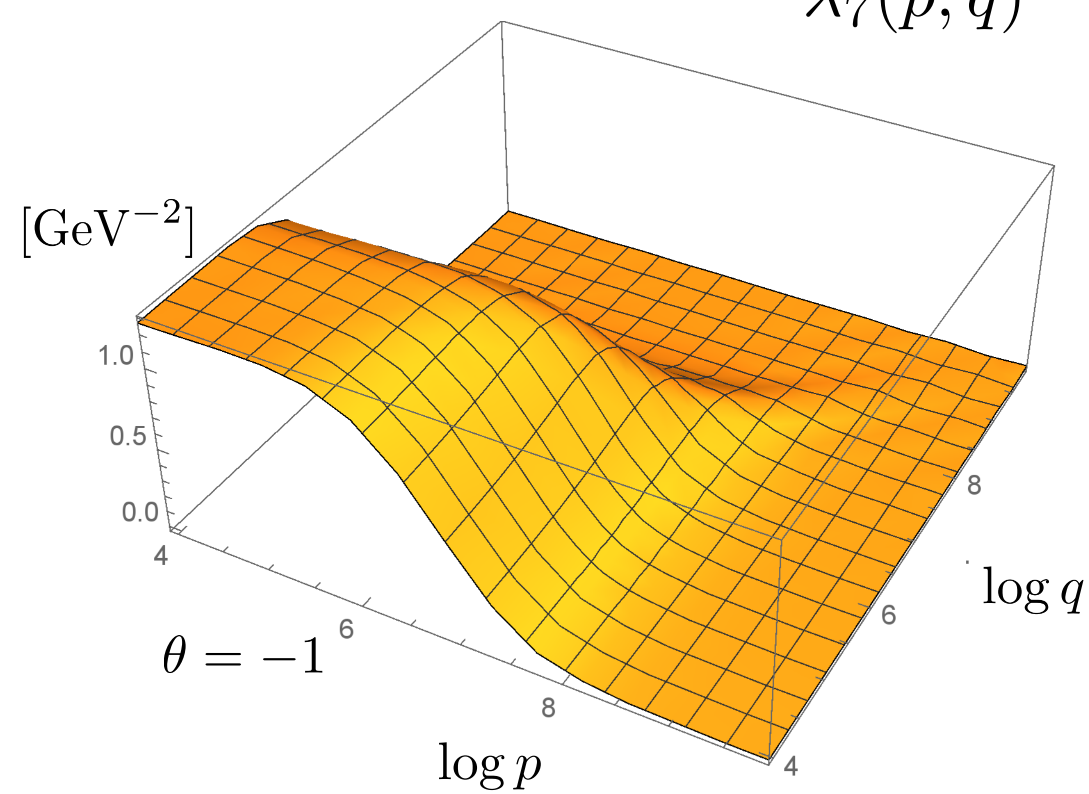
$\lambda_1(p,q)$



$\lambda_4(p,q)$



$\lambda_7(p,q)$



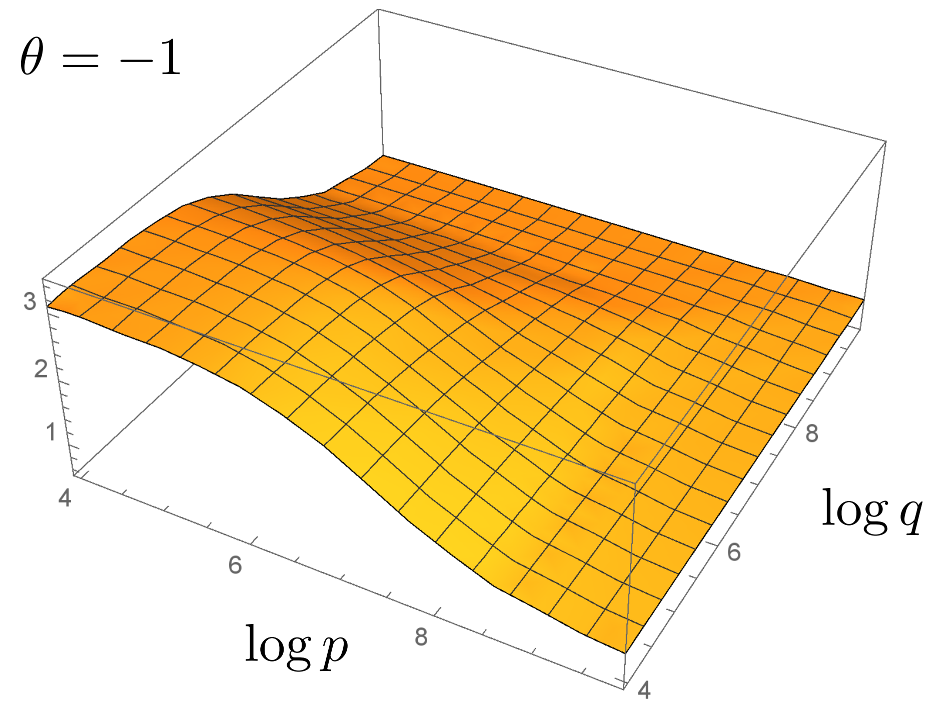
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



$\lambda_1(p, q)$

up-to-date 1st principles works:

FunMethods:

Williams, EPJ A51 (2015) 57
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026

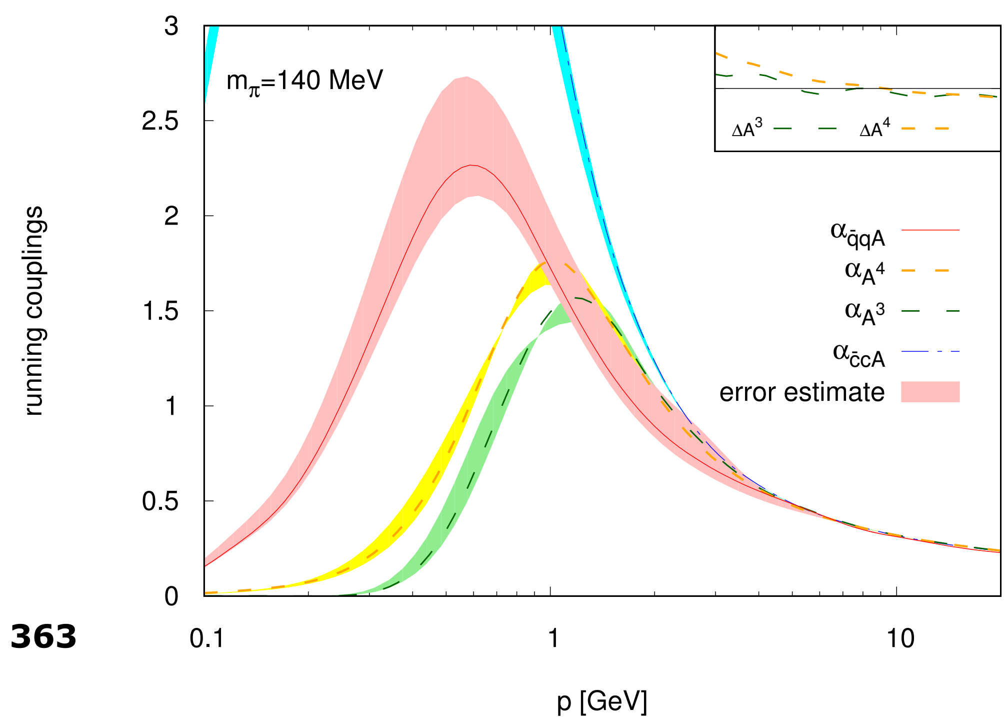
Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, arXiv:1610.06158

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**



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Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

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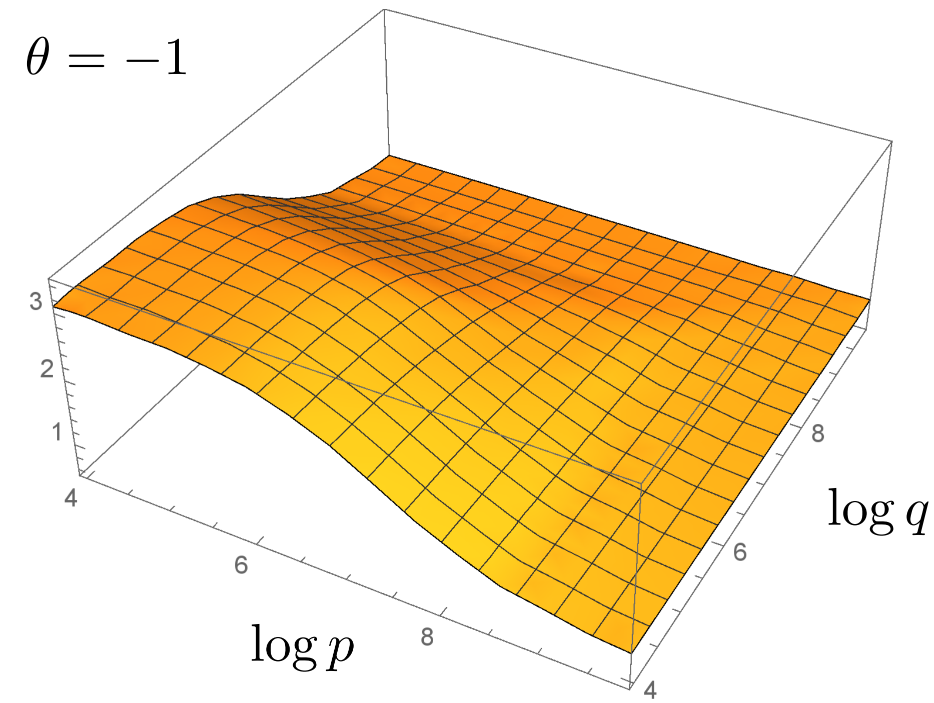

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Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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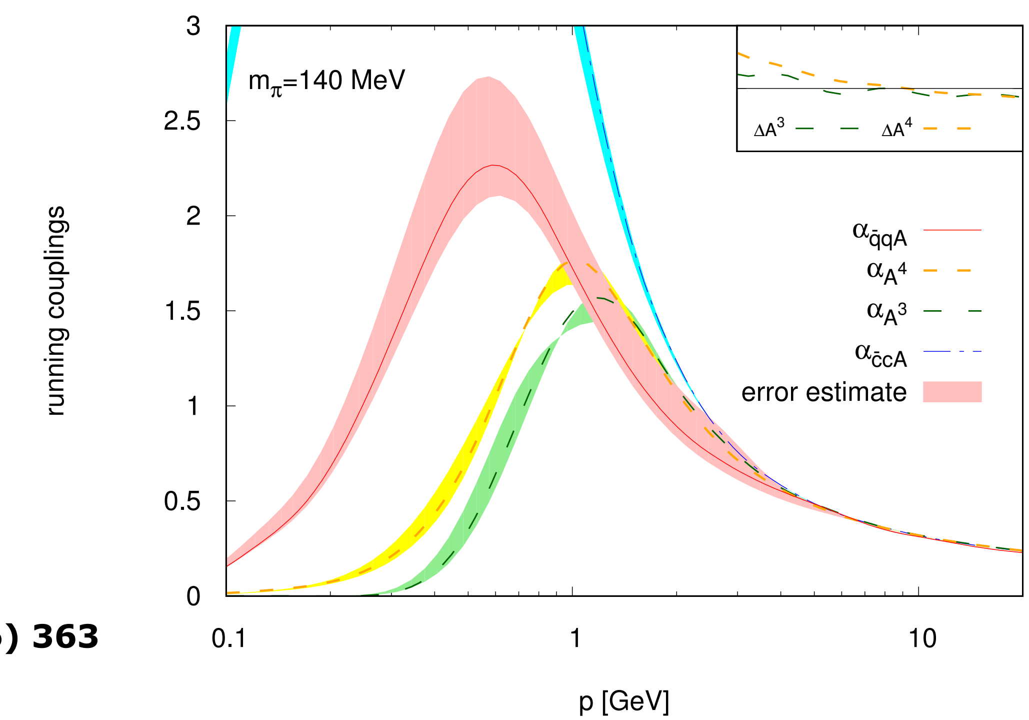
Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**

Beware of BRST



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

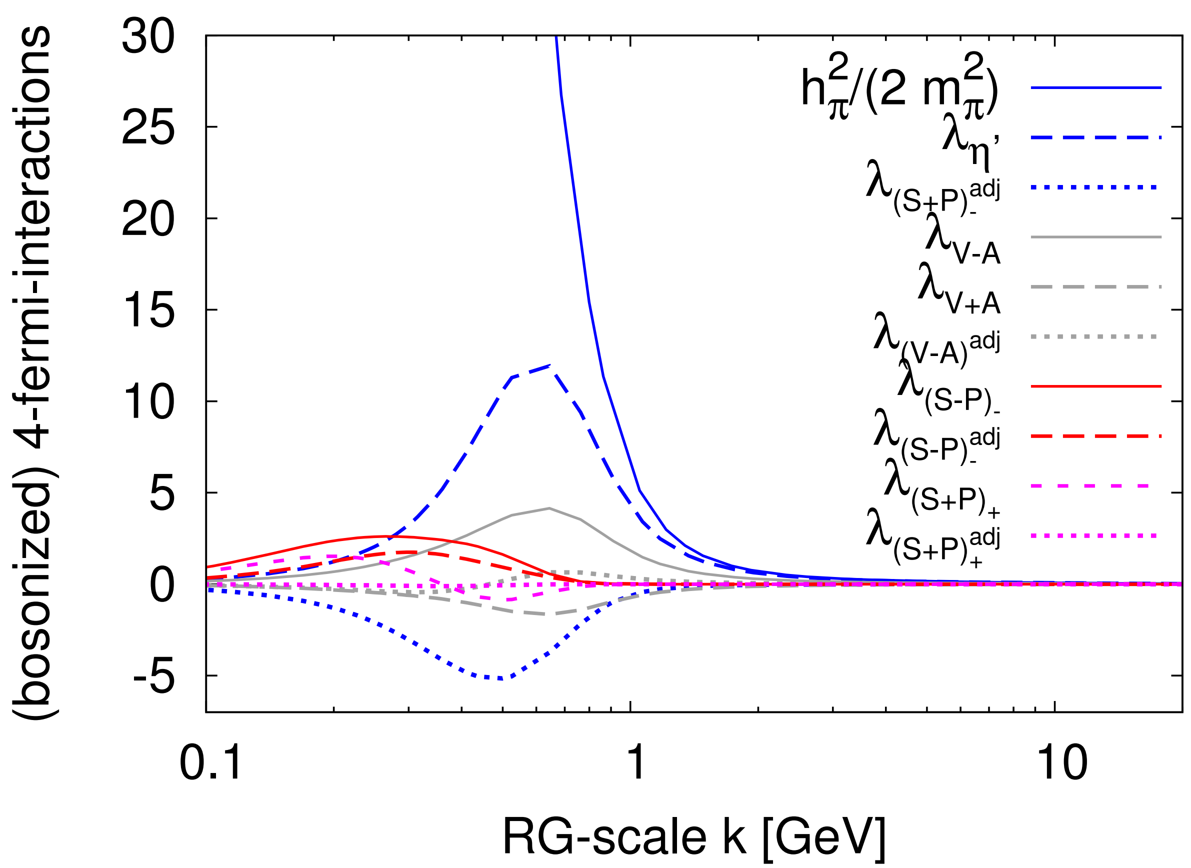
A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \;\; \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \to \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$$



Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\boxed{\begin{array}{c}\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \\ \Big\Downarrow \\ \Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p)\end{array}} \quad \to \quad \boxed{\begin{array}{c}\dfrac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \\ \Big\Downarrow \\ \dfrac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}\end{array}} \quad + \quad \text{finite terms}$$

Mitter, JMP, Strodthoff, in preparation

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preliminary

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pion decay constant $\mathbf{f}_\pi$ **via normalisation of** $\mathbf{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f}_\pi \simeq \mathbf{99\,MeV}$$

quenched QCD

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\boxed{\begin{array}{c}\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \\ \Big\downarrow \\ \Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p)\end{array}} \to \boxed{\begin{array}{c}\dfrac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \\ \Big\downarrow \\ \dfrac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}\end{array}} + \quad \text{finite terms}$$

pion decay constant $\mathbf{f}_\pi$ **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f}_\pi \simeq \mathbf{99\,MeV}$$

quenched QCD

$$\mathbf{f}_\pi \simeq \mathbf{89\,MeV}$$

unquenched QCD

lattice **Davies et al., PRL 92 (2004) 022001** $\dfrac{\mathbf{f}_\pi^{\text{quenched}}}{\mathbf{f}_\pi^{\text{unquenched}}} \simeq 1.1$

Mitter, JMP, Strodthoff, in preparation

A glimpse at the hadron spectrum

preliminary

four-fermi scattering amplitude at pion pole

$$\boxed{\begin{array}{c}\langle \bar{q}\vec{\sigma}\gamma_5 q(p)\quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \\ \Big\Downarrow \\ \Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p)\end{array}} \to \boxed{\begin{array}{c}\dfrac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2-m_\pi^2} \\ \Big\Downarrow \\ \dfrac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2-m_\pi^2}\end{array}} + \quad \text{finite terms}$$

pion decay constant f_π via normalisation of $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f}_\pi \simeq \mathbf{99\,MeV}$$

quenched QCD

$$\mathbf{f}_\pi \simeq \mathbf{89\,MeV}$$

unquenched QCD

lattice Davies et al., PRL 92 (2004) 022001

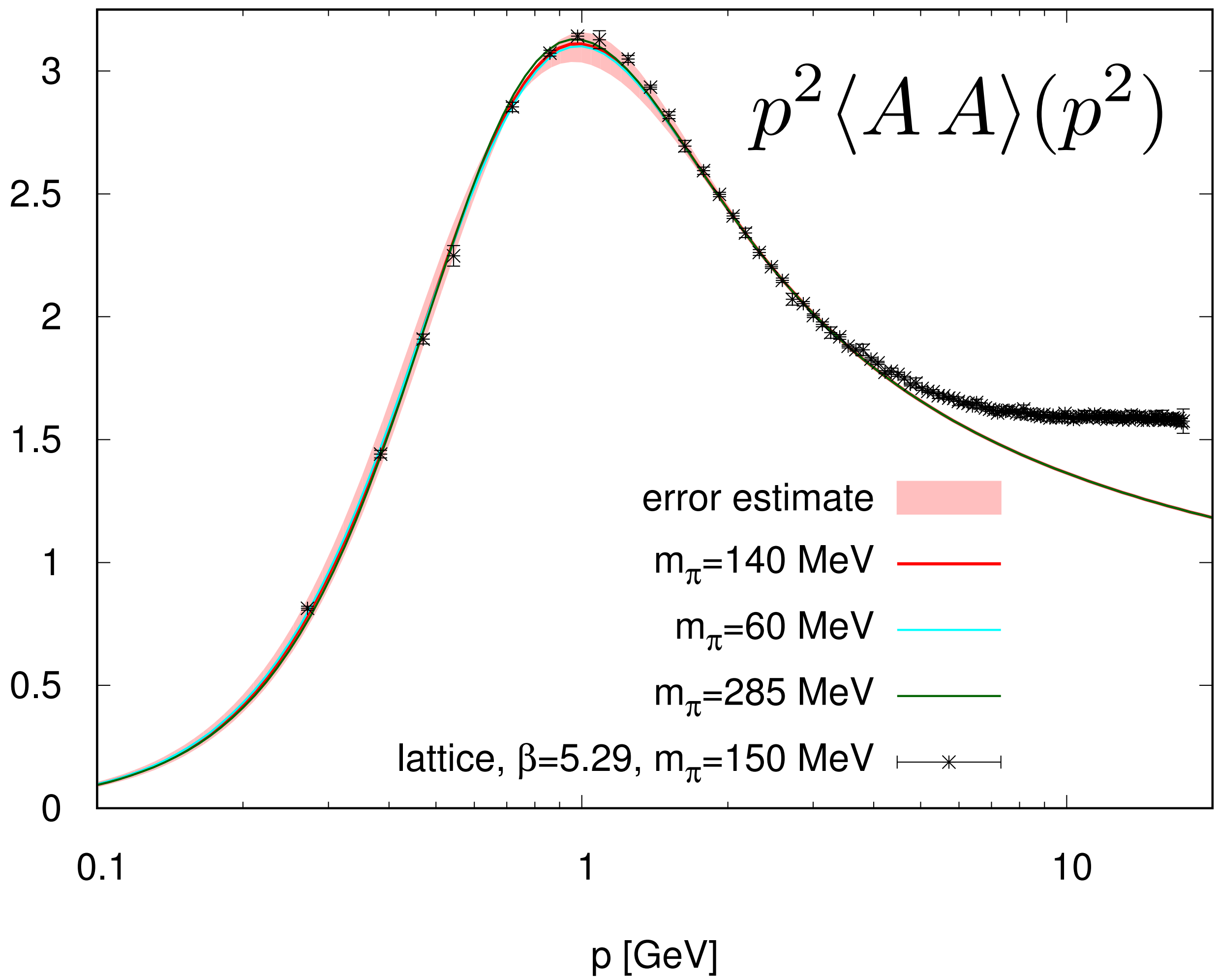
unquenched e.g. Horsley et al., PLB 732, 41 (2014) $\mathbf{f}_\pi^{\mathrm{lattice}} \simeq 89\,\mathrm{MeV}$

Mitter, JMP, Strodthoff, in preparation

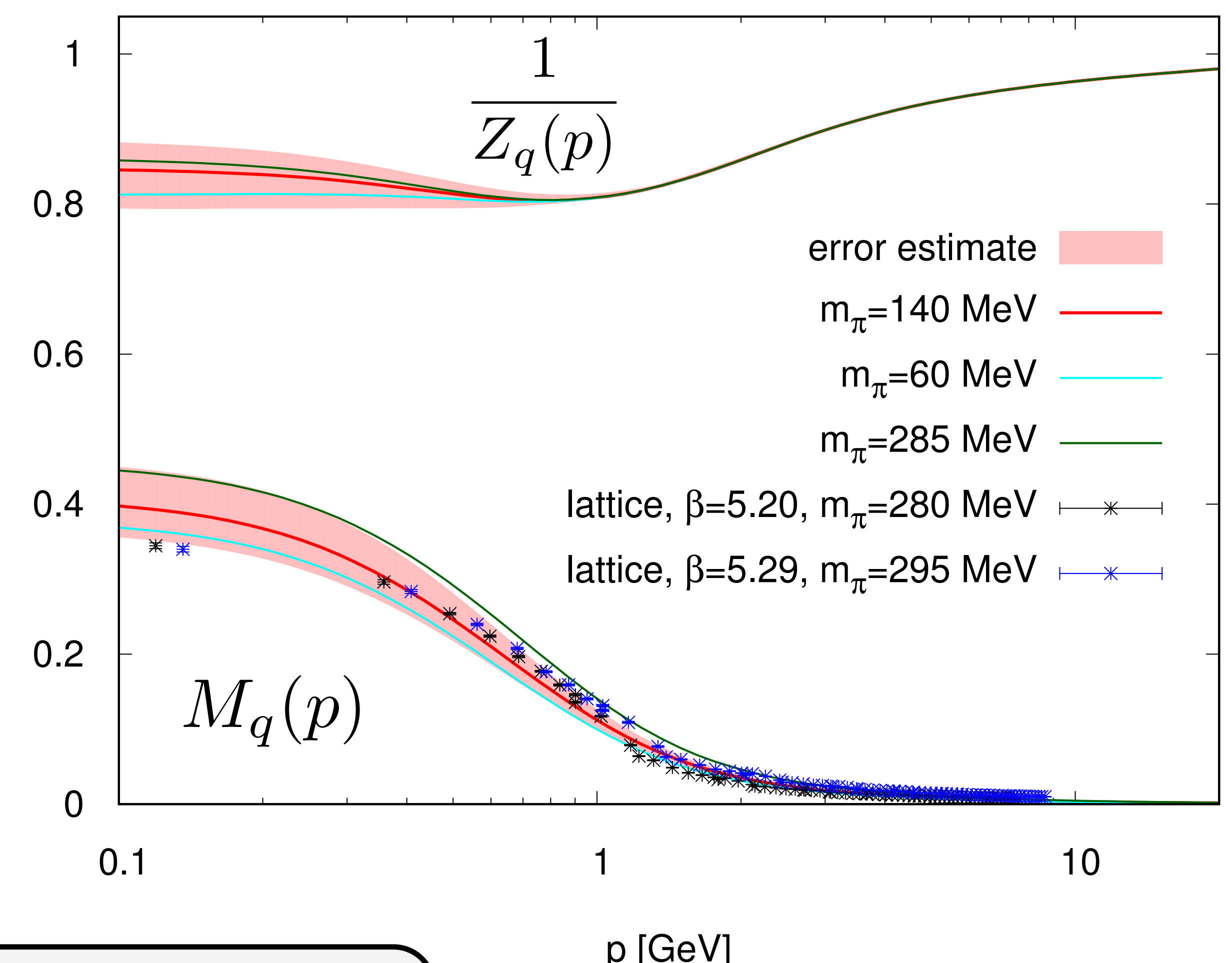
Outline

- Introduction
- Phase structure of QCD
    - Confinement & chiral symmetry breaking
    - Finite temperature correlation functions
    - QCD at finite density & fluctuations
- QCD transport
    - Real time correlation functions
    - Single particle spectral functions
    - transport coefficients
- Summary & outlook

QCD: Euclidean propagators



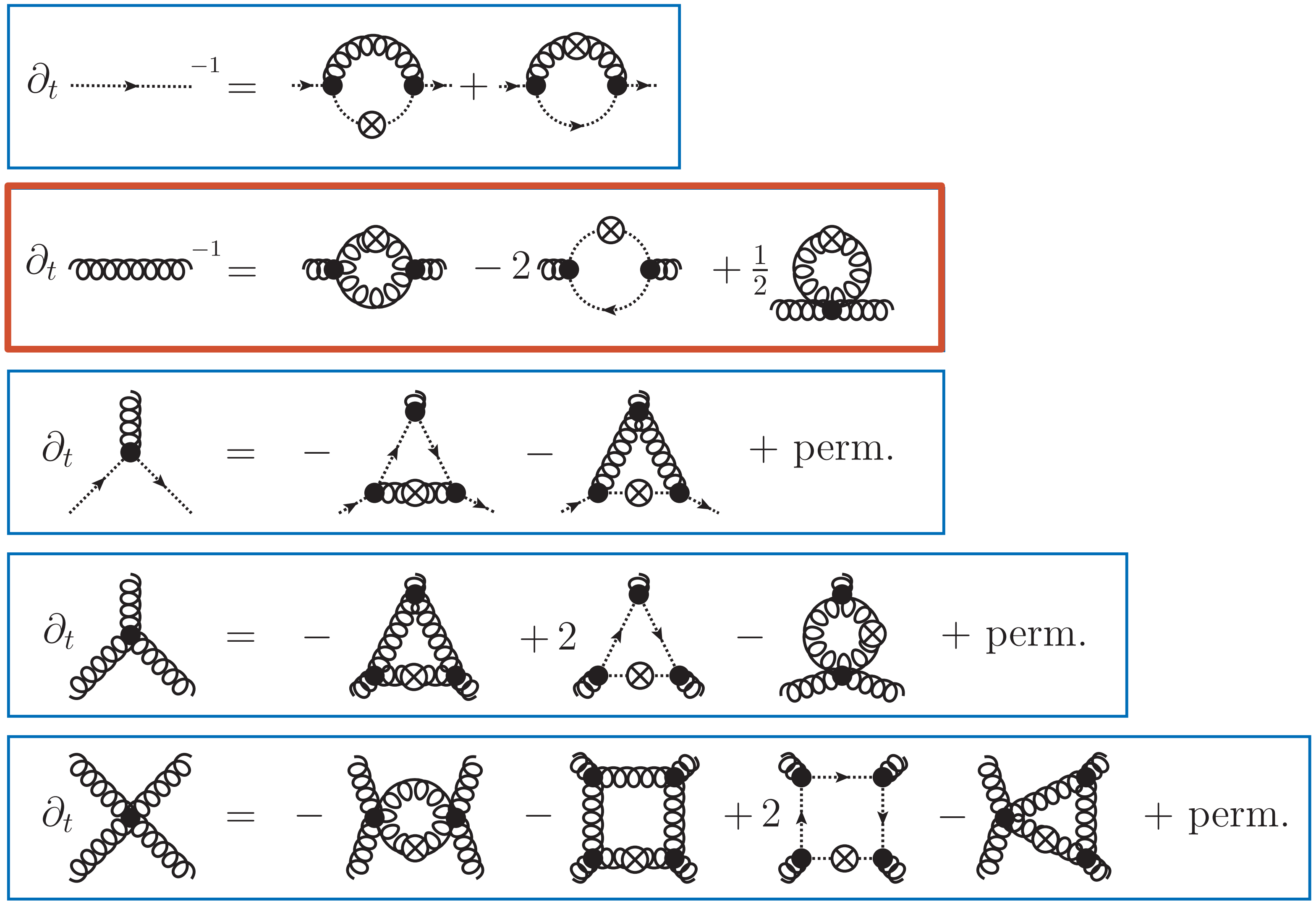
$$\frac{1}{Z_q(p)}\,\frac{1}{i\,\not{p} + M_q(p)}$$

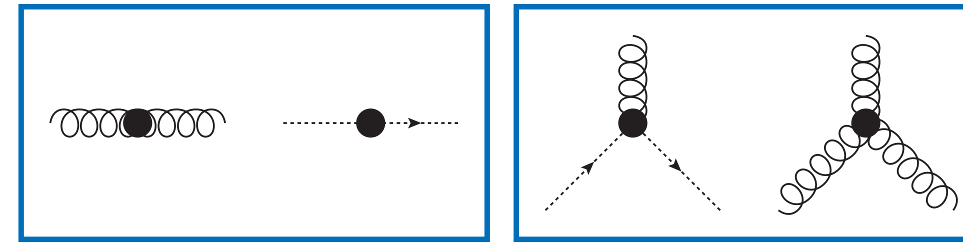


Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, arXiv:1706.06326

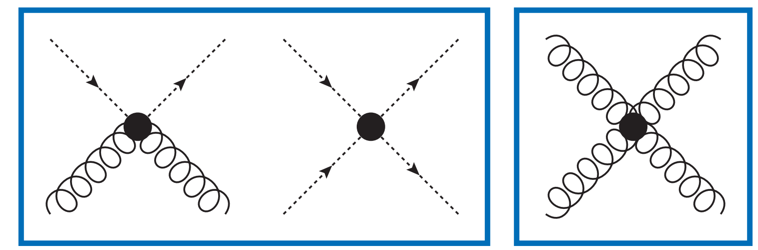
YM-theory: gluonic correlation functions





full. mom. dep.

full. mom. dep.
classical tensor structures



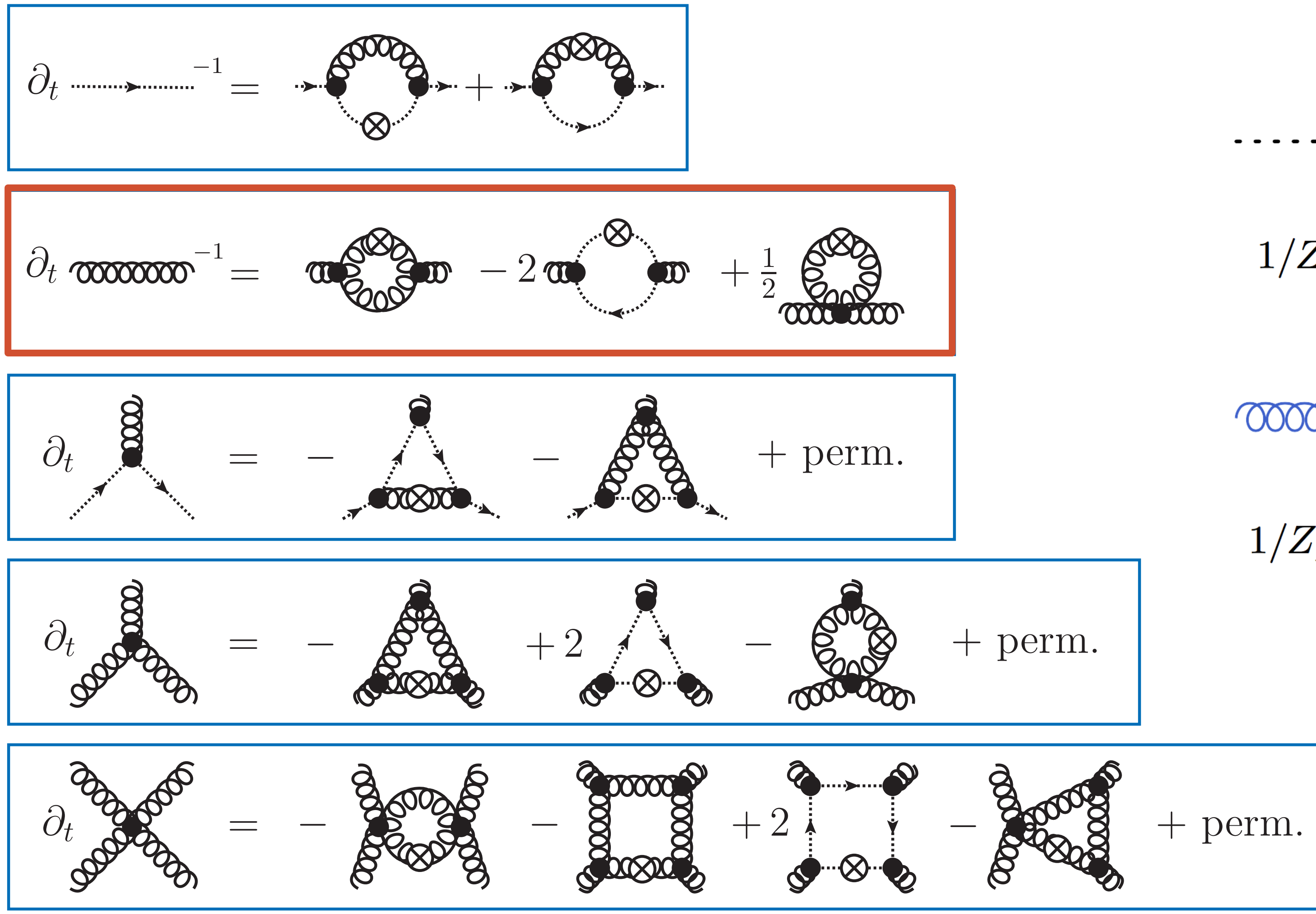
mom. dep. needed by tadpoles
full tensor basis

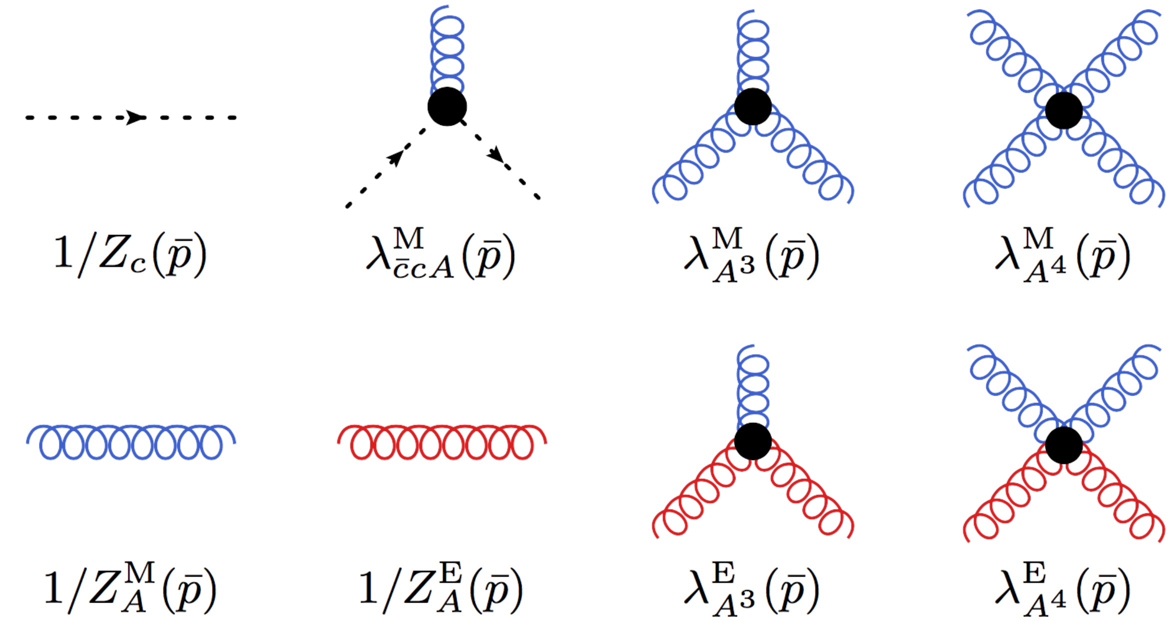
sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions





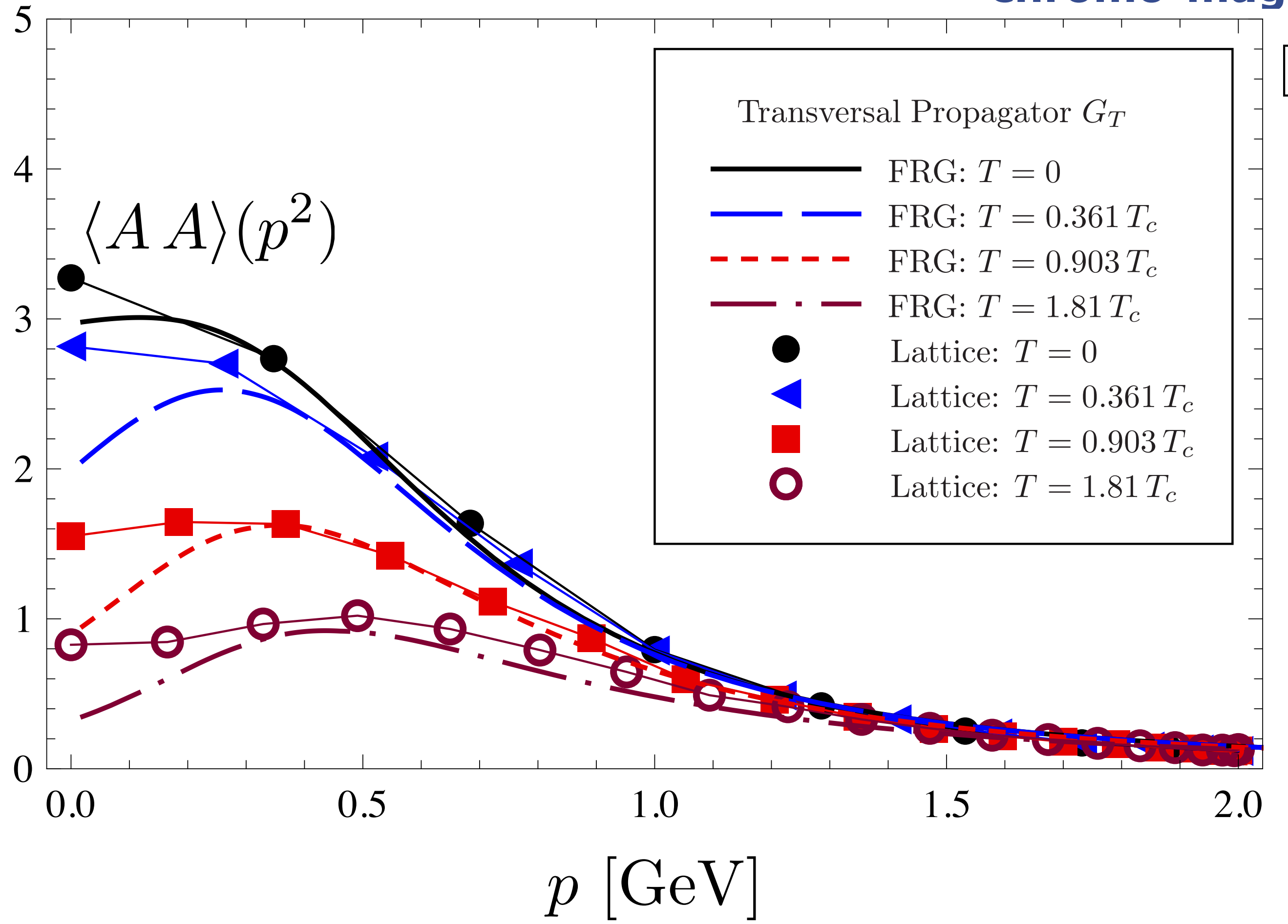
Aiming at apparent convergence

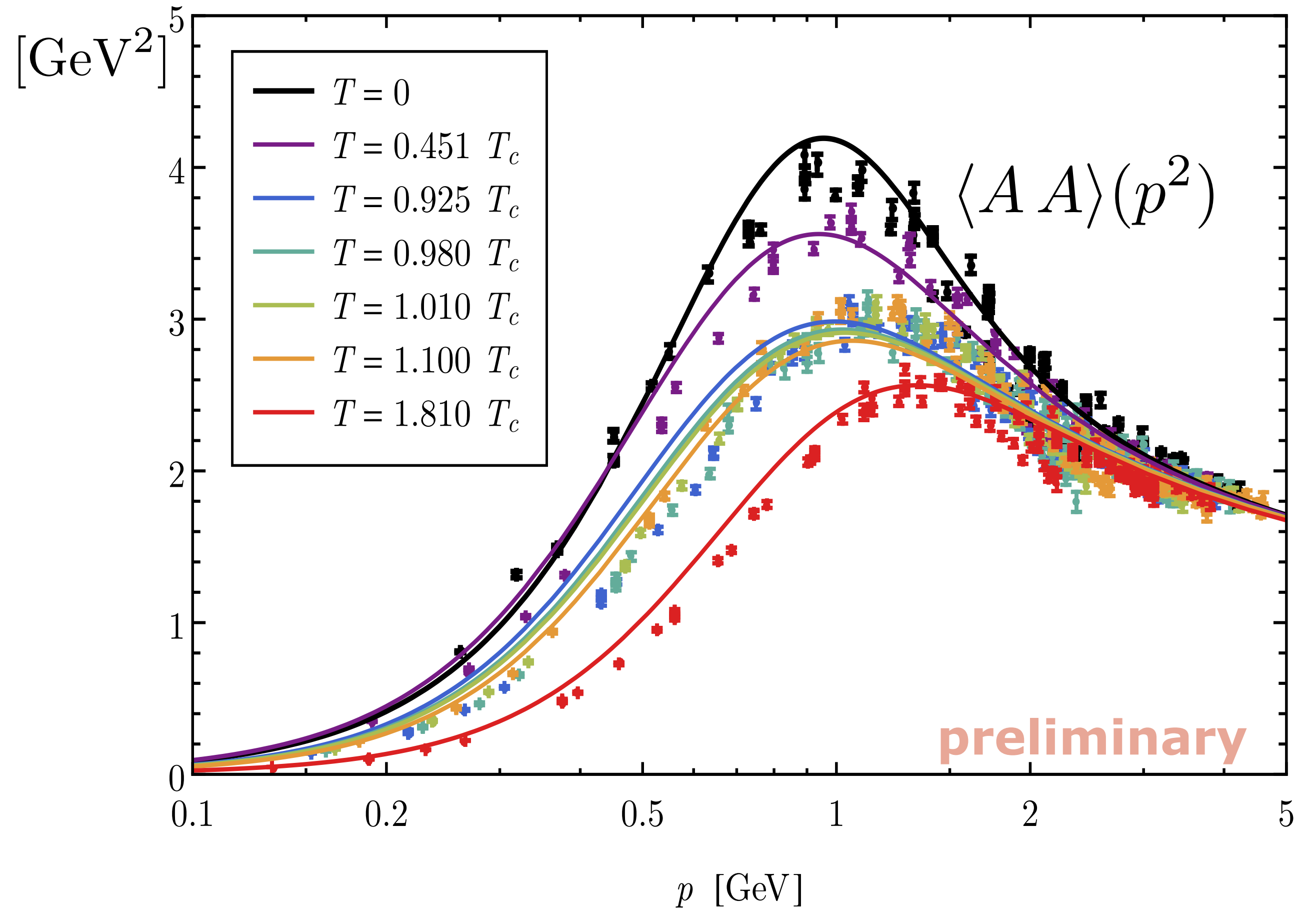
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Euclidean gluon propagator at finite T

Yang-Mills propagators, finite T

chromo-magnetic propagator





Fister, JMP, arXiv:1112.5440

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

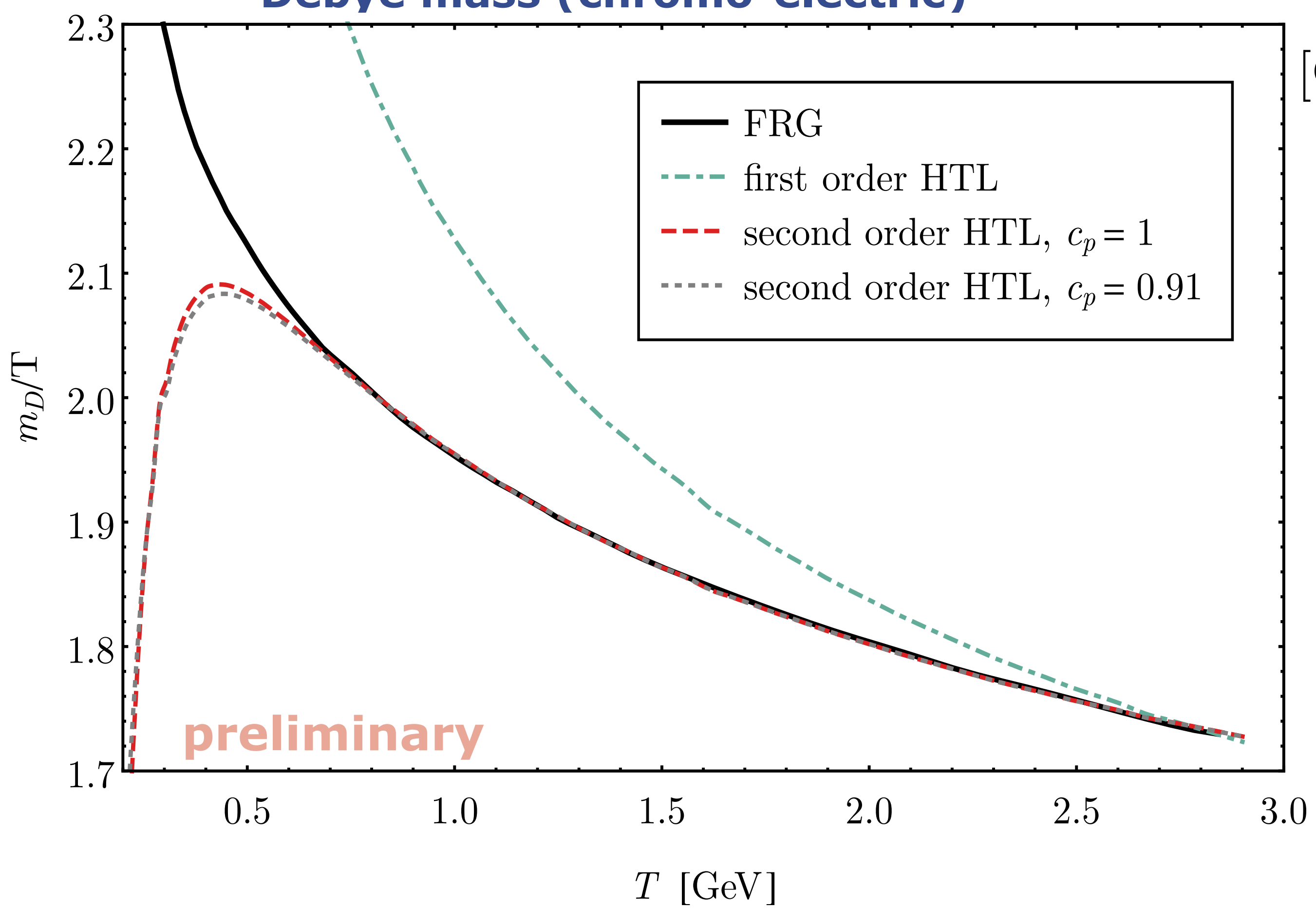
Cyrol, Mitter, JMP, Strodthoff, in preparation

Lattice: Silva, Oliviera, Bicudo, Cardoso, PRD89 (2014) 7, 074503

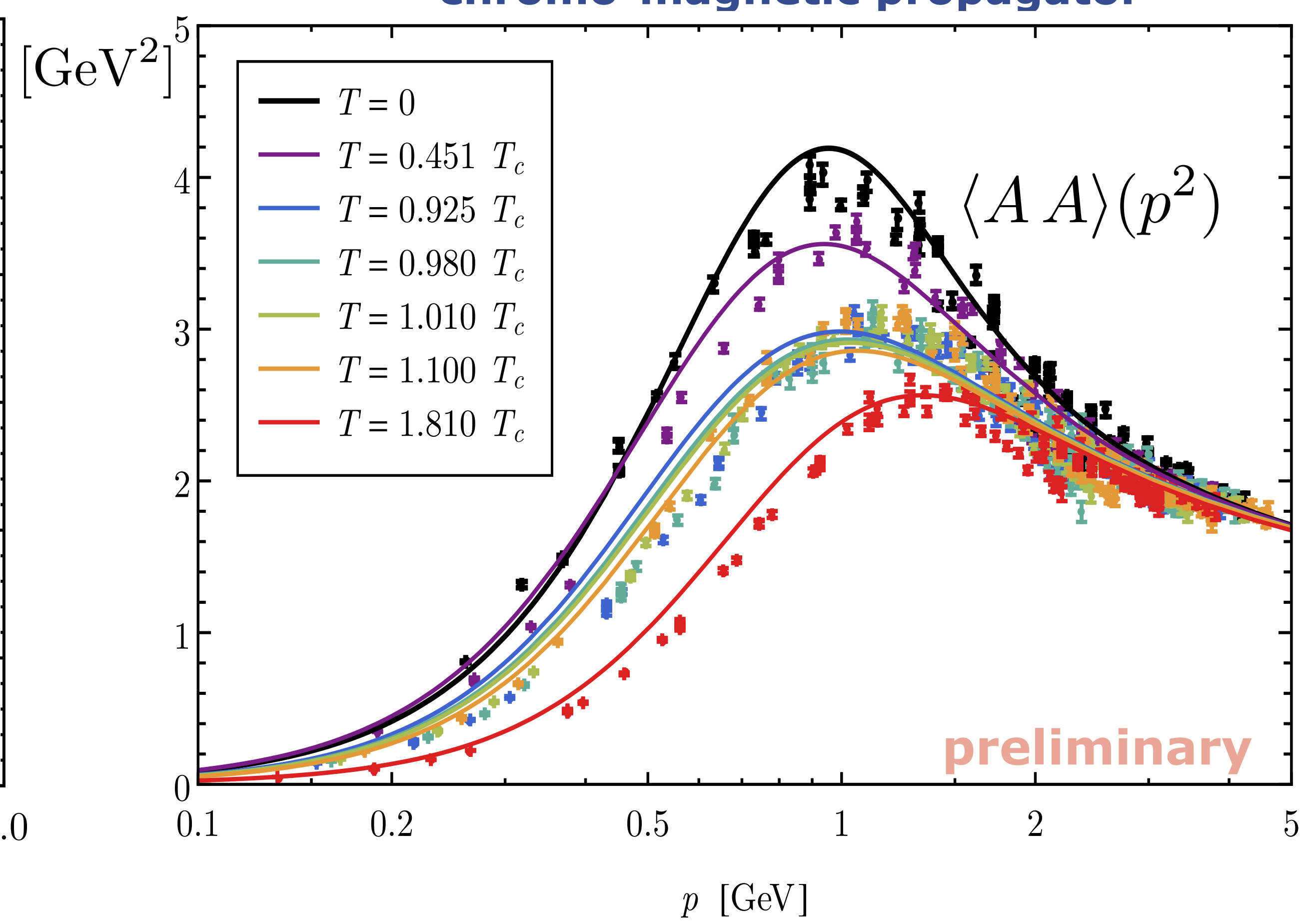
Euclidean gluon propagator at finite T

Yang-Mills propagators, finite T

Debye mass (chromo-electric)



chromo-magnetic propagator



Cyrol, Mitter, JMP, Strodthoff, in preparation

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

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Euclidean gluon propagator at finite T

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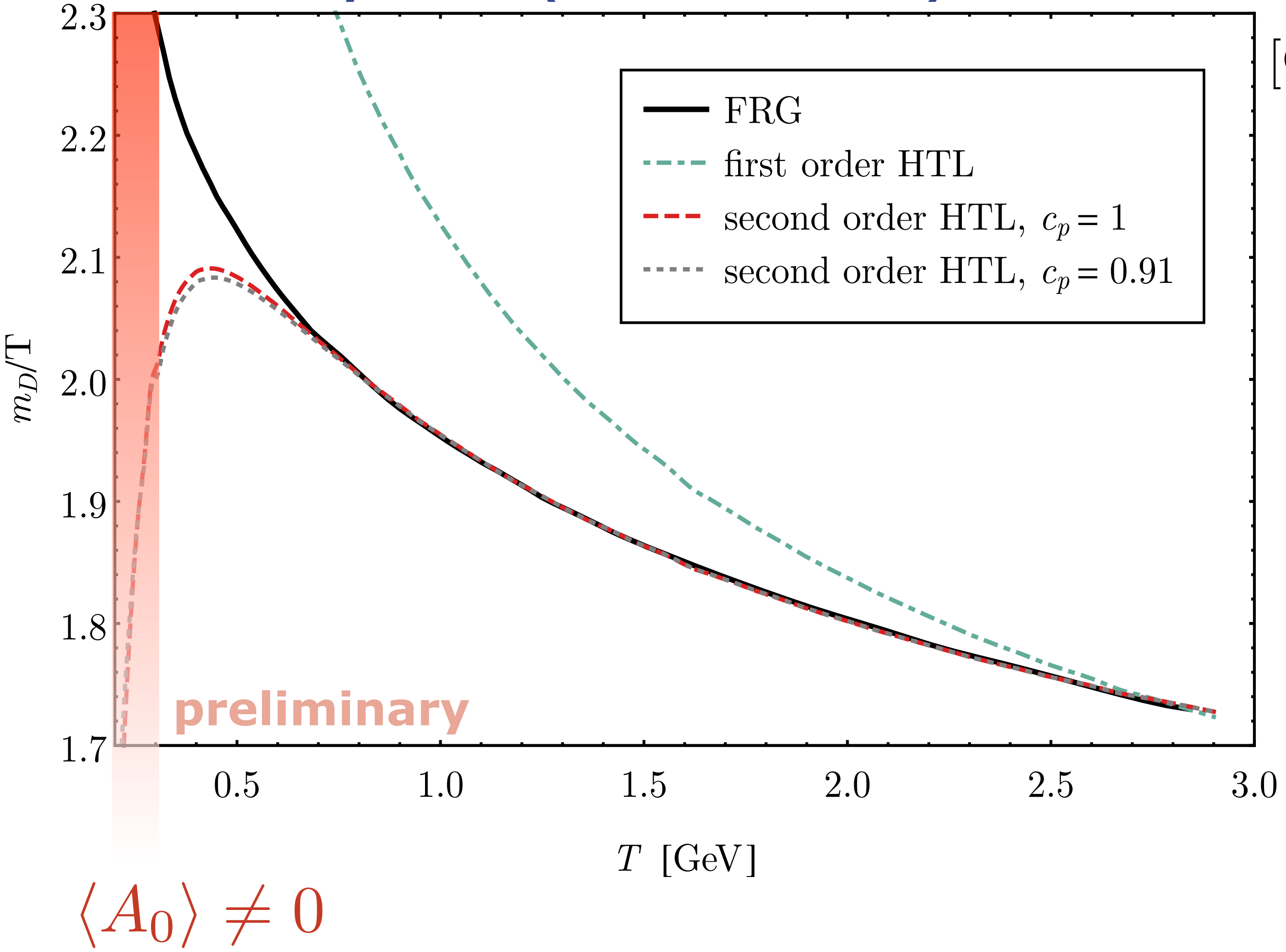
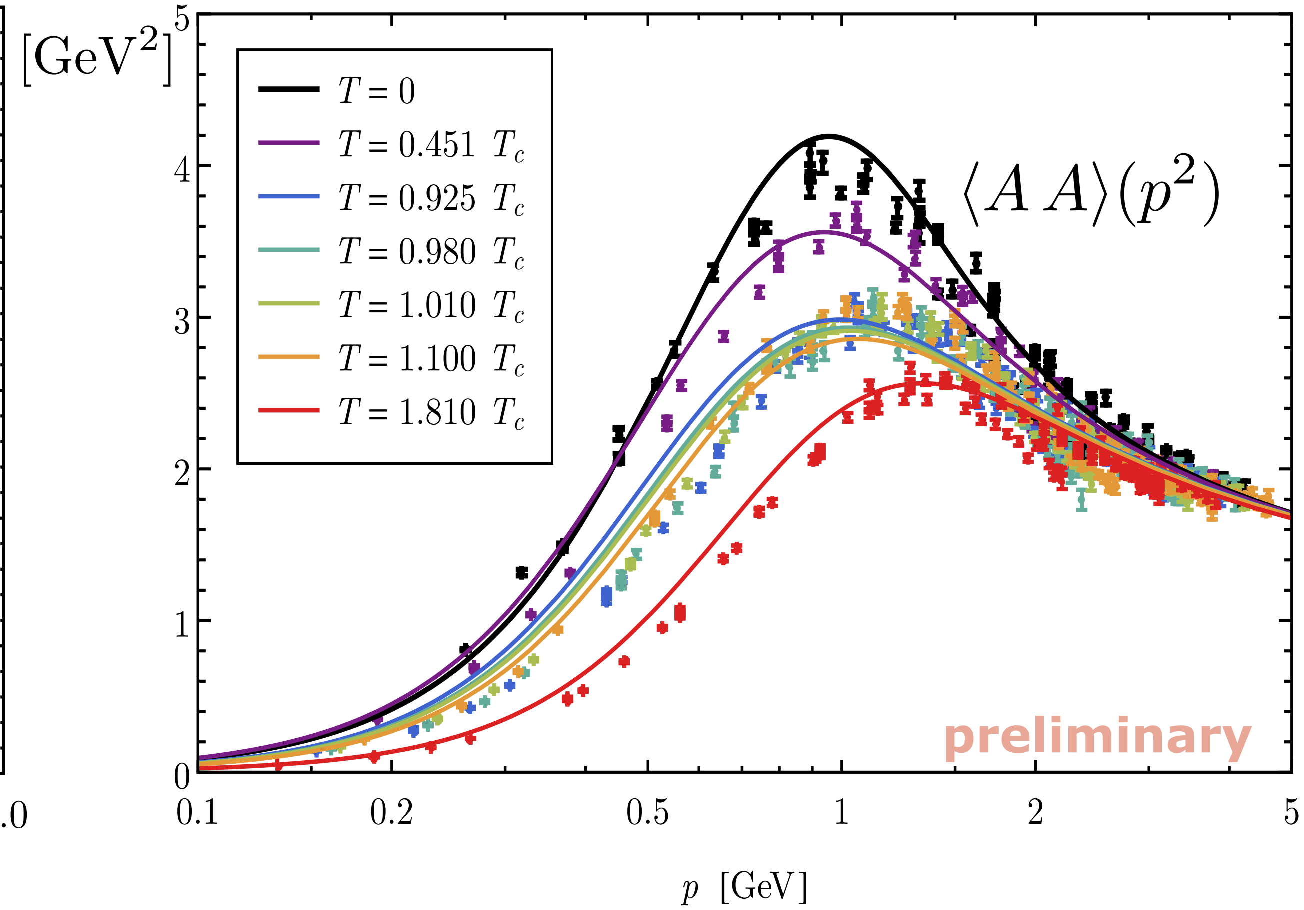


chromo-magnetic propagator



Cyrol, Mitter, JMP, Strodthoff, in preparation

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

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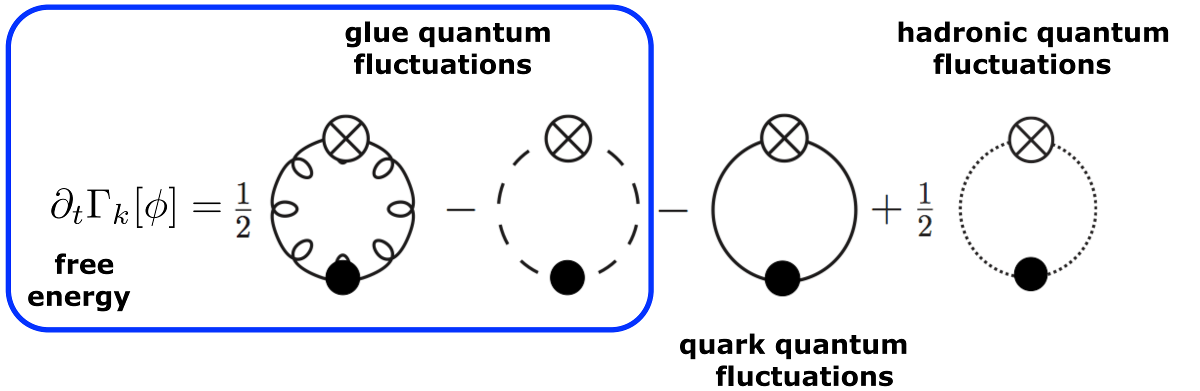
CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$\boxed{L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}}\,\mathbf{tr}\,\mathcal{P}\mathbf{e}^{\mathbf{i\,g}\int_0^\beta \mathbf{A_0(x)}}}$$



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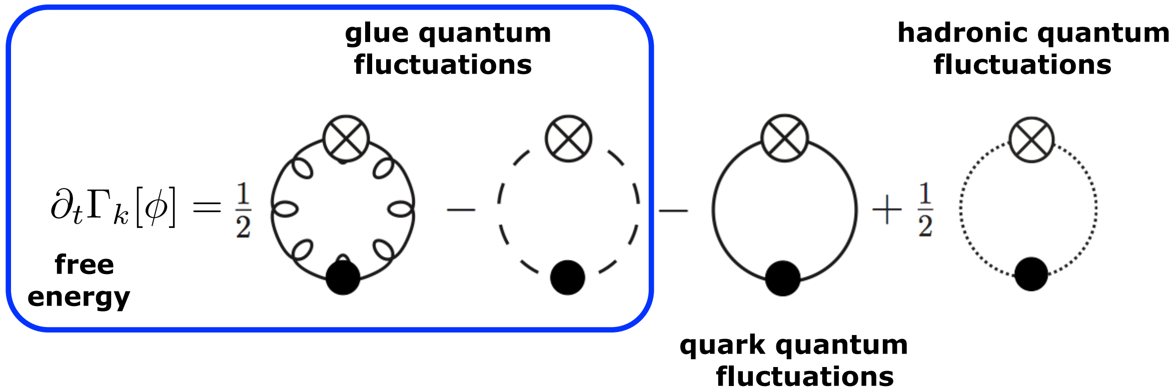
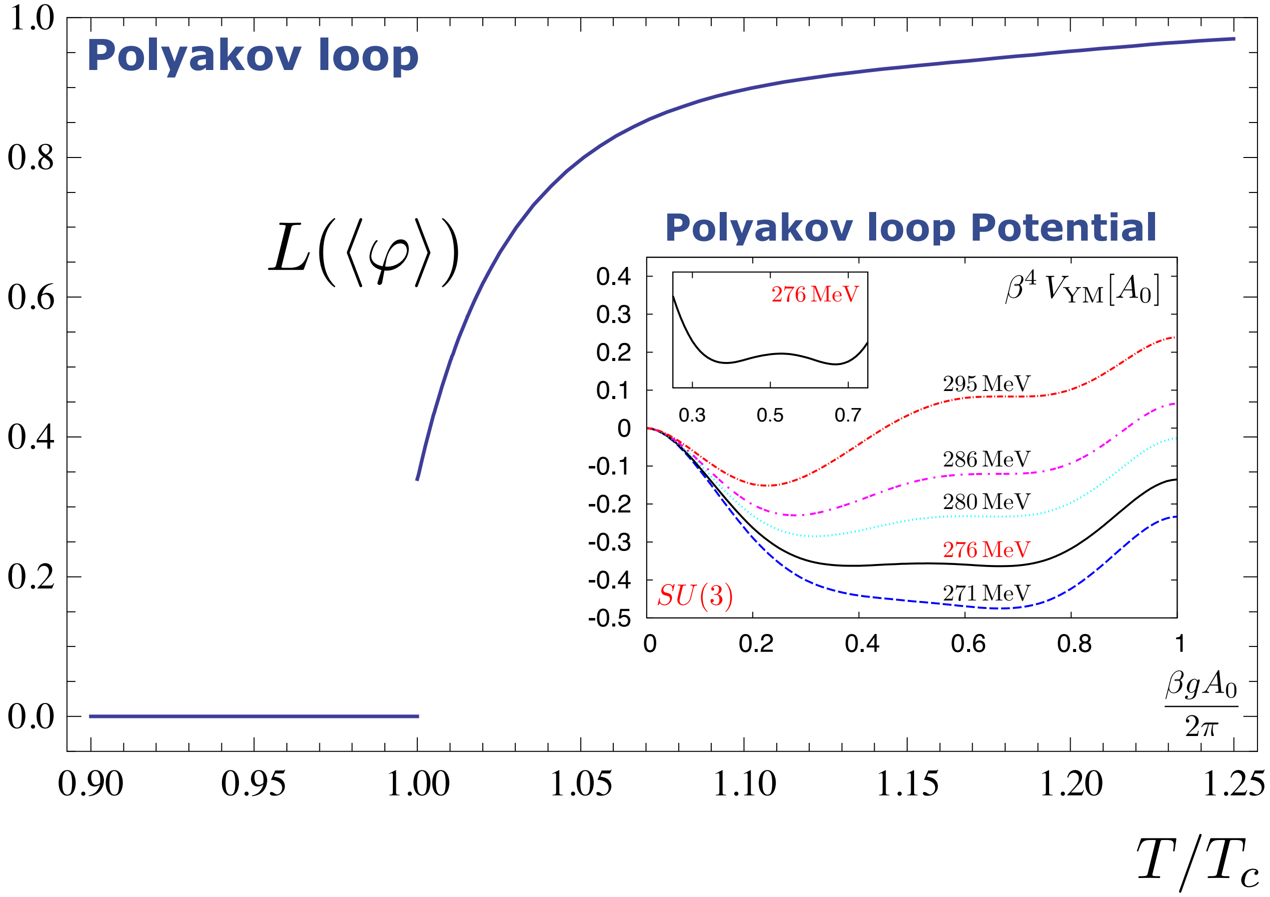


$$\mathcal{P}e^{i\,g\,\int_0^\beta A_0(x)} = e^{i\varphi}$$



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

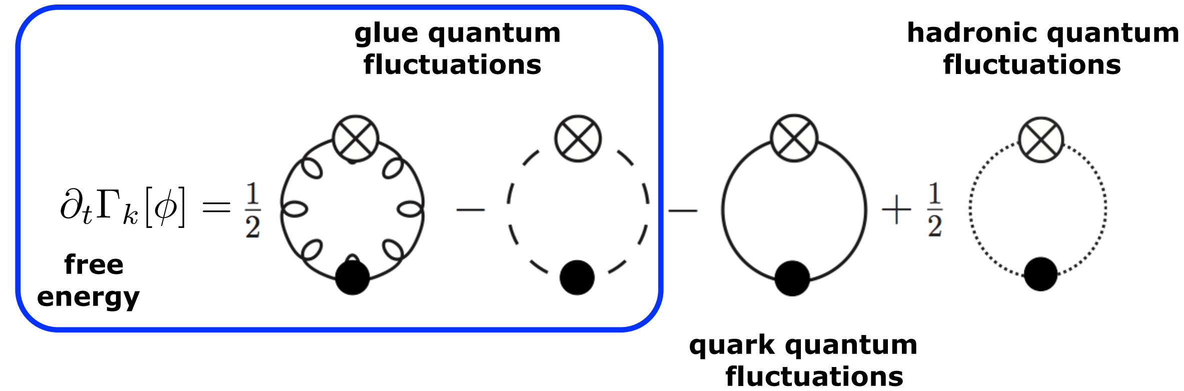
$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{\mathbf{1}}{\mathbf{N_c}}\,\mathbf{tr}\,\mathcal{P}e^{\mathbf{i\,g}\int_0^\beta \mathbf{A_0(x)}}$$



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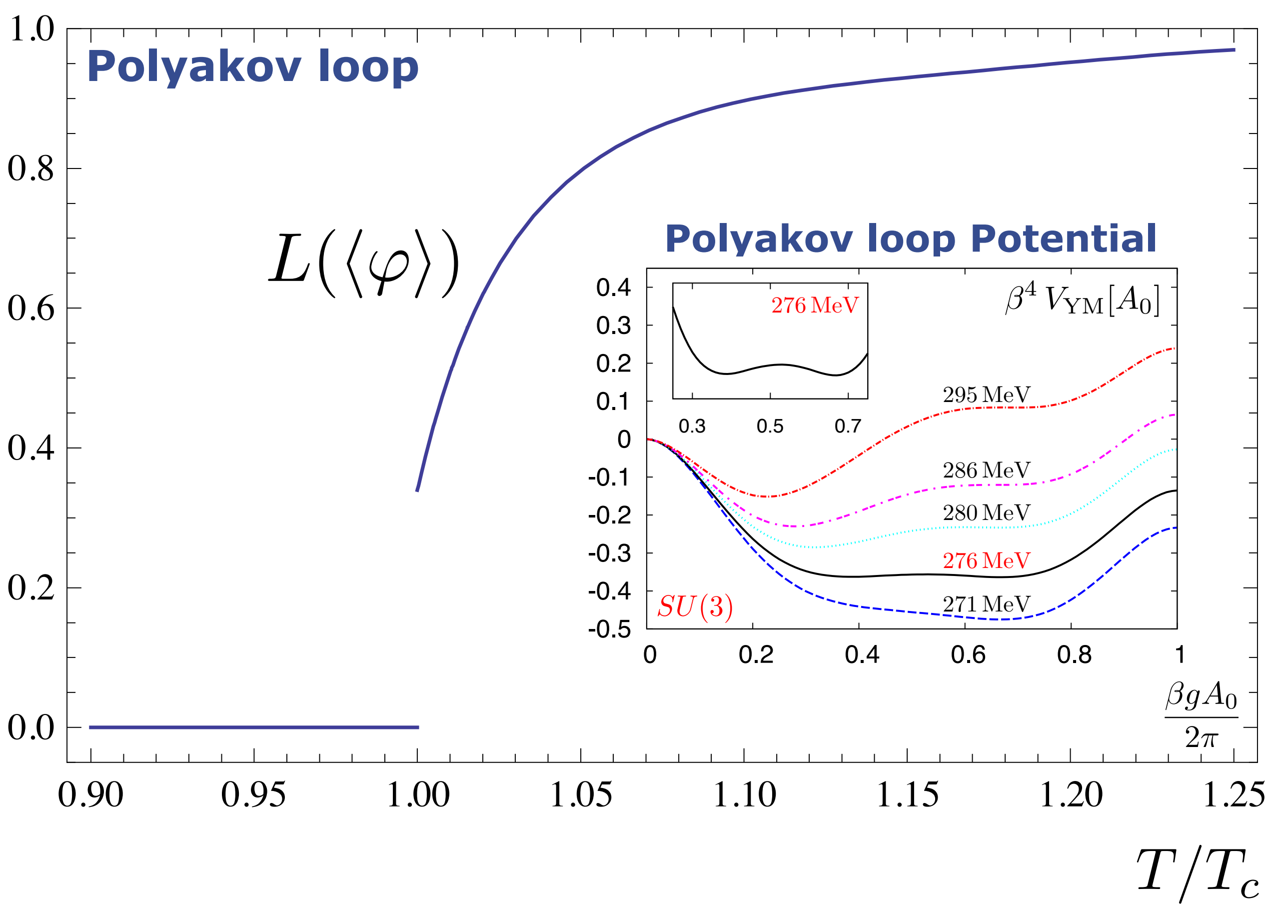
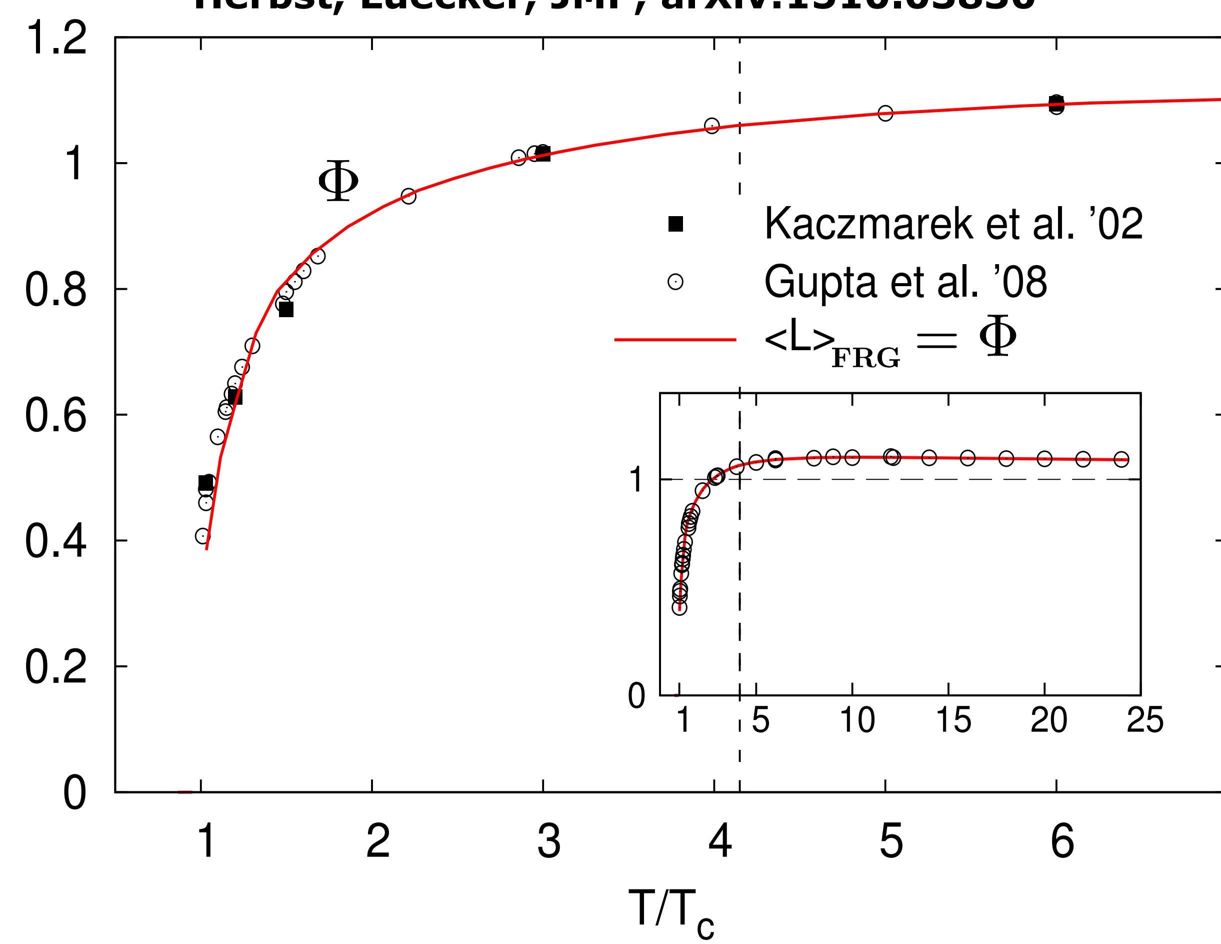


Herbst, Luecker, JMP, arXiv:1510.03830

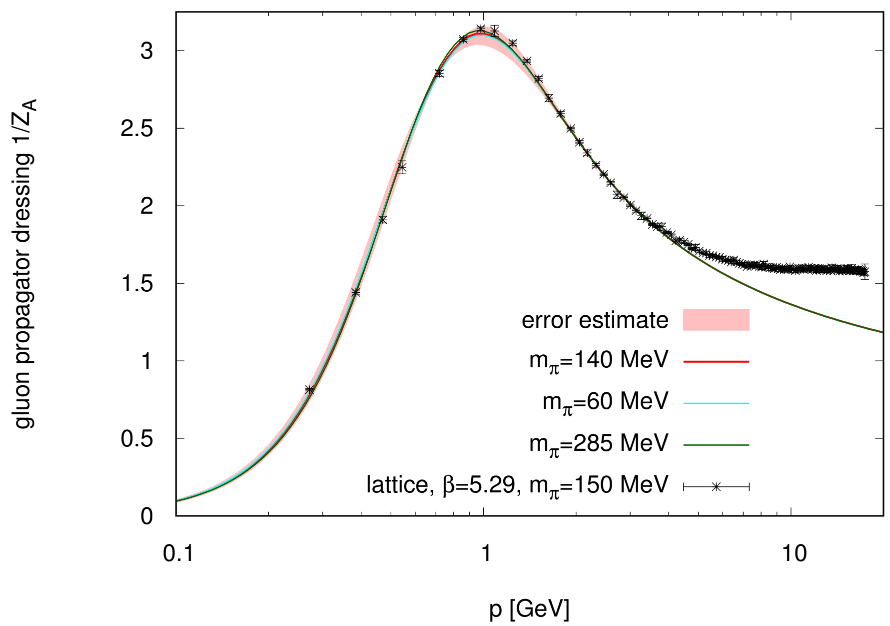


Summary I

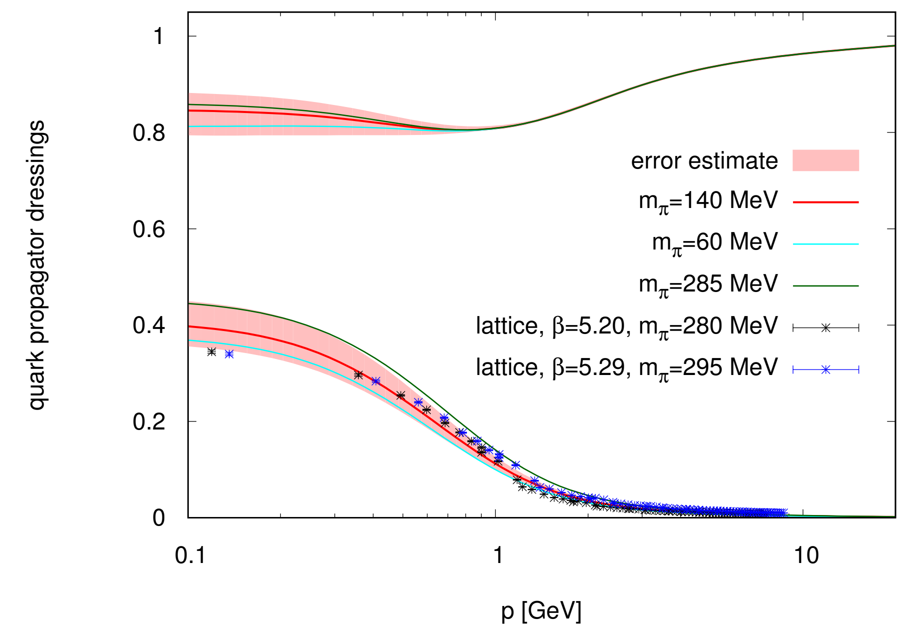
- **Chiral Symmetry Breaking and Confinement**

$$\boxed{\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99}$$

Gluon correlations

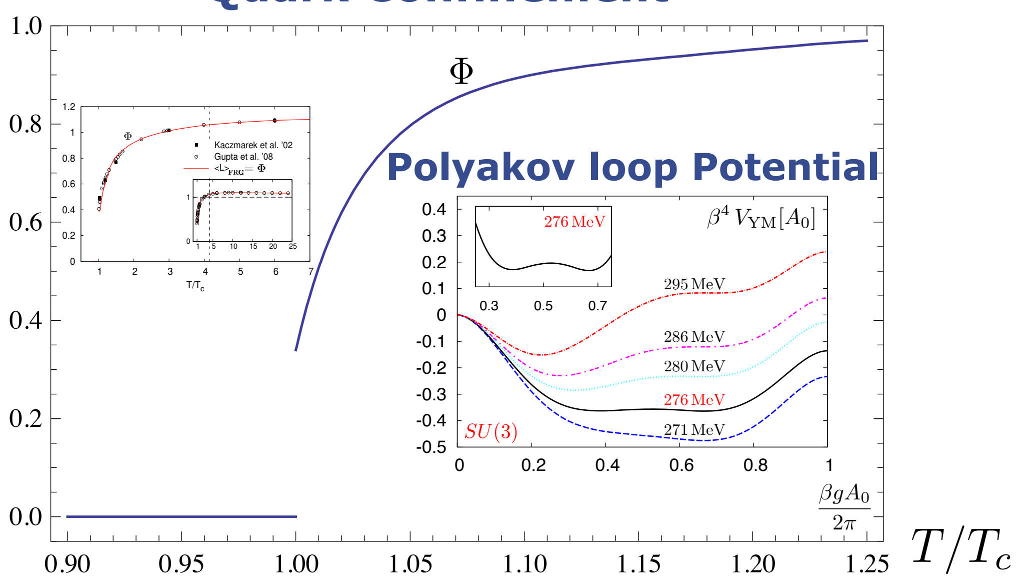


chiral symmetry breaking

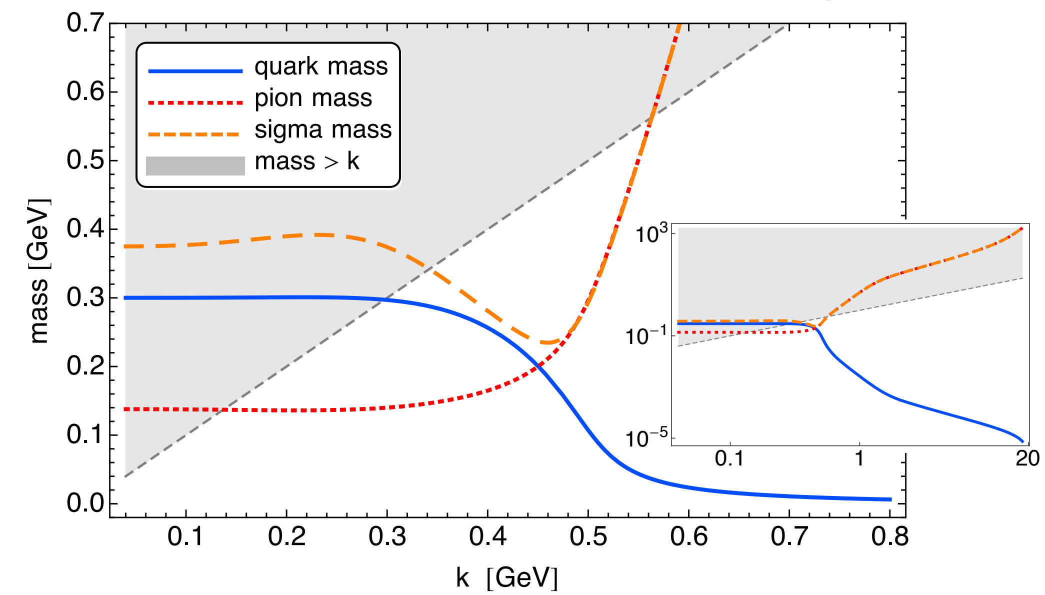


fQCD

Quark Confinement



meson masses & decoupling



Outline

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    - $\boxed{\text{QCD at finite density \& fluctuations}}$
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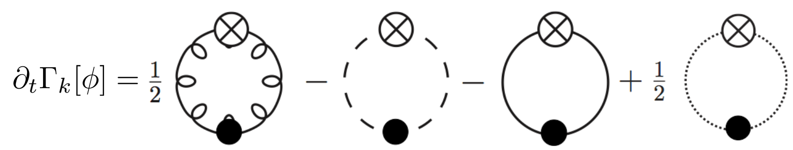
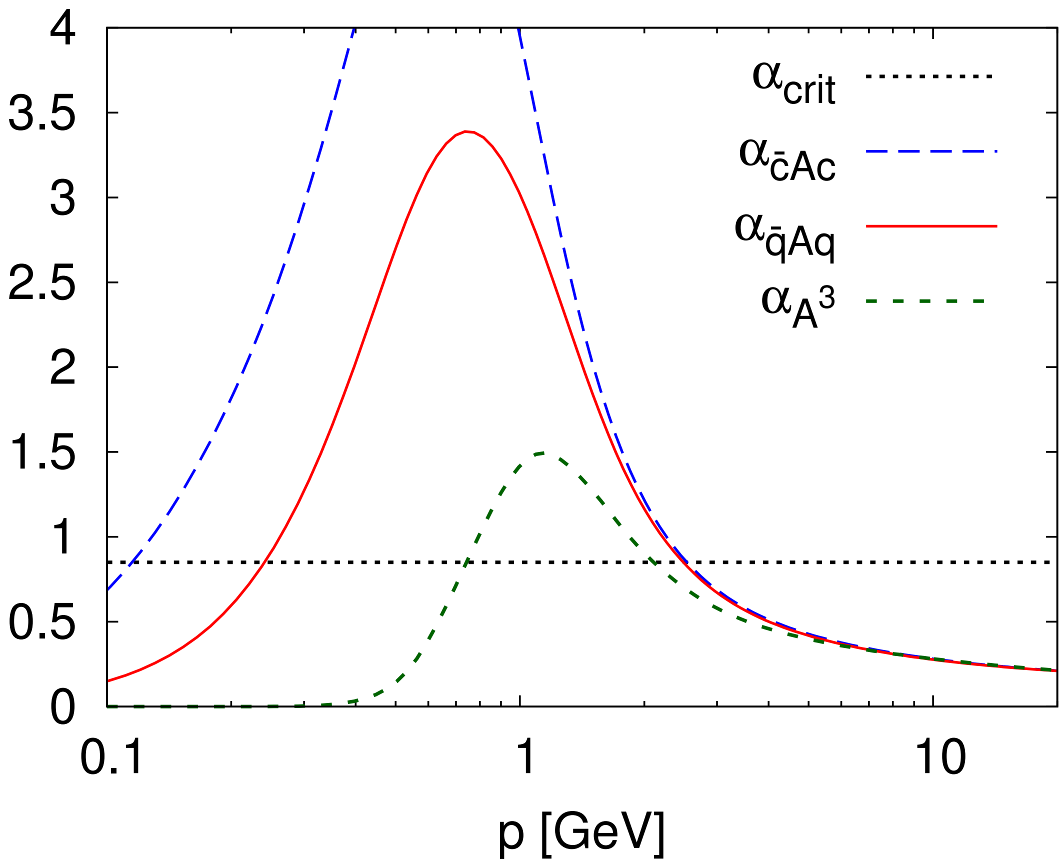
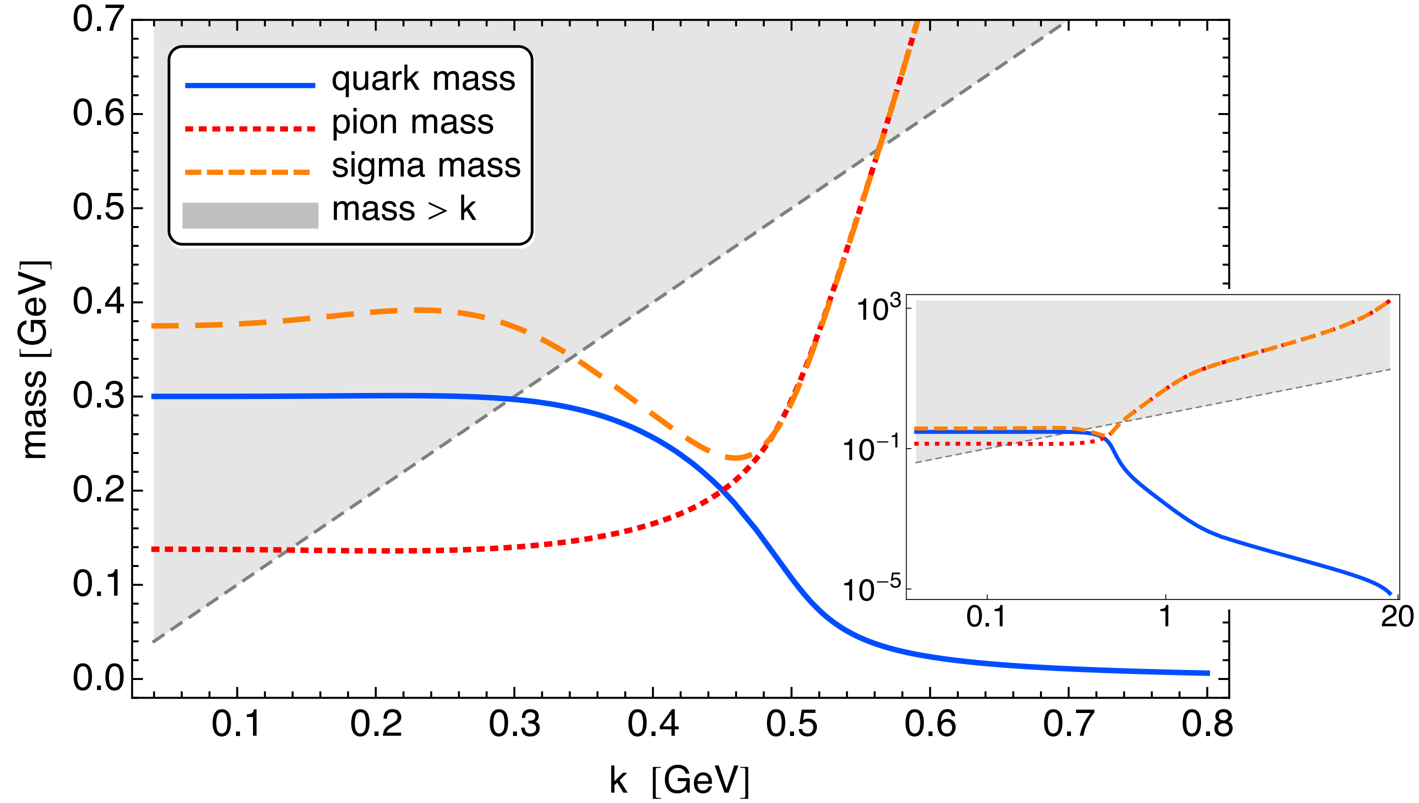
QCD phase structure



Sequential decoupling of gluon, quark, sigma, pion fluctuations



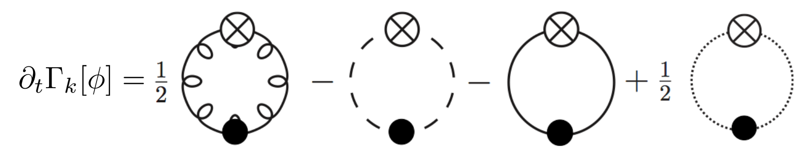
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035



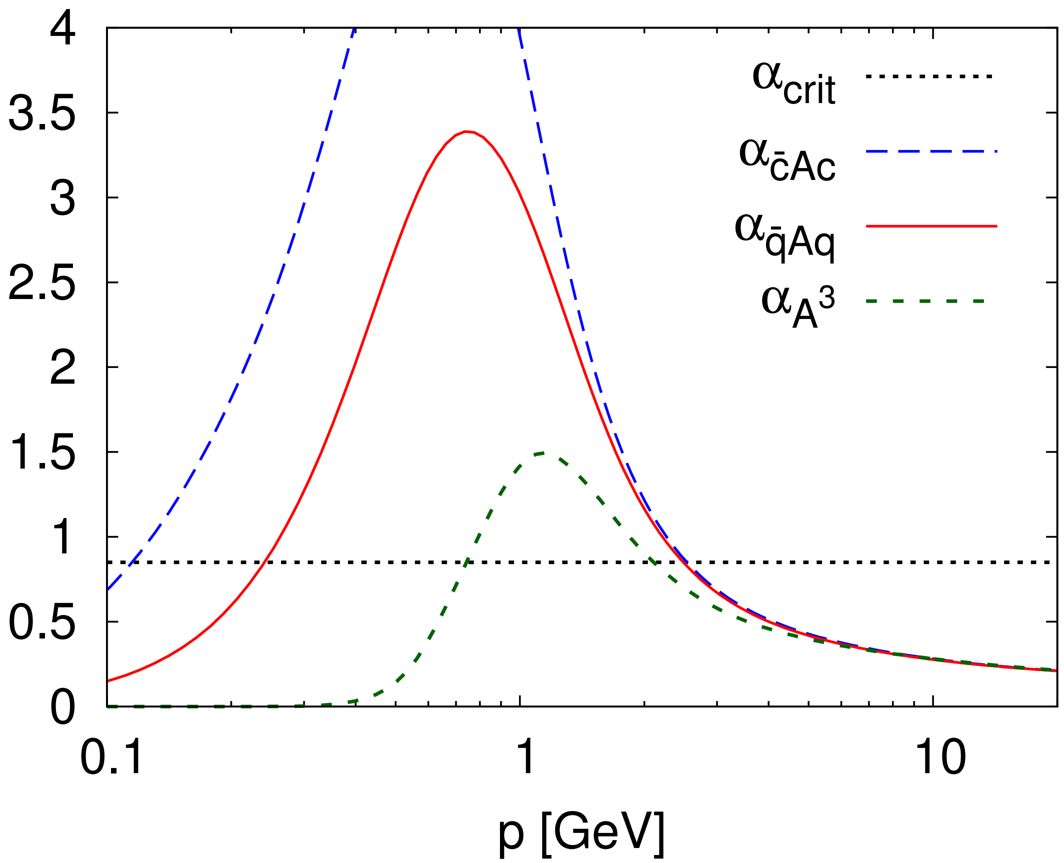
Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

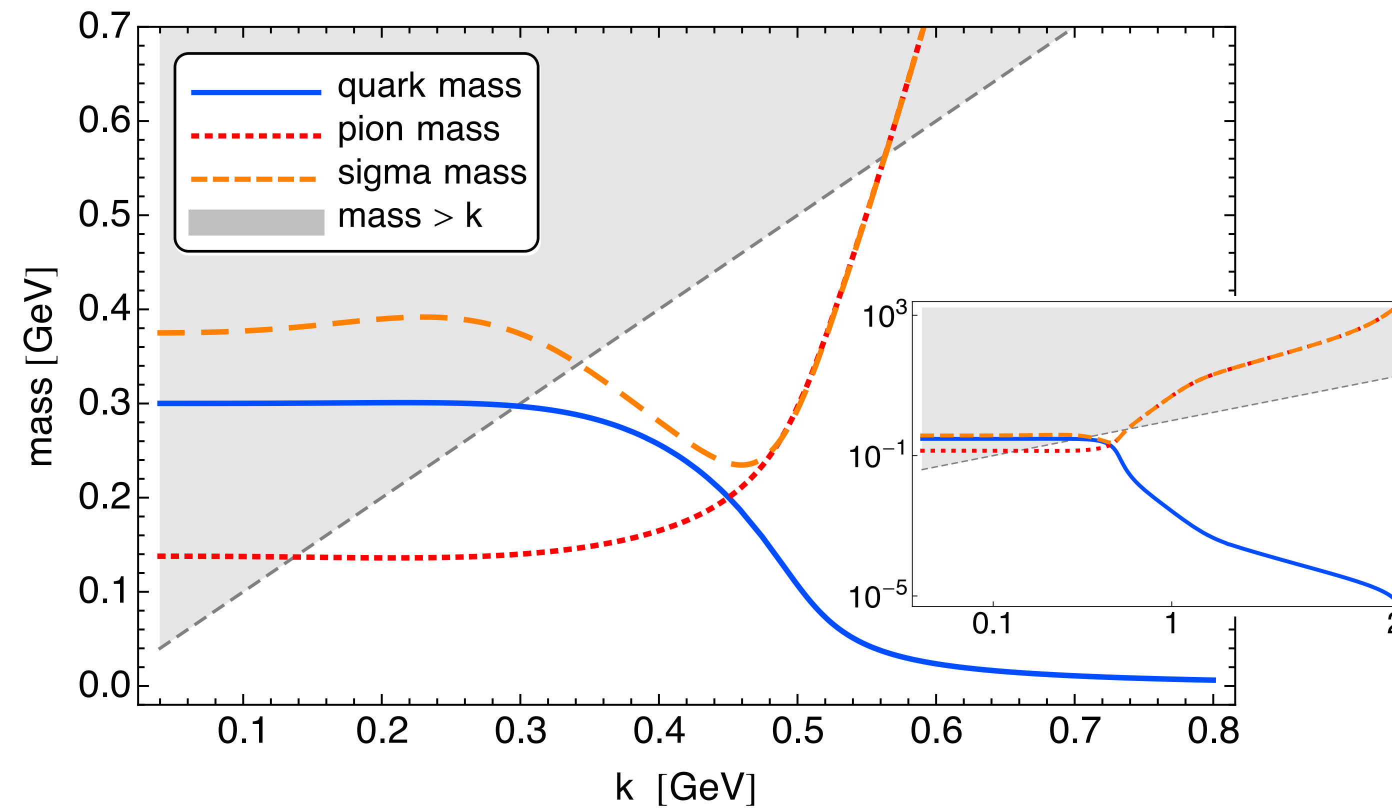
Rennecke, PRD 92 (2015) 076012

QCD phase structure

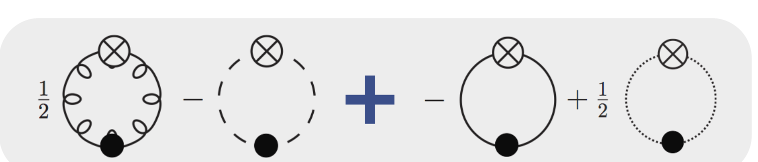


Sequential decoupling of gluon, quark, sigma, pion fluctuations

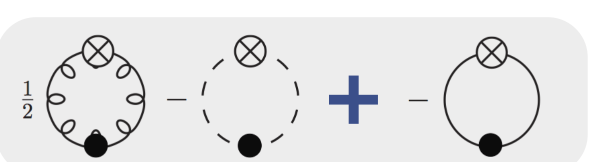




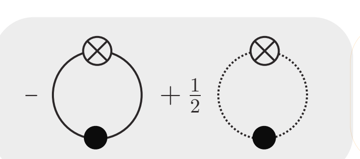
PQM-model



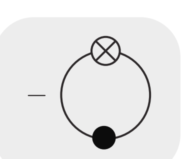
PNJL-model



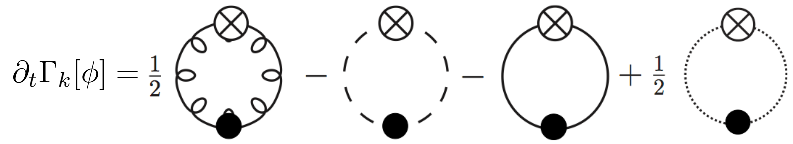
QM-model



NJL-model

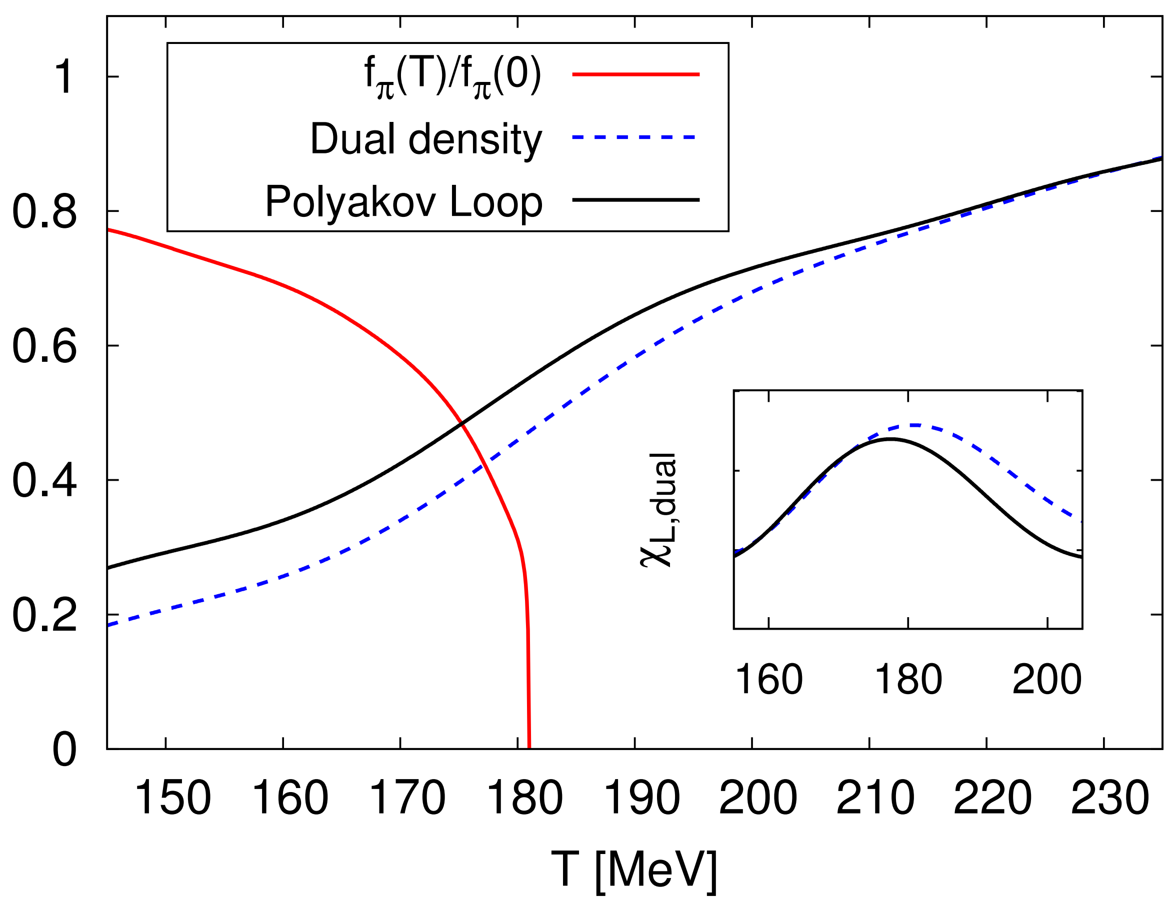


QCD

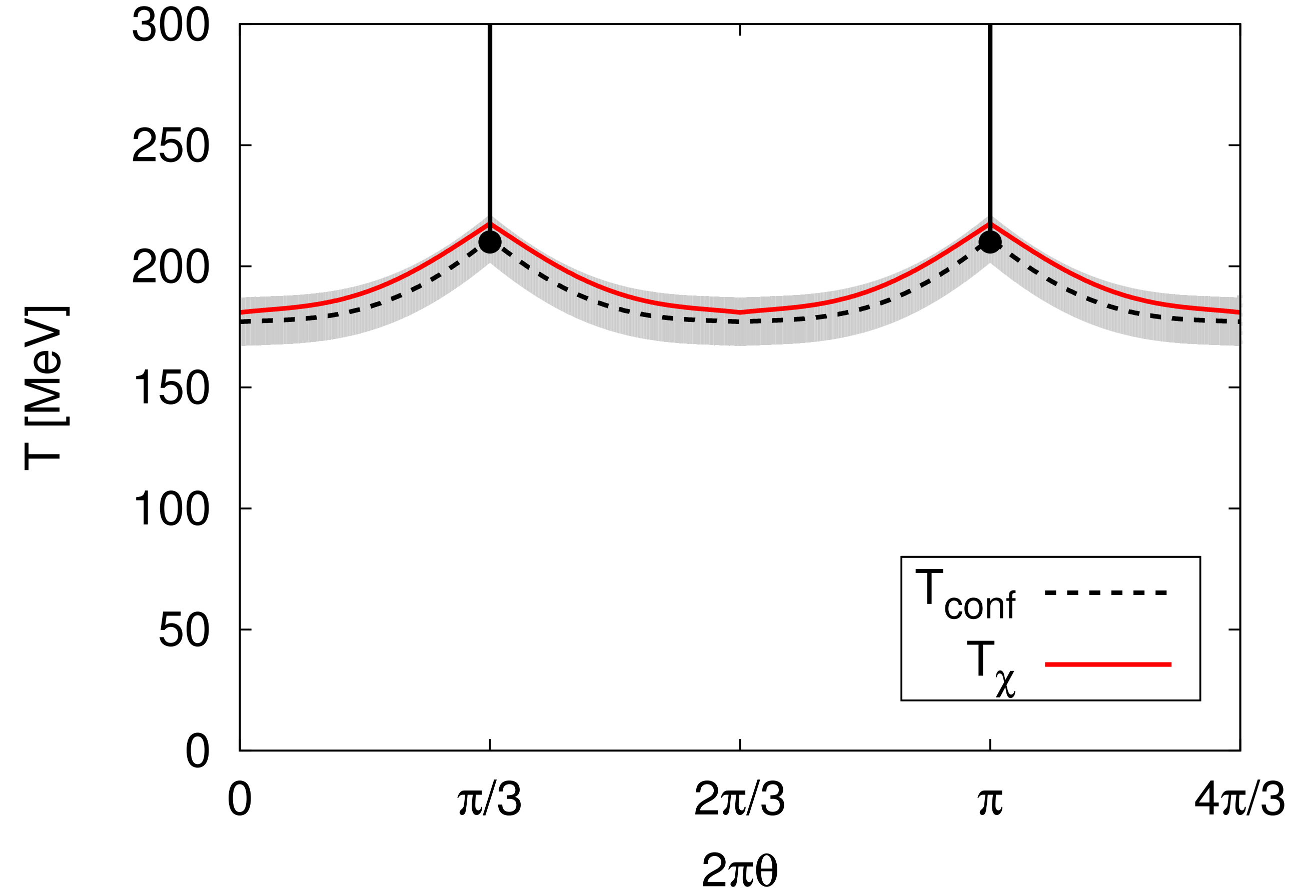


2 flavors & chiral limit

vanishing density



imaginary chemical potential



Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

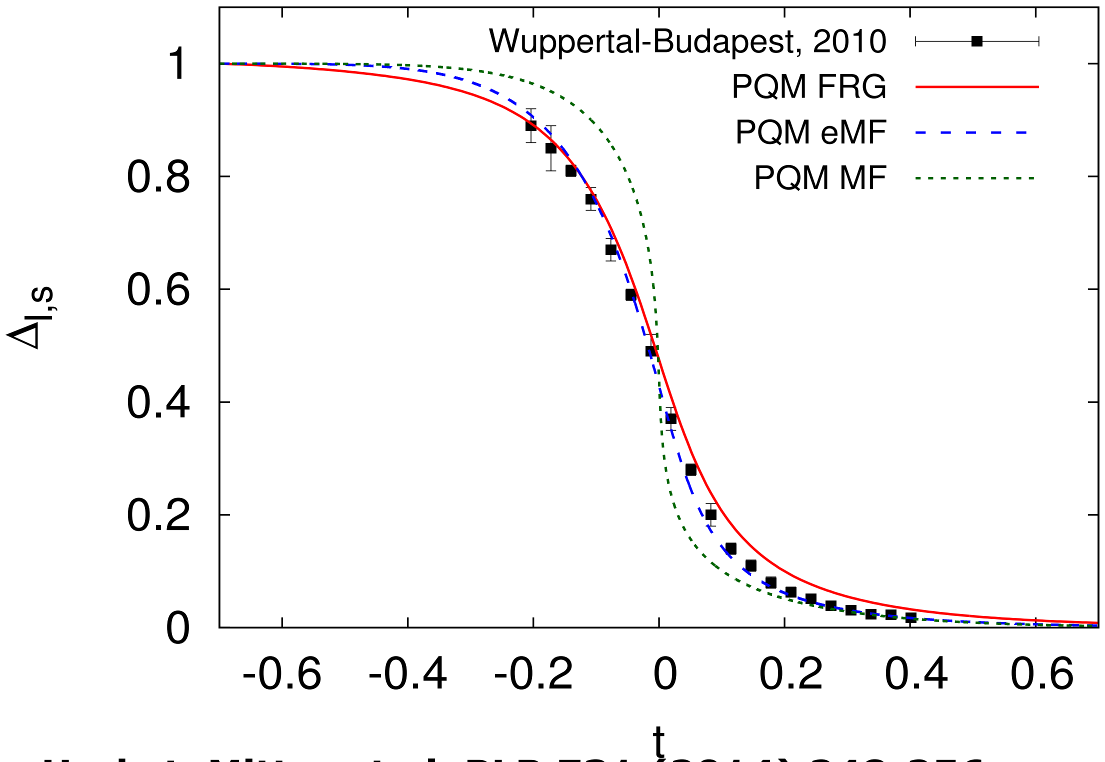
Flows towards the fluctuating PQM model for low energies

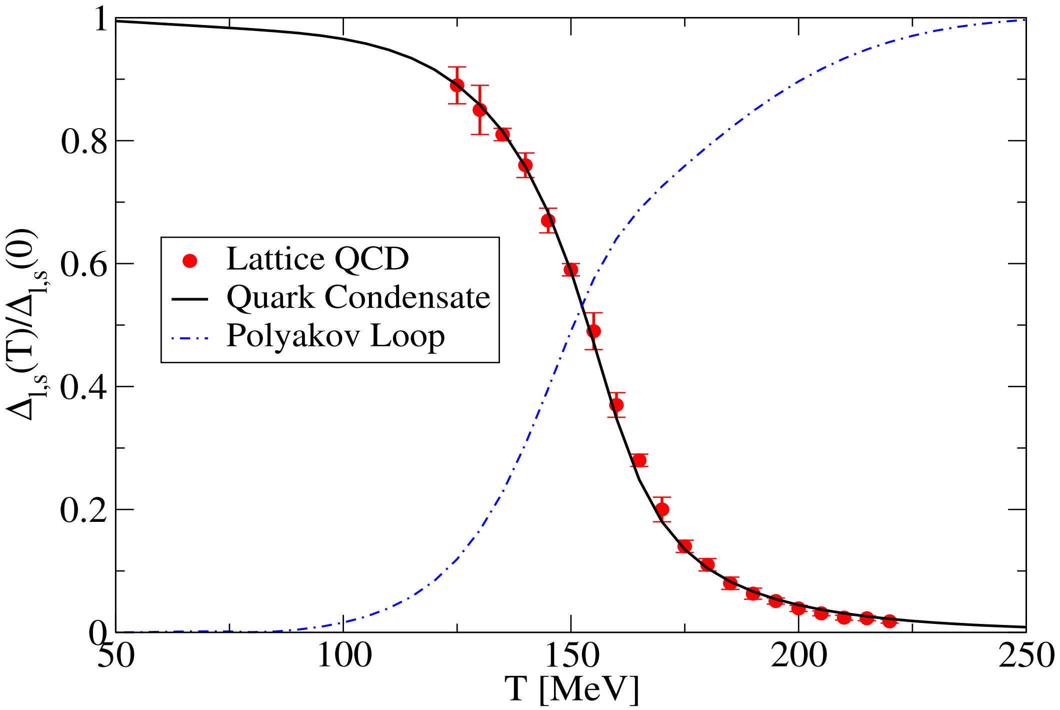
Thermodynamics and condensates

2+1 flavor QCD - enhanced PQM-model

2+1 flavor DSE

reduced chiral condensate





Herbst, Mitter et al, PLB 731 (2014) 248-256

Fischer, Luecker, Welzbacher, PRD 90 (2014), 034022

Fischer, Fister, Luecker, JMP, PLB 732 (2014) 273-277

Glue potential from QCD-computation with FRG

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

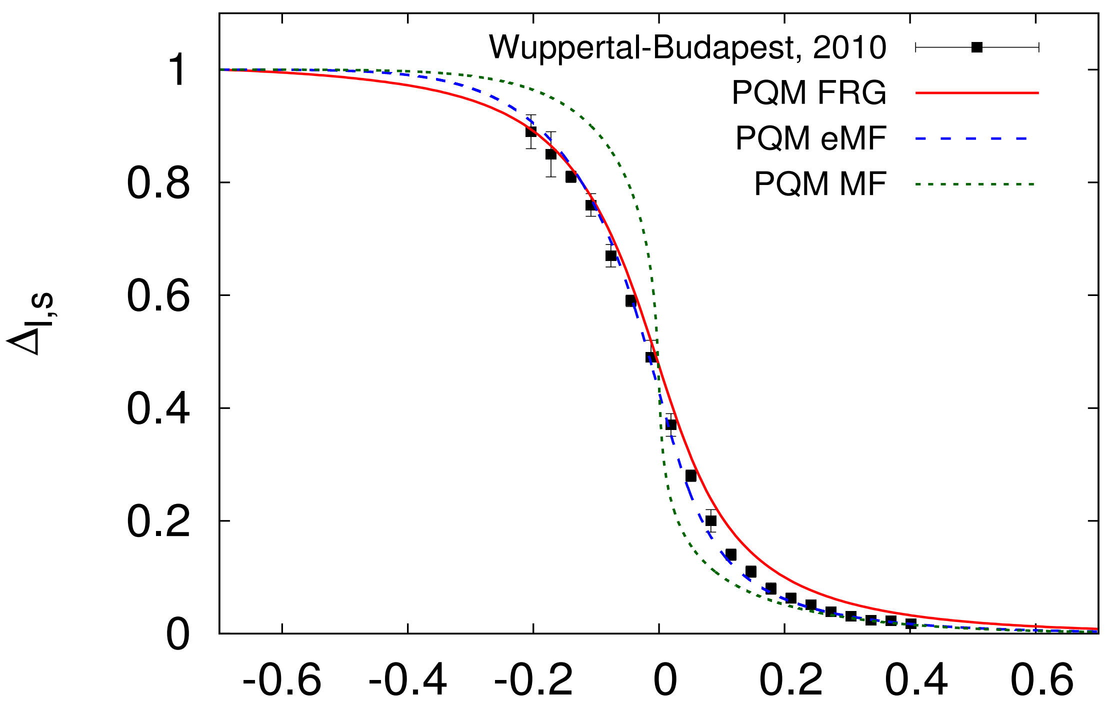
Approximations of infrared dynamics involved

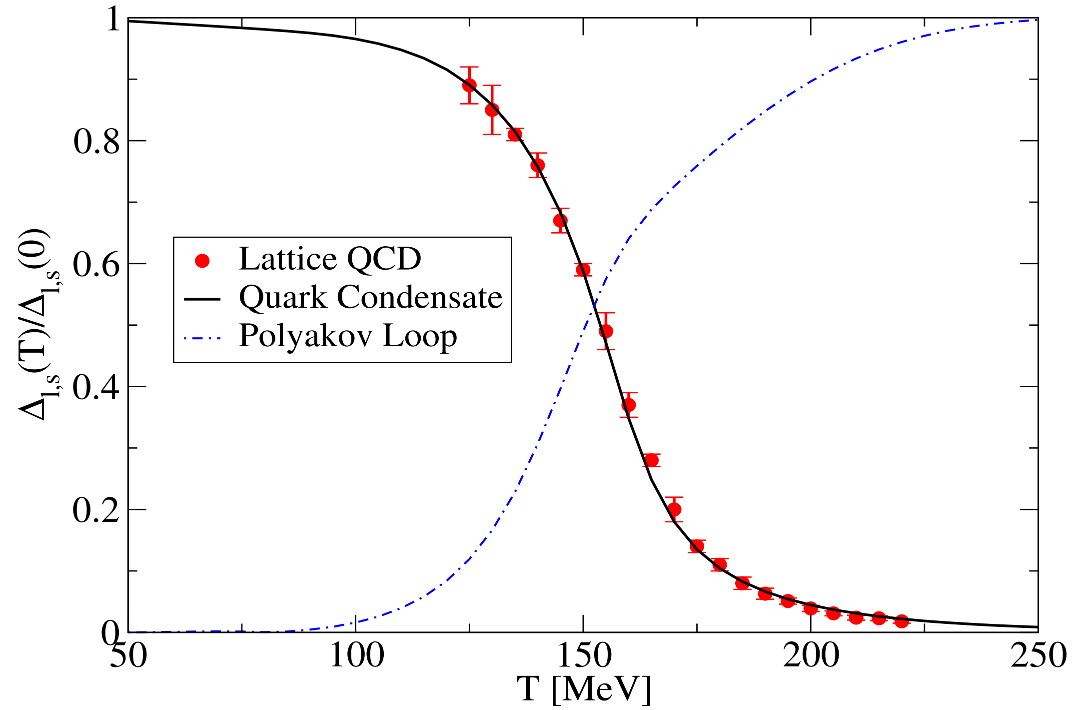
Thermodynamics and condensates

2+1 flavor QCD - enhanced PQM-model

2+1 flavor DSE

reduced chiral condensate





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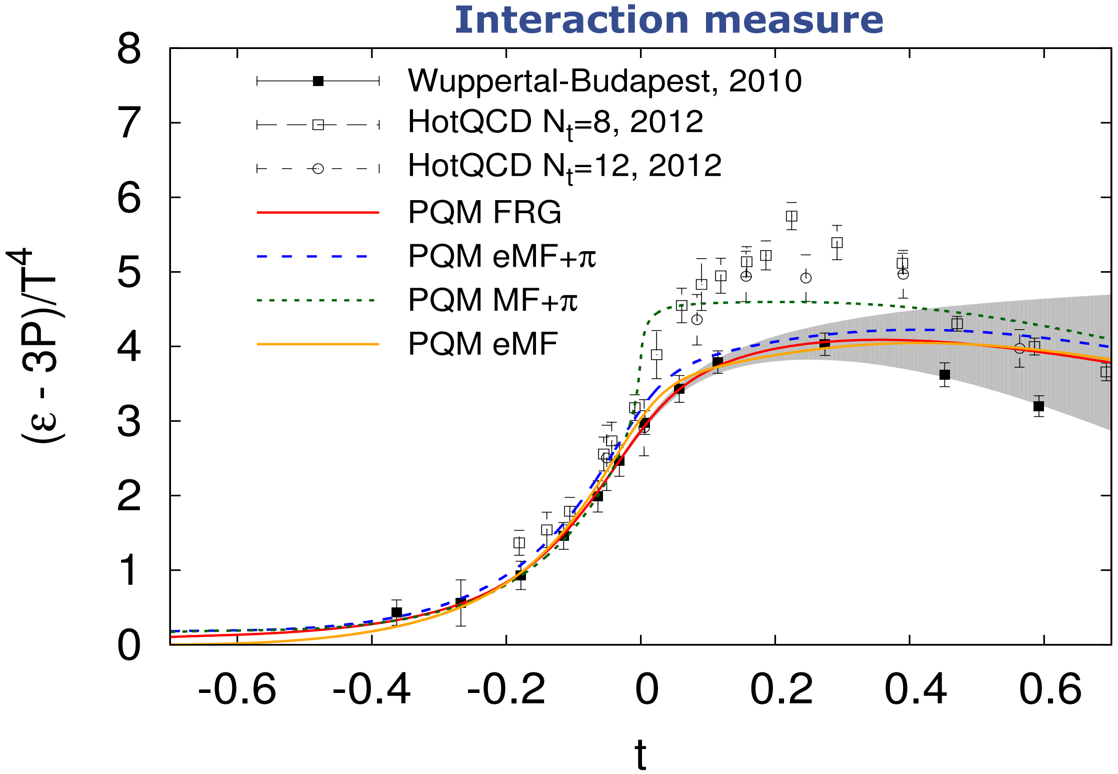
Fischer, Luecker, Welzbacher, PRD 90 (2014), 034022

Fischer, Fister, Luecker, JMP, PLB 732 (2014) 273-277

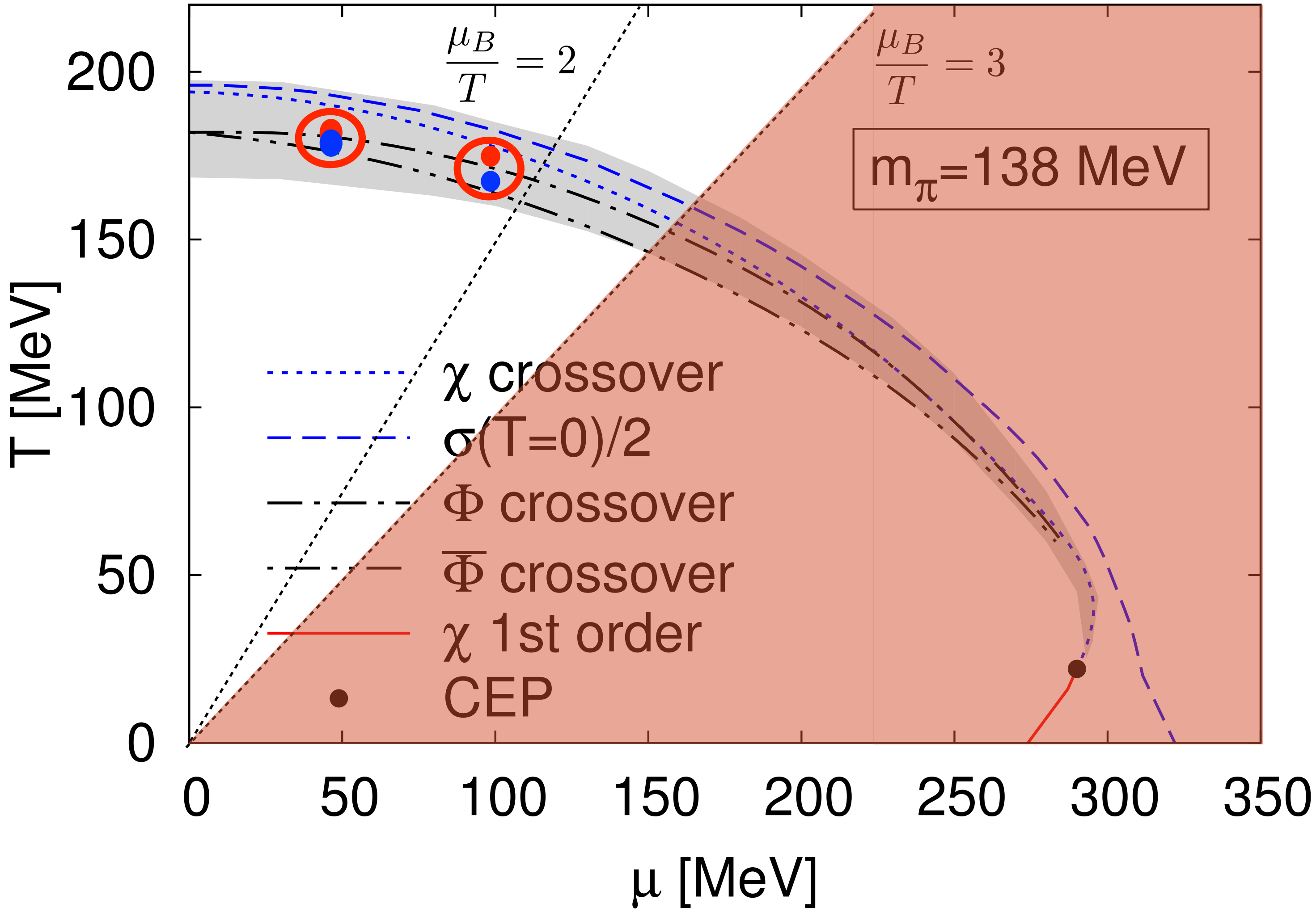
Interaction measure



Shaded area:
systematic error estimate
due to low initial UV scale 1 GeV

QCD at finite density

Phase diagram of the QCD-enhanced PQM model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

FRG QCD results at finite density

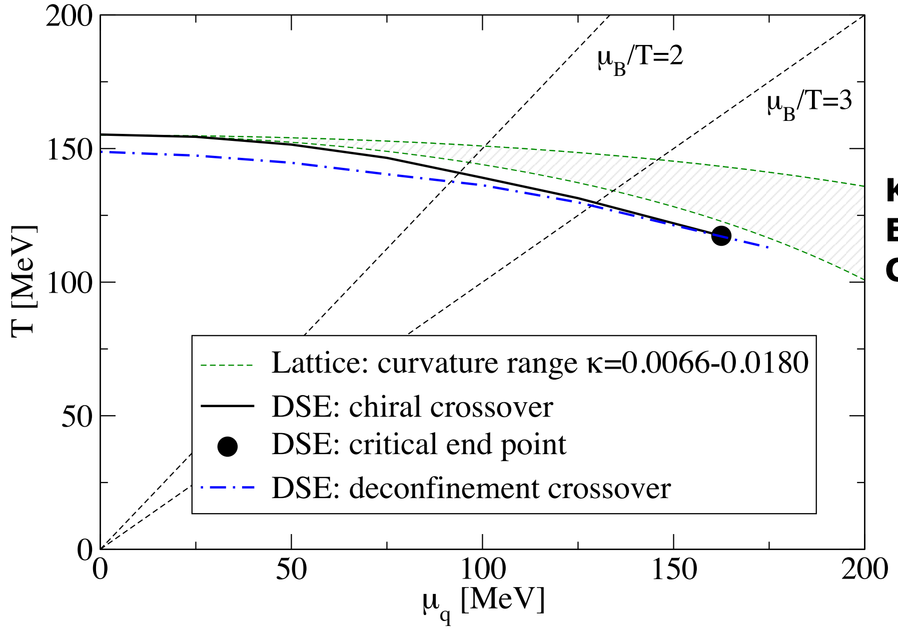
Haas, Braun, JMP ’09, unpublished

Extension of FRG QCD results at imaginary chemical potential

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

Phase structure at finite density

Phase diagram of 2+1 flavor QCD



Kaczmarek at al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

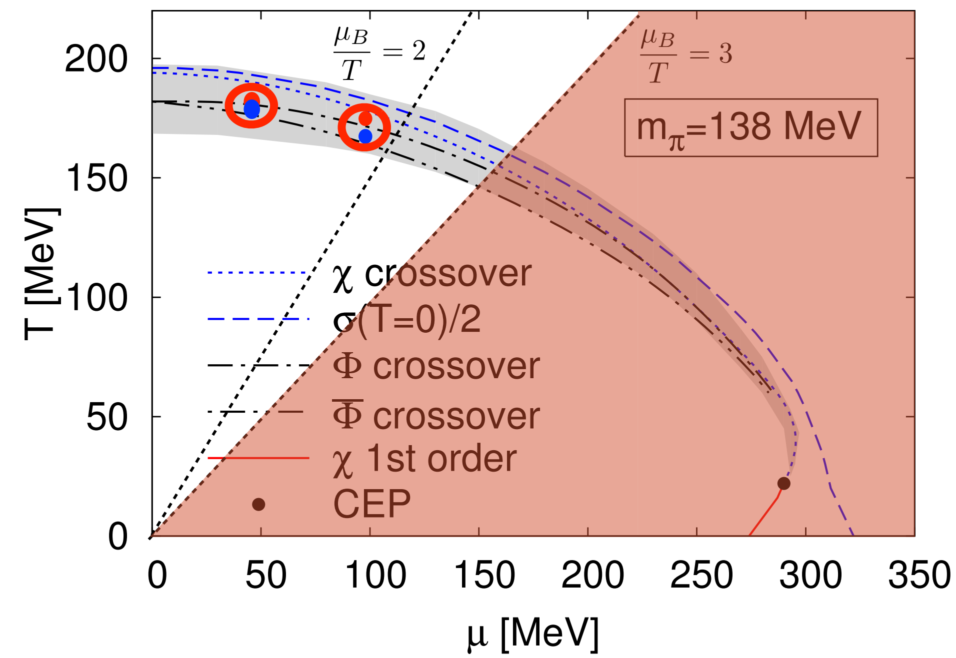
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, Fischer, Welzbacher, PRD 93 (2014) 034013

Chiral phase structure

Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301

Phase diagram of QCD-enhanced 2-flavor PQM-model



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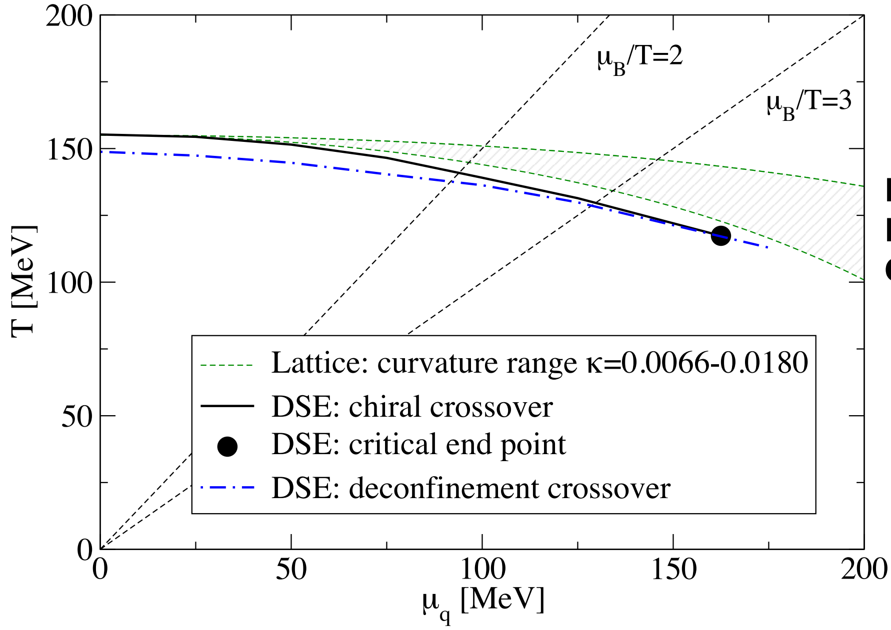

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Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

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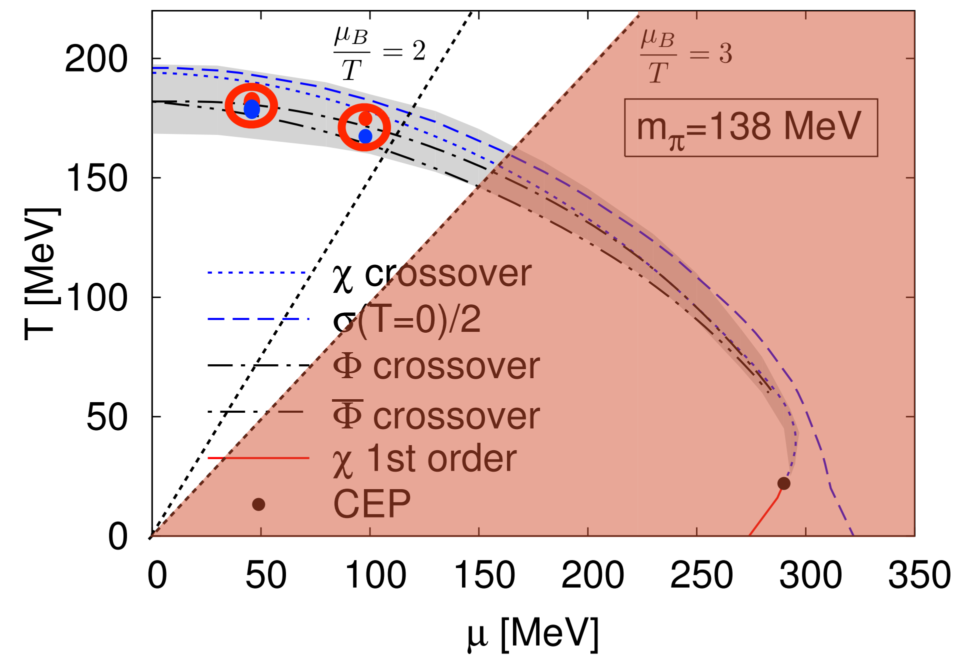
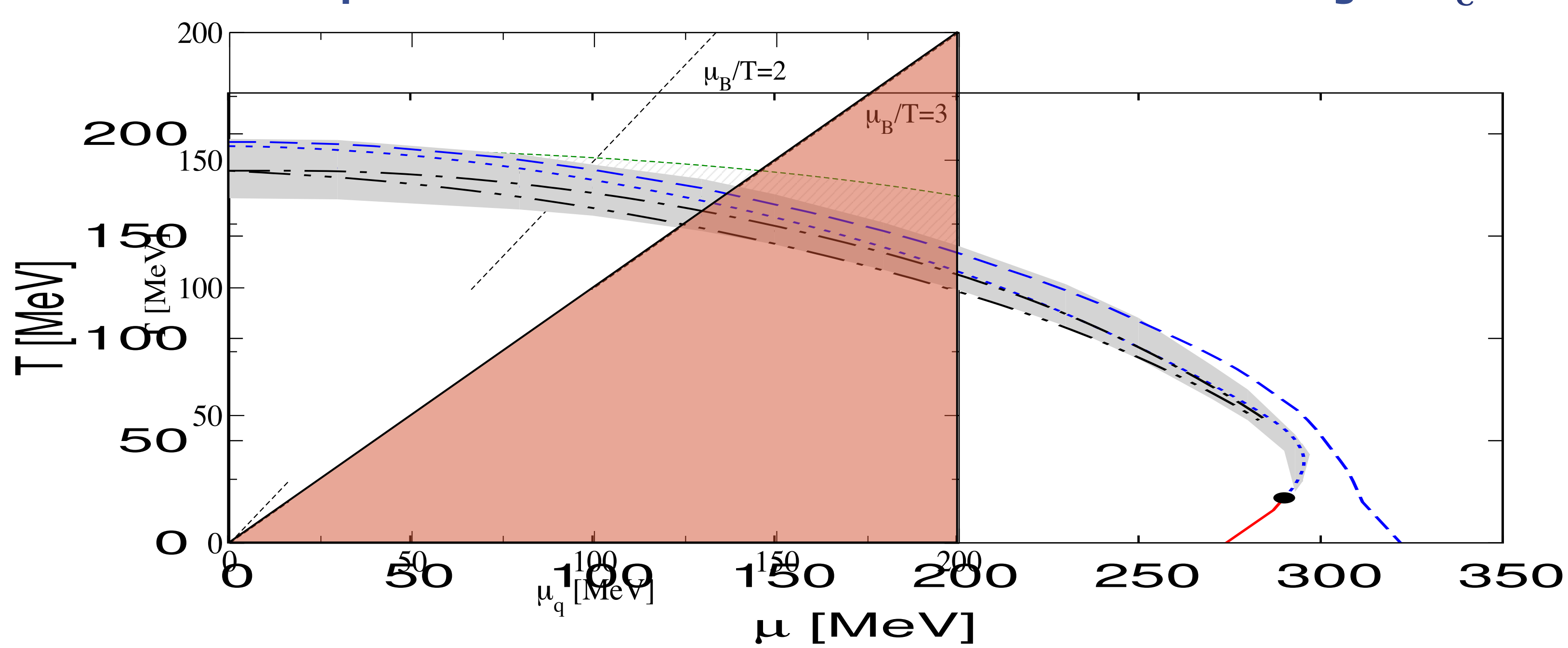


Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

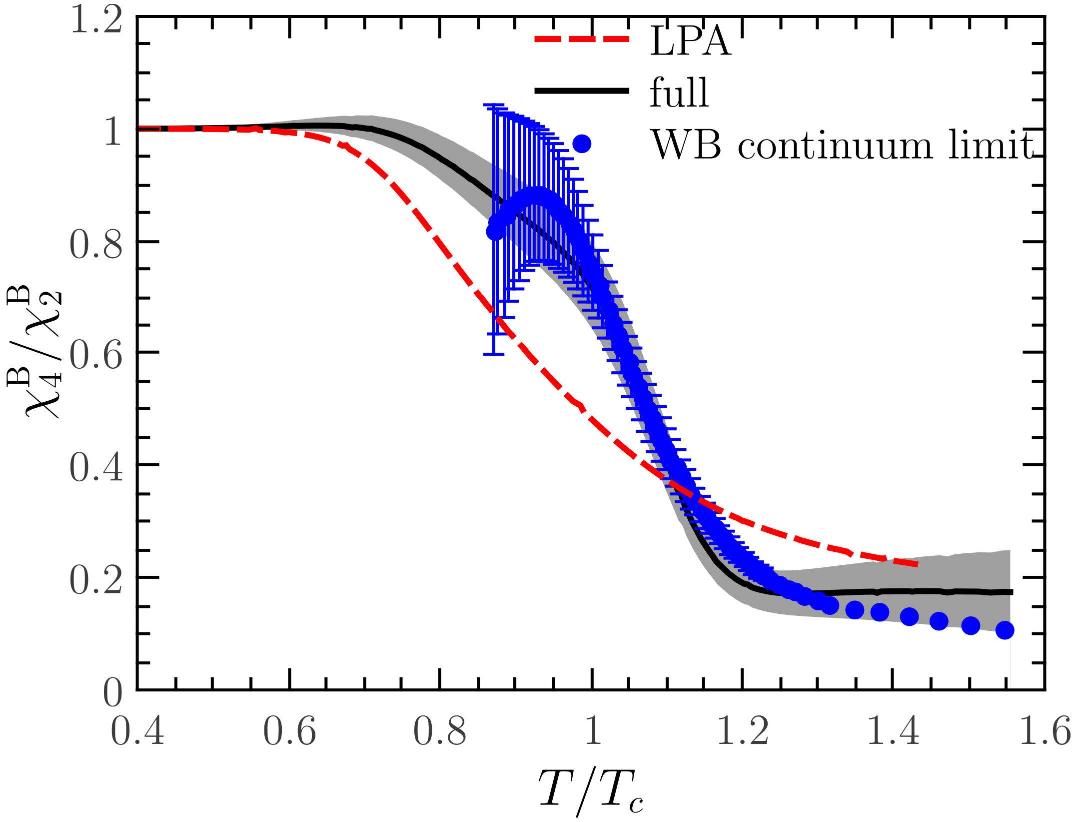
FRG QCD results at finite density

Haas, Braun, JMP ’09, unpublished

Comparison with 2 flavor vs 2+1 flavor scale matching of T_c



Fluctuations as a measure of confinement



Fu, JMP, PRD 92 (2015) 116006

$$\chi_n^{\mathrm{B}} = \frac{\partial^n}{\partial(\mu_{\mathrm{B}}/T)^n}\frac{p}{T^4}$$

Skewness, Kurtosis

$$\sigma^2 = VT^3\chi_2^{\mathrm{B}} \qquad S = \frac{\chi_3^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma} \qquad \kappa = \frac{\chi_4^{\mathrm{B}}}{\chi_2^{\mathrm{B}}\sigma^2}$$

Karsch, Schaefer, Wagner, Wambach, PLB 698 (2011) 256

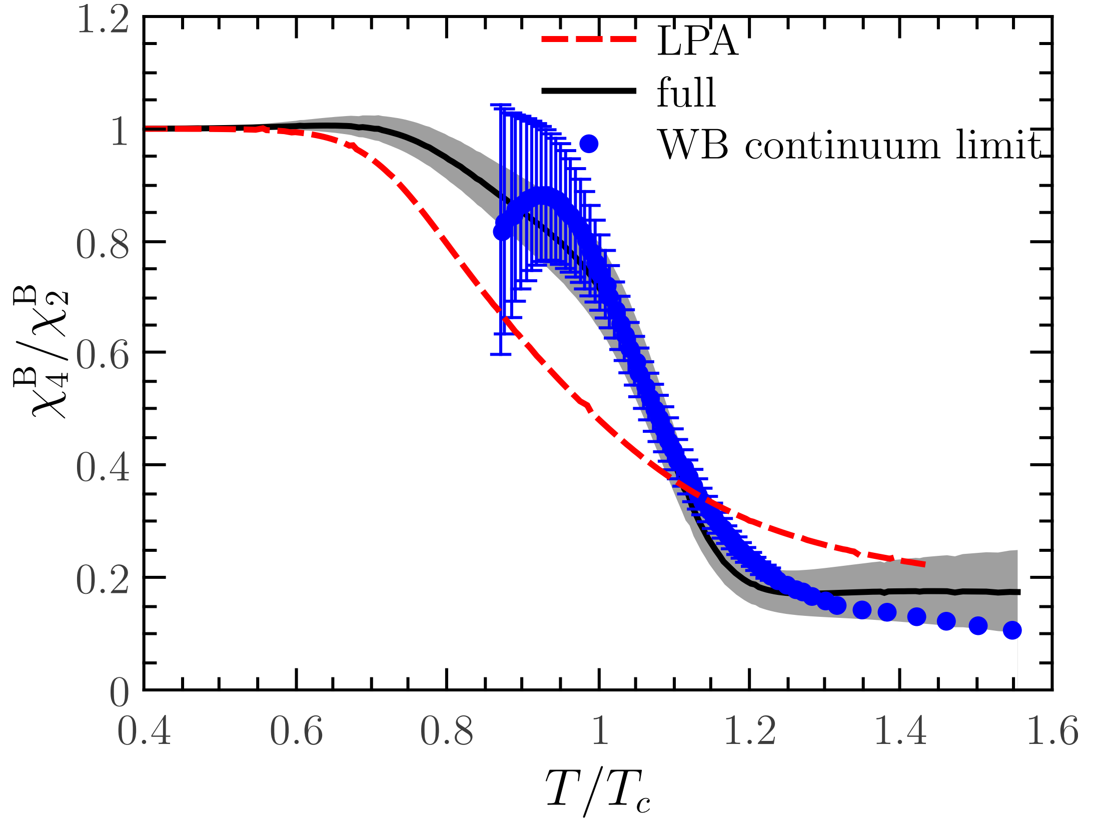
Friman, Karsch, Redlich, Skokov, EPJ C71 (2011) 1694

Schaefer, Wagner, PRD 85 (2012) 034027

Skokov, Friman, Redlich, PRC 88 (2013) 034911

Almasi, Friman, Redlich, Nucl.Phys. A956 (2016) 356-359

Fluctuations as a measure of confinement



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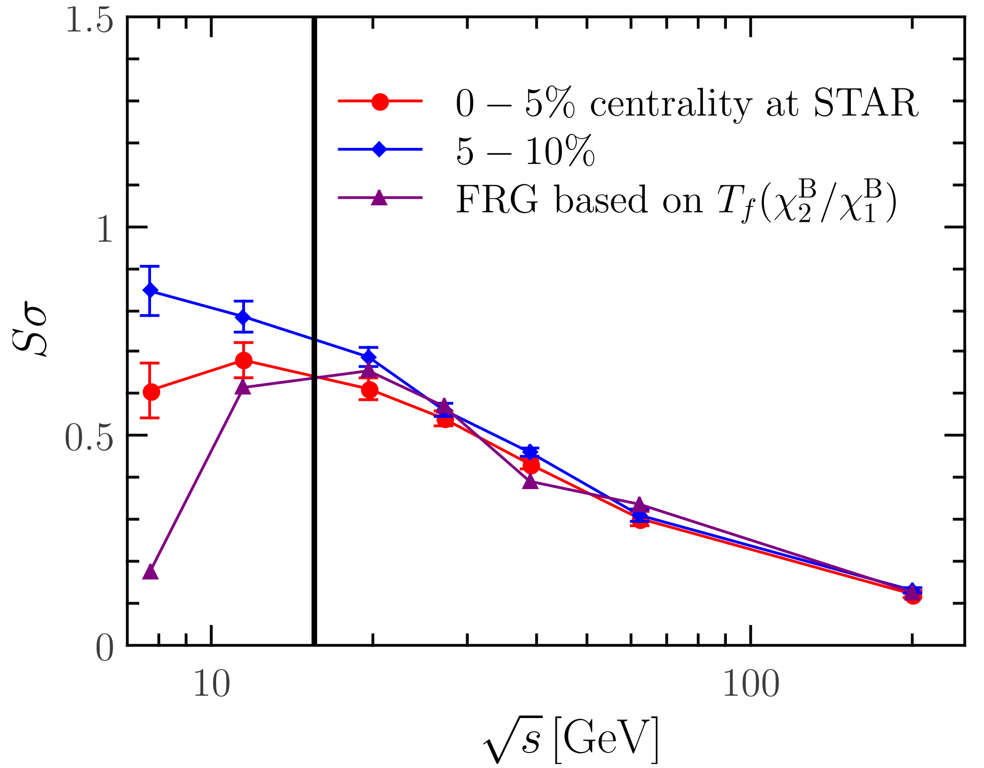
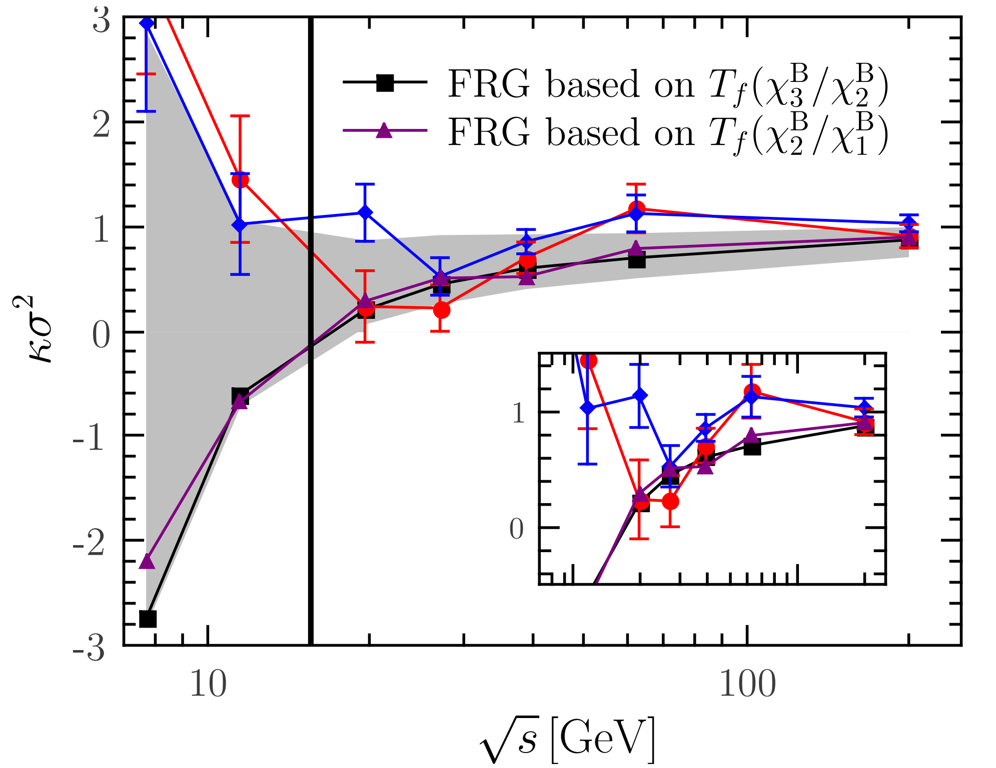
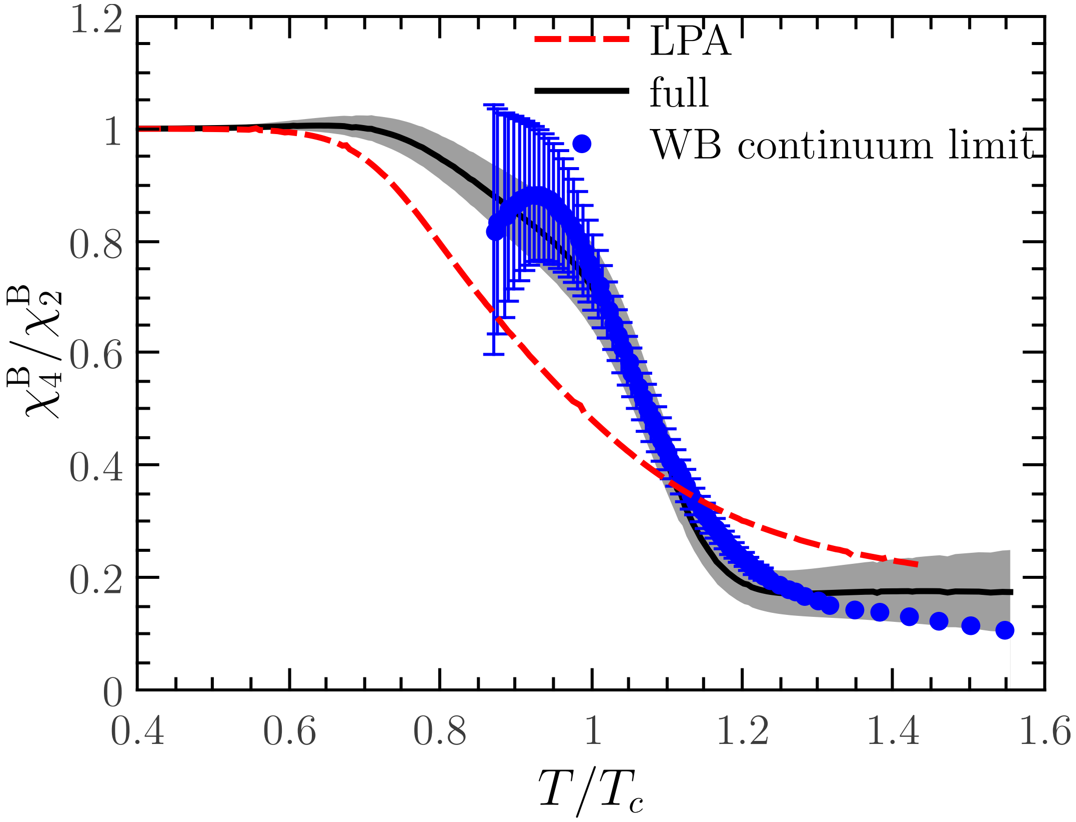




Fu, JMP, PRD 93 (2016 091501

Fluctuations as a measure of confinement



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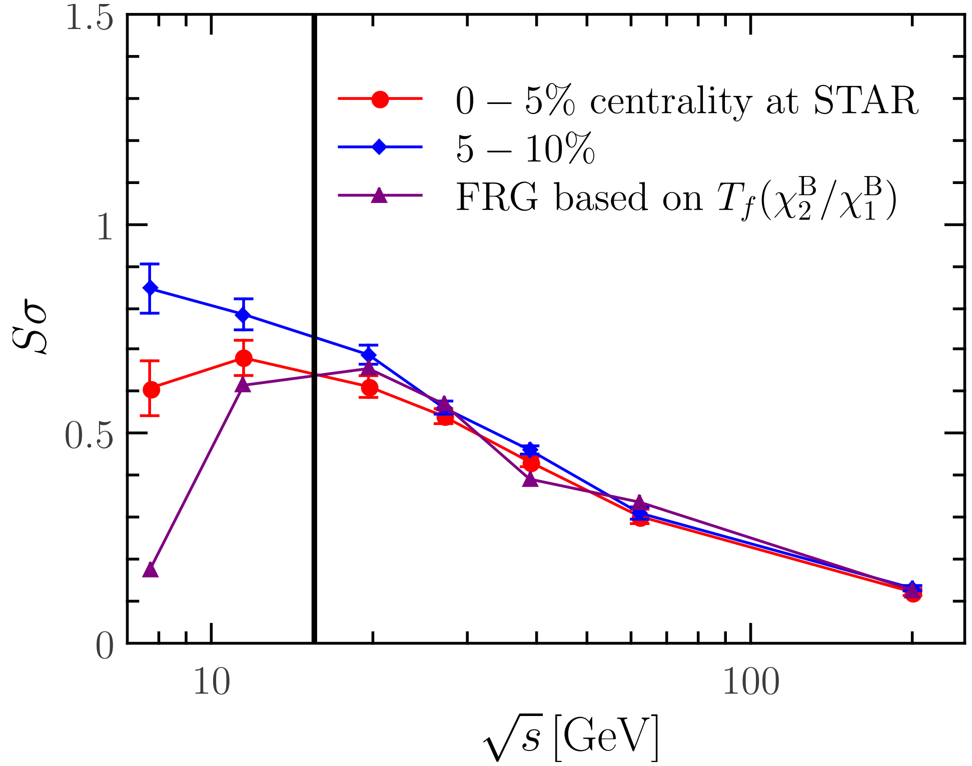

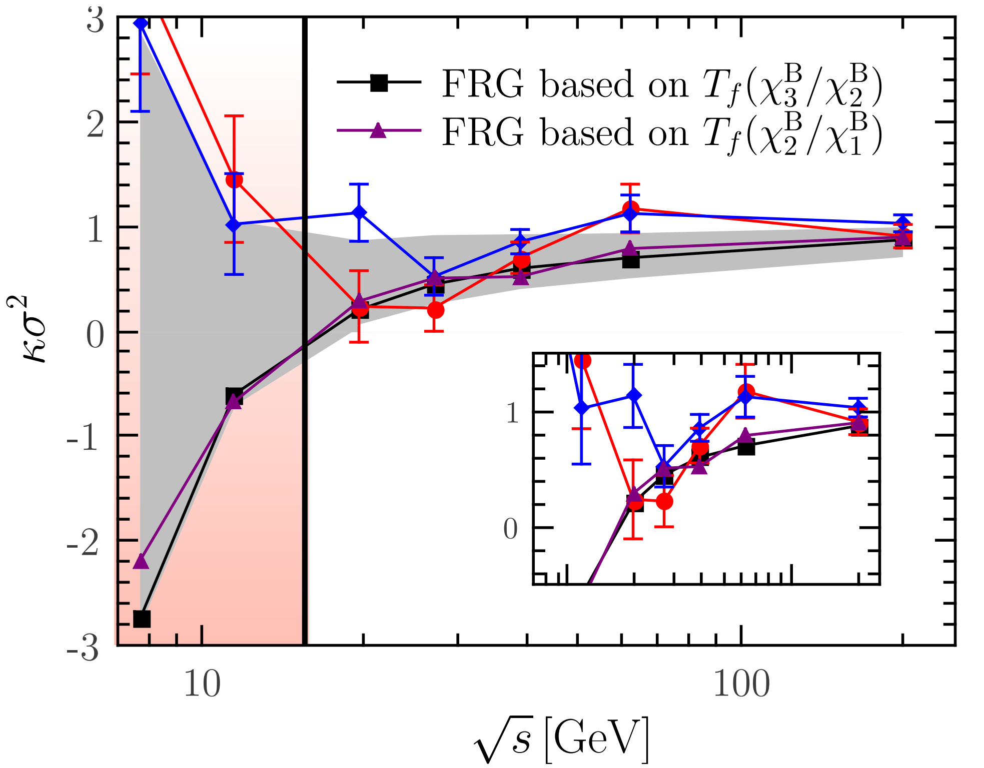

Fu, JMP, PRD 93 (2016 091501

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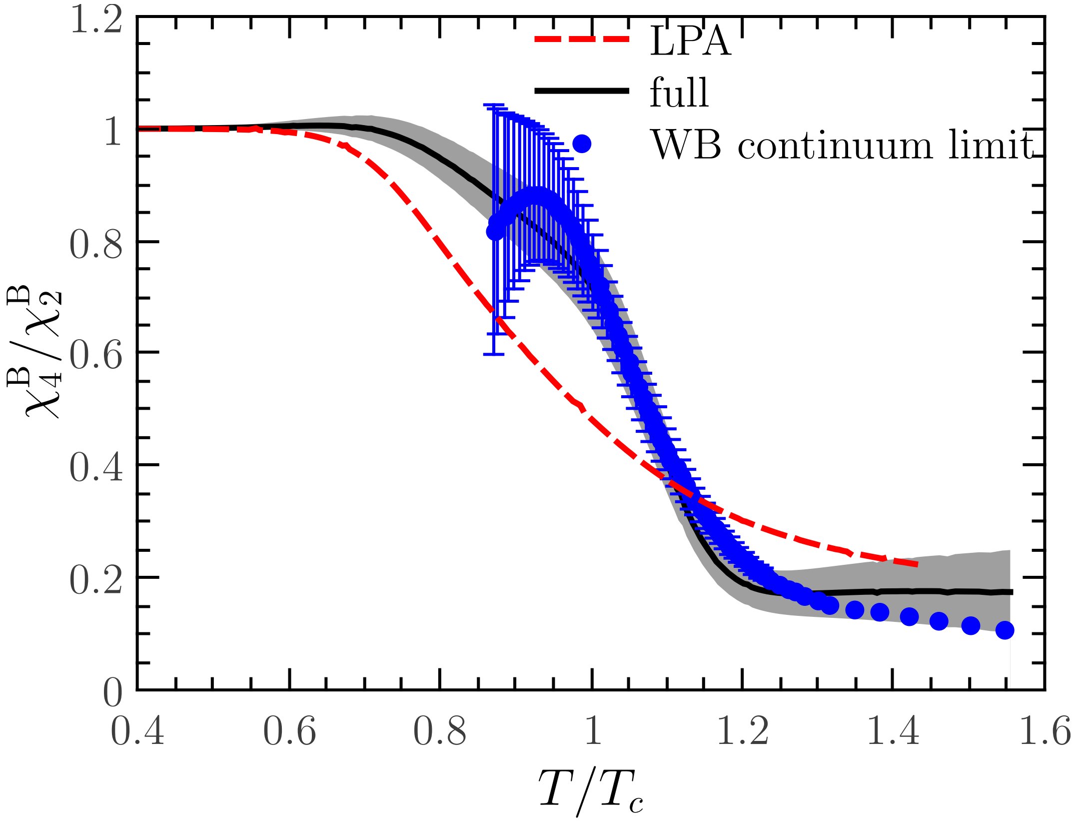

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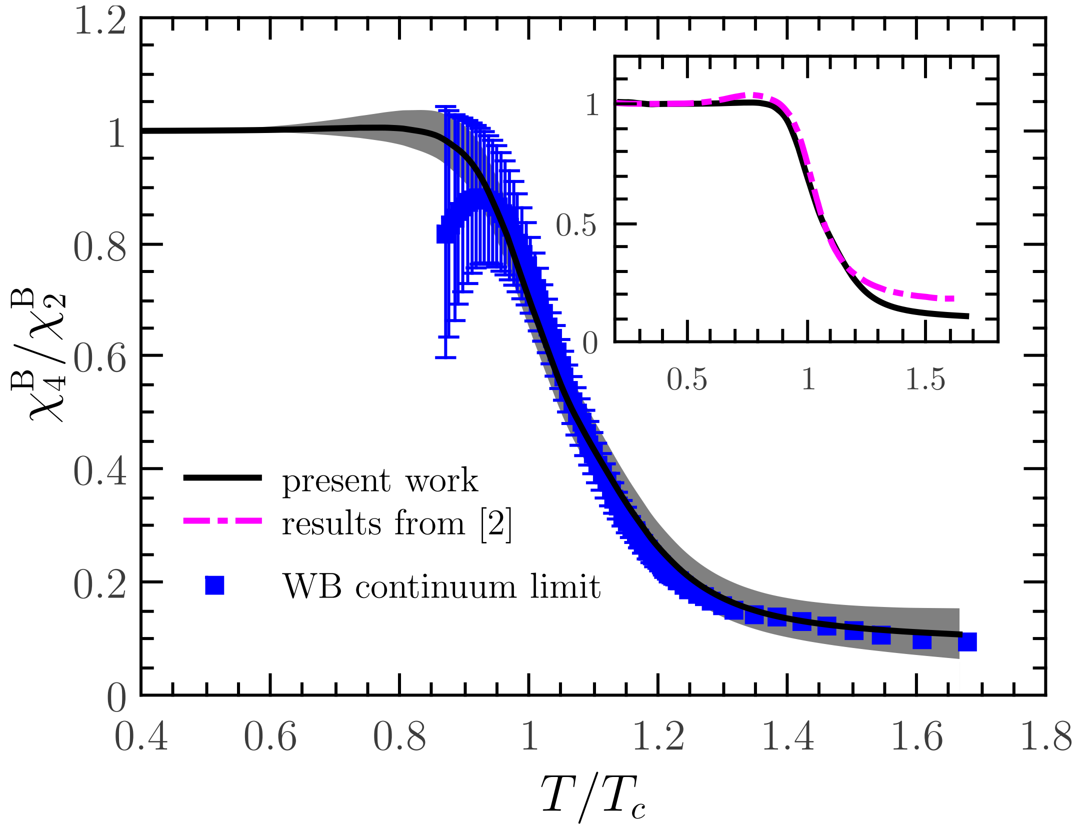
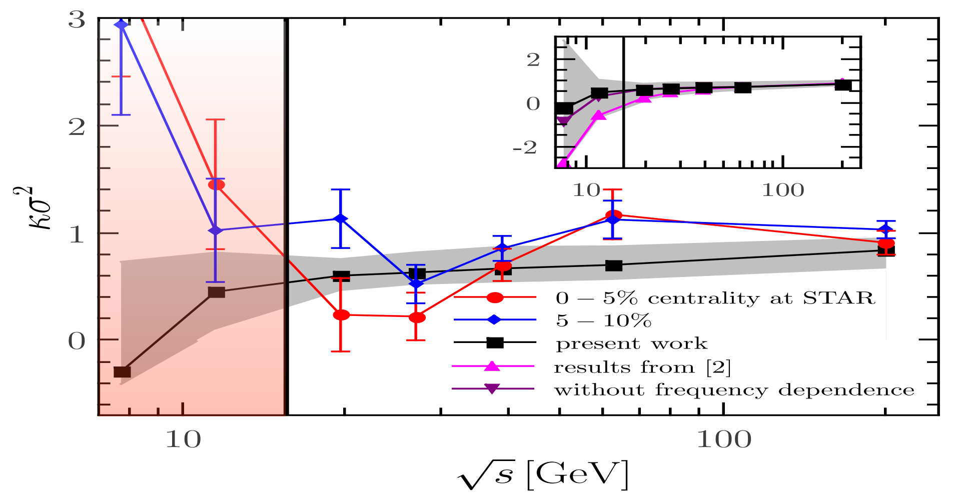




Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 11, 116020

Outline

- Introduction
- Phase structure of QCD
  - Confinement & chiral symmetry breaking
  - Finite temperature correlation functions
  - QCD at finite density & fluctuations
- QCD transport
  - Real time correlation functions
  - Single particle spectral functions
  - transport coefficients
- Summary & outlook

Threefold way to transport

Real-Time

MEM & Real-time diagrams

MEM & lattice

Threefold way to transport

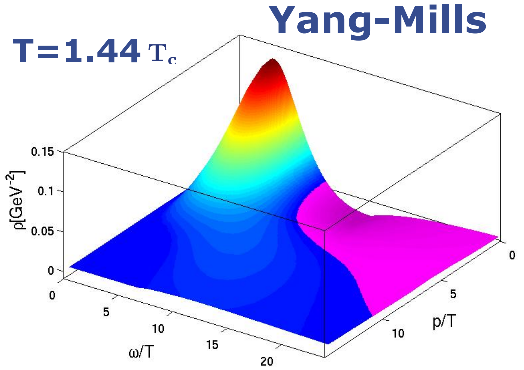
Real-Time

MEM & Real-time diagrams

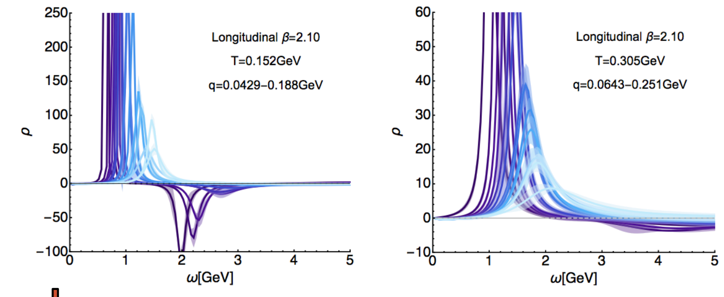
MEM & lattice

single particle spectral functions

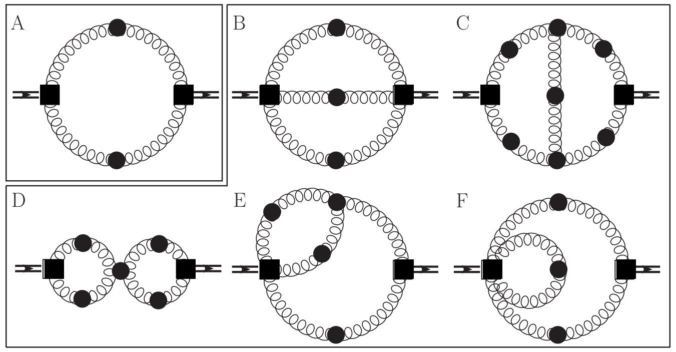
T=1.44 T_c **Yang-Mills**



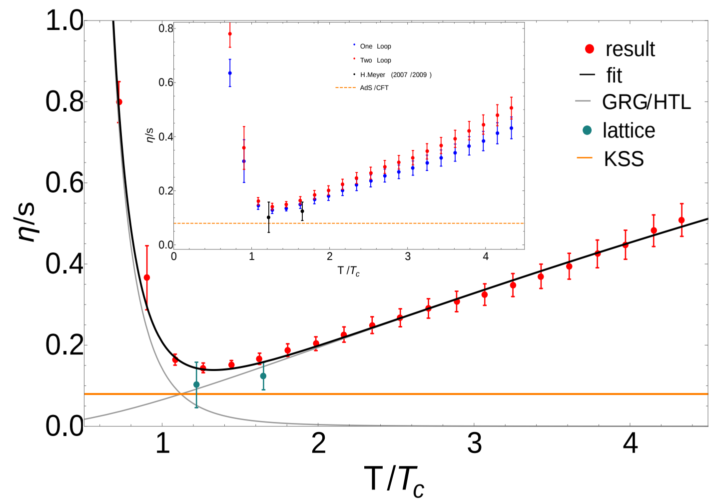
2+1+1 flavour QCD



diagrammatic representation of transport coefficients



transport coefficients



Transport

Haas, Fister, JMP, PRD 90 (2014) 9, 091501
Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002
JMP, Rosenblüh, Strodthoff; in prep.

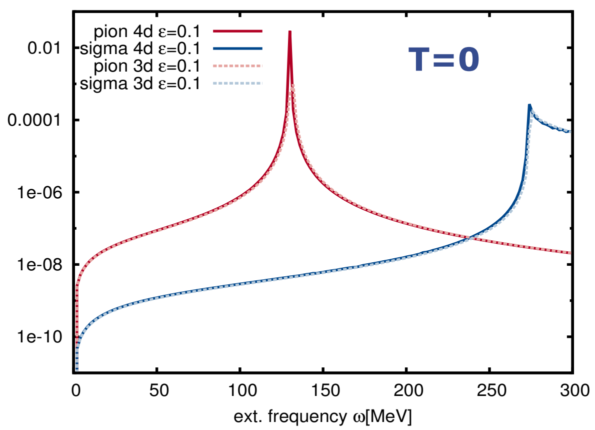
MEM-type reconstruction

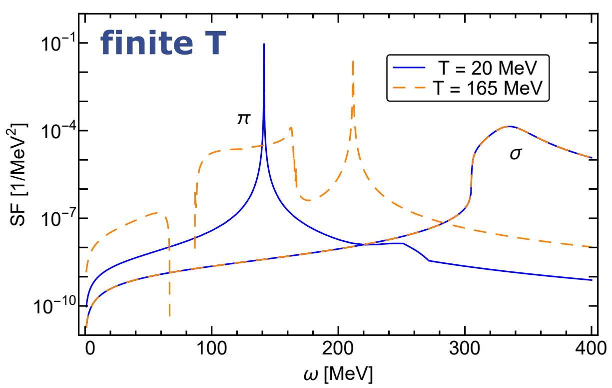
Rothkopf, arXiv:1611:00482
Ilgenfritz, JMP, Rothkopf, Trunin; arXiv:1701.08610
Fischer, JMP, Rothkopf, Welzbacher, arXiv:1705.03207

Threefold way to transport

Real-Time

Real time correlation functions

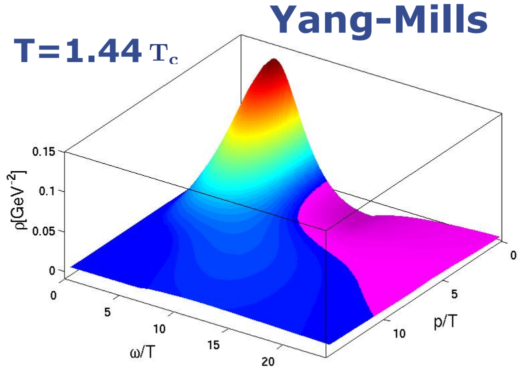




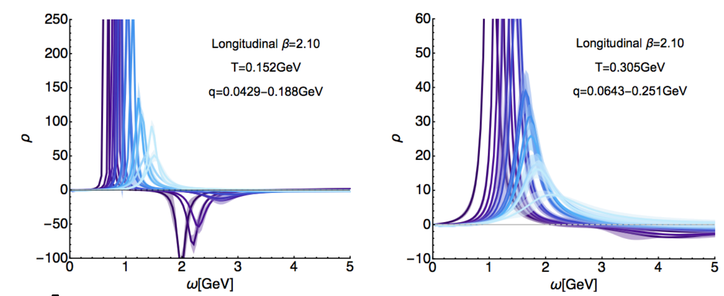
MEM & Real-time diagrams

single particle spectral functions

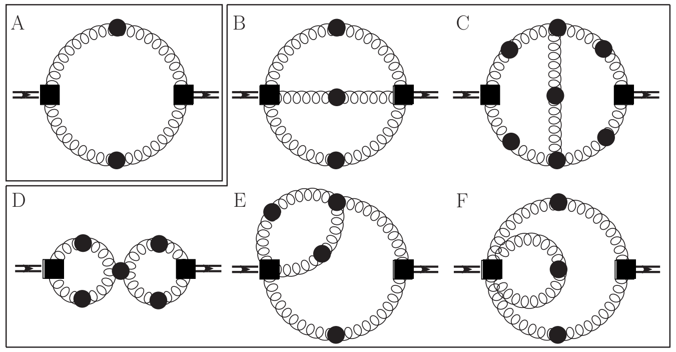
Yang-Mills



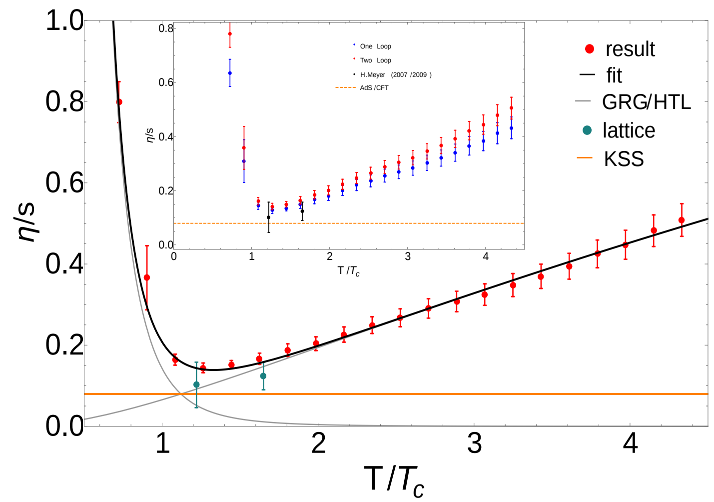
2+1+1 flavour QCD



diagrammatic representation of transport coefficients



transport coefficients



MEM & lattice

Real time:

JMP, Strodthoff, PRD 92 (2015) 094009
Strodthoff, Phys.Rev. D95 (2017) no.7, 076002
JMP, Strodthoff, Wink; in prep.

Transport

Haas, Fister, JMP, PRD 90 (2014) 9, 091501
Christiansen, Haas, JMP, Strodthoff,
PRL 115 (2015) 11, 112002
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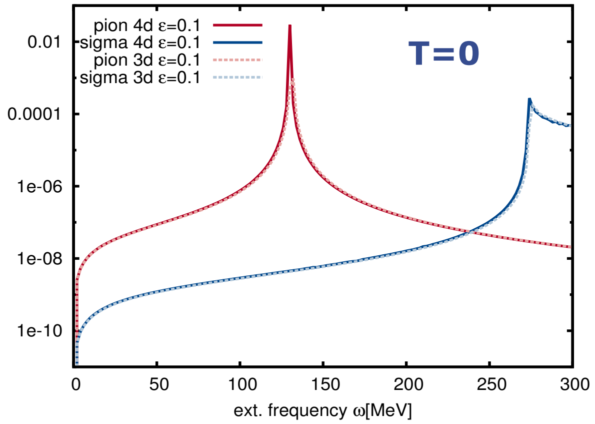
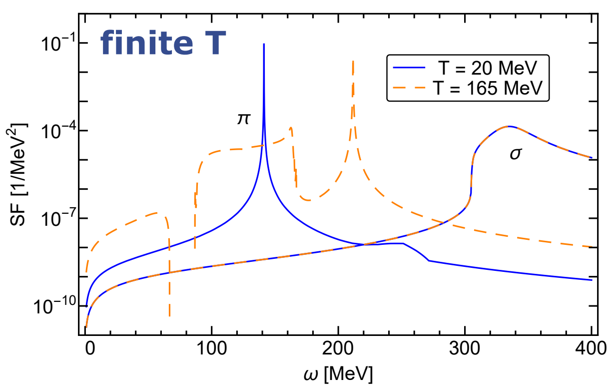
Threefold way to transport

Real-Time

Real time correlation functions

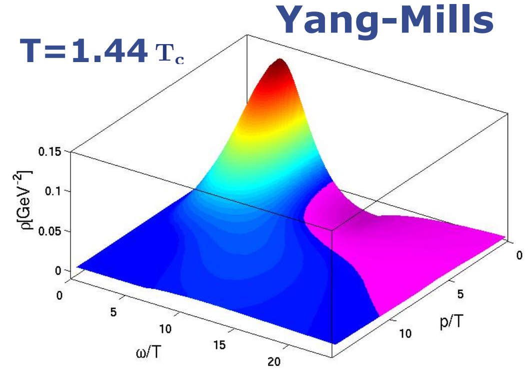




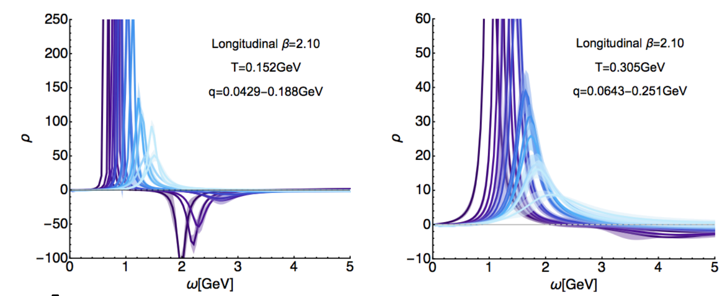
MEM & Real-time diagrams

single particle spectral functions

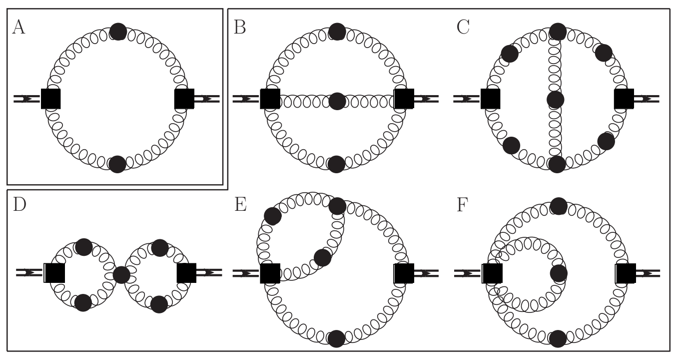
Yang-Mills



2+1+1 flavour QCD



diagrammatic representation of transport coefficients



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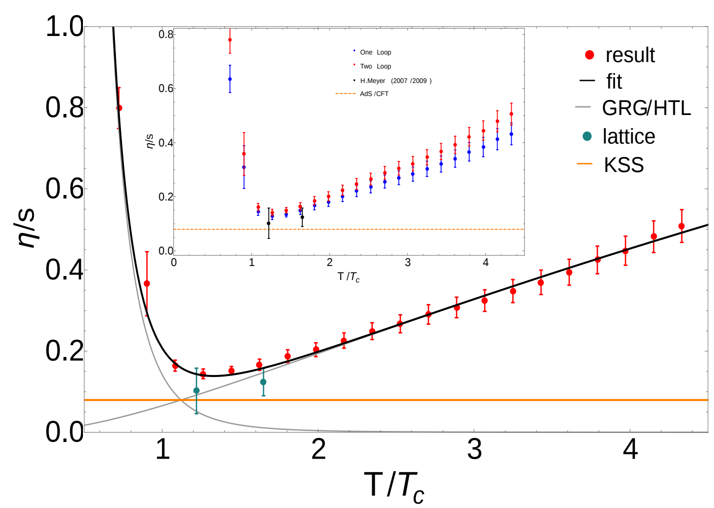
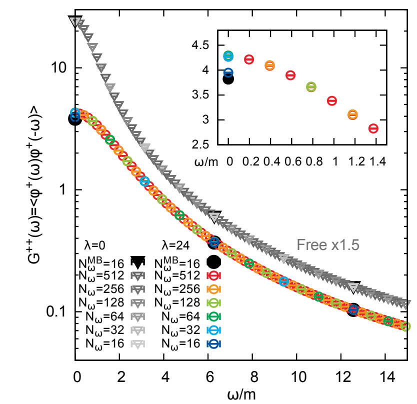
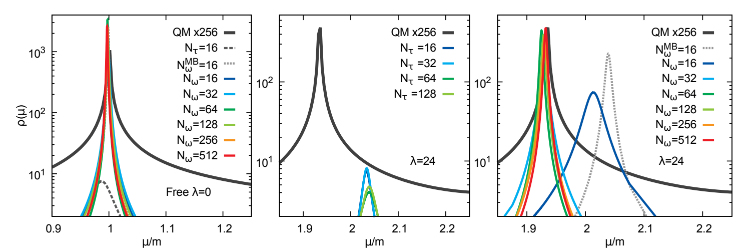


MEM & lattice

exponentially improved MEM

finite T spectral functions





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Christiansen, Haas, JMP, Strodthoff,
PRL 115 (2015) 11, 112002
JMP, Rosenblüh, Strodthoff; in prep.

Imag. time lattice at finite T

JMP, Rothkopf, arXiv:1610:09531
JMP, Rothkopf, work in progress
JMP, Rothkopf, Ziegler, work in progress

MEM-type reconstruction

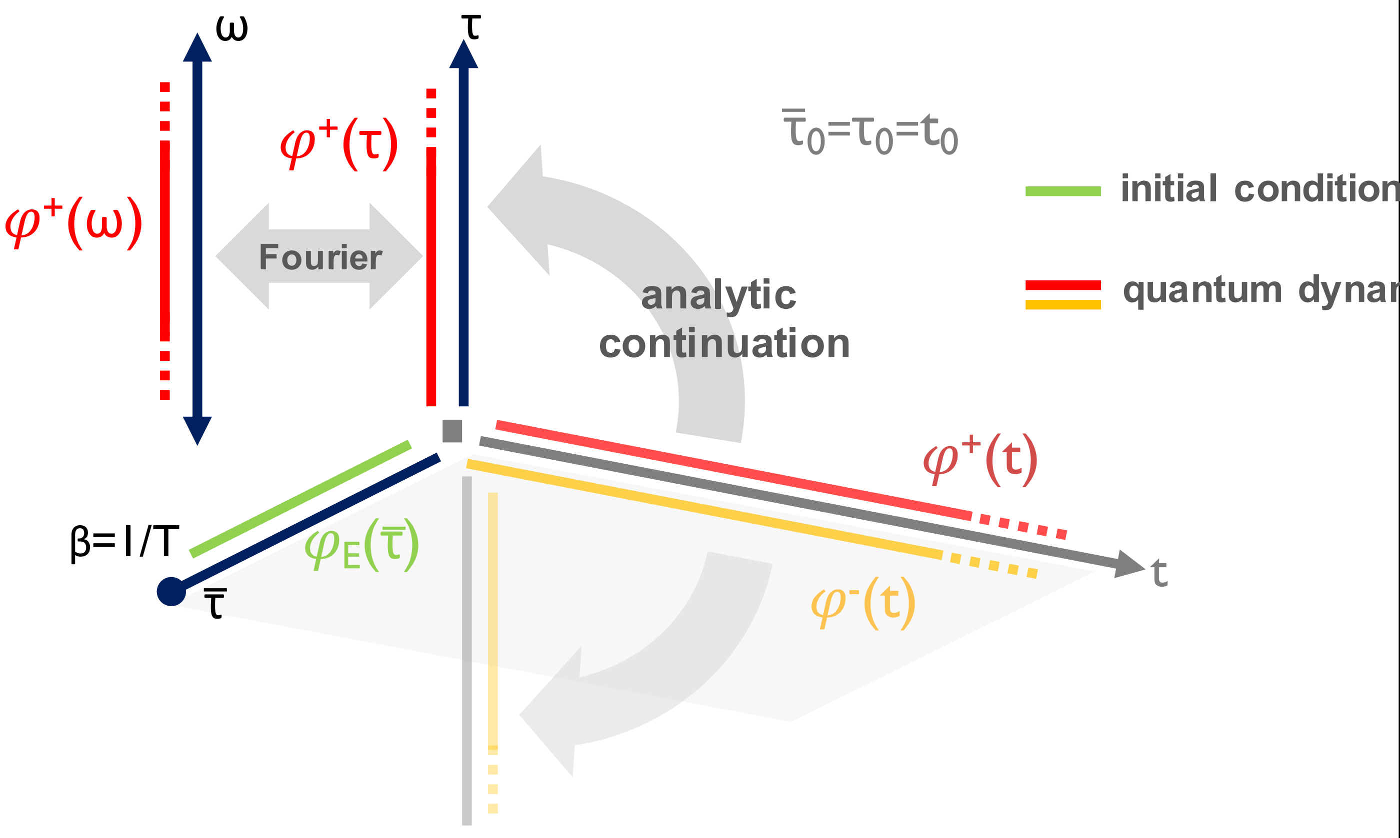
Rothkopf, arXiv:1611:00482
Ilgenfritz, JMP, Rothkopf, Trunin; arXiv:1701.08610
Fischer, JMP, Rothkopf, Welzbacher, arXiv:1705.03207

thermal spectral functions on the lattice

$$\underbrace{\int [d\varphi_0^+][d\varphi_0^-]\langle\varphi_0^+|e^{-\beta\hat{H}}|\varphi_0^-\rangle}_{\text{initial conditions}} \underbrace{\int_{\varphi_0^+}^{\varphi_0^-} \mathcal{D}\varphi e^{iS_M[\varphi^+]-iS_M[\varphi^-]}}_{\text{quantum dynamics}}$$

Stochastic quantisation

$$\partial_{t_5}\varphi^+(\omega_l) = -\frac{\delta S_0}{\delta\varphi^+(\omega_l)} - \frac{\delta S_E^{\rm int}}{\delta\varphi^+(\tau_j)}\frac{\delta\varphi^+(\tau_j)}{\delta\varphi^+(\omega_l)} + \eta(\omega_l)$$



JMP, Rothkopf, arXiv:1610:09531

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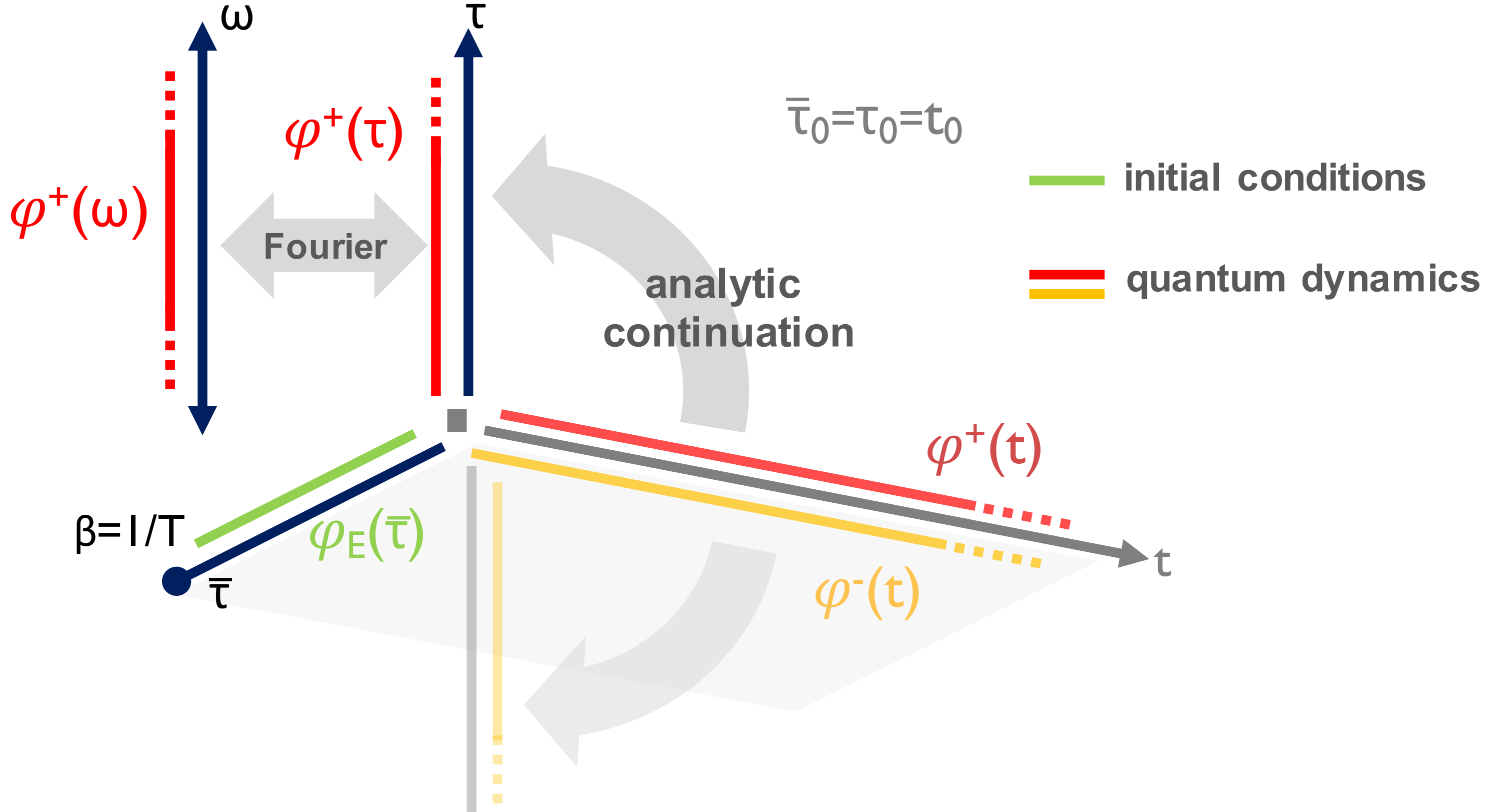
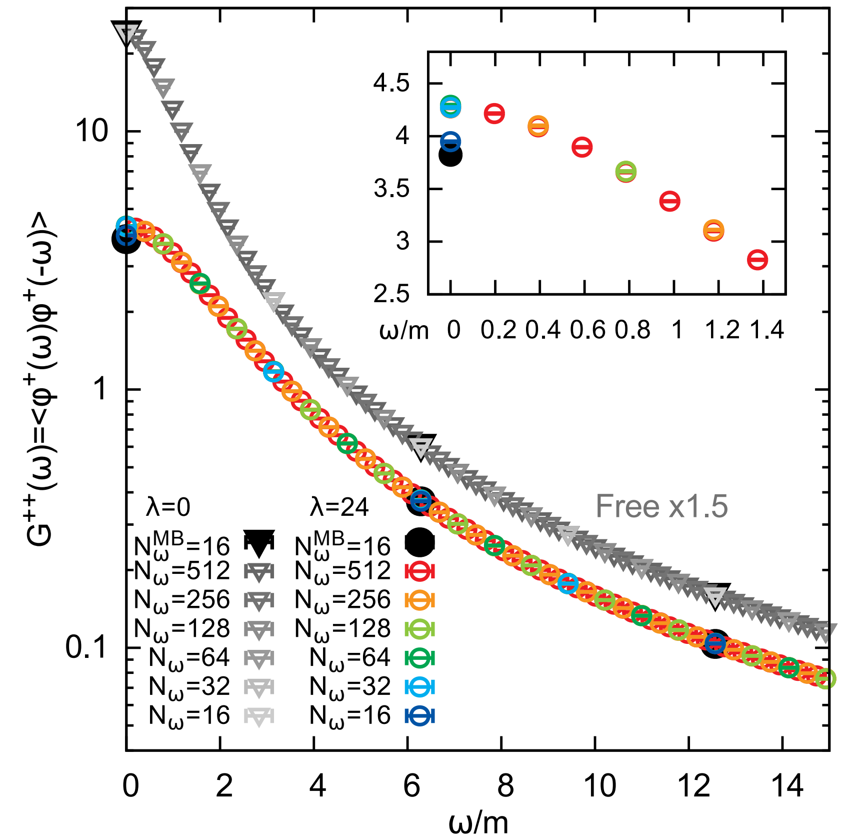
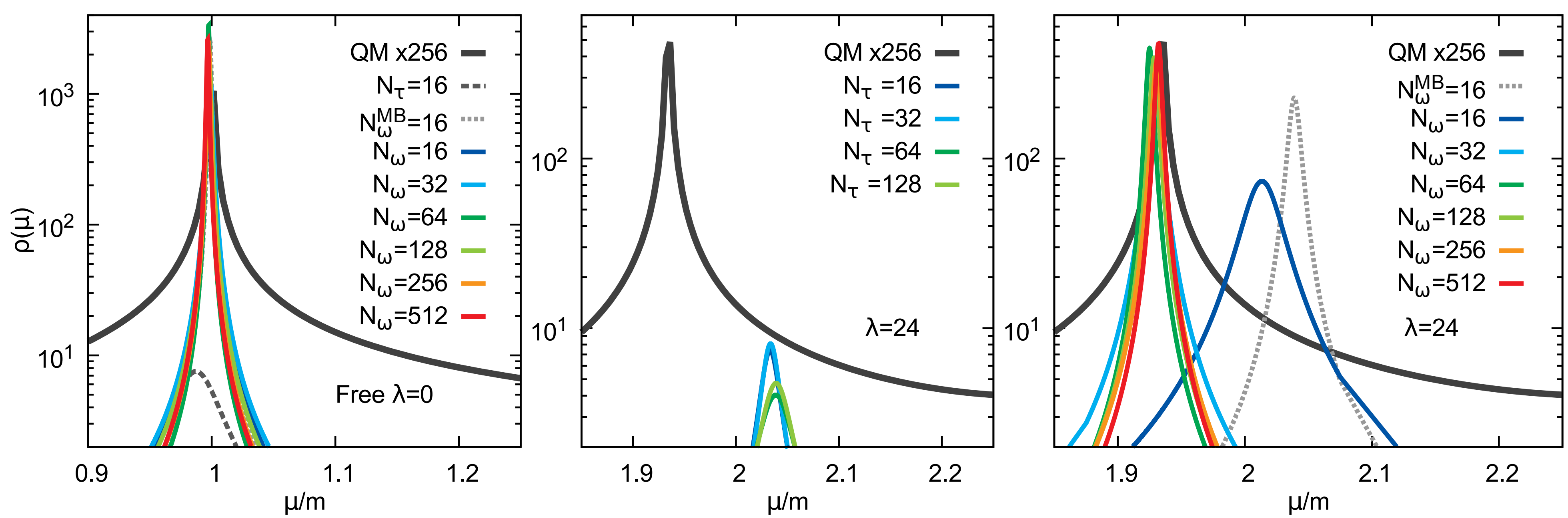


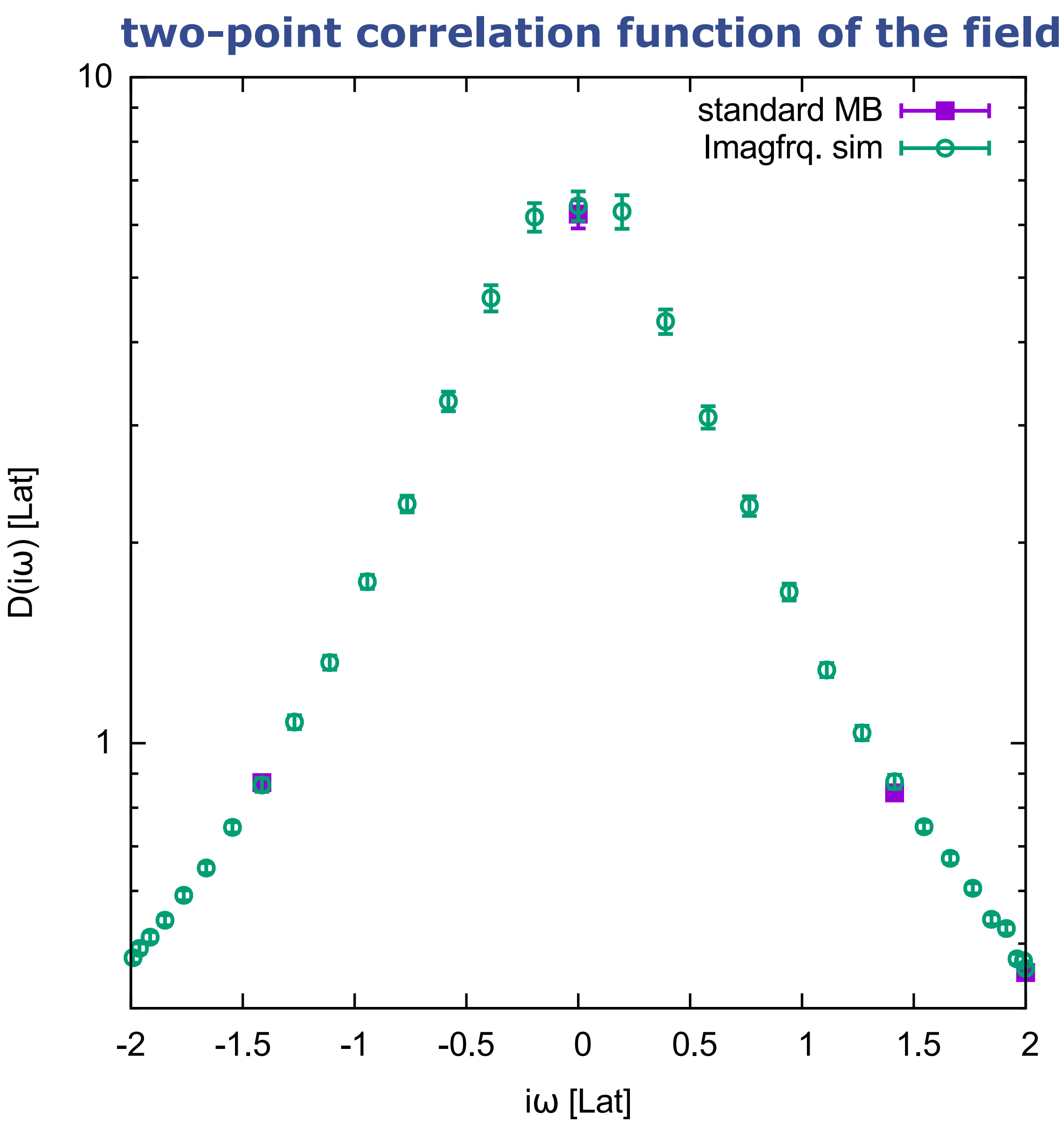


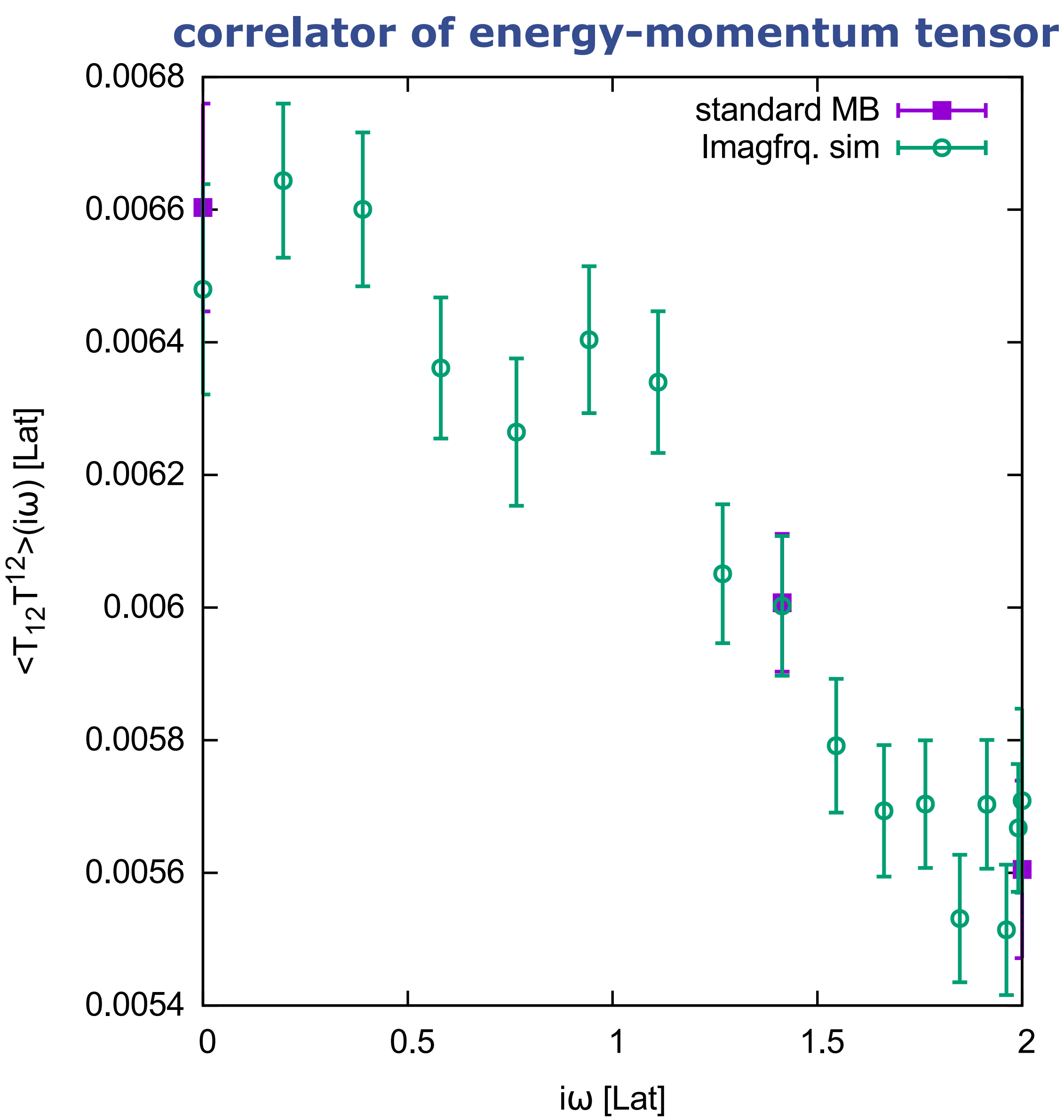
Test case: 1+0 dimensional scalar theory



JMP, Rothkopf, arXiv:1610:09531

3+1-dimensional complex scalar field

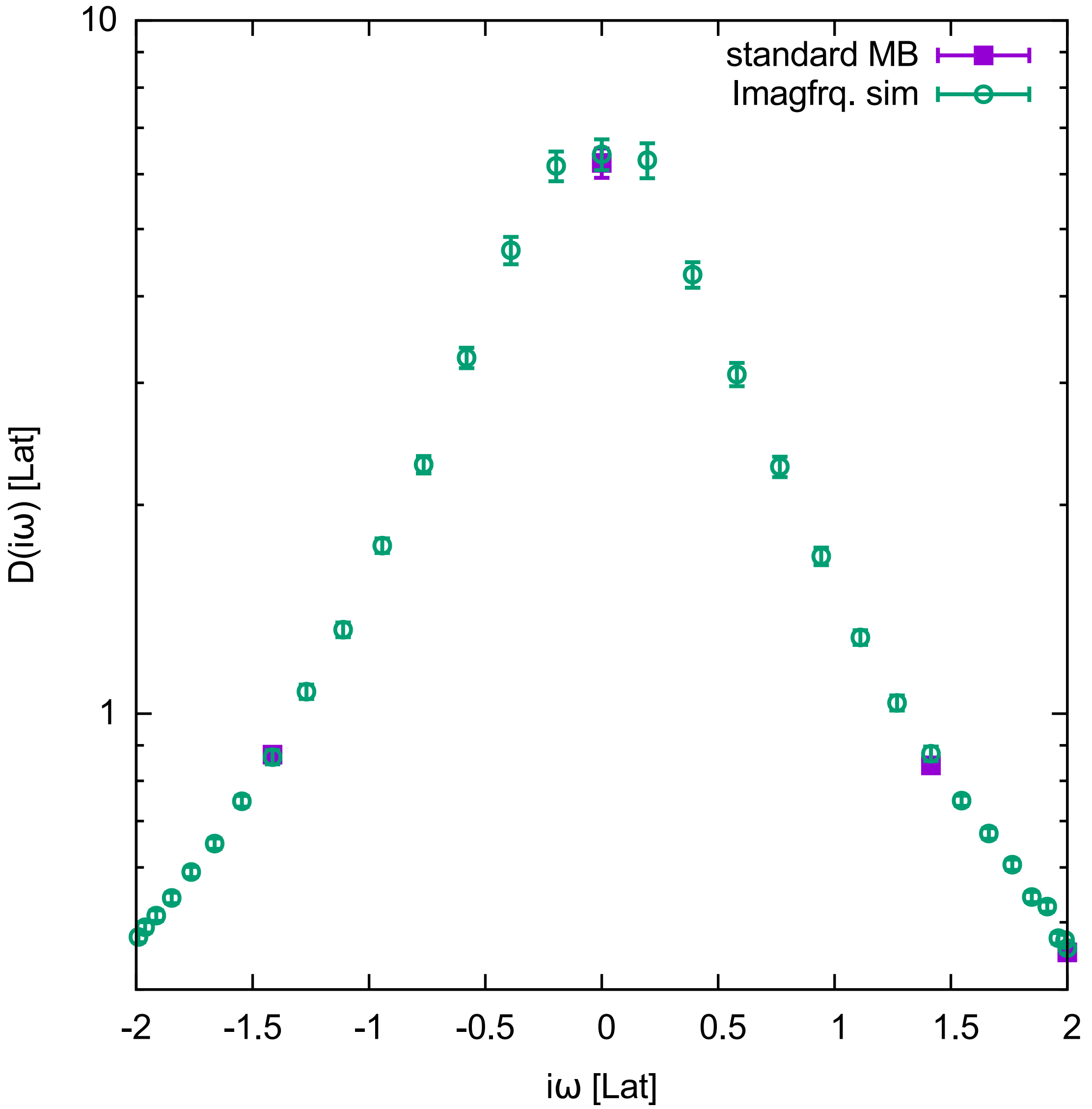




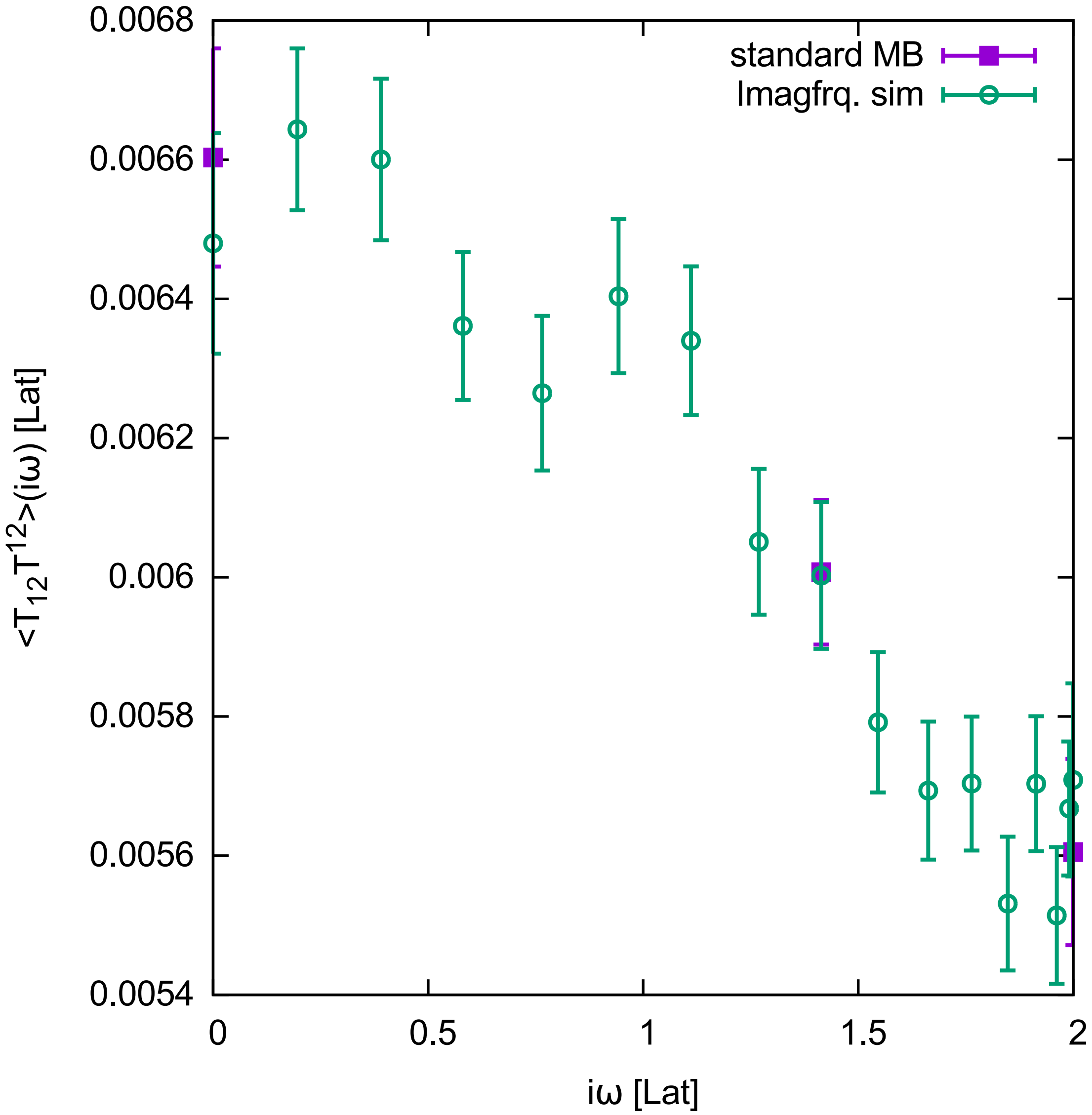
JMP, Rothkopf, work in progress

3+1-dimensional complex scalar field

two-point correlation function of the field



correlator of energy-momentum tensor



'Those are my methods (principles), and if you don't like them...well, I have others'

direct computation

Groucho Marx

JMP, Rothkopf, work in progress

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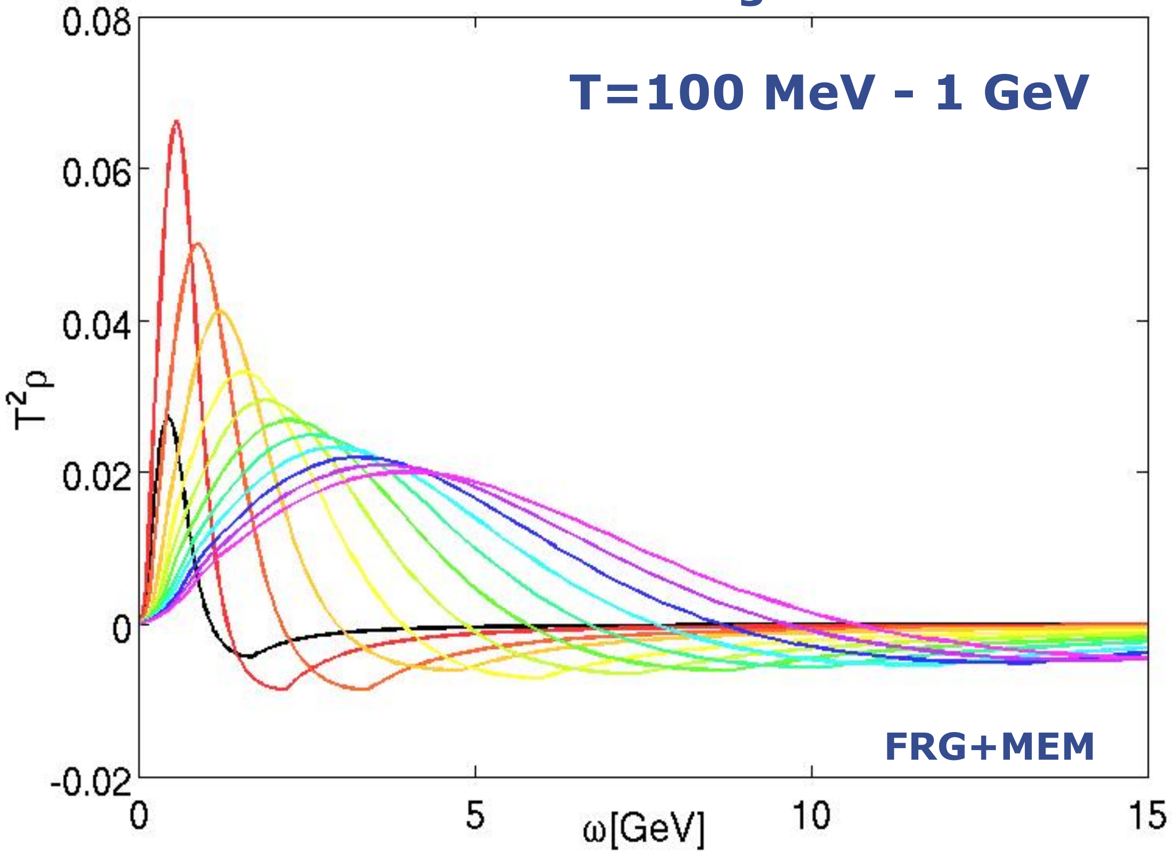
Single particle spectral functions

$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

Single particle spectral functions

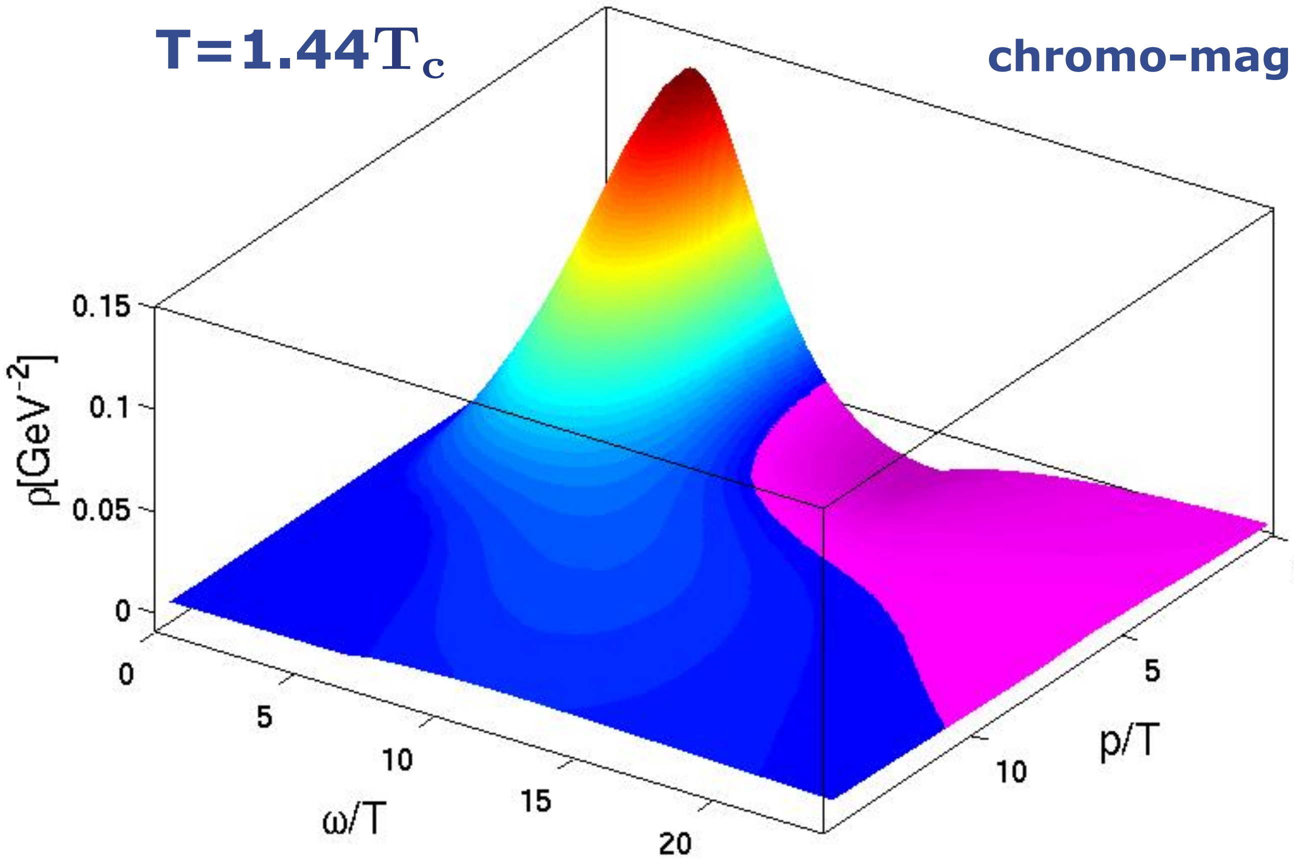
chromo-magnetic



$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

T=1.44$\mathrm{T_c}$

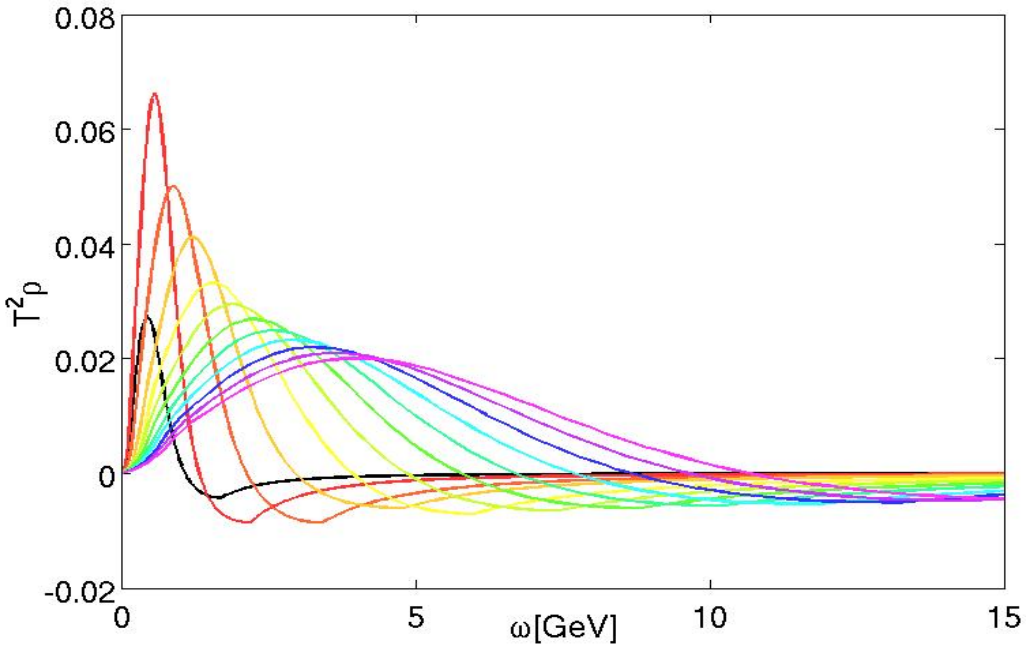
chromo-magnetic



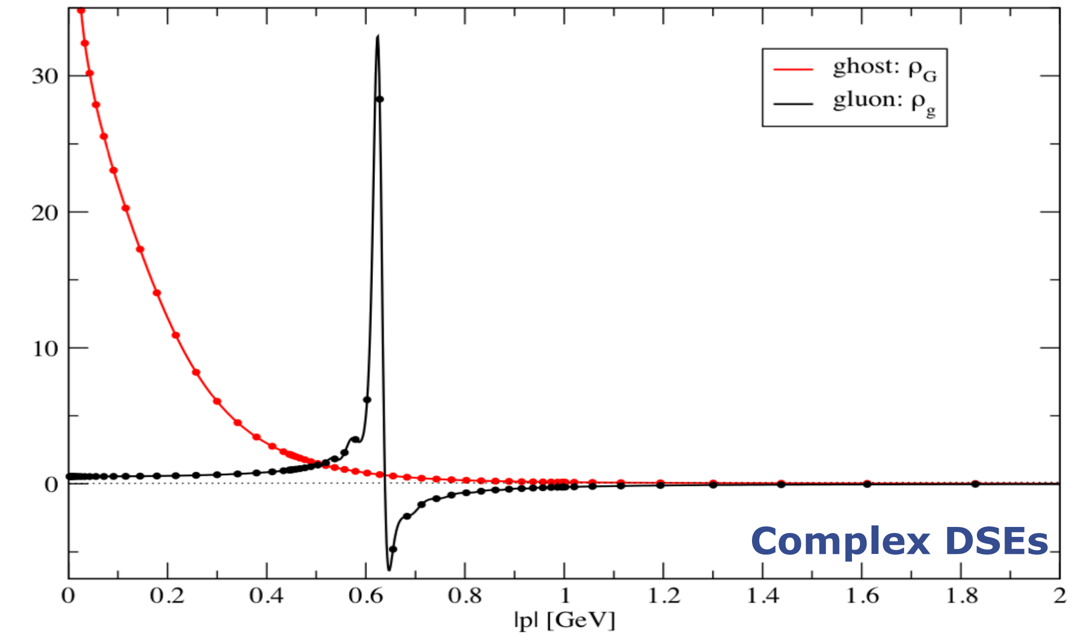
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Transport

gluon spectral functions

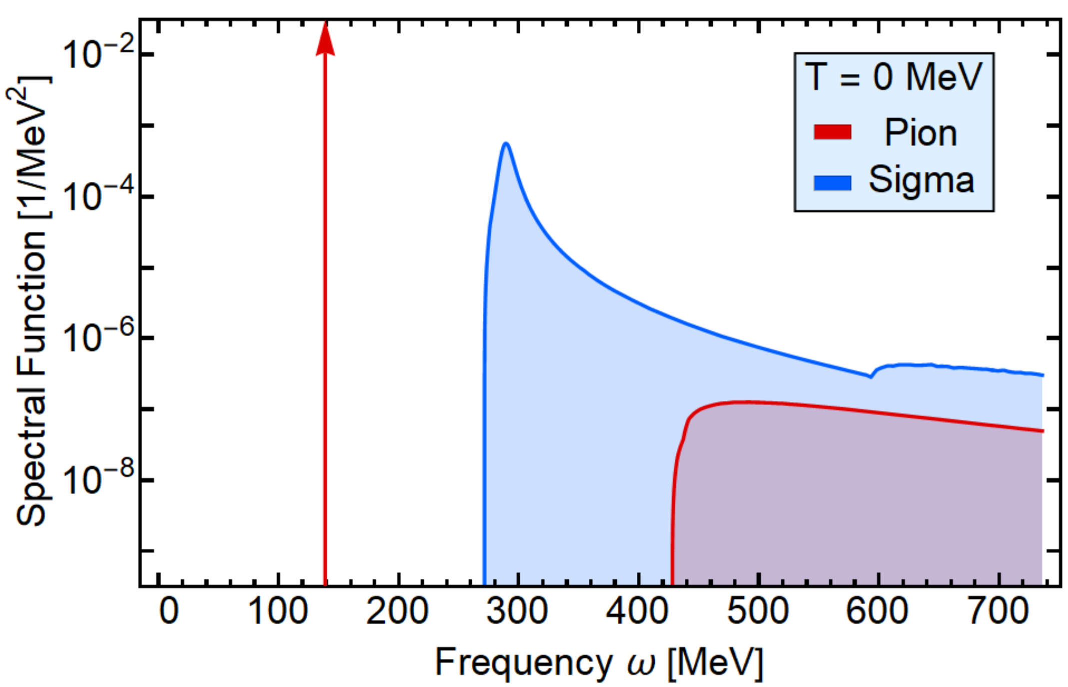


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

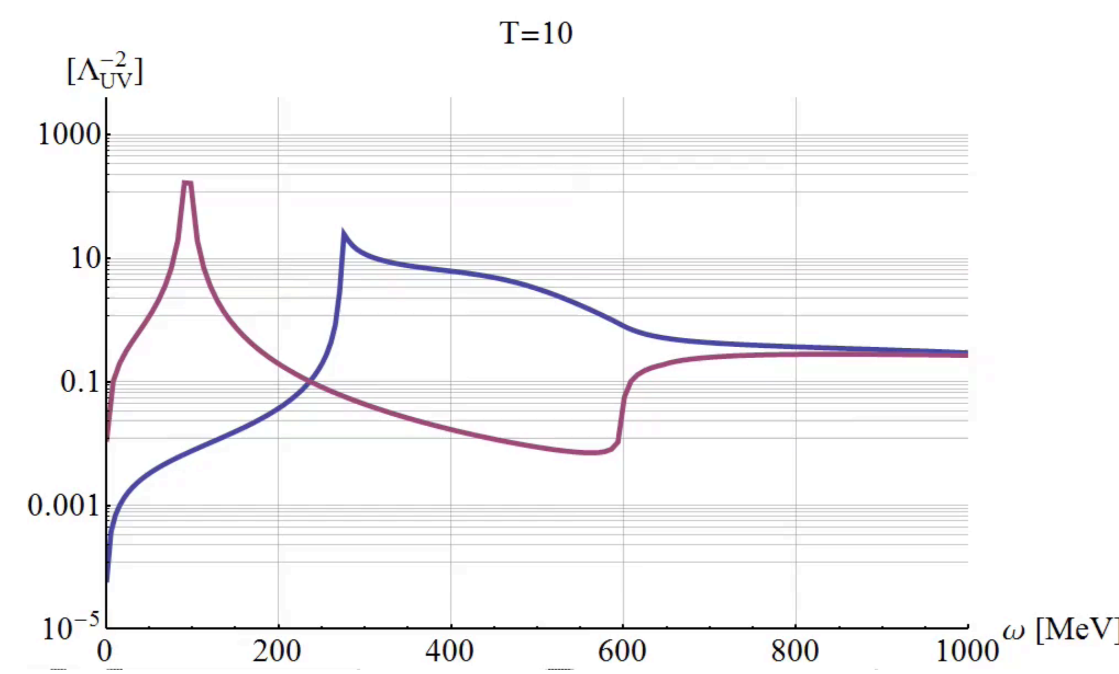


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

full complex FRG/real time **pion and sigma spectral functions** **analytic complex FRG**



JMP, Strodthoff, PRD 92 (2015) 094009
Strodthoff, PRD 95 (2017) 076002
JMP, Strodthoff, Wink, in preparation



Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806
Kamikado, Strodthoff, von Smekal, Wambach, PRD 95 (2017) 036020

Outline

- Introduction
- Phase structure of QCD
    - Confinement & chiral symmetry breaking
    - Finite temperature correlation functions
    - QCD at finite density & fluctuations
- QCD transport
    - Real time correlation functions
    - Single particle spectral functions
    - transport coefficients
- Summary & outlook

Transport

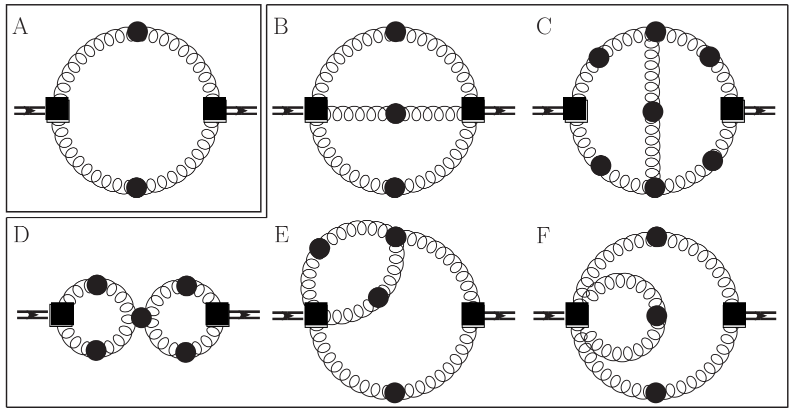
transport coefficients

Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

'3-loop' exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



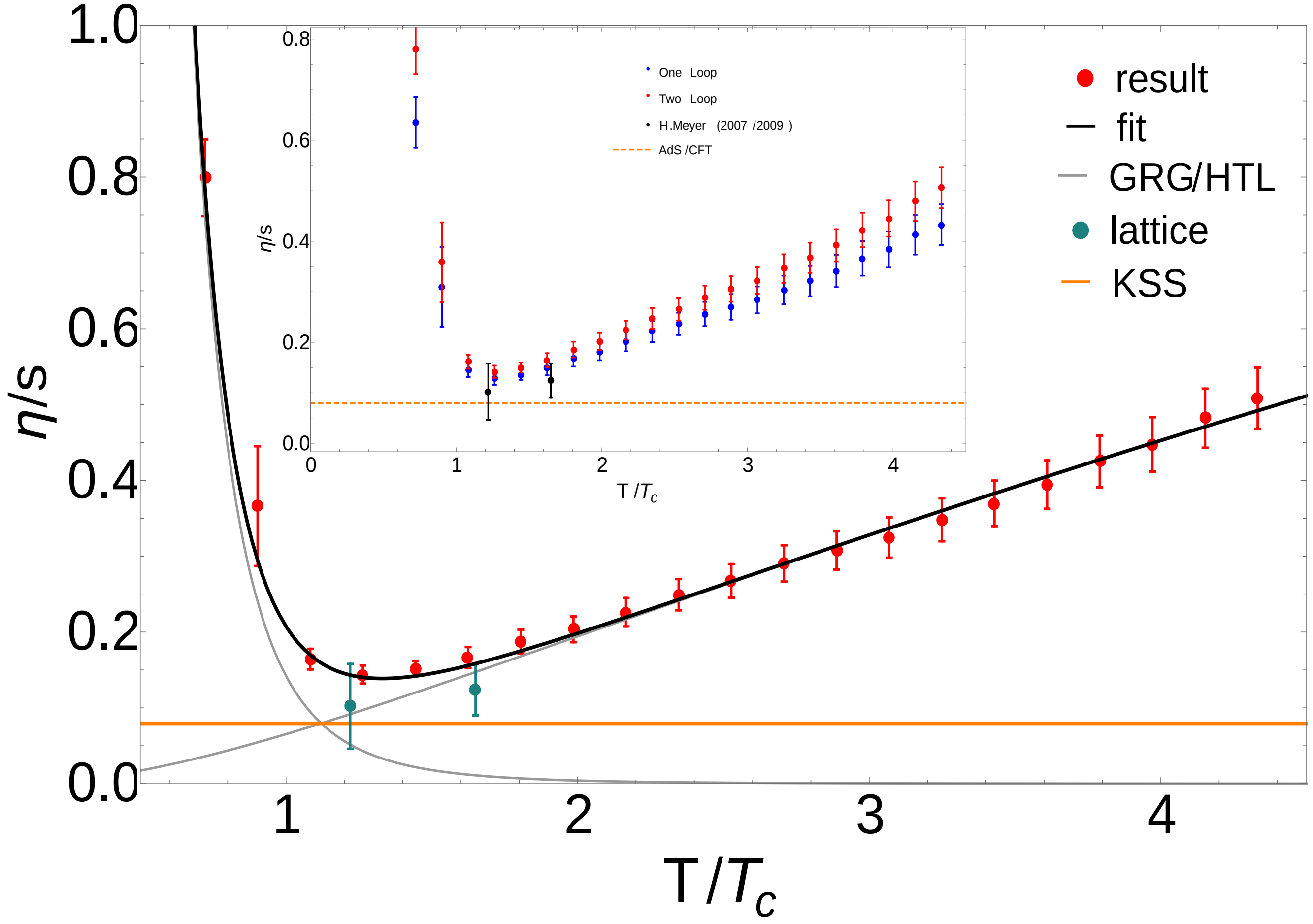
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

transport coefficients

Yang-Mills viscosity over entropy ratio



Aiming at apparent convergence

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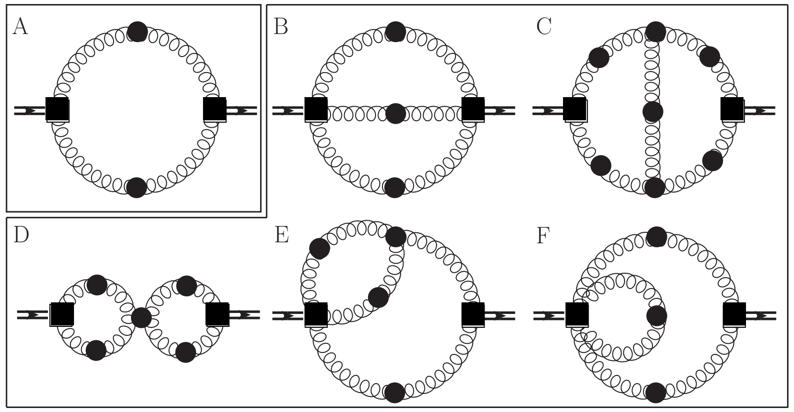

Haas, Fister, JMP, PRD 90 (2014) 9, 091501

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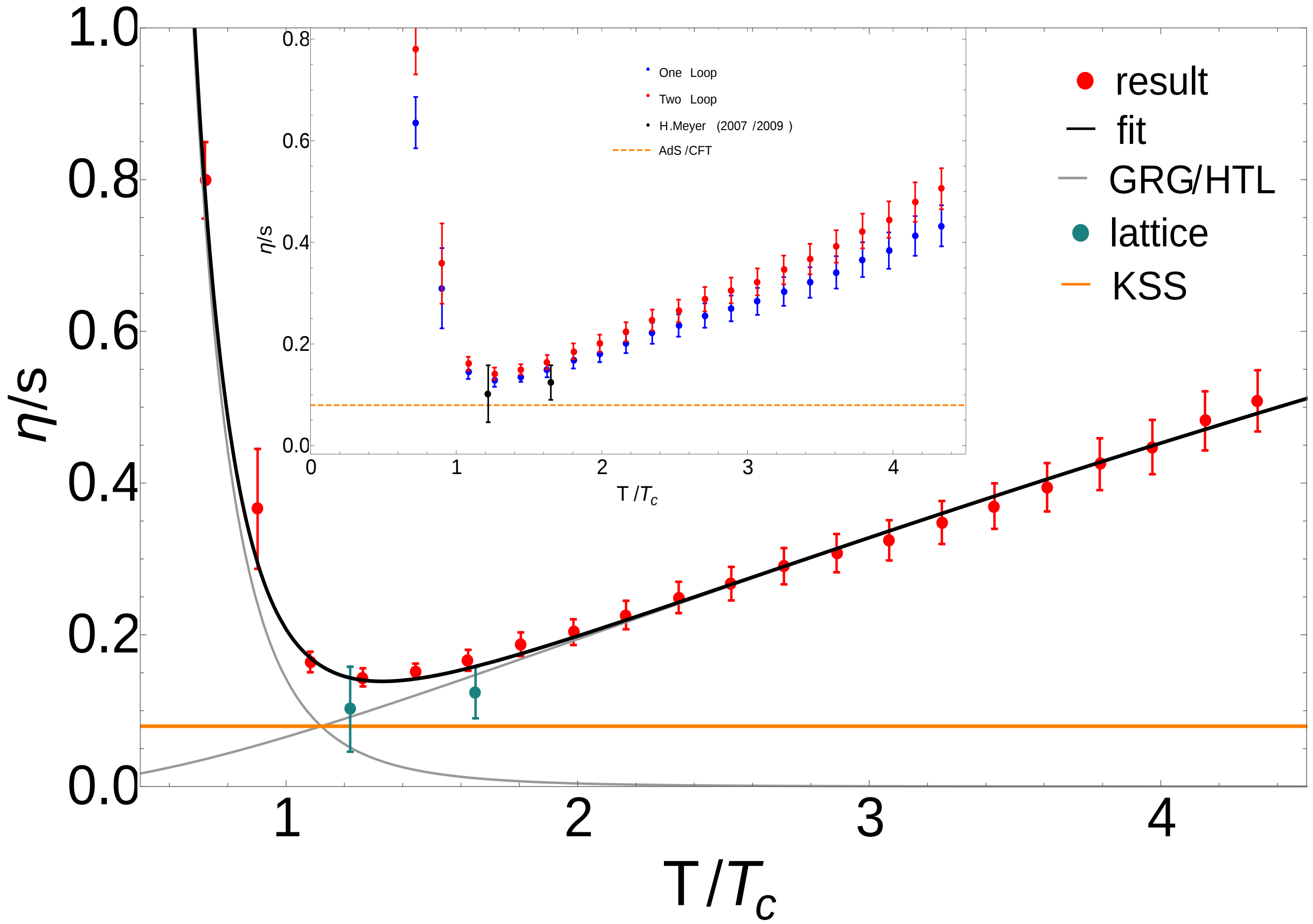
Transport

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\gamma_{\mathrm{grg}} \approx 5$$

$$\gamma_{\mathrm{qgp}} \approx 1.6$$

pure glue

$$\frac{\eta}{s}(T) = \frac{a_{\mathrm{qgp}}}{\alpha_s^{\gamma_{\mathrm{qgp}}}(cT/T_c)} + \frac{a_{\mathrm{grg}}}{(T/T_c)^{\gamma_{\mathrm{grg}}}}$$

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Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

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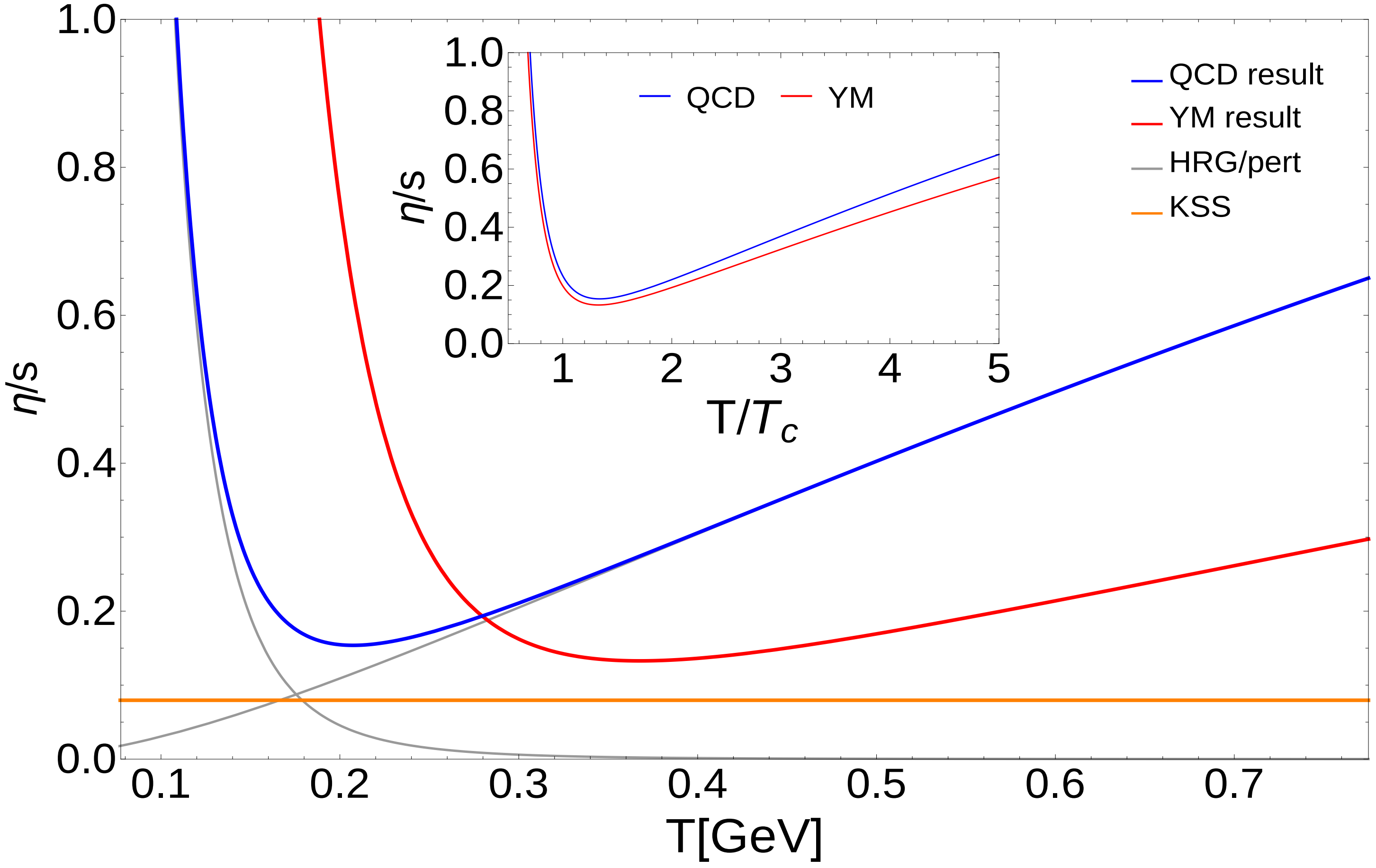

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Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

transport coefficients

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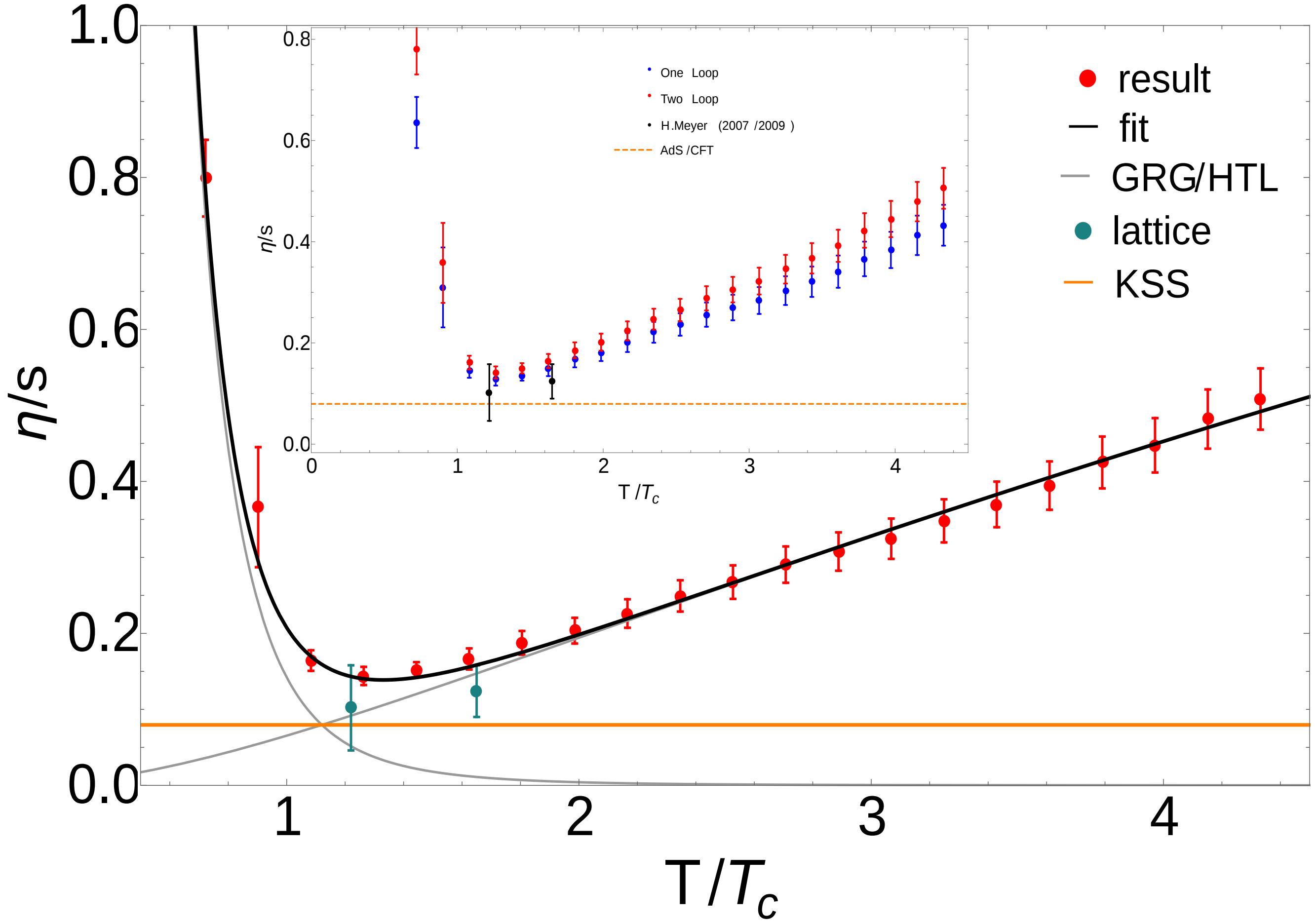

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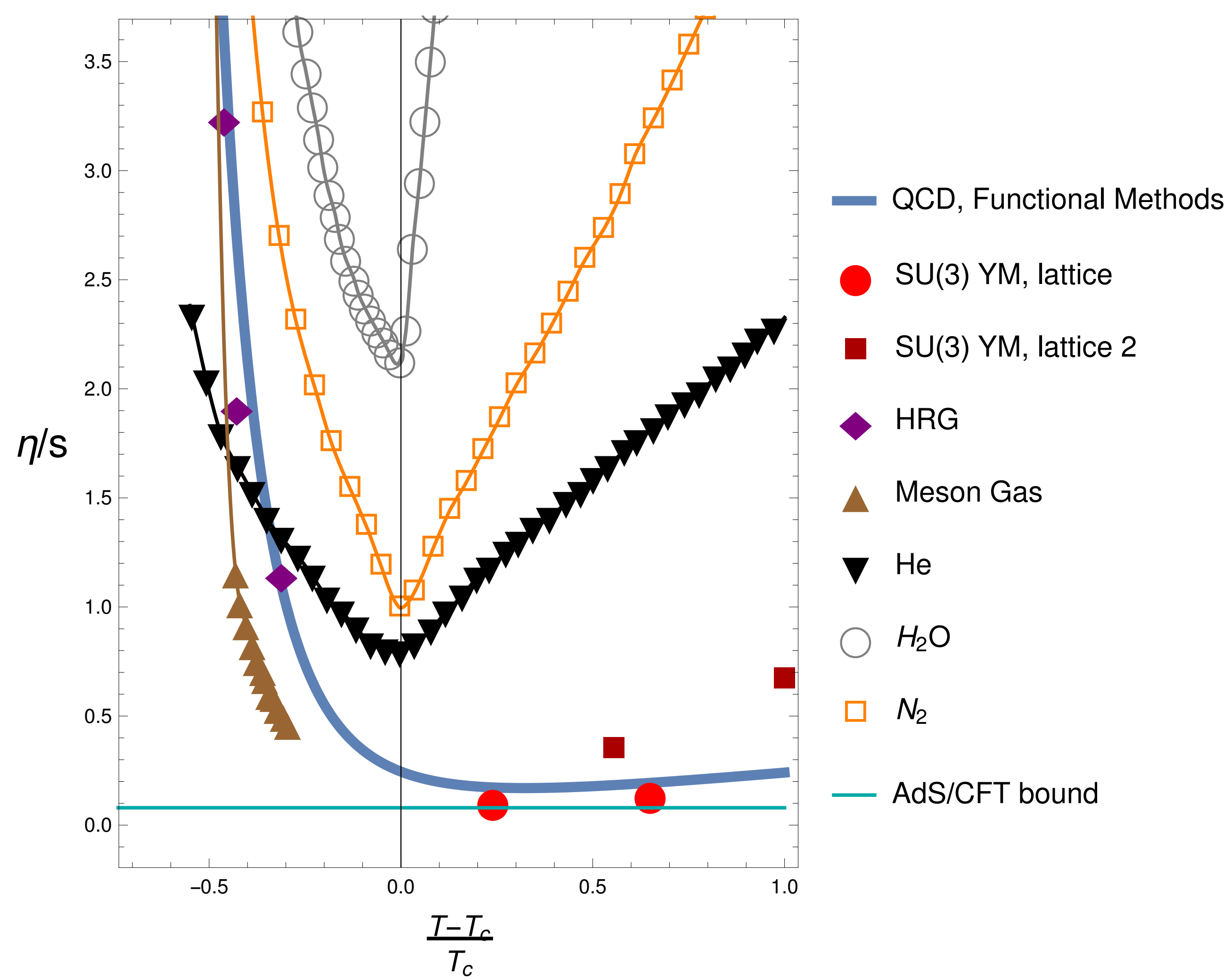

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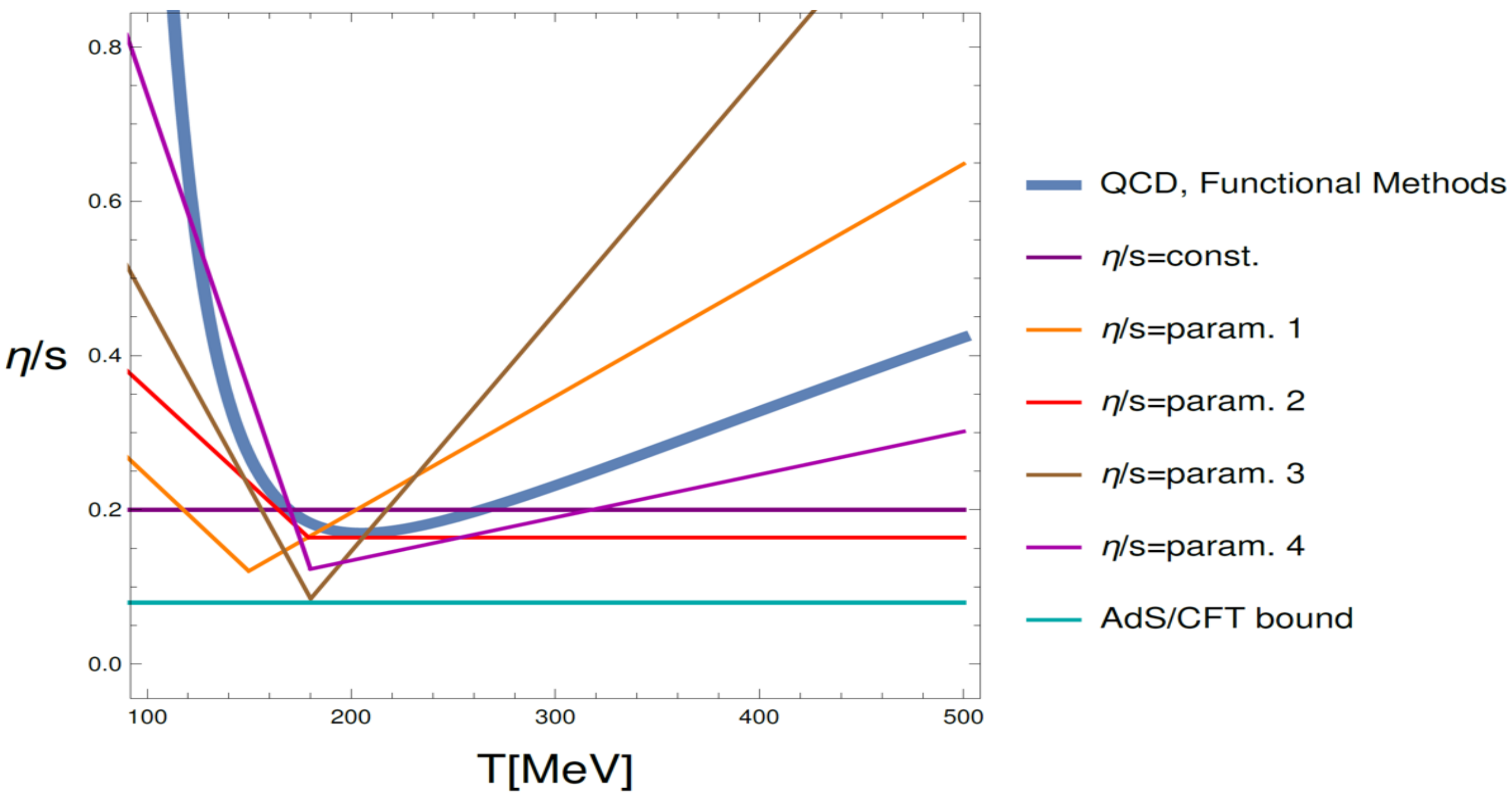
Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

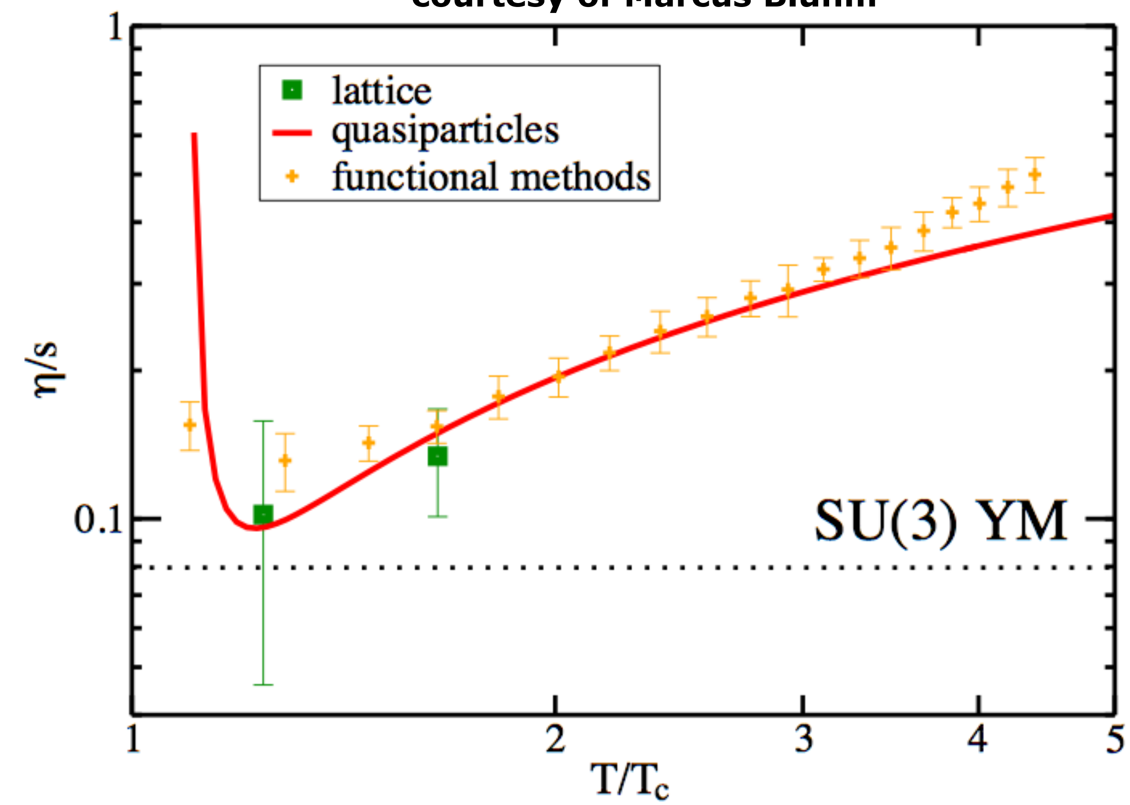
QCD transport & transport models

courtesy of Nicolai Christiansen



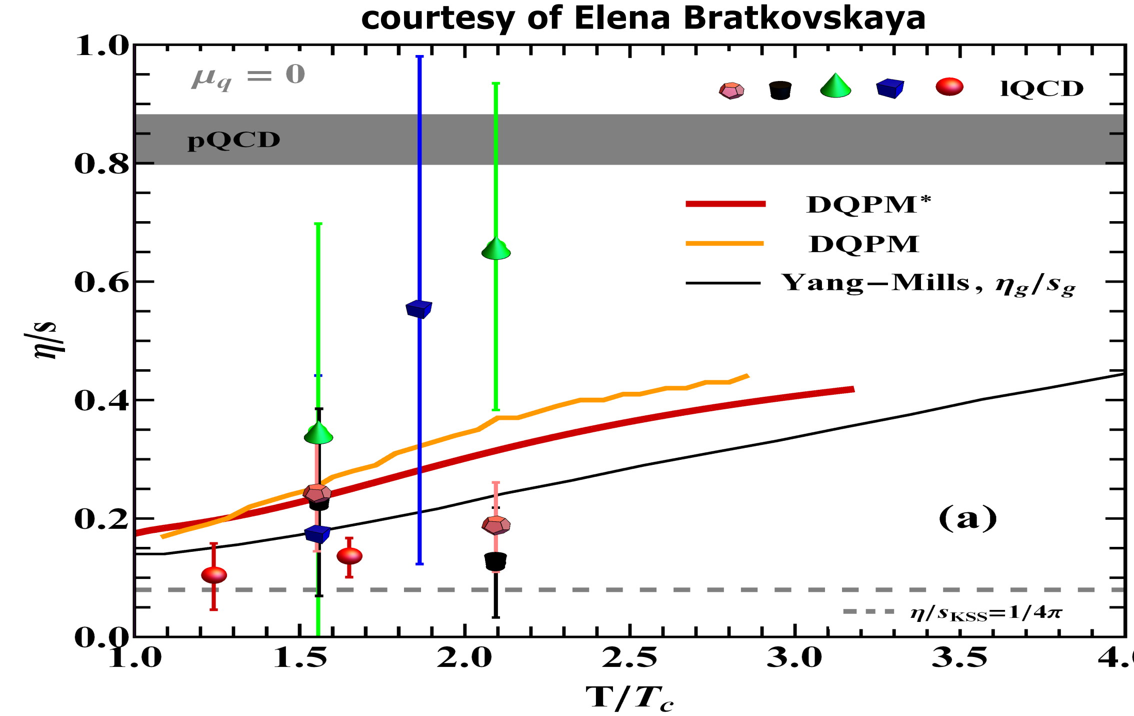
Niemi, Eskola, Paateleinen, PRC 93 (2016) 024907

courtesy of Marcus Bluhm



Bluhm, Kaempfer, Redlich, PRC 84 (2011) 025201

courtesy of Elena Bratkovskaya



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Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Berrehrah, Cassing, Bratkovskaya, Steinert, PRC 93 (2016) 044914

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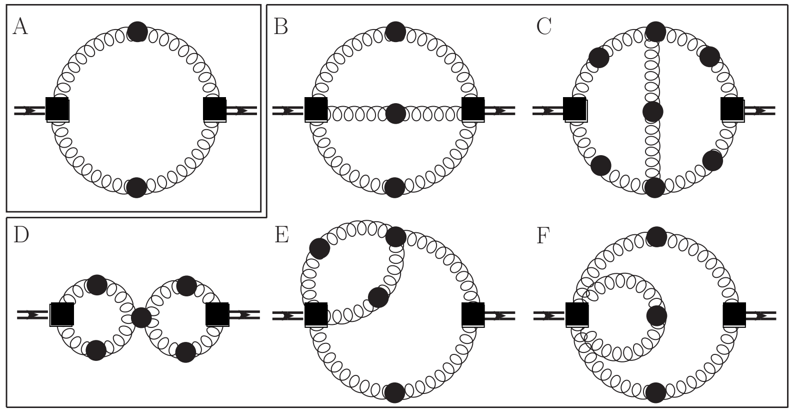

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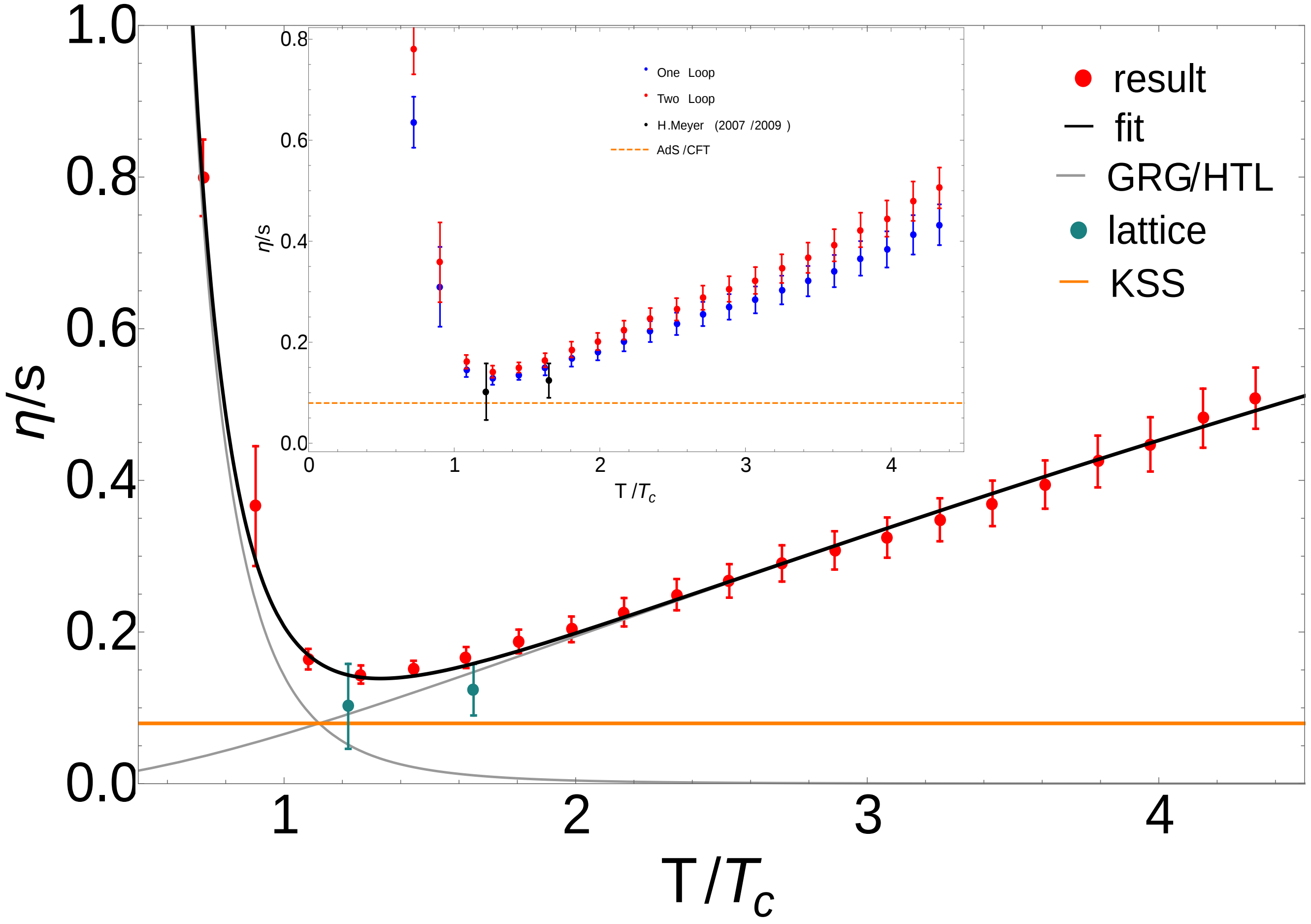
transport coefficients

Yang-Mills viscosity over entropy ratio



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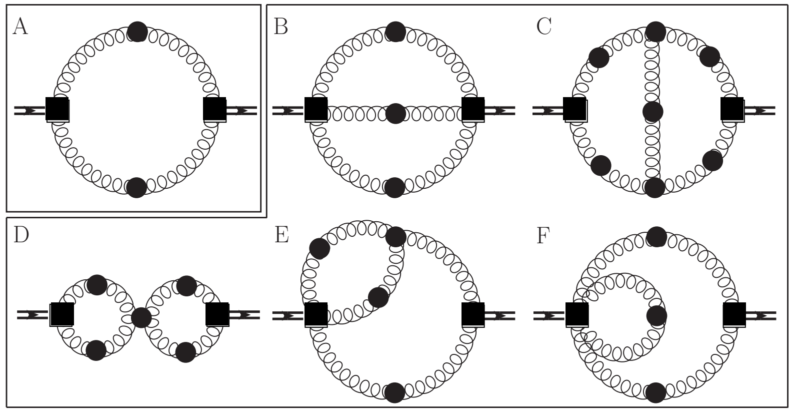

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Transport

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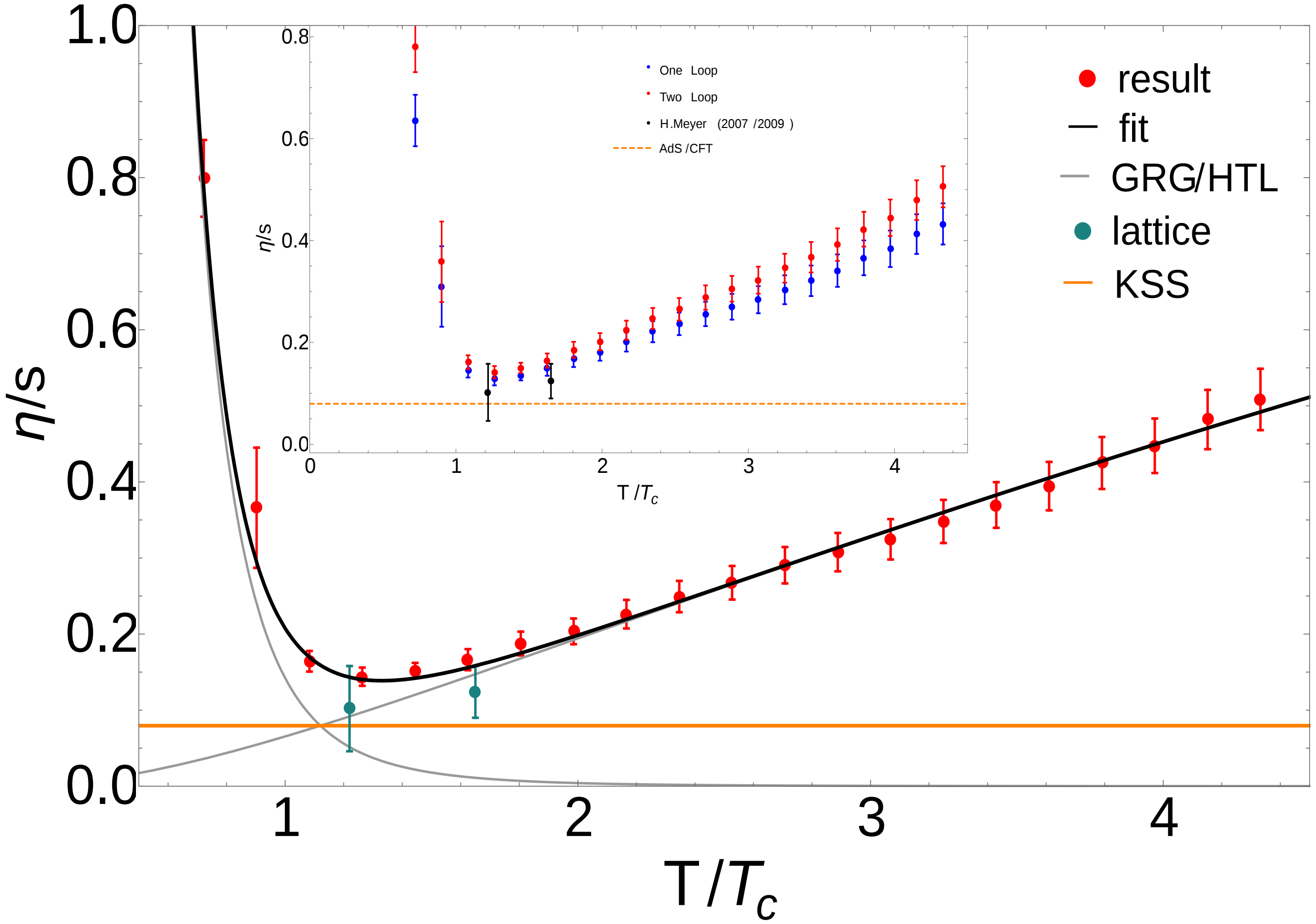
viscosity over entropy ratio



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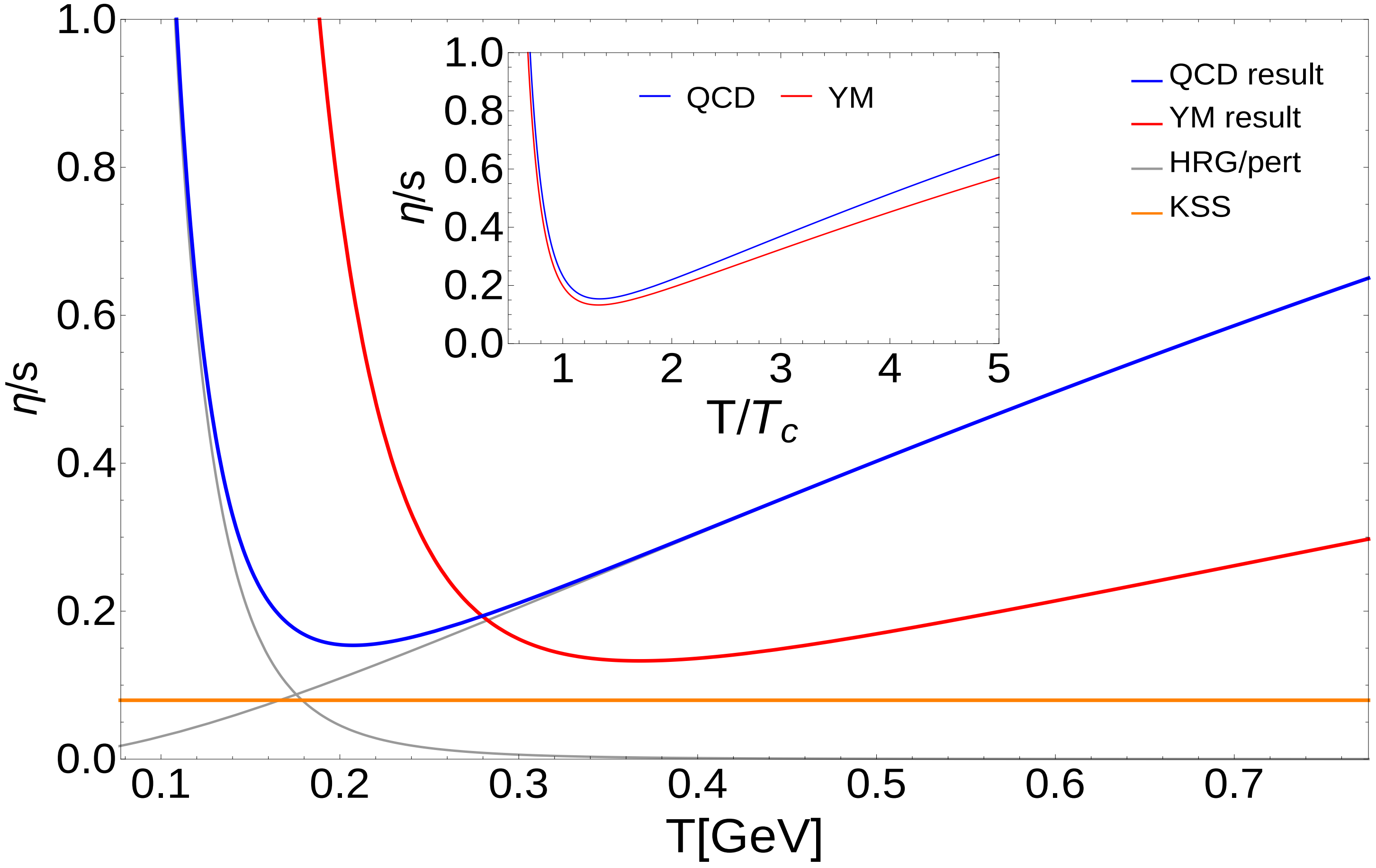

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transport coefficients

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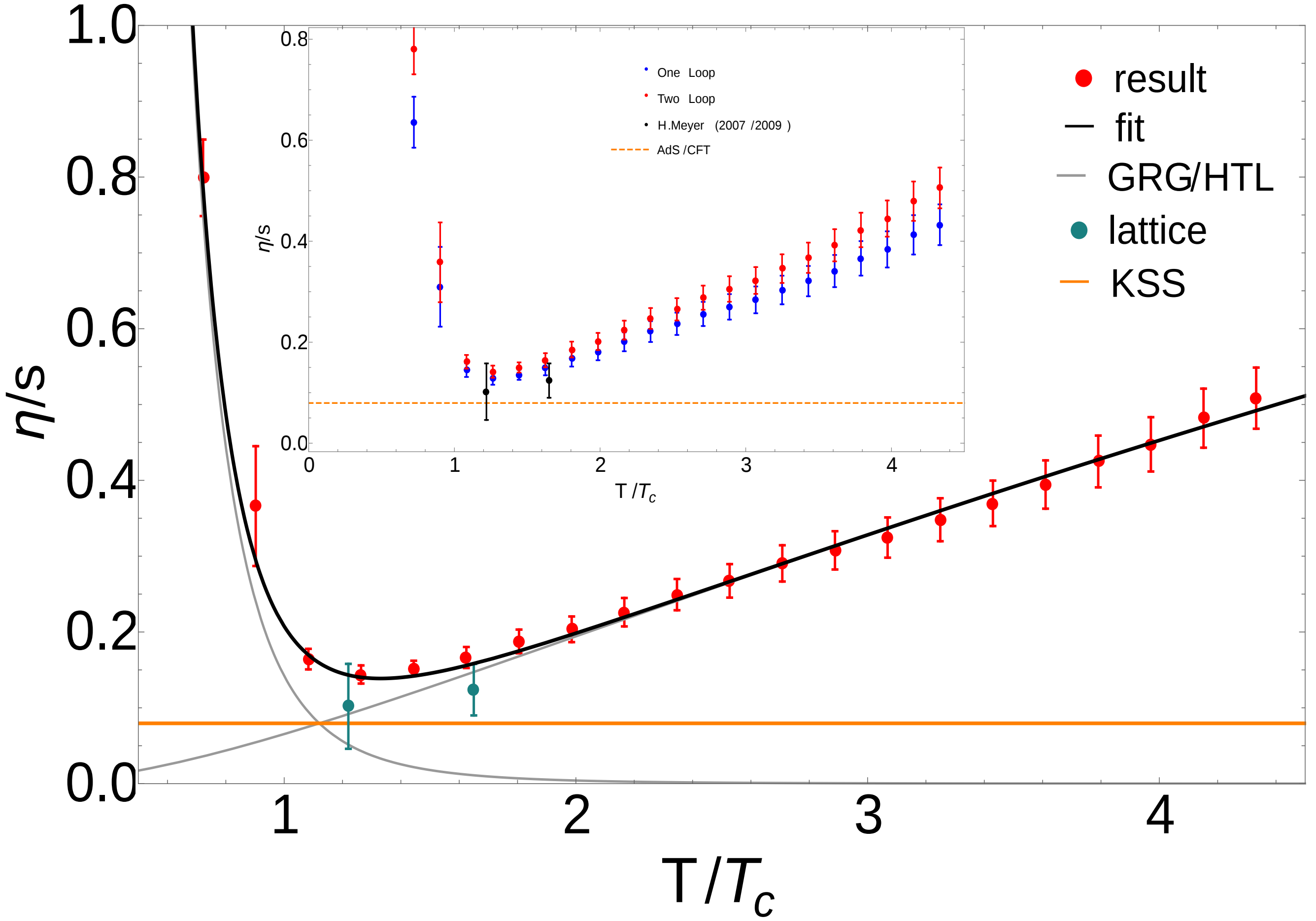

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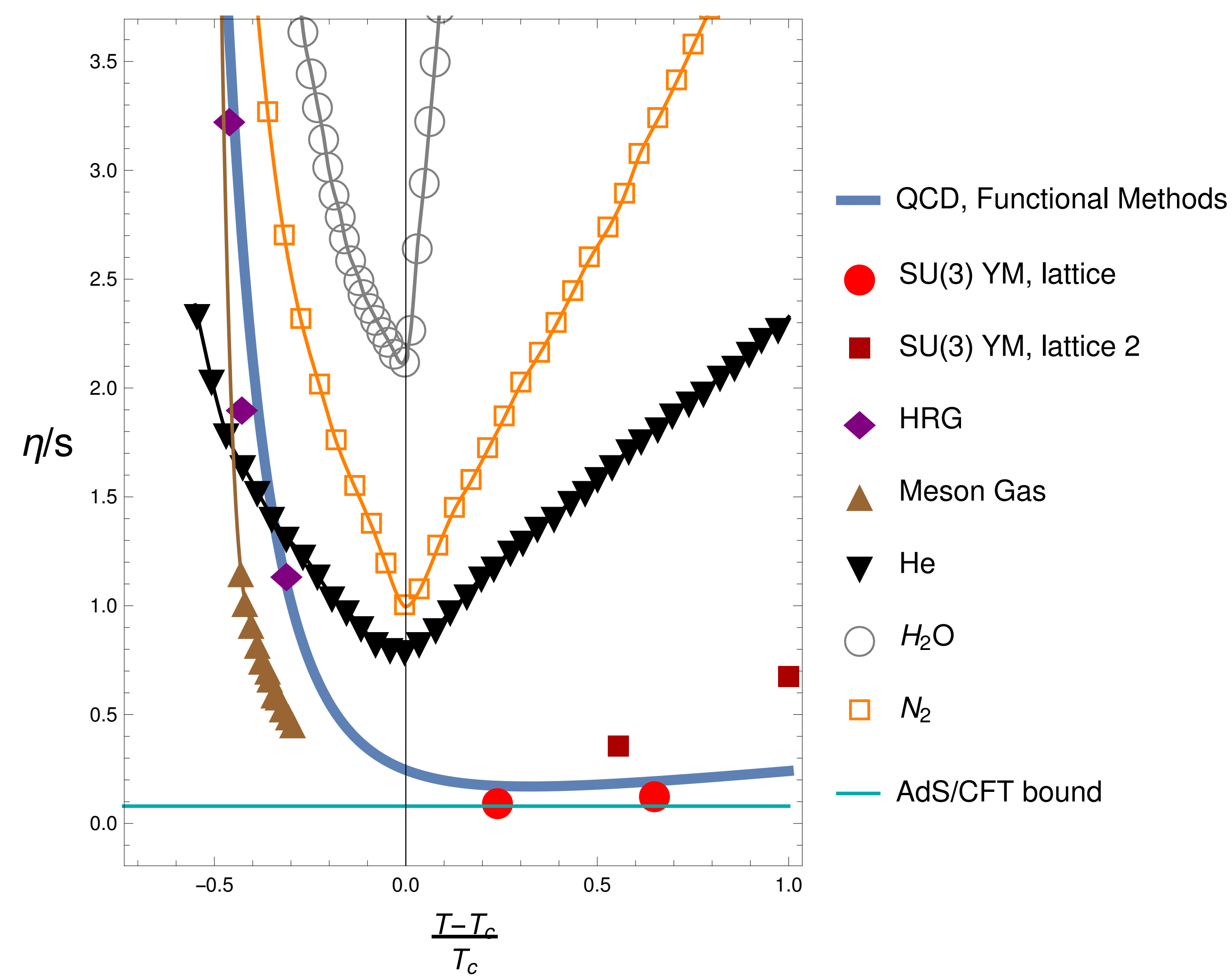

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Transport

QCD transport & transport models

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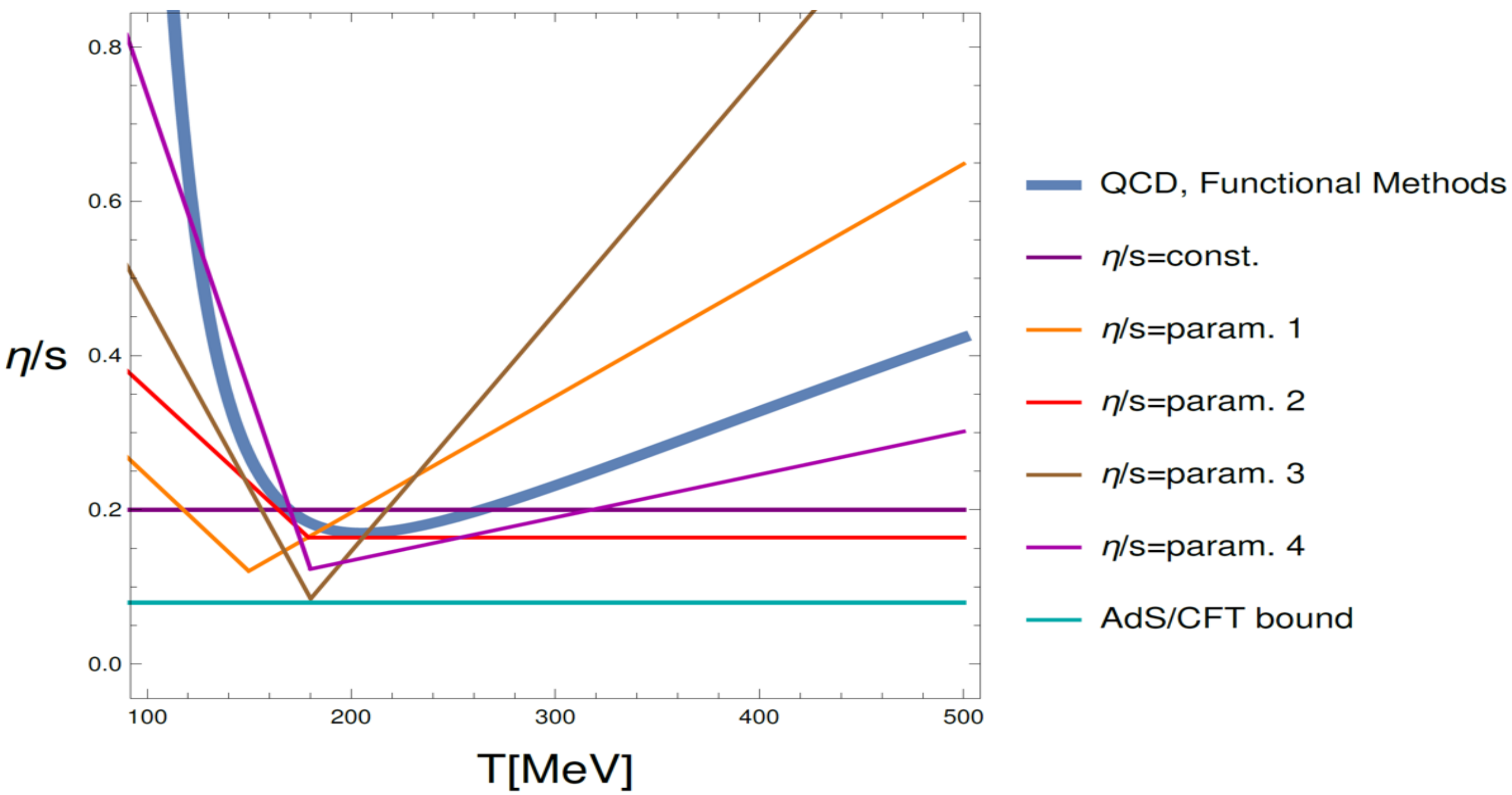

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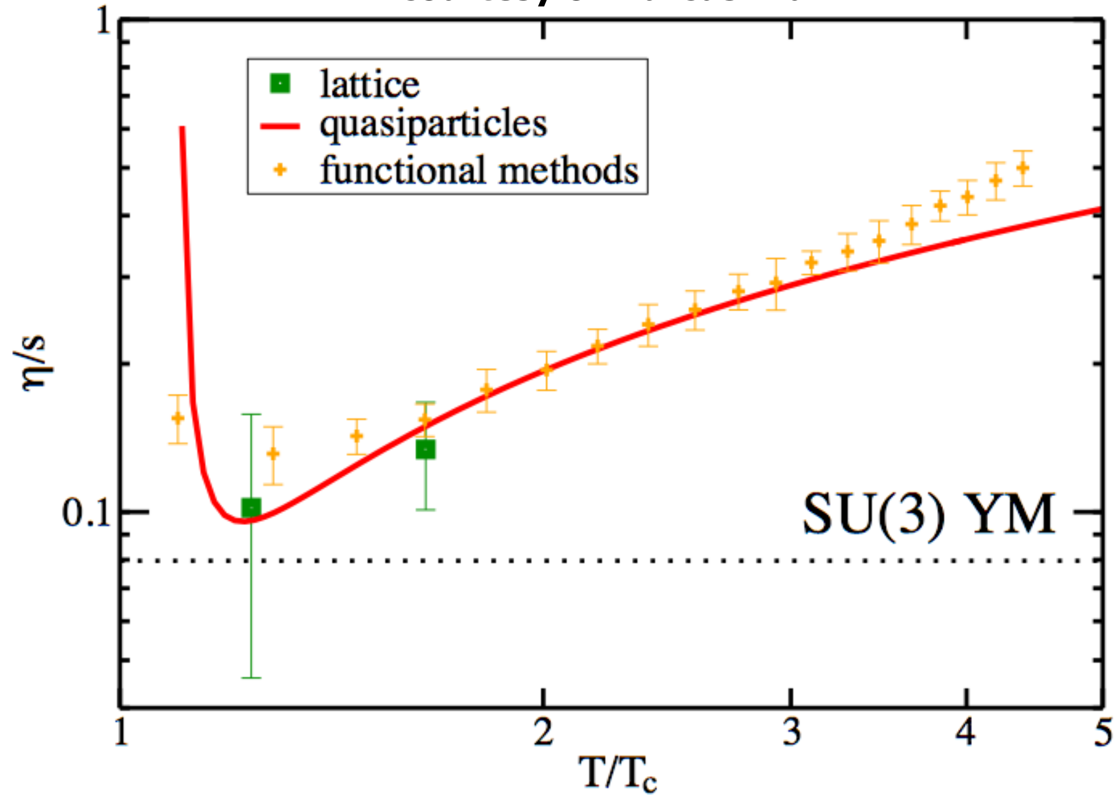

Bluhm, Kaempfer, Redlich, PRC 84 (2011) 025201

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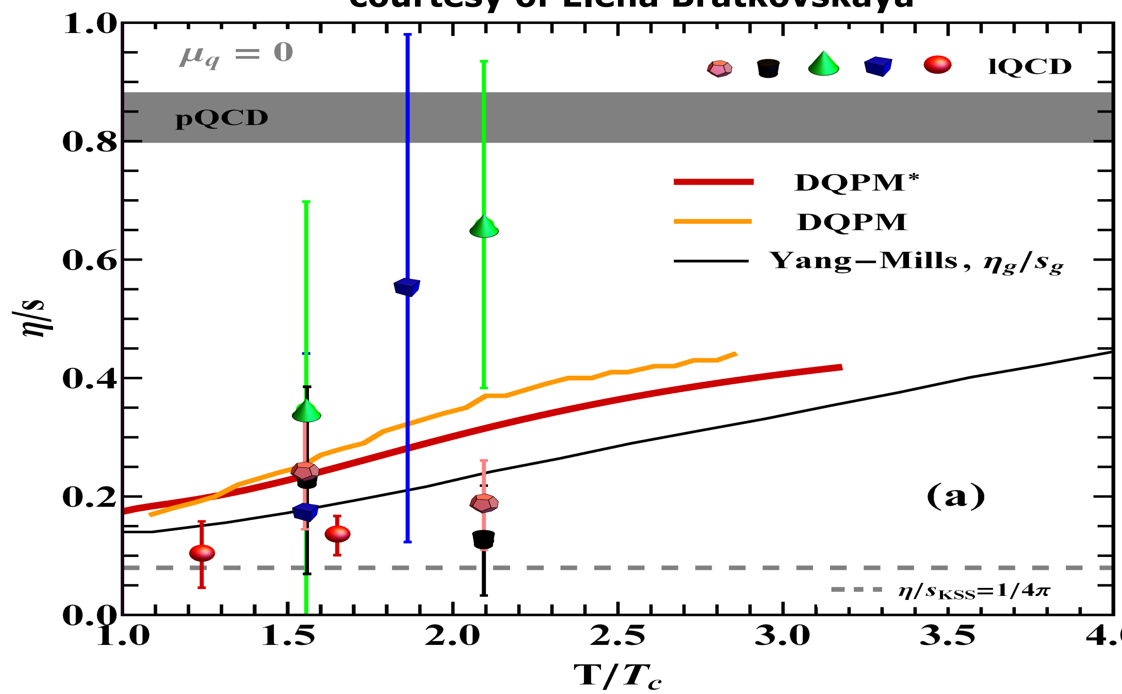

Berrehrah, Cassing, Bratkovskaya, Steinert, PRC 93 (2016) 044914

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Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

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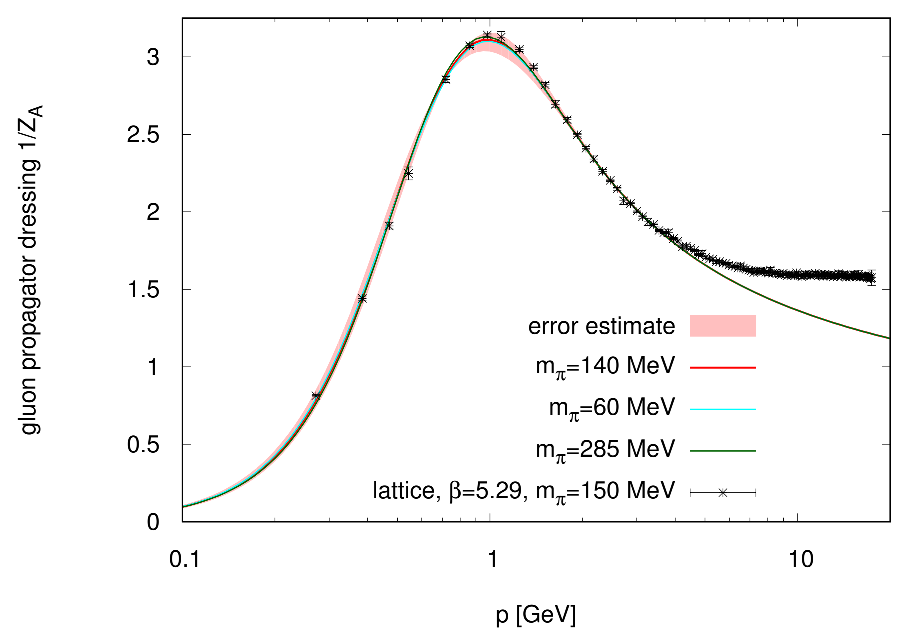
- Summary & outlook

Summary & Outlook

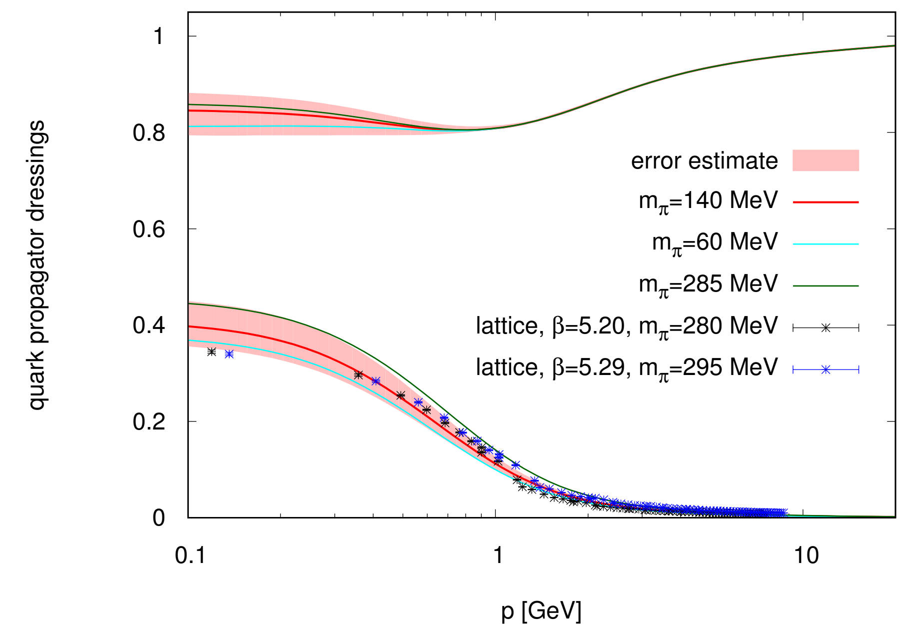
- **Chiral Symmetry Breaking and Confinement**

$$\boxed{\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99}$$

Gluon correlations

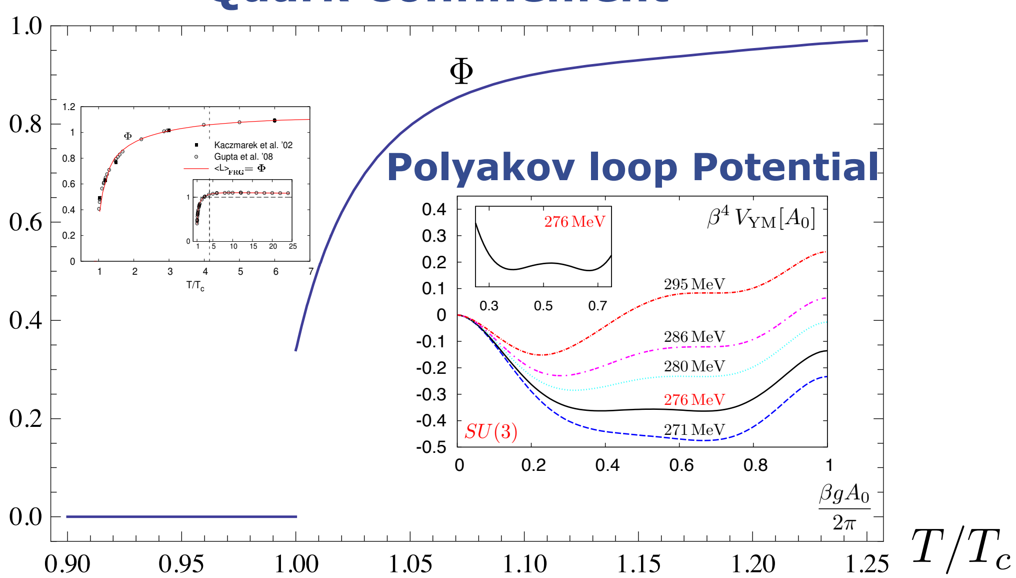


chiral symmetry breaking

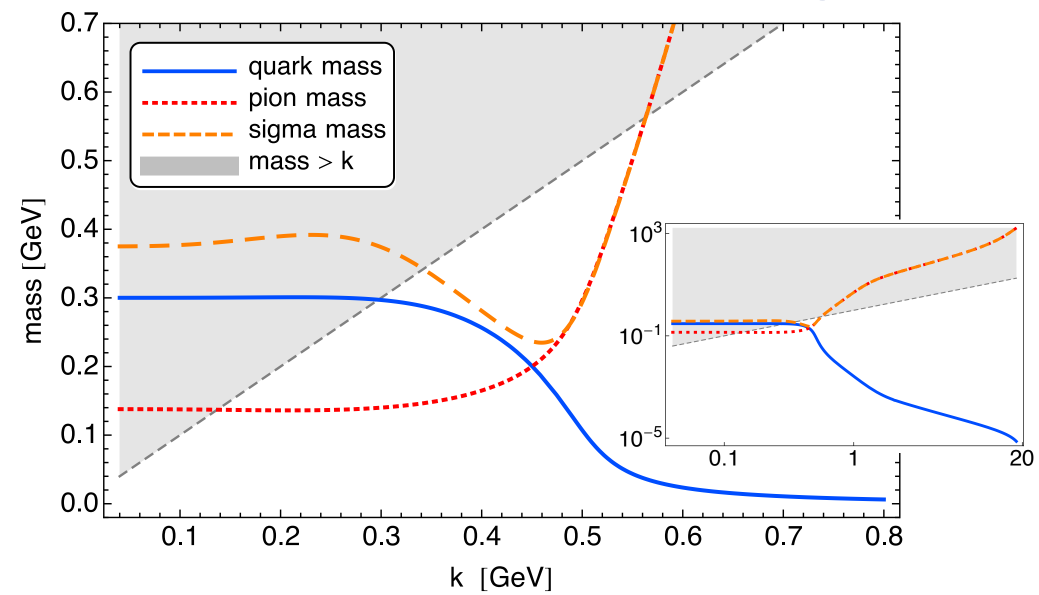


fQCD

Quark Confinement

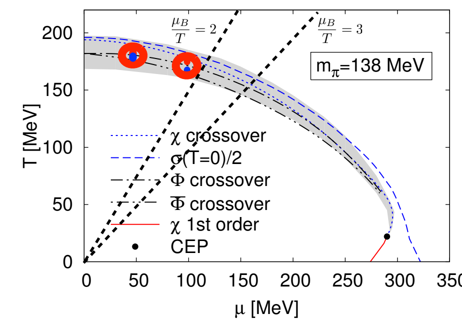


meson masses & decoupling

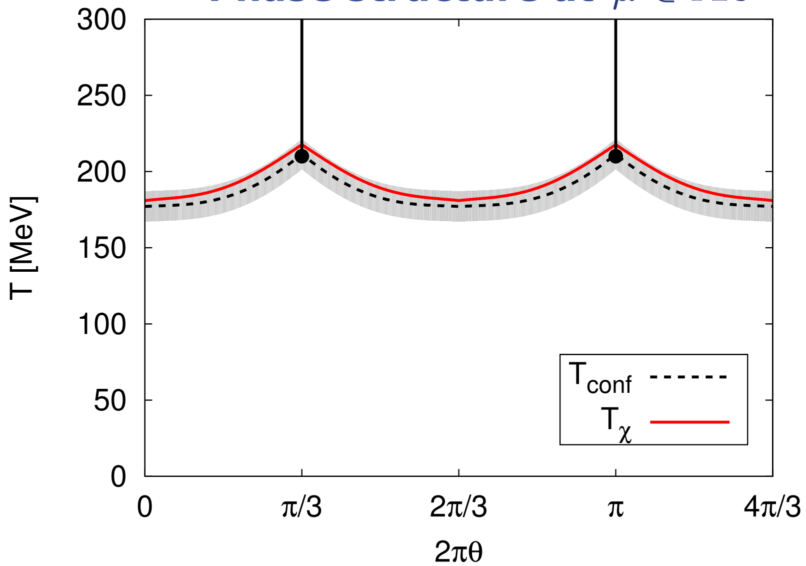


Summary & Outlook

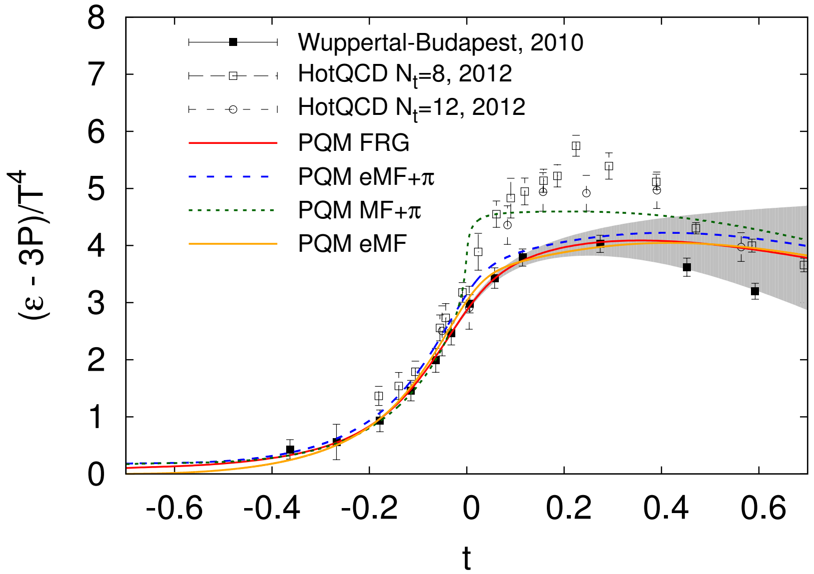
- **Phase structure and Transport**



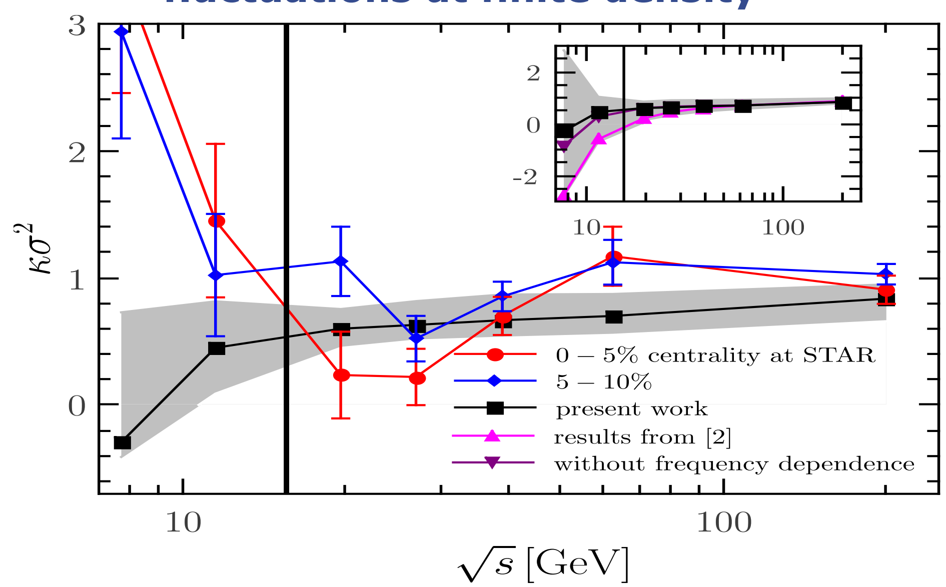
Phase structure at $\mu \in i\mathbb{R}$



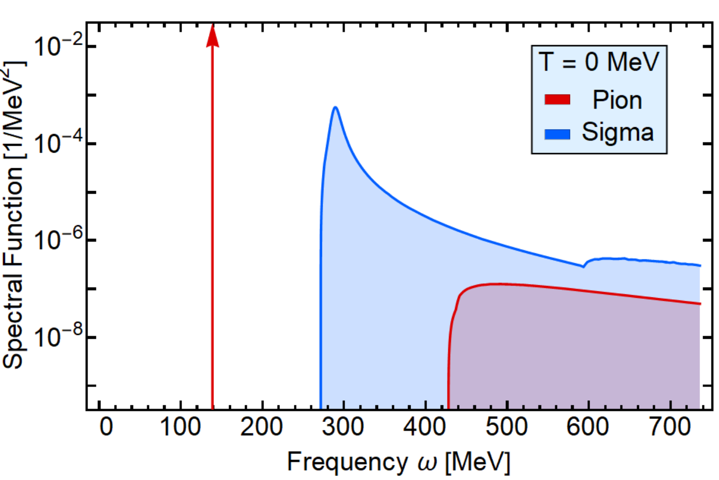
Interaction measure



fluctuations at finite density



meson spectral fcts.



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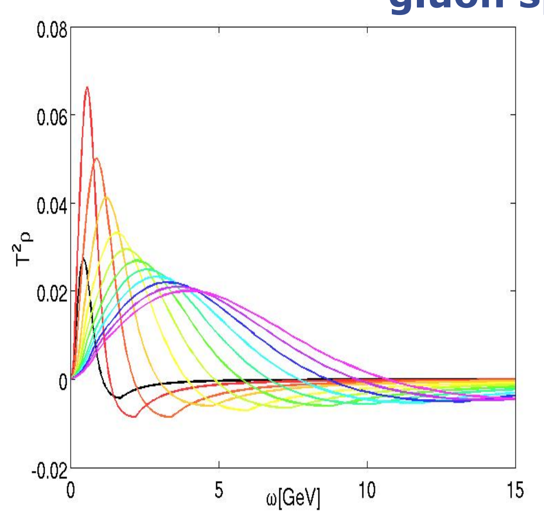
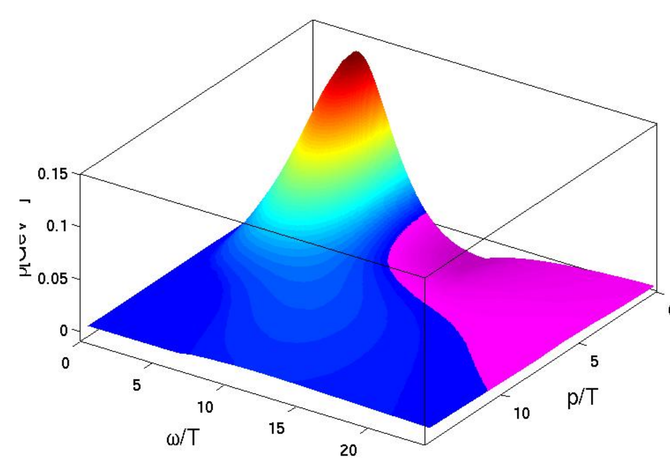
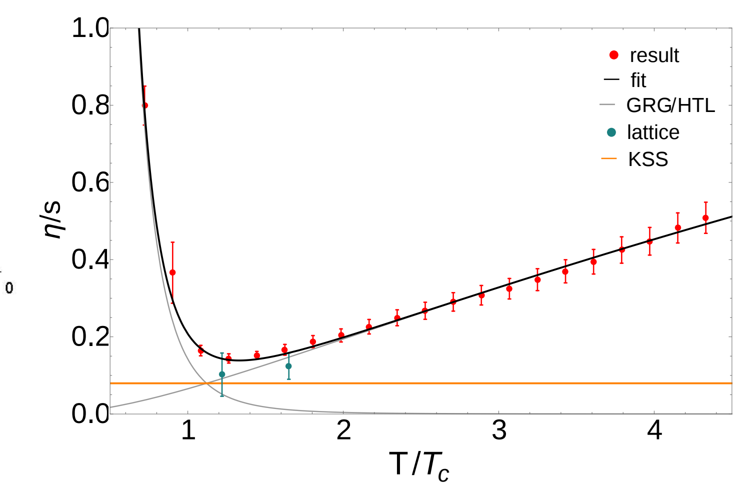




glue viscosity/entropy



Summary & Outlook

- **Chiral Symmetry Breaking and Confinement**

- **Phase Structure and Transport**

Summary & Outlook

- Chiral Symmetry Breaking and Confinement

- Phase Structure and Transport

- Towards quantitative precision

- Baryons, high density regime & CEP, dynamics

- Hadronic properties
    - hadron spectrum & in medium modifications
    - low energy constants