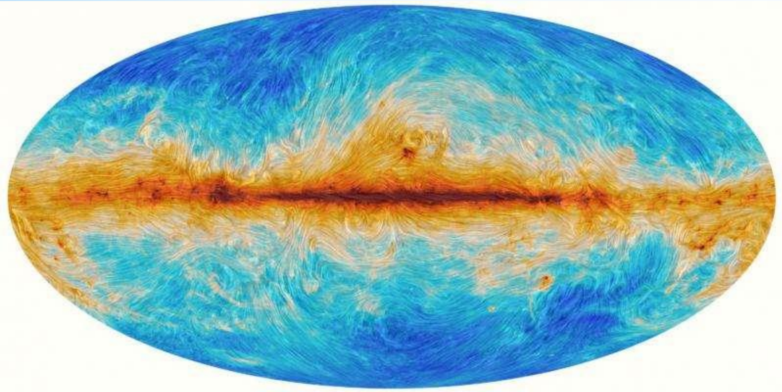
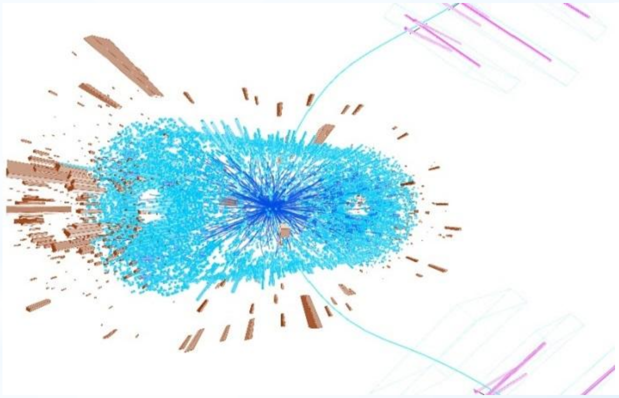
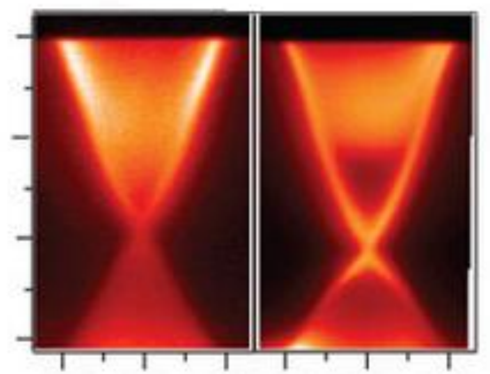
Real-time dynamics of chiral plasma







Pavel Buividovich
(Regensburg)

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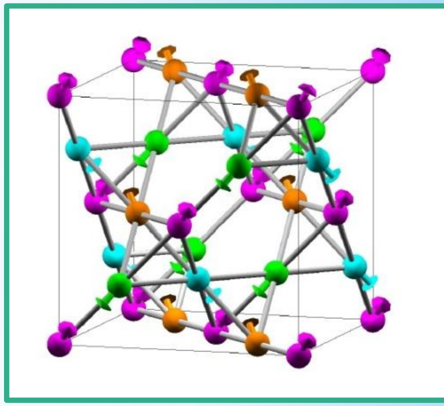
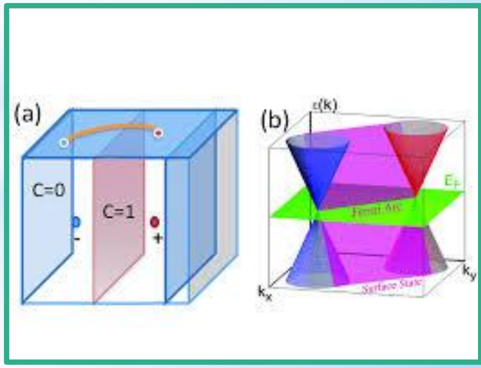
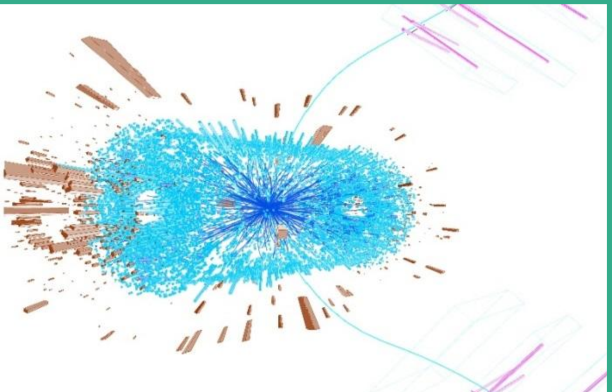
Why chiral plasma?

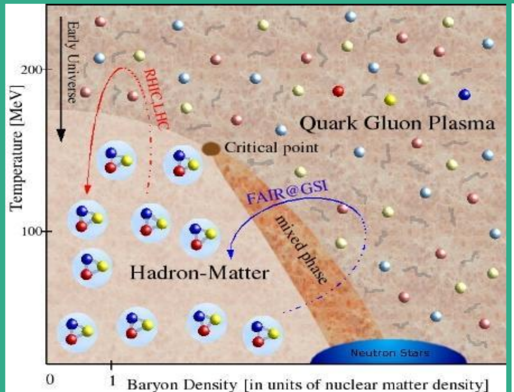
Collective motion of chiral fermions

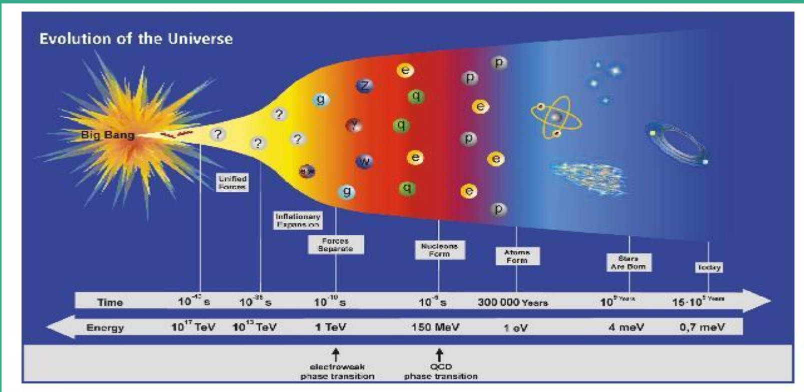
- **High-energy physics:**
  - ✓ Quark-gluon plasma
  - ✓ Hadronic matter
  - ✓ Neutrinos/leptons in Early Universe
- **Condensed matter physics:**
  - ✓ Weyl semimetals
  - ✓ Topological insulators
  - ✓ Liquid Helium [G. Volovik]











Anomalous transport: Hydrodynamics

Classical conservation laws for chiral fermions

- Energy and momentum
- Angular momentum
- Electric charge
- Axial charge

⟷ No. of left-handed / No. of right-handed

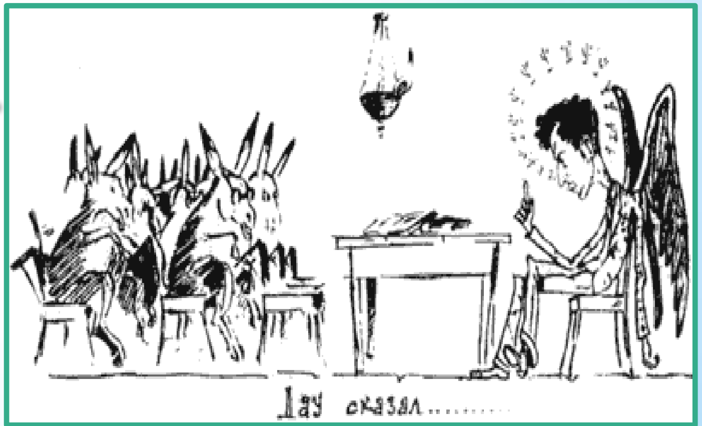
Hydrodynamics:

- Conservation laws
- Constitutive relations

Axial charge violates parity

↓

New parity-violating transport coefficients



Anomalous transport: CME, CSE, CVE

Chiral Magnetic Effect
[Kharzeev, Warringa, Fukushima]

$$j^i_V = \sigma^{\mathcal{B}}_{VV}\, B^i = \frac{N_c e\, \mu_A}{2\pi^2}\, B^i$$

Chiral Separation Effect
[Zhitnitsky,Son]

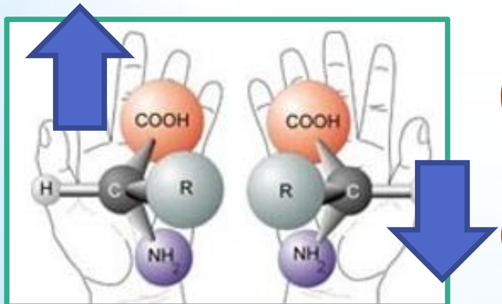
$$j^i_A = \sigma^{\mathcal{B}}_{AV}\, B^i = \frac{N_c e\, \mu_V}{2\pi^2}\, B^i$$

Chiral Vortical Effect
[Erdmenger *et al.*, Teryaev, Banerjee *et al.*]

$$j_V = \sigma^{\mathcal{V}}_V w = \frac{N_c e}{2\pi^2}\, \mu_A\, \mu_V\, w$$

$$j_A = \sigma^{\mathcal{V}}_A w = N_c e \left(\frac{\mu_V^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12} \right) w$$

Flow vorticity



Origin in quantum anomaly!!!

Observable signatures of anomalous transport?

As such, anomalous transport effects are difficult to see directly – CP breaking terms vanish on average …

Isobar run @ RHIC in 2018

Indirect signatures:

- New hydro excitations, chiral (shock) waves
- Electric conductivity in magnetic field
- Hall-type anomalous effects
- **Plasma instabilities**

Anomalous Maxwell equations (now a coarse approximation…)

Maxwell equations + ohmic conductivity + CME

$$\partial_t \vec{B} = -\mathrm{rot}\, \vec{E}, \quad \partial_t \vec{E} = \mathrm{rot}\, \vec{B} - \sigma \vec{E} - \chi \vec{B}$$

Ohmic conductivity (σ)

Chiral magnetic conductivity (χ)

Assumption: $\sigma\left(w, \vec{k}\right) = \mathrm{const}$ $\quad \chi\left(w, \vec{k}\right) = \mathrm{const}$

Plane wave solution

$$iw\vec{B} = -i\vec{k} \times \vec{E} \quad iw\vec{E} = i\vec{k} \times \vec{B} - \sigma \vec{E} - \chi \vec{B}$$

Chiral plasma instability

Dispersion relation

$$w = i\sigma/2 \pm \sqrt{k^2 - \chi k - \sigma^2/4}$$

At $k < \chi = \mu_A/(2\pi^2)$: Im(w) < 0

Unstable solutions!!!

Cf. [Hirono, Kharzeev, Yin 1509.07790]

Real-valued solution:

$$E_1 = f e^{\kappa t} \cos(k x_3),\ E_2 = -f e^{\kappa t} \sin(k x_3),$$
$$B_1 = -f \tfrac{k}{\kappa} e^{\kappa t} \cos(k x_3),\ B_2 = f \tfrac{k}{\kappa} e^{\kappa t} \sin(k x_3),$$

$$\kappa \equiv -iw = -\frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} - k^2 + \chi k}$$

Helical structure

$$E_1 = f e^{\kappa t} \cos(kx_3),\; E_2 = -f e^{\kappa t} \sin(kx_3),$$
$$B_1 = -f \frac{k}{\kappa} e^{\kappa t} \cos(kx_3),\; B_2 = f \frac{k}{\kappa} e^{\kappa t} \sin(kx_3),$$

Helical structure of unstable solutions
Helicity only in space - no running waves
E || B - ``topological" density

Note: E || B not possible for oscillating ``running wave" solutions, where *E•B=0*

What can stop the instability?

What can stop the instability?

$$\partial_t Q_A = \frac{g^2}{2\pi^2} \int d^3x \vec{E} \cdot \vec{B}$$

For our unstable solution with $\mu_A > 0$:

$$\vec{E} \cdot \vec{B} = -f^2 \frac{k}{\kappa} e^{2\kappa t} \Rightarrow \partial_t Q_A < 0$$

Instability depletes Q_A

μ_A and chi decrease, instability stops

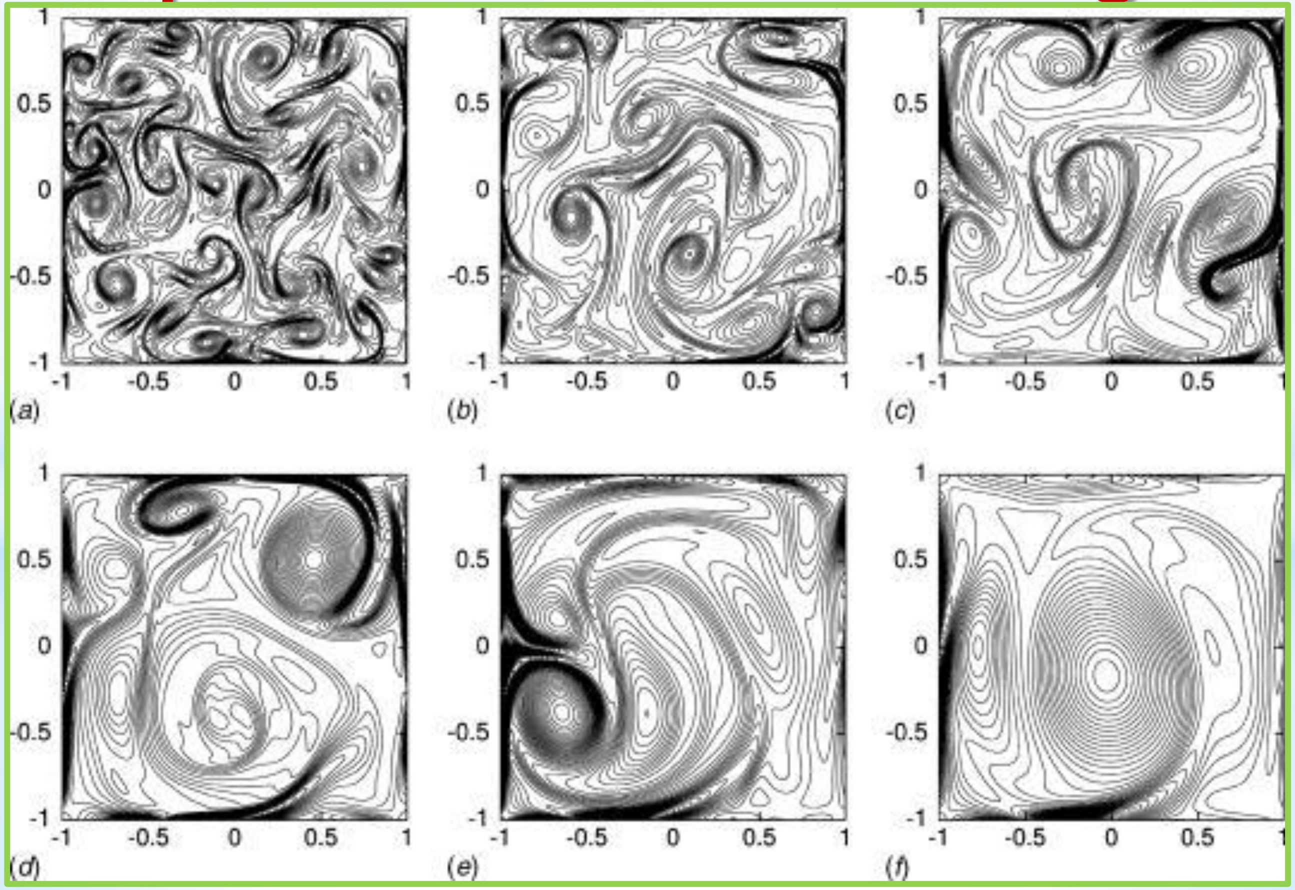
Energy conservation:

$$\partial_t \int d^3\vec{x} \left(\vec{E}^2 + \vec{B}^2\right) = \int d^3\vec{x} \left(-\sigma \vec{E}^2 - \chi \vec{E} \cdot \vec{B}\right)$$

Keeping constant μ_A requires work!!!

Chiral instability and Inverse cascade

Energy of large-wavelength modes grows … at the expense of short-wavelength modes!

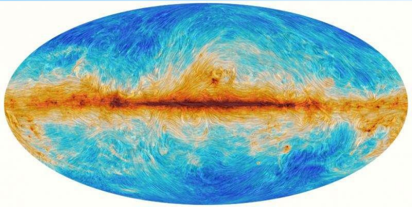
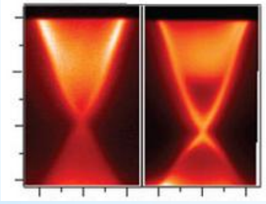
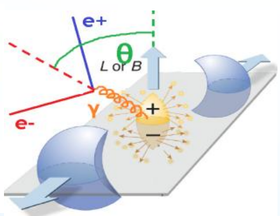


2D turbulence, from H. J. H. Clercx and G. J. F. van Heijst *Appl. Mech. Rev* 62(2), 020802

Chiral instability and Inverse cascade

Energy of large-wavelength modes grows
… at the expense of short-wavelength modes!

- Generation of cosmological magnetic fields [Boyarsky, Froehlich, Ruchayskiy, 1109.3350]
- Circularly polarized, anisotropic soft photons in heavy-ion collisions [Hirono, Kharzeev,Yin 1509.07790][Torres-Rincon,Manuel,1501.07608
- Spontaneous magnetization of topological insulators [Ooguri,Oshikawa,1112.1414]
- THz circular EM waves from Dirac/Weyl semimetals [Hirono, Kharzeev, Yin 1509.07790]

Real-time dynamics of chiral plasma

Approaches used so far:

- Anomalous Maxwell equations
- Hydrodynamics (long-wavelength)
- Holography (unknown real-world system)
- Chiral kinetic theory (linear response, relaxation time, long-wavelength…)

What else can be important:

- Nontrivial dispersion of conductivities
- Developing (axial) charge inhomogeneities
- Nonlinear responses

Let's try to do numerics!!!

Real-time simulations: classical statistical field theory approach

[Son'93, Aarts&Smit'99, J. Berges&Co]

- Full real-time quantum dynamics of fermions
- Classical dynamics of electromagnetic fields
- Backreaction from fermions onto EM fields

$$\partial_t \vec{A} = -\vec{E}$$
$$\partial_t \vec{E} = \vec{\nabla} \times \vec{B} - \langle \vec{j} \rangle - \vec{j}_{ext}$$

$$\partial_t \hat{\psi} = i \left[\hat{H} \left[\vec{A} \right], \hat{\psi} \right]$$

$$\langle \vec{j} \rangle = \langle \psi^\dagger j \psi \rangle$$

Vol X Vol matrices,

Bottleneck for numerics!

$$\partial_t U = ih \left[\vec{A} \right] U$$

$$\langle \vec{j} \rangle = \mathrm{Tr} \left(\rho_0 U \vec{j} U^\dagger \right)$$

Overlap fermions for real-time

[Creutz,Horvath,Neuberger hep-lat/0110009]
[PB,Valgushev 1611.05294] [Mace et al. 1612.02477]

$$h_{ov} = \gamma_0 + \gamma_0 \gamma_5 \, \mathrm{sign} \, \left(\gamma_5 \gamma_0 \, h_{wd} \left(-M\right)\right)$$

$$\vec{j}_x = \frac{\partial h_{ov} \left[\vec{A}_x\right]}{\partial \vec{A}_x}$$

Wilson-Dirac Hamiltonian
Negative mass term

- Zolotarev/polynomial approximation of Sign
- Dynamically adjust approximation range
- Use ARPACK to find eigenspectrum support
- Deflation does not pay off too much (complicated for current)

Calculation of electric current

[Creutz,Horvath,Neuberger hep-lat/0110009]
[PB,Valgushev 1611.05294] [Mace et al. 1612.02477]

$$\langle \vec{j}(x) \rangle \sim \langle 1 | \frac{\partial \,\mathrm{sign}\,(K)}{\partial \vec{A}} | 2 \rangle, \quad K = \gamma_5 \gamma_0 h_{wd} \left[\vec{A} \right]$$

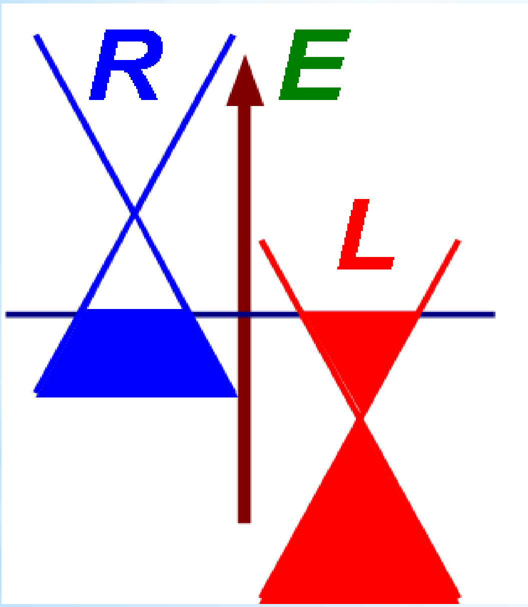
$$\mathrm{sign}\ (K) = K \sum_i \frac{a_i}{b_i + K^2}$$

$$\frac{\partial}{\partial \vec{A}} \langle 1 | \,\mathrm{sign}\ (K)\, | 2 \rangle = \sum_i a_i \langle 1 | \frac{1}{b_i + K^2} \frac{\partial}{\partial \vec{A}} K^2 \frac{1}{b_i + K^2} | 2 \rangle = $$
$$= \sum_i a_i \langle 1_i | \frac{\partial}{\partial \vec{A}} K^2 | 2_i \rangle$$

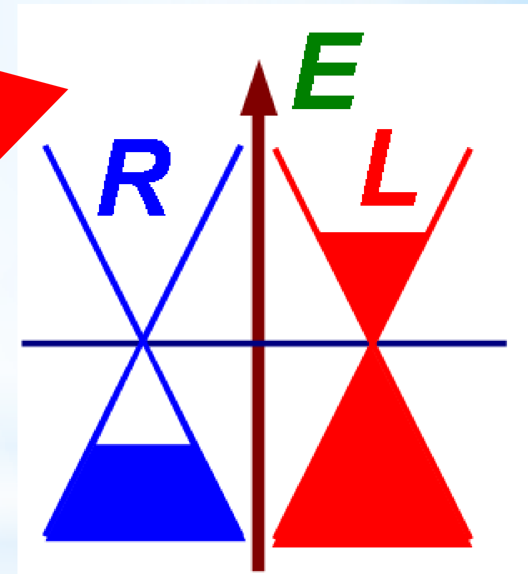
- **Vectors |1i>, |2i> : use multishift CG**
- **Contractions with dK^2/dA_{xi} volume-independent**
- **Number of operations ~ V^2, as for fermion evol.**

Options for initial chiral imbalance

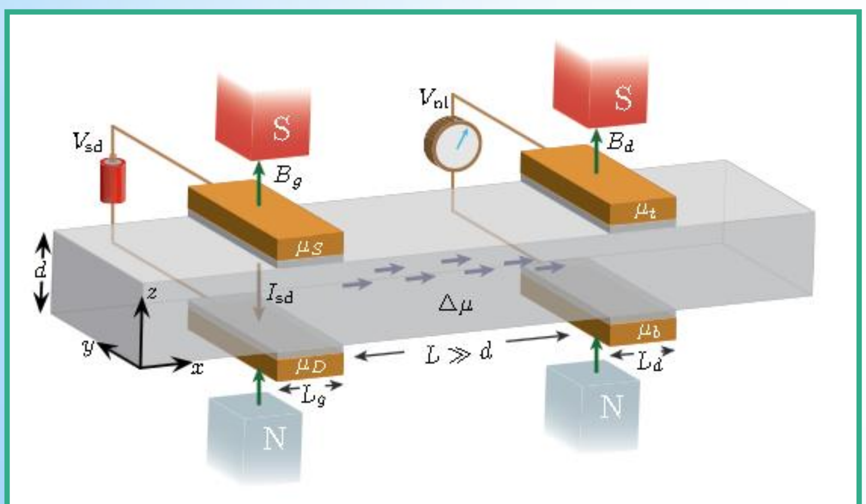
Chiral chemical potential



Excited state with chiral imbalance



Hamiltonian is CP-symmetric, State is not!!!



Pumping of chirality

$$\frac{d}{dt} Q_A = \frac{e^2}{2\pi^2} \int d^3x \, \vec{E} \cdot \vec{B}$$

Electric field is switched off at some time

Numerical setup

μ_A < ~*1* on the lattice (van Hove singularities)

To reach $k < \mu_A/(2\pi^2)$:

- ***200x20x20*** **lattices, MPI parallelisation**
- **Translational invariance in 2 out of 3 dimensions**

To detect instability and inverse cascade:

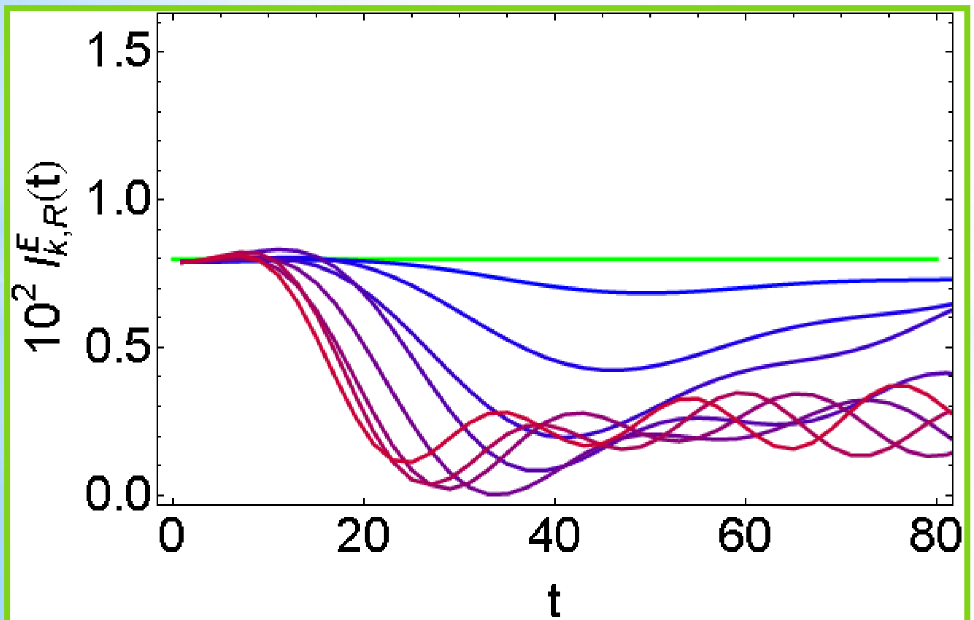
- **Initially n modes of EM fields with equal energies and random polarizations**

Power spectrum and inverse cascade

Fourier transform the fields

$$E_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} E_{x,i}(t),$$

$$B_{k,i}(t) = \frac{1}{\sqrt{L_3}} \sum_{x_3} e^{ikx_3} B_{x,i}(t),$$



Basis of helical components

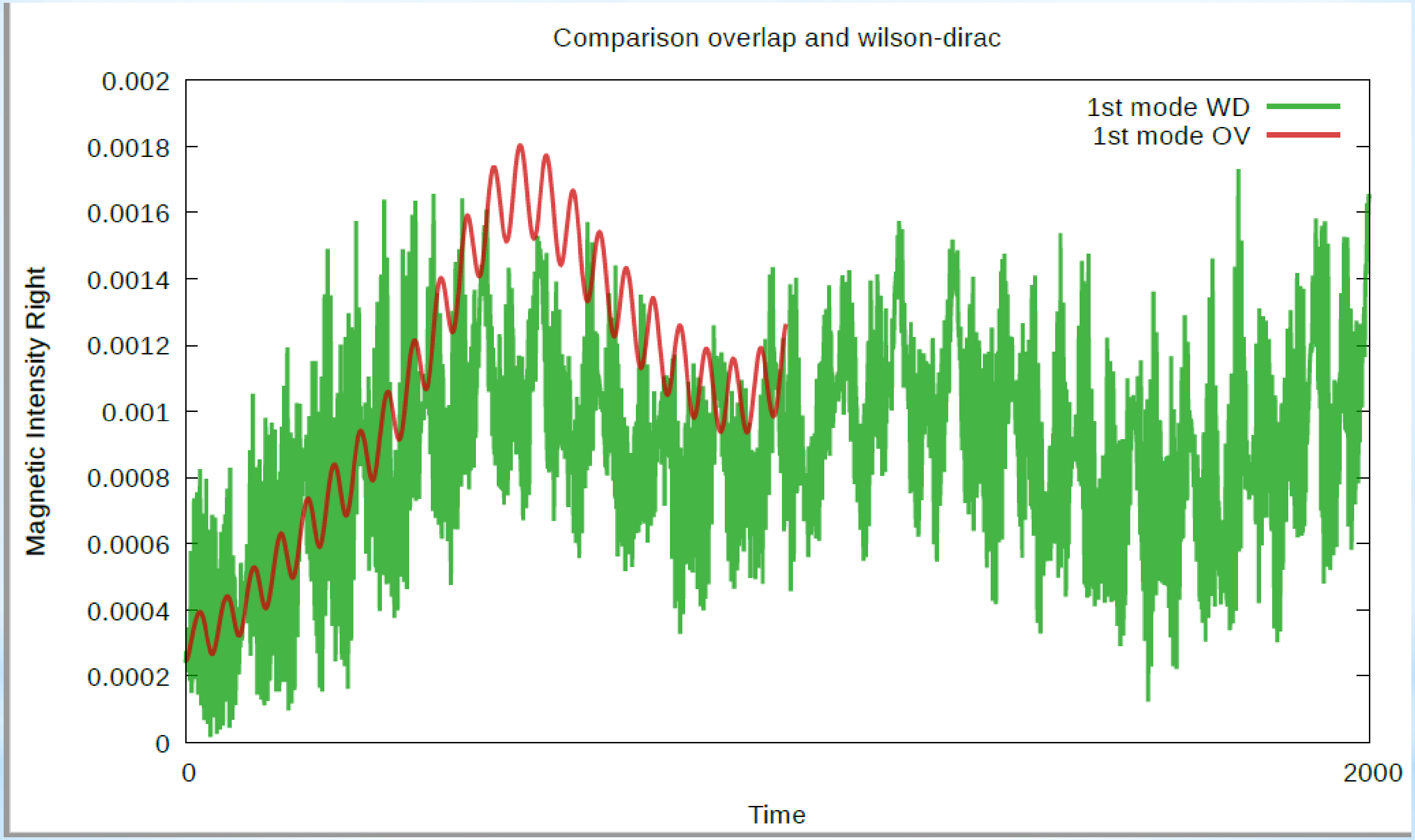
$$B_{k,R}(t) = \frac{1}{2}\left(B_{k,1}(t) + B_{-k,1}(t)\right) + \\ + \frac{1}{2i}\left(B_{k,2}(t) - B_{-k,2}(t)\right),$$

$$B_{k,L}(t) = \frac{1}{2i}\left(B_{k,1}(t) - B_{-k,1}(t)\right) + \\ + \frac{1}{2}\left(B_{k,2}(t) + B_{-k,2}(t)\right).$$

Smearing the short-scale fluctuations

$$\bar{I}^{E,B}_{k,R/L}(t) = \frac{1}{T} \int\limits_{t-T/2}^{t+T/2} dt' I^{E,B}_{k,R/L}(t').$$

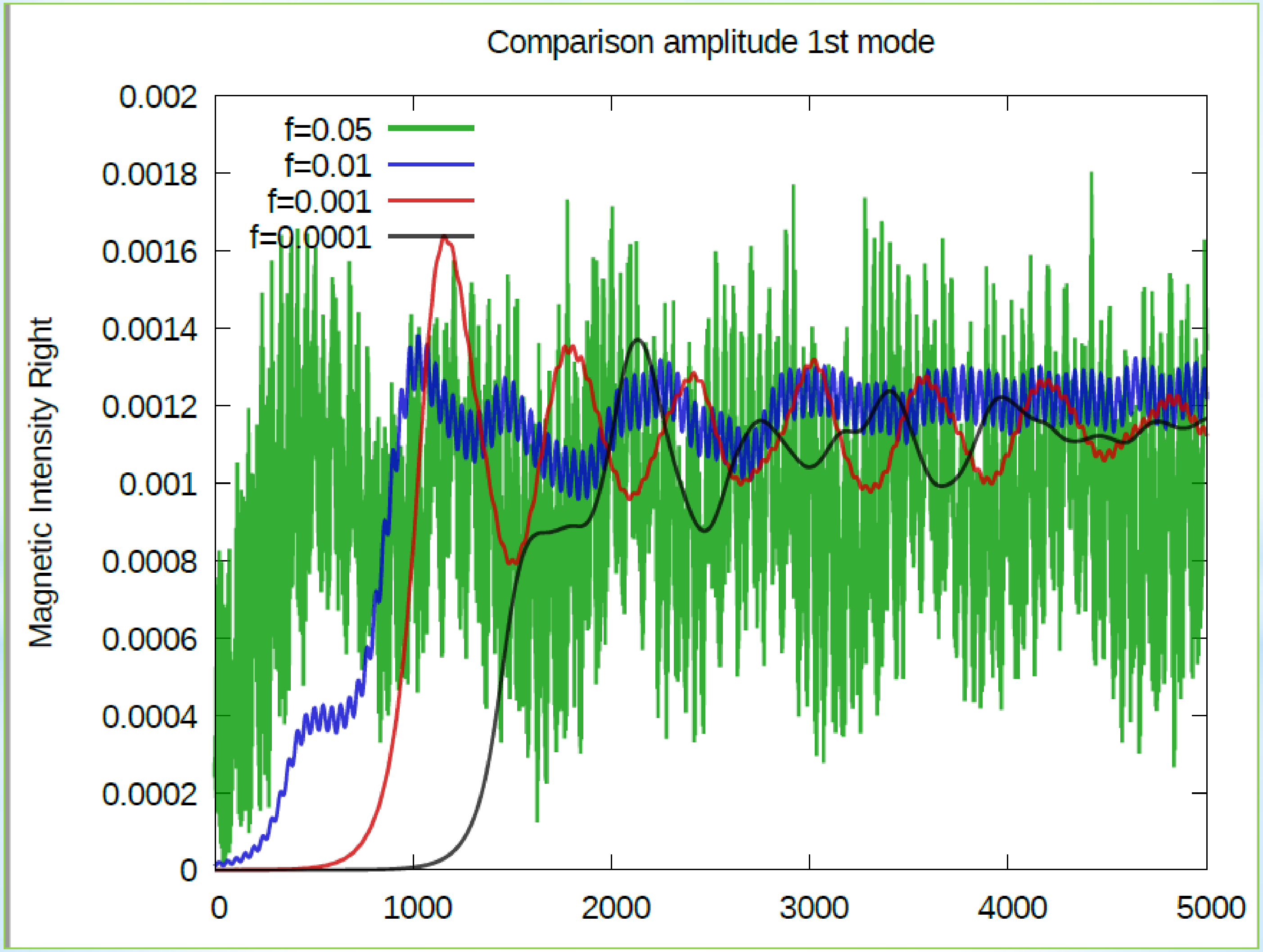
Comparison of Overlap and Wilson-Dirac



Very similar dynamics for both fermions…

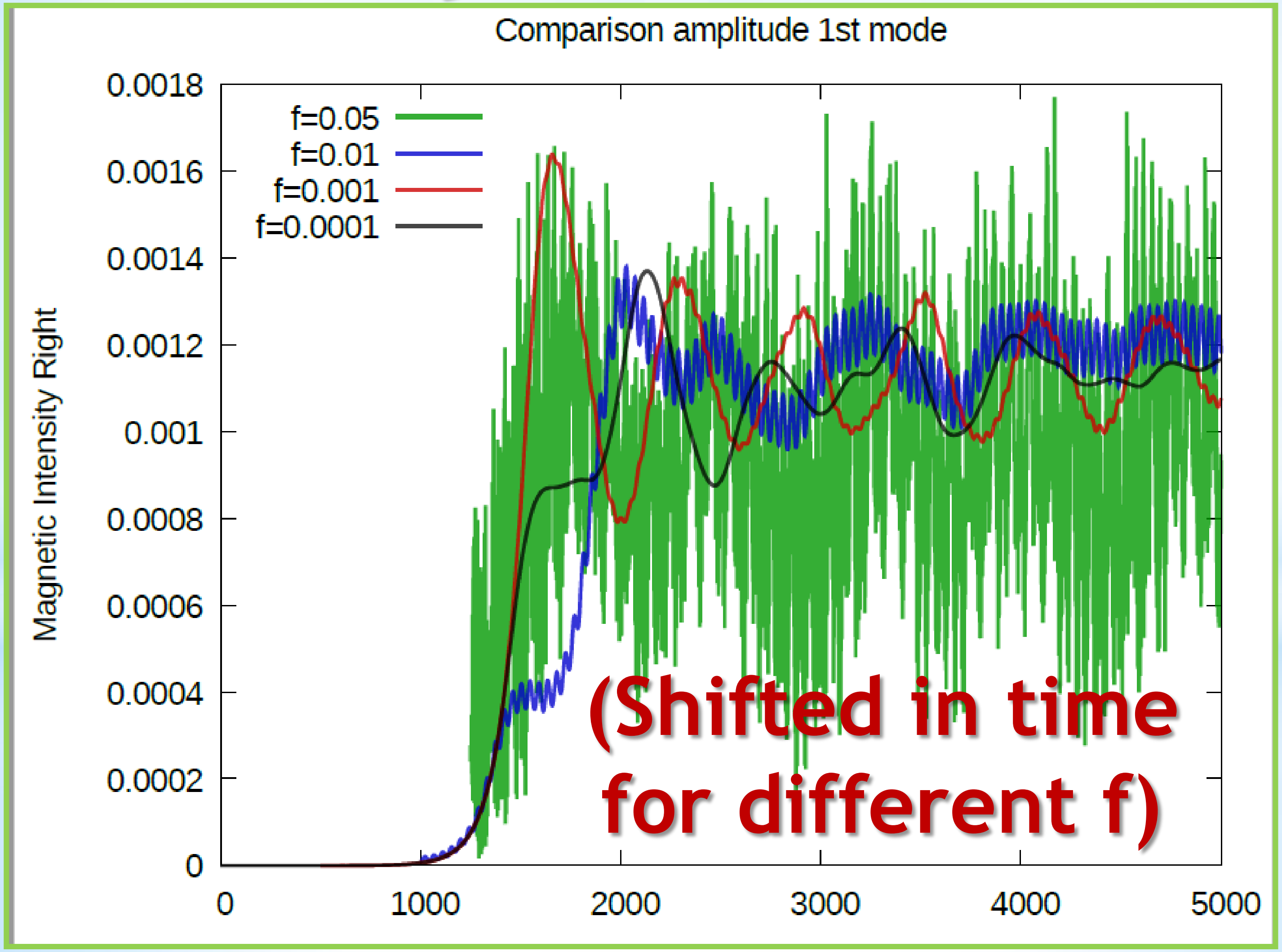
Use Wilson-Dirac and Overlap for control

Instability of helical modes



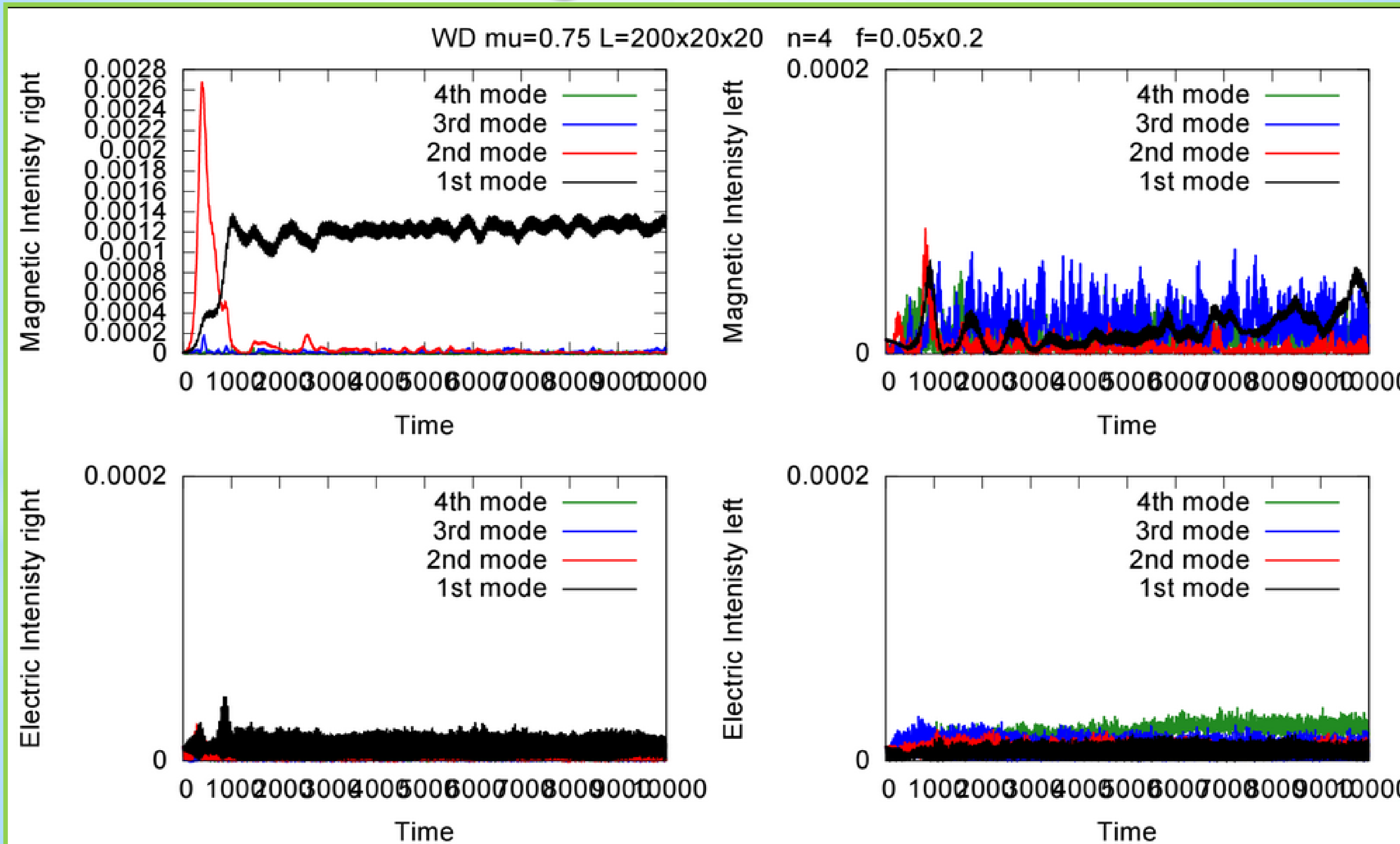
μA = 0.75, L = 200 - only one mode (should be) unstable "Chiral Laser"

Instability of helical modes



Universal features:
exponential growth + late-time stabilization

Electric + magnetic helical modes



Only two right-handed modes are important

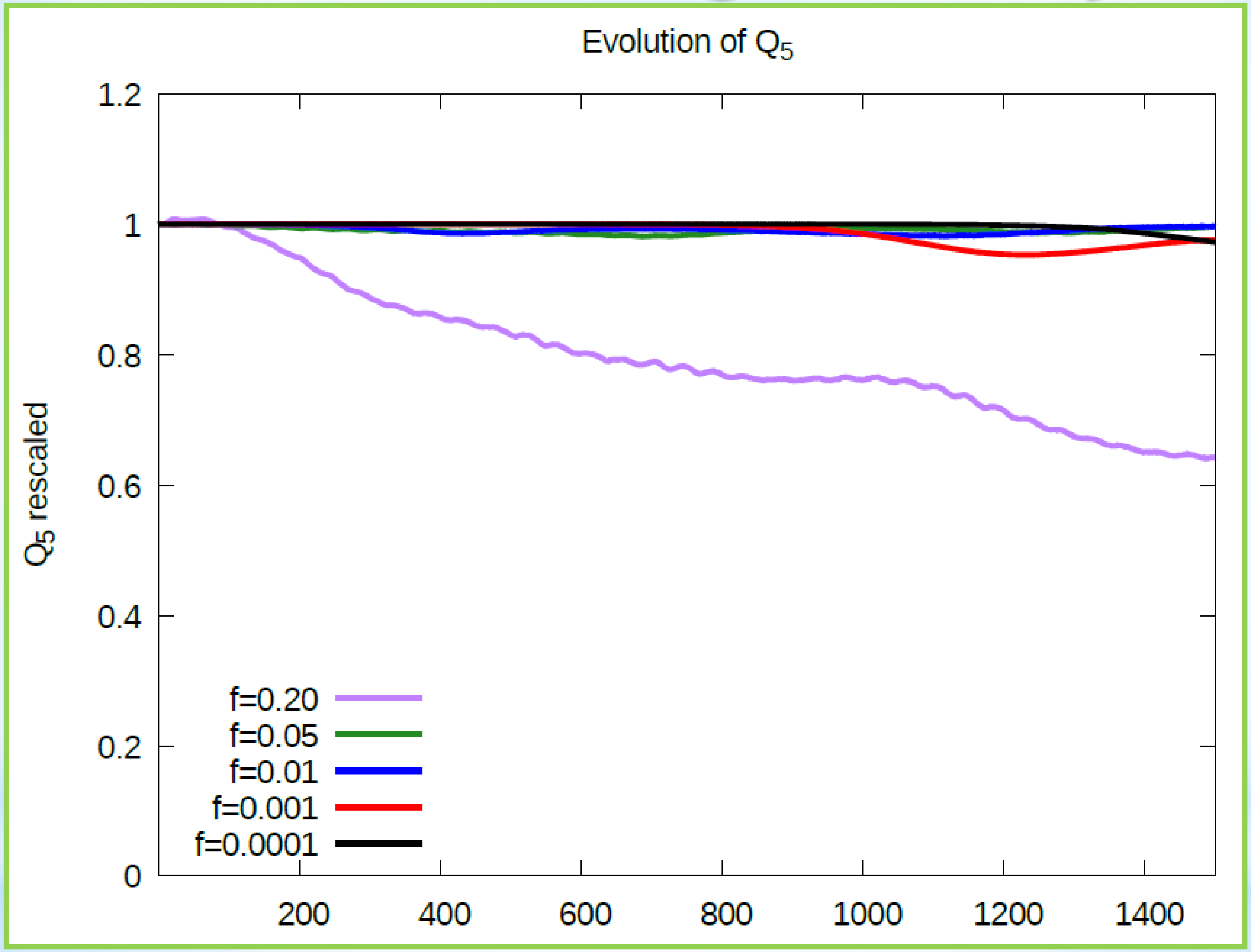
What stops the instability?

At the time of saturation, axial charge is not changed + very homogeneous …

Differences with Kinetic Theory/Maxwell:

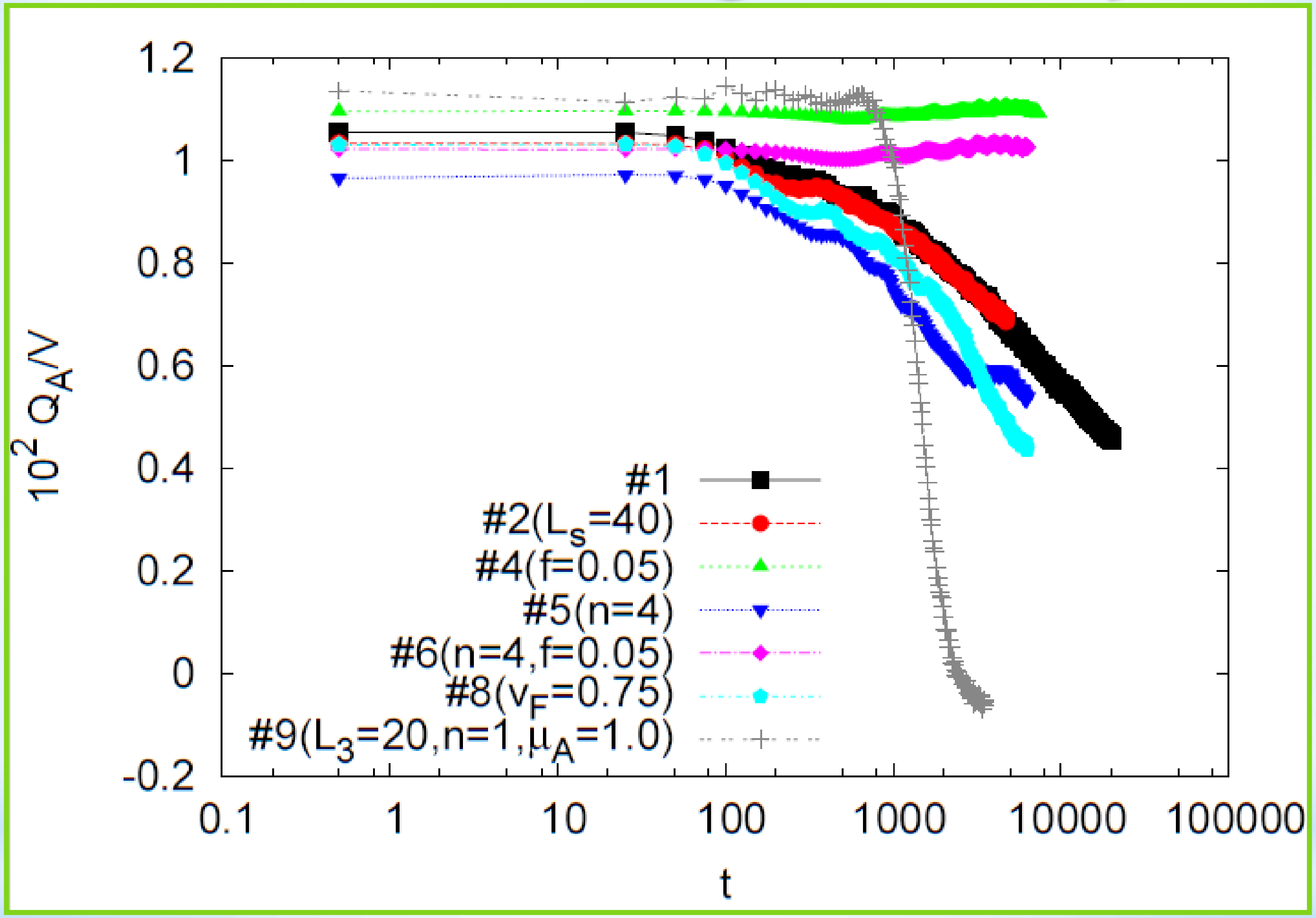
- **Backreaction from anomaly plays no role**
- **Only helical magnetic field is important**
- **Transient second mode excitation**
- **Electric field strongly suppressed**

Axial charge decay



No chirality decay in linear regime

Axial charge decay

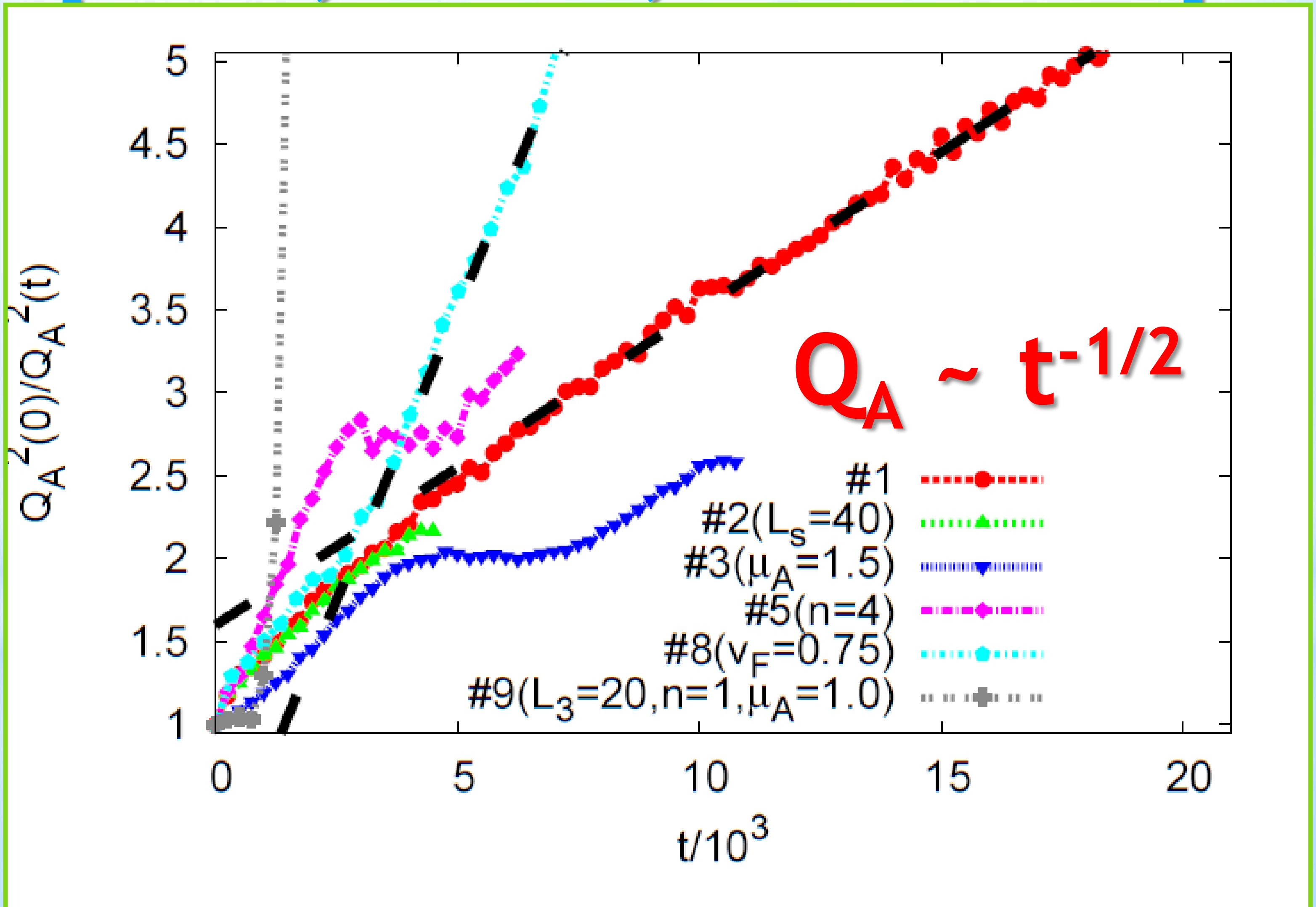


200 x 20 x 20 lattice, μ_A= 0.75

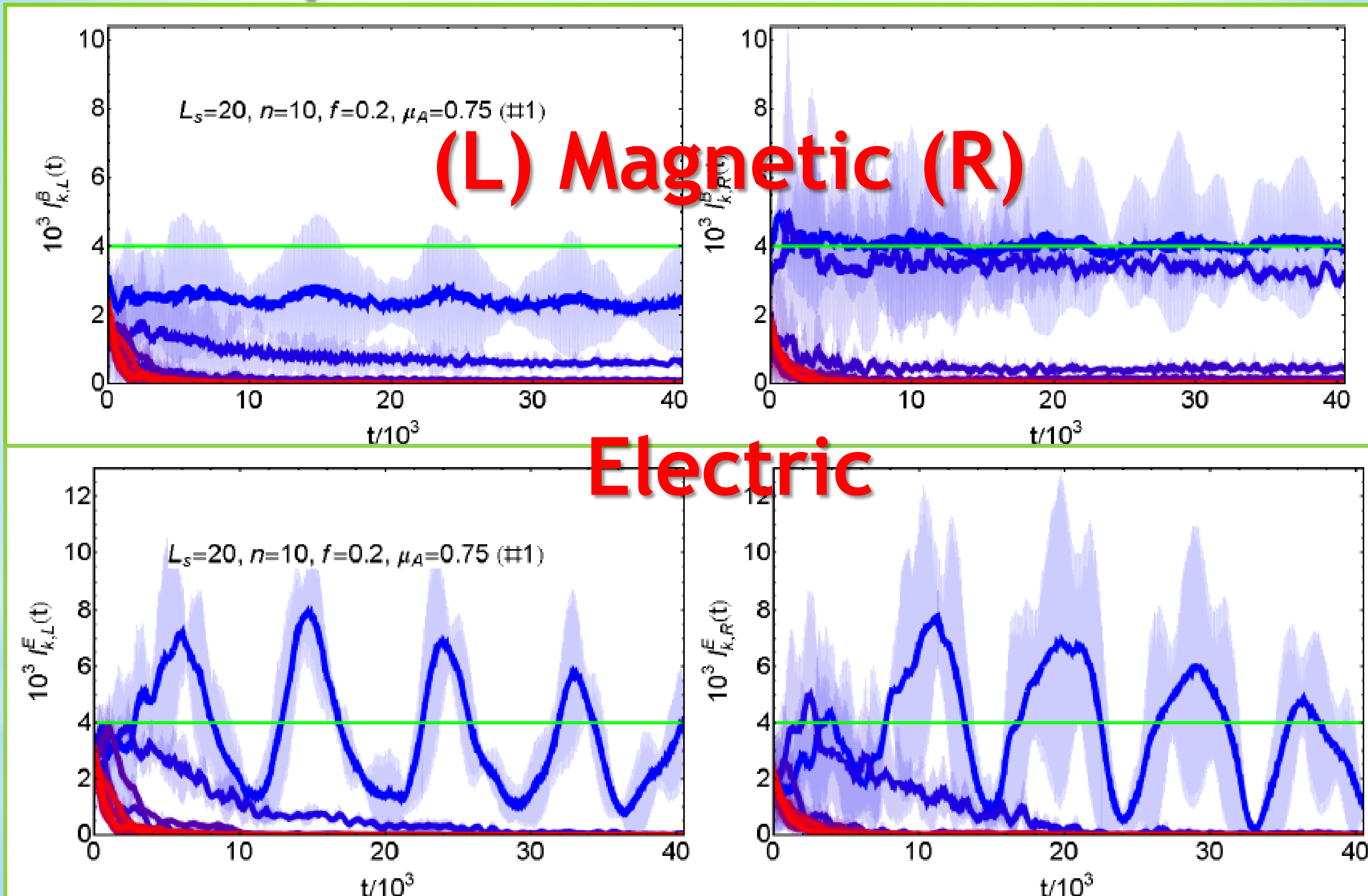
If amplitude small, no decay

Universal late-time scaling

[Yamamoto 1603.08864], [Hirono,Kharzeev,Yin1509.07790]

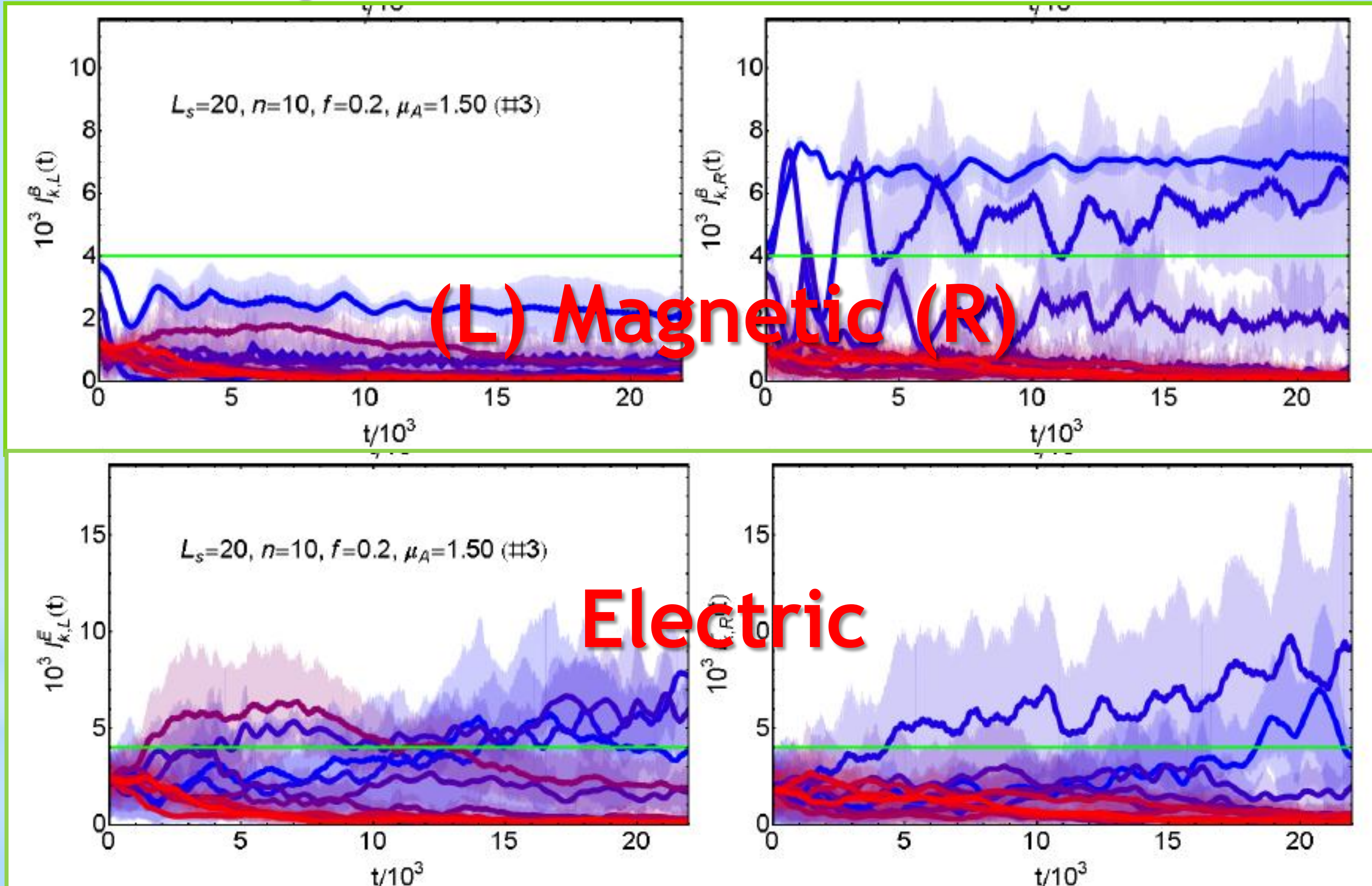


Power spectrum and inverse cascade



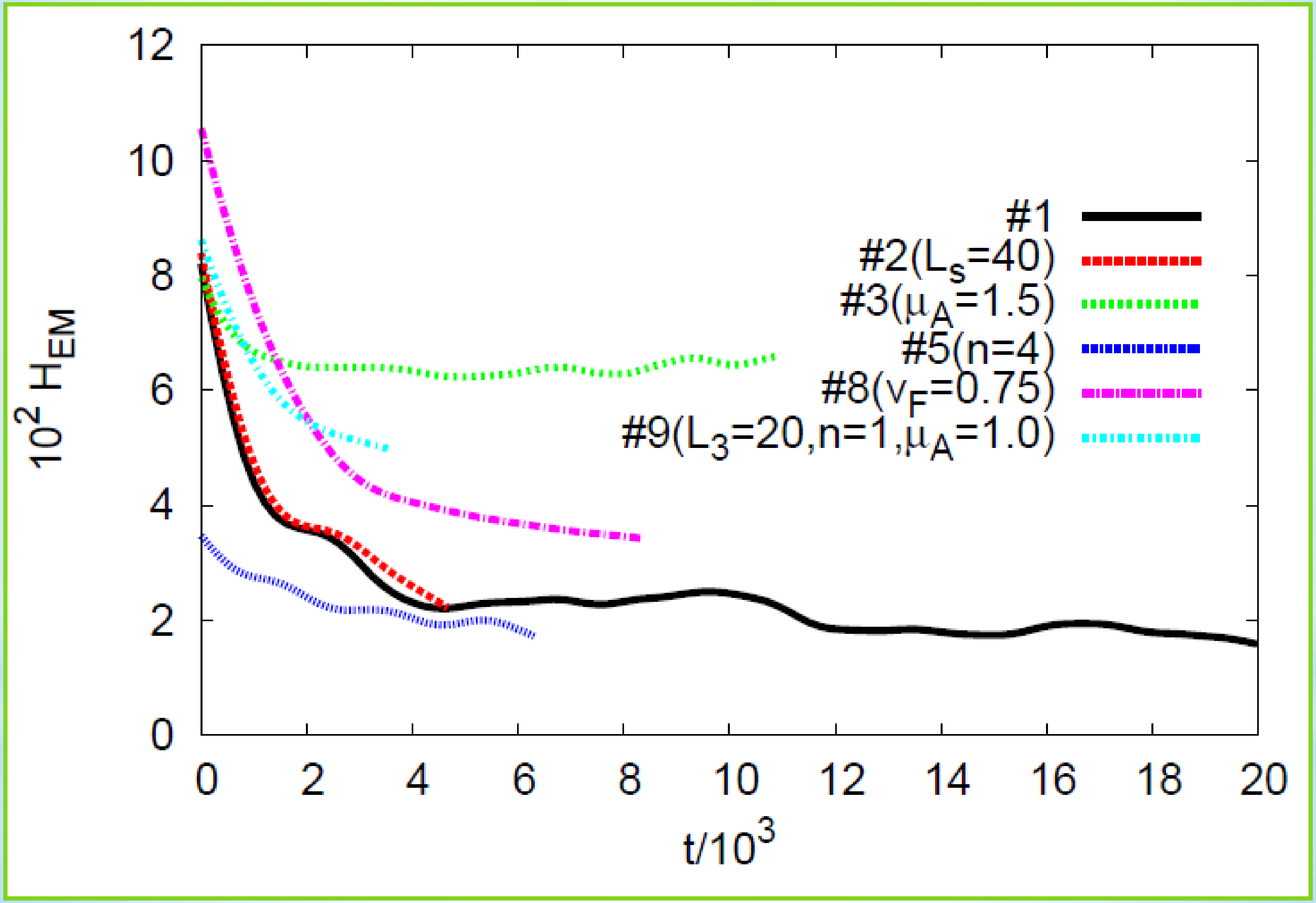
Large amplitude, Q_A decays

Power spectrum and inverse cascade



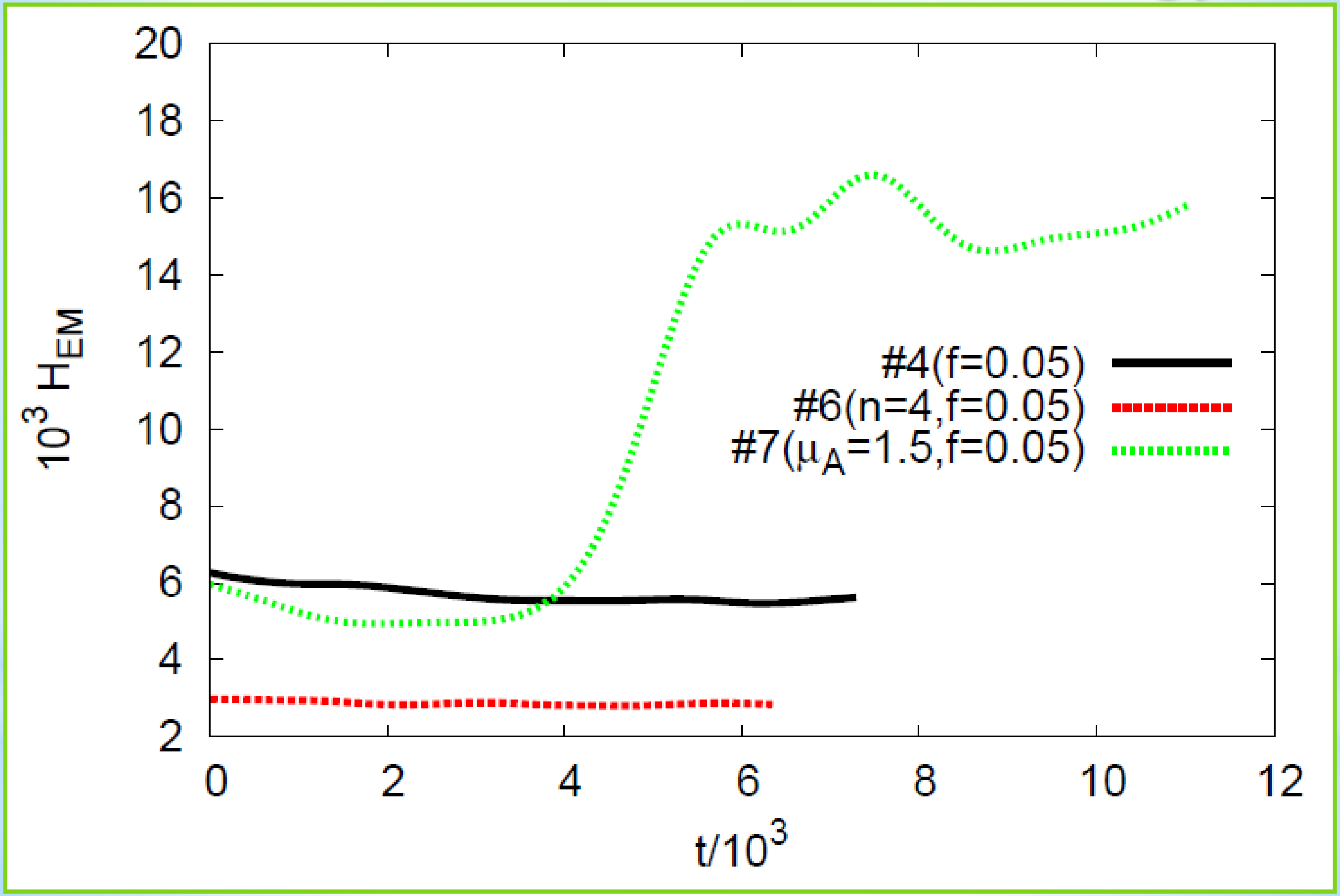
Large amplitude, large μ_A, Q_A decays

Overall transfer of energy



Amplitude *f = 0.2*

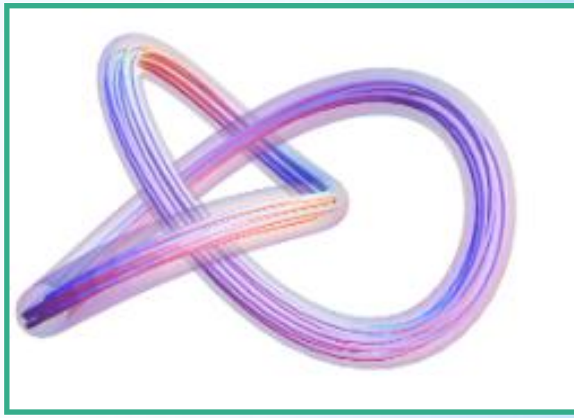
Overall transfer of energy



Amplitude *f = 0.05*

Discussion and outlook

- **Axial charge decays with time**
  (nature doesn't like fermion chirality)
- **Large-scale helical EM fields**
- **Short EM waves decay**
- **Non-linear mechanism!**
- **Instability stops much earlier than predicted by anomalous Maxwell eqns. !!!**



Discussion and outlook

- How to capture non-linear effects?
- Three-photon vertex zero even with μ_A
- Four-photon vertex complicated beyond Euler-Heisenberg
- Non-linear effects within Kinetic Theory?

Chiral Separation Effect in QCD

$$J_i^A = (1 - g_{\pi\gamma\gamma}) \frac{q N_c \mu}{2\pi^2} B_i$$

- Important for Chiral Magnetic Wave
- Can induce large chirality imbalance
- Truly equilibrium phenomenon

[Zhitnitsky,Metlitski, hep-ph/0505072]

$g_{\pi\gamma\gamma}=0$, CSE is purely topological

[Son,Newman, hep-ph/0510049]

$$g_{\pi\gamma\gamma} = \frac{7\zeta(3) m^2}{4\pi^2 T^2} \sim 1$$ (*m*~*300* MeV, *T*~*150* MeV)

From linear sigma model (chiral symmetry spontaneously broken)

Numerical setup

- **Finite-density overlap fermions**

$$\mathcal{D}_{ov} = 1 + \gamma_5 \mathrm{sign}\left(\gamma_5 \mathcal{D}_{WD}\right),$$
$$\left(\gamma_5 \mathcal{D}_{WD}(\mu)\right)^{\dagger} = \gamma_5 \mathcal{D}_{WD}(-\mu)$$

- **Special algorithm for currents**

$$J^{A}_{x,\mu} = \mathrm{Tr}\left(\gamma_5 \mathcal{D}_{ov}^{-1} \frac{\partial}{\partial \theta_{x,\mu}} \mathcal{D}_{ov}\right)$$

(Derivatives of sign of non-Hermitian matrix)

For the first time, transport with strongly coupled, dense, exactly chiral lattice fermions

[M. Puhr, PhD early 2017]

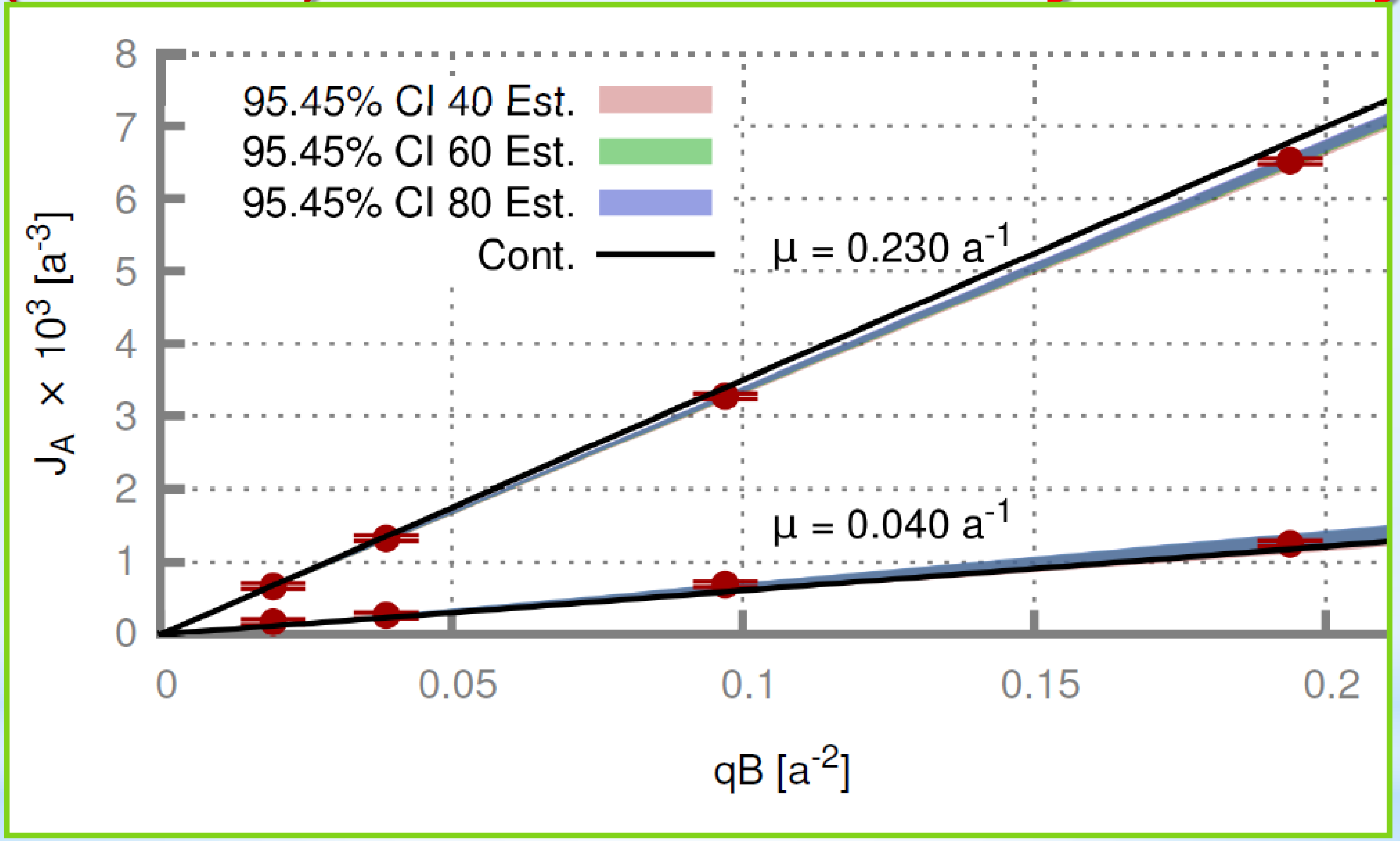
Numerical setup

- Sign problem in finite-density QCD,
- But also with G2, SU(2) gauge theory or with isospin/chiral chemical potential if magnetic field added
- We use quenched SU(3) gauge theory

$$\langle O \rangle = Z^{-1} \int dA_\mu O\left[A_\mu\right] \det\left(\not{D}\left[A_\mu\right]\right)^{N_f} e^{-S_{YM}\left[A_\mu\right]}$$

- Exactly zero mass for zero topology *Q*
- Very small mass ~ *3.2* MeV at *Q≠0*
- High-temperature and low-temperature phases

High-temperature phase
(almost) restored chiral symmetry

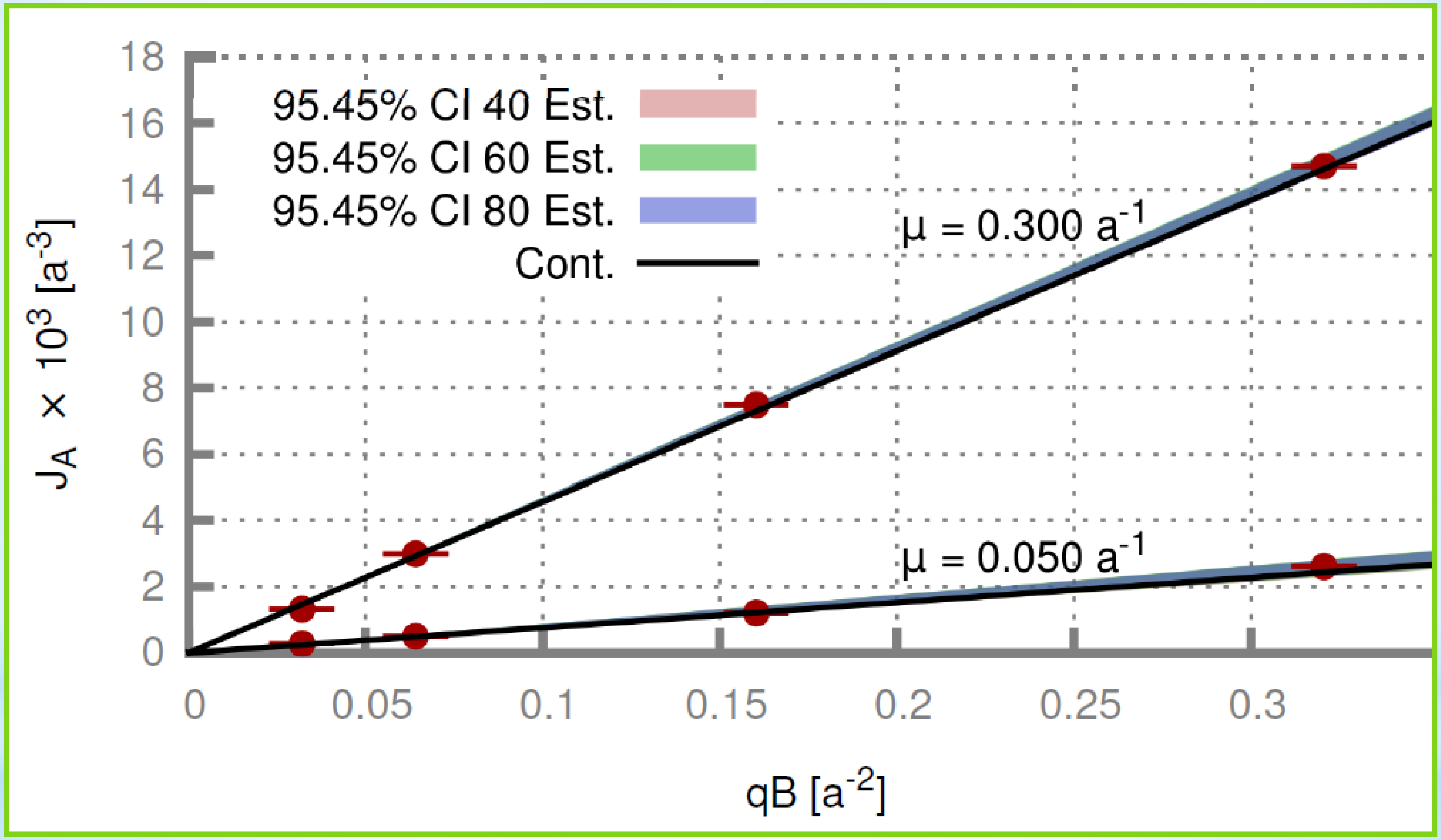


$\mu B/2\pi^2$ for Q=0 and Q=±1

vs. [Yamamoto'1105.0385] CME,5x difference

Low-temperature phase

Spontaneosly broken chiral symmetry

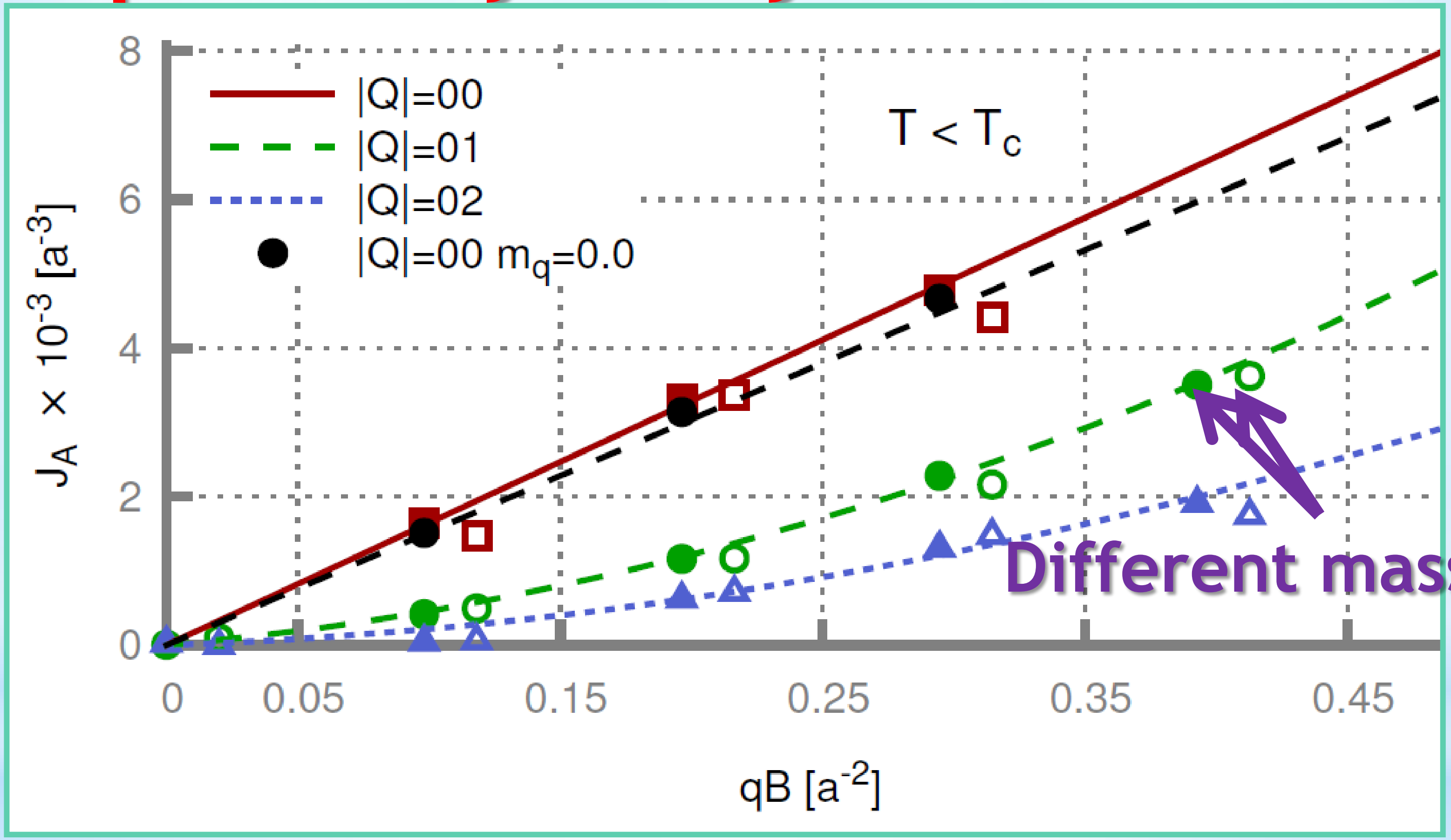


Excellent agreement with free fermions for Q=0

Low-temperature phase: discussion

- Even with Q=0, chiral symmetry is spontaneously broken
- Lowest Dirac modes effectively decoupled from topological modes
- Corrections due to spontaneous chiral symmetry breaking are very small or vanishing, at least with quenching
- Sharp contrast with predictions of [Son,Newman, hep-ph/0510049]

Effects of nonzero topology (exploratory study on 8x8 lattice)

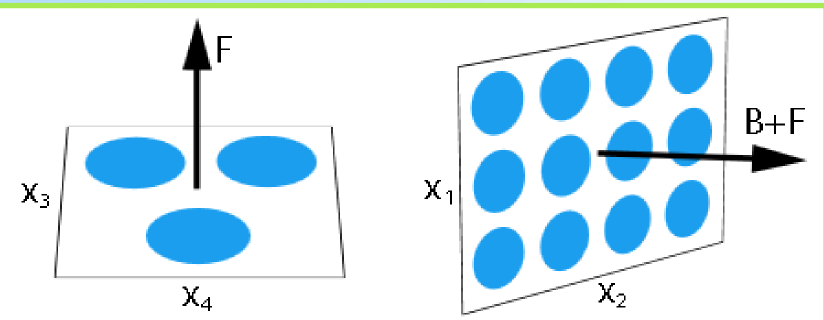


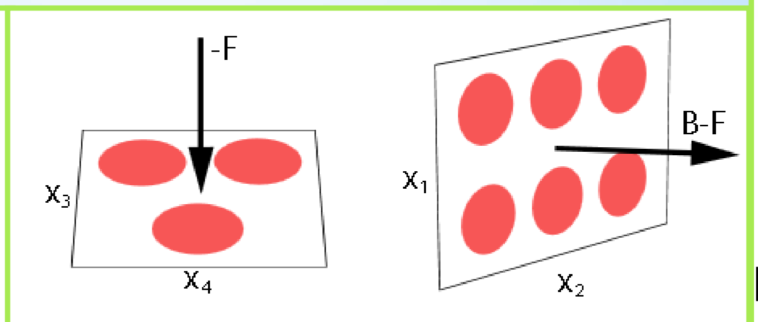
Strong suppression in Q≠0 sectors
Nonlinear dependence on B

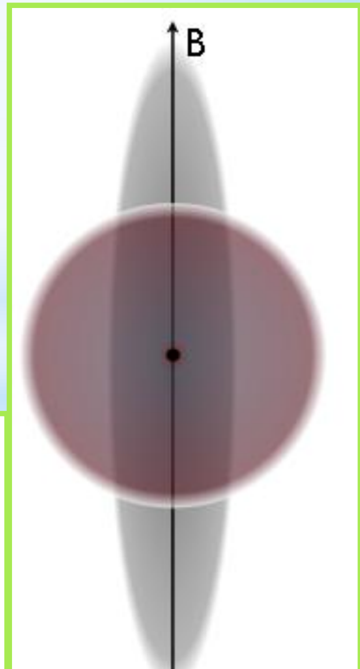
Suppression of CSE in topological backgrounds?

Large instanton/large B limit of [Basar,Dunne,Kharzeev 1112.0532]:

- **Self-dual, constant non-Abelian field strength tensor**
- **Landau quantization in (x,y) and (z,t) planes**







From 1112.0532]

Landau quantization at finite density

Dirac operator with finite chemical potential and mass

$$\mathcal{D} = \gamma_\mu \nabla_\mu + \mu\gamma_0 + m = \begin{pmatrix} m & -iW^- \\ -iW^+ & m \end{pmatrix}$$

$$W^\pm = \begin{pmatrix} w_E a_E^\pm & \pm w_B a_B^\dagger \\ \mp w_B a_B & w_E a_E^\mp \end{pmatrix}$$

$$w_E = \sqrt{2E}, \quad w_B = \sqrt{2B}$$

$$a_E^\pm = \frac{\partial_3}{w_E} \mp \frac{w_E}{2}\left(x_3 - \frac{k_0 - i\mu}{E}\right)$$

$$a_B = \frac{\partial_1}{w_B} + \frac{w_B}{2}\left(x_1 - \frac{k_2}{B}\right)$$

$$\left[a_E^-, a_E^+\right] = 1$$

$$\left[a_B, a_B^\dagger\right] = 1$$

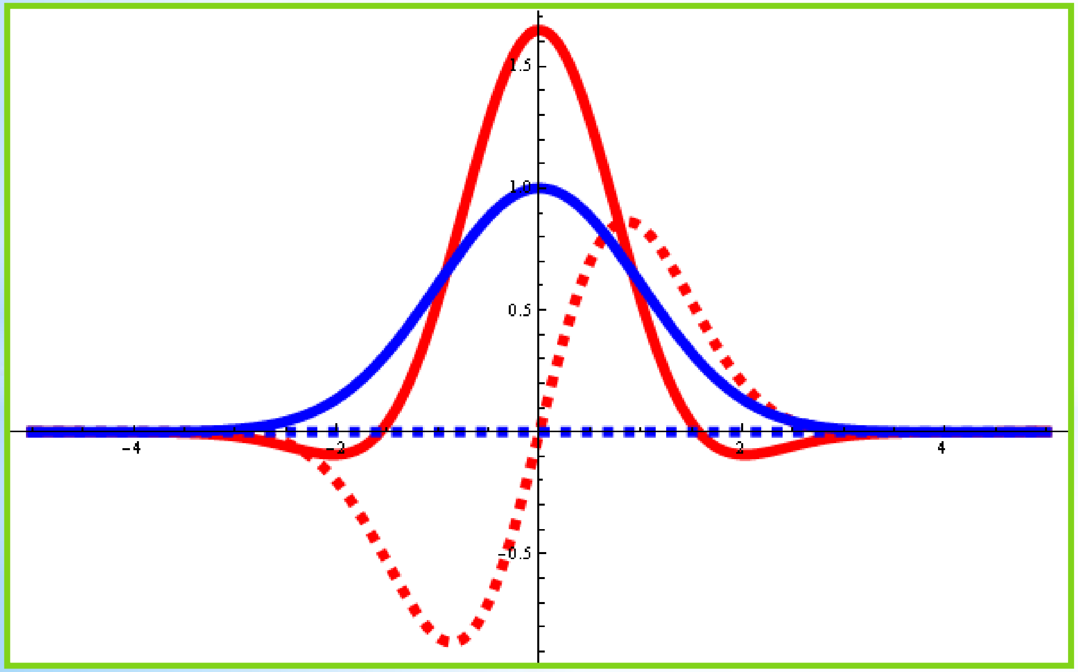
$$\left(a_E^-\right)^\dagger \neq a_E^+$$

Landau quantization in (zt) plane at finite density

Still eigenstates of harmonic oscillator

…with a complex shift $x_3 \to x_3 - \frac{k_0 - i\mu}{E}$

$$\langle L_0|\,|x_3\rangle = \langle x_3|\,|R_0\rangle = (E/\pi)^{1/4} \exp\left(-\frac{E}{2}\left(x_3 - \frac{k_0 - i\mu}{E}\right)^2\right)$$



$$|R_n\rangle = \frac{\left(a_E^+\right)^n}{\sqrt{n!}} |R_0\rangle$$

$$\langle L_n| = \langle L_0| \frac{\left(a_E^-\right)^n}{\sqrt{n!}}$$

$$\langle L_n|\,|R_m\rangle = \delta_{nm}$$

Landau quantization in (zt) plane at finite density

$$J_{A3} = \mathrm{Tr}\,\left(\mathcal{D}^{-1}\gamma_5\gamma_3\right) =$$
$$= i\mathrm{Tr}\,\left(\left(W^+\sigma_3 + \sigma_3 W^-\right)\left(m^2 + W^+W^-\right)^{-1}\right) =$$
$$= iw_E \mathrm{Tr}\,\left(a_E^+ + a_E^-\right)\left(G\left(n_E, n_B\right) - G\left(n_E + 1, n_B + 1\right)\right)$$

$$n_B = a_B^\dagger a_B \qquad n_E = a_E^+ a_E^-$$

$$G\left(n_E, n_B\right) = \frac{1}{m^2 + w_E^2 n_E + w_B^2 n_B}$$

$$J_{A3} = iw_E \sum_{n_E, n_B} \langle L_n|\,\left(a_E^+ + a_E^-\right)\,|R_n\rangle F\left(n_E, n_B\right) = 0$$

Completely analogous to zero-density result
All dependence on μ went into global shifts

CSE and topology: conclusions

- Constant Euclidean electric field eats up all the dependence on density
- Somewhat similar to QHE – flat bands!
- Topology is not the full story, it has (seemingly) no effect at high temperatures
- Perfect agreement with anomaly shows the advantage of overlap (cf. [A. Yamamoto 1105.0385], ~100% corrections to CME in both phases)

Brief summary

- Chirality pumping: backreaction makes axial charge and CME current oscillating, $Q_A \sim B^{1/2}$ scaling vs. $Q_A \sim B$
- Chiral plasma instability stops earlier than chiral imbalance is depleted
- Possible corrections to CSE due to global topology

Thank you for your attention!!!